Labor Market Shocks and Monetary Policy

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The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or Bank of Canada.
Motivation

- Literature focuses on the unemployment rate as a measure of slack.
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- U.S. employer-to-employer (EE) transition rate is procyclical and persistent.
  - Important driver of wage and productivity growth.
    - Moscarini and Postel-Vinay (2017), Karahan et al. (2017), Haltiwanger et al. (2018), Faberman et al. (2022)
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- Relative strength of wage vs productivity growth over the cycle can determine inflation.
  
  - Potential implications for monetary policy.
Comovement of Unemployment and EE Flows

Weakening correlation between unemployment and EE rates between 2016–2019.
1. **Positive:** How do fluctuations in EE flows affect inflation dynamics?

2. **Normative:** What is the optimal monetary policy accounting for EE dynamics?
This Paper

- Develop HANK model with frictional labor market and on-the-job search (OJS).
  - income risk cyclicality and corr. with MPC. Acharya and Dogra (2020) and Patterson (2022)
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○ Positive analysis: Analyze macro implications of EE fluctuations.
  
  - Document a significant weakening of $corr(u, EE)$ post Great-Recession (2016–19).
  - Quantify magnitude of “missing inflation” due to muted EE response as 0.23 pp.
  - Decompose channels through which EE fluctuations affect marginal cost (inflation).
    
    ○ Labor market: direct (e.g. wage rebargaining), GE effect on tightness (e.g. labor demand).
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    - Labor market: direct (e.g. wage rebargaining), GE effect on tightness (e.g. labor demand).

- **Normative analysis:** Study optimal monetary policy within a class of Taylor rules.
  - Including the EE rate in the Taylor rule
    - reduces inflation and unemployment volatility.
    - provides non-monotonic welfare gains across worker productivity.
Roadmap

Motivation

Model

Solution

Results
Overview: HANK + Labor Search with OJS

- Households:
  - Experience labor market transitions over lifecycle with stochastic retirement and death.
  - Heterogeneity in wealth, employment status, skill, match productivity, and wage.
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  - Labor service: hire workers in a frictional labor market and sell labor services at \( p^l_t \).
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  - Final good: assemble differentiated goods into final consumption good.

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- Monetary authority controls nominal interest rate through Taylor rule.

**Details**

**Timing**
Households

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- Wages $w(h, x, \alpha)$: Depend on skill $h$, match productivity $x$, and bargained piece rate $\alpha$. 
**Wage Determination**

- Wage is an endogenous piece rate $\alpha$ of output $F(h, x) = hx$: $w(h, x, \alpha) = \alpha \phi^E F(h, x)$.
  - $\phi^E \in (0, 1)$: maximum share of output a worker can receive with full piece rate $\alpha = 1$.
  - $\alpha$ determined by Bertrand competition over flows. Bagger et al. (2014), Graber and Lise (2015)

**Diagram:**

- Draw offer $x'$
  - $(x, \alpha)$
  - $x' > x$ (switch)
  - $x' \leq x$ (stay)
    - $x' \leq x\alpha$
      - (discard)
    - $x\alpha < x' \leq x$
      - (rebargain)
  - $(x', \alpha' = x/x')$
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More offers $\Rightarrow$ More frequent EE or rebargaining $\Rightarrow$ Price of labor services $\Rightarrow$ Inflation
Search and Matching

- Random search with worker and firm contact rates $f(\theta)$ and $q(\theta)$ per unit search efficiency.
  
  - Labor market tightness $\theta = v/S$: Ratio of vacancies to aggregate effective search.
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  - Unemployed search with efficiency $\zeta$ and accept all offers.
  - Employed search with on-the-job search efficiency $\nu$ and accept if offer is better.
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  - Labor service firms take into account entire distribution of workers when posting vacancies
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- Vacancy creation: Service firm
  - Labor service firms take into account entire distribution of workers when posting vacancies

- Worker-firm match terminates due to:
  - (exogenous) job separation shock.
  - (exogenous) retirement.
  - (endogenous) worker quitting to take another job.
Roadmap

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**Directed Acyclic Graph (DAG) Model Representation**

**Shocks and unknown variables:**
- **Exogenous:** $z, \nu, \beta$
- **Endogenous:** $\pi, Y, p^l, b, u, \theta, \Gamma^S, \epsilon^2e$

**Targets:**
1. Phillips curve
2. Labor market clearing
3. Asset market clearing
4. Government budget balance
5. Consistency of unemployment rate
6. Free-entry condition
7. Consistency of service firm profits
8. Consistency of job switching rate

**Solution:** Sequence-space Jacobian method (Auclert et al., 2021) + worker distribution
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Positive Analysis

Post-Great Recession Case Study
How do labor market dynamics affect the comovement of inflation and unemployment?

