A Macroeconomic Framework for Quantifying Systemic Risk*

Zhiguo He   Arvind Krishnamurthy

First Draft: November 20, 2011
This Draft: March 13, 2015

Abstract

Systemic risk arises when shocks lead to states where a disruption in financial intermediation adversely affects the economy and feeds back into further disrupting financial intermediation. We present a macroeconomic model with a financial intermediary sector subject to an equity capital constraint. The novel aspect of our analysis is that the model produces a stochastic steady state distribution for the economy, in which only some of the states correspond to systemic risk states. The model allows us to examine the transition from "normal" states to systemic risk states. We calibrate our model and use it to match the systemic risk apparent during the 2007/2008 financial crisis. We also use the model to compute the conditional probabilities of arriving at a systemic risk state, such as 2007/2008. Finally, we show how the model can be used to conduct a macroeconomic "stress test" linking a stress scenario to the probability of systemic risk states.

JEL Codes: G12, G2, E44

Keywords: Liquidity, Delegation, Financial Intermediation, Crises, Financial Friction, Constraints.

*University of Chicago, Booth School of Business and NBER, zhiguo.he@chicagobooth.edu; Stanford University, Graduate School of Business and NBER, a-krishnamurthy@stanford.edu. We thank seminar participants at the Banque de France, Bank of Canada, Chicago Booth (Finance and Macro workshops), Federal Reserve Board, Federal Reserve Bank at Atlanta, EPFL at Lausanne, Monetary Economics Conference at the Bank of Portugal, INET Conference, INSEAD, MIT-Sloan, NBER Summer Institute EFG meeting, Northwestern University, Princeton University, Riksbank, Rising Star Conference, SED 2013 at Seoul, Swiss Finance Institute, UC-Berkeley, UCLA, UC-Irvine, UCSD, University of North Carolina, University of Rochester, Washington University in St. Louis, and the Yale 2012 GE conference for comments. We thank Viral Acharya, Mark Gertler, Pete Kyle, John Leahy, Matteo Maggiori, Adriano Rampini, Alp Simsek and Wei Xiong for helpful suggestions. We also thank Valentin Haddad and Tyler Muir for their suggestions and research assistance, and Simon Gilchrist and Egon Zakrajsek for their EBP data.

1 Introduction

It is widely understood that a disruption in financial intermediation, triggered by losses on housing-related investments, has played a central role in the recent economic crisis. Figure 2 plots the market value of equity for the financial intermediary sector, along with a credit spread, investment, and a land price index. All variables have been normalized to one in 2007Q2. The figure illustrates the close relation between reductions in the value of financial intermediary equity, rising spreads, and falling land prices and aggregate investment.

In the wake of the crisis, understanding *systemic risk*, i.e., the risk that widespread financial constraints in the financial intermediation sector trigger adverse effects for the real economy (see, e.g., Bernanke, 2009; Brunnermeier, Gorton and Krishnamurthy, 2010), has been a priority for both academics and policy-makers. To do so, it is important to not only embed a financial intermediary sector in a macroeconomic setting, but also to study a model in which financial constraints on the intermediary sector only bind in some states ("systemic states"). This is a necessary methodological step in order to study systemic risk because systemwide financial disruptions are rare, and in most cases we are interested in understanding the transition of the economy from non-systemic states into systemic states.

The first part of our paper develops such a model. The model's equilibrium is a stochastic steady state distribution for the economy, in which systemic states where constraints on the financial sector bind correspond to only some of the possible realizations of the state variables. Moreover, in any given state, agents anticipate that future shocks may lead to constraints tightening, triggering systemic risk. As the economy moves closer to a systemic state, these anticipation effects cause banks to reduce lending and hence investment falls even though capital constraints are not binding. Relative to other papers in the literature (e.g., Bernanke, Gertler, and Gilchrist, 1999, Kiyotaki and Moore, 1997, Gertler and Kiyotaki, 2010), our approach enables us to study the global dynamics of the system, not just the dynamics around a non-stochastic steady state. Our paper belongs to a growing literature studying global dynamics in models with financial frictions (see He and Krishnamurthy (2012, 2013), Brunnermeier and Sannikov (2012), Adrian and Boyarchenko (2012), and Maggiori (2012)). Our contribution relative to these papers is quantitative: we show that our model (and by extension, this class of models) can successfully match key macroeconomic and asset pricing data. The literature thus far has explored modeling strategies that generate qualitative insights.

The second part of the paper confronts the model with data. The key feature of the model is a non-linearity. When constraints on the intermediary sector are binding or likely to bind in the near future, a negative shock triggers a substantial decline in intermediary equity, asset prices and investment. When constraints on the intermediary sector are slack and unlikely to bind in the near future, the same size negative shock triggers only a small decline in intermediary equity, asset prices and investment. In short, the model generates conditional amplification, where the state variable determining conditionality is the incidence of financial constraints in the intermediary sector. We establish that this non-linearity is present in the data. Based on U.S. data from 1975 to 2010, we compute covariances between growth in the equity capitalization of the financial intermediary sector, Sharpe ratios (i.e. economic risk premia), growth in aggregate investment, and growth in land prices, *conditional* on intermediary "distress" and "non-distress" (defined more precisely below). We choose parameters of our model based on *unconditional* moments of asset pricing and macroconomic data. We then simulate the model and compute the model counterpart of the data covariances, again conditioning on whether the intermediary sector is at or near distress or in a non-distress period. We show that the conditional covariances produced by the model match their data counterparts.

We should note that our model misses quantitatively on other dimensions. To keep the model tractable and analyze global dynamics, the model has only two state variables. One cost of this simplicity is that there is no labor margin in the model, and thus we are unable to address measures such as hours worked. We stress that the key feature of the model is a nonlinear relationship between financial variables and real investment, and that is the dimension on which our model is successful.

In our sample from U.S. data, the only significant financial crisis is the 2007-2009 crisis. We show that our model can replicate data patterns in this crisis. We choose a sequence of underlying shocks to match the evolution of intermediary equity from 2007 to 2009. Given this sequence, we then compute the equilibrium values of the Sharpe ratio, aggregate investment and land prices. The analysis shows that the model's equity capital constraint drives a quantitatively significant amplification mechanism. That is, the size of the asset price declines produced by the model are much larger than the size of the underlying shocks we consider. In addition, the analysis shows that focusing only on shocks to intermediary equity results in an equilibrium that matches the behavior of aggregate investment, the Sharpe ratio, and land prices. This analysis lends further weight to explanations of the 2007-2009 crisis that emphasize shocks to the financial intermediary sector.

We also study smaller financial crises including the 1988-1991 Savings and Loan Crisis, the 1998 Hedge Fund crisis and the 2002 corporate bond market crisis. The model captures some important patterns in these episodes, but on the whole is a poorer fit of these events compared to the 2007-2009 crisis. It is likely that there are other important factors at work in these episodes that do not operate through the financial sector ("non-financial real shocks") and that our model omits.

We then apply our model to assessing the likelihood of a systemic crisis. Our model allows us to compute counterfactuals. In early 2007, what is the likelihood of reaching a state where constraints on the financial intermediary sector bind over the next T years? What scenarios make this probability higher? We find that the odds of hitting the crisis states over the next 2 years, based on an initial condition chosen to match credit spreads in 2007Q2, is 3.57%. When we expand the horizon these probabilities rise to 17.3% for 5 years. While these numbers are small, it should be noted that most financial market indicators in early 2007, such as credit spreads or the VIX (volatility index), were low and did not anticipate the severity of the crisis that followed. That is, without the benefit of hindsight, in both the model and data the probability of the 2007-2009 crisis is low. A lesson from our analysis is that it is not possible to construct a model in which spreads are low ex-ante, as in the data, and yet the probability of a crisis is high.

The utility of our structural model is that we can compute these probabilities based on alternative scenarios, as under a stress test. That is, the model helps us to understand the type of information that agents did not know ex-ante but which was important in subsequently leading to a crisis. With the benefit of hindsight, it is now widely understood that the financial sector had embedded leverage through off-balance sheet activities, for example, which meant that true leverage was higher than the measured leverage based on balance sheets. In our baseline calibration, financial sector leverage is 3. We perform a computation that incorporates shadow banking (structured-investment vehicles and repo financing) onto bank balance sheets, and find that leverage may be as high as 3.45. We then conduct a stress test where we increase true leverage from 3 to 3.45, but assume that the agents in the economy think that leverage is 3. The latter informational assumption captures the notion that it is only with hindsight that the extent of leveraging of the financial system has become apparent (i.e., consistent with the evidence that credit spreads and VIX were low prior to the crisis). Thus, we suppose that agents' decisions rules, equilibrium prices and asset returns are all based on an aggregate intermediary leverage of 3, but that actually shocks impact intermediary balance sheets with a leverage that is 3.45. We then find that the probability of the crisis over the next 2 years rises from from 3.57% to 23.45%, and for 5 years it rises from 17.3% to 57.95%. This computation shows how much hidden leverage contributed to the crisis.

Similarly, the model allows us to ask how a stress scenario to capital, similar to the Federal Reserve's stress test, increases the probability of systemic risk. The endogenous feedback of the economy to the stress scenario is the key economics of our model that cannot be captured in a scenario-type analysis such as the Fed's stress tests. That is, conditional on a scenario triggering a significant reduction in the equity capital of financial firms, it is likely that the endogenous response of the economy will lead to a further loss on assets and further reduction in equity capital. Additionally, the model allows us to translate the stress test into a probability of systemic risk, which is something that the Fed's current methodology cannot do. We illustrate through an example how to compute the probability of systemic risk based on a hypothetical stress test.

The papers that are most similar to ours are Mendoza (2010) and Brunnermeier and Sannikov (2012). These papers develop stochastic and non-linear financial frictions models to study financial crises. Mendoza is interested in modeling and calibrating crises, or sudden stops, in emerging markets. From a technical standpoint, Mendoza relies on numerical techniques to solve his model, while we develop an analytically tractable model whose equilibrium behavior can be fully characterized by a system of ordinary differential equations. Our approach is thus complementary to his. Brunnermeier and Sannikov also take the differential equation approach of our paper. Their model illustrates the non-linearities in crises by showing that behavior deep in crises regions is substantially different than that in normal periods and underscores the importance of studying global dynamics and solving non-linear models. In particular, their model delivers a steady state distribution in which the economy can have high occupation time in systemic risk states. The principal difference relative to these paper is that we aim to quantitatively match the non-linearities in the data and use the model to quantify systemic risk. Finally, both Mendoza and Brunnermeier-Sannikov study models with an exogenous interest rate, while the interest rate is endogenous in our model.

The model we employ is closely related to our past work in He and Krishnamurthy (2012, 2013). He and Krishnamurthy (2012) develop a model integrating the intermediary sector into a general equilibrium asset pricing model. The intermediary sector is modeled based on a moral hazard problem, akin to Holmstrom and Tirole (1997), and optimal contracts between intermediaries and households are allowed.[1] We derive the equilibrium intermediation contracts and asset prices in closed form. He and Krishnamurthy (2013) assume the form of intermediation contracts derived in He and Krishnamurthy (2012), but enrich the model so that it can be realistically calibrated to match asset market phenomena during the mortgage market financial crisis of 2007 to 2009. In the present paper, we also assume the structure of intermediation in reduced form. The main innovation relative to our prior work is that the present model allows for a real investment margin with capital accumulation and lending, and includes a housing price channel whereby losses on housing investments affect intermediary balance sheets. Thus the current paper speaks to not only effects on asset prices but also real effects on economic activity.

The paper is also related to the literature on systemic risk measurement. The majority of this literature motivates and builds statistical measures of systemic risk extracted from asset market data. Papers include Hartmann, Straetmans and De Vries (2005), Huang, Zhou, and Zhu (2010), Acharya, Pedersen, Philippon, and Richardson (2010), Adrian and Brunnermeier (2010), Billio, Getmansky, Lo, and Pelizzon (2010), and Giglio, Kelly and Pruitt (2013). Our line of inquiry is dif-

[1]Our paper belongs to a larger literature, which has been growing given the recent crisis, on the macro effects of disruptions to financial intermediation. Papers most closely related to our work include Adrian and Shin (2010), Gertler and Kiyotaki (2010), Kiley and Sim (2011), Rampini and Viswanathan (2011), Bigio (2012), Adrian and Boyarchenko (2012), He and Kondor (2012), Maggiori (2012) and Dewachter and Wouters (2012).

ferent from this literature in that we build a macroeconomic model to understand how economic variables relate to systemic risk. Acharya, Pedersen, Philippon, and Richardson (2010) is closest to our paper in this regard, although the model used in that paper is a static model that is not suited to a quantification exercise. It is ultimately important that our model-based approach meets the data-oriented approaches.

The paper is laid out as follows. Section 2 describes the model. Section 3 goes through the steps of how we solve the model. Section 4 presents our choice of parameters for the calibration. Sections 5, 6, and 7 present the results from our model. Figures and an appendix with further details on the model solution are at the end of the paper.

2 Model

Time is continuous and indexed by t. The economy has two types of capital: productive capital K_t and housing capital H. We assume that housing is in fixed supply and normalize $H \equiv 1$. We denote by P_t the price of a unit of housing, and q_t the price of a unit of capital; both will be endogenously determined in equilibrium. The numeraire is the consumption good. There are three types of agents: equity households, debt households, and bankers.

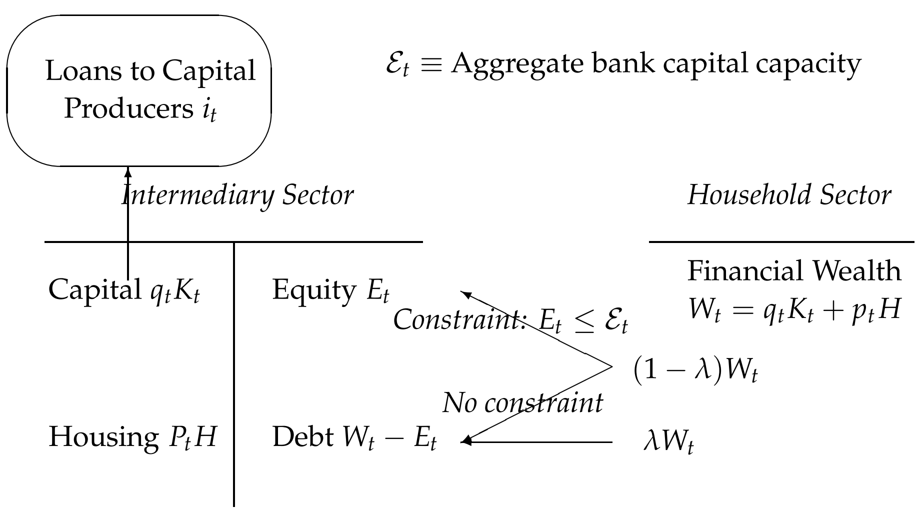


Figure 1: Model Schematic

We begin by describing the production technology and the household sector. These elements of the model are a slight variant on a standard stochastic growth model. We then describe bankers and intermediaries, which are the non-standard elements of the model. We assume that all of the housing and capital stock are owned by intermediaries that are run by bankers. Intermediaries also fund new investments. Households are assumed to not be able to directly own the housing and capital stock. Instead, the intermediaries raise equity and debt from households and use these funds to purchase housing and capital. The key assumption we make is that intermediaries face an equity capital constraint. Figure 1 presents the main pieces of the model, which we explain in detail over the next sections.

2.1 Production and Households

There is an "AK" production technology that generates per-period output Y_t:

$$Y_t = AK_t, \tag{1}$$

where A is a positive constant. The evolution of capital is given by:

$$\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t. \tag{2}$$

The term i_t is the amount of new capital installed at date t. Capital depreciates by δdt, where δ is constant. The last term σdZ_t is a capital quality shock, following Gertler and Kiyotaki (2010). For example, K_t can be thought of as the effective quality/efficiency of capital rather than the amount of capital outstanding. The capital quality shock is a simple device to introduce an exogenous source of variation in the value of capital. Note that the price of capital q_t and the price of housing P_t are endogenous. Thus, we will be interested in understanding how the exogenous capital quality shock translates into endogenous shocks to asset prices. Finally, the shock σdZ_t is the only source of uncertainty in the model ($\{Z_t\}$ is a standard Brownian motion, while σ is a positive constant).

Commonly, RBC models introduce shocks to the productivity parameter A rather than the quality shocks we have introduced. Introducing shocks to A will add another state variable and greatly complicate solutions to the model. We assume shocks directly in the evolution of the capital stock, K_t, because capital will be one of the state variables in the solution. But, note that a shock to A and the direct shock to $\frac{dK_t}{K_t}$ will work similarly. That is imagine a model with A shocks and consider a -10% drop in A. In this case Y_t falls by 10% and, for a fixed price/dividend ratio, the drop in the dividend on capital will lead to a -10% return to owners of capital. Now consider the shock we model as a direct -10% shock to $\frac{dK_t}{K_t}$. The shock also leads output to fall by 10%. Owners of capital "lose" 10% of their capital so that, for a fixed price/dividend ratio, they experience a -10% return to capital. These aspects thus appear similar across the two ways of modeling the shock. The main difference will be in the price of capital, q. With a shock to A, we would expect that q will fall through a direct effect of approximately 10% (ignoring the general equilibrium effects), while with the shock to $\frac{dK_t}{K_t}$, there is no direct effect on q (only general equilibrium effects cause q to fall).

We assume adjustment costs so that installing $i_t K_t$ new units of capital costs $\Phi(i_t, K_t)$ units of consumption goods where,

$$\Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2}(i_t - \delta)^2 K_t.$$

That is, the adjustment costs are assumed to be quadratic in net investment.

There is a unit measure of households. Define a consumption aggregate as in the Cobb-Douglas form,

$$C_t = \left(c_t^y\right)^{1-\phi}\left(c_t^h\right)^{\phi},$$

where c_t^y is consumption of the output good, c_t^h is consumption of housing services, and ϕ is the expenditure share on housing. The household maximizes utility,

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\frac{1}{1-\gamma}C_t^{1-\gamma}dt\right],$$

(i.e. CRRA utility function, with the log case when $\gamma = 1$), and the constant ρ is the discount rate.

Given the intratemporal preferences, the optimal consumption rule satisfies:

$$\frac{c_t^y}{c_t^h} = \frac{1-\phi}{\phi}D_t, \tag{3}$$

where D_t is the endogenous rental rate on housing to be determined in equilibrium. In equilibrium, the parameter ϕ affects the relative market value of the housing sector to the goods producing sector.

2.2 Bankers and Equity Capital Constraint

We assume that all productive capital and housing stock can only be held directly by "financial intermediaries." When we go to the data, we calibrate the intermediaries to include not only commercial banks, but also investment banks and hedge funds. There is a continuum of intermediaries. Each intermediary is run by a single banker who has the know-how to manage investments. That is, we assume that there is a separation between the ownership and control of an intermediary, and the banker make all investment decisions of the intermediary.

