SENTIMENT AND THE U.S. BUSINESS CYCLE

FABIO MILANI

UNIVERSITY OF CALIFORNIA, IRVINE

ABSTRACT. Psychological factors are commonly believed to play a role on cyclical economic fluctuations, but they are typically omitted from state-of-the-art macroeconomic models.

This paper introduces "sentiment" in a medium-scale DSGE model of the U.S. economy and tests the empirical contribution of sentiment shocks to business cycle fluctuations.

The assumption of rational expectations is relaxed. The paper exploits, instead, observed data on expectations in the estimation. The observed expectations are assumed to be formed from a *near*-rational learning model. Agents are endowed with a perceived law of motion that resembles the model solution under rational expectations, but they lack knowledge about the solution's reduced-form coefficients. They attempt to learn those coefficients over time using available time series at each point in the sample and updating their beliefs through constant-gain learning. In each period, however, they may form expectations that fall above or below those implied by the learning model. These deviations capture excesses of optimism and pessimism, which can be quite persistent and which are defined as sentiment in the model. Different sentiment shocks are identified in the empirical analysis: undue optimism and pessimism may refer to future expected consumption, future investment, or future inflationary pressures.

The results show that exogenous variations in sentiment are responsible for a sizable (about forty percent) portion of historical U.S. business cycle fluctuations. Sentiment shocks related to investment decisions, which bring to mind Keynes' animal spirits, play the largest role. When the model is estimated imposing the rational expectations hypothesis, instead, the role of structural investment-specific and neutral technology shocks significantly expands to capture the omitted contribution of sentiment.

Keywords: Sentiment, Animal Spirits, Learning, DSGE Model, Sources of Business Cycle Fluctuations, Observed Survey Expectations.

JEL classification: E32, E50, E52, E58.

[PRELIMINARY & INCOMPLETE DRAFT]

1. Introduction

Economists have always recognized the importance of expectations for aggregate economic behavior. Some of the most influential economic thinkers of the past century attributed explicitly to the volatility of expectations a large role in explaining the existence and depth of business cycles.

Keynes emphasized in the *General Theory* the importance of changes in expectations that are not necessarily driven by rational probabilistic calculations, but which are rather motivated by what he famously labeled "animal spirits". In particular, entrepreneurs' animal spirits related to their investment decisions were theorized of being a major determinant of economic fluctuations. Pigou (1927) also thought of business cycles as being largely driven by expectations and he stressed entrepreneurs' errors of optimism and pessimism as key drivers of fluctuations in real activity.

Expectations also play an important role in modern state-of-the-art general equilibrium models. Expectations are almost universally modeled as formed according to the rational expectations hypothesis. As a result, at least in models with determinate equilibria, expectational errors can be solved out as a function of fundamental shocks and they disappear as autonomous sources of dynamics. Hence, there is typically no scope for fluctuations in expectations in the spirit of those emphasized by Keynes, which are driven by animal spirits, market psychology, sentiment, or by any expectational shift that cannot be reconnected to original structural disturbances.[1]

In state-of-the-art DSGE models, the main sources of fluctuations are typically shocks to demand, such as exogenous shifts in preferences, risk-premia, and monetary and fiscal policies, shocks related to technology, such as Hicks-neutral or investment specific technology shocks, or to market power, such as price and wage markup shocks. While the empirical DSGE macro literature disagrees on the relative contributions of each shock, most of it implicitly agrees on assigning a nil role to explanations based on non-fundamental expectational shifts, such as swings in sentiment that are not necessarily motivated by fundamentals.

This paper aims to fill this gap in the literature. It revisits a benchmark DSGE model that is often used to characterize the dynamics of the U.S. economy at business cycle frequencies. But the model is extended to incorporate "sentiment", which represents waves of optimism and pessimism that are exogenous to the state of the economy.

---

[1]Animal spirits may, instead, be reintroduced under rational expectations by assuming indeterminacy: the expectational errors in that case are not only a function of structural disturbances, but also of exogenous sunspot variables. There is a large literature, surveyed in Benhabib and Farmer (1999), which is focused on studying indeterminacy and sunspots in macroeconomic models. The work in this area has, however, been more often theoretical than empirical (Lubik and Schorfheide, 2004, is an econometric analysis of sunspots in a general equilibrium model).

The strong informational requirements of rational expectations are relaxed. In their place, I will exploit observed data on expectations, obtained from the Survey of Professional Forecasters, in the estimation.

The observed expectations are assumed to be, on average, the outcome of a near-rational expectation formation process, which allows for learning by economic agents. Agents form expectations based on a linear model that has the same structural form of the system solution under rational expectations (i.e., the model used by agents is correctly specified, since it contains the same regressors). The paper, however, relaxes the assumption that economic agents in the model have an informational advantage over the econometrician estimating the model. Here, at each point in the sample, economic agents can observe only historical data up to that point and they form beliefs about the reduced-form model coefficients by estimating simple regressions. At least in the baseline estimation, they are not endowed with knowledge about the unobserved disturbances. The framework allows for deviations from rational expectations, but the deviation is meant to be small: agents still use a correctly specified model. Such small deviations set the learning literature apart from more drastic alternatives that abandon rational expectations to assume, for example, simple heuristic rules.

Although expectations are formed, on average, from the learning model, economic agents can form expectations in every period that deviate from the point forecast that their learning model yields. In each period, their expectations regarding aggregate variables can be above or below the forecasts that are justified by their perceived model of the economy. These deviations of actual expectations from their levels that can be explained by a near-rational model with learning are interpreted as denoting exogenous waves of undue optimism or pessimism, and define the "sentiment" terms in the model. Sentiment shocks are, therefore, identified from the dynamic interaction between observed expectations and realized macroeconomic time series.

The DSGE model is estimated using Bayesian techniques and adding data on observed expectations about consumption, investment, and inflation to the set of observables to match. The main scope in the empirical analysis lies in studying the empirical contribution of these newly defined sentiment shocks to macroeconomic fluctuations.

Main Results. The empirical results show that sentiment shocks explain a sizable portion of U.S. business cycle fluctuations. Sentiment explains roughly more than forty percent of the variability of output and consumption at horizons between one year and six years, and up to sixty percent of the variability of investment and inflation. The most important component of sentiment

consists of sentiment related to future investment expectations, which is found to be the single main driver of business cycle movements. Inflation is driven by structural price markup shocks in the short-run; their transmission is, however, very quick, and market participants' sentiment about inflationary pressures becomes predominant at frequencies above one year.

If learning and sentiment are shut down and the conventional assumption of rational expectations is re-imposed, technology shocks become the dominant source of aggregate fluctuations (in both forms of investment-specific and neutral technology shocks), as theorized by the RBC literature. The contribution of technological changes for booms and busts significantly rises to close the gap induced by the omitted role of households and firms' sentiment.

When the model is estimated under observed expectations, allowing for learning and sentiment, the degrees of some real and nominal frictions necessary to fit the data is diminished. In particular, moving away from rational expectations attenuates the degree of habit formation in consumption and sharply reduces the magnitude of adjustment costs in investment. The estimated autocorrelation of several structural disturbances is largely reduced and the responses of macroeconomic variables to some structural shocks become faster than usually estimated. Sentiment, on the other hand, is responsible for more sluggish adjustment in the economy.

Related Literature. In previous related work (Milani, 2011), I have modeled expectation shocks and shown that they potentially play a large role as drivers of business cycles. That paper used a stylized three-equation New Keynesian model. The current paper extends the analysis to a more comprehensive and empirically relevant model of the U.S. economy. While the previous paper abstracted from capital accumulation, this work includes capital and investment, allowing for adjustment costs and variable capital utilization, in addition to features such as wage stickiness, and it exploits expectations about future consumption and investment in the estimation. In this way, the paper can disentangle the role of sentiment related to consumers' expenditure decisions and to firm's investment choices, which was the channel emphasized in Keynes' theories.

