Collateral Advantage: Exchange Rates, Capital Flows, and Global Cycles

Michael B. Devereux∗
Charles Engel†
Steve Pak Yeung Wu‡

This version: 4 Nov 2022

Abstract

This paper explores the effect of global shocks in a two-country New Keynesian model in which US government debt has an advantage as a superior collateral asset in the balance sheets of banks. We show that the model can account for the observed response of the US dollar and US bond returns to a global downturn. Our model predicts that the U.S. enjoys an “exorbitant privilege” as its government bonds are desired by banks both in the U.S. and abroad as superior collateral. In times of global stress, the dollar appreciates and the “convenience yield” earned by U.S. government bonds increases. There is “retrenchment” - each country reduces its holdings of foreign assets - a critical determinant of which is the endogenous response of prices and returns. In addition, the model displays a U.S. real exchange rate appreciation despite that domestic absorption in the US falls relative to the rest of the world during a global downturn, thus addressing the “reserve currency paradox” highlighted by Maggiori (2017).

JEL Classification Codes: F3, F4, G1

Key words: Exchange rates, financial frictions, liquidity, convenience yield, dollar specialness

∗E-mail: michael.devereux@ubc.ca. University of British Columbia, NBER and CEPR.
†E-mail: cengel@ssc.wisc.edu. University of Wisconsin, NBER and CEPR.
‡E-mail: steve.py.wu@gmail.com. University of California, San Diego
1 Introduction

During the global financial crisis of 2007-2009, the U.S. dollar appreciated strongly. This appreciation of the dollar has been described as the “exorbitant duty” of the U.S. currency – foreign investors are rewarded during times of global stress by an increase in the value of their U.S. assets, while U.S. investors, because of the dollar depreciation find the value of their foreign-currency denominated assets declining. This duty is the complement of the “exorbitant privilege” the U.S. enjoys during quiescent times, as the average return on U.S. investments abroad exceeds that of foreign investments in the U.S.\(^1\)

Such behavior of the dollar implies that the dollar is a hedge during times of global downturn, so it seems plausible that the low rate of return on U.S. assets is attributable at least in part to their value as insurance. There is little consensus on why the dollar appreciates during downturns. Gourinchas and Rey (2022) provide a model in which global demand for dollar assets as insurance increases as the potential for a global crisis increases. Bianchi et al. (2021), on the other hand, attribute the appreciation to liquidity demand arising from financial intermediaries. A disproportionate amount of volatile, short-term funding globally is in dollars, and as funding becomes more uncertain, the demand by intermediaries for liquid dollar assets increases, thus appreciating the dollar. Kekre and Lenel (2020) and Jiang et al. (2021a) construct models in which the dollar appreciates because it earns a higher “convenience yield” during times of global stress. The convenience yield arises in those frameworks because agents directly derive utility from dollar assets, though this is meant to be an expedient way to capture the various factors such as liquidity services that are provided by dollar assets.\(^2\)

Here we posit that U.S. Treasury dollar liabilities play a special role in the balance sheet of financial intermediaries. These intermediaries face constraints on their ability to leverage, as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). Markets or regulators constrain the banks from acquiring too large a balance sheet, because intermediaries (which we shall call “banks”) have private information about the value of their assets that is not observed by private agents or financial regulators. In order to prevent excessive risk taking, the leverage of banks is constrained. However, there is less private information about the value of government debt, and especially U.S. government debt that is widely traded globally in very deep markets. In our set-up, U.S. and foreign banks are less constrained in their holdings of government debt, and U.S. Treasury debt has a “comparative advantage” in the sense that the relative constraint on this debt is lower globally than on foreign government debt.

The fact that the value of U.S. Treasury debt is easily assessed in global markets is one sense in which these assets are very liquid. We draw a connection between liquidity and “safety”. As Gorton (2017) states, “A safe asset is an asset that is (almost always) valued at face value without expensive and prolonged analysis. By design, there is no benefit to producing (private) information

\(^1\)Gourinchas and Rey (2007a,b, 2022) and Eichengreen (2011) originate this idea, provide empirical evidence to support it, and build a model to account for it.

\(^2\)See also Engel (2016) and Engel and Wu (2018) for New Keynesian models in which the convenience yield comes about because bonds are in the utility function.
about its value, and this is common knowledge.” This is precisely the motivation for our modeling of the special role of U.S. Treasury assets in bank balance sheets.

Our model is a two-country, New Keynesian DSGE model. In each country, households consume, provide labor services, and save by putting deposits in banks located in their own country. Firms in each country produce output using labor and capital. Banks in each country rent capital to firms in their own country, supply deposits to households, and hold government bonds from both countries. Governments issue debt and make transfers to households. Monetary policy in each country is modeled as a simple inflation-targeting rule.

The banks face constraints requiring that a weighted average of the value of their assets must be less than some multiple of the equity value of the bank. However, the weights on the government bonds are lower than the weight on capital, implying that banks are less constrained in their holdings of government bonds. Importantly, though, banks are less constrained in their holdings of U.S. government bonds, which are considered to be better collateral. Globally, therefore, U.S. debt is especially safe/liquid. However, we assume the advantage of U.S. government bonds relative to foreign government bonds is relatively greater for U.S. financial intermediaries than for foreign intermediaries.

Even abstracting from the insurance premium that U.S. Treasury debt might earn, the model can explain the exorbitant privilege the U.S. enjoys. During normal times, U.S. debt pays a low rate of monetary return because it constrains banks’ balance sheets less than other assets. In fact, the model is capable of accounting for the fact that while the U.S. is a net international debtor, its net investment income is positive, a noted feature of the U.S. balance of payments.

Moreover, the model explains the appreciation of the dollar during times of global downturn as an increase in demand by banks, both in the U.S. and abroad, for dollar Treasury debt. This appreciation is coincident with an increase in the convenience yield on U.S. government debt, measured as a deviation from UIP. The global downturn may be caused either by a financial crisis (modeled here as a uniform tightening of borrowing constraints) hitting all countries equally, or a global negative total factor productivity shock.

However, our model demonstrates an important feature of equilibrium adjustment in the global economy. While the foreign bank, ceteris paribus, demands more of the home bond, we find in our baseline calibration that both the value of U.S. bonds held by the foreign bank and the share of home bonds in its portfolio of assets declines. This finding highlights the importance of understanding the equilibrating process when demand for assets change. Here, there are two important features that lead the foreign bank to shed some of its holdings of U.S. bonds. First, because of the appreciation, the foreign currency value of the bank’s holdings of U.S. bonds increases even without a change in the quantity that it holds. Second, the increase in U.S. demand for U.S. bonds is somewhat stronger, as it has a greater incentive to shift out of non-governmental assets. As we will explain, the advantage of U.S. government bonds relative to equity capital is greater than the equivalent for foreign bonds. The global increase in demand drives down the relative rate of return on these U.S. bonds (that is, there is an increase in the convenience yield), which ultimately leads foreign banks...
to decrease their holdings of U.S. bonds and increase their holdings of foreign bonds. Indeed, there is retrenchment: banks both in the U.S. and abroad shed assets of the other country (bonds and equities) during times of global stress, a prediction which is consistent with the empirical findings of Milesi-Ferretti and Tille (2011), Forbes and Warnock (2012) and, with a particular focus on banks, Wang (2018).³

The “reserve currency paradox” of Maggiori (2017) notes that during these times in which the dollar appreciates, the net international investment position of the U.S. deteriorates as the value of U.S. holdings of foreign assets falls relative to the value of foreign holdings of U.S. assets. The paradox is that if the real value of the dollar is determined through home bias in preferences (or through the presence of non-traded goods) by a rise in the price of goods favored by U.S. households, then a real appreciation of the dollar should be associated with an increase in relative demand by U.S. households. It is difficult to reconcile an increase in demand by U.S. households with a fall in their wealth. This paradox is resolved in our model because nominal goods prices are sticky in the currency of consumers (local-currency pricing, or LCP.) A real appreciation is not immediately associated with an increase in the relative price of U.S. goods.

We also consider the effects of monetary shocks in this framework. As the work of Rey (2015), Miranda-Agrippino and Rey (2020), and others have emphasized, there are repercussions from U.S. monetary shocks on the rest of the world that work through financial channels, and not just through the traditional channels of import demand. In addition, we consider the effects of an equal monetary contraction in the U.S. and the rest of the world. The insight from this exercise is that even when monetary policy changes are identical across countries, there is an asymmetric effect on the dollar exchange rate. If the U.S. and other countries were to follow uniform monetary contractions (for example, in response to an inflationary shock), the dollar would nonetheless appreciate. The contraction lowers aggregate demand, and tightens balance sheet constraints on banks, precipitating a global increase in demand for dollar bonds and an appreciation.

Figure 1 shows some stylized facts around time of the global financial crisis that helps to motivate the model. The upper panel shows the strong relationship of liquidity returns and the U.S. dollar exchange rate. The blue line represents the U.S. dollar price of the euro in the left panel and the U.S. dollar price of the average of the rest of the G9 currencies in the right panel (converted into real exchange rates by adjusting for relative consumer prices.) The red line in both graphs presents the liquidity yield measure in Engel and Wu (2018), defined as the difference between a market rate of return and the rate of return on short-term government bonds for the U.S. relative to the other country.⁴ The measure captures the liquidity or convenience services of government bonds. The panel clearly illustrates that the sharp appreciation of the U.S. dollar in the 2008 period is associated with a large increase in the US liquidity yield, which implies an increase in demand for U.S. Treasuries during the crisis.

³Tabova and Warnock (2021) in fact show that foreign holdings of U.S. Treasury bonds fell during the Global Financial Crisis, though their focus is on long-term bonds and much of that change in portfolios was driven by central bank holdings.
⁴This measure is used by Engel and Wu (2018) but is nearly identical to the ones that are used in the studies of Du et al. (2018a) and Jiang et al. (2021a). The exchange rate and price data is from DataStream. The “rest of the G10 means the G10 countries excluding the U.S. and euro area.
The middle panel shows retrenchment during the crisis. The left (right) panel plots the ratio of capital inflows (outflows) to GDP respectively for the U.S., Germany and the rest of the G10 average.\textsuperscript{5} Before 2008, there are regular inflow and outflow of roughly 10% of GDP. (Note that a positive amount of outflows from a country is plotted as a negative number in the right-hand-side panel.) During the global financial crisis period, highlighted in gray, there is a dramatic difference. Inflows into these countries turn from positive to negative, implying that investors from other countries are shedding their holdings of foreign (i.e., U.S., German, or G10, respectively) assets. The right panel tells a similar story, that U.S., German, or G10 investors are reducing their holdings of foreign assets, as signified by the fact that the outflow numbers become positive.

The last panel reports the dynamics of real variables of the US relative to the rest of the world. We look at three different ratios: the U.S. GDP to world GDP ratio, the U.S. consumption to world consumption ratio and the U.S. investment to world investment ratio. The figure plots the first difference of these ratios. During the financial crisis, U.S. GDP fell relative to the rest of the world (blue line). U.S. consumption also fell relative to the rest of the world but less than the drop of the GDP ratio. Finally, there is a big drop in the relative U.S. investment.

In addition to the well-known data on net capital flows, our model will offer some insights into these empirical regularities.

\textsuperscript{5}The capital flow data is from Bluedorn et al. (2013).
Figure 1: Stylized facts during the Global Financial Crisis

Upper panel

Exchange rate and liquidity yield: U.S. vs Germany

U.S. vs rest of G10

Note: Liquidity yield is defined as in Engel and Wu (2018), which is home to foreign market interest rate indifferential minus home to foreign government interest rate differential \((i_m - i^* - (i_g - i_g^*))\).

Middle panel

Capital inflow to GDP

Capital outflow to GDP

Note: Following the convention, a capital outflow is defined as a negative value when outflow occurs.