Consider a single intermediary run by a banker. This banker invests some of the households' wealth, W_t, in the capital and housing stock on behalf of the households. The banker raises funds from households in two forms, equity and debt. To draw an analogy, think of equity raised as the assets under management of a hedge fund and think of debt financing as money borrowed in the repo market. At time t, a given banker has a type of ϵ_t that measures an equity capital constraint. The banker can issue equity upto ϵ_t at zero issuance cost, but faces infinite marginal issuance cost in issuing equity above ϵ_t. Thus, faced with an ϵ_t-banker, households invest up to ϵ_t to own the equity of that intermediary. Any remaining funds raised by the intermediary are in the form of short-term (from t to $t + dt$) debt financing (see Figure 1).[2]

[2]Note that we place no restriction on the raising of debt financing by the intermediary. Debt is riskless and is always over-collateralized so that a debt constraint would not make sense in our setting. It is clear in practice that there are times in which debt or margin constraints are also quite important. Our model sheds light on the effects of limited equity capital (e.g., limited bank capital) and its effects on intermediation.

Denote the realized profit-rate on the intermediary's assets (i.e. holdings of capital and housing) from t to $t + dt$, net of any debt repayments, as $d\tilde{R}_t$. This is the return on the shareholder's equity of the intermediary. The profit is stochastic and depends on shocks during the interval $[t, t + dt]$.

Our key assumption is that the equity capital capacity of a given banker evolves based on the banker's returns:

$$\frac{d\epsilon_t}{\epsilon_t} = m d\tilde{R}_t, \tag{4}$$

where $m > 0$ is a constant. Poor investment returns reduce ϵ_t and thus reduce the maximum amount of equity a given intermediary can raise going forward.

There are different ways of interpreting equation (4), which we assume in reduced form. In macroeconomic models such as Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997), the "net worth" of productive agents corresponds to the inside equity capital of these agents and plays a key role in macroeconomic dynamics. In these papers, net worth fluctuates as a function of the past performance and profits of the productive agent, just as in (4). This interpretation may be natural in applying the model to the commercial banking sector where equity capital largely varies with earnings. In He and Krishnamurthy (2012) we consider a setting where bankers have preferences over consumption, and households write incentive contracts in order to solve a moral hazard problem with bankers who manage intermediaries. We derive an incentive contract between bankers and households and find that the banker's net worth plays a similar role as ϵ_t in our current setting. In particular, we find that bankers' equity capital constraint is similarly a function of their past performance.[3]

Equation (4) can also be interpreted in terms of behavior in the asset management industry. Equation (4) is a contemporaneous relationship between the flows into an intermediary and the performance of the intermediary. This sort of flow-performance relationship is a well documented empirical regularity among mutual funds (see Warther, 1995, or Chevalier and Ellison, 1997), for which there is substantial data on returns and equity inflows/outflows.[4] The flow-performance relationship has also been documented for hedge funds (Getmansky, 2012) and private equity funds (Kaplan and Schoar, 2005). The leading explanation for the flow-performance relationship is based on investors' learning the skill of the fund manager (Berk and Green, 2004). Although

[3]The modeling leads to two changes relative to He and Krishnamurthy (2012, 2013). First, we do not have to keep track of the bankers' consumption decisions which simplifies the model's analysis somewhat. More substantively, in our previous work we find that, in crisis states, the interest rate diverges to negative infinity. In the present modeling, the interest rate is determined purely by the household's Euler equation (since the bankers do not consume goods), which leads to a better behaved interest rate.

[4] Warther (1995) documents a positive contemporaneous correlation between aggregate monthly flows into stock funds and stock returns over a sample from 1984 to 1992. His baseline estimate is that a 5.7% stock return is associated with a 1% contemporaneous unexpected inflow into funds. He also shows that flows are AR(1) with parameter of 0.6, so that the cumulative effect on inflows due to a 1% increase in stock returns is $\frac{1}{5.7} \times \frac{1}{1-0.6} = 0.43\%$. In terms of (4), consider a 1% stock return, which increases assets in a fund by 1%, and further generates cumulative new inflows of 0.43%, so that total assets rise by 1.43%. This means that $m = 1.43$.

we do not model learning, this type of explanation is a motivation for equation (4). That is, one can give a rational underpinning for a loss of equity capital of an intermediary following bad past returns. The thorny issue for such an explanation is that indexing or benchmarking the returns of one manager to another manager, which is typically optimal in a learning setting, can substantially reduce aggregate effects. We note that this type of indexation issue arises in many macroeconomic models, including those of collateral constraints (see Krishnamurthy, 2003).[5]

We apply our model to the entire sophisticated financial sector. Importantly, the application is appropriate because equation (4), interpreted as either net worth or skill, captures the evolution of equity capital in both asset management and banking sectors.

We refer to ϵ_t as the bankers "reputation," and assume that a banker makes investment decisions to maximize his future reputation. Bankers do not consume goods (a feature which is convenient when clearing the goods market).[6] A given banker may die at any date at a constant Poisson intensity of $\eta > 0$. When the banker dies he consumes his reputation. Thus, a banker makes investment decisions to maximize,

$$\mathbb{E}\left[\int_0^\infty e^{-\eta t} \ln \epsilon_t dt\right].$$

Given the log form objective function and equation (4), it is easy to show that the time t decision of the banker is chosen to maximize,

$$\mathbb{E}_t[d\tilde{R}_t] - \frac{m}{2} Var_t[d\tilde{R}_t]. \tag{5}$$

The constant m thus parameterizes the "risk aversion" of the banker.

To summarize, a given intermediary can raise at most ϵ_t of equity capital. If the intermediary's investments perform poorly, then ϵ_t falls going forward, and the equity capital constraint tightens. The banker in charge of the intermediary chooses the intermediary's investments to maximize the mean excess return on equity of the intermediary minus a penalty for variance multiplied by the "risk aversion" m.

[5]The finance literature has explored the effects of the flow-performance relationship on asset prices in limits-to-arbitrage models. An equation like (4) underlies the influential analysis of Shleifer and Vishny (2004). More recently, Vayanos and Woolley (2012) have studied such a model to explain the momentum effect in stock returns. Dasgupta and Prat (2011), Dasgupta, Prat, and Verardo (2012), and Guerreri and Kondor (2012) present theoretical papers showing how career concerns of fund managers, or their desire to maintain reputations, affects asset market equilibrium. In this paper, we consider the macroeconomic implications of the flow-performance relationship.

[6]As bankers do not consume goods, we also need to discuss what happens to any profits made by bankers. We assume that a given intermediary-banker is part of larger intermediary-conglomerate (i.e., to draw an analogy, think of each intermediary as a mutual fund, and the conglomerate as a mutual fund family). In equilibrium, the intermediary-bankers make profits which then flow up to the conglomerate and are paid out as dividends to households, who are the ultimate owners of the conglomerates. It will be clearest to understand the model under the assumption that the households' ownership interest in these conglomerates is not tradable. That is, it is not a part of the household's investable wealth (which we denote as W_t). This assumption turns out not to make any difference.

2.3 Aggregate Intermediary Capital

Consider now the aggregate intermediary sector. We denote by $\mathcal{E}_t$ the maximum equity capital that can be raised by this sector, which is just the aggregate version of individual banker's capital constraint ϵ_t. The maximum equity capital $\mathcal{E}_t$ will be one of the state variables in our analysis, and its dynamics are given by,

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m d\tilde{R}_t - \eta dt + d\psi_t. \tag{6}$$

The first term here reflects that all intermediaries are identical, so that the aggregate stock of intermediary reputation/capital constraint evolves with the return on the intermediaries' equity.[7] The second-term, $-\eta dt$, captures exit of bankers who die at the rate η. Exit is important to include; otherwise, $d\mathcal{E}_t/\mathcal{E}_t$ will have a strictly positive drift in equilibrium, which makes the model non stationary. In other words, without exit, intermediary capital will grow and the capital constraint will not bind. The last term, $d\psi_t \geq 0$ reflects entry. We describe this term more fully below when describing the boundary conditions for the economy. In particular, we will assume that entry occurs when the aggregate intermediary sector has sufficiently low capital, because the incentives to enter are high in these states.

2.4 Capital Goods Producers

Capital goods producers, owned by households, undertake real investment. As with the capital stock and the housing stock, we assume that capital goods must be sold to the intermediary sector. Thus, q_t, based on the intermediary sector's valuation of capital also drives investment. Given q_t, i_t is chosen to solve,

$$\max_{i_t} \; q_t i_t K_t - \Phi(i_t, K_t) \;\Rightarrow\; i_t = \delta + \frac{q_t - 1}{\kappa}. \tag{7}$$

Recall that $\Phi(i_t, K_t)$ reflects a quadratic cost function on investment net of depreciation.

2.5 Household Members and Portfolio Choices

We make assumptions so that a minimum of λW_t of the household's wealth is invested in the debt of intermediaries. We may think of this as reflecting household demand for liquid transaction balances in banks, although we do not formally model a transaction demand. The exogenous constant λ is useful to calibrate the leverage of the intermediary sector, but is not crucial for the qualitative properties of the model.

The modeling is as follows. Each household is comprised of two members, an "equity household" and a "debt household." At the beginning of each period, the household consumes, and

[7] The model can accommodate heterogeneity in reputations, say ϵ_t^i where i indexes the intermediary. Because the optimal decision rules of a banker are linear in ϵ_t^i, we can aggregate across bankers and summarize the behavior of the aggregate intermediary sector with the average reputation, which is equivalent to $\mathcal{E}_t$.

then splits its W_t between the household members as $1-\lambda$ fraction to the equity household and λ fraction to the debt household. We assume that the debt household can only invest in intermediary debt paying the interest rate r_t, while the equity household can invest in either debt or equity. Thus households collectively invest in at least λW_t of intermediary debt. The household members individually make financial investment decisions. The investments pay off at period $t+dt$, at which point the members of the household pool their wealth again to give wealth of W_{t+dt}. The modeling device of using the representative family follows Lucas (1990).

Collectively, equity households invest their allocated wealth of $(1-\lambda)\,W_t$ into the intermediaries subject to the restriction that, given the stock of banker reputations, they do not purchase more than $\mathcal{E}_t$ of intermediary equity. When $\mathcal{E}_t > W_t(1-\lambda)$ so that the intermediaries reputation is sufficient to absorb the households' maximum equity investment, we say that the capital constraint is not binding. But when $\mathcal{E}_t < W_t(1-\lambda)$ so that the capital constraint is binding, the equity household restricts its equity investment and places any remaining wealth in bonds. In the case where the capital constraint does not bind, it turns out to be optimal – since equity offers a sufficiently high risk-adjusted return – for the equity households to purchase $(1-\lambda)W_t$ of equity in the intermediary sector. We verify this statement when solving the model. Let,

$$E_t \equiv \min\left(\mathcal{E}_t, W_t(1-\lambda)\right)$$

be the amount of equity capital raised by the intermediary sector. The households' portfolio share in intermediary equity, paying return $d\tilde{R}_t$, is thus, $\frac{E_t}{W_t}$.

The debt household simply invests its portion λW_t into the riskless bond. The household budget constraint implies that the amount of debt purchased by the combined household is equal to $W_t - E_t$.

2.6 Riskless Interest Rate

Denote the interest rate on the short-term bond as r_t. Given our Brownian setting with continuous sample paths, the short-term debt is riskless. Consider at the margin a household that cuts its consumption of the output good today (the envelope theorem allows us to evaluate all of the consumption reduction in terms of the output good), investing this in the riskless bond to finance more consumption tomorrow.[8] The marginal utility of consumption of the output good is $e^{-\rho t}(1-$

[8]There are some further assumptions underlying the derivation of the Euler equation. If the household reduces consumption today, a portion of the foregone consumption is invested in riskless bonds via the debt member of the household and a portion is invested in equity via the equity member of the household. We assume that equity households are matched with bankers to form an intermediary, and that bankers have a local monopoly with the equity households, such that the households receive their outside option, which is to invest in the riskless bond at rate r_t, on any marginal funds saved with intermediaries. Thus, the Euler equation holds for riskless bonds paying interest rate r_t, and equation (8) has the appealing property that it is a standard expression for determining interest rates. It is straightforward to derive an expression for interest rates under the alternative assumption that the equity household receives an excess return from his investment in the intermediary. In this case, households will have an extra incentive to delay consumption, and the equilibrium interest rate will be lower than that of (8).

$\phi)\left(c_t^y\right)^{(1-\phi)(1-\gamma)-1}\left(c_t^h\right)^{(\phi)(1-\gamma)}$, which, in equilibrium, equals $e^{-\rho t}(1-\phi)\left(c_t^y\right)^{(1-\phi)(1-\gamma)-1}$ as $c_t^h = H \equiv 1$ in equilibrium. Let, $\xi \equiv 1-(1-\phi)(1-\gamma)$. The equilibrium interest rate r_t satisfies:

$$r_t = \rho + \xi \mathbb{E}_t\left[\frac{dc_t^y}{c_t^y}\right] - \frac{\xi(\xi+1)}{2} Var_t\left[\frac{dc_t^y}{c_t^y}\right]. \tag{8}$$

Here, $1/\xi$ can be interpreted as the elasticity of intertemporal substitution (EIS).[9]

2.7 Intermediary Portfolio Choice

Each intermediary chooses how much debt and equity financing to raise from households, subject to the capital/reputation constraint, and then makes a portfolio choice decision to own housing and capital. The return on purchasing one unit of housing is,

$$dR_t^h = \frac{dP_t + D_t dt}{P_t}, \tag{9}$$

where P_t is the pricing of housing, and D_t is the equilibrium rental rate given in (3). Let us define the risk premium on housing as $\pi_t^h \equiv \mathbb{E}_t[dR_t^h]/dt - r_t$. That is, by definition the risk premium is the expected return on housing in excess of the riskless rate. Then,

$$dR_t^h = (\pi_t^h + r_t)dt + \sigma_t^h dZ_t.$$

Here, the volatility of investment in housing is σ_t^h, and from (9), σ_t^h is equal to the volatility of dP_t/P_t.

For capital, if the intermediary buys one unit of capital at price q_t, the capital is worth q_{t+dt} next period and pays a dividend equal to Adt. However, the capital depreciates at the rate δ and is subject to the capital quality shocks σdZ_t. Thus, the return on capital investment, accounting for the Ito quadratic variation term, is as follows:

$$dR_t^k = \frac{dq_t + Adt}{q_t} - \delta dt + \sigma dZ_t + \left[\frac{dq_t}{q_t}, \sigma dZ_t\right]. \tag{10}$$

We can also define the risk premium and risk on capital investment suitably so that,

$$dR_t^k = (\pi_t^k + r_t)dt + \sigma_t^k dZ_t.$$

We use the following notation in describing an intermediary's portfolio choice problem. Define α_t^k (α_t^h) as the ratio of an intermediary's investment in capital (housing) to the equity raised by an intermediary. Here, our convention is that when the sum of αs exceed one, the intermediary is shorting the bond (i.e., raising debt) from households. For example, if $\alpha_t^k = \alpha_t^h = 1$, then

[9] Note that with two goods, the intratemporal elasticity of substitution between the goods enters the household's Euler equation. Piazzesi, Schneider and Tuzel (2007) clarify how risk over the composition of consumption in a two-goods setting with housing and a non-durable consumption good enters into the Euler equation.

an intermediary that has one dollar of equity capital will be borrowing one dollar of debt (i.e. $1-\alpha_t^k-\alpha_t^h=-1$) to invest one dollar each in housing and capital. The intermediary's return on equity is,

$$d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h + (1-\alpha_t^k-\alpha_t^h) r_t dt. \tag{11}$$

From the assumed objective in (6), a banker solves,

$$\max_{\alpha_t^k,\alpha_t^h} \mathbb{E}_t[d\tilde{R}_t] - \frac{m}{2} Var_t[d\tilde{R}_t]. \tag{12}$$

The optimality conditions are,

$$\frac{\pi_t^k}{\sigma_t^k} = \frac{\pi_t^h}{\sigma_t^h} = m\left(\alpha_t^k\sigma_t^k + \alpha_t^h\sigma_t^h\right). \tag{13}$$

The Sharpe ratio is defined to be the risk premium on an investment divided by its risk (π/σ). Optimality requires that the intermediary choose portfolio shares so that the Sharpe ratio on each asset is equalized. Additionally, the Sharpe ratio is equal to the riskiness of the intermediary portfolio, $\alpha_t^k\sigma_t^k + \alpha_t^h\sigma_t^h$, times the "risk aversion" of m. This latter relation is analogous to the CAPM. If the intermediary sector bears more risk in its portfolio, and/or has a higher m, the equilibrium Sharpe ratio will rise.

2.8 Market Clearing and Equilibrium

1. In the goods market, the total output must go towards consumption and real investment (where we use capital C to indicate aggregate consumption)

$$Y_t = C_t^y + \Phi(i_t, K_t). \tag{14}$$

Note again that bankers do not consume and hence do not enter this market clearing condition. Households receive all of the returns from investment.

2. The housing rental market clears so that

$$C_t^h = H \equiv 1. \tag{15}$$

3. The intermediary sector holds the entire capital and housing stock. The intermediary sector raises total equity financing of $E_t = \min\left(\mathcal{E}_t, W_t(1-\lambda)\right)$. Its portfolio share into capital and housing are α_t^k and α_t^h.[10] The total value of capital in the economy is q_tK_t, while the total value of housing is P_t. Thus, market clearing for housing and capital are:

$$\alpha_t^k E_t = K_t q_t \text{ and } \alpha_t^h E_t = P_t. \tag{16}$$

These expressions pin down the equilibrium values of the portfolio shares, α_t^k and α_t^h.

[10] Keep in mind that while we use the language "portfolio share" as is common in the portfolio choice literature, the shares are typically larger than one because in equilibrium the intermediaries borrow from households.

4. The total financial wealth of the household sector is equal to the value of the capital and housing stock:

$$W_t = K_t q_t + P_t.$$

An equilibrium of this economy consists of prices, (P_t, q_t, D_t, r_t), and decisions, $(c_t^y, c_t^h, i_t, \alpha_t^k, \alpha_t^h)$. Given prices, the decisions are optimally chosen, as described by (3), (7), (8) and (12). Given the decisions, the markets clear at these prices.