The paper adds to the expanding literature on bounded rationality and learning in macroeconomics (e.g., Evans and Honkapohja, 2001, Sargent, 1993). It exploits direct data on expectations to inform the estimation of the best-fitting learning process over the sample. Moreover, it shows that, in addition to the role of learning, a different component of expectations, which the paper defines as sentiment, is key to understand business cycles. A particularly related paper in the learning literature is Bullard et al. (2009), who introduce judgment in economic agents' learning model, showing that it leads near-rational exuberance equilibria.

On the methodological side, the recent literature has started to include data on expectations more and more in the estimation of DSGE models. Del Negro and Eusepi (2011) use data on inflation expectations to test whether rational expectations DSGE models can successfully explain the expectation series; Ormeno (2010) add information from observed inflation expectations to discipline the estimation of models with learning. Milani (2011) uses data on expectations regarding output, inflation, and interest rates. Aruoba and Schorfheide (2011) exploit long-run inflation expectations to help extract the implicit Federal Reserve's inflation target. Milani and Rajbhandari (2012b) exploit a large set of expectations series at different horizons to enhance the identification of news shocks.[2]

The paper can be connected to the literature on multiple equilibria, sunspots, and animal spirits, although most of the modeling choices differ. The paper can be seen as an econometric evaluation of the importance of animal spirits, here defined somewhat differently, since they arise in a model in which rational expectations have been relaxed. An advantage of the approach suggested in this paper is that the existence of self-fulfilling fluctuations in expectations are not conditional on indeterminacy of the equilibrium, which in a model as the one considered in this paper, would be mostly due to a failure by monetary policy to satisfy the Taylor principle. Self-fulfilling fluctuations and expectations-driven business cycles may arise here in a model in which monetary policy still responds aggressively toward inflation and there's a unique determinate equilibrium.

The paper has also points of contact with the literature on news shocks (e.g., Beaudry and Portier, 2006). That literature has mostly emphasized news about future technology as sources of fluctuations, although recently other types of news have been considered (e.g., Khan and Tsoukalas, 2011, Schmitt-Grohe and Uribe, 2012, and Milani and Treadwell, 2011). The sentiment variables identified here do not represent news about future improvements in technology or future monetary or fiscal policies, but they are intended to capture unjustified and possibly persistent waves of optimism and pessimism that are orthogonal to the past and current state of the economy. The paper shows that the identified sentiment in the model is indeed correlated to innovations obtained from available survey data on sentiment, supporting the interpretation given in the paper.

Finally, other recent studies have, instead, focused on second-moment, rather than first-moments shocks, by investigating the potential for uncertainty shocks as a source of economic fluctuations (e.g., Bloom, 2009, Fernandez-Villaverde et al., 2011). I abstract from those here.

---

[2]Milani and Rajbhandari (2012a) study various models of expectation formation and include survey forecasts in their econometric evaluation. Milani (2012) offers a survey of the treatment of expectations in empirical DSGE models.

2. MODEL

The current generation of DSGE models joins elements from the RBC tradition (explicit microfoundations, dynamic optimization, capital accumulation, and technology shocks) and elements from the New Keynesian tradition (imperfect competition, sticky prices, sticky wages, an interest rate rule for monetary policy). Economic agents are typically assumed to form model-consistent rational expectations. To be taken to the data, the model needs to incorporate a variety of real and nominal frictions, along with a combination of serially-correlated exogenous shocks.

This paper follows in this tradition by assuming that fluctuations in the U.S. economy at business cycle frequencies can be summarized by a medium-scale DSGE model, based on Smets and Wouters (2007), and which has been used as benchmark in several studies.Similar models have been developed and fitted to U.S. data by Christiano et al. (2005), Justiniano et al. (2010), Del Negro and Schorfheide (2007), among countless others. Since later we will relax the assumption of rational expectations, we replace the mathematical expectation operator E_t with the indicator for subjective expectations $\widehat{E}_t$. We report the loglinearized equations here.[3]

$$y_t = c_y c_t + i_y i_t + u_y u_t + \varepsilon_t^g \tag{2.1}$$

$$c_t = c_1 c_{t-1} + (1 - c_1)\widehat{E}_t c_{t+1} + c_2(l_t - \widehat{E}_t l_{t+1}) - c_3(r_t - \widehat{E}_t \pi_{t+1} + \varepsilon_t^b) \tag{2.2}$$

$$i_t = i_1 i_{t-1} + (1 - i_1)\widehat{E}_t i_{t+1} + i_2 q_t + \varepsilon_t^i \tag{2.3}$$

$$q_t = q_1 \widehat{E}_t q_{t+1} + (1 - q_1)\widehat{E}_t r_{t+1}^k - (r_t - \widehat{E}_t \pi_{t+1} + \varepsilon_t^b) \tag{2.4}$$

$$y_t = \Phi_p(\alpha k_t^s + (1 - \alpha)l_t + \varepsilon_t^a) \tag{2.5}$$

$$k_t^s = k_{t-1} + u_t \tag{2.6}$$

$$u_t = u_1 r_t^k \tag{2.7}$$

$$k_t = k_1 k_{t-1} + (1 - k_1)i_t + k_2 \varepsilon_t^i \tag{2.8}$$

$$\mu_t^p = \alpha(k_t^s - l_t) + \varepsilon_t^a - w_t \tag{2.9}$$

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 \widehat{E}_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p \tag{2.10}$$

$$r_t^k = -(k_t - l_t) + w_t \tag{2.11}$$

$$\mu_t^w = w_t - \left(\sigma_l l_t + \frac{1}{1 - h/\gamma}\left(c_t - \frac{h}{\gamma}c_{t-1}\right)\right) \tag{2.12}$$

$$w_t = w_1 w_{t-1} + (1 - w_1)\widehat{E}_t(w_{t+1} + \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w \tag{2.13}$$

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)\left[\chi_\pi \pi_t + \chi_y(y_t - \Phi_p \varepsilon_t^a)\right] + \varepsilon_t^r \tag{2.14}$$

---

[3]The reader is referred to the extensive appendix in Smets and Wouters (2007) for a step-by-step derivation of these equations.

The composite coefficients are given by:

$$
\begin{aligned}
&c_y = 1 - i_y - g_y; \quad i_y = \delta k_y; &&u_y = r_k^* k_y; \\
&c_1 = h/(1+h); \quad c_2 = (\sigma_c - 1)(W_*^h L_*/C_*)/\sigma_c(1+h); \quad c_3 = (1-h)/[\sigma_c(1+h)]; \\
&i_1 = 1/(1+\beta); \quad i_2 = 1/[(1+\beta)\varphi]; \\
&q_1 = \beta(1-\delta); \\
&z_1 = \psi/(1+\psi); \\
&k_1 = 1 - \delta; \quad k_2 = \delta(1+\beta)\varphi; \\
&\pi_1 = \iota_p/(1+\beta\iota_p); \quad \pi_2 = \beta/(1+\beta\iota_p); \\
&\pi_3 = [1/(1+\beta\iota_p)][(1-\beta\xi_p)(1-\xi_p)/(\xi_p(\phi_p-1)\varepsilon_p+1)]; \\
&w_1 = 1/(1+\beta); \quad w_2 = (1+\beta\iota_w)/(1+\beta); \quad w_3 = \iota_w/(1+\beta); \\
&w_4 = [1/(1+\beta)][(1-\beta\xi_w)(1-\xi_w)/(\xi_w(\phi_w-1)\varepsilon_w+1)];
\end{aligned}
\tag{2.15}
$$

Equation (2.1) is the economy's aggregate resource constraint: output is denoted by y_t, consumption by c_t, investment by i_t, and variable capacity utilization by u_t. The term ε_t^g denotes an exogenous government spending shock. The coefficients c_y, i_y, and u_y, denote the steady-state shares of consumption, investment, and resources used to vary capital utilization, expressed as a fraction of steady-state output.