**Case study**: Significant weakening of $\text{corr}(u, EE)$ post-Great Recession, 2016–19.
Labor Markets Under Demand and OJS Shocks

Common unemployment path,
Labor Markets Under Demand and OJS Shocks

Common unemployment path, but different EE dynamics.
Identical unemployment and similar output dynamics,
Identical unemployment and similar output dynamics, but muted inflation response.
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Annual inflation rate 0.23pp lower due to lower EE.
Positive Analysis

Decomposing Effects of OJS Shocks on Inflation
NKPC implies that inflation $\pi$ is driven by marginal cost $p^l$ to a first order approximation. An increase in OJS efficiency $\nu$ leads to an increase in $p^l$. 
Decomposing Effects of OJS: Labor Market

More frequent rebargaining and shorter matches lower expected match value. For free-entry to hold, $p^l$ increases.
Decomposing Effects of OJS: Labor Market

Improved productivity distribution raises supply of labor services. For labor market to clear, $\theta$ decreases. For free-entry to hold, $p^l$ decreases.
Decomposing Effects of OJS: Labor Market

Crowd-out increases $u$—reducing $C$ and $Y$—and hence reduces demand for labor services. For labor market to clear, $\theta$ decreases. For free-entry to hold, $p^l$ decreases.
Decomposing Effects of OJS: Discount Rate

Inflation rises in equilibrium, which increases $r$ through MP.
For free-entry to hold, $p^l$ increases.
Decomposing Effects of OJS: Discount Rate

Unemployment rises in equilibrium, which reduces $r$ through MP. For free-entry to hold, $p^l$ decreases.
Heterogeneous consumption responses to higher OJS

- **PE:** Results
  - positive consumption response stronger for wealth-poor among E but *wealth-rich* among U,
  - as raising current $c$ upon shock affecting *future* income requires some wealth or income.

- **GE:** Results
  - negative consumption response stronger for *wealth-rich* regardless of employment,
  - due to lower dividends and share prices caused by an increase in real rate.

- **Results**
  - Implications for our quantitative results: In a complete-markets model,
    - Aggregate consumption response would follow response of wealth-rich.
    - Overstatement of decline in aggregate demand leads to larger GE effect on tightness.
    - This would lead to smaller increase in marginal cost $p_l$ and inflation.
    - Thus, we would attribute smaller role to job ladder shocks in explaining missing inflation.
HETEROGENEOUS CONSUMPTION RESPONSES TO HIGHER OJS

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Normative Analysis

Optimal Monetary Policy
Optimal Monetary Policy

- Under a central bank loss function

\[ \mathcal{W} = \text{var}(\pi_t - \pi^*) + \Psi \text{var}(Y_t - Y^*), \]

evaluate the performance of a generalized Taylor rule for \( \Phi_u, \Phi_{EE} \) combinations:

\[ i_t = i^* + \Phi_\pi (\pi_t - \pi^*) + \Phi_u (u_t - u^*) + \Phi_{EE} (EE_t - EE^*). \]

- Computational challenge:

  - Each \( \Phi_u, \Phi_{EE} \) combination corresponds to a new set of Jacobians.

  - Costly in our model because Jacobians involve the worker distribution over many states.

  - **Key:** Use policy shocks to compute IRFs to structural shock under alternative Taylor rule.

McKay and Wolf 2022
Optimal Monetary Policy

- Optimal MP prescribes $\Phi_u^* = -3.18$ and $\Phi_{EE}^* = 2.22$.

- Relative to baseline MP, optimal MP reduces central bank loss by close to 80 percent.
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- Macroeconomic outcomes under baseline and optimal policies:
  - volatilities of output, tightness, consumption, and price of labor services reduce to one-third,
  - volatilities of real interest rate and price of shares double.
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- Macroeconomic outcomes under baseline and optimal policies:
  - volatilities of output, tightness, consumption, and price of labor services reduce to one-third,
  - volatilities of real interest rate and price of shares double.

- Ignoring job mobility dynamics, i.e., $\Phi_{EE} = 0$, yields:
  - less aggressive response on unemployment gap $\Phi_u = -2.71$
  - 12 percent higher central bank loss relative to optimal MP.
Heterogeneous Welfare Gains

- Non-monotonic welfare gains across match productivity distribution:
  - Low productivity workers gain from stable job ladder during recessions.
  - High productivity workers benefit from faster recovery of productivity upon job loss.

- Monotonically declining welfare gains across wealth distribution:
  - Wealth-poor benefit from less volatile unemployment risk.
  - Wealth-rich exposed to larger fluctuations in price of shares.

- Large welfare gains even among employed:
  - Employed workers benefit from improved job-ladder stability.
  - Unemployed workers, in addition, benefit from less severe downturns and faster recoveries.
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CONCLUSION

○ Develop a HANK model featuring a frictional labor market with on-the-job search.

○ Analyze macro implications of an EE shock.
  - Quantify magnitude of missing inflation post-Great Recession to be 0.23pp.
  - Decompose channels through which an EE shock affects inflation.

○ Evaluate monetary policy under a dual-mandate objective function.
  - Including EE rate in reaction function reduces overall inflation and unemployment volatility.