3 Model Solution

We derive a Markov equilibrium where the state variables are K_t and $\mathcal{E}_t$. That is, we look for an equilibrium where all the price and decision variables can be written as functions of these two state variables. We can simplify this further and look for price functions of the form $P_t = p(e_t)K_t$ and $q_t = q(e_t)$ where e_t is the aggregate reputation/capital-capacity of the intermediary sector scaled by the outstanding physical capital stock:

$$e_t \equiv \frac{\mathcal{E}_t}{K_t}.$$

That P_t is linear in K_t is an important property of our model and greatly simplifies the analysis (effectively the analysis reduces to one with a single state variable). To see what assumptions lead to this structure, consider the following. In equilibrium, aggregate consumption of the non-housing good is,

$$C_t^y = Y_t - \Phi(i_t, K_t) = K_t \left[A - i_t + \frac{\kappa}{2} (i_t - \delta)^2 \right],$$

given the adjustment cost specification. From the Cobb-Douglas household preferences, we have derived in equation (3) that $\frac{C_t^y}{C_t^h} = \frac{1-\phi}{\phi} D_t$. Since $C_t^h = H = 1$, the rental rate D_t can be expressed as

$$D_t = \frac{\phi}{1-\phi} \left[A - i_t + \frac{\kappa}{2} (i_t - \delta)^2 \right] K_t.$$

As the price of housing is the discounted present value of the rental rate D_t, and this rental rate is linear in K_t, it follows that P_t is also linear in K_t.

In summary, K_t scales the economy while e_t describes the equity capital constraint of the intermediary sector. The equity capital constraint, e_t, evolves stochastically. The appendix goes through the algebra detailing the solution. We show how to go from the intermediary optimality conditions, (13), to a system of ODEs for $p(e)$ and $q(e)$.

3.1 Capital Constraint, Amplification, and Anticipation Effects

The solution of the model revolves around equation (13) which is the optimality condition for an intermediary. The equation states that the required Sharpe ratio demanded by an intermediary to own housing and capital is linear in the total risk borne by that intermediary, $m\left(\alpha_t^k\sigma_t^k + \alpha_t^h\sigma_t^h\right)$. If intermediaries hold more risky portfolios, which can happen if α_t^k and α_t^h are high, and/or if σ_t^h and σ_t^k are high, they will require a higher Sharpe ratio to fund a marginal investment. Equilibrium conditions pin down the αs (portfolio shares) and the σs (volatilities). Consider the αs as they are the more important factor. The variable α_t^k is the ratio of the intermediary's investment in capital to the amount of equity it raises. Market clearing dictates that the numerator of this ratio must be equal to q_tK_t across the entire intermediary sector, while the denominator is the equity capital raised by the intermediary sector, E_t (see (16)).

Let us first consider the economy without a reputation/equity constraint. Then, the household sector would invest $(1-\lambda)W_t$ in equity and λW_t in debt. That is, from the standpoint of households and given the desire for some debt investment on the part of households, the optimal equity/debt mix that households would choose is $(1-\lambda)W_t$ of equity and λW_t of debt. In this case, α_t^k is equal to $\frac{q_tK_t}{(1-\lambda)W_t}$. Moreover, because $W_t = K_t(q_t + p_t)$, i.e., the aggregate wealth is approximately proportional to the value of the capital stock, this ratio is near constant. A negative shock that reduces K_t also reduces W_t proportionately with no effects on α_t^k. A similar logic applies to α_t^h. This suggests that the equilibrium Sharpe ratio would be nearly constant if there was no equity capital constraint. While we have not considered the σs in this argument (they are endogenous objects that depend on the equilibrium price functions), they turn out to be near constant as well without a capital constraint. Thus, without the capital constraint, shocks to K_t just scale the entire economy up or down, with investment, consumption, and asset prices moving in proportion to the capital shock.

Now consider the effect of the capital constraint. If $\mathcal{E}_t < W_t(1-\lambda)$, then the intermediary sector only raises $E_t = \mathcal{E}_t$ of equity. In this case, α_t^k and α_t^h must be higher than without capital constraint. In turn, the equilibrium Sharpe ratios demanded by the intermediary sector must rise relative to the case without capital constraint because the amount of risk borne in equilibrium by intermediaries, $m\left(\alpha_t^k\sigma_t^k + \alpha_t^h\sigma_t^h\right)$, rises. In this state, consider the effect of negative shock. Such a shock reduces W_t, but reduces $E_t = \mathcal{E}_t$ more through two channels. First, since the intermediary sector is levered (i.e. in equilibrium the sum of αs are larger than one simply because some households only purchase debt which is supplied by the intermediary sector), the return on equity is a multiple of the underlying return on the intermediary sector's assets. Second, we parameterize the model so that the speed in the flow-performance relationship, m, is larger than one, which implies that $\mathcal{E}_t$ moves more than one-for-one with the return on equity (see (4)). Thus negative shocks are amplified and cause the equilibrium αs to rise when the capital constraint binds. The higher αs imply a higher Sharpe ratio on capital and housing investment, which in turn implies that the price of capital and housing must be lower in order to deliver the higher expected returns implied by the higher Sharpe ratios. This means in turn that the capital constraint is tighter, further reducing equity capital. This effect also amplifies negative shocks. There is a further amplification mechanism: since the price of housing and capital are more sensitive to aggregate equity capital when such capital is low, the equilibrium volatility (i.e, σs) of housing and capital are higher, further increasing Sharpe ratios and feeding through to asset prices and the equity capital constraint. All of these effects reduce investment, because investment depends on q_t which is lower in the presence of the equity capital constraint.

Next consider how the economy can transit from a state where the equity capital constraint does not bind to one where the constraint binds. Even when the constraint is not active, returns realized by the intermediaries affect the capital capacity $\mathcal{E}_t$, as in equation (4). If there is a series of negative shocks causing low returns, $\mathcal{E}_t$ falls, and as described above, the fall is larger than the fall in W_t. Thus, a series of negative shocks can cause $\mathcal{E}_t$ to fall below $W_t(1-\lambda)$, leading to a binding capital constraint.

Last consider how the effect of an anticipated constraint may affect equilibrium in states where the constraint is not binding. Asset prices are the discounted presented value of future dividends. As the economy moves closer to the constraint binding, the discount rates (i.e. required expected returns) rise, causing asset prices to fall. That is asset prices fall to anticipate the possibility that the constraint may bind in the future. Through this channel, the equilibrium is affected by $\mathcal{E}_t$ even in cases where it is larger than $W_t(1-\lambda)$. This is an anticipation effect that emerges from solving for the global dynamics of the model.

The anticipation effect is important in empirically verifying the model. It is likely that widespread financial constraints in the intermediary sector were only present during the 2007-2009 crisis. Our analysis shows that even when such constraints are not binding, if agents anticipate that they are likely to bind in the near future (what we label below as "distress"), then financial friction effects will be present.

3.2 Boundary Conditions

The equilibrium prices $p(e_t)$ and $q(e_t)$ satisfy a system of ODEs based on (13), which are solved numerically subject to two boundary conditions. First, the upper boundary is characterized by the economy with $e \to \infty$ so that the capital constraint never binds. We derive exact pricing expressions for the economy with no capital constraint and impose these as the upper boundary. The Appendix provides details.

The lower boundary condition is as follows. We assume that new bankers enter the market when the Sharpe ratio reaches B, which is an exogenous parameter in the model. This captures the idea that the value of entry is high when the Sharpe ratio of the economy is high. We can also think of entry as reflecting government intervention in the financial sector in a sufficiently adverse state.

Entry alters the evolution of the state variables e and K. In particular, the entry point $\underline{e}$ is endogenous and is a reflecting barrier. We assume that entry increases the aggregate intermediary reputation (and therefore the aggregate intermediary equity capital), but is costly. In order to increase $\mathcal{E}_t$ by one unit, the economy must destroy $\beta > 0$ units of physical capital. Thus, we adjust the capital evolution equation (2) at the entry boundary.

Since the entry point is a reflecting barrier it must be that the price of a unit of capital, $q(e)$, and the price of a unit of housing, $p(e_t)K_t$, have zero derivative with respect to e at the barrier (if not, an investor can make unbounded profits by betting on an almost sure increase/decrease in the asset price). This immediately implies $q'(\underline{e}) = 0$. For the housing price, imposing that $p(e_t) K_t$ has zero derivative implies the lower boundary condition $p'(\underline{e}) = \frac{p(\underline{e})\beta}{1+\underline{e}\beta} > 0$. The derivative is positive, as entry uses up capital, K_t falls at the entry boundary, and hence p must rise in order to keep pK constant. In economics terms, the positive derivative $p'(\underline{e}) > 0$ implies that at the entry point $\underline{e}$ a negative shock lowers the land price. Intuitively, a falling K_t reduces the aggregate housing rental income which is proportional to the aggregate consumption, leading to a lower land price. See the appendix for the exact argument and derivation.

4 Calibration

The parameters, ρ (household time preference), δ (depreciation), and κ (adjustment cost) are relatively standard. We use conventional values for these parameters (see Table 1). Note that since our model is set in continuous time, the values in Table 1 correspond to annual values rather than the typical quarterly values one sees in discrete time DSGE parameterizations.

The most important parameter in the model is σ which governs the exogenous uncertainty in this model. Increasing σ increases the volatility of all quantities and prices in the model. We choose $\sigma = 3\%$ as our baseline. The baseline generates a volatility of investment growth in the model of 4.48% and a volatility of consumption growth of 2.31%. In the data, the volatility of investment growth from 1973 to 2010 is 7.78% while the volatility of consumption growth is 2.17%. We will also present results for a variation with higher σ.

The main intermediation parameters are m and λ. The parameter m governs the "risk aversion" of the banker. As we vary m, the Sharpe ratio in the model changes proportionately (see (13)). The choice of $m = 2$ gives an average Sharpe ratio from the model of 38%, which is in the range of typical asset pricing calibrations. If we look to the flow-performance relationship for mutual funds as a guide, the results of Warther (1995) imply a value of $m = 1.43$ (see footnote 4). The parameter λ is equal to the financial intermediary sector's debt/assets ratio when the capital constraint does not bind. The main challenge in choosing λ is that it represents the leverage across the entire and heterogenous sophisticated intermediary sector, encompassing commercial banks,

Table 1: Parameters and Unconditional Moments

	Parameter	Choice	Target
Panel A: Intermediation Parameters			
m	Performance sensitivity	2	Average Sharpe ratio
λ	Debt ratio	0.67	Average intermediary leverage
η	Banker exit rate	13%	Probability of crisis
B	Entry barrier	6.5	Highest Sharpe ratio
β	Entry cost	2.43	Land price volatility
Panel B: Technology Parameters			
σ	Capital quality shock	3%	Consumption volatility
δ	Depreciation rate	10%	Literature
κ	Adjustment cost	3	Literature
A	Productivity	0.133	Investment-to-capital ratio
Panel C: Other Parameters			
ρ	Time discount rate	2%	Literature
ξ	1/EIS	0.15	Interest rate volatility
ϕ	Housing share	0.5	Housing-to-wealth ratio
Panel D: Unconditional Moments from Simulation			
Probability of Crisis			3%
Mean Sharpe Ratio			38%
Mean Interest Rate			2%
Mean Intermediary Leverage			3.07
Mean $\left(\frac{\text{Land Value}}{\text{Total Wealth}}\right)$			34%
Volatility(Cons. Growth)			2.3%
Volatility(Inv. Growth)			4.6%
Volatility(Output Growth)			3.05%
Volatility(Land Price Growth)			14.3%
Volatility(Equity Growth)			20.3%
Volatility(Interest Rate)			0.92%

investment banks, hedge funds, and venture capital/private equity funds. We base our calibration on the following numbers. The Federal Reserve's Flow of Funds (December 8, 2011 release) for calendar year 2007 reports commercial bank (L.110) assets of \$8.84tn and debt of \$7.33tn. For the broker/dealer sector, assets are \$3.09tn and debt is \$2.46tn. The total assets under management of the hedge fund sector is \$1.98tn as of December 2007 (Barclay Hedge, Hedge Fund Flow Reports 2009). Ang, Gorovyy and van Inwegen (2011) report average hedge fund leverage of 2.1. Summing across these three sectors, total assets is \$15.7tn and total debt is \$10.55tn, for a leverage ratio $\left(\frac{\text{assets}}{\text{assets - debt}}\right)$ of 3.04. Our choice of $\lambda = 0.67$ produces an average leverage ratio in the simulation of 3.07. This calibration is a rough effort to represent leverage across a heterogeneous intermediation sector.

We set η (the bankers' death rate, or equivalently the depreciation in banker reputation stock; see equation (6)) equal to 13% based on considerations of the historical incidence of financial crises.

The choice of η affects the mean of the steady state distribution of e. A high value of η implies that banker reputation falls faster and thus increases the probability mass for low values of e in the steady state distribution. We choose η so that the probability of the capital constraint binding in the steady state is 3%, which is chosen as a target based on observing three major financial crises in the US over the last 100 years. In later comparative static analyses when we vary parameters we also vary η so as to keep the crisis probability at 3%.

We set $\phi = 0.5$. The parameter ϕ governs the dividend on housing which in turns drives the total value of the housing assets relative to wealth. From Flow of Funds (December 8, 2011) release, Table B100, the total net worth of the household sector in 2007 is 65.1tn. Of this wealth, real estate accounts for 23.2tn, or 35.6%. In our simulation, the choice of $\phi = 0.5$ yields that the mean ratio $\frac{p}{p+q}$ is 34.2%.

We set $\xi = 0.15$. This choice implies an EIS of 6.66 which is unusually high. Our parameterization is based on attempting to match the empirical volatility of real interest rates (1%). More conventional values of the EIS produces an overly volatile interest rate.

The entry boundary condition (i.e. lower boundary) is determined by B and β. We set $B = 6.5$, so that new entry occurs when the Sharpe ratio is 650%. Based on movements in credit spreads, as measured by Gilchrist and Zakrajsek (2010)'s excess bond premium (see the data description in Section 6.1), we compute that Sharpe ratio of corporate bonds during the 2008 crisis was roughly 17 times the average. Since in our simulation the average Sharpe ratio is around 38%, we set the highest Sharpe ratio to be 650%. Although a high entry threshold is crucial for our model, the exact choice of B is less important because the probability of reaching the entry boundary is almost zero. Our choice is principally motivated by setting B sufficiently high that it does not affect the model's dynamics in the main part of the distribution. The value of β is far more important because it determines the slope of the housing price function at the entry boundary, and therefore the slope all through the capital constrained region. The volatility of land prices is closely related to the slope of the price function (see equation (18)). In the data, the empirical volatility of land price growth from 1975 to 2009 is 14.47%. The choice of $\beta = 2.43$ produces unconditional land price volatility of 14.3%.[11]

We set $A = 0.133$ to target the average investment to capital ratio in the data. From 1973 to 2010, this average is 11% in the data and 10% in the simulation.

Finally, as should be clear from the preceding discussion, our calibration is based on matching *unconditional* targets only. In the results of the next section, we evaluate how this calibration performs in matching conditional moments.

[11]This choice of β leads to a slope of $p'(\underline{e}) = 0.419$ at the endogenous entry point $\underline{e}$. Also note that it is tautological within our model that at the entry barrier the household sector is willing to pay exactly βK units of capital to boost wealth (i.e. P and q) by increasing e. That is, the value of β cannot be independently pinned down from this sort of computation.

5 Results

5.1 Price and Policy Functions: "Anticipation" Effects

Figures 4 and 5 plot the price and policy functions for the baseline parameterization and a variation with a higher σ. Consider the baseline in Figure 4 first. The X-axis in all of the graphs is the scaled intermediary capital capacity/reputation $e = \mathcal{E}/K$. The equity capital constraint binds for points to the left of 0.435. The lower-right panel plots the steady-state distribution of the state variable e. Most of the weight is on the part of the state space where the capital constraint does not bind. That is, a systemic crisis, defined as periods where the capital constraints bind, is rare in the model (probability 3%).

The top row, third panel is the Sharpe ratio. The Sharpe ratio is about 32% in the unconstrained region and rises rapidly upon entering the constrained region. The interest rate (second row, left panel) also falls sharply when the economy enters the constrained region. Both effects reflect the endogenous increase in "risk aversion," operating through the binding constraint rather than preferences, of the intermediary sector during a systemic crisis.

The first two panels on the top row are $p(e)$ (housing price divided by capital stock) and $q(e)$. Both price functions are increasing in equity capital as one would expect. It is worth noting that going from right-to-left, prices fall before entering the capital constrained region. This occurs through *anticipation effects*. As the economy moves closer to the constraint, the likelihood of falling into the constrained region rises and this affects asset prices immediately. Moreover, note that if the model had no capital constraint, these price functions $p(e)$ and $q(e)$ would be flat lines. The crisis-states, even though unlikely, affect equilibrium across the entire state space. This result is similar to results from the rare-disasters literature (Rietz, 1988; Barro, 2006).

Comparing the first two panels for $p(e)$ and $q(e)$, the main difference is that the range of variation for q is considerably smaller than that for p. This is because housing is in fixed supply while physical capital is subject to adjustment costs. With the $\kappa = 3$ parameterization, the adjustment costs are sufficiently small that capital prices do not vary much. It may be possible to arrive at higher volatility in q if we consider higher adjustment costs or flow adjustment costs as in Gertler and Kiyotaki (2010). As noted earlier, q will also vary more if we allowed for shocks to A instead of directly shocking K_t.[12] The graph illustrates that the aggregate asset price volatility in the economy is substantially driven by housing volatility. The middle and right panel of the second row are for return volatility of q and p. Housing volatility is much higher than q volatility. Note also

[12] In investigating the model, we have also found that increasing the intertemporal elasticity of substitution (IES) for the household increases the range of variation of q. This appears to be through an effect on the interest rate. In the current calibration, interest rates fall dramatically the constrained region, which through a discount rate effect supports the value of q. Dampening this effect by increasing the IES increase the range of variation of q. This observation also suggests to us that introducing nominal frictions that bound the interest rate from falling below zero will increase the range of variation of q.

that the actual price of housing is equal to p times K, and since K is also volatile, housing prices are more volatile than just p.

The first panel in the bottom row graphs the investment policy function. Since investment is driven by $q(e)$, investment also falls before the intermediary sector is constrained. The second panel in the bottom row graphs the consumption policy function. Investment-to-capital falls as q falls. The resource constraint implies that $C/K + \Phi(I,K)/K = A$. Thus, consumption-to-capital rises as the constraint becomes tighter. Note that aggregate consumption depends on this policy function and the dynamics of capital. In the constrained region, capital falls so that while C/K rises, K falls, and the net effect on aggregate consumption depends on parameters. For our baseline parameters, consumption growth in the non-distress region averages 0.7% while it is -1.6% in the distress region.

Figure 5 plots the baseline plus a variation with higher sigma ($\sigma = 4\%$). The results are intuitive. With higher exogenous volatility, Sharpe ratios, return volatility and risk premia are higher (the Sharpe ratio rises in the unconstrained region from 32% to 48%, but given the range of variation in the Sharpe ratio, it hard to make this out in the figure). Thus asset prices and investment are lower.