Equation (2.2) is the Euler equation for consumption. Consumption depends on expected future consumption, on past consumption, through the assumption of habit formation in households' preferences, on current and expected hours of work l_t, on the ex-ante real interest rate ($r_t - E_t\pi_{t+1}$, and on a risk-premium shock ε_t^b. It is perhaps more common in the macro literature to assume a preference shock with the power of shifting the Euler equation; the risk-premium shock has similar implications, but with the advantage of helping the model match the comovement between consumption and investment by entering also in (2.4). The main coefficients of interest in this equation are h, the degree of habit formation in consumption, and σ_c, the inverse of the intertemporal elasticity of substitution in consumption.

Equation (2.3) represents the first-order condition for investment. Current investment depends on lagged and expected investment, and on the value of capital stock q_t. The term ε_t^i represents a disturbance that accounts for investment-specific technological change. The dynamics of the value of capital q_t is characterized by equation (2.4): it depends on its future expected value, on expectations about the rental rate on capital $E_t r_{t+1}^k$, and on the ex-ante real interest rate, adjusted for the risk-premium disturbance. The elasticity of investment to q is governed by the coefficient φ, which represents adjustment costs in investment.

Equation (2.5) denotes a Cobb-Douglas aggregate production function: the technology to produce output requires capital services k_t^s and labor hours, which enter with shares α and $(1-\alpha)$. The term ε_t^a denotes the neutral technology shock, while the coefficient Φ_p accounts for the existence

of fixed costs in production. The model assumes variable capital utilization. As equation (2.6) shows, capital services used to produce output are, therefore, a function of the whole capital stock in the previous period (given the assumption that capital becomes effective after a one-quarter lag) and the capital utilization rate u_t. The degree of capital utilization is varied depending on the rental rate of capital (equation (2.7)); the relation depends on the parameter $0 \leq \psi \leq 1$, which is a positive function of the elasticity of the capital adjustment cost function, but normalized to be between 0 and 1.

The capital accumulation equation (2.8) shows that capital, net of depreciation, changes due to new investment and to the efficiency of these new investment, captured by the investment-specific technology process ε_t^i.

Equations (2.9) and (2.10) summarize the equilibrium in the goods market. Inflation π_t is a function of both lagged and expected inflation, and it depends on the time-varying price mark-up μ_t^p and on the price mark-up shock ε_t^p. The price mark-up μ_t^p equals the difference between the marginal product of labor $(\alpha(k_t^s - l_t) + \varepsilon_t^a)$ and the real wage w_t.

Equation (2.11) shows that the rental rate of capital is a function of the capital to labor ratio and of the real wage.

Equations (2.12) and (2.13) describe the labor market. The wage mark-up μ_t^w captures the difference between the real wage and the marginal rate of substitution between consumption and leisure, given by $\left(\sigma_l l_t + \frac{1}{1-h} \left(c_t - h c_{t-1} \right) \right)$, where σ_l is the inverse of the Frisch elasticity of labor supply. The real wage depends on its past and expected future values, on current, past, and expected inflation, and on the wage mark-up. Wage dynamics is also affected by the wage mark-up disturbance ε_t^w. The importance of the backward-looking terms in the inflation and wage equations are driven by the indexation to past inflation coefficients ι_p and ι_w; the slopes of the curves are an inverse function of the Calvo price and wage stickiness coefficients ξ_p and ξ_w.

Finally, equation (2.14) serves as an approximation of monetary policy decisions in the economy. The monetary authority follows a Taylor rule with partial adjustment, moving the short-term nominal interest rate r_t in response to changes in inflation and the output gap. The Taylor rule is simplified with respect to the one used in Smets and Wouters (2007). Potential output is not defined here as the level of output in the same economy, but under flexible prices. Given that the estimation is complicated by the addition of sentiment shocks, learning, and so forth, I avoid augmenting the state-space with extra equations for the flexible price economy: the output gap is

simply defined here as the deviation of output from a potential level of output driven exclusively by technology.

All exogenous shocks, except the monetary policy shock, which is *i.i.d.*, are assumed to evolve as AR processes as in Smets and Wouters (2007).[4] The government spending shock is allowed to respond to innovations in technology as $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \tilde{\varepsilon}_t^g + \rho_{ga} \tilde{\varepsilon}_t^a$, where $\tilde{\varepsilon}_t^g$ and $\tilde{\varepsilon}_t^a$ are spending and technology innovations and ρ_{ga} is a coefficient to be estimated.

Therefore, the model summarizes the dynamics for fourteen endogenous variables. Smets and Wouters (2007) use observables for seven of the variables. Moreover, there are seven structural disturbances that are unobserved and are obtained by filtering. To the observables and shocks in Smets and Wouters (2007), I will add available observable data on expectations about consumption, investment, and inflation, and expectational, or sentiment, shocks, which are defined in the next section.

3. Relaxing Rational Expectations: Learning and Sentiment

Economic agents in the model form expectations about future aggregate consumption, investment, hours of work, inflation, real wages, the rental rate of capital, and the value of capital. The literature typically assumes that such expectations are formed according the rational expectations hypothesis. Here, we relax the strong informational assumptions imposed by rational expectations to exploit direct, observed, data on expectations and to investigate the role of sentiment on the economy.

Agents are assumed to form expectations using their perceived model of the economy, which is correctly specified, i.e., it has the same structural form as the minimum state variable (MSV) solution under rational expectations. The departure from rational expectations consists of agents' lacking knowledge about the reduced-form model coefficients (for example, they lack knowledge about Calvo coefficients and, as a result, they cannot recover the reduced-form coefficients in the model solution of the system) and of the realizations of the unobserved structural disturbances. Therefore, while the model departs from rational expectations, the departure is usually interpreted as a 'minimal' deviation and the model can be defined as *near*-rational.

---

[4]Smets and Wouters (2007) assume MA(1) components in the price and wage markup shocks. We assume that they also follow AR processes here.

Economic agents use historical data to infer the unknown coefficients over time. They do so by estimating the following perceived law of motion

$$Y_t = a_t + b_t Y_{t-1} + \epsilon_t \tag{3.1}$$

where $Y_t = [c_t, i_t, q_t, l_t, k_t, r_t^k, \pi_t, w_t, r_t]'$, and a_t and b_t are vectors and matrices of coefficients. Restrictions with ones and zeros are used to select variables that do or do not enter the MSV solution (q_t, l_t, and r_t^k do not enter the model in lags and, hence, the corresponding coefficients on their lagged values in b_t equal 0).[5] I assume that agents use data on the endogenous variables to form expectations, but they do not observe the structural disturbances that would also enter the PLM, but that are typically unobserved to the econometrician. I regard this as the most empirically realistic description of the information available to forecasters.

In each period t, agents are assumed to observe values of the endogenous variables up to $t-1$. This assumption is mainly motivated by the need to be consistent with the timing in the Survey of Professional Forecasters: when survey participants are asked in period t for their forecasts for period $t+1$, they can observe historical data only up to $t-1$. The assumption is also typical in the theoretical learning literature as a means to avoid simultaneity issues in self-referential models. Therefore, agents, in each period t, form expectations using observations up to $t-1$ along with their beliefs, which they have previously updated by running regressions of $(t-1)$-dated variables on $(t-2)$-dated variables.

The beliefs are recursively updated following a constant-gain learning algorithm as

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \overline{\mathbf{g}} R_t^{-1} X_t (Y_t - X_t' \widehat{\phi}_{t-1}) \tag{3.2}$$

$$R_t = R_{t-1} + \overline{\mathbf{g}}(X_t X_t' - R_{t-1}) \tag{3.3}$$

where $X_t \equiv \{1, Y_{t-1}\}$, and $\widehat{\phi}_t = [a_t', b_t']'$. Equation (3.2) describes the updating of beliefs regarding the model solution coefficients, while equation (3.3) describes the updating of the corresponding precision matrix R_t.