○ Future work:
  - Heterogeneous labor market shocks.
  - Estimate model with and without labor market shocks and evaluate its performance.
  - Fiscal and monetary policy interactions accounting for rich labor market dynamics.
Comovement of Unemployment and Labor Costs

Introduction
1. **Missing disinflation/inflation around Great Recession:**
   **This paper:** Quantify role of job mobility on inflation

2. **Effects of job mobility on inflation:**
   *RANK:* Moscarini and Postel-Vinay (2019), Faccini and Melosi (2022); *HANK:* Alves (2019)
   **This paper:** Rich labor market heterogeneity, decomposition, and optimal MP

3. **HANK with labor search:**
   Ravn and Sterk (2016), Gornemann, Kuester, and Nakajima (2021)
   **This paper:** Incorporate job mobility dynamics

4. **Computational methods:** Auclert, Bardoczy, Rognlie, and Straub (2021)
   **This paper:** Incorporate discretized worker distribution into the SSJ method
Final and Intermediate Goods Firms

Final good firm:
  ◦ Combines differentiated intermediate goods using a CES technology and sells goods at $P_t$.

Intermediate goods firms:
  ◦ Firms are monopolistically competitive.
  ◦ Operate linear technology using labor services $l^t(j)$ to produce differentiated variety $y^t(j)$.
    - Production function is $y^t(j) = z^t l^t(j)$ with aggregate productivity $z^t$.
    - Price of labor services $p^l_t$ is determined in the labor market by worker flows.
  ◦ Pricing subject to quadratic costs $⇒$ New Keynesian Phillips Curve: Rotemberg (1982)
    \[
    \frac{\log (1 + \pi^t - \pi^∗)}{1 + \pi^t - \pi^∗} = \vartheta \left( p^l_t z^t - \eta - 1 \eta \right) + 1 + r_t \frac{\log (1 + \pi^{t+1} - \pi^∗)}{1 + \pi^{t+1} - \pi^∗} \left( \frac{Y^t+1}{Y^t} \right).
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○ Pricing subject to quadratic costs $\Rightarrow$ New Keynesian Phillips Curve: Rotemberg (1982)

$$\log \left( \frac{1 + \pi_t - \pi^*}{1 + \pi_t - \pi^*} \right) \left( \frac{1 + \pi_t}{1 + \pi_t - \pi^*} \right) = \vartheta \left( \frac{p^l_t}{z_t} - \frac{\eta - 1}{\eta} \right) + \frac{1}{1 + r_{t+1}} \log \left( \frac{1 + \pi_{t+1} - \pi^*}{1 + \pi_{t+1} - \pi^*} \right) \left( \frac{1 + \pi_{t+1}}{1 + \pi_{t+1} - \pi^*} \right) \frac{Y_{t+1}}{Y_t}.$$
The central bank controls the short-term nominal interest rate following a Taylor rule:

\[ i_t = i^* + \Phi_\pi (\pi_t - \pi^*) + \Phi_u (u_t - u^*). \]
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The fiscal authority:

- Consumption tax \( \tau_c \), income tax \( (\tau_t, \Upsilon) \), UI, pensions \( \phi^R \), and exogenous spending \( G_t \).
- Finances deficits by issuing nominal debt \( B_t \).
**Monetary and Fiscal Authority**

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Mutual fund:

- Owns all firms and holds government debt.
- Issues shares and pays dividends.
Incomes Taxes

High average tax rate (low $\tau$)

Net income $= \tau \omega^{1-\gamma} \Rightarrow$ average tax rate $= 1 - \tau \omega^{-\gamma}$
Government Budget Constraint

\[ B_{t-1} + G_t + P_t \int UI(h) d\lambda^U_t(s,h) + P_t \int \phi^R d\lambda^R_t(s) = \frac{B_t}{1 + i_t} \]

\[ + P_t \tau_c \int c(s, h, x, \alpha) d\lambda_t(s, h, x, \alpha) \]

\[ + P_t \int \left( UI(h) - \tau_t UI(h)^{1-\Upsilon} \right) d\lambda^U_t(s, h) \]

\[ + P_t \int \left( w(h, x, \alpha) - \tau_t w(h, x, \alpha)^{1-\Upsilon} \right) d\lambda^E_t(s, h, x, \alpha) \]

\[ + P_t \int \left( \phi^R - \tau_t (\phi^R)^{1-\Upsilon} \right) d\lambda^R_t(s) \]

- Consumption tax \( \tau_c \) and progressive income tax \((\tau_t, \Upsilon)\).
- Unemployment benefit \( UI(h) \), retirement pension \( \phi^R \), government expenditures \( G_t \).
- Nominal debt \( B_t \).
- \( \lambda^X_t(.) \) worker distribution over relevant states.
- Nominal price level \( P_t \).
Mutual Fund