Lower panel

Change in US relative to the world real dynamics

Note: The three ratios are US GDP/World GDP, US consumption/World consumption and US investment/World investment. Data is annual and the first difference (\(\Delta\)) is year-on-year difference.
Several recent papers have found a relationship between “convenience yields” and either exchange rates or deviations from uncovered interest rate parity. These include Jiang et al. (2021a), Jiang et al. (2018), Jiang et al. (2021b), Krishnamurthy and Lustig (2019) Engel and Wu (2018), Kekre and Lenel (2020), Valchev (2020), and Bianchi et al. (2021). These papers provide evidence that deviations from uncovered interest rate parity (u.i.p.) may be attributable in part to liquidity or convenience yields, and that this return to liquidity also influences the level of the exchange rate. Much of the literature has noted especially the nexus between convenience yields on U.S. government bonds and the level of the dollar exchange rate as well as deviations from u.i.p. for the dollar. These general equilibrium models follow earlier literature that takes the deviation from u.i.p. due to the convenience yield either as exogenous, or because some assets are in the utility function, or from exogenously given bond demand functions. An exception is Bianchi et al. (2021), which models the endogenous demand for assets from financial intermediaries during times of global stress, with emphasis on the liquidity return of dollar short-term assets. That model, however, is too stylized to take to a realistic quantitative open-economy macro setting.

The “exorbitant privilege” of the United States - that it earns a greater rate of return on its foreign investments than foreigners earn on investments in the U.S. - in conjunction with the persistent U.S. trade balance and current account deficits has been intensively investigated. Gourinchas and Rey (2007b) and Gourinchas and Rey (2007a) perhaps first noted the importance of these excess returns in the global financial adjustment process. Mendoza et al. (2009), Caballero et al. (2008), and Caballero et al. (2016) build models to account for this global pattern of portfolio returns.

The role of global financial intermediation is the focus of much contemporary research into exchange rates and capital flows. Notable contributions include Maggiori (2017), Gabaix and Maggiori (2015), Itskhoki and Mukhin (2021), and Gopinath and Stein (2021).

Some other recent studies have provided potential explanations for how dollar bonds can earn on average a lower return than foreign bonds, and how the dollar can appreciate during global downturns, and yet provide resolutions to the reserve currency paradox. In Maggiori (2017), foreign banks face balance sheet constraints, while U.S. banks are unconstrained. This effectively makes foreigners more risk averse than investors in the U.S. In equilibrium, the U.S. borrows from abroad and invests in equities, while the foreign country buys U.S. debt which acts as insurance during global downturns. However, since the real exchange rate is determined by the relative price of goods produced in the U.S. compared to those in the foreign country due to home bias in preferences, there

---

6 Earlier papers that model the convenience yield as arising from assets in the utility function include Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2016), and, in an international model, Engel (2016). A prominent recent example of a model with an exogenous u.i.p. shock is Itskhoki and Mukhin (2021). Greenwood et al. (2020) and Gourinchas et al. (2022) specify international portfolio choice models in which a “preferred habitat” as well as risk and return play a role in asset demand. Du et al. (2018a) provide measures of the convenience yield on U.S. bonds relative to government bonds of other countries.


8 See He and Krishnamurthy (2013) for a general survey of how financial constraints affect and magnify the effective risk aversion of investors.
has to be a channel that makes this relative price increase. Maggiori assumes that there are costs of exporting the foreign good that rise during bad times. This leads to a switch in demand toward U.S. goods, which generates the rise in their relative price.

Gourinchas and Rey (2022) build a model in which the global economy can be in one of three states at any time: normal, fragile, and disaster. That study associates times of global stress with the fragile state. Investors in both countries can trade in equities and real bonds that pay off in terms of each country’s consumption basket. U.S. bonds are less risky because of their payoffs in the fragile and disaster states, so they earn on average a lower rate of return. During disaster periods, which are rare, there is a large drop in global output, and in addition, there is partial default on foreign bonds. When the economy enters into a fragile state from a normal state, the probability of disaster increases. To hedge the risk that occurs from holding foreign bonds in the disaster state, investors purchase home bonds during fragile times. The trade deficit of the U.S. increases during fragile times, allowing the U.S. to increase demand for its own non-traded goods, leading to a real appreciation.

Jiang et al. (2020) posit that there is an exogenously given demand from abroad for U.S. bonds because they are valued for their liquidity or some other special property. Bonds are issued by firms in the U.S., but these firms face a borrowing limit. Bonds are used to finance productive activity. The model can, for example, account for dynamics of real exchange rates in response to a monetary tightening in the U.S. As the U.S. raises interest rates, the debt issuing capacity of firms falls, reducing their output but also reducing their supply to the rest of the world of the liquid asset. The dollar appreciates both because of the increase in the interest rate and the higher liquidity return on the liquid bonds forced by the contraction in their supply. The study also considers the effects of a flight to dollars, which would also lead to an increase in the convenience yield and appreciation of the dollar. The paper emphasizes that if such a shock is accompanied by a monetary easing in the U.S., that the U.S. trade balance could run into deficit even while the dollar was appreciating. This is driven by the increase in U.S. wealth from the greater seignorage arising from the increase in the liquidity yield on bonds. In another experiment, the study considers a shock to productivity for U.S. firms. This reduces the firms’ borrowing capacity, reduces the supply of debt, and thus raises the convenience yield and appreciates the dollar. Foreign firms which hold some dollar debt have an adverse balance sheet effect, which is contractionary for their output.

Kekre and Lenel (2020) present a model of convenience yields and risk premia in an general equilibrium model of the global economy. The convenience yield arises from an ad hoc model in which agents have some preference for bonds issued by the U.S. The main drivers of global downturns are a disaster shock that is also correlated with a “safety shock” that changes the relative demand for U.S. assets. In addition, there is nominal wage stickiness and monetary policy set by a Taylor rule. The model is quite rich, but some intuition can be gleaned by considering the effect of the safety shock alone, as global demand for U.S. bonds tends to increase during global crises. The shift in demand toward U.S. bonds tends to lower their rate of return, which is expansionary in the U.S. (relative to the foreign country). This leads to higher inflation in the U.S. (again, relative
to the rest of the world), which induces a monetary policy response that raises the real interest rate, and in turn leads to a real appreciation of the dollar. On the goods market side, the drop in U.S. consumption induced by the real interest rate increase raises the real wage (given the nominal wage stickiness), which in turn induces producers to supply less. The drop in supply outstrips the drop in consumption, so the relative price of U.S. goods rises, consistent with the real appreciation given home bias in preferences.

In Akinci et al. (2022), during times of global stress, there is an increase in volatility of the productivity process. In the version of their model that aims to explain the real appreciation of the dollar during times of uncertainty, there is a financial intermediary in the U.S. that takes in saving from U.S. households and invests in U.S. equities and foreign debt. This intermediary faces balance sheet constraints, and, as in our model, the bank faces lower constraints on its holdings of debt than on capital, though there is no special role for U.S. government debt. The foreign financial sector is unconstrained, so we note that the set-up is the precise opposite of that in Maggiori (2017) in which foreign intermediaries are constrained and the U.S. financial sector operates freely. Akinci et al. (2022) consider an increase in the volatility of U.S. productivity, holding foreign volatility unchanged, which leads to an increase in risk aversion in the U.S. relative to the rest of the world. The shock causes U.S. intermediaries to lower demand for foreign bonds, and they shed some of their holdings of foreign debt, contributing to a depreciation of the foreign currency. Since the foreign country must borrow less, its consumption level declines. With home bias in preferences, it appears that the drop in foreign demand for their goods helps deliver the real depreciation of their currency.

Dahlquist et al. (2022) build a model in which agents have “deep habits” - an external habit for each good in the consumption basket. The key properties of these preferences is that they are not homothetic, and expenditure shares may vary as wealth and consumption levels change. In particular, the study assumes that U.S. households are wealthier, and thereby less risk averse than those in other countries. During periods in which income and global consumption takes a turn downward, U.S. consumption falls less than that in other countries, and because of home bias in consumption, the relative demand for goods produce in the U.S. increases. This leads to a U.S. real appreciation during bad times, which also makes U.S. bonds a good hedge against global shocks.

Our model differs from much of the literature in that we examine a global shock that hits the U.S. and the rest of the world equally. Our main focus is on a global financial tightening. The asymmetry in the model arises not from the shock, but from the property that U.S. government bonds are considered to be better collateral on banks’ balance sheets. We emphasize the increase in the liquidity yield on these bonds, and the appreciation of the dollar that occurs because these bonds become more valuable during global downturns. In most respects, our model is a standard open-economy New Keynesian model, so it does not require the introduction of any new features, other than the special role of dollar debt, to account for the puzzles. Moreover, we find that in equilibrium there is retrenchment, which accounts for another possibly puzzling feature of the data.

We find that during times of global stress, the U.S. banks are more incentivized to switch their
portfolios away from capital toward their country’s safe bonds than are foreign banks. In turn, since output is demand-determined in the short run in the model, the downturn in U.S. output is greater that for the rest of the world. In terms of the Maggiori (2017) reserve currency paradox, it might seem trivial that if the U.S. has a greater downturn, then the dollar must appreciate in real terms. In a flexible-price model in which output is exogenous, if U.S. output falls more, the relative price of U.S. goods will rise more, which will lead to a real appreciation if there is consumption home bias. But in our model, the mechanism is entirely different. Output is not exogenous, but determined endogenously, and it is demand-determined. The real appreciation does not require an increase in the relative price of U.S.-produced goods under LCP. And, importantly, we are considering global shocks that hit the U.S. and the foreign country equally. Moreover, we can find under some unrealistic calibrations that the global shock can lead to a dollar appreciation even with output falling less in the U.S., so the drop in relative U.S. output is not intrinsically necessary to deliver the real appreciation.

Section presents the model. The calibrated parameters are described in section 3 and section 4 describes the steady state. Section 5 examines the responses to a global financial tightening, to a global productivity shock, and to a monetary shock originating in the U.S.. Section 6 concludes.

2 Model

We describe a two country model, denoted Home and Foreign, where agents supply labour and consume goods from both countries. The world is populated with a unit mass of agents and Home has share $n$ of these, with Foreign share $1 - n$. We assume that firms set prices in domestic currency (PCP) for home sales and foreign currency (LCP) for exports, and adjust prices constrained by Rotemberg-style price adjustment costs. Agents in the Home country have preferences over consumption and hours given by

$$U = \frac{(C_t - hC_{t-1})^{1-\sigma} - 1}{1 - \sigma} - \frac{\chi}{1 + \psi} H_t^{1+\psi}$$  \hspace{1cm} (1)

Financial markets are restricted for households. Households can interact only with domestic banks in the form of non-contingent home currency denominated bonds. Banks in turn hold domestic and foreign currency denominated bonds as well as domestic equity. The environment for banks is described further below.

2.1 Households

The Home country budget constraint is

$$P_t C_t + B_t = W_t H_t + R_t B_{t-1} + \Pi_t + T R_t - T_{s,t}$$  \hspace{1cm} (2)

where $P_t$ is the CPI in Home currency, $B_t$ represents households deposits of domestic currency in the home banking system, $\Pi_t$ represents the net receipts that households receive from production
firms and banks, and $TR_t$ is a transfer made from the home government to the households. Here, $R_t$ is the domestic home currency interest rate received by households on bank deposits. Finally, $T_{s,t}$ is the startup capital transferred to new banks at time $t$.

It is assumed that

$$C_t = \left( \omega \frac{1}{\lambda} C_{h,t}^{\frac{1}{\lambda}} + (1 - \omega) \frac{1}{\lambda} C_{f,t}^{\frac{1}{\lambda}} \right)^{\frac{1}{1-\lambda}}$$

where $\omega \geq n$, representing the possibility of home bias in preferences.\(^9\)

Optimal consumption of Home and Foreign goods for the Home consumer is

$$C_{h,t} = \omega \left( \frac{P_{h,t}}{P_t} \right)^{-\lambda} C_t$$

$$C_{f,t} = (1 - \omega) \left( \frac{P_{f,t}}{P_t} \right)^{-\lambda} C_t$$

Optimal labour supply is described by

$$W_t = \chi P_t (C_t - h\bar{C}_{t-1})^\sigma H_t^\psi$$

Given $R_{t+1}$ (known at time $t$), the return on deposits, Home household’s Euler equation is

$$1 = E_t R_{t+1} \beta \left( \frac{C_{t+1} - h\bar{C}_t}{C_t - h\bar{C}_{t-1}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \equiv E_t R_{t+1} \Omega_{t+1}$$

The preferences, budget constraints, and optimal choices for the Foreign economy are analogous. The presence of home bias in Foreign preferences then implies that the price index for the Foreign economy is

$$P^*_t = \left( \omega^* P_{f,t}^{1-\lambda} + (1 - \omega^*) P_{h,t}^{1-\lambda} \right)^{\frac{1}{1-\lambda}}$$

### 2.2 Firms

A measure $n$ of firms in the Home economy produce differentiated goods. The aggregate Home good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is denoted as $\varepsilon > 1$. The production function for firm $i$ in the Home country is

$$Y_{i,t} = A_t (L_{i,t}^{1-\alpha} K_{i,t}^{\alpha})$$

where $A_t$ is an aggregate productivity term, $K_{i,t}$ is the firm’s use of capital, and $L_{i,t}$ the use of labor.