Given knowledge of the endogenous variables in $(t-1)$ and given the state of recently updated beliefs, observed expectations are assumed to be formed as follows

$$\widehat{E}_{t-1} Y_{t+1} = \left(I + \widehat{b}_{t-1}\right) \widehat{a}_{t-1} + \widehat{b}_{t-1}^2 Y_{t-1} + \widehat{d}\alpha_t, \tag{3.4}$$

where α_t is the vector collecting the different sentiment shocks $\alpha_t = [\alpha_t^c, \alpha_t^i, \alpha_t^\pi]'$, and d is a selection matrix with elements equal to 1 for expectations for which an observable is available and 0 otherwise.

---

[5]No restrictions are imposed on the variance-covariance matrix of ϵ_t.

Expectations can, therefore, be decomposed in two parts. One consists of the endogenous reaction of expectations to the state of the economy, given the agents' learning beliefs: this is the forecast implied by the near-rational learning model (i.e., the right-hand side except $d\alpha_t$). The other consists of the component of expectations that cannot be rationalized as derived as the outcome of a near-rational model. This second components accounts for exogenous movements in expectations, which are unrelated to current and past fundamentals.

Expectations in the model about consumption, investment, and inflation, will be matched to the corresponding observable expectation variables in the empirical analysis. Those observed expectations are assumed to be formed from the near-rational learning model specified above. But in each period, agents are allowed to deviate from the point forecasts that arise from their near-rational model: they can form forecasts that are unduly optimistic or pessimistic, given the state of the economy and their most recent updated beliefs. These deviations, which are the component of expectations that cannot be explained as the outcome of the near-rational learning model, are defined as "sentiment" in the model. Sentiment, therefore, captures exogenous waves of optimism and pessimism, which cannot be explained by existing economic conditions.[6]

4. Sentiment and the Business Cycle: Estimation Approach

4.1. **Observed Expectations and Real-Time Data.** The model is estimated using Bayesian methods to match the following set of observables: Real GDP, Real Consumption, Real Investment, Hours worked, Real Wage, Inflation, Federal Funds rate, Expected Real Consumption, Expected Real Investment, and Expected Inflation. Therefore, to extract and investigate the role of sentiment shocks, I add to the same variables that are used in Smets and Wouters (2007), information on expectations about future consumption, investment, and inflation (I do not exploit forecasts on output and interest rates as these variables don't enter the model in expectations).[7] The structural "deep" parameters, the shock parameters, the learning, and sentiment parameters will all be jointly estimated.

The data on expectations are obtained from the Survey of Professional Forecasters (SPF), hosted by the Federal Reserve Bank of Philadelphia. I use mean expectations across forecasters for levels of consumption, investment, and for the inflation rate. In each period t, agents in the model, in the same way as forecasters in the survey, form expectations about variables in $t + 1$, knowing the

---

[6]For variables for which a corresponding observable series is not available, the expectations in the empirical analysis will be simply equal to those implied by the learning model.

[7]In principle, they could still be used in the estimation, but this is not the approach that I follow here.

values of endogenous variables up to $t - 1$ (forecasters at each t are also asked for their estimate of variables in $t - 1$ and the vast majority of them simply reports the latest BEA data release for variables in $t - 1$). In the SPF, the forecasts that will be used in the empirical analysis correspond to the column *'dVariable3'*, i.e., expectations about values of the variable one-quarter-ahead.

Expectations about consumption correspond to the series "(*Forecasts for the quarterly and annual level of real personal consumption expenditures (RCONSUM)*)". Expectations about investment are obtained by summing over data for nonresidential and residential investment: "(*Forecasts for the quarterly and annual level of real nonresidential fixed investment (RNRESIN)*)" and "(*Forecasts for the quarterly and annual level of real residential fixed investment (RRESINV)*)". These forecasts are available starting from 1981:Q3, which, therefore, is chosen as sample starting date for the main estimation in the paper. Expectations about inflation are calculated from the price level series "(*Forecasts for the quarterly and annual level of the GDP Price Index (PGDP)*)"; the series is available from 1968:Q3. The series have been transformed to maintain the same base year across the full sample.

Given the focus on identifying the learning process of economic agents in real-time and on disentangling the components of expectations that can be rationalized as the outcome of a learning model and those that can be attributed to exogenous sentiment, it is important to mirror as closely as possible the information set of agents available at each point in the sample. Therefore, I choose to use real-time data in the estimation.

For each variable being forecasted, the SPF provides a link to "Real-time data for this variable". I use those to better approximate the information set available to forecasters in real time and as the observable series in the model. The realized data series for each variable are hence obtained from the corresponding Real Time Data Set for Macroeconomists' website, also hosted by the Federal Reserve of Philadelphia, with the exception of the Federal Funds rate (since it's never revised), which is obtained from FRED, the Federal Reserve Economic Database, made available by the Federal Reserve of St. Louis.

For consumption, investment, and inflation, therefore, I use the real-time data series corresponding to the forecasts described above. I use the real-time real GDP series (ROUTPUT) as measure of output. Hours are computed using the total aggregate weekly hours index (H) divided by civilian noninstitutional population (POP). I compute real wages as total wage and salary disbursements, private industries (WSD) divided by total aggregate hours and by the GDP deflator. The definition for wages is somewhat different from the one used in Smets and Wouters (2007). I choose to use a

related definition, for which real-time data are available, rather than the exact series they use, for which real-time data do not exist.

Finally, as measure of the short-term nominal interest rate, I use the Federal Funds rate (FED-FUNDS) from FRED. The annual series is expressed into quarterly rates for the estimation.

The estimation sample spans the years from 1981:III to 2011:I; the starting date is chosen due to the availability of expectations data (available only from 1981:III for consumption and investment expectations). All variables are at quarterly frequency.

The raw variables, before any detrending, that will be used in the estimation are shown in Figure 1.

4.2. **Trends and State-Space System.** The approach departs from most of the literature by estimating directly the model on the levels of the variables, rather than on detrended variables or variables in growth rates: in its most general form, this approach lets the estimation endogenously separate between trend and cycles for all variables. The estimation on raw data is in the spirit of Canova (2012).

I will, however, consider different detrending procedures, some allowing for considerable flexibility in the trend, some much more restrictive, but simpler and more widespread. I will then present as benchmark results those corresponding to the best-fitting option, evaluated using marginal likelihoods.

The state space system can be written as (see Harvey, 1989):

$$Y_t^{OBS} = \bar{H} + T_t + \xi_t \tag{4.1}$$

$$T_t = T_{t-1} + \tau_{t-1} + \nu_t^T \tag{4.2}$$

$$\tau_t = \tau_{t-1} + \nu_t^\tau \tag{4.3}$$

$$\xi_t = A_t + F_t \xi_{t-1} + G_t \omega_t \tag{4.4}$$

where $\xi_t = [Y_t, \widehat{E}_t Y_{t+1}, \varepsilon_t, \alpha_t, Y_{t-1}]'$, $\nu_t^T \sim N(0, \sigma_T^2)$, $\nu_t^\tau \sim N(0, \sigma_\tau^2)$, $\omega_t \sim N(0, \sigma_\omega^2)$.