- Owns intermediate and labor service firms, and all government bonds.
- Issues shares at price $P_s$ and holds government bonds to earn a gross return of $1 + i$.
- No-arbitrage implies returns on stock and bonds are equalized:
  \[
  \frac{P_{t+1}^s + D_{t+1}}{P_t^s} = 1 + i_t.
  \]
- Cannot retain any funds. All balances are distributed to share owners as dividends:
  \[
  D_t = B_{t-1} - \frac{B_t}{1 + i_t} + P_t \Gamma_t^I + P_t \Gamma_t^S,
  \]
  where $\Gamma^I$ and $\Gamma^S$ are per-period real profits of intermediate and service firms:
  \[
  \Gamma_t^I = \left(1 - \frac{p^I_t}{z_t} - \frac{\eta}{2\vartheta} \log(1 + \pi_t - \pi^*)^2\right)Y_t
  \]
  \[
  \Gamma_t^S = \int (p^S_t F(h, x) - w(h, x, \alpha)) \, d\lambda_t^E(s, h, x, \alpha).
  \]
Timing

1. **Aggregate shocks** are realized.

2. **Monetary authority** sets the nominal rate.

3. **Idiosyncratic shocks** are realized.
   - Life cycle and job destruction shocks.
   - Worker skills evolve.

4. **Labor search:** Service firms post vacancies, workers search, new matches are formed.

5. **Production:** Labor services, intermediate goods, and final goods are produced.

6. **Consumption:**
   - Profits are realized, wages, dividends and transfers are paid out.
   - Consumption-saving decisions are made.
**Unemployed Worker**

Value of unemployment:

\[
V_t^U(s, h) = \max_{s' \geq 0, c} u(c) + \beta (1 - \psi^R) \mathbb{E}_{h' \mid h} \left[ \Omega_{t+1}^U(s', h') \right] + \beta \psi^R V_{t+1}^R(s')
\]

s.t. \( P_t c (1 + \tau_c) + P_t^s s' = P_t \tau_t UI(h)^{1-\gamma} + (P_t^s + D_t) s \)

- \( \psi^R \): Probability of retirement
- \( V_t^R(s) \): Value of retirement
- \( UI(h) \): Unemployment benefits
- \( P_t \): Price of final good
- \( P_t^s \): Price of mutual fund shares
- \( D_t \): Dividends
**Unemployed Worker**

Value of unemployment:

\[
V_t^U(s, h) = \max_{s' \geq 0, \ c} u(c) + \beta(1 - \psi^R)\mathbb{E}_{h'|h}[\Omega_{t+1}^U(s', h')] + \beta\psi^RV_{t+1}(s')
\]

\[
\text{s.t. } P_tc(1 + \tau_c) + P_ts' = P_t\tau_tUI(h)^{1-\gamma} + (P_ts + Dt)s
\]

Value of job search:

\[
\Omega_t^U(s, h) = \zeta f(\theta_t) \mathbb{E}_x V_t^E(s, h, x, x/x) + (1 - \zeta f(\theta_t)) V_t^U(s, h)
\]

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- \(UI(h)\): Unemployment benefits
- \(P_t\): Price of final good
- \(P_ts\): Price of mutual fund shares
- \(D_t\): Dividends
**Employed Worker**

Value of employment:

\[
V_t^E(s, h, x, \alpha) = \max_{s' \geq 0, c} \left[ u(c) + \beta(1 - \psi^R)\mathbb{E}_{h'|h} \left\{ (1 - \delta)\Omega_{t+1}^E(s', h', x, \alpha) + \delta\Omega_{t+1}^U(s', h') \right\} \right. \\
+ \left. \beta\psi^R V_{t+1}^R(s') \right]
\]

s.t. \[ P_t c (1 + \tau_c) + P_t^s s' = P_t \tau_t w(h, x, \alpha)^{1-\gamma} + (P_t^s + D_t) s \]

- \( \delta \): Probability of match separation
- \( \nu \): On-the-job search efficiency
**EMPLOYED WORKER**

Value of employment:

\[
V_t^E(s, h, x, \alpha) = \max_{s' \geq 0, c} u(c) + \beta(1 - \psi^R)\mathbb{E}_{h' | h} \left\{ (1 - \delta)\Omega_{t+1}^E(s', h', x, \alpha) + \delta\Omega_{t+1}^U(s', h') \right\} \\
+ \beta\psi^R V_{t+1}^R(s') \\
\text{s.t. } P_t c(1 + \tau_c) + P_t^s s' = P_t \tau_t w(h, x, \alpha)^{1 - \gamma} + (P_t^s + D_t)s
\]

Value of on-the-job search:

\[
\Omega_t^E(s, h, x, \alpha) = \nu f(\theta_t) \mathbb{E}_{\tilde{x}} \left[ \max \left\{ V_t^E(s, h, \tilde{x}, x/\tilde{x}), \right. \right. \\
\left. \left. V_t^E(s, h, x, \max\{\alpha, \tilde{x}/x\}) \right\} \right]
\]

\[
+ (1 - \nu f(\theta_t)) V_t^E(s, h, x, \alpha)
\]

- \(\delta\): Probability of match separation
- \(\nu\): On-the-job search efficiency
Value of retirement:

\[ V_t^R(s) = \max_{s' \geq 0, c} u(c) + \beta (1 - \psi^D) V_{t+1}^R(s') \]

subject to

\[ P_t c (1 + \tau_c) + P_s s' = P_t \tau_t (\phi^R)^{1 - \gamma} + (P_s + D_t) s \]

Retired only face mortality risk.