5.2 Model Nonlinearity and Impulse Response Function

An important feature of the model, apparent in the figures, is its nonlinearity. A reduction to intermediary equity, conditional on a low current value of intermediary equity, has a larger effect on the economy than the same size shock, conditional on a high value of intermediary equity. Figure 6 illustrates this feature. We study the effect of -1% shock in σdZ_t, so that the fundamental shock leads capital to fall exogenously by 1%. We consider the effect of this shock in a "crisis" state ($e = 0.435$, which is the boundary of the constrained region) and a "normal" state ($e = 15.44$). We trace out the effect on investment (first panel), the Sharpe ratio (second panel), and the price of land (third panel). Because the impact of a shock depends on future shocks in a nonlinear model, and our stochastic economy is always subject to shocks, we adopt the following procedure to calculate impulse response functions. First, we compute the benchmark path of these variables without any shocks, but still subject to the endogenous drift of the state variable in our model. In other words, we calculate the benchmark path for the realizations of $dZ_{t+s} = 0$ for $s \geq 0$. Second, we compute the "shocked" path of these variables given this initial shock $\sigma dZ_t = -1\%$, but setting future realizations of shocks to be zero, i.e. $dZ_{t+s} = 0$ for $s > 0$. We then calculate and plot the (log) difference between the path with the shock and the benchmark path without any shocks. This computation is meant to mimic a deviation-from-steady-state computation that is typically plotted in impulse response functions in linear-non-stochastic models. Therefore, the effect illustrated in Figure 6 should be thought of as the marginal effect of the shock on the *mean* *path* for the variables plotted.[13]

The dashed line in the first panel indicates that a shock in a normal state causes investment to fall by a little over 1% (relative to the mean path of investment) on incidence of the shock, and this effect continues out to 8 quarters. The effect does not revert back to zero because K_t is permanently lower by about 1% and our economy scales with K_t. The solid line is the effect of the same shock in the crisis state. Now investment falls by 1.9%, with the effect dying out over three quarters. The second panel shows that the Sharpe ratio is completely unaffected by the shock in the normal region, while it rises by 40% in the crisis region. The effect also dies out after three quarters. The last panel plots the price of land. Land prices fall by a little over 1.5% in the normal region indicating some amplification even when the constraint is not active, while it falls by 8% in the crisis region indicating significant amplification in the crisis region.

6 Matching Nonlinearity in Data

Guided by the nonlinearity present in the model, we first ask if such nonlinearity is present in historical data, and second, we ask how well our model can quantitatively match the nonlinearity in the data.

6.1 Data

We compute covariances in growth rates of intermediary equity, investment, consumption, the price of land, as well as the level of a credit risk spread, using quarterly data from 1975Q1 to 2010Q4. We sample the data quarterly but compute annual log changes in the series. We focus on annual growth rates because there are slow adjustment mechanisms in practice (e.g., flow adjustment costs to investment) that our model abstracts from. We thus sample at a frequency where these adjustment mechanisms play out fully. The intermediary equity measure is the sum across all financial firms (banks, broker-dealers, insurance and real estate) of their stock price times the number of shares from the CRSP database.[14,15] The consumption and investment data are from NIPA. Consumption is non-housing services and nondurable goods. Investment is business investment in software, equipment, structures, and residential investment. Land price

[13]Another reasonable way to calculate impulse response functions in our stochastic nonlinear models is to calculate the expected impact of the initial shock $\sigma dZ_t = -1\%$ on the $t+s$ variable by integrating over all possible future paths. Here, we only focus on the mean path by shutting future shocks to zero. Note that traditional linear models are free from this issue, as the impulse response functions in linear models are independent of future shocks. For more on the difference between impulse responses in linear models with a non-stochastic steady state and those non-linear models with a stochastic steady state, see Koop, Pesaran, and Potter (1996) and a recent contribution by Borovička, Hansen, Hendricks, and Scheinkman (2011).

[14]Muir (2011) shows that this measure is useful for predicting aggregate stock returns as well as economic activity. Moreover, intermediary equity is a priced factor in the cross-section of stock returns.

[15]We have also considered an alternative equity measure based only on banks and broker-dealers and the results are quite similar to the ones we report.

data is from the Lincoln Institute (http://www.lincolninst.edu/subcenters/land-values/price-and-quantity.asp), where we use *LAND_PI* series based on Case-Shiller-Weiss. These measures are expressed in per-capita terms and adjusted for inflation using the GDP deflator. The credit risk spread is drawn from Gilchrist and Zakrajsek (2010). There is a large literature showing that credit spreads (e.g., the commercial paper to Treasury bill spread) are a leading indicator for economic activity (see Philippon (2010) for a recent contribution). Credit spreads have two components: expected default and an economic risk premium that lenders charge for bearing default risk. In an important recent paper, Gilchrist and Zakrajsek (2010) show that the spread's forecasting power stems primarily from variation in the risk premium component (the "excess bond premium"). The authors also show that the risk premium is closely related to measures of financial intermediary health. Our model has predictions for the link between intermediary equity and the risk premium demanded by intermediaries, while being silent on default.[16] We convert the Gilchrist and Zakrajsek's risk premium into a Sharpe ratio by scaling by the risk of bond returns, as the Sharpe ratio is the natural measure of risk-bearing capacity in our model.[17] The Sharpe ratio is labeled EB in the table.

6.2 Conditional Moments

Table 2 presents covariances depending on whether or not the intermediary sector is in a "distress" period. (Annual growth rates are centered around the quarter classified as distress). Table 3 lists the distress classification. Ideally, we would like to split the data based on observations of e_t, which measures the equity capacity of the intermediary sector. However, e_t is not directly observable in data. Instead, the model suggests that there is a one-to-one link between the Sharpe ratio and e_t. Thus, we consider as distress periods the highest one-third of realizations of the EB Sharpe ratio, but requiring that the distress or non-distress periods span at least two contiguous quarters. In choosing the distress/non-distress classification, we face the tradeoff that if we raise cutoff to define distress (say, worst 10% of observations as opposed to 33%), then the data is more reflective of the crisis effects suggested by the model but we have too little data on which to compute meaningful statistics. After experimenting with the data, we have settled on the one-third/two-thirds

[16] There is no default in the equilibrium of the model. Of course, one can easily price a defaultable corporate bond given the intermediary pricing kernel, where default is chosen to match observables such as the correlation with output. We do not view having default in the equilibrium of the model as a drawback of our approach.

[17] Suppose that the yield on a corporate bond is y^c, the yield on the riskless bond is y^r and the default rate on the bond is $E[d]$. The expected return on the bond is $y^c - y^r - E[d]$, which is the counterpart to the excess bond premium of Gilchrist and Zakrajsek (2010). To compute the Sharpe ratio on this investment, we need to divide by the riskiness of the corporate bond investment. Plausibly, the risk is proportional to $E[d]$ (for example, if default is modeled as the realization of Poisson process, this approximation is exact). Thus the ratio $\frac{y^c - y^r - E[d]}{E[d]}$ is proportional to the Sharpe ratio on the investment, and this is how we construct the Sharpe ratio.

split.[18]

The distress periods roughly correspond to NBER recession dates, with one exception. We classify distress periods in 1985Q4-1987Q3, 1988Q4-1990Q1, and again 1992Q3-1993Q2. The NBER recession over this period is in 1990 to 1991. The S&L crisis and falling real estate prices in the late 80s put pressure on banks which appear to result in a high EB and hence leads us to classify these other periods as distress.

Table 2: Covariances in Data

The table presents standard deviations and covariances for intermediary equity growth (Eq), investment growth (I), consumption growth (C), land price growth (PL), and Sharpe ratio (EB). Suppose quarter t is classified as a distress quarter. We compute growth rates as annual changes in log value from $t-2$ to $t+2$. The Sharpe ratio is the value at t. The first column is using the distress classification of Table 1. The second uses NBER recession dates, from Table 1. The third uses these recession dates, plus two adjoining quarters at the start and end of the recession. The last is based on the distress dates from Table 1 but drops the last period (the recent crisis).

	EB	NBER Recession	NBER+/-2Qs	EB, Drop Crisis
Panel A: Distress Periods				
vol(Eq)	31.48	32.40	31.78	22.19
vol(I)	8.05	8.79	7.44	4.56
vol(C)	1.71	1.54	1.59	0.95
vol(PL)	21.24	23.34	21.07	7.91
vol(EB)	60.14	93.59	74.57	28.69
cov(Eq, I)	1.31	1.08	0.84	0.37
cov(Eq, C)	0.25	0.16	0.13	0.04
cov(Eq, PL)	4.06	5.61	4.39	-0.63
cov(Eq, EB)	-6.81	-10.89	-7.57	-2.12
Panel B: Non-distress Periods				
vol(Eq)	17.54	19.42	17.11	17.26
vol(I)	6.61	5.97	4.91	6.60
vol(C)	1.28	0.98	0.91	1.28
vol(PL)	9.79	10.00	8.46	9.34
vol(EB)	12.72	30.93	30.42	12.78
cov(Eq, I)	0.07	0.09	-0.06	0.03
cov(Eq, C)	0.03	0.01	0.01	0.03
cov(Eq, PL)	0.12	0.07	-0.31	-0.01
cov(Eq, EB)	-0.14	-0.81	-0.78	-0.19

Table 2 shows that there is an asymmetry in the covariances across the distress and non-distress periods, qualitatively consistent with the model. There is almost no relation between equity and

[18]We present a table in the appendix with results from the model and data, using a distress classification of 10% of observations (16 distress data points) and 20% of observations (29 distress data points). Both data and model display asymmetry across distress and non-distress periods, consistent with the results we present in the text. If we focus on the covariance between equity and investment, the data show an increasing covariance in the distress region as we focus on successively worse classifications of the distress episode. The model is able to match this increasing covariance. There is one oddity of the data that is likely due to the small samples in these splits. The volatility of equity is roughly constant across the distress episodes, as opposed to rising with severity of the distress episode.

Table 3: Distress Classification

Distress Periods	NBER Recessions
1974Q3 - 1975Q4	11/73 - 3/75
1982Q3 - 1982Q4	7/81 - 11/82
1985Q4 - 1987Q3	
1988Q4 - 1990Q1	
	7/90 - 3/91
1992Q4 - 1993Q2	
2001Q2 - 2003Q1	3/01 - 11/01
2007Q3 - 2009Q3	12/07 - 6/09

the other variables in the non-distress periods, while the variables are closely related in the distress periods. Volatilities are much higher in the distress periods than the non-distress periods. The table also presents results for alternative classifications of the distress periods. All of the classifications display the pattern of asymmetry so that our results are not driven by an arbitrary classification of distress. The only column that looks different is the last one where we drop the recent crisis. For this case, most of the covariances in the distress period drop in half, as one would expect. In addition, the land price volatility drops substantially while the covariance goes to zero. This is because it is only the recent crisis which involve losses on real estate investments and financial intermediaries.

6.3 Simulated Conditional Moments

We compare the results from simulating the model to quarterly data from 1975 to 2010, as presented in Table 2. When simulating the model we follow the one-third/two-thirds procedure as when computing moments in historical data and label distress as the worst one-third of the sample realizations. Importantly therefore our definitions are consistent and comparable across both model and data. From Figure 4, points on the x-axis where $e < e_{distress} = 1.27$ are classified as distressed.

We simulate the model, quarterly, for 2000 years. To minimize the impact of the initial condition, we first simulate the economy for 2000 years, and then record data from the economy for the next 2000 years. We then compute sample moments and the probability of distress region accordingly. We run the simulation 5000 times and report the sample average.

Table 4 provides numbers from the data and the simulation. When reading these numbers it is important to keep in mind that our calibration targets are neutral and we have not explicitly targeted the asymmetry across distress and non-distress periods. Thus one criterion for the success of our work is whether the non-linearity imposed by the theoretical structural of the model can match the asymmetry in the data.

Table 4: Model Simulation and Data

The table presents standard deviations and covariances for intermediary equity growth (Eq), investment growth (I), consumption growth (C), land price growth (PL), and Sharpe ratio (EB). Growth rates are computed as annual changes in log value from t to $t+1$. The Sharpe ratio is the value at $t+1$. The column labeled data are the statistics for the period 1975 to 2010. The Sharpe ratio is constructed from the excess bond premium, and other variables are standard and defined in the text. The next four columns are from the model, reflecting different parameter choices. Numbers are presented conditional on being in the distress period or non-distress period. For the data, the classification of the periods follows Table 1. For the model simulation, the distress period is defined as the 33% worst realizations of the Sharpe ratio.

	Data	Baseline	$\sigma = 4\%$	$\phi = 0$	$m = 1.8$	$\lambda = 0.6$
Panel A: Distress Periods						
vol(Eq)	31.48%	34.45	48.65	11.03	23.78	28.27
vol(I)	8.05%	5.30	10.11	3.35	4.32	4.62
vol(C)	1.71%	3.54	5.89	2.47	2.78	3.28
vol(PL)	21.24%	21.04	38.15		9.85	16.90
vol(EB)	60.14%	74.20	121.61	14.32	42.36	55.83
cov(Eq, I)	1.31%	1.05	3.33	0.22	0.57	0.74
cov(Eq, C)	0.25%	-0.96	-2.10	-0.10	-0.56	-0.77
cov(Eq, PL)	4.06%	5.87	16.19		1.68	3.88
cov(Eq, EB)	-6.81%	-14.95	-38.62	-0.50	-5.00	-8.48
Panel B: Non-distress Periods						
vol(Eq)	17.54%	5.40	7.85	2.99	4.23	4.92
vol(I)	6.61%	4.19	7.78	3.00	3.44	3.69
vol(C)	1.28%	1.19	3.52	2.93	1.14	0.84
vol(PL)	9.79%	9.24	14.66		5.56	7.58
vol(EB)	12.72%	7.97	17.22	0.04	3.97	5.40
cov(Eq, I)	0.07%	0.23	0.58	0.09	0.15	0.18
cov(Eq, C)	0.03%	-0.05	-0.19	0.09	0.03	-0.01
cov(Eq, PL)	0.12%	0.50	1.10		0.23	0.37
cov(Eq, EB)	-0.14%	-0.13	-0.67	0.00	-0.03	-0.07

In the data, the covariance between equity and investment is 1.31% in distress and 0.07% in non-distress. In the simulation, these numbers are 1.05% and 0.23%. The model also comes close to matching the asymmetry in land price volatility and covariance with land prices and equity. In the data, the volatility numbers are 21.24% and 9.79%; while the corresponding land price volatilities from the model are 21% and 9.24%. Recall that our parameters (particularly β) are chosen to match the unconditional volatility of 14.47% in the data. Therefore, matching the asymmetry across distress and non-distress periods should be considered as a success of the model. The land-equity covariances in the data are 4.06% and 0.12%; while in the model, they are 5.87% and 0.5%. The model is also quite close in matching the asymmetry patterns in the Sharpe ratio, although asymmetry in the covariance with intermediary equity is too high in the model.

The model misses substantially in a few dimensions. Most importantly, while not immediately apparent from the table, the volatility of output (consumption plus investment) is constant at roughly 3%. Over a short period of time, output is equal to AK and thus the volatility of output is driven by the exogenous volatility in K, which is 3%. This creates the following problem: negative shocks reduce investment through the financial intermediation channel, but for given output, consumption has to rise. We can see this effect when comparing the covariance of equity and consumption to the covariance of equity and investment. The investment covariance is positive, while the consumption covariance is negative. The second problem created is that investment volatility is uniformly too low while consumption volatility is nearer the data. It seems clear that more work needs to be done in order to better match both investment and consumption dynamics. For example, introducing endogenous labor supply can lead the endogenous output volatility to differ significantly from σ.

The last four columns in the table consider variations where in different ways we change the volatility of the economy. In each of these variations, we vary η to ensure that the probability of being constrained remains at 3%.

The variation with σ raised to 4% from 3% increases the volatility of most variables considerably. The increase is larger in the distress period than the non-distress period which should be expected given the non-linearity in the model. An interesting point from this case is that the volatility of investment rises more than the volatility of consumption. This comparison makes clear that the main effect of the constraint we have introduced is on investment. Increasing σ raises the effects of the non-linear constraint and particularly affects investment.

The variation with $\phi = 0$ is interesting in that it reveals the workings of the model. When $\phi = 0$, land drops out of the model. From Figure 6 note that land price volatility rises in the constrained region while the volatility of q remains roughly constant. Thus, when land is removed from the economy, the volatility of intermediary equity in the distress region falls from 34.45% to 11.03%. The intermediary pricing kernel is far less volatile which in turn greatly reduces the non-linearity in the model. Recall in the baseline model with land, reduced demand for assets in the constrained region causes land prices to fall sharply as land is in fixed supply, while physical capital is subject to adjustment costs so that reduced demand both reduces quantity and price. This distinction is what drives the high volatility of land relative to physical capital in our baseline. Eliminating land thereby reduces the non-linear effects produced by the model.

The variation where we reduce m to 1.8 effectively reduces volatility in most parameters. Here again we see a different effect on investment and consumption, as consumption volatility rises while investment volatility falls. Reducing m reduces the strength of the intermediation friction which explains these effects.

The last column in the table considers a variation with a lower λ. Reducing the leverage of intermediaries in the unconstrained region reduces the amount of risk borne by intermediary equity and thus reduces risk premia and the intermediation effects of our model. That is, this variation is qualitatively similar to the effect of reducing m.

7 Systemic Risk

We now turn our attention to quantifying systemic risk, which we define as the probability that the economy can transit into a state where the capital constraint binds. Figure 7 plots the stock market value of intermediary equity and the EB-Sharpe ratio from 2006 to 2010. The financial crisis is evident as the spike in EB and fall in equity, reaching a climax in the fall of 2008. The equity and EB variables show some sign of stress beginning in late 2006/early 2007, but the movements are small in this period compared to the crisis. That is, the financial crisis is a non-linear phenomenon, which is exactly what our model attempts to capture.

We are interested in using our model to say something about the likelihood of the impending crisis in mid-2007, at a date before the financial crisis. Figure 7 hints at the challenges presented by this exercise. Both the EB and the equity variables in early 2007 show little sign of the crisis that followed (the VIX, which is not plotted, follows a similar pattern as the EB). A purely statistical exercise which uses this data to forecast the crisis will have little chance of predicting the crisis. The first part of the section formalizes this observation within our model, showing that even with our non-linear model, the likelihood of the crisis given 2007 initial conditions is low. This is a negative result, but should not be surprising: the initial conditions are chosen to be consistent with the early 2007 EB measure, and given that this measure shows little stress in the data, our rational expectations model produces a low likelihood of the crisis. A lesson from our analysis is that it is not possible to construct a model in which spreads are low ex-ante, as in the data, and yet the probability of a crisis is high.