We assume that the firm in each country sets two prices, one for sales in the domestic market in the domestic currency, and one for sales in the export market in the local currency of the importer. Thus, both countries engage in ‘local currency pricing’ (LCP).

\(^9\)Letting $0 \leq x \leq 1$ represent the degree of home bias in preferences, where $x = 0$ ($x = 1$) represents zero (full) home bias, we can define $\omega = n + x(1 - n)$. 

11
The profits of the Home firm $i$ are then represented as

$$\Pi_{i,t} = \left(1 + s_{i,t}\right)\left(\Pi_{i,t}^{1/\alpha} - MC_i^{\alpha}(Y_{i,t} - Y_{i,t}^{*})\right)$$

(10)

where $P_{i,h,t}$ is the price set in domestic currency for Home sales, and $P_{i,h,t}^{*}$ is the Foreign currency price, with $S_t$ being the exchange rate (Home price of Foreign Currency). Also, $MC_i$ denotes the firm’s marginal cost, and $s_{i,t}$ represents a subsidy that may be given to the firm to offset the monopoly distortion in pricing. Cost minimization by the firm implies:

$$A_t(1 - \alpha)(1 - \alpha)K_{i,t}^{\alpha}MC_i = W_t L_{i,t}$$

(11)

$$A_t\alpha(L_{i,t}^{1/\alpha} - \alpha K_{i,t}^{\alpha})MC_i = R_{K,t}K_{i,t}$$

(12)

The firm chooses its Home and Foreign price to maximize the present value of expected profits, net of price adjustment costs

$$E_t \sum_{j=0}^{\infty} \Omega_t \left( \Pi_{i,t} - \xi_t \left( \frac{P_{i,h,t}}{P_{i,h,t-1}} \right) P_{h,t} Y_{h,t} - \xi_t \left( \frac{P_{i,h,t}}{P_{i,h,t-1}} \right) S_t P_{h,t} Y_{h,t}^{*} \right)$$

(13)

where $\Omega_t$ is the firm’s nominal stochastic discount factor, and $\xi_t(\cdot)$ represents a price adjustment cost function for the firm. We assume that $\xi'(\cdot) > 0$, and $\xi''(\cdot) > 0$. Price adjustment costs are proportional to the nominal value of Home sales to each of the Home and Foreign markets, to be consistent with the nominal profit objective function of the firm.

The first order conditions for profit maximization for the Home firm $i$ can be described as

$$\left(1 + s_{i,t}\right)Y_{i,t,k} = \epsilon\left(P_{i,h,t}(1 + s_{i,t}) - MC_i Y_{i,t,k}\right) + \xi_t \left( \frac{P_{i,h,t}}{P_{i,h,t-1}} \right) \frac{1}{P_{i,h,t}} P_{h,t} Y_{h,t}$$

$$-E_t \Omega_t + 1 \xi_t \left( \frac{P_{i,h,t+1}}{P_{i,h,t}} \right) \frac{1}{P_{i,h,t+1}} P_{h,t+1} Y_{h,t+1}$$

(14)

$$\left(1 + s_{i,t}\right)S_t Y_{i,t,k}^{*} = \epsilon\left(S_t P_{i,h,t}^{*}(1 + s_{i,t}) - MC_i Y_{i,t,k}^{*}\right) + \xi_t \left( \frac{P_{i,h,t}}{P_{i,h,t-1}} \right) \frac{1}{P_{i,h,t-1}} S_t P_{h,t}^{*} Y_{h,t}^{*}$$

$$-E_t \Omega_t + 1 \xi_t \left( \frac{P_{i,h,t+1}}{P_{i,h,t}} \right) \frac{1}{P_{i,h,t+1}} S_t P_{h,t+1}^{*} Y_{h,t+1}^{*}$$

(15)

### 2.3 Banks

Banks are modeled as in Gertler and Karadi. A fraction of household members $\theta$ become bankers in any period, continue as bankers with probability $\theta$, and revert to being consumers with probability $1 - \theta$. When starting up, a bank receives some start up capital from households to establish its net worth, and borrow from households at fixed rates to invest in claims to capital, home and foreign currency denominated government bonds. Besides government, only banks operate in international

---

10In the calibration, $\xi_t = \varphi^2 \left( \frac{P_{i,h,t}}{P_{i,h,t-1}} - 1 \right)^2$. 

---

12
financial markets. A Home banker $i$’s balance sheet in period $t$ is

$$B_{i,t} + N_{i,t} = Q_i K_{h,i,t+1} + S_i Q_t^* K_{f,i,t+1} + D_{h,i,t} + S_i D_{f,i,t}$$  \hspace{1cm} (16)$$

where $B_{i,t}$ represents domestic currency deposits from households, $N_{i,t}$ is beginning net worth, $Q_i$ and $Q_t^*$ are the price of a unit of domestic and foreign capital, $K_{h,i,t+1}$ and $K_{f,i,t+1}$ are the holding of domestic and foreign capital, $D_{h,i,t}$ is the banks purchase of home denominated government bonds, $D_{f,i,t}$ is their purchase of foreign currency denominated government bonds. This says that bankers use their net worth and new debt to invest in the home and foreign capital and home and foreign bonds. We calibrate so that banks will hold a positive position in all assets.

Banker $i$ chooses $K_{h,i,t+1}$, $K_{f,i,t+1}$, $D_{h,i,t}$, and $D_{f,i,t}$ to maximize her value evaluated using the SDF of home households, $\Omega_{t+1} = \beta \frac{(C_{t+1} - hB_t)}{(C_{t} - hC_{t-1})} - \sigma \frac{P}{S}$. Following Gertler and Karadi, conjecture that the value function of the bank is a time varying linear function of her net worth, so that $V_{i,t} = \nu_t N_{i,t}$

The banker’s value function then satisfies:

$$V_{i,t} = E_t \Omega_{t+1} \left((1 - \theta)N_{i,t+1} + \theta V_{i,t+1}\right)$$  \hspace{1cm} (17)$$

This captures the fact that the banker will revert to being a consumer with probability $1 - \theta$ and consume it’s net worth, and continue to be a banker with probability $\theta$. The net worth dynamics must satisfy

$$N_{i,t+1} = \bar{R}_{K,t+1} Q_i K_{h,i,t+1} + \bar{R}_{Q,t+1}^* Q_t^* S_i K_{f,i,t+1} + R_{h,t+1} D_{h,i,t} + S_i R_{f,t+1} D_{f,i,t} - R_{t+1} B_{i,t}$$

$$= (\bar{R}_{K,t+1} - R_{t+1}) Q_i K_{h,i,t+1} + \frac{S_i}{S_t} R_{h,t+1} K_{h,t+1} + (R_{h,t+1} - R_{t+1}) D_{h,i,t} + \frac{S_i}{S_t} R_{f,t+1} K_{f,t+1} + (R_{f,t+1} - R_{t+1}) D_{f,i,t} + R_{t+1} N_{i,t}$$  \hspace{1cm} (18)$$

Here, $\bar{R}_{K,t+1} = \frac{R_{K,t+1} + (1 - \delta) Q_{t+1}}{Q_t}$ and $\bar{R}_{Q,t+1}^* = \frac{R_{Q,t+1}^* + (1 - \delta) Q_t^*}{Q_t^*}$ are the net return on the home capital where $\delta$ is the depreciation rate on capital, $R_{h,t+1}$ is the return on the domestic currency government bond and $R_{f,t+1}$ is the analogous return on the foreign government bond.

Banks maximize this subject to (16) and subject to the participation constraint:

$$V_{i,t} \geq \vartheta_t \left( (\kappa_{h1,t} + \kappa_{h2,t} D_{h,i,t}) D_{h,i,t} + (\kappa_{f1,t} + \kappa_{f2,t} S_i D_{f,i,t}) S_i D_{f,i,t} \right)$$

$$+ \vartheta_t \left( (\kappa_{Kh1,t} + \kappa_{Kh2,t} Q_i \bar{R}_{K,t+1}) Q_i K_{h,i,t+1} + (\kappa_{Kf1,t} + \kappa_{Kf2,t} S_i Q_t^* \bar{R}_{f,t+1}) S_i Q_t^* K_{f,i,t+1} \right)$$  \hspace{1cm} (19)$$

We introduce a set of asset-specific constraint parameters to allow for differential pledgeability as a collateral across assets. Specifically, we have $\kappa_{Kh1,t}$, $\kappa_{Kf1,t}$, $\kappa_{h1,t}$ and $\kappa_{f1,t}$ as the constraint parameters for home capital, foreign capital, home bonds and foreign bonds respectively.\textsuperscript{11} As we discuss at length below, we posit that holdings of capital are more constrained than government bonds, and in turn, foreign government bonds holdings are more constrained than home bonds.

\textsuperscript{11}We also introduce $\kappa_{Kh2,t}$, $\kappa_{Kf2,t}$, $\kappa_{h2,t}$ and $\kappa_{f2,t}$ so the constraint depends on the aggregate bank holding of the assets (denoted with: $D_{h,i,t}, D_{f,i,t}, K_{h,i,t+1}$ and $K_{f,i,t+1}$). The idea is that the monitoring cost is increasing with the asset size. We set these parameter values very small (0.025) and the main purpose of these parameters is to introduce stationarity in the linearized model.
implies that the price of capital is a function of the capital stock. Banks sell the new capital to banks at price \( \tilde{f}_{i,t+1} \).

Capital goods producers buy the unused capital from banks, and engage in new investment, and sell the new capital to banks at price \( Q_t \).

### 2.4 Capital Goods Producers

Capital goods producers buy the unused capital from banks, and engage in new investment, and sell the new capital to banks at price \( Q_t \). The representative capital goods producer has the profit function

\[
Q_t I_t - P_t (I_t + K_t \psi(\frac{I_t}{K_t}))
\]

where \( \psi(.) \) is an adjustment cost function, satisfying \( \psi'(.) > 0 \) and \( \psi''(.) > 0 \), with \( \psi(\delta) = 0 \). This implies that the price of capital is

\[
Q_t = P_t (1 + \psi'(\frac{I_t}{K_t}))
\]
2.5 Monetary policy

We assume a Taylor rule where the Central Bank targets the CPI inflation rate and use the government interest rate as an instrument:

\[ R_{h,t+1} = \frac{1}{R_{h,ss}} \left( \frac{P_t}{P_{t-1}} \right) \eta^\pi (R_{h,t})^{\rho_R} \]  

(28)

2.6 Fiscal policy

The Home and Foreign governments make transfers to the households and subsidize firms by issuing government debt. For the Home country we have

\[ \tilde{D}_{h,t} = R_{h,t} \tilde{D}_{h,t-1} + s_t (P_{h,t}Y_{h,t} + S_t P^*_h Y^*_h) + TR_t \]  

(29)

where \( \tilde{D}_{h,t} \) is the total outstanding debt of the Home government and is assumed to be exogenous.