- \( \psi^D \): Probability of death
- \( \phi^R \): Retirement pensions
- \( P_s \): Price of mutual fund shares
- \( D_t \): Dividends
**LABOR SERVICE FIRMS**

Value of matched firm:

\[ J_t(h, x, \alpha) = p_t^l F(h, x) - w(h, x, \alpha) + \frac{1}{1 + r_{t+1}} (1 - \psi^R) (1 - \delta) \]

\[ \times \mathbb{E}_{h'|h} \left\{ (1 - \nu f(\theta_{t+1})) J_{t+1}(h', x, \alpha) + \nu f(\theta_{t+1}) \int_{\tilde{x}}^x J(h', x, \max\{\alpha, \tilde{x}/x\}) d\Gamma^x(\tilde{x}) \right\} \]

- \( p_t^l \): Price of labor services
- \( r_t \): Real interest rate
- \( \kappa \): Cost of posting vacancy
- \( F(h, x) = h.x \): Match output
- \( S_t \): Aggregate search effort
- \( \Gamma^x \): Sampling distribution for match productivity
**Value of matched firm:**

\[ J_t(h, x, \alpha) = p_t^l F(h, x) - w(h, x, \alpha) + \frac{1}{1 + r_{t+1}} (1 - \psi^R) (1 - \delta) \]

\[ \times \mathbb{E}_{h'|h} \left\{ (1 - \nu f(\theta_{t+1})) J_{t+1}(h', x, \alpha) + \nu f(\theta_{t+1}) \int_x J(h', x, \max\{\alpha, \tilde{x}/x\}) d\Gamma^x(\tilde{x}) \right\} \]

**Value of posting vacancy:**

\[ V_t = -\kappa + q(\theta_t) \frac{1}{S_t} \left[ \zeta \int_{s,h} \int_{\tilde{x}} J_t(h, \tilde{x}, x/\tilde{x}) d\Gamma^x(\tilde{x}) d\mu^U_t(s, h) \right. \]

\[ \left. + \nu \int_{s,h,x,\alpha} \int_x J_t(h, \tilde{x}, x/\tilde{x}) d\Gamma^x(\tilde{x}) d\mu^E_t(s, h, x, \alpha) \right] \]

- \( p_t^l \): Price of labor services
- \( r_t \): Real interest rate
- \( \kappa \): Cost of posting vacancy
- \( F(h, x) = h.x \): Match output
- \( S_t \): Aggregate search effort
- \( \Gamma^x \): Sampling distribution for match productivity
**Labor Service Firms**

Value of matched firm:

\[
J_t(h, x, \alpha) = p_t^l F(h, x) - w(h, x, \alpha) + \frac{1}{1 + r_{t+1}} (1 - \psi^R) (1 - \delta) \\
\times \mathbb{E}_{h'|h} \left\{ (1 - \nu f(\theta_{t+1})) J_{t+1}(h', x, \alpha) + \nu f(\theta_{t+1}) \int_{x}^{x} J(h', x, \max\{\alpha, \tilde{x}/x\}) d\Gamma^{x}(\tilde{x}) \right\}
\]

Value of posting vacancy:

\[
V_t = -\kappa + q(\theta_t) \frac{1}{S_t} \left[ \zeta \int_{s,h} \int_{\tilde{x}} J_t(h, \tilde{x}, x/\tilde{x}) d\Gamma^{x}(\tilde{x}) d\mu_t^U (s, h) \\
+ \nu \int_{s,h,x,\alpha} \int_{x}^{x} J_t(h, \tilde{x}, x/\tilde{x}) d\Gamma^{x}(\tilde{x}) d\mu_t^E (s, h, x, \alpha) \right]
\]

Free-entry implies \( V_t = 0 \).

- \( p_t^l \): Price of labor services
- \( r_t \): Real interest rate
- \( \kappa \): Cost of posting vacancy

\( p_t^l \), \( F(h, x) = h.x \): Match output
\( S_t \): Aggregate search effort
\( \Gamma^{x} \): Sampling distribution for match productivity
Labor Service Firms

- Post vacancies $v$ at cost $\kappa$ to hire workers in a frictional labor market.
  - Labor market tightness $\theta = v/S$, where $S = \int \zeta d\mu^U(s,h) + \int \nu d\mu^E(s,h,x,\alpha)$.
  - CRS matching function $M(v,S)$ determines number of new worker-firm contacts.
  - Worker and firm contact rates, $f(\theta) = \frac{M(v,S)}{S}$ and $q(\theta) = \frac{M(v,S)}{v}$, pinned down by $\theta$.

- Produce $F(h,x) = hx$ labor services using one unit of labor.

- Sell labor services to intermediate firms in a competitive market at nominal price $P^l$. 
Solution of Model with Aggregate Shocks

Follow Auclert, Bardoczy, Rognlie, Straub (2021) to compute IRFs and model simulation.