The second part of the section shows how our model can be used to understand systemic risk. The utility of our structural model is that we can compute these probabilities based on alternative scenarios, as under a stress test. That is, the model helps us to understand the type of information that agents did not know ex-ante but which was important in subsequently leading to a crisis. The model is also useful is identifying the types of stress scenarios that most significantly threaten financial stability.

7.1 Simulation of 2007-2009 Crisis

We first use our model to attempt to replicate the crisis of 2007-2009, as reflected in Figure 2. To do so, we need to pick an initial condition in terms of e and a sequence of shocks that can reflect events in 2007-2009.

The choice of initial condition is important for the exercise that follows. The red dashed line in Figure 7 indicates the cutoff we have used in classifying distress and non-distress periods (i.e. 33% of the distribution of the EB over the sample lies above the cutoff). The economy transits into the distress region at some point between 2007Q2 and 2007Q3. The 33% threshold in our model simulation is a value of $e_{distress} = 1.27$. Thus, we assume that the economy in 2007Q2 is at $e = 1.27$. Note we are using information from the EB to pin down the initial condition. In principle more data (VIX, equity, etc.) can also be used to choose the initial condition. Since all of these variables suggest little stress in early 2007, our initial condition is unlikely to change based on these additional datapoints.

Starting from the $e = 1.27$ state, we impose a sequence of exogenous quarterly shocks, $\sigma(Z_{t+0.25} - Z_t)$ to the capital dynamics equation (2). The shocks are chosen so that the model implied intermediary equity matches the data counterpart during 2007Q3 to 2009Q4 in Figure 2. These shocks are in units of percentage change in capital. From 2007Q3 to 2009Q4, the shocks are $(-2.5\%, -4.2\%, -1.1\%, -1.1\%, -0.7\%, -1.6\%, -1.8\%, -1.8\%, -0.9\%, -0.9\%)$ which totals about -15.5% (geometric sum). We compute the values of all endogenous variables, intermediary equity, land prices, investment, and the Sharpe ratio, after each shock.

By matching the intermediary equity data, our model focuses on shocks in the world that most directly affect the intermediary sector. Note also that a given shock, say the -2.5% first shock does not only reflect losses by banks for 2007Q3, but also reflects losses anticipated by investors over the future, which is then impounded in the current market value of equity. That is, as the world is evolving over 2007Q3 to 2009Q4, investors receive information that cause them to anticipate losses to the intermediary sector which then immediately reduces the market value of the intermediary sector. We pick the shock in a given quarter to match the reduction in the market value of the intermediary equity over that quarter.

Figure 3 plots the values of the endogenous variables from the model simulation at each quarter (all variables are normalized to one in 2007Q2). The exogenous shocks total 15.5% while intermediary equity and land prices fall by about 70% in the trough of the model. Thus, there is clearly an amplification of shocks. The equity capital constraint comes to bind after the first three shocks, totaling -7.6%, and corresponding to 2008Q1. The Sharpe ratio rises dramatically after 2008Q1. Also, note that from that point on, the shocks are smaller but the responses of endogenous variables are larger, reflecting the non-linearity of the model.

Figure 3 should be compared to Figure 2. It is apparent that the model can replicate important features of the crisis with a sequence of shocks that plausibly reflects current and anticipated losses on bank mortgage investments. In addition, the analysis shows that focusing only on shocks to intermediary equity results in an equilibrium that matches the behaviors of aggregate investment, the Sharpe ratio, and land prices. This result suggests that an intermediary-capital-based mechanism, as in our model, can be a successful explanation for the macroeconomic patterns from 2007 to 2009.

7.2 Probability of Systemic Crisis and A Leverage Counterfactual

We use our model to compute the probability of falling into a systemic crisis. Consider the sequence of shocks as in Figure 3 that leads the capital constraint to bind in 2008Q1. We ask, what is the probability of the capital constraint binding any time over the next T years, given the initial condition of being on the distress boundary ($e_{distress} = 1.27$). These probabilities are 0.32% for 1 year, 3.57% for 2 years, and 17.30% for 5 years. This result confirms the intuition offered earlier. Since our initial condition is based on financial market measures that showed little sign of stress prior to the summer of 2007, our model offers little advance warning of the crisis that followed. That is, without the benefit of hindsight, in both the model and data the probability of the 2007-2009 crisis is low.

As many observers have pointed out, it is clear with the benefit of hindsight that there was a great deal of leverage "hidden" in the system. For example, many were unaware of the size of the structured investment vehicles (SIVs) that commercial banks had sponsored and the extent to which these assets were a call on bank's liquidity and capital. As Acharya, Schnabl and Suarez (2013) have documented, much of the assets in SIVs came back onto bank balance sheets causing their leverage to rise. Likewise, hedge funds and broker/dealers were carrying high leverage in the repo market, but this was not apparent to observers given the opacity of the repo market. As Gorton and Metrick (2011) have argued, this high leverage was a significant factor in the crisis. However, in early 2007, this high leverage was hidden in financial markets, and is perhaps one reason why financial market indicators did not signal a crisis.

We consider a counterfactual to see how accounting for the hidden leverage in the system may change the probability of the crisis. The experiment we lay out should be thought of as stress test that asks how much higher the crisis probability would be if a regulator had known that the leverage was higher than widely understood. In our baseline calibration, financial sector leverage is 3. Recall that the return on equity produced by an intermediary is,

$$d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h + (1 - \alpha_t^k - \alpha_t^h) r_t dt.$$

where,

$$\alpha_t^k = \frac{1}{1-\lambda}\frac{q_t K_t}{W_t} \quad \text{and,} \quad \alpha_t^k = \frac{1}{1-\lambda}\frac{P_t}{W_t},$$

when the capital constraint does not bind. The leverage parameter, λ, enters by affecting the αs and thus the exposure of intermediary equity to returns on housing and capital.

We recompute financial sector leverage in the data in 2007 accounting for two other types of leverage. We assume that the financial sector carries an additional \$1.2 trillion of assets with zero capital. This is based on the amount of SIVs pre-crisis and the results from Acharya, Schnabl and Suarez (2013) that these structures succeeded in evading all capital requirements. We also assume that the financial sector carries \$1 trillion of repo assets at a 2% haircut (capital requirement). These numbers are based on data on the repo market from Krishnamurthy, Nagel and Orlov (2013). These computations result in an increase in financial sector leverage from 3 to 3.45.[19] Translating this into our calibration, we replace $\lambda = 0.67$ with $\widehat{\lambda} = 0.71$ in the expressions above. This increases the leverage of the intermediaries, causing the αs to rise.

We assume, however, that this increase in leverage is "hidden," in the sense that agents continue to make decisions as if $\lambda = 0.67$ so that the equilibrium decisions rules, prices, and returns correspond to the baseline calibration. But when returns are realized, the hidden leverage leads to a larger-than-expected effect (i.e. $\widehat{\lambda} = 0.71$) on the return to intermediary equity. Thus our experiment is trying to hold fixed agents decisions rules and equilibrium prices and returns, and only allowing these returns to have a levered effect on the dynamics of intermediary equity. With the higher leverage, one can expect that shocks will be amplified and thus the crisis state will be more likely. We compute exactly how much more likely by simulating the model. The appendix describes the simulation procedure in detail.

The probability of the crisis over the next year rises from 0.32% to 6.73%, while for 2 years it rises from 3.57% to 23.45%, and for 5 years it rises from 17.3% to 57.95%. This computation quantifies how important a factor hidden leverage was in contributing the crisis. The exercise also shows how stressing the financial system, because of the non-linearity in the model, can have a large impact on crisis probabilities.

7.3 Stress Tests

The hidden leverage exercise is an example of a stress test. The fact that financial market indicators offered a poor signal of the crisis has led regulators to emphasize stress testing as a tool to uncover vulnerabilities in the financial system. Current regulatory stress tests are exercises which measure the impact of a stress event on the balance sheet of a bank. The typical stress test maps a scenario into a loss to equity holders. For example, a stress-test may assess how much equity capital a given bank will lose in the event that loss rates on mortgage loans double. Our model offers two ways in which to improve the stress test methodology. First, we have learned from the 2007/08 crisis that banks with shrinking capital base will be more reluctant to extend new housing related loans, which may further exacerbate the losses on existing mortgage assets. This further hurts bank equity capital, and so on (this point was identified by Brunnermeier, Gorton and Krishnamurthy (2011)). That is these exercises miss the general equilibrium feedback effect of the stress on aggregate bank balance sheets to the real sector and back to bank balance sheets. Our general

[19]Here are the details. In our baseline, we choose $\lambda = 0.67$ based on total intermediary assets of \$15.7tn and debt of \$10.5tn. In the variation which consolidates SIVs and repo, assets rise to \$17.9tn and debt rises to \$12.7tn.

equilibrium model allows us to compute the fixed point of this feedback mechanism Second, our model allows us to translate the result of the stress test into likely macroeconomic outcomes and particularly the probability of a crisis. This is likely a more useful metric for evaluating financial stability than a loss to bank capital.

The following thought experiment illustrates how the partial equilibrium approach ignoring the feedback amplification effect could give misleading answers. Suppose that we are at 2007Q2 with $e = e_{distress} = 1.27$, and the hypothetical stress scenario is a -30% loss on the bank equity. What is the underlying fundamental shock? Since the leverage in our model is three, the answer from a partial equilibrium perspective is a shock of $\sigma dZ_t = -30\%/3 = -10\%$. However, at $e = 1.27$, once feeding this -10% shock into our model over a quarter, we find that the full feedback effect in general equilibrium leads the return on equity to be -115%, which is far greater than the initial assumed shock of -30%.

Our model is useful to help determine the size of the dZ_t shock that generates a given equity loss. In practice, the stress test scenarios considered by the Federal Reserve were over six quarters rather than over one quarter. Hence, starting at $e = 1.27$, we consider feeding in negative shocks equally over six quarters, so that the resulting 6-quarter return on equity matches a particular stress test scenarios, as shown in the left-hand-side column of Table 6. We report the geometric sum of six-quarter fundamental dZ_t shocks in the middle column. The right-hand-side column in Table 6 gives the probability of a crisis within the next 2 years after experiencing those six-quarter losses. We find that if the bank equity suffers a loss of -2%, the probability of a crisis within the next 2 years rises modestly to 5.25%. But a -30% loss on bank equity pushes the economy into the crisis state, and this is why with probability 100% there will be a crisis in the next 2 years. From Table 6 we can also see the non-linearity in amplification. When the targeted equity return is small (-2%), the required fundamental shock is similar (-1.16%); but when the targeted equity return is large (-30%), we need a relatively smaller fundamental shock of -8.72%.

Table 6: Probability of Crisis and Return on Equity

The table gives the probability of a crisis within the next 2 years for a number of different six-quarter scenarios. The scenarios are chosen based on feeding in shocks over six quarters to match a given return on equity (left hand column).

Return on Equity	6 QTR Shocks	Prob(Crisis within 2 years)
-2%	-1.16%	5.25 %
-5	-2.53	8.90
-10	-4.69	22.88
-15	-6.71	48.90
-30	-8.72	100.00

Our analysis in this section should be viewed as illustrative. We can consider other shocks, scenarios, and initial conditions. The model can be used by mapping these scenarios into the dynamics of the state variable e_t which is the key to understanding crisis probabilities.

8 Simulating Smaller Crises

We repeat the crisis-matching exercise with other crisis episodes: the 1988-1991 Savings and Loan crisis, the 1998 Hedge Fund crisis and the 2002 corporate bond market meltdown. None of these episodes approach the severity of the 2007-2009 financial crisis. The exercise is nevertheless interesting as it highlights some of the challenges for our model.

8.1 Savings and Loan Crisis

Caprio and Klingbiel (2003) date the Savings and Loan crisis as beginning in 1988 and ending in 1991. They report that during this episode more than 1,400 Savings and Loan institutions and 1,300 banks failed, and the cost to taxpayers of resolving the failed institutions totaled $180 billion (3% of GDP). We begin our analysis of this episode in 1988Q3 and trace it through until 1990Q1. The initial date corresponds to the transition into the distress regime, following our classification.

Figure 8, Panel A plots three data series: intermediary equity (blue dash), aggregate investment (red dash), EB (green dash). We first note that the movements in the spread and equity series are much smaller than we have seen in the 2007-2009 episode. There is a small increase in the spread and fall in equity in 1988Q4. The equity series reverses and rises by 1989, while the spread falls by 1989. Aggregate investment is relatively stable through the entire episode.

We next repeat our model matching exercise. We draw shocks to match the intermediary equity data series within our model, assuming an initial condition of $e = 1.27$ in 1988Q3. We plot three series from the model: investment (red solid), Sharpe ratio (green solid), and probability of crisis in the next 2 years (orange solid). The model interprets the data to reflect a low likelihood of a financial crisis. The crisis probability rises as high as 12% in late 1988, coincident with a reduction in equity and rise in spreads. Otherwise, the crisis probability remains low. In short, the model interprets the 1988-1991 Savings and Loan crisis as an episode that is at most a mild financial crisis. Additionally, the match between investment in the model and that of the data is weak.

What does our model miss about this episode? The Savings and Loans episode, to the extent that it involves financial frictions effects, is a crisis that plays out very slowly. As Kane (1989) has noted, the problems of the Savings and Loans were known to many observers as far back as 1982. Regulators chose not to close down insolvent institutions (regulatory forbearance), and thus the period of 1988-1991 corresponds only to the period when the regulators finally decide to take action. Because the crisis is slow-moving, equity prices, which react only to unexpected news, do not signal a crisis. This is why there is no sudden increase in crisis probabilities over the 1988-1991 period. Second, to the extent that there are real or financial effects of the crisis, they likely play out over a longer period and are mixed with other macroeconomic factors. We have followed Caprio and Klingbiel and started our analysis in 1988, but one could as well begin in 1982. In either case, since our model is designed to primarily represent comovement conditional on a crisis and the crisis effects in this episode are considerably diluted by other macroeconomic forces that our model omits, it is unsurprising that our model offers a poor description of the data over this episode.

8.2 1998 Hedge Fund Crisis

The 1998 crisis is typically viewed as beginning with the Russian default in August 1998 and culminating in the near failure and government bailout of Long-term Capital Management (Scholes, 2000). The crisis is noteworthy for the comovement in asset prices across previously uncorrelated assets, such as U.S mortgage-backed securities and emerging market bonds. Many researchers tie this comovement to stress in the hedge fund sector, which caused hedge funds to liquidate their investments in mortgage-backed securities and emerging market bonds (e.g., Kyle and Xiong, 2000). This episode is thus an intermediary crisis that fits our model.

We begin our analysis of the hedge fund episode in 1998Q2. The EB indicator crosses the distress threshold between 1998Q2 and Q3.[20] Figure 8, Panel B plots intermediary equity (blue dash), aggregate investment (red dash), and the EB (green dash). The spread rises sharply from 1998Q2 to 1998Q3 while intermediary equity falls. That is the asset market measures indicate a financial crisis (again smaller than the 2007-2009 episode). There is little action however in aggregate investment. The crisis also resolves quickly: by 1999Q1 the spread is back to a normal level and equity has rebounded.

The model interprets this event as a financial crisis. Consider the orange line in the graph which is the 2-year crisis probability. This probability jumps to nearly 90% in 1998Q3 before falling back to 12.5% by 1999Q1. In the model, investment falls and rises in line with crisis probability. Investment in the model responds to the crisis while in the data investment appears unaffected by the crisis.

The model in this episode is informative by providing a counterfactual. We could have had a significant financial and real crisis, but a favorable set of shocks (e.g., good news, government intervention) helped avoid the crisis. Our model misses in one important dimension: it predicts that real quantities should have been contemporaneously affected by the crisis, while in practice

[20]We have not classified this episode as one of our distress periods because it only lasts one quarter. We classify distress periods as ones which last at least two quarters.

they were not. There is another version of the slow adjustment of the real sector at work here. The crisis occurs and dissipates so quickly that quantities do not have a chance to react. It is possible that including mechanisms that slow down the adjustment of real quantities, such as flow adjustment costs of investment (e.g., Gertler and Kiyotaki, 2010) can improve the fit of the model.

8.3 2002 Corporate Bond Market Crisis

The bankruptcies of Enron in late 2001 and WorldCom in the summer of 2002 resulted in large losses to holders of corporate bonds. Corporate bond spreads rose sharply in the summer of 2002 before returning to more normal levels by the summer of 2003. Berndt, Douglas, Duffie, Ferguson and Schranz (2005) study credit default swap rates and estimate that the risk premium demanded by investors to bear default risk more than doubled over that period (see Figure 2 of their paper). They suggest that these risk premium movements are consistent with the large losses and limited intermediation capital of corporate bond investors (e.g., banks, insurance companies, asset managers, etc), in line with our model.

We examine this episode through the lens of our model. According to our classification scheme, the economy crosses the distress threshold between 2001Q1 and 2001Q2. Note that this is one year before the turmoil in corporate bonds in the summer of 2002. There is an NBER recession that begins in March 2001, and our classification scheme places the economy on the distress boundary roughly at the start of this recession. It is also worth noting that the corporate bond market turmoil is two years after the bursting of the technology bubble, which many argue to have caused the 2001 recession.

Figure 8, Panel C plots intermediary equity (blue dash), aggregate investment (red dash), and the EB (green dash). There is little movement in spreads or equity over the 2001 recession. Investment falls, consistent with a recession. The financial turmoil is visible in the summer of 2002 when the EB rises and intermediary equity falls by nearly 20%. There is little movement in investment during this period. The EB returns to normal by 2003Q1, which is the end of the distress period according to our classification. The equity series does not agree with this classification, as equity remains low even into 2003Q1.

The model correctly identifies a financial turmoil episode, but gives it too much importance. The 2-year crisis probability rises to nearly 90% in the summer of 2002 and remains there to the end of the episode. Since the equity series remains low into 2003Q1 and our shocks are chosen to match equity, our model interprets this episode as one of continuing turmoil and the crisis probability remains high. This is likely incorrect. The corporate bond spread falls back to normal by 2003Q1, and since this turmoil originated in the corporate bond market, it is likely that the spread is a more informative indicator of financial stress than changes in equity. Moreover, the Dow Jones

Industrial average falls by about 25% in the summer of 2002, suggesting that shocks other than the corporate bond market turmoil were at work in determining asset prices. Our model omits these other non-financial shocks and thus over-interprets the reduction in intermediary equity prices.

8.4 Lessons

Here is the main lesson from our study of other crises. To better fit the data, it is important to introduce other sources of macroeconomic disturbances that do not operate through the financial sector ("non-financial real shocks"). Our model is designed to represent comovement between asset prices and real quantities during financial crisis episodes. The model has a harder time matching smaller crises because there are other factors at play during these smaller episodes that our model omits. In the context of our calibration where we match conditional moments, this observation suggests that by interpreting all of the movements in quantities through the lens of a financial frictions model, we have assigned too much weight to financial factors.[21]

A secondary lesson from this exercise is that more work is needed on the asset price signals of financial turmoil. Particularly in the corporate bond episode, the equity and spread series offer conflicting signals regarding the end of the turmoil. There has been a growing empirical literature on systemic risk measurement. This literature has considered a variety of systemic risk indicators, although because the literature is still in its infancy, there is no single agreed upon metric for financial turmoil. As this empirical literature advances, our modeling approach and calibration can be refined to better accord with findings from this work.