2.7 Balance of Payments

The profit of the Home production firms is

\[ \Pi^p_t = \left( 1 + s_t \right) (P_{h,t}Y_{h,t} + S_t P^*_h Y^*_h) - MC_{h,t} (Y_{h,t} + Y^*_h) - \xi_t \left( \frac{P_{h,t}}{P_{h,t-1}} \right) P_{h,t} Y_{h,t} - \xi_t \left( \frac{P^*_h}{P^*_h} \right) S_t P^*_h Y^*_h \]  

(30)

Given constant returns to scale, we can write \( MC_{h,t} (Y_{h,t} + Y^*_h) \) as

\[ W_t L_{h,t} + r_{K,t} K_t \]

In equilibrium, labour supply must equal labour demand, so that

\[ H_t = L_t \]

The profit of Home capital producing firms is:

\[ \Pi^K_t = Q_t I_t - P_t (I_t + K_t \psi \left( \frac{I_t}{K_t} \right)) \]  

(31)

In addition, the capital stock accumulation equation must satisfy

\[ K_{t+1} = I_t + (1 - \delta) K_t \]

where \( \delta \) is the depreciation rate on capital.

Total profits from the corporate non-financial sector are then

\[ \Pi^p_t + \Pi^K_t \]
In addition, the return on deposits to Home households may be expressed as

\[ R_t B_{t-1} = r_{K,t} K_{h,t} + (1 - \delta)Q_t K_{h,t} + r_{K,t}^* K_{f,t} + (1 - \delta)S_t Q_t^* K_{f,t} + R_{h,t} D_{h,t-1} + S_t R_{f,t} D_{f,t-1} - N_t^e \]

(32)

where \( N_t^e \) represents the net worth of existing banks. The startup capital transferred from households to banks is

\[ T_{s,t} = \varphi (Q_t K_{t+1} + D_{h,t} + S_t D_{f,t}) \]

(33)

So total net worth of the banking sector at time \( t \) is \( N_t = N_t^e + T_{s,t} \)

Finally, government transfers are:

\[ T R_t = \bar{D}_{h,t} - R_{h,t} \bar{D}_{h,t-1} - s_t (P_{h,t} Y_{h,t} + S_t P_t^* Y^*_{h,t}) \]

(34)

Note that the net deposits from households to financiers can be defined as

\[ B_t = Q_t K_{h,t+1} + S_t Q_t^* K_{f,t+1} + D_{h,t} + S_t D_{f,t} - N_t \]

Now putting together (30), (31), (32), (33), (34), into the home budget constraint (2), we get the balance of payments condition:

\[
P_t (C_t + X_t + I_t + K_t \phi (\frac{K_t}{K_t^*})) + D_{h,t} - \bar{D}_{h,t} + S_t D_{f,t} + Q_t (K_{h,t} - K_t) + S_t Q_t^* K_{f,t} = P_{h,t} Y_{h,t} - \bar{P}_{h,t} Y_{h,t} + S_t P_t^* Y^*_{h,t} - \bar{P}_{h,t} P_t^* Y^*_{h,t} + R_{h,t} (D_{h,t-1} - \bar{D}_{h,t-1}) + S_t R_{f,t} D_{f,t-1} + \bar{R}_{k,t+1} (K_{h,t-1} - K_{t-1}) + S_t \bar{R}_{k,t+1} K_{f,t-1}
\]

(35)

where \( \bar{D}_{h,t} \) is the total outstanding debt of the Home government.

### 2.8 Adjusted UIP condition

By combining equation 22 and 23 we get:

\[
E_t N_{t+1} (\frac{S_{t+1}}{S_t} R_{f,t+1} - R_{h,t+1}) = \eta_t \vartheta (\kappa_f - \kappa_h)
\]

(36)

Log linearizing the equation, we have an UIP condition adjusted for the balance sheet friction:

\[ E_t s_{t+1} - s_t = (r_{h,t+1} - r_{f,t+1}) - \bar{\eta}_t \]

(37)

Forward iterating the equation gives:

\[
s_t = -E_t \sum_{j=1}^{\infty} (r_{h,t+j} - r_{f,t+j} - (r_h - r_f)) - E_t \sum_{j=0}^{\infty} (\bar{\eta}_{t+j} - (\bar{\eta})) + \lim_{k \to \infty} (E_t s_{t+k} - k(\bar{s} + 1 - s))
\]

(38)
3 Calibration

The model frequency is annual. We calibrate the model to match U.S. moments. We think of Home as the U.S. and Foreign as the rest of the World. To highlight the important role of the asymmetric collateral value, we set all other parameters the same across the Home and the Foreign. The parameter values are summarized in Table 1.

The parameters can be partitioned into two blocs. The first bloc is externally set and mostly set to values that are in line with standard literature values. The second bloc is calibrated to match some long-term averages in the data.

On the household side, we set the discount factor ($\beta$) to be 0.97 and the CRRA coefficient ($\sigma$) to be 5. The habit parameter ($h$) is set at 0.8, slightly above estimates from Justiniano et al. (2010), Smets and Wouters (2007) and slightly below that in Gertler and Karadi (2015). Home bias ($\omega, \omega^*$), the cross-country elasticity of substitution of goods ($\lambda$) and the within-country elasticity of substitution of goods ($\epsilon$) are set at 0.8, 0.8 and 6 respectively. The inverse of the Frisch elasticity ($\psi$) is set at 1 and we calibrate the disutility of labor $\chi$ to match a steady-state $H^*$ of 0.33.

On the production side, the country mass ($n$) is set at 0.5 to preserve symmetry. The Rotemberg price adjustment cost ($\phi_\pi$) is set at 72.83, matching an annual probability of price change of 0.78 in a Calvo-type model of sticky prices. The capital adjustment cost ($\phi_k$) is set at 4. The capital share ($\alpha$) is set at 0.35, slightly higher than the standard of 0.33 to have a more realistic consumption to investment ratio. The capital depreciation rate ($\delta$) is set at 0.1.

On the government side, the Taylor coefficient ($\eta_\pi$) is set at 1 with a smoothing parameter ($\rho^R$) equal to 0.4 (taken from Justiniano et al. (2010)). Government debt is fixed ($\bar{D}_h$ and $\bar{D}_f$) at a constant value of 0.4, resulting in a steady state home debt to GDP ratio ($\bar{D}_h/GDP$) of 77%, matching the long-run average of the U.S.. The monopoly subsidy is set at 0.2 ($s = \frac{1}{1+\epsilon}$) to eliminate the steady state mark-up distortion. For simplicity, gross steady state inflation ($\bar{\pi}$) is set to zero.

On the banking side, we calibrate the bond constraint parameters to match the U.S. external positions. This is the main source of asymmetry between the two countries. We calibrate the four bond constraint parameters ($\kappa_{h1}, \kappa_{f1}, \kappa_{h1}^*, \kappa_{f1}^*$) to match a steady state convenience yield of 1% (home minus foreign interest rate differential of -1%), a steady state positive income account of $+0.0013 \ (\text{current account - trade balance)/GDP of US from 1990-2019}$), a steady state negative NFA position and a steady state foreign holding of U.S. government bond ($D_{h}^*/\bar{D}_h$) of 27% (average of 1990-2019). While these are calibrated jointly, lowering the home bond constraint parameter ($\kappa_{h1}$ and $\kappa_{h1}^*$) relative to the foreign bond constraint parameter ($\kappa_{f1}$ and $\kappa_{f1}^*$) generates a positive convenience yield. Since the net foreign bond position is defined as $-D_{h,t}^* + S_tD_{f,t}$, a relatively low value of $\kappa_{h1}^*$ to $\kappa_{f1}$ is useful to generate a negative NFA position. A higher $\kappa_{h1}^*$ (relative to $\kappa_{h1}$) helps to match the 27% share of foreign holdings of U.S. Treasury obligations. Finally, the relative values of $\kappa_{h1}$ and $\kappa_{f1}$ are useful for pinning down the positive income account. On the capital side, we maintain an agnostic view about the relative collateral values in generating the asymmetry of external positions. Instead, we calibrate the model such that $\kappa_{K,h} = \kappa_{K,f}^* < \kappa_{K,f} = \kappa_{K,h}^*$. That is, the Home capital serves as a better collateral than Foreign capital for Home banks and the opposite.
is true for Foreign banks. We set $\kappa_{Kf} = \kappa_{Kh} = 0.6$ and $\kappa_{Kf} = \kappa_{Kh} = 0.48$ to match a net equity return of 5.2% and equity home share of 85% (average of 1990-2016 from Hnatkovska (2019)), which are in the range of estimates from the equity premium puzzle and home equity bias puzzle literatures (Victoria). We normalize the country specific collateral constraint parameters ($\vartheta$, $\vartheta^*$) to be 1. Finally, we set the bank survival probability ($\theta$) to be 0.878 and start-up capital injection rate ($\varphi$) of 0.001 to match a steady state leverage of around 6.

We solve the model assuming six shocks; Home and Foreign productivity shocks $A_t, A_t^*$, Home and Foreign balance sheet constraint values, $\vartheta_t, \vartheta_t^*$, and Home and Foreign monetary shocks. In the current draft, we focus on one-time shocks beginning in the deterministic steady state, and we assume all the shocks follow an AR1 process with a persistence of 0.7.
### Table 1: Parameter values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Meaning / description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Households</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 $\beta$</td>
<td>0.97</td>
<td>discount factor</td>
</tr>
<tr>
<td>2 $\sigma$</td>
<td>5</td>
<td>CRRA risk coefficient</td>
</tr>
<tr>
<td>3 $h$</td>
<td>0.8</td>
<td>habit persistence</td>
</tr>
<tr>
<td>4 $\omega = \omega^*$</td>
<td>0.8</td>
<td>home bias</td>
</tr>
<tr>
<td>5 $\chi$</td>
<td>0.33</td>
<td>labor disutility</td>
</tr>
<tr>
<td>6 $\psi$</td>
<td>1</td>
<td>inverse of Frisch elasticity</td>
</tr>
<tr>
<td>7 $\lambda$</td>
<td>0.8</td>
<td>home and foreign goods elasticity of substitution</td>
</tr>
<tr>
<td>8 $\epsilon$</td>
<td>6.0</td>
<td>within country elasticity of substitution</td>
</tr>
<tr>
<td>Firms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 $n$</td>
<td>0.5</td>
<td>home country mass</td>
</tr>
<tr>
<td>10 $\alpha$</td>
<td>0.35</td>
<td>capital share</td>
</tr>
<tr>
<td>11 $\phi_\pi$</td>
<td>72.83</td>
<td>price adjustment cost</td>
</tr>
<tr>
<td>12 $\phi_k$</td>
<td>4</td>
<td>capital adjustment cost</td>
</tr>
<tr>
<td>13 $\delta$</td>
<td>0.1</td>
<td>depreciation rate</td>
</tr>
<tr>
<td>Governments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 $\bar{\pi}$</td>
<td>1</td>
<td>steady state inflation</td>
</tr>
<tr>
<td>14 $s = s^*$</td>
<td>0.2</td>
<td>monopoly subsidy ($\frac{1}{\epsilon - 1}$)</td>
</tr>
<tr>
<td>15 $D_h = D_f$</td>
<td>0.4</td>
<td>total government debt</td>
</tr>
<tr>
<td>16 $\eta^\pi$</td>
<td>1</td>
<td>Taylor rule coefficient</td>
</tr>
<tr>
<td>17 $\rho^R$</td>
<td>0.4</td>
<td>interest rate smoothing parameter</td>
</tr>
<tr>
<td>Banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18 $\vartheta = \vartheta^*$</td>
<td>1</td>
<td>country-specific constraint value</td>
</tr>
<tr>
<td>19 $\kappa_Kh$</td>
<td>0.48</td>
<td>home bank constraint value on home capital</td>
</tr>
<tr>
<td>20 $\kappa_Kf$</td>
<td>0.65</td>
<td>home bank constraint value on foreign capital</td>
</tr>
<tr>
<td>21 $\kappa_{K^*h}$</td>
<td>0.65</td>
<td>foreign bank constraint value on home capital</td>
</tr>
<tr>
<td>22 $\kappa_{K^*f}$</td>
<td>0.48</td>
<td>foreign bank constraint value on foreign capital</td>
</tr>
<tr>
<td>23 $\kappa_{h1}$</td>
<td>0.075</td>
<td>home bank constraint value on home bond</td>
</tr>
<tr>
<td>24 $\kappa_{f1}$</td>
<td>0.4141</td>
<td>home bank constraint value on foreign bond</td>
</tr>
<tr>
<td>25 $\kappa_{h1}^*$</td>
<td>0.0848</td>
<td>foreign bank constraint value on home bond</td>
</tr>
<tr>
<td>26 $\kappa_{f1}^*$</td>
<td>0.41</td>
<td>foreign bank constraint value on foreign bond</td>
</tr>
<tr>
<td>27 $\theta$</td>
<td>0.878</td>
<td>bank survival prob.</td>
</tr>
<tr>
<td>28 $\varphi$</td>
<td>0.001</td>
<td>bank starting networth</td>
</tr>
</tbody>
</table>

### 4 Steady state values

The model is solved by linearizing around the non-stochastic steady state. The table below present some steady state values using the parameterization listed above.\(^{12}\)

The only asymmetry of the model is the bond constraint parameters. As we noted above, we set the home bond to be better collateral for both Home and Foreign banks ($\kappa_{h1} , \kappa_{h1}^* < \kappa_{f1} , \kappa_{f1}^*$). With only this asymmetry, we are able to generate three important features of the US external position. First, the US has a government rate that is lower than the Foreign government rate by 1%, This can

\(^{12}\)A more comprehensive list of the steady state values is provided in the Appendix.
be understood by looking at steady state version of equation 37. Since \( E_t s_{t+1} - s_t = 0 \) at the steady state, the excess monetary return of foreign bond reflects the additional non-pecuniary balance sheet cost of holding it. This demonstrates a convenience yield due to the better pledgeability and acts like a “seigniorage” effect.