Equation (4.1) is the measurement equation that relates observed data series to the variables in the model and it endogenously separates between noncyclical, or trend, (T_t), and cyclical components (ξ_t). The vector $\bar{H}$ may contain steady state parameters or simply the sample mean of the variables. The trend is computed through a local linear trend model, which is a simplified version of the even more flexible detrending used in Canova (2012), as exemplified by equations (4.2) and (4.3). The trend is therefore jointly estimated with the rest of the system. Equation (4.2) accounts for changes in the level of the trend, while equation (4.3) accounts for changes in the

slope. Equation (4.4), instead, contains the DSGE model for the cyclical components of the series. Under rational expectations, the equation corresponds to the rational expectation solution of the system (2.1)-(2.14), which has constant coefficients $A_t = A = 0$ and $F_t = F$. With observed expectations and learning, it is obtained by replacing rational expectations with survey expectations, and allowing survey expectations to derive from the near-rational learning model as in (3.4). The vectors and matrices of coefficients are possibly time-varying as a result of agent's learning process, as modeled in (3.2)-(3.3).

Given the uncertainty on how to best model the non-stationary components in macroeconomic variables, I will consider a variety of options. If the standard deviations for ν_T and ν_τ are both zero, the trend specification collapses to linear and deterministic, which could be equivalently expressed as $T_t = \delta_0 + \delta_1 t$. Besides its simplicity, an advantage of the linear trend is that it is probably more likely than more sophisticated alternatives to mimic the trend and cycle decomposition that forecasters had in mind when communicating their survey forecasts over the sample. An extension of the linear trend specification, intended to even better capture the trend estimation by forecasters in real time, consists of adopting a recursive linear trend. In this case, the trend coefficients are estimated using only information from $t = 1$ up to $t = \tau$, at each point τ in the sample. In line with the spirit of the learning approach, when forming expectations, agents use the trend coefficient they have estimated on time series available up to $t - 1$ to forecast variables in $t + 1$. This is the second approach that will be considered in the estimation.

The third option will consist of assuming a stochastic rather than deterministic trend. This case is also nested in the local linear trend model above by setting $\sigma_\tau = 0$. The observables are, therefore, assumed to follow a random walk with drift and the estimation corresponds to an estimation on variables in first-difference.

Finally, I will repeat the estimation for the flexible local linear trend model written above. The unrestricted trend model carries a number of new parameters related to the standard deviations σ_T and σ_τ, which will now be estimated.

In all cases described above, I allow the trends to differ across each variable. In the robustness section, I will impose the restriction that a common trend exists among real variables, as often assumed in the macroeconomic literature. Also to minimize a priori assumptions, I allow trends to potentially matter for each observable variable, including inflation and the interest rates (which actually display declining trends over the sample, but are often treated as stationary in DSGE

estimations). Expectation series are also detrended, and their trends are allowed to differ from those of the corresponding realized variables.

In the unrestricted trend estimations, I will use both a diffuse initialization and a dogmatic one for the trend parameters. In the diffuse case, to minimize the effect of initial conditions, I drop the first three years of data in the calculation of the likelihood.

Finally, the state space above can be written more compactly as:

$$Y_t^{OBS} = \bar{H} + H\Upsilon_t \tag{4.5}$$

$$\Upsilon_t = \mathbf{A_t} + \mathbf{F_t}\Upsilon_{t-1} + \mathbf{G_t}\nu_t \tag{4.6}$$

where $\Upsilon_t = [\xi_t, T_t, \tau_t]'$, $\nu_t = [\omega_t, \nu_t^T, \nu_t^\tau]$.

4.3. **Bayesian Estimation and Priors.** The priors for the model coefficients are shown in Table 1. The majority of prior choices follow Smets and Wouters (2007). There are, however, some differences. The prior for the intertemporal elasticity of substitution is a Gamma with mean 2 and standard deviation 0.5. The degree of habit formation has prior mean 0.5, rather than the higher 0.7 used by Smets and Wouters. The priors for the Calvo coefficients here are Beta with mean 0.7, rather than 0.5, to be more consistent with the recent micro-level evidence on price stickiness (Nakamura and Steinsson, 2008). The shock autoregressive coefficients all follow Beta prior distributions with means equal to 0.5 and standard deviations 0.2. Inverse Gamma distributions are used for shock standard deviations: they have prior means equal to 0.3 in all cases, except the shocks related to investment, which have a mean of 1, given the a priori expectation that exogenous shifts in investment may have higher volatility. For the main learning parameter, I assume a Beta prior with mean 0.025 and standard deviation 0.01, which spans the range of calibrated constant gain parameters used in the theoretical adaptive learning literature.

The model is estimated using full-information Bayesian techniques. Draws are generated using the Metropolis-Hastings algorithm. I run 400,000 draws, discarding the initial 40% as burn-in. The parameter posterior distributions are usually well-behaved. When bimodality exists, I will point it out in the discussion of the results.

4.4. **Initialization of the Agents' Learning Process.** Besides detrending details, another factor that may potentially affect the results is the initialization of the agents' learning process. Again, I consider various alternatives. The preferred initialization can again be chosen on the basis of its ability to fit the data; moreover, I will show later in the paper, that the empirical conclusions are robust to different choices of initial beliefs at the beginning of the sample.

To minimize the influence of initial values on the main estimation results, I first estimate the model for a presample period. The initialization requires full-information Bayesian estimation, since also some unobservable variables, which need to be obtained by filtering, enter the MSV solution. The model is therefore estimated on the 1964:I-1981:II sample (with initial date chosen since labor hours are available from 1964). I report the results for two main options (I considered other related alternatives, without effects on the results). In the first, I estimate the model in the presample period under the assumption of rational expectations. Under this approach, when moving to the main estimation on the second sample (1981-2011), I set the initial beliefs as equal to their rational expectations equilibrium obtained from the presample period, i.e. $\phi_{t=0} = \phi_{RE}$. The precision matrix is similarly initialized as $R_{t=0} = XX'_{RE}$, which is also the value obtained in the presample estimation under rational expectations. Both $\phi_{t=0}$ and $R_{t=0}$ are obtained as means across MH draws in the rational expectations DSGE estimation. The interpretation of this initialization is as follows: agents living in the 1964-1981 period are assumed to have had enough time to converge to the rational expectations equilibrium. The post-1981 sample may be interpreted as a new regime: agents start from their beliefs that they have formed by living in the pre-1981 regime and gradually learn about the new structure of the economy in the second sample.[8]

The second option is more agnostic. I estimate also the model in the presample 1964-1981 period under non-fully rational expectations and learning. The initialization in 1964 is left as uninformative as possible: all variables in the perceived law of motion are assumed to evolve as AR(1) with an autoregressive coefficient equal to 0.9. This choice assigns agents the knowledge that macroeconomic variables are persistent, but it doesn't endow them with information on more complicated dynamic interactions among variables. The learning process is, therefore, given time to update in the presample estimation, and the state of beliefs at the end of the presample (1981:III) is then set as the initial set of beliefs for the main post-1981 estimation. The 70 quarterly periods in the presample estimation provide sufficient time to remove the most severe effects of initial conditions (although whether this is actually happening will be tested in the robustness section).

The benchmark results described in the next section will refer to the estimated model version that delivers the highest marginal likelihood (i.e., the case with linear detrending and initial beliefs derived from presample estimation under learning). Posterior estimates and other results for the

---

[8]We do not follow the practice of starting from RE estimates, since those require estimation over the full sample, which cannot be in the agents' information set in 1964.

full set of estimated specifications will discussed in the robustness section (most estimations have been done, but the results still need to be added in the current version of the paper).

5. Sentiment and the Business Cycle: Empirical Results

5.1. **Business Cycle Evidence under Rational Expectations.** For the sake of comparison, I start by estimating the model under the conventional assumption of rational expectations. Agents have perfect knowledge regarding the model parameters, other agents' preferences and constraints, the distribution of the shocks, and so forth. Expectational errors in this scenario (given that the equilibrium exists and is unique) are simply a function of structural innovations and do not represent an autonomous source of fluctuations in the model.

The estimation under rational expectations is similar to the one in Smets and Wouters (2007), but with the difference that here I use real-time data, rather than revised data. Moreover, I consider various detrending procedures, the sample is limited to the post-1981 period and extended to 2011, and some series definitions differ, given the need here to match the real-time series on realized variables and their forecasts (for example, the wage series is different form the one in Smets and Wouters). There are some minor differences in priors and model specifications.