9 Conclusion

We presented a fully stochastic model of a systemic crisis in which the main friction is an equity capital constraint on the intermediary sector. We first showed that the model offers a good quantitative representation of the U.S. economy. In particular, the model is able to replicate behavior in non-distress periods, distress periods, and extreme systemic crisis, quantitatively matching the nonlinearities that distinguish patterns across these states. We then used the model to evaluate and quantify systemic risk, defined as the probability of reaching a state where capital constraints bind across the financial sector. We showed how the model can be used to evaluate the macroeconomic impact of a stress scenario on the systemic risk probability.

[21]If other shocks are orthogonal to the financial shocks (which is unlikely to be true), then since we are matching covariances, our calibration will not be biased.

References

Acharya, V., L. Pedersen, T. Philippon, and M. Richardson, 2010, "Measuring Systemic Risk," NYU Working Paper

Acharya, V., P. Schnabl, and G. Suarez, 2013, "Securitization without Risk Transfer," *Journal of Financial Economics*, 107(3), pp. 515-536.

Adrian, Tobias and Boyarchenko, Nina, 2012, "Intermediary Leverage Cycles and Financial Stability," FRB of New York Staff Report No. 56.

Adrian, T. and H. Shin (2010), Liquidity and leverage. *Journal of Financial Intermediation*, 19, p.418-437.

Adrian, Tobias and Markus Brunnermeier, 2008, "CoVaR," working paper, Federal Reserve Bank of New York.

Berk, J. and R. Green, 2004, "Mutual Fund Flows and Performance in Rational Markets", *Journal of Political Economy* 112, 1269–1295.

Bernanke, Ben, 2009, Financial Reform to Address Systemic Risk, Speech at the Council on Foreign Relations.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, John Taylor and Michael Woodford editors.

Berndt, Antje, Rohan Douglas, Darrell Duffie, Mark Ferguson, and David Schranz. 2005. "Measuring Default Risk Premia from Default Swap Rates and EDFs." Bank for International Settlements Working Paper 173.

Barro, Robert, 2006, "Rare disasters and asset markets in the twentieth century," *The Quarterly Journal of Economics*, 121(3), pp. 823-866.

Bigio, Saki, Financial Risk Capacity, working paper, NYU.

Billio, M., M. Getmansky, A. Lo, and L. Pelizzon, 2010, "Measuring Systemic Risk in the Finance and Insurance Sectors," MIT Working Paper.

Borovička, J., L. P. Hansen, M. Hendricks, and J. Scheinkman, 2011, "Risk-price Dynamics," Journal of Financial Econometrics 9 (1): 3-65

Brunnermeier, Markus and Yuliy Sannikov, 2013, "A Macroeconomic Model with a Financial Sector," *American Economic Review*.

Brunnermeier, Markus, Arvind Krishnamurthy, and Gary Gorton, 2011, "Risk Topography," in Acemoglu, D., and Woodford, M. (Eds.), *NBER Macro Annual 2011*.

Caprio, Gerard and Daniela Klingebiel (2003), "Episodes of systematic and borderline financial crises," World Bank.

Chevalier, J., and G. Ellison, 1997, "Risk Taking by Mutual Funds as a Response to Incentives," *Journal of Political Economy* 105, 1167–1200.

Christiano, Larry, Martin Eichenbaum, and Sergio Rebelo, 2010, "When is the Government Spending Multiplier Large?", *Journal of Political Economy*.

Dasgupta, Amil and Prat, Andrea and Verardo, Michela, 2011,"The price impact of institutional herding," *Review of Financial Studies*, 24 (3). pp. 892-925.

Dewachter, Hans and Raf Wouters, 2012, "Endogenous risk in a DSGE model with capital-constrained Financial intermediariees," NBB working paper.

Gertler, Mark, and Kiyotaki, Nobuhiro, 2010, "Financial Intermediation and Credit Policy in Business Cycle Analysis." In Friedman, B., and Woodford, M. (Eds.), *Handbook of Monetary Economics*. Elsevier, Amsterdam, Netherlands.

Getmansky, Mila, 2012, The Life Cycle of Hedge Funds: Fund Flows, Size, Competition, and Performance *Quarterly Journal of Financial Economics*

Guerreri, Veronica and Peter Kondor, 2012, "Fund Managers, Career Concerns, and Asset Price Volatility," *American Economic Review,* 102(5), August 2012, 1986–2017

Kane, Edward, 1989, "The High Cost of Incompletely Funding the FSLIC Shortage of Explicit Capital," *The Journal of Economic Perspectives*, 3(4), pp. 31-47

Kaplan, S. and A. Schoar, 2005, "Private Equity Performance: Returns, Persistence and Capital Flows", *The Journal of Finance* 60, 1791–1823.

Kiyotaki, Nobuhiro and John Moore, 1997, "Credit Cycles," *Journal of Political Economy*.

Kyle, Albert and Wei Xiong, 2001, "Contagion as a Wealth Effect," *Journal of Finance*, 56 (4), 1401-1440.

Giglio, Stefano, Bryan Kelly and Seth Pruitt, 2013, "Systemic Risk and the Macroeconomy: An Empirical Evaluation," working paper, University of Chicago.

Gorton, Gary and Andrew Metrick, 2012, "Securitized Banking and the Run on Repo," *Journal of Financial Economics*, forthcoming.

Gray, Dale F., Robert C. Merton, and Zvi Bodie, 2008, "New Framework for Measuring and Managing Macrofinancial Risk and Financial Stability," Working Paper, Harvard Business School.

Gilchrist, Simon, and Egon Zakrajsek, 2010, "Credit Spreads and Business Cycle Fluctuations," *American Economic Review*.

Hartmann, P, S. Straetmans and C. de Vries, 2005, "Banking System Stability: A Cross- Atlantic Perspective"

He, Zhiguo and Arvind Krishnamurthy, 2012. "A Model of Capital and Crises," *Review of Economic Studies* 79(2): pp. 735-777.

He, Zhiguo and Arvind Krishnamurthy, 2013. "Intermediary Asset Pricing," *American Economic Review*, 103(2): pp. 732-770.

He, Zhiguo and Peter Kondor, 2012. "Inefficient Investment Waves," Working paper, University of Chicago.

Holmstrom, Bengt and Jean Tirole, 1997, "Financial Intermediation, Loanable Funds and the Real Sector," *Quarterly Journal of Economics*.

Huang, Xin, Hao Zhou, and Haibin Zhu, 2009, "A Framework for Assessing the Systemic Risk of Major Financial Institutions," *Journal of Banking and Finance*, 33(11) 2036 - 2049

Kiley, Michael, and Jae Sim, 2011, Financial Capital and the Macroeconomy: A Quantitative Framework, Federal Reserve Board Working Paper 2011-27.

Koop, G., Pesaran, H., Potter, S., 1996. Impulse Response Analysis in Nonlinear Multivariate Models. *Journal of Econometrics* 74, 119-148.

Lui, Zheng, Pengfei Wang and Tao Zha, 2011, "Land-Price Dynamics and Macroeconomic Fluctuations," NBER working paper #17045.

Lucas, Robert E, 1990, "Liquidity and Interest Rates," *Journal of Economic Theory.*

Maggiori, Matteo, 2012, "Financial Intermediation, International Risk Sharing, and Reserve Currencies," working paper, NYU.

Mendoza, Enrique, 2010, "Sudden Stops, Financial Crises and Leverage," *American Economic Review*

Muir, Tyler, 2012, "Financial Crises, Risk Premia, and the Term Structure of Risky Assets," working paper, Yale University.

Piazzesi, Monika, Martin Schneider and Selale Tuzel, 2007, "Housing, Consumption, and Asset Pricing", *Journal of Financial Economics* 83, pp. 531-569.

Philippon, Thomas, 2010, "The Bond Market's Q," *Quarterly Journal of Economics*.

Rampini, Adriano and S. Viswanathan, "Financial Intermediary Capital," working paper, Duke University.

Rietz, Thomas, 1988, "The equity risk premium: A solution," *Journal of Monetary Economics,* 2(1):117-131.

Scholes, Myron, 2000, Crisis and risk management, *American Economic Review* 90, 17-21

Shleifer, A. and R. Vishny, 1997, "The Limits of Arbitrage,' *Journal of Finance*, 52, 35-55.

Vayanos, Dimitris, and Paul Woolley, 2012, An Institutional Theory of Momentum and Reversal, working paper, London School of Economics.

Warther, Vincent, 1995, Aggregate mutual fund flows and security returns, *Journal of Financial Economics* 39, 209-236

A Derivation of ODE

A.1 Asset returns and Intermediary Optimality

We write the evolution of e_t in equilibrium as

$$de_t = \mu_e dt + \sigma_e dZ_t,$$

The functions μ_e and σ_e are state-dependent drift and volatility to be solved in equilibrium.

The terms in equation (13) can be expressed in terms of the state variables of the model. Consider the risk and return terms on each investment. We can use the rental market clearing condition $C_t^h = H = 1$ to solve for the housing rental rate D_t:

$$D_t = \frac{\phi}{1-\phi} C_t^y = \frac{\phi}{1-\phi} K_t (A - i_t - \frac{\kappa}{2}(i_t - \delta)^2),$$

where we have used the goods market clearing condition in the second equality. Note that i_t, as given in (7), is only a function of $q(e_t)$. Thus, D_t can be expressed as a function of K_t and e_t.

Given the conjecture $P_t = p(e_t)K_t$, we use Ito's lemma to write the return on housing as,

$$\begin{aligned} dR_t^h &= \frac{dP_t + D_t dt}{P_t} = \frac{K_t dp_t + p_t dK_t + [dp_t, dK_t] + D_t dt}{p_t K_t} \\ &= \left[\frac{p'(e)(\mu_e + \sigma\sigma_e) + \frac{1}{2}p''(e)\sigma_e^2 + \frac{\phi}{1-\phi}\left(A - i_t - \frac{\kappa}{2} i_t^2\right)}{p(e)} + i_t - \delta \right] dt + \sigma_t^h dZ_t, \end{aligned} \tag{17}$$

where the volatility of housing returns is,

$$\sigma_t^h = \sigma + \sigma_e \frac{p'(e)}{p(e)}.$$

The return volatility has two terms: the first term is the exogenous capital quality shock and the second term is the endogenous price volatility due to the dependence of housing prices on the intermediary reputation e (which is equal to equity capital, when the constraint binds). In addition, when e is low, prices are more sensitive to e (i.e. $p'(e)$ is high), which further increases volatility.

Similarly, for capital, we can expand (10):

$$dR_t^k = \left[-\delta + \frac{(\mu_e + \sigma\sigma_e)\, q'(e) + \frac{1}{2}\sigma_e^2 q''(e) + A}{q(e)}\right] dt + \sigma_t^k dZ_t,$$

with the volatility of capital returns,

$$\sigma_t^k = \sigma + \sigma_e \frac{q'(e)}{q(e)}$$

The volatility of capital has the same terms as that of housing. However, when we solve the model, we will see that $q'(e)$ is far smaller than $p'(e)$ which indicates that the endogenous component of volatility is small for capital.

The supply of housing and capital via the market clearing condition (16) pins down α_t^k and α_t^h. We substitute these market clearing portfolio shares to find an expression for the equilibrium volatility of the intermediary's portfolio,

$$\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h = \frac{K_t}{E_t} \left(\sigma_e(q' + p') + \sigma(p+q)\right). \tag{18}$$

From the intermediary optimality condition (13), we note that:

$$\frac{\pi_t^k}{\sigma_t^k} = \frac{\pi_t^h}{\sigma_t^h} = m\frac{K_t}{E_t} \left(\sigma_e(q' + p') + \sigma(p+q)\right) \equiv \text{Sharpe ratio}. \tag{19}$$

When K_t/E_t is high, which happens when intermediary equity is low, the Sharpe ratio is high. In addition, we have noted earlier that p' is high when E_t is low, which further raises the Sharpe ratio.

We expand (19) to find a pair of second-order ODEs. For capital:

$$(\mu_e + \sigma\sigma_e)\, q' + \frac{1}{2}\sigma_e^2 q'' + A - (\delta + r_t) q = m\left(\sigma q + \sigma_e q'\right) \frac{K_t}{E_t} \left(\sigma_e(q' + p') + \sigma(p+q)\right); \tag{20}$$

and for housing:

$$\begin{aligned} & (\mu_e + \sigma\sigma_e)\, p' + \frac{1}{2}\sigma_e^2 p'' + \frac{\phi}{1-\phi}\left(A - i_t - \frac{\kappa}{2}(i_t - \delta)^2\right) - (\delta + r_t - i_t)\, p \\ = \; & m\left(\sigma p + \sigma_e p'\right) \frac{K_t}{E_t} \left(\sigma_e(q' + p') + \sigma(p+q)\right). \end{aligned} \tag{21}$$

A.2 Dynamics of State Variables

We derive equations for μ_e and σ_e which describe the dynamics of the capital constraint. Applying Ito's lemma to $\mathcal{E}_t = e_t K_t$, and substituting for dK_t from (2), we find:

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = \frac{K_t de_t + e_t dK_t + \sigma_e \sigma K dt}{e_t K_t} = \frac{\mu_e + \sigma_e\sigma + e\,(i_t - \delta)}{e} dt + \frac{\sigma_e + e\sigma}{e} dZ_t. \tag{22}$$

We can also write the equity capital dynamics directly in terms of intermediary returns and exit, from (6). When the economy is not at at a boundary (hence $d\psi = 0$), equity dynamics are given by,

$$\begin{aligned} \frac{d\mathcal{E}_t}{\mathcal{E}_t} &= m\alpha_t^k \left(dR_t^k - r_t\right) + m\alpha_t^h \left(dR_t^h - r_t\right) + (mr_t - \eta)dt \\ &= m\alpha_t^k \left(\pi_t^k dt + \sigma_t^k dZ_t\right) + m\alpha_t^h \left(\pi_t^h dt + \sigma_t^h dZ_t\right) + (mr_t - \eta)dt. \end{aligned}$$

We use (13) relating equilibrium expected returns and volatilities to rewrite this expression as,

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m^2\left(\alpha_t^k\sigma_t^k + \alpha_t^h\sigma_t^h\right)^2 dt + m\left(\alpha_t^k\sigma_t^k + \alpha_t^h\sigma_t^h\right) dZ_t + (mr_t - \eta)dt \tag{23}$$

where the portfolio volatility term is given in (18). We match drift and volatility in both equations (22) and (23), to find expressions for μ_e and σ_e. Matching volatilities, we have,

$$m\frac{K_t}{E_t}\left(\sigma_e(q'+p') + \sigma(p+q)\right) = \frac{\sigma_e}{e} + \sigma$$

while matching drifts, we have,

$$\left(m\frac{K_t}{E_t}\left(\sigma_e(q'+p') + \sigma(p+q)\right)\right)^2 + mr_t - \eta = \frac{\mu_e + \sigma_e\sigma + e\left(i_t - \delta\right)}{e}.$$

Because

$$\frac{E_t}{K_t} = \frac{\min\left(\mathcal{E}_t, (1-\lambda)\, W_t\right)}{K_t} = \min\left(e_t, (1-\lambda)\left(p\left(e\right) + q\left(e\right)\right)\right),$$

these equations can be rewritten to solve for μ_e and σ_e in terms of e, $p\left(e\right)$, $q\left(e\right)$,and their derivatives.

A.3 Interest Rate

Based on the household consumption Euler equation, we can derive the interest rate r_t. Since

$$C_t^y = Y_t - i_t K_t - \frac{\kappa K_t}{2}(i_t - \delta)^2 = \left(A - \delta - \frac{q_t - 1}{\kappa} - \frac{(q_t-1)^2}{2\kappa}\right) K_t,$$

we can derive $\mathbb{E}_t\left[dC_t^y/C_t^y\right]$ and $Var_t\left[dC_t^y/C_t^y\right]$ in terms of $q\left(e\right)$ (and its derivatives), along with μ_e and σ_e. Then using (8) it is immediate to derive r_t in these terms as well.