Second, the US has a negative NFA position, meaning that it has a net liability to the rest of the world. Third, despite the net liability, the US has a steady state trade balance deficit and positive income account. This is because while it owes the rest of the world repayment, it pays a lower interest rate on its liability to the rest of the world than the rest of the world pays to the US. Overall, this characterizes an exorbitant privilege that follows from the convenience yield.

The asymmetry in pledgeability or acceptability as collateral has some interesting macroeconomic implications. The U.S. has a higher steady-state consumption than the rest of the world (0.3985 vs 0.3980). While the steady state capital is the same, the U.S. has lower steady state employment and a higher steady state wage than the rest of the world. Taken together, this implies that while the U.S. produces less than the rest of the world, it lives with a higher consumption and wage levels purely due to the “seigniorage” from the exorbitant privilege. On the financial side, we observe that the U.S. banks have a higher leverage than the rest of the world (5.92 vs 5.50) but less tight constraint, reflected by a lower \( \eta \) in the steady state (0.079 vs 0.080).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Steady state value</th>
<th>Symbol</th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>0.3985</td>
<td>( Y )</td>
<td>0.516</td>
</tr>
<tr>
<td>( C^* )</td>
<td>0.3980</td>
<td>( Y^* )</td>
<td>0.518</td>
</tr>
<tr>
<td>( K )</td>
<td>1.1895</td>
<td>( d_h )</td>
<td>0.292</td>
</tr>
<tr>
<td>( K^* )</td>
<td>1.1917</td>
<td>( d_f )</td>
<td>0.127</td>
</tr>
<tr>
<td>( L )</td>
<td>0.3295</td>
<td>( \eta )</td>
<td>0.079</td>
</tr>
<tr>
<td>( L^* )</td>
<td>0.3310</td>
<td>( \eta^* )</td>
<td>0.080</td>
</tr>
<tr>
<td>( W )</td>
<td>1.012</td>
<td>( r_h )</td>
<td>1.034</td>
</tr>
<tr>
<td>( W^* )</td>
<td>1.017</td>
<td>( r_f )</td>
<td>1.044</td>
</tr>
<tr>
<td>( RER )</td>
<td>0.9975</td>
<td>( r = r^* )</td>
<td>1.031</td>
</tr>
</tbody>
</table>
5 Impulse responses

5.1 Global Financial Shock

In this section, we look at the impulse responses from the simulated model to understand the model mechanism and also how it could explain the dynamics around the global financial crisis. As a baseline, in Figure 2 we look at a case where there is a uniform increase of one percent in $\vartheta$ and $\vartheta^*$, the tightness parameter on the capital market constraint (see equation 19). This represents a negative shock to the banking system in each country, requiring a higher bank value for all types of collateral and forcing the banks to de-lever as well as adjust their investment across asset classes.

This baseline case (in Figure 2) represents a global tightening of the financial constraint faced by banks, which may be precipitated by a sudden loss in confidence in the value of banks’ collateral. Bernanke (2018) and Gorton and Metrick (2012), for example, make the case that in the early stages of the Global Financial Crisis, financial intermediaries began to mistrust the value of collateral put up by other financial intermediaries, firms, and households as the housing bubble in the U.S. began to burst. As the downturn began, credit conditions tightened even more.

The shock to the financial constraint leads to an immediate appreciation of the US real exchange rate, followed by an expected depreciation. The top right panel of Figure 2 shows that this is associated with a rise in convenience yield (defined as negative of $\tilde{r}_{ht} - \tilde{r}_{ft} - (rer_{t+1} - rer_t)$, the blue line in the top right panel), where $\tilde{r}_{ht}$ represents the home CPI real interest rate, and analogously for the foreign country. Thus, there is an increase in the deviation from UIP; the excess expected return on foreign relative to US government bonds rises in response to the global financial shock. To understand the mechanics of this response, take the difference between equation (22) and equation (23) for the home bank, which gives the condition

$$E_t \Lambda_{i,t+1} \left( R_{h,t+1} - R_{f,t+1} \frac{S_{t+1}}{S_i} \right) = \eta_{i,t} \vartheta_i (\kappa_{h1,t} - \kappa_{f1,t})$$  \hspace{1cm} (39)

The term inside the parenthesis on the right hand side is negative, given that the home bond is better collateral than the foreign bond. Then, abstracting from second-order terms, the expected return on the home bond is less than that of the foreign bond, so that on average, UIP will not hold, as in the data. But then it follows that given a negative financial shock (a rise in $\vartheta_t$), the impact on the left-hand side of this equation must be negative, so that the the expected return on the home bond must fall relative to that on the foreign bond - hence the convenience yield on US government bonds rises further given a global negative financial shock. While this explanation uses the first-order conditions for the home bank alone, a similar explanation holds for the foreign bank. We see therefore that a direct implication of the asymmetric collateral value of US relative to foreign government debt is that a uniform tightening of global financial conditions leads to an increase in the convenience yield on US bonds. In fact, given the monetary rule, the interest rates on home and foreign bonds do not change very much, as can be seen in Figure 2 top right panel. Instead,

\textsuperscript{13}In this description, we abstract from the quadratic terms in the collateral constraint and the first order conditions 22 and 23, since in practice these terms are very small, and do not affect the argument.
the increase in the convenience yield on US bonds is achieved by an immediate appreciation and therefore an expected depreciation of the home real exchange rate.

We can examine Figure 2 in more detail to track the macroeconomic responses to the negative financial shock. The response of $\eta$ and $\eta^*$ represent the endogenous increase in Lagrange multipliers on the bank’s collateral constraints. While this increases for both countries, it increases more for the home bank than the foreign bank. This is because the home bank disinvests in physical capital more than the foreign bank, given the advantage of home bonds over foreign bonds, and the fact that the relative balance sheet cost of home bonds to capital for the home bank is less than the equivalent relative cost for the foreign bank. Again, we can see the intuition behind this response by focusing on equations (20) and (22) above, and the equivalent conditions for the foreign bank. This gives us the conditions:

$$E_t \Lambda_{i,t+1} \left( \tilde{R}_{k,t+1} - R_{ht+1} \right) = \eta_{i,t} \vartheta_t (\kappa_{Kh1,t} - \kappa_{h1,t})$$  \(40\)

$$E_t \Lambda_{i,t+1}^* \left( \tilde{R}_{k,t+1}^* - R_{ft+1} \right) = \eta_{i,t} \vartheta_t (\kappa_{Kh1,t}^* - \kappa_{f1,t}^*)$$  \(41\)

Equation (40) describes the home bank’s trade off between home government bonds and home capital (or equity), while equation (41) describes the analogous trade-off for the foreign bank between foreign government bonds and foreign capital. The negative financial shock reduces investment in both countries, but investment falls by more in the home country. The term in parentheses on the right hand side of (40) is larger than the equivalent term in (41) precisely because home government bonds represent better collateral, relative to home capital, than foreign government bonds relative to foreign capital. Hence the expected excess return on home capital relative to home bonds must rise more than that on foreign capital relative to foreign bonds, and the end result is that home investment must fall relative to foreign investment, translating into a greater fall in home output relative to foreign output.14

Since home investment and consumption fall relative to the foreign country, Figure 2 (row 2 column 4, and row 4 column 3, respectively) shows that the home trade balance improves after the financial shock. We can interpret this as the US effectively making a transfer to the rest of the world as part of the ’exorbitant duty’ during a financial crisis. In addition, the real exchange rate appreciation and the fall in the value of home capital translates into a greater fall in the net worth of home banks relative to foreign banks, which can be interpreted as another way in which the US acts as an implicit ’insurer’ to the rest of the world in times of global crisis, given that banks are owned by domestic residents. The asymmetric response of home and foreign banks also has a striking implication for gross asset holding positions. Given that the home bank increases its demand for home bonds as it substitutes away from capital, there is an external portfolio retrenchment - home banks increase their holding of home bonds and decrease their holdings of foreign bonds, and

14While this argument seems incomplete because both home and foreign banks invest in bonds and capital in each others country, the fact that in both equity capital and bond holdings banks have relative larger positions in their home market ensures that the financial shock hits home investment more than it does foreign investment.
the opposite applies to the foreign banks. As noted in the introduction, this retrenchment is a common feature of financial crises and was clearly a feature of the Global Financial Crisis. The detailed aspects of this retrenchment are discussed further below.

Note also that if we measure the relative convenience yield as in Engel and Wu (2018), the difference between the deposit rate in the U.S. and the government bond interest rate, relative to the same interest rate differential in the foreign country, that also increases at the time of the financial shock (row 1, column 4.) This is in agreement with the empirical regularities plotted in 1.

Another striking feature of the response of the model is that the real exchange rate appreciation is almost wholly driven by deviations from the law of one price. As shown in bottom right panel of Figure 2, there is almost no response of the terms of trade to the financial shock. This offers a potential answer to the Maggiori (2017) ‘reserve currency paradox’, which questions how the US dollar country can appreciate during times when US relative wealth falls, and US consumers are biased towards home produced goods (so the terms of trade should deteriorate). In our model, this does not present a puzzle, since the movement in the real exchange rate can be entirely divorced from the terms of trade, when prices are sticky firms set prices according to LCP.

To provide a clean illustration, we also look at the response to the same financial shocks, except in the case when there is no capital trade across countries in figure 3. The qualitative and quantitative results of the financial shock are very similar to those in figure 2, although the expected depreciation of the real exchange rate (and the consequent rise in the U.S. convenience yield) is somewhat less in this case.