The parameters estimated for the DSGE model under rational expectations are shown in Table 1. The table reports the results for the detrending choice that provided the best fit of the data, as indicated by the highest marginal likelihood (the simple linear trend). The estimation reveals significant degrees of real frictions, such as investment adjustment costs (with a posterior estimate for $\varphi = 5.96$, which updates the prior toward larger values) and habit formation in consumption ($h = 0.70$), which are necessary to fit the sluggishness of macroeconomic data. Nominal rigidities are also essential: the posterior mean estimate for the Calvo coefficient in price-setting falls on the high side at 0.88, probably as a result of a less restrictive prior (while Smets and Wouters impose a prior with mean 0.5, I allow let here the data free to move to regions with higher price stickiness), and for the Calvo wage-stickiness coefficient is equal to 0.80. Indexation to past inflation is important in wage-setting ($\iota_w = 0.58$), but less so in price-setting ($\iota_p = 0.14$).

Structural disturbances related to government spending and technology are very persistent, with autoregressive coefficients above 0.9. The investment-specific technology shock and the price markup shock are also persistent with autoregressive coefficients equal to 0.68 and 0.73. The wage markup shock has only a limited serial correlation, a result that differs from the corresponding

estimate in Smets and Wouters and that is in large part due to the choice of relaxing the assumption of a common trend between the real wage and other real variables. The posterior mean for the autocorrelation of the risk-premium disturbance is quite low (0.23). The estimation, however, reveals a clear bimodality: one mode is characterized by a very large degree of habit formation in consumption, but a low serial correlation of the exogenous risk-premium shock, the other by a more moderate degree of habit formation, but by a substantially serially-correlated risk premium. Bivariate posterior scatter plots indicate a strong negative relation between the two coefficients. The high habits-low autocorrelation mode, however, achieves higher probability and is, therefore, visited much more often by the MCMC sampler.

Figures 2 and 3 show the impulse responses of output and inflation to selected shocks. Many impulse responses show the usual hump-shaped patterns. Output responds sluggishly to investment-specific, technology, wage markup, and monetary policy shocks. The peak effect for the risk-premium shock happens two quarters after the initial impact, whereas peaks are more delayed for the previous shocks, ranging from four quarters for the investment-specific to eight/ten quarters for the technology and wage markup shocks. Inflation adjusts somewhat more quickly to the shocks.

The variance decomposition for the model with rational expectations is shown in Table 2 (the shares for rational expectations are those shown in brackets under the shares for the learning and sentiment model).

The shock that is responsible for the largest portion of fluctuations is the investment-specific shock, which is the dominant shock at high frequencies, explaining 63% of output variability at horizons below one year, and it is also important at business-cycle frequencies, with a share of the forecast error variance for output of 38.7%; a predominant role for this shock has been found also in Justiniano et al. (2011). Technology shocks are the main contributors at business-cycle horizons: in addition to the investment-specific shock, the Hicks-neutral technology shock accounts for another 40% of fluctuations.

The variance of inflation is mostly driven by the price markup shock at high frequencies, and by investment-specific shocks at lower frequencies, with technology, price, and wage markup shocks also playing a major role.

5.2. **Learning and Sentiment.** I now move to estimate the version of the model that relaxes the stringent informational assumptions imposed by rational expectations. Economic agents form subjective expectation from a near-rational model and can deviate from near-rational forecasts

because of exogenous changes in "sentiment". Observed expectations are used to better identify the economic agents' learning process over the sample and the expectation components that can be attributed to sentiment.

Table 1 shows the posterior estimates for the best-fitting version, which is the one with linear detrending and initial agents' beliefs set in 1981 to those obtained from the presample estimation under learning. To gauge the sensitivity of results to the various assumptions, I will present the estimates for the other detrending approaches and learning initializations in Table 3 (table to be added).

There are three areas in which the results under learning and sentiment provide insights that go beyond traditional results under rational expectations: the role of real frictions, or of the so-called "mechanical" sources of persistence, the response of macroeconomic variables to structural innovations, and the sources of business cycles.

5.2.1. *__Mechanical sources of persistence__*. When direct data on expectations are used to replace rational expectations, the estimation points toward smaller degrees of real frictions that are necessary to fit the persistence in the data. In particular, the posterior mean for the elasticity of the investment adjustment cost function is considerably reduced from $\varphi = 5.96$ under rational expectations to $\varphi = 2.67$ with observed expectations and learning. The lower magnitude of adjustment costs removes some of the delays and sluggishness in the responses of output and investment to shocks. The estimated degree of habit formation in consumption h also falls from 0.70 to 0.48 (the decline is larger, from 0.75 to 0.35, if one looks at posterior modes, rather than posterior means).[9]

Turning to nominal rigidities, the level of price stickiness remains similar between rational and subjective expectations estimations, whereas the estimated wage stickiness is reduced to 0.71, indicating wages that are re-optimized on average every ten months. Wage indexation to past inflation is modestly lower under subjective expectations.

The elasticity of intertemporal substitution in consumption moves closer to unity under learning. The mean estimates for the elasticity of labor supply vary between rational expectations and learning, but, as indicated by the wide 95% credible sets, the uncertainty surrounding their estimation

---

[9]The habit formation estimates vary across detrending choices: the required degree of habit formation under learning and sentiment in this case still remains high, given that the stationary component of consumption remains very persistent if a linear trend is assumed.

is substantial. A key parameter in models with learning is the constant gain: here the gain is estimated equal to 0.013, suggesting that learning by economic agents takes place rather slowly.[10]

One of the main differences in terms of estimation results concerns the properties of some of the shocks: the estimated persistence for the investment-specific shock is reduced from 0.68 to 0.14 and for the price markup shock falls from 0.73 to 0.09. The risk-premium shock is close to i.i.d., with an autoregressive coefficient equal to 0.10. Sentiment shocks are, instead, identified as quite persistent with autocorrelations in the 0.7-0.85 range.

The estimation results are suggestive that subjective expectations and learning help in capturing some of the persistence in macroeconomic data. Figure 4 helps in summarizing the evidence. The figure shows the posterior distribution for selected endogenous and exogenous sources of persistence. The first panel compares the posterior distributions for the investment adjustment cost coefficient obtained for the model under rational expectations and under learning. Given that endogenous and exogenous sources of persistence can be interchangeable for some variables, the second and third panels show the posterior distributions for the sum of the coefficient capturing the endogenous mechanism and the autoregressive coefficient for the exogenous shock, instead of distributions for single coefficients. The second panel refers to sources of persistence in consumption (habits plus the serial correlation of the risk-premium disturbance) and the third to sources of persistence in inflation (endogenous indexation to past inflation plus serial correlation in the price markup shock). The posterior distributions indicate that large degrees of structural and exogenous persistence are needed to fit the data under the assumption of rational expectations. If rational expectations are replaced by observed expectations in a model with learning, there is less need for additional sources of persistence: the relevant posterior distributions all shift to the left.[11]

5.2.2. ***Responses to structural and sentiment shocks***. Figures 4 and 5 overlap the responses of output and inflation to some of the most influential structural and sentiment disturbances (given that impulse responses are time-varying under learning, to simplify the presentation in the graph, I report average impulse responses over the sample). The figures show the mean impulse responses across the last 50,000 MCMC draws, along with error bands corresponding to the 5th and 95th percentiles.

---

[10]A constant gain equal to 0.013 means that agents weigh the current observation as 1, the $t-1$ observation as 0.987, the $t-2$ observation as 0.987^2, and so forth.

[11]I will need to insert the posterior distribution for the investment-specific shock serial correlation in this graph as well.