A.4 The System of ODEs

Here we give the expressions of ODEs, expecially write the second-order terms p'' and q'' in terms of lower order terms. For simplicity, we ingore the argument for $p\left(e\right)$, $q\left(e\right)$ and their derivateives. Let

$$\begin{aligned} x\left(e\right) &= A - \delta - \widehat{i}\left(e\right) - \frac{\kappa\left[\widehat{i}\left(e\right)\right]^2}{2}, w\left(e\right) \equiv p\left(e\right) + q\left(e\right), F\left(e\right) \equiv \frac{w\left(e\right)}{e} - m\theta\left(e\right) w'\left(e\right), \\ \text{and } G\left(e\right) &\equiv x\left(e\right)\kappa F\left(e\right) + qq'm\left(1 - \theta\left(e\right)\right) w, \end{aligned}$$

where

$$\theta\left(e\right) \equiv \max\left[\frac{w\left(e\right)}{e}, \frac{1}{1-\lambda}\right] \text{ and } \widehat{i}\left(e\right) \equiv \frac{q\left(e\right) - 1}{\kappa}.$$

We have

$$\sigma_e = \frac{ew\left(e\right)\sigma\left(m\theta\left(e\right) - 1\right)}{w\left(e\right) - em\theta\left(e\right) w'\left(e\right)}.$$

Define

$$\begin{aligned} a_{11} &\equiv p'\left(\frac{x\left(e\right)\kappa}{G\left(e\right)}\left(-m\left(1-\theta\left(e\right)\right)\xi\left(\frac{\frac{1}{2}q\sigma_e^2}{x\left(e\right)\kappa}\right) w\left(e\right) + m\theta\left(e\right)\frac{1}{2}\sigma_e^2\right)\right) + \frac{p\xi}{G\left(e\right)}\left(qq'm\theta\left(e\right)\frac{1}{2}\sigma_e^2 + \frac{F\left(e\right)}{2}q\sigma_e^2\right), \\ a_{12} &\equiv p'\left(\frac{x\left(e\right)\kappa}{G\left(e\right)} m\theta\left(e\right)\frac{1}{2}\sigma_e^2\right) + \frac{1}{2}\sigma_e^2 + \frac{p\xi}{G\left(e\right)}\left(qq'm\theta\left(e\right)\frac{1}{2}\sigma_e^2\right), \\ a_{21} &\equiv q'\left(\frac{x\left(e\right)\kappa}{G\left(e\right)}\left(\left[-m\left(1-\theta\left(e\right)\right)\xi\frac{q\sigma_e^2}{2x\left(e\right)\kappa}\right] w\left(e\right) + \frac{1}{2}m\theta\left(e\right)\sigma_e^2\right)\right) + \frac{1}{2}\sigma_e^2 + \frac{q\xi}{G\left(e\right)}\left(qq'm\theta\left(e\right)\frac{1}{2}\sigma_e^2 + \frac{F\left(e\right)}{2}q\sigma_e^2\right), \\ a_{22} &\equiv q'\frac{x\left(e\right)\kappa}{G\left(e\right)} m\theta\left(e\right)\frac{1}{2}\sigma_e^2 + \frac{q^2\xi}{G\left(e\right)} q'm\theta\left(e\right)\frac{1}{2}\sigma_e^2, \end{aligned}$$

and

$$
\begin{aligned}
b_1 \quad \equiv \quad & \left(p\sigma + p'\sigma_e\right)\sigma m\theta\left(e\right)\frac{w\left(e\right) - ew'\left(e\right)}{eF\left(e\right)} - \frac{p'x\left(e\right)\kappa}{G\left(e\right)}\left(\begin{array}{c} \left[m\left(1-\theta\left(e\right)\right)\left(\rho + \xi\left(\widehat{\imath} - \frac{\left(q'\right)^2\sigma_e^2}{2x(e)\kappa}\right) - \frac{\xi(1+\xi)\left[x(e)\sigma - \frac{qq'\sigma_e}{\kappa}\right]^2}{2x(e)^2}\right) - \widehat{\imath} - \eta\right]w\left(e\right) \\ +m\theta\left(e\right)\left[\frac{A-\delta}{\phi} + \left(p - \frac{1-\phi}{\phi}\right)\widehat{\imath} - \frac{1-\phi}{\phi}\frac{\kappa}{2}\widehat{\imath}^2 + \delta\left(1-q\left(e\right)\right)\right] \end{array}\right) \\
& -\frac{1-\phi}{\phi}x\left(e\right) - p\widehat{\imath} + \frac{p}{G\left(e\right)}\left(\begin{array}{c} \left(\rho + \xi\widehat{\imath}\right)x\left(e\right)\kappa F\left(e\right) - \xi\frac{F(e)}{2}\left(q'\right)^2\sigma_e^2 - \frac{\xi(1+\xi)}{2}\frac{F(e)\kappa\left[x(e)\sigma - \frac{qq'\sigma_e}{\kappa}\right]^2}{c^y} \\ -\xi qq'\left(\left[-\widehat{\imath} - \eta\right]w\left(e\right) + m\theta\left(e\right)\left[\frac{A-\delta}{\phi} + \left(p - \frac{(1-\phi)}{\phi}\right)\widehat{\imath} - \frac{1-\phi}{\phi}\frac{\kappa}{2}\widehat{\imath}^2 + \delta\left(1-q\left(e\right)\right)\right]\right) \end{array}\right), \\
b_2 \quad \equiv \quad & \left(\sigma_e q' + q\sigma\right)\sigma m\theta\left(e\right)\frac{w\left(e\right) - ew'\left(e\right)}{eF\left(e\right)} - \frac{q'x\left(e\right)\kappa}{G\left(e\right)}\left(\begin{array}{c} \left[m\left(1-\theta\left(e\right)\right)\left(\rho + \xi\left(\widehat{\imath} - \frac{\left(q'\right)^2\sigma_e^2}{2x(e)\kappa}\right) - \frac{\xi(1+\xi)\left[x(e)\sigma - \frac{qq'\sigma_e}{\kappa}\right]^2}{2x(e)^2}\right) - \widehat{\imath} - \eta\right]w\left(e\right) \\ +m\theta\left(e\right)\left[\frac{A-\delta}{\phi} + \left(p - \frac{1-\phi}{\phi}\right)\widehat{\imath} - \frac{1-\phi}{\phi}\frac{\kappa}{2}\widehat{\imath}^2 + \delta\left(1-q\left(e\right)\right)\right] \end{array}\right) \\
& -A + q\delta + \frac{q}{G\left(e\right)}\left(\begin{array}{c} \left(\rho + \widehat{\imath}\right)x\left(e\right)\kappa F\left(e\right) - \frac{F(e)}{2}\left(q'\right)^2\sigma_e^2 - \frac{F(e)\kappa\left[x(e)\sigma - \frac{qq'\sigma_e}{\kappa}\right]^2}{x(e)} \\ -qq'\left(\left[-\widehat{\imath} - \eta\right]w + m\theta\left(e\right)\left[\frac{A-\delta}{\phi} + \left(p - \frac{\phi}{1-\phi}\right)\widehat{\imath} - \frac{1-\phi}{\phi}\frac{\kappa}{2}\widehat{\imath}^2 + \delta\left(1-q\right)\right]\right) \end{array}\right).
\end{aligned}
$$

Then the second-order terms can be solved as

$$
\left[\begin{array}{c} q'' \\ p'' \end{array}\right] = \left[\begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array}\right]^{-1}\left[\begin{array}{c} b_1 \\ b_2 \end{array}\right] = \frac{1}{a_{11}a_{22} - a_{12}a_{21}}\left[\begin{array}{c} a_{22}b_1 - a_{12}b_2 \\ -a_{21}b_1 + a_{11}b_2 \end{array}\right].
$$

A.5 Boundary Conditions and Numerical Methods

A.5.1 When $e \to \infty$ without capital constraint

When $e \to \infty$, we have q and p as constants. Let $\widehat{\imath} = \frac{q-1}{\kappa}$, and since

$$
C_t^y = \left(A - \delta - \widehat{\imath} - \frac{\kappa\widehat{\imath}^2}{2}\right)K_t,
$$

we have $dC_t^y/C_t^y = dK_t/K_t = \widehat{\imath}dt + \sigma dZ_t$. As a result, both assets have the same return volatility $\sigma_R^k = \sigma_R^h = \sigma$, and the interest rate is

$$
r = \rho + \xi\widehat{\imath} - \frac{\xi\left(1+\xi\right)}{2}\sigma^2.
$$

Because the intermediary's portfolio weight $\theta = \frac{1}{1-\lambda}$, the banker's pricing kernel is $\sigma m\theta\left(e\right) = \frac{m\sigma}{1-\lambda}$. Therefore

$$
\frac{\mu_R^k - r}{\sigma_R^k} = \frac{m\sigma}{1-\lambda} \Rightarrow \mu_R^k = \frac{m\sigma^2}{1-\lambda} + \rho + \xi\widehat{\imath} - \frac{\xi\left(1+\xi\right)}{2}\sigma^2 = \rho + \xi\widehat{\imath} + \frac{2m - \xi\left(1+\xi\right)\left(1-\lambda\right)}{2\left(1-\lambda\right)}\sigma^2.
$$

Because $\mu_R^k = -\delta + \frac{A}{q}$ by definition, we can solve for

$$
q = \frac{A}{\rho + \delta + \xi\widehat{\imath} + \frac{2m-\xi(1+\xi)(1-\lambda)}{2(1-\lambda)}\sigma^2}.
$$

Because $\widehat{\imath} = \frac{q-1}{\kappa}$, plugging in the above equation we can solve for

$$
q = \frac{-\left(\rho + \delta + \frac{2m-\xi(1+\xi)(1-\lambda)}{2(1-\lambda)}\sigma^2 - \frac{\xi}{\kappa}\right) + \sqrt{\left(\rho + \delta + \frac{2m-\xi(1+\xi)(1-\lambda)}{2(1-\lambda)}\sigma^2 - \frac{\xi}{\kappa}\right)^2 + \frac{4A\xi}{\kappa}}}{\frac{2\xi}{\kappa}} \tag{24}
$$

which gives the value of q and $\widehat{i}$ when $e=\infty$.

Now we solve for p. Using $\frac{\mu_R^h - r}{\sigma_R^h} = \frac{m\sigma}{1-\lambda}$ we know that $\mu_R^h = \rho + \xi\widehat{i} + \frac{2m-\xi(1+\xi)(1-\lambda)}{2(1-\lambda)}\sigma^2$. Since $\frac{\frac{\phi}{1-\phi}\left(A-\delta-\widehat{i}-\frac{\kappa\widehat{i}^2}{2}\right)}{p} + \widehat{i} = \mu_R^h$ by definition, we have

$$p = \frac{\frac{(1-\phi)}{\phi}\left(A-\delta-\widehat{i}-\frac{\kappa\widehat{i}^2}{2}\right)}{\rho + (\xi-1)\,\widehat{i} + \frac{2m-\xi(1+\xi)(1-\lambda)}{2(1-\lambda)}\sigma^2}. \tag{25}$$

Numerically, instead of (24) and (24) we impose the slope conditions $p'(\infty) = q'(\infty) = 0$ which give more stable solutions.

A.5.2 Lower entry barrier

Consider the boundary condition at $\underline{e}$ which is a reflecting barrier due to linear technology of entry. More specifically, at the entry boundary $\underline{e}$, we have

$$dE = m\theta(e_t)\left[dR_t^{agg} - r_t dt\right]E_t dt + dU_t$$

where dU_t reflects E_t at $\underline{e}K_t$. Heuristically, suppose that at $\underline{E} = \underline{e}K$, a negative shock ϵ sends E to $\underline{e}K - \epsilon$ which is below $\underline{e}K$. Then immediately there will be βx unit of physical capital to be converted into x units of E, so that the new level $\widehat{E} = \underline{e}K - \epsilon + x = \underline{e}\widehat{K} = \underline{e}(K-\beta x)$. This implies that the amount of capital to be converted to E is $x = \frac{\epsilon}{1+\underline{e}\beta} > 0$, and the new capital is $\widehat{K} = K - \beta x = K - \beta\frac{\epsilon}{1+\underline{e}\beta}$.

Now we give the boundary conditions for p and q. First, although entry reduces physical capital K, since q is measured as per unit of K, the price should not change during entry. Therefore we must have $q'(\underline{e}) = 0$. For scaled housing price p, there will be a non-zero slope. Intuitively, entry lowers the aggregate physical capital K, hence future equilibrium consumption as well as future equilibrium housing rents are lower, translating to a lower P directly. Formally, right after the negative shock described above, the housing price is $p\left(\frac{E}{K}\right)K$ can be rewritten as $p\left(\frac{E}{K}\right)K = p\left(\underline{e} - \frac{\epsilon}{K}\right)K$, which must equal the housing price $p(\underline{e})\widehat{K} = p(\underline{e})\left(K - \beta\frac{\epsilon}{1+\underline{e}\beta}\right)$ right after the adjustment (otherwise there will be an arbitrage). Hence,

$$p(\underline{e})\left(K - \beta\frac{\epsilon}{1+\underline{e}\beta}\right) = p\left(\underline{e} - \frac{\epsilon}{K}\right)K = p(\underline{e})K - p'(\underline{e})\epsilon \Rightarrow p'(\underline{e}) = \frac{p(\underline{e})\beta}{1+\underline{e}\beta} > 0$$

here we have used the fact that ϵ can be arbitrarily small in the continuous-time limit. In sum, the boundary condtions are

$$p'(\underline{e}) = \frac{p(\underline{e})\beta}{1+\underline{e}\beta} \text{ and } q'(\underline{e}) = 0. \tag{26}$$

A.5.3 Numerical method

Given (26), the following results is useful. Define $\chi \equiv \frac{p(\underline{e})\beta}{1+\underline{e}\beta}$. We know that at $\underline{e}$ the equilibrium Sharpe ratio is (recall $w(e) = p(e) + q(e)$)

$$B = \sigma m\theta(\underline{e})\frac{w(\underline{e}) - \underline{e}w'(\underline{e})}{w(\underline{e}) - \underline{e}m\theta(\underline{e})w'(\underline{e})} = \sigma m\frac{w(\underline{e})}{\underline{e}}\frac{w(\underline{e}) - \underline{e}w'(\underline{e})}{w(\underline{e}) - \underline{e}m\frac{w(\underline{e})}{\underline{e}}\chi} = \sigma m\frac{w(\underline{e}) - \underline{e}\chi}{\underline{e}(1-m\chi)},$$

which implies that

$$p(\underline{e}) + q(\underline{e}) = w(\underline{e}) = \frac{B\underline{e}(1-m\chi)}{\sigma m} + \underline{e}\chi. \tag{27}$$

Based on (27) numerically we use the following 2-layer loops to solve the ODE system in (A.4) with endogenous entry boundary $\underline{e}$.

1. In the inner loop, we fix $\underline{e}$. Consider different trials of $q(\underline{e})$; given $q(\underline{e})$, we can get $p(\underline{e}) = \frac{B\underline{e}(1-m\chi)}{\sigma m} + \underline{e}\chi - q(\underline{e})$. Then based on the four boundary conditions
$$p(\underline{e}), q(\underline{e}), p'(\infty) = q'(\infty) = 0,$$
we can solve this 2-equation ODE system with boundary conditions using the *Matlab* builtin ODE solver *bvp4c*. We then search for the right $q(\underline{e})$ so that $p'(\underline{e}) - q'(\underline{e}) = \chi$ holds.

2. In the outer loop, we search for the endogenous $\underline{e}$. For each trial of $\underline{e}$, we take the inner loop, and keep searching until $q'(\underline{e}) = 0$.

B Derivation for hidden leverage case

The dynamics of the state variable are, $de_t = \mu_e dt + \sigma_e dZ_t$. We recompute μ_e and σ_e based on the higher leverage. The reputation dynamics are:
$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m\alpha_t^k\left(\pi_t^k dt + \sigma_t^k dZ_t\right) + m\alpha_t^h\left(\pi_t^h dt + \sigma_t^h dZ_t\right) + (mr_t - \eta)dt,$$
where $\alpha_t^k = \frac{1}{1-\widehat{\lambda}}\frac{q_t K_t}{W_t}$ and $\alpha_t^k = \frac{1}{1-\widehat{\lambda}}\frac{P_t}{W_t}$ are larger than the baseline equilibrium portfolio shares to reflect the higher leverage based on $\widehat{\lambda}$. For illustration here we focus on the case where capital constraint is not binding and the leverage is simply the intermeidary leverage is simply $\frac{1}{1-\widehat{\lambda}}$. When the capital constraint is binding, the leverage is determined by $\frac{W}{\mathcal{E}_t}$ as the baseline model.

We assume that the interest rate (r_t) and ex-ante risk premia (π_t^k, π_t^h) are the functions of e_t that solve the model based on λ rather than $\widehat{\lambda}$. That is we hold expected returns and interest rates fixed in the experiment. Recall that,
$$\sigma_t^h = \sigma + \sigma_e\frac{p'(e)}{p(e)} \quad \text{and,} \quad \sigma_t^k = \sigma + \sigma_e\frac{q'(e)}{q(e)}.$$
We also assume that the price functions, $p(e)$ and $q(e)$, solve the model based on λ rather than $\widehat{\lambda}$. We account for the fact that higher leverage implies a more volatile σ_e which in turn means that σ_t^h and σ_t^k rises. That is, a given shock dZ_t causes e_t to fall which feeds back into a further fall in asset prices and a larger fall in e_t. It is essential to account for this amplification since it is the non-linearity of the model. Thus,
$$\begin{aligned}\frac{d\mathcal{E}_t}{\mathcal{E}_t} &= m\alpha_t^k\left(\pi_t^k dt + \left(\sigma + \sigma_e\frac{q'(e)}{q(e)}\right)dZ_t\right) + m\alpha_t^h\left(\pi_t^h dt + \left(\sigma + \sigma_e\frac{p'(e)}{p(e)}\right)dZ_t\right) + (mr_t - \eta)dt \\ &= m\left(\alpha_t^k\pi_t^k + \alpha_t^h\pi_t^h + r_t - \eta\right)dt + m\frac{1}{1-\widehat{\lambda}}\left[\sigma + \frac{q'(e)+q'(e)}{p(e)+q(e)}\sigma_e\right]dZ_t \end{aligned} \quad (28)$$
where the second equality uses the fact that $\alpha_t^k = \frac{1}{1-\widehat{\lambda}}\frac{q_t K_t}{W_t}$, $\alpha_t^k = \frac{1}{1-\widehat{\lambda}}\frac{P_t}{W_t}$, and $W_t = K_t(p(e) + q(e))$. From (22), we can also write,
$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = \frac{\mu_e + \sigma_e\sigma + e(i_t - \delta)}{e}dt + \frac{\sigma_e + e\sigma}{e}dZ_t. \quad (29)$$
By matching (28) and (29), we can solve for μ_e and σ_e with hidden leverage. For instance, for σ_e, we have
$$m\frac{1}{1-\widehat{\lambda}}\left[\sigma + \frac{q'(e)+q'(e)}{p(e)+q(e)}\sigma_e\right] = \frac{\sigma_e}{e} + \sigma \Rightarrow \sigma_e = \sigma\frac{m\frac{1}{1-\widehat{\lambda}} - 1}{\frac{1}{e} - m\frac{1}{1-\widehat{\lambda}}\frac{q'(e)+q'(e)}{p(e)+q(e)}}$$

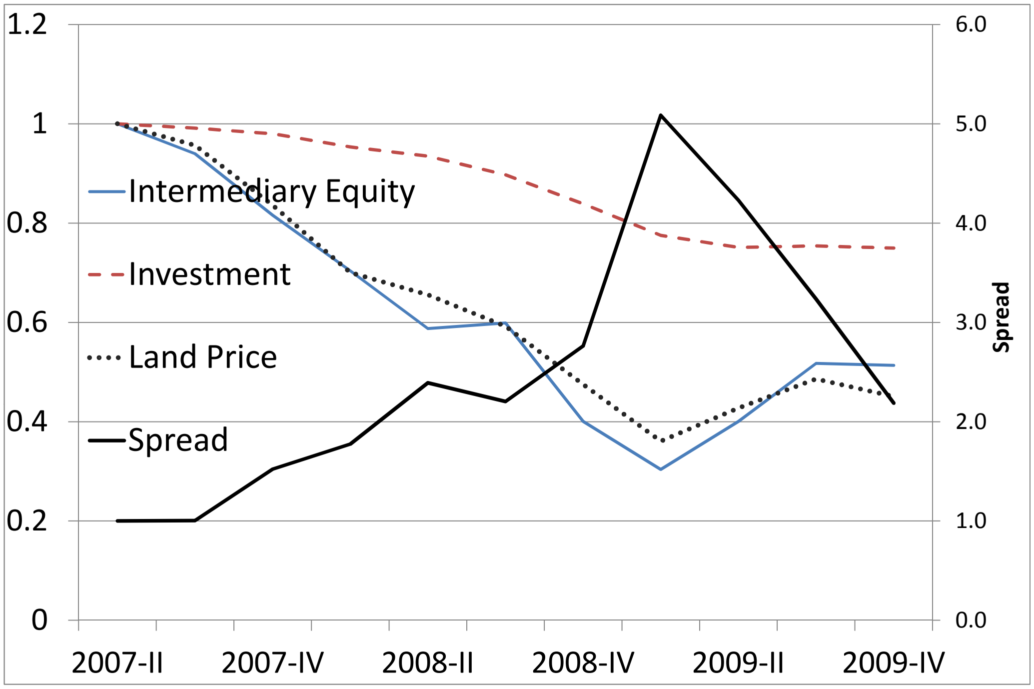


Figure 2: Data from 2007 to 2009. Intermediary equity, investment, land price index are on left-axis. Excess bond premium (labeled spread) is on right-axis. Variables are scaled by their initial values in 2007Q2.