1. Cast the model in sequence space for \( t \in \{0, ..., T\} \) as a directed acyclic graph (DAG).
   - simple blocks
   - heterogeneous-agent (HA) block

2. For each block, compute partial Jacobians of each output with respect to each input.
   - Automatic differentiation for simple blocks.
   - Numerical differentiation for HA block (fake-news algorithm).

3. Forward accumulate partials along topological sort of DAG to get total derivatives.

4. Use implicit function theorem to get the GE response of endogenous variables to shocks.

5. The GE Jacobian is sufficient to compute IRFs to aggregate shocks.

6. Simulate the model subject to aggregate shocks (IRF \( \equiv \) MA).
### Externally Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Value</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Curvature in utility function</td>
<td>2</td>
<td>Standard</td>
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<tr>
<td>$\psi^R$</td>
<td>Retirement probability</td>
<td>0.00625</td>
<td>40 years of work stage</td>
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<tr>
<td>$\psi^D$</td>
<td>Death probability</td>
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<td>20 years of retirement stage</td>
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<td>$\Delta h$</td>
<td>Skill appreciation/depreciation amount</td>
<td>0.275</td>
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<td>$\pi^E$</td>
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<td>Wage growth for job stayers</td>
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<td>$\xi$</td>
<td>Matching function elasticity</td>
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<td>Normalization</td>
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<td>$\eta$</td>
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<td>20 percent markup</td>
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<tr>
<td>$\vartheta$</td>
<td>Price adjustment cost parameter</td>
<td>0.021</td>
<td>Slope of Phillips curve, Gali and Gertler (1999)</td>
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<tr>
<td>$x_G$</td>
<td>Government spending/GDP ratio</td>
<td>0.19</td>
<td>Total net federal outlay/ GDP</td>
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<tr>
<td>$x_B$</td>
<td>Debt/GDP ratio</td>
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<td>Total public debt/GDP</td>
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<td>$\tau_c$</td>
<td>Consumption tax rate</td>
<td>0.0312</td>
<td>Sales tax receipt/consumption exp.</td>
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<td>$\Upsilon$</td>
<td>Progressivity of income tax</td>
<td>0.151</td>
<td>Heathcote et al. (2014)</td>
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<td>$\rho_{\tau}$</td>
<td>Responsiveness of income tax to debt level</td>
<td>0.10</td>
<td>Auclert et al. (2020)</td>
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<td>Steady-state inflation rate</td>
<td>0.00496</td>
<td>2% annual inflation rate</td>
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<td>$\Phi_{\pi}$</td>
<td>Responsiveness of interest rate to inflation</td>
<td>1.5</td>
<td>Taylor (1993) and Gali (2015)</td>
</tr>
<tr>
<td>$\Phi_u$</td>
<td>Responsiveness of interest rate to unemployment</td>
<td>-0.25</td>
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<td>$\beta$</td>
<td>Discount factor</td>
<td>0.981</td>
<td>Fraction with non-positive liquid wealth</td>
<td>0.16</td>
<td>0.11</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Vacancy creation cost</td>
<td>0.670</td>
<td>Unemployment rate</td>
<td>0.051</td>
<td>0.052</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Job separation probability</td>
<td>0.091</td>
<td>EU rate</td>
<td>0.038</td>
<td>0.033</td>
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<tr>
<td>$\nu$</td>
<td>Search efficiency of employed</td>
<td>0.108</td>
<td>EE rate (w/o non-employment spell)</td>
<td>0.02</td>
<td>0.02</td>
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<td>$\pi^U$</td>
<td>Skill depreciation probability</td>
<td>0.022</td>
<td>Earnings drop upon job loss</td>
<td>-0.35</td>
<td>-0.36</td>
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<tr>
<td>$\sigma_x$</td>
<td>Standard deviation of match productivity</td>
<td>0.063</td>
<td>Wage growth of job switchers</td>
<td>0.09</td>
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<td>$\phi^E$</td>
<td>Maximum share of output as wages</td>
<td>0.823</td>
<td>Labor share</td>
<td>0.67</td>
<td>0.74</td>
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<tr>
<td>$\phi^U$</td>
<td>UI replacement rate</td>
<td>0.385</td>
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<td>0.40</td>
<td>0.44</td>
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<td>$\phi^R$</td>
<td>Retirement benefit amount</td>
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Estimation of Shocks

- Estimate AR(1) processes for discount factor $\beta$, productivity $z$, OJS efficiency $\nu$.