The first set of impulse responses shows that output responds rather quickly to structural innovations. The response to the government spending and risk premium shocks reach their peak effects on impact, while the investment-specific shock generates a peak after only one quarter. Particularly for the case of the risk-premium and investment-specific shocks, the effects are transmitted more quickly to the economy in the estimation that uses subjective rather than rational expectations.

Sentiment shocks, instead, produce longer adjustments. The sentiment shock related to investment leads to a larger and more persistent response of output compared with the corresponding investment-specific structural shock. The output response is hump-shaped with stronger effects between one and two years after the initial impact. The magnitude of the effect for the consumption sentiment shock is roughly similar to the magnitude for the risk-premium shock, except for short-horizons, where the risk-premium dominates. Both sentiment shocks lead to sluggish adjustment in output with more forceful effects that are delayed by at least one year.

Figure 5 displays the impulse response functions for inflation. The top panel shows the responses to the cost-push (price-markup and wage-markup) and inflationary sentiment shocks. The bottom panel shows the responses to the neutral and investment-specific technology shocks and to the two demand-related sentiment shocks. The price markup shock leads to a large immediate response in inflation, but the adjustment is very quick. Fluctuations in inflation over the medium term are mostly driven by the sentiment about future inflation pressures shock, with the wage markup shock playing a role at longer horizons. Technology shocks lead to a negative sluggish response in inflation. Sentiment about aggregate demand, however, likely plays an even more important role over the business cycle by producing persistent adjustments in inflation.

5.2.3. **_Sources of Business Cycles_**. What are the main drivers of business cycle fluctuations? The literature is divided between explanations focused on technology shocks and explanations based on demand shocks. On the other hand, shifts in expectations that are unrelated to fundamentals, psychological forces and market sentiment, waves of optimism and pessimism, typically receive a zero weight as drivers of fluctuations in state-of-the-art general equilibrium macroeconomic models.

By relaxing the assumption of rational expectations and using data on observed expectations, this paper can test the contribution of sentiment to aggregate fluctuations.

Table 2 shows the forecast error variance decomposition for short-run (here 0 to 4 quarters) and business cycle frequencies (here 4 to 24 quarters, but results were similar for a definition based on 6 to 32 quarters).[12]

---

[12][Full table with the addition of wage, hours, interest rates, to be added in the next draft].

Sentiment shifts are indeed a major contributor of business cycle fluctuations. The ensemble of sentiment shocks explains 43.7% of output fluctuations. The most important driver of output fluctuations at business cycle frequencies appears to be the sentiment shock related to investment expectations, which accounts by itself for 35% of the variance. The structural investment-specific technology shock is dominant, among the remaining shocks, accounting for about 20% of fluctuations. The key role of sentiment linked to investment decisions is clearly reminiscent of Keynes' animal spirits, which he also discussed in association to entrepreneurs' investment behavior.Sentiment shocks explain 60% of the variability of investment and less than 40% of the variability in consumption.

Inflation is also largely driven by sentiment shifts. Inflation sentiment is dominant over business cycle horizons, accounting for about a third of the inflation forecast error variance.

While sentiment shocks are particularly important at business cycle frequencies, they also play a role in creating noise at higher frequencies. Sentiment explains between 21% and 35% of short-run fluctuations in the same variables, with investors' sentiment again playing the largest role, among the sentiment shocks, for movements in output. Consumers' sentiment also matters, accounting for a third of consumption variability in the short-run. Some of the structural shocks have become less persistent in the model with observed expectations. As a result, they are mostly important at horizons below one year: the risk premium shock is the main determinant of short-run consumption movements, the investment-specific shock is the main determinant of short-run investment, and the price markup shock is the main determinant of short-run inflation. Government spending and investment-specific innovations are the main drivers of output variability at horizons below one year.

The empirical results seem to suggest that structural shocks are important, but they have a large and immediate impact on the economy, rather than a prolonged one. At business cycle frequencies, sentiment becomes a major source of fluctuations.

One of the most striking differences between the conclusions in the model with rational expectations and in the model with subjective expectations and sentiment is given by the role of technology shocks. When exogenous shifts in expectations due to sentiment are permitted in the model, sentiment accounts for a large share of cyclical fluctuations in consumption, investment, and output. The two technology shocks account for about a quarter or less of their changes. But when we follow the previous literature by shutting down sentiment and imposing rational expectations (and hence implicitly assuming that any learning that may have taken place has already converged to

the rational expectations equilibrium), the contribution of technology jumps to levels around 80%
of fluctuations, in line with the RBC literature assertion, to capture the now omitted role of sentiment. For inflation, technology and markup shocks rise to close the gap created by the omission
of sentiment.

Overall, the results show that macroeconomic models may miss an important channel by removing, by assumption, sentiment, or similar psychological forces, from their analyses.

5.3. **But Is It Really Sentiment?** In the estimation, we have identified sentiment as the component of expectations that cannot be rationalized as coming from a near-rational forecasting model,
which allows for learning by economic agents. But sentiment in the model is obtained without
using any data and information that may reflect actual sentiment, optimism or pessimism, degree
of confidence, and so forth, in the economy. Can the new disturbances be really interpreted as
sentiment then? In this section, we provide evidence that the identified sentiment shocks are really
related to excess optimism and pessimism in expectations about the future state of the economy,
even if no sentiment data were used in their calculation.[13]

The sentiment shocks about future expected consumption and investment are shown in Figure
7. The shaded areas denote NBER recession dates. The graphs show that there are large shifts
toward pessimism in correspondence to recession periods (in most cases, the sentiment indicator's
change of direction toward pessimism leads the corresponding NBER recession). An interesting
period in the graph corresponds to the dot-com boom years: the investor sentiment disturbance
persistently shifts toward increased excess optimism over this period.

More importantly, is there any evidence that our identified disturbances reflect aggregate market
sentiment, rather than unrelated factors?

Figure 8 shows that our sentiment shocks indeed capture exogenous shifts in aggregate optimism
and pessimism. Using the available survey indicators of sentiment can be informative. The figure
shows scatter plots between the i.i.d. innovation to consumption sentiment (left panel) and to
investment sentiment (right panel) obtained from the DSGE model estimation, and the i.i.d. innovations obtained by regressing survey data on consumer sentiment (University of Michigan Consumer Sentiment) on a vector of endogenous variables (detrended output, inflation, interest rates,
left panel) and by regressing a variable given by the Conference Board indicator of respondents
expecting better business conditions in the six months ahead minus the indicator of respondents

---

[13]In the robustness section, the estimation will be repeated using observable sentiment indicators from surveys to
further discipline the extraction of sentiment shocks.

expecting worse business conditions in the six months ahead (right panel) on the same vector. The innovations from the survey sentiment regressions have been sometimes interpreted, when included in a VAR, as animal spirits, or confidence, shocks.

The scatter plots show that there is a strong positive relation between our model-based sentiment shocks and the sentiment innovations obtained using survey data, purified from their contemporaneous dependence on macroeconomic variables. The correlation coefficients between the two series are in the 0.4-0.5 range.

6. Robustness Analysis

I will add estimation and variance decomposition results for the following sensitivity checks:

- Add Measurement error in expectations. Test measurement error versus structural interpretation as sentiment.
- Different wage and hour series (not constructed from real time data, but more similar to SW)
- Common trend among real variables (much worse fit, but more comparable with literature).
- Different initial beliefs for learning.
- Endogenize Sentiment (add dependence on structural innovations)
- Use survey sentiment index data to inform estimation of sentiment disturbances.
- VAR(2) to better approximate unknown MSV solution
- Agents observe structural disturbances as under RE (MSV solution)
- Assume forecasters have time-t information set rather than time-$t-1$.
- Use data on government spending to better identify govt spending shock. Now I believe it picks up a lot of what should be measurement error in the national accounting identity.