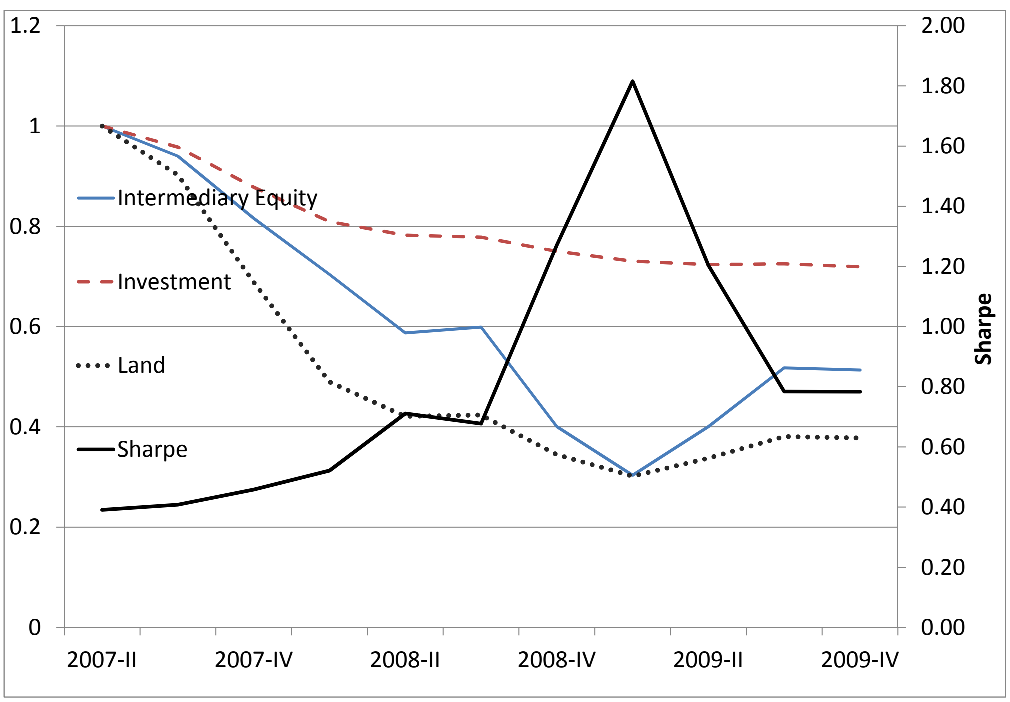


Figure 3: Model simulation matching data from 2007 to 2009. Intermediary equity, investment, land price index are on left-axis. Sharpe ratio is on right-axis. Variables are scaled by their initial values in 2007Q2.

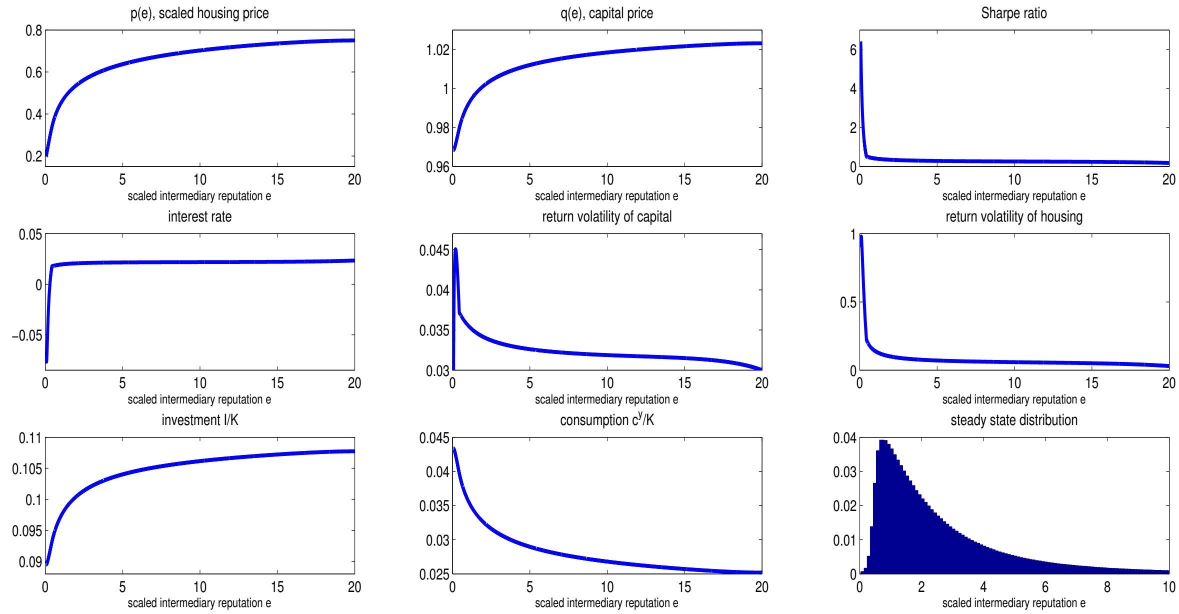


Figure 4: Price and Policy Functions for Baseline Parameters

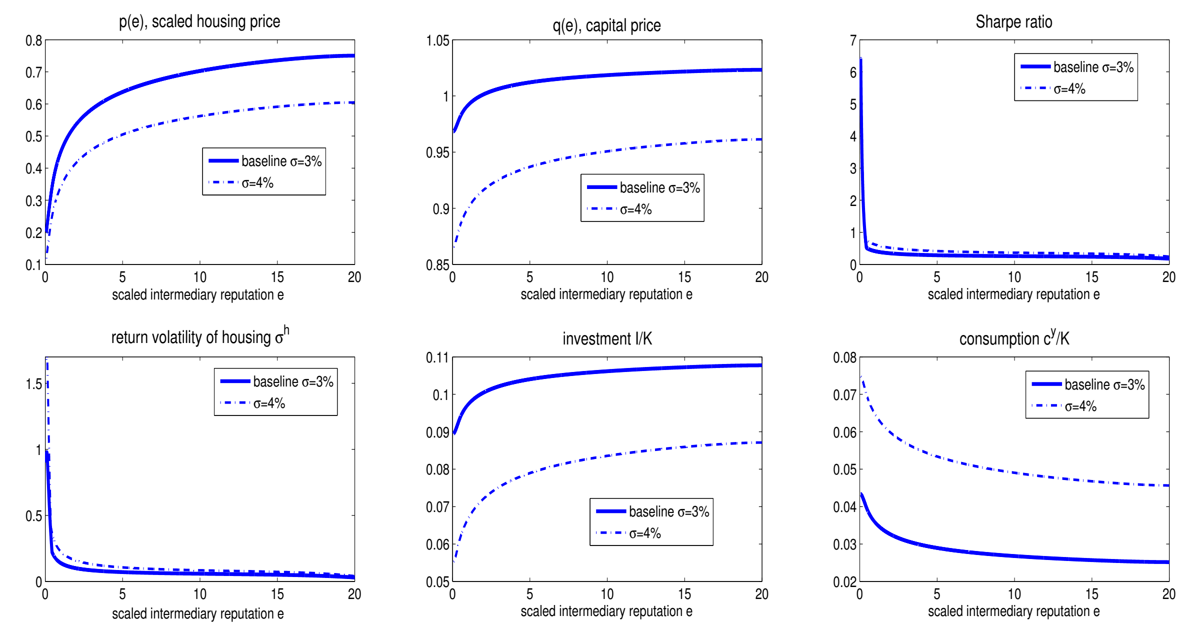


Figure 5: Price and Policy Functions for $\sigma = 4\%$ Case

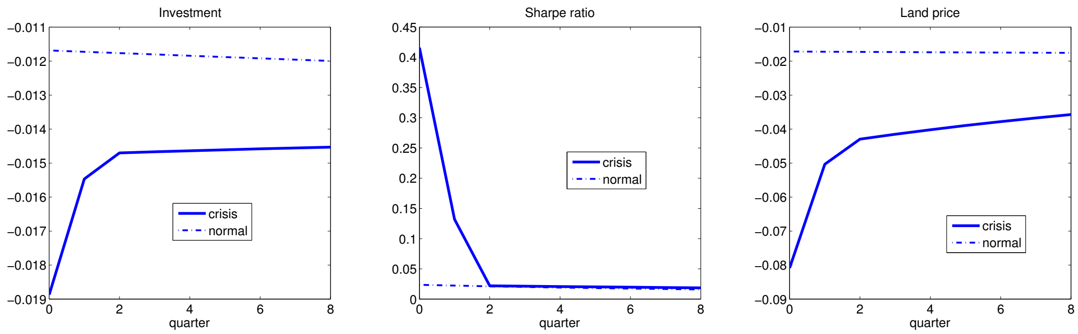


Figure 6: Effect of a −1% shock on investment, Sharpe ratio, and land prices, conditional on crisis (solid line) and normal states (dashed line)

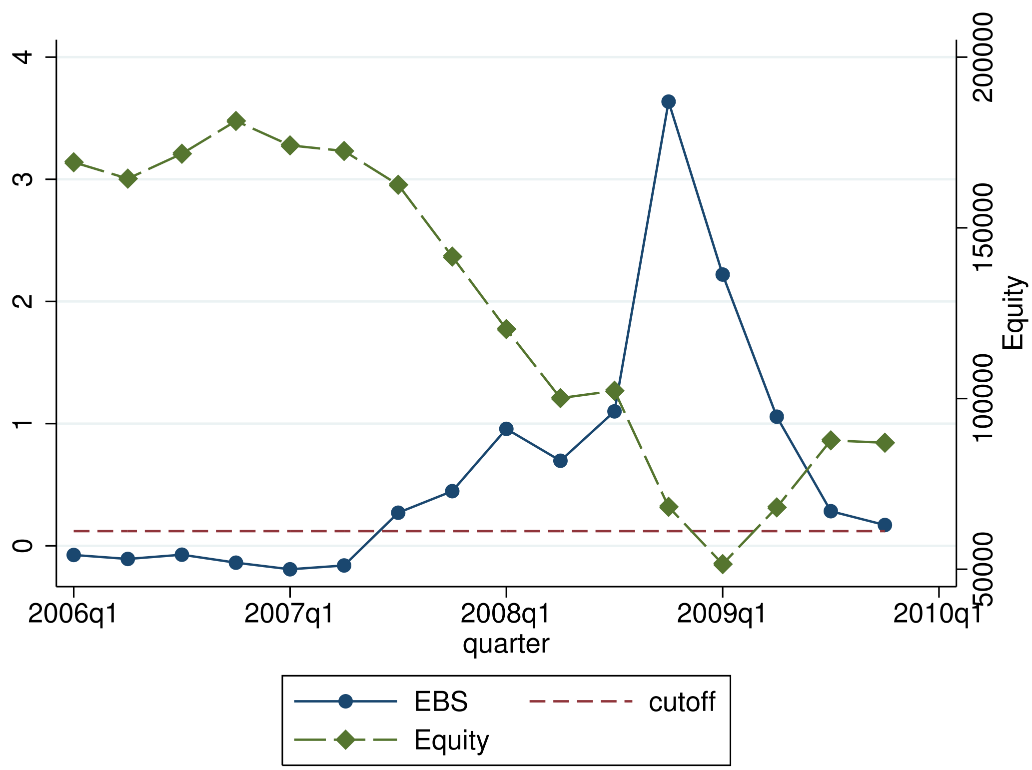
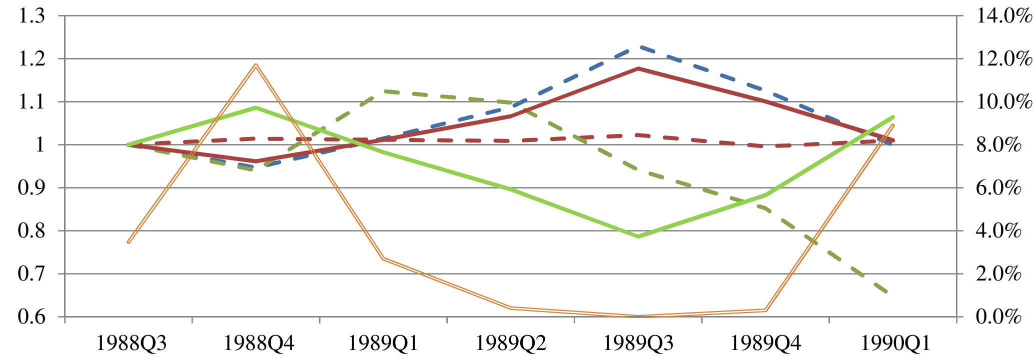
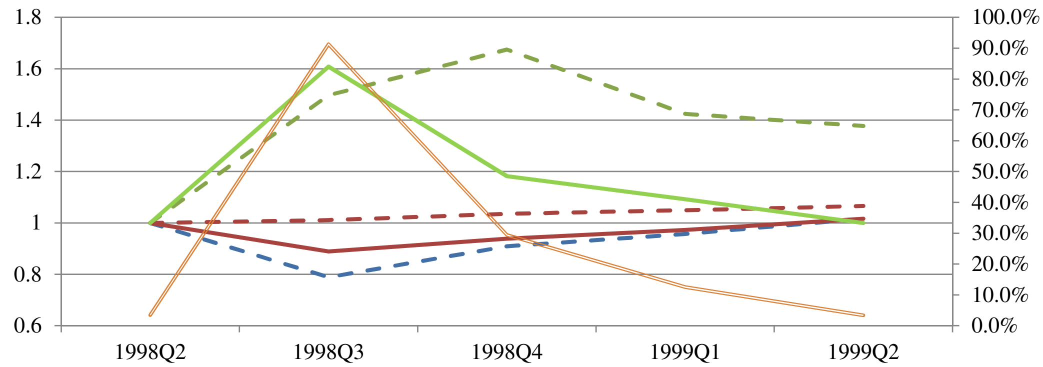


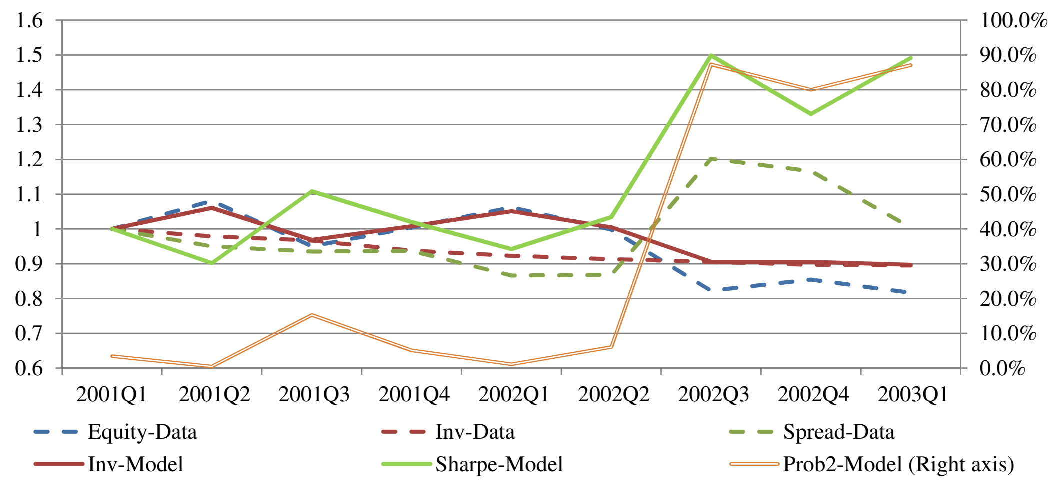
Figure 7: EB and Intermediary Equity are graphed from 2006 to 2010. The dashed horizontal line is the cutoff for our classification of distress and non-distress periods.



Panel A: Savings and Loan Crisis



Panel B: 1998 Hedge Fund Crisis



Panel C: 2002 Corporate Bond Market Crisis

Figure 8: Model simulation and data from three financial crisis episodes. Data for intermediary equity, aggregate investment and EB spread are plotted in dashed lines, normalized to one at the initial date of the episode. Data from the model simulation is in solid lines, normalized to one at the initial date. The model based 2-year probability of falling into a systemic crisis when the capital constraints for the financial sector bind is plotted in orange and on the right axis.

Online Appendix

Table Appendix: Model Simulation and Data

The table presents standard deviations and covariances for intermediary equity growth (Eq), investment growth (I), consumption growth (C), land price growth (PL), and Sharpe ratio (EB). Growth rates are computed as annual changes in log value from t to $t+1$. The Sharpe ratio is the value at $t+1$. The columns labeled data are the statistics for the period 1975 to 2010. The Sharpe ratio is constructed from the excess bond premium, and other variables are standard and defined in the text. The data columns correspond to distress classification of the 10% worst observations and the 20% worst observations. For the model simulation, the distress period is defined as the 10% and 20% worst realizations of the Sharpe ratio.

	Data 10	Model 10	Data 20	Model 20
Panel A: Distress Periods				
vol(Eq)	33.45%	61.33	27.88	43.53
vol(I)	9.74%	5.89	8.35	5.54
vol(C)	2.27%	5.14	1.98	4.11
vol(PL)	29.01%	29.89	24.98	24.34
vol(EB)	76.06%	127.15	64.14	92.29
cov(Eq, I)	1.85%	2.56	1.26	1.50
cov(Eq, C)	0.49%	-2.73	0.29	-1.47
cov(Eq, PL)	8.13%	16.50	5.50	8.92
cov(Eq, EB)	-5.39%	-48.55	-3.85	-24.12
Panel B: Non-distress Periods				
vol(Eq)	20.46%	6.45	19.79	5.69
vol(I)	6.50%	4.42	6.49	4.31
vol(C)	1.20%	1.59	1.22	1.41
vol(PL)	11.15%	11.18	10.36	10.17
vol(EB)	23.32%	12.87	15.90	9.88
cov(Eq, I)	0.17%	0.27	0.05	0.24
cov(Eq, C)	0.03%	-0.08	0.03	-0.06
cov(Eq, PL)	0.39%	0.69	0.31	0.57
cov(Eq, EB)	-0.76%	-0.33	0.21	-0.17