- Target autocorrelations and standard deviations of $Y$, $u$ and $EE$.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
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<tbody>
<tr>
<td>$Y$</td>
<td>0.024</td>
<td>0.963</td>
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<tr>
<td>$u$</td>
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<td>$EE$</td>
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<td>$\pi$</td>
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<td>0.388</td>
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# Variance Decomposition

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<tr>
<th>Variable</th>
<th>Share of Variance Explained by $z$</th>
<th>Share of Variance Explained by $\nu$</th>
<th>Share of Variance Explained by $\beta$</th>
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<tbody>
<tr>
<td>$Y$</td>
<td>0.008</td>
<td>0.031</td>
<td>0.961</td>
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<tr>
<td>$u$</td>
<td>0.111</td>
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<tr>
<td>$EE$</td>
<td>0.070</td>
<td>0.787</td>
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<tr>
<td>$\theta$</td>
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<tr>
<td>$\pi$</td>
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<th>Shock</th>
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<td>$z$</td>
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<tr>
<td>$\nu$</td>
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<tr>
<td>$\beta$</td>
<td>0.909</td>
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</table>
How do labor market dynamics affect the comovement of inflation and unemployment?

Case study: Significant weakening of \( \text{corr}(u, EE) \) post-Great Recession, 2016–19.
How do labor market dynamics affect the comovement of inflation and unemployment?

Case study: Significant weakening of $\text{corr}(u, EE)$ post-Great Recession, 2016–19.
Model Simulation of Post-Great Recession


Consider two economies starting from the same steady state.

Economy 1: Counterfactual economy
- Shocks to discount factor to match decline in unemployment post-Great Recession
  \[
  \beta_t = (1 - \rho) \beta^* + \rho \beta_{t-1} + \varepsilon_{\beta,t}.
  \]

Economy 2: Post-Great Recession
- Flat EE rate
  - Shocks to discount factor to match decline in unemployment post-Great Recession
    \[
    \beta_t = (1 - \rho) \beta^* + \rho \beta_{t-1} + \varepsilon_{\beta,t}.
    \]
  - Shocks to OJS efficiency to match path of EE rate
    \[
    \nu_t = (1 - \rho) \nu^* + \rho \nu_{t-1} + \varepsilon_{\nu,t}.
    \]
Model Simulation of Post-Great Recession


Consider two economies starting from the same steady state.
Model Simulation of Post-Great Recession

Goal: Quantify magnitude of “missing inflation” due to weakening \( corr(u, EE) \) in 2016–2019.

Consider two economies \textbf{starting from the same steady state}.

\textbf{Economy 1:} \textit{Counterfactual economy} — EE moves with unemployment

\begin{itemize}
  \item Shocks to discount factor to match decline in unemployment post-Great Recession
  \[ \beta_t = (1 - \rho_\beta)\beta^* + \rho_\beta \beta_{t-1} + \varepsilon_{\beta,t}. \]
\end{itemize}
MODEL SIMULATION OF POST-GREAT RECESSION

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Consider two economies starting from the same steady state.

**Economy 1:** Counterfactual economy — EE moves with unemployment
- Shocks to discount factor to match decline in unemployment post-Great Recession
  \[
  \beta_t = (1 - \rho_\beta) \beta^* + \rho_\beta \beta_{t-1} + \varepsilon_\beta,t.
  \]

**Economy 2:** Post-Great Recession — flat EE rate
- Shocks to discount factor to match decline in unemployment post-Great Recession
  \[
  \beta_t = (1 - \rho_\beta) \beta^* + \rho_\beta \beta_{t-1} + \varepsilon_\beta,t.
  \]
- Shocks to OJS efficiency to match path of EE rate
  \[
  \nu_t = (1 - \rho_\nu) \nu^* + \rho_\nu \nu_{t-1} + \varepsilon_\nu,t.
  \]
Details on Model Simulation

- Targets over transition horizon \( t \in \{0, \ldots, T\} \):
  1. Path of unemployment rate during post-Great Recession:
     - 15% decline by \( T = 16 \) quarters and revert back to steady state with
       \[
       u_t = (1 - \rho_u)u^* + \rho_u u_{t-1},
       \]
       where \( \rho_u = 0.85 \).
  2. Path of EE rate during post-Great Recession:
     - Remain at steady state level of EE rate despite declining unemployment rate.

- Two economies:
  1. Economy 1: positive demand shocks
     - Shocks modeled as innovations to discount factor \( \beta \).
     - Assume at each \( t = 0, \ldots, T \), shock \( \varepsilon_\beta < 0 \) hits economy.
  2. Economy 2: positive demand and negative OJS shocks
     - Shocks modeled as innovations to discount factor \( \beta \) and OJS efficiency \( \nu \).
     - Assume at each \( t = 0, \ldots, T \), shocks \( \varepsilon_\beta < 0 \) and \( \varepsilon_\nu < 0 \) hit economy.
Negative OJS efficiency shocks limit the rise in average $x$ and $\alpha$. 
Negative OJS efficiency shocks lead to leftward shifts in distributions of $x$ and $\alpha$. 
Higher EE and higher inflation.
Flatter Phillips curve under negative OJS efficiency shocks.
Decomposing Effects of OJS Shock on Inflation

Exogenous: $z, \beta, \nu$
Endogenous: $\pi, Y, p_l, b, u, \theta, \Gamma^S, e2e$
Decomposing Effects of OJS Shock on Inflation