Finally, figure 4 illustrates the case where $\kappa_{h1} = \kappa^*_{f1}$ and $\kappa_{f1} = \kappa^*_{f1}$. In this case, while the US government bond still has a collateral advantage, the collateral constraints are identical for both home and foreign banks, so there is no home bias in bond holdings among banks. This figure helps us to understand the forces driving retrenchment. In this case, because the banks in each country face identical constraints on U.S. and foreign bonds, in essence at steady state their portfolios of U.S. bonds and foreign bonds are nearly identical. The tightening of the credit constraint raises demand for U.S. bonds relative to foreign bonds because U.S. bonds are preferred collateral. If all investors were identical and evaluated assets in the same currency, given that the supplies of bonds from both countries are fixed in the short run, the increase in demand would lower the expected return on U.S. bonds relative to foreign bonds until investors were satisfied to hold the existing stock of both bonds. However, in reality, the appreciation of the dollar raises the foreign currency value of U.S. bonds, and by itself increases the share of those bonds in the foreign bank’s portfolio. Given that the foreign bank has increased its share of U.S. bonds in this way, the U.S. bank can, in equilibrium, increase the share of U.S. bonds in its portfolio by buying some from the foreign bank. The fact that the U.S. bank has a stronger incentive to switch its portfolio out of capital and into home bonds further reinforces the retrenchment. The relative reduction in demand for foreign bonds works the opposite way. The depreciation of the foreign currency automatically reduces the share of foreign bonds in the U.S. bank’s portfolio, so the foreign bank balances the market by

15There is a slight difference arising from the fact that U.S. wealth is somewhat higher in steady state because of the seignorage it earns from its exorbitant privilege.
The contribution here is two-fold. First, we see an explanation for the appreciation of the dollar during times of financial contraction. If there were only risk motives for holding bonds, the reserve currency paradox arises. If the dollar is a good asset to hold because it appreciates during times of global stress, what factors lead to its appreciation? In the first place, it cannot be that investors buy more insurance at that point (that is, buy more U.S. bonds) because once the downturn happens, it is too late to buy insurance. The paradox is that the U.S. must make its insurance payment to foreigners during these global financial recessions, but that implies a reduction in U.S. wealth. In a standard model in which the real exchange rate is linked to the terms of trade, and there is home bias in preferences, this relative reduction in U.S. wealth ought to lead to a reduction in demand for U.S. goods, and therefore a real depreciation. In our model, these difficulties are resolved. The demand for dollar assets increases in bad times because of the increase in demand for the asset that provides better collateral when financial constraints are tightened. That indeed may mean the U.S. is making an insurance payment to foreigners, though our linear approximation has simplified away the risk motives for holding assets (which is not to say they are unimportant.) But with local-currency pricing, the link between the terms of trade and the real exchange rate is broken, and so even if U.S. relative wealth falls, a real appreciation is still possible. Of course, the exchange rate must satisfy both goods and asset market equilibrium, but in a sense, because prices are sticky, in the short run it is the asset markets that play the dominant role in exchange rate determination, and goods markets adjust as in Keynesian models through demand-determined changes in output.

The second contribution is a better understanding of asset flows. The literature has puzzled over how to reconcile the empirical finding of global retrenchment during times of financial stress with the observation that the dollar appreciates. Retrenchment seems to imply that while the U.S. is shifting its demand away from foreign assets, the rest of the world is shifting its demand away from dollar assets. But as we have seen, that understanding of retrenchment mixes the equilibrium change in asset holdings with the change in demand. We see that foreign banks do increase their demand for U.S. assets, as do U.S. financial intermediaries, and that contributes to the appreciation of the dollar. But it is the change in the exchange rate itself as well as the influence of changes in demand for other assets that leads to the equilibrium portfolio adjustment - with foreign banks reducing their holdings of U.S. bonds, while U.S. banks increase theirs.

5.2 Global Productivity Shock

Figure 5 presents the effects of a 1% global decline in TFP, which illustrates how a global slowdown arising from a productivity slowdown may have similar effects on the value of the dollar and the dollar liquidity premium. The effects on exchange rates, asset prices, and capital flows is mostly very similar to the effects of a tightening of the financial constraint. That is because the drop in productivity reduces the profitability of the bank, and so endogenously tightens the lending constraint. As in the literature on the “financial accelerator” (e.g., Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke et al. (1999), Gertler and Kiyotaki (2010), Gertler and Karadi (2011)), the
financial squeeze reduces investment and exacerbates the effects of the original drop in productivity on real output and employment. As in the case of an increase in $\vartheta$ and $\vartheta^*$, the drop in global productivity (a 1% initial decline in $A_t$ and $A_t^*$) leads to a tightening of the lending constraint as can be seen by the increase in the multipliers, $\eta_t$ and $\eta_t^*$.

Because the U.S. bond is relatively less constrained, demand for it increases, which lowers the expected return on the bond. This is mainly accomplished through an appreciation of the dollar, leading to an expectation of a depreciation. There is a larger decline in investment in the U.S. for the same reason described above for a financial shock, leading to a larger decline in U.S. output.

One difference compared to the financial tightening is that retrenchment does not occur with a global productivity decline. In steady state, under the baseline calibration, the home bank’s portfolio is more heavily tilted toward U.S. bonds because they are relatively more pledgeable for the U.S. bank. The foreign bank, in turn, holds a larger portfolio share in equities, so the global decline in productivity leads to a larger drop in the value of foreign banks and more tightening of their balance-sheet constraint. In this case, the asymmetric effect of the shock on the constraints of the two banks leads the foreign bank to actually acquire more U.S. bonds. The finding that retrenchment does not necessarily occur when there is a global productivity decline is consistent with the empirical observation that the global retrenchment phenomenon is more robust during recessions driven by global financial contractions.

### 5.3 U.S. Monetary Contraction

Figure 6 shows the response to a surprise monetary tightening in the U.S. Rey (2015), Rey (2016), and Miranda-Agrippino and Rey (2020) have emphasized the importance of spillovers from U.S. monetary policy in driving the business cycle and monetary policy choices of other countries. In this figure, we see the usual effects on the U.S. of a monetary contraction - increased real interest rates, a drop in investment, consumption and output. The monetary contraction leads to an appreciation through the usual channel of tight money, but also there is an increase in the convenience yield on U.S. Treasury bonds. As investment in capital becomes less attractive, U.S. banks switch their demand toward domestic bonds, which lowers their expected return relative to foreign bonds - that is, an increase in the liquidity yield on U.S. bonds. We can also see that the gap between deposit rates and liquid bond rates in the U.S. widens relative to that in the rest of the world. There is an increase in the U.S. trade balance, though that is not driven primarily by an expenditure switching response (because of local-currency pricing) but rather through the expenditure reducing effects of lower investment and consumption. The contraction in the U.S. spills over to the foreign country through conventional channels, but also through financial channels that are unconventional. As U.S. banks demand switches from equities to U.S. Treasury bonds, lowering the return on those bonds, they acquire U.S bonds from foreign banks. There is a tightening of financial constraints in the foreign country, which leads to a drop in investment demand there, and an increase in demand for local government bonds. However, the effects on the real economy are smaller in the foreign country than in the U.S.
5.4 Global Monetary Contraction

Figure 7 illustrates an equal unexpected monetary contraction in the U.S. and Foreign country. Even though the direct effect of this shock is for both countries to raise their interest rates equally, the effects on the exchange rate, expected returns, and the policy instrument itself are not equal in equilibrium. The tightening monetary policy reduces aggregate demand and lowers the value of capital. It also lowers the value of the banks that own capital, tightening the balance sheet constraint. As with the global financial shock, the effect of the more stringent constraint is to raise the demand for U.S. dollar bonds globally. The liquidity return on these bonds increases, and the dollar appreciates. The disinflationary effect is stronger in the U.S. because banks there are more incentivized to switch out of equities investments and into U.S. bonds, thus lowering aggregate demand to a greater extent. As a result, monetary policy must tighten more in the Foreign country - its policy rate must relative to the policy rate in the U.S.

5.5 Alternative Assumptions on Price Setting

Figure 8 shows how the results are dependent on the assumption that retail prices are set in the currency of the buyer, i.e. LCP. The figure illustrates the impact of the same financial shock as in Figure 2 but now assuming that all goods prices are set in the seller’s currency, (PCP), so that the firm in each country sets only one price in the domestic currency. The most notable difference from the baseline case is the much smaller response of the real exchange rate. With PCP, real exchange rate appreciation can occur only due to terms of trade appreciation, in combination with home bias in consumer preferences. In Figure 8, despite that home consumption falls relative to foreign consumption, we do observe terms of trade appreciation in response to the financial shock. This is due to the larger drop in U.S. output. But the resultant real exchange rate appreciation is much smaller than in the baseline case. The Figure also shows that the real convenience yield is mostly driven by the differential in real interest rates across countries rather than expected real exchange rate depreciation, as in the baseline case. Of course, in this case, there is no deviation from the law of one price in traded goods.

Figure 9 looks at the case with purely flexible prices. While the financial shock still raises the convenience yield, which is driven by the banking block of the system, the real exchange rate now moves in the wrong direction. The Figure shows a very small real exchange rate depreciation. This is driven by a small terms of trade deterioration.

We can conclude from these two Figures that a complete analysis of the impact of a global financial shock that can account for the response of the U.S. convenience yield, the real appreciation of the U.S. dollar, and the feature of global portfolio retrenchment requires the combination of capital constrained banks, an advantage in collateral value for U.S. bonds, as well as sticky prices with limited exchange rate pass-through, captured by a local currency pricing rule for exporting firms.
6 Conclusions

The special features of the US dollar, and US financial assets in the world financial system has generated enormous academic interest over the last decade. The literature has established that the US benefits from an “exorbitant privilege”, with US dollar denominated liabilities offering low returns to foreign investors in normal times, but on the flip side there is an “exorbitant duty” associated with a large US dollar appreciation during global crises, particularly global financial crises. We take a very standard New Keynesian open economy model with balance sheet constrained banks and make a minimal additional assumption by letting US government assets have a higher collateral value than those of the rest of the world. We believe this is a highly realistic assumption quite accurately characterizing the special liquidity features of US treasuries in the global financial system. In a steady state, this model captures the “exorbitant privilege” in the sense that returns on US treasuries are below those of foreign governments, the US is a net debtor, but has a negative trade balance due to a excess income flows on its foreign assets relative to liabilities. In response to a global financial shock coming from a sudden tightening of balance sheet constraints for all banks, we show that the model accurately captures the empirical observations discussed in the introduction. Notably, the US dollar appreciates strongly, and the appreciation is associated with a spike in the convenience yield on US treasuries relative to foreign government assets. Moreover, this appreciation is achieved even though the US trade balance improves during a crisis, and US households have a diminished share of world wealth. The appreciation of the US dollar and the improvement in the trade balance gives the “exorbitant duty”. In addition, although the US government bond represents better collateral, and a global financial crisis leads to an increase in demand for the ‘safe asset’, in equilibrium we see an external retrenchment in capital flows, as in the data. We show that these results are robust to alternative calibrations of the model, and most results carry over to a case where a global downturn is precipitated by a uniform negative shock to all country’s productivity.
Figure 2: Baseline with 1% global financial shock ($\vartheta$, $\vartheta^*$)
Figure 3: 1% global financial shock ($\vartheta$, $\vartheta^*$) with no capital trade
Figure 4: 1% global financial shock ($\vartheta$, $\vartheta^*$) with $\kappa_{h1} = \kappa_{h1}^* = 0.075 < \kappa_{f1} = \kappa_{f1}^* = 0.41$
Figure 5: 1% TFP shock ($A, A^*$) with baseline calibration
Figure 6: 25 basis point Home monetary shock with baseline calibration
Figure 7: 25 basis point both Home and Foreign monetary shock with baseline calibration
Figure 8: 1% global financial shock ($\vartheta$, $\vartheta^*$) with baseline calibration in PCP case
Figure 9: 1% global financial shock ($\vartheta$, $\vartheta^*$) with baseline calibration in flexible price case
Figure 10: Portfolio dynamics under baseline case with 1% global financial shock ($\theta$, $\theta^*$)
References


Appendix

A Collecting all the equations and rewriting the system in real terms

Define real debt holdings: $d_{h,t} = \frac{D_{h,t}}{P_t}, \quad d_{f,t} = \frac{D_{f,t}}{P_t}$

Terms of Trade: $\mathcal{S}_t = \frac{P_{f,t}}{P_{h,t}}$, $\mathcal{S}^*_t = \frac{P^*_{f,t}}{P^*_{h,t}}$

Deviations from LOOP: $\mathcal{D}_t = \frac{S_t P_{h,t}^*}{P_{h,t}}$, $\mathcal{D}^*_t = \frac{S^*_t P_{f,t}^*}{P_{f,t}^*}$

Real Returns: $r_{h,t} = \frac{R_{h,t} P_{t+1}}{P_t}$, $r_{f,t} = \frac{R_{f,t}^* P_{t+1}}{P_t}$

Real marginal product of capital: $r_{K,t} = \frac{R_{K,t}^* P_{t+1}}{P_t}$, $r_{K,t}^* = \frac{R_{K,t}^* P_{t+1}}{P_t}$

Real net worth $n_t = \frac{N_t}{P_t}$, $n_t^* = \frac{N_t^*}{P_t}$

Real equity prices: $q_t = \frac{Q_t}{P_t}$, $q_t^* = \frac{Q_t^*}{P_t}$

Real marginal costs: $mc_t = \frac{MC_t}{P_{h,t}}$, $mc_t^* = \frac{MC_t^*}{P_{f,t}^*}$

Price indices: $P(1, \mathcal{S}_t) = \frac{P_t}{P_{h,t}}$, $P^*(1, \mathcal{S}^*_t) = \frac{P_t^*}{P_{f,t}^*}$

PPI inflation rates: $\pi_{h,t} = \frac{P_t}{P_{h,t-1}}$, $\pi_{h,t}^* = \frac{P_t^*}{P_{h,t-1}}$, $\pi_{f,t} = \frac{P_t}{P_{f,t-1}}$, $\pi_{f,t}^* = \frac{P_t^*}{P_{f,t-1}}$

Also impose bond market clearing: $d_{h,t}^* = d_{h,t} - d_{h,t}$, $d_{f,t}^* = d_{f,t} - d_{f,t}^*$

Labour market clearing: $L_t = H_t$, $L_t^* = H_t^*$

Real wage $w_t = \frac{W_t}{P_t}$, $w_t^* = \frac{W_t^*}{P_t}$

Real deposit (policy) rate $r_{t+1} = \frac{R_{t+1}}{P_{t+1}}$, $r_{t+1}^* = \frac{R_{t+1}^*}{P_{t+1}^*}$

Note we don’t explicitly define the real exchange rate, but it is implied by $\frac{S_t P_{h,t}^*}{P_t} = \frac{1}{P(1, \mathcal{S}_t)} P^*(1, \mathcal{S}^*_t)$. That is, the real exchange rate depends on both deviations from LOOP as well as movements in the terms of trade. Also, set $\tilde{\pi} = 0$

Balance of payments:

$$(C_t + I_t + K_t \phi(\frac{K_t}{K_t})) + d_{h,t} - d_{h,t} + \mathcal{D}_t P(1, \mathcal{S}_t) d_{f,t} = \frac{C_t - \sigma}{C_t} \frac{Y_t}{P(1, \mathcal{S}_t)} (1 - \bar{\xi}(\pi_{h,t})) + \frac{\mathcal{D}_t}{P(1, \mathcal{S}_t)} Y_t^* (1 - \bar{\xi}(\pi_{h,t}^*)) + r_{h,t}(d_{h,t-1} - d_{h,t-1}) + r_{f,t} d_{f,t-1}$$

Home Euler equation:

$$1 = E_t \beta r_{t+1} \frac{C_{t+1}^\sigma}{C_t^{\sigma}}$$

Foreign Euler equation:

$$1 = E_t \beta r_{t+1}^* \frac{C_{t+1}^\sigma}{C_t^{\sigma}}$$

Profit max home:
\[(1 + s_t)Y_{h,t} - \varepsilon((1 + s_t) - mc_t)Y_{h,t} = 0 \quad (45)\]
\[-\xi'(\pi_{h,t})\pi_{h,t}Y_{h,t} + E_t\beta C_t^{\sigma - \alpha} P_t^{(1, \mathcal{F}_t)} \xi'(\pi_{h,t+1})\pi_{h,t+1}Y_{h,t+1} = 0 \quad (46)\]

Factor markets home:

\[A_t(1 - \alpha)\zeta(L_t^{1-\alpha} K_t^\alpha)mc_t = w_t P(1, \mathcal{F}_t)L_t \quad (47)\]

\[A_t\alpha\zeta(L_t^{1-\alpha} K_t^\alpha)mc_t = r_{K,t} P(1, \mathcal{F}_t)K_t \quad (48)\]

\[w_t = \chi C_t^\sigma L_t^\psi \quad (49)\]

Capital and Price of capital at Home:

\[K_{t+1} = I_t + (1 - \delta)K_t \quad (50)\]

\[q_t = (1 + \psi'(I_t/K_t)) \quad (51)\]

\[Y_{h,t} + Y_{h,t}^* = A_t(L_t^\alpha K_t^{1-\alpha}) \quad (52)\]

Profit Max Foreign:

\[(1 + s_t^*)Y_{f,t}^* - \varepsilon((1 + s_t) - mc_t^*)Y_{f,t}^* = 0 \quad (53)\]

\[-\xi'(\pi_{f,t})\pi_{f,t}^*Y_{f,t}^* + E_t\beta C_t^{\sigma - \alpha} P_t^{(1, \mathcal{F}_t)} \xi'(\pi_{f,t+1})\pi_{f,t+1}^*Y_{f,t+1}^* = 0 \quad (54)\]

Foreign factor markets:

\[A_t^*(1 - \alpha)\zeta(L_t^{*(1-\alpha)} K_t^{*\alpha})mc_t^* = P_t^*(1, \mathcal{F}_t^*)w_t^* L_t^* \quad (55)\]

\[A_t^*\alpha\zeta(L_t^{*(1-\alpha)} K_t^{*\alpha})mc_t^* = K_t^* P_t^*(1, \mathcal{F}_t^*) r_{K,t}^* \quad (56)\]

\[w_t^* = \chi C_t^{\sigma} L_t^{*\psi} \quad (57)\]

Capital and Price of capital Foreign:
\[ K_{t+1}^* = I_t^* + (1 - \delta)K_t^* \]  
\[ q_t^* = (1 + \psi'(I_t/K_t)) \]  
\[ Y_{f,t}^* + Y_{f,t} = A_t^* (L_t^* K_t^{(1-\alpha)}) \]  

**Market Clearing:**

**Bond market clearing:**
\[ d_{h,t}^* = d_{h,t}^* - d_{f,t}, \quad d_{f,t} = d_{f,t}^* - d_{f,t}^* \]

**Home good:**
\[ Y_{h,t}^* (1 - \xi(\pi_{h,t})) = \omega \left( \frac{1}{P(1,\mathcal{S})} \right)^{-\lambda} (C_t + I_t + K_t \phi(I_t/K_t)) \]  
\[ Y_{h,t}^* (1 - \xi(\pi_{h,t})) = \frac{(1 - n)}{n} (1 - \omega^*) \left( \frac{1}{P^*(1,\mathcal{S}^*)} \right)^{-\lambda} (C_t^* + I_t^* + K_t^* \phi(I_t/K_t)) \]  

**Foreign good:**
\[ Y_{f,t}^* (1 - \xi(\pi_{f,t})) = \frac{n}{1 - n} (1 - \omega) \left( \frac{\mathcal{S}_t}{P(1,\mathcal{S})} \right)^{-\lambda} (C_t + I_t + K_t \phi(I_t/K_t)) \]  
\[ Y_{f,t}^* (1 - \xi(\pi_{f,t}^*)) = \omega^* \left( \frac{\mathcal{S}_t^*}{P^*(1,\mathcal{S}^*)} \right)^{-\lambda} (C_t^* + I_t^* + K_t^* \phi(I_t/K_t)) \]  

**Home Bank:**
\[ E_t \tilde{\Lambda}_{t+1} (\tilde{r}_{k,t+1} - r_{t+1}) = \eta_t \phi_t (\kappa_{Kh1,t} + \kappa_{Kh2,t} K_{h,t}) \]  
\[ E_t \tilde{\Lambda}_{t+1} \left( \frac{\text{RER}_{t+1}}{\text{RER}_t} \tilde{r}_{k,t+1} - r_{t+1} \right) = \eta_t \phi_t (\kappa_{Kf1,t} + \kappa_{Kf2,t} \text{RER}_t K_{f,t}) \]  
\[ E_t \tilde{\Lambda}_{t+1} (r_{h,t+1} - r_{t+1}) = \eta_t \phi_t (\kappa_{h1,t} + \kappa_{h2,t} d_{h,t}) \]  
\[ E_t \tilde{\Lambda}_{t+1} \left( \frac{\text{RER}_{t+1}}{\text{RER}_t} r_{f,t+1} - r_{t+1} \right) = \eta_t \phi_t (\kappa_{f1,t} + \kappa_{f2,t} \text{RER}_t d_{f,t}) \]  
\[ \tilde{\Lambda}_{t+1} = \Omega_{t+1} \frac{\pi_{h,t+1} P(1,\mathcal{S}_{t+1})}{P(1,\mathcal{S}_t)} ((1 - \theta) + \theta v_{t+1}) \]  
\[ \tilde{r}_{K,t+1} = \frac{r_{K,t+1} + (1 - \delta) q_{t+1}}{q_t} \]

**Home envelope condition:**
\[ \nu_t = \frac{E_t \Omega_{t+1} \pi_{h,t+1} P(1, \mathcal{Q}_{t+1}) ((1 - \theta) + \theta \nu_{t+1}) r_{t+1}}{1 - \eta_t} \]  

(69)

Home participation constraint:

\[ \nu_t n_t = \partial_t \left( (\kappa_{Kh1,t} + \kappa_{Kh2,t} q_t K_{h,t+1}) q_t K_{h,t+1} + (\kappa_{Kf1,t} + \kappa_{Kf2,t} RER_t q_t^* K_{h,t+1}^*) RER_t q_t^* K_{h,t+1}^* + (\kappa_{h1,t} + \kappa_{h2,t} d_{h,t} t) d_{h,t} \right) \]  

(70)

Home net worth dynamics:

\[ n_{t+1} = \theta (\tilde{r}_{k,t+1} - r_{t+1}) q_t K_{t+1} + (r_{h,t+1} - r_{t+1}) d_{h,t} \]  

\[ + \left( \frac{\pi_{h,t+1} P(1, \mathcal{Q}_{t+1}) \rho(1, \mathcal{Q}_{t+1})}{\rho(n_{t+1} P(1, \mathcal{Q}_{t+1}))} d_{f,t} + r_{t+1} n_t \right) \]  

\[ + \phi \frac{\pi_{h,t+1} \rho(1, \mathcal{Q}_{t+1})}{\rho(n_{t+1} P(1, \mathcal{Q}_{t+1}))} \left( q_t K_{t+1} + d_{h,t} + \frac{\rho(n_{t+1} P(1, \mathcal{Q}_{t+1}))}{\rho(n_{t+1} P(1, \mathcal{Q}_{t+1}))} d_{f,t} \right) \]  

(71)

Foreign Bank:

\[ E_t \tilde{\Lambda}_{t+1}^* \left( \frac{RER_t}{RER_{t+1}} \tilde{r}_{k,t+1} - r_{t+1}^* \right) = \eta_t^* \partial_t^* \left( \kappa_{Kh1,t}^* + \kappa_{Kh2,t}^* K_{h,t+1}^*/RER_t \right) \]  

(72)

\[ E_t \tilde{\Lambda}_{t+1}^* \left( \tilde{r}_{k,t+1} - r_{t+1} \right) = \eta_t^* \partial_t^* \left( \kappa_{Kf1,t}^* + \kappa_{Kf2,t}^* K_{f,t+1}^* \right) \]  

(73)

\[ E_t \tilde{\Lambda}_{t+1}^* \left( \frac{RER_t}{RER_{t+1}} r_{h,t+1} - r_{t+1}^* \right) = \eta_t^* \partial_t^* \left( \kappa_{h1,t}^* + \kappa_{h2,t}^* d_{h,t}^*/RER_t \right) \]  

(74)

\[ E_t \tilde{\Lambda}_{t+1}^* \left( r_{f,t+1} - r_{t+1}^* \right) = \eta_t^* \partial_t^* \left( \kappa_{f1,t}^* + \kappa_{f2,t}^* d_{f,t}^* \right) \]  

(75)

\[ \tilde{\Lambda}_{t+1}^* = \Omega_{t+1}^* \pi_{h,t+1}^* \frac{P(1, \mathcal{Q}_{t+1})}{P(1, \mathcal{Q}_{t+1})} ((1 - \theta) + \theta \nu_{t+1}^*) \]  

\[ \tilde{r}_{k,t+1}^* = \frac{r_{K,t+1}^* + (1 - \delta) q_{t+1}^*}{q_t^*} \]  