Exogenous: $z, \beta, \nu$
Endogenous: $\pi, Y, p^l, b, u, \theta, \Gamma^S, e^2e$

Diagram:
- **monetary policy**
  - Inputs: $\pi, u$
- **service firm**
  - Inputs: $p^l, \theta, \nu$

Results:
- $H_6$

Free entry, $H_6$
Decomposing Effects of OJS Shock on Inflation

Exogenous: $z, \beta, \nu$
Endogenous: $\pi, Y, p^l, b, u, \theta, \Gamma^S, e2e$
Details for Inflation Decomposition

1. Start from total Jacobians of a block of interest, e.g., H6 (free-entry condition).

2. Apply IFT to H6 to get the derivative of $p^l$ w.r.t. endogenous and exogenous variables.

3. Multiply the total derivative of $p^l$ with GE IRFs of variables w.r.t $\nu$.

4. This gives the response of $p^l$ components $\nu$, $\theta$, $\pi$, $u$ w.r.t $\nu$ (direct and indirect).

5. Can further decompose these components, e.g., $\theta$ using other related blocks, e.g., H2.
GE Effects of $\nu$ on Model Outcomes

Tightness

Unemployment

Output

Inflation

Impulse responses to a unit increase in $\nu$. 
HETEROGENEOUS CONSUMPTION RESPONSES

Consumption of employed: PE

Higher future income raises consumption,
Heterogeneous consumption responses

Consumption of employed: PE

Consumption of unemployed: PE

Higher future income raises consumption, except for wealth-poor unemployed
In GE, consumption declines due to real rates.
Decline more prominent for wealth-rich due to lower dividends and share prices.
Heterogeneous consumption responses

Consumption of employed: GE
Consumption of unemployed: GE

Complete markets would overstate aggregate demand decline.
GE Effects of $\nu$ on Model Outcomes

Real dividends

Real share price

Real interest rate

Real total firm profits

Impulse responses to a unit increase in $\nu$. 
Monetary Policy

- Nominal interest rates typically react to inflation and a measure of the output gap.

Our claim:
- Unemployment rate is not a sufficient statistic to gauge the health of the labor market.
- Including EE rate in the reaction function improves inflation and unemployment stability.

Agenda:
Study monetary policy under a dual-mandate central bank objective function.
MONETARY POLICY

- Nominal interest rates typically react to inflation and a measure of the output gap.
- A standard measure used for the output gap is the unemployment rate.

Our claim:
- The unemployment rate is not a sufficient statistic to gauge the health of the labor market.
- Including the Employment to Population Ratio (EE rate) in the reaction function improves inflation and unemployment stability.
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- **Agenda:** Study monetary policy under a dual-mandate central bank objective function.
Evaluating Alternative Taylor Rules

- Use policy shocks to compute IRFs to non-policy shocks under alternative Taylor rules.
  - **Key**: Firms and households do not care about the systematic component of monetary policy, but care about the current and future path of interest rates. McKay and Wolf (2022)
  - No need to recompute Jacobians. Derivatives under the baseline policy are enough.

- Solving the system for policy news shocks \( \mu = \{\mu_t\}_{t=1}^T \) given non-policy shock \( \varepsilon \)

\[
\begin{align*}
  \text{IRF of } i \text{ under baseline} & = \tilde{\Phi}_i (\pi \Phi_\pi, \Phi_u (\varepsilon) + \Theta_{\pi,\mu} \Phi_\pi, \Phi_u \mu) + \tilde{\Phi}_u (u \Phi_\pi, \Phi_u (\varepsilon) + \Theta_{\pi,\mu} \Phi_\pi, \Phi_u \mu) + \tilde{\Phi}_{EE} (EE \Phi_\pi, \Phi_u (\varepsilon) + \Theta_{EE,\mu} \Phi_\pi, \Phi_u \mu),
\end{align*}
\]

allows for computing the IRF to \( \varepsilon \) under alternative Taylor rule \( \tilde{\Phi}_\pi, \tilde{\Phi}_u, \tilde{\Phi}_{EE} \).

- IRF to \( \varepsilon \) under alternative rule \( \equiv \) IRF to \( \varepsilon \) and \( \{\mu_t\}_{t=1}^T \) under baseline rule.
Impulse responses to a unit increase in $\nu$ under baseline (blue) and optimal (red) MP.
## Outcomes under Optimal Policy

- Volatilities of macroeconomic outcomes under baseline and optimal policies:

<table>
<thead>
<tr>
<th></th>
<th>$\pi$</th>
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<td>0.0013</td>
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<td>0.0019</td>
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<td>0.0047</td>
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<td>0.0203</td>
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- Heterogeneous welfare gains under optimal policy:

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<tr>
<th></th>
<th>Bottom</th>
<th>Middle</th>
<th>Top</th>
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<th>Middle</th>
<th>Top</th>
<th>$E$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Match quality $x$</td>
<td>0.24</td>
<td>0.13</td>
<td>0.16</td>
<td>0.13</td>
<td>0.18</td>
<td>0.10</td>
<td>0.15</td>
<td>0.20</td>
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