Foreign envelope condition:

\[ \nu_t^* = \frac{E_t \Omega_{t+1}^* \pi_{h,t+1}^* P(1, \mathcal{Q}_{t+1}^*) ((1 - \theta) + \theta \nu_{t+1}^*) r_{t+1}^*}{1 - \eta_t^*} \]  

(76)

Foreign participation constraint:

\[ \nu_t^* n_t^* = \partial_t \left( \left( \kappa_{Kf1,t}^* + \kappa_{Kf2,t}^* q_t^* K_{f,t+1}^* \right) q_t^* K_{f,t+1}^* + \frac{1}{RER_t} q_t K_{f,t+1}^* \frac{1}{RER_t} q_t K_{f,t+1}^* \right) + \frac{1}{RER_t} q_t K_{f,t+1}^* + \frac{1}{RER_t} q_t K_{f,t+1}^* \]  

(77)

Foreign net worth dynamics:
\[ n_{t+1}^* = \theta((\bar{r}_{t+1}^* - r_{t+1}^*)q_{t}^*K_{t+1}^*) \]
\[ + (\frac{\partial}{\partial r_{t+1}^*} P(1, J_t^*) \frac{P(1, J_{t+1}^*)}{P(1, J_{t+1}^*)} \bar{r}_{t+1}^* - r_{t+1}^*) \frac{P(1, J_t^*)}{P(1, J_{t+1}^*)} d_{h,t}^* \]
\[ + (r_{f,t+1} - r_{t+1}^*)d_{f,t}^* + r_{t+1}^* n_{t+1}^* \]
\[ + \varphi \frac{P(1, J_t^*)}{P(1, J_{t+1}^*)} \left( q_{t}^* K_{t+1}^* + \frac{P(1, J_t^*)}{P(1, J_{t+1}^*)} d_{h,t}^* + d_{f,t}^* \right) \]

(78)

Home monetary Rule:
\[ \frac{P(1, J_t^*)}{P(1, J_{t-1}^*)} \pi_{h,t} = \frac{1}{\beta} \left( \frac{P(1, J_t^*)}{P(1, J_{t-1}^*)} \right)^{\eta} \]

(79)

Foreign monetary Rule:
\[ \frac{P(1, J_{t-1}^*)}{P(1, J_{t-1}^*)} \pi_{f,t} = \pi_{h,t} \frac{J_{t-1}^*}{J_t^*} \]

(80)

Definitions:
\[ \pi_{h,t} = \pi_{f,t} \frac{J_{t-1}^*}{J_t^*} \]

(81)

\[ \pi_{f,t}^* = \pi_{h,t}^* \frac{J_t^*}{J_{t-1}^*} \]

(82)

\[ \Psi_t = \frac{D_t^*}{J_t^*} \]

(83)

Equations (42) - (83) give 42 equations in
\[ C_t, C_t^*, L_t, L_t^*, l_t, l_t^*, K_t, K_t^*, X_t, X_t^*, \]
\[ \pi_{h,t}, \pi_{h,t}^*, \pi_{f,t}, \pi_{f,t}^*, w_t, w_t^*, q_t, q_t^*, mc_t, mc_t^* \]
\[ r_{K,t}, r_{K,t}^*, r_t, r_t^*, h_{h,t}, h_{h,t}^*, Y_{h,t}, Y_{h,t}^* \]
\[ Y_{f,t}, Y_{f,t}^*, u_t, u_t^*, \eta_t, \eta_t^* \]
\[ d_{h,t}, d_{f,t}, J_t, J_t^*, \Psi_t, \Psi_{t-1}, J_{t-1}^* \]

Endogenous state variables:
\[ n_t, n_t^*, K_t, K_t^*, d_{h,t}, d_{f,t}, J_{t-1}, J_{t-1}^* \]
B Steady state equations

Define real debt holdings: $d_h, d_f$,

Terms of Trade: $\mathcal{S} = \mathcal{S}^*$,

Deviations from LOOP: $\mathcal{D} = 1, \mathcal{D}^* = 1$

Real Returns: $r_h, r_f$

Real equity prices: $q = 1, q^* = 1$

Real marginal product of capital: $r_K = R_K, r_{K}^* = R_{K}^*$

Real net worth $n, n^*$

Real marginal costs: $mc = mc^* = 1$ (Setting $A = A^* = 1$)

Price indices: $P(1, \mathcal{S}), P^*(1, \mathcal{S})$

PPI inflation rates: $\pi_h = 1, \pi_{h}^* = 1, \pi_f = 1, \pi_{f}^* = 1$

Also impose bond market clearing: $d_{h}^* = \bar{d}_h - d_h, d_{f}^* = \bar{d}_f - d_f$

Labour market clearing: $L = H, L^* = H^*$

Real wage $w, w^*$

Real deposit (policy) rate $r, r^*$

Subsidy rate $s = \frac{1}{\varepsilon - 1}$

Define the steady state real exchange rate as $\mathcal{D} = \frac{P^*(1, \mathcal{S})}{P(1, \mathcal{S})}$. So it depends only on the terms of trade and home bias in consumption aggregators.

Balance of payments:

$$C + X + \delta K + d_h - \bar{d}_h + \mathcal{D}d_f = \frac{Y_h + Y_h^*}{P(1, \mathcal{S})} + r_h(d_h - \bar{d}_h) + \mathcal{D}r_fd_f$$

(84)

Home Euler equation:

$$1 = \beta r$$

(85)

Foreign Euler equation:

$$1 = \beta r$$

(86)

Profit max home:

$$1 = mc$$

(87)

$$1 = mc$$

(88)

Factor markets home:

$$(1 - \alpha)\zeta(L^{1-\alpha}K^{\alpha})^{\xi}X^{1-\xi} = \chi C^\sigma L^{1+\psi}P(1, \mathcal{S})$$

(89)
\[ \alpha \zeta (L^{1-\alpha} K^\alpha)^\zeta X^{1-\zeta} = r_K P(1, \mathcal{J}) \]  
\[ (1 - \zeta) (L^{1-\alpha} K^\alpha)^\zeta X^{1-\zeta} = P(1, \mathcal{J}) X \]  

Capital and Price of capital at Home:

\[ K \delta = I \]  
\[ q = 1 \]  
\[ Y_h + Y_{h}^* = (L^{\alpha} K^{1-\alpha})^\zeta X^{1-\zeta} \]  

Profit Max Foreign:

\[ 1 = mc^* \]  
\[ 1 = mc^* \]  

Foreign factor markets:

\[ (1 - \alpha) \zeta (L^{*(1-\alpha)} K^{*\alpha})^\zeta X^{*(1-\zeta)} = \frac{P^*(1, \mathcal{J})}{\mathcal{J}} \chi C^{*\sigma} L^{*1+\psi} \]  
\[ \alpha \zeta (L^{*(1-\alpha)} K^{*\alpha})^\zeta X^{*(1-\zeta)} = \frac{P^*(1, \mathcal{J})}{\mathcal{J}} r_K^* \]  
\[ (1 - \zeta) (L^{*(1-\alpha)} K^{*\alpha})^\zeta X^{*(1-\zeta)} = \frac{P^*(1, \mathcal{J})}{\mathcal{J}} X^* \]  

Capital and Price of capital Foreign:

\[ K \delta^* = I^* \]  
\[ q^* = 1 \]  
\[ Y_{f}^* + Y_{f} = (L^{*\alpha} K^{*(1-\alpha)})^\zeta X^{*(1-\zeta)} \]  

Market Clearing:

Home good:

\[ Y_h + Y_{f} = \omega \left( \frac{1}{P(1, \mathcal{J})} \right)^{-\lambda} (C + X + \delta K) + \frac{(1-n)}{n} (1 - \omega^*) \left( \frac{1}{P^*(1, \mathcal{J})} \right)^{-\lambda} (C^* + X^* + \delta K^*) \]  

Foreign good:
\[ Y_f + Y_d^* = \frac{n}{1-n} (1-\omega) \left( \frac{\mathcal{J}}{P(1,\mathcal{J})} \right)^{-\lambda} (C+X+\delta K) + \omega \left( \frac{\mathcal{J}}{P^*(1,\mathcal{J})} \right)^{-\lambda} (C^*+X^*+\delta K) \] (104)

Home Bank:

\[ \tilde{\Lambda} (\tilde{r}_k - r) = \eta \vartheta (\kappa_{K_{h1}} + \kappa_{K_{h2}} K_h) \] (105)

\[ \tilde{\Lambda} (\tilde{r}_f^* - r) = \eta \vartheta (\kappa_{K_f1} + \kappa_{K_f2} K_f) \] (106)

\[ \tilde{\Lambda} (r_h - r) = \eta \vartheta (\kappa_{h1} + \kappa_{h2} d_h) \] (107)

\[ \tilde{\Lambda} (r_f - r) = \eta \vartheta (\kappa_{f1} + \kappa_{f2} d_f) \] (108)

\[ \tilde{\Lambda} = \beta ((1 - \theta) + \theta \nu) \] (109)

\[ \tilde{r}_K = r_K + (1 - \delta) \] (110)

Home envelope condition:

\[ \nu = \frac{((1 - \theta) + \theta \nu)}{1 - \eta} \] (111)

Home participation constraint:

\[ \nu n = \vartheta (\kappa_{K}K + (\kappa_{h1} + \kappa_{h2} d_h)d_h + (\kappa_{f1} + \kappa_{f2} d_f) \vartheta d_f) \] (112)

Home net worth dynamics:

\[ n = \theta (\tilde{r}_k - r)K + (r_h - r)d_h + (r_f - r) \vartheta d_f + \nu n \]
\[ + \varphi (K + d_h + \vartheta d_f) \] (113)

Foreign Bank:

\[ \tilde{\Lambda}^* (\tilde{r}_k - r^*) = \eta^* \vartheta^* (\kappa_{K_{h1}}^* + \kappa_{K_{h2}}^* K_h^*) \] (114)

\[ \tilde{\Lambda}^* (\tilde{r}_f^* - r^*) = \eta^* \vartheta^* (\kappa_{K_f1}^* + \kappa_{K_f2}^* K_f^*) \] (115)

\[ \tilde{\Lambda}^* (r_h - r^*) = \eta^* \vartheta^* (\kappa_{h1}^* + \kappa_{h2}^* d_h^*) \] (116)

\[ \tilde{\Lambda}^* (r_f - r^*) = \eta^* \vartheta^* (\kappa_{f1}^* + \kappa_{f2}^* d_f^*) \] (117)

\[ \tilde{\Lambda}^* = \beta ((1 - \theta) + \theta \nu^*) \] (118)

\[ \tilde{r}_K^* = r_K^* + (1 - \delta) \] (119)
Foreign envelope condition:

\[ \nu^* = \frac{(1 - \theta) + \theta \nu^*}{1 - \eta^*} \]  

(120)

Foreign participation constraint:

\[ \nu^* n^* = \theta \left( \kappa_K^* K^* + \kappa_h \frac{d_h^*}{\vartheta} + \kappa_f d_f^* \right) \]  

(121)

Foreign net worth dynamics:

\[ n^* = \theta \left( (\tilde{r}_K^* - r^*) K^* + (r_h - r^*) \frac{d_h^*}{\vartheta} + (r_f - r^*) d_f^* + r^* n^* \right) + \varphi \left( K^* + \frac{d_h^*}{\vartheta} + d_f^* \right) \]  

(122)

Home monetary Rule:

\[ R_t = \frac{1}{\beta} \]  

(123)

Foreign monetary Rule:

\[ R_t^* = \frac{1}{\beta} \]  

(124)

Take \( m_c, r, q, I, \tilde{A} \) as given (and same for foreign). Then we have (84), (89), (90), (91), (94), (97), (98), (99), (102), (103), (104), (105), (107), (108), (111), (112), (113), (115), (116), (117), (120), (121), (122),

Solve for \( C, C^*, K, K^*, X, X^*, L, L^*, Y, Y^* \) 
(Note just add \( Y_h + Y_h^* = Y \) together)

\( \mathcal{J}, d_h, d_f, n, n^*, \nu, \nu^*, \eta, \eta^*, r_c, r_c^*, r_h, r_f \)