7. Conclusions

The role of psychological factors in booms and busts has been emphasized in the early stages of economic theory by economists of the caliber of Keynes and Pigou and it still prominently features in discussions about business cycles by economic observers. Yet, current macroeconomic theory, and particularly empirical work in macroeconomics, have taken another route and typically abstract from psychology almost entirely.

This paper suggested an approach to reintroduce psychology at the center of macroeconomic analysis, by modeling 'sentiment' in a microfounded DSGE model of the U.S. economy. The paper's

main objective was to investigate whether the typically omitted sentiment matters for aggregate fluctuations.

The empirical results indeed show that the literature should probably take sentiment and psychological elements more seriously. Sentiment shocks are found to explain roughly forty percent of U.S. output fluctuations at business cycle horizons. The main contributor to fluctuations is, in particular, sentiment associated to expectations regarding future investment decisions. Sentiment also explains a large portion of the variability in inflation rates.

References

[1] Beaudry, P., and F. Portier, 2006. "Stock Prices, News, and Economic Fluctuations," *American Economic Review*, vol. 96(4), pages 1293-1307.

[2] Benhabib, J. and R.E.A. Farmer, 1999. 'Indeterminacy and Sunspots in Macroeconomics', in (J.B. Taylor and M. Woodford, eds.), *Handbook of Macroeconomics*, vol. 1, part 1, pp. 387-448, Amsterdam: Elsevier.

[3] Bullard, J., Evans, G. and Honkapohja, S. (2008). 'Monetary policy, judgment and near-rational exuberance', *American Economic Review*, vol. 98, pp. 1163-77.

[4] Canova, F., 2012. "Bridging cyclical DSGE models and the raw data", mimeo, UPF.

[5] Christiano, L.J., Eichenbaum, M., and C.L. Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113, 1-45.

[6] Del Negro, M. and S. Eusepi, 2011. "Fitting Observed Inflation Expectations", *Journal of Economic Dynamics and Control*, 35(12), 2105-2131.

[7] Evans, G. W. and Honkapohja, S. (2001). *Learning and Expectations in Economics.* Princeton: Princeton University Press.

[8] Harvey, A., 1989. *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press.

[9] Jaimovich, N., and S. Rebelo, 2009. "Can News about the Future Drive the Business Cycle?," *American Economic Review*, vol. 99(4), 1097-1118.

[10] Justiniano, A., Primiceri, G.E., and A. Tambalotti, 2010. "Investment Shocks and Business Cycles," *Journal of Monetary Economics*, vol. 57(2), pages 132-145.

[11] Keynes, J.M., 1936. *The General Theory of Employment, Interest and Money.* London: MacMillan and Co.

[12] Mansky, C.F., 2011. "Interpreting and Combining Heterogeneous Survey Forecasts", in M. Clements and D. Hendry (editors), *Oxford Handbook on Economic Forecasting*, Chapter 16, 457-472, Oxford: Oxford University Press.

[13] Milani, F., 2007. "Expectations, Learning and Macroeconomic Persistence", *Journal of Monetary Economics*, 54(7), 2065-2082.

[14] Milani, F., 2011. "Expectation Shocks and Learning as Drivers of the Business Cycle", *Economic Journal*, 121(552), 379-401.

[15] Milani, F., 2012a. "The Modeling of Expectations in Empirical DSGE Models: a Survey", *Advances in Econometrics*, 28, 3-38.

[16] Milani, F., and A. Rajbhandari, 2012a. "Expectation Formation and Monetary DSGE Models: Beyond the Rational Expectations Hypothesis", *Advances in Econometrics*, 28, 253-288.

[17] Milani, F., and A. Rajbhandari, 2012b. "Observed Expectations, News Shocks, and the Business Cycle", Working Paper 12-13-05, UC Irvine.

[18] Milani, F., and J. Treadwell, 2012. "The Effects of Monetary Policy "News" and "Surprises"", *Journal of Money, Credit and Banking*, 44(8), 1667-1692.

[19] Nakamura, E., and J. Steinsson, 2008. "Five Facts about Prices: A Reevaluation of Menu Cost Models," *The Quarterly Journal of Economics*, 123(4), 1415-1464.

[20] Ormeno, A., 2011. 'Disciplining Expectations: Using Survey Data in Learning Models', Working Paper, Universitat Pompeu Fabra.

[21] Pesaran, M.H., and M. Weale, 2006. "Survey Expectations," *Handbook of Economic Forecasting*, Amsterdam: Elsevier.

[22] Pigou, A.C., 1927. *Industrial Fluctuations*, London: MacMillan.

[23] Schmitt-Grohé, S., and M. Uribe, 2012, "What's News in Business Cycles?", *Econometrica*, 80(6), 27332764.

[24] Smets, F., and R. Wouters, 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 1(5), 1123-1175.

[25] Smets, F., and R. Wouters, 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586-606.

	Prior Distributions	**Posterior Distributions**	
Param.		Rational Expectations	Sentiment Shocks
φ	$\Gamma(4, 1.5)$	5.96 [3.83,8.41]	2.67 [1.94,3.74]
σ_c	$\Gamma(2, 0.5)$	1.65 [1.26,2.10]	1.02 [0.84,1.22]
h	$B(0.5, 0.15)$	0.70 [0.44,0.81]	0.48 [0.31,0.66]
σ_l	$\Gamma(2, 0.75)$	1.27 [0.47,2.70]	1.59 [0.57,2.95]
ξ_w	$B(0.7, 0.1)$	0.80 [0.69,0.93]	0.71 [0.54,0.87]
ξ_p	$B(0.7, 0.1)$	0.88 [0.78,0.99]	0.89 [0.81,0.95]
ι_w	$B(0.5, 0.15)$	0.58 [0.36,0.80]	0.47 [0.24,0.72]
ι_p	$B(0.5, 0.15)$	0.14 [0.04,0.43]	0.23 [0.08,0.47]
ψ	$B(0.5, 0.15)$	0.84 [0.71,0.94]	0.84 [0.71,0.94]
$\Phi_p - 1$	$\Gamma(0.25, 0.12)$	0.38 [0.20,0.66]	0.55 [0.29,0.83]
ρ_r	$B(0.75, 0.1)$	0.83 [0.78,0.87]	0.83 [0.75,0.90]
χ_π	$N(1.5, 0.25)$	1.74 [1.24,2.13]	1.49 [1.09,1.95]
χ_y	$N(0.125, 0.05)$	0.06 [0.02,0.10]	0.06 [0.02,0.11]
ρ_g	$B(0.5, 0.2)$	0.95 [0.92,0.97]	0.95 [0.91,0.98]
ρ_b	$B(0.5, 0.2)$	0.23 [0.04,0.78]	0.10 [0.02,0.22]
ρ_i	$B(0.5, 0.2)$	0.68 [0.54,0.81]	0.14 [0.03,0.31]
ρ_a	$B(0.5, 0.2)$	0.97 [0.93,0.99]	0.95 [0.90,0.98]
ρ_p	$B(0.5, 0.2)$	0.73 [0.14,0.91]	0.09 [0.02,0.23]
ρ_w	$B(0.5, 0.2)$	0.18 [0.05,0.35]	0.13 [0.03,0.26]
ρ_{ga}	$N(0.5, 0.25)$	0.35 [0.17,0.52]	0.34 [0.15,0.52]
σ_g	$\Gamma^{-1}(0.3, 1)$	0.61 [0.53,0.68]	0.60 [0.53,0.69]
σ_b	$\Gamma^{-1}(0.3, 1)$	0.30 [0.14,0.38]	0.68 [0.60,0.77]
σ_i	$\Gamma^{-1}(1, 5)$	0.74 [0.59,0.93]	2.00 [1.77,2.30]
σ_a	$\Gamma^{-1}(0.3, 1)$	0.59 [0.52,0.68]	0.58 [0.51,0.66]
σ_p	$\Gamma^{-1}(0.3, 1)$	0.12 [0.08,0.19]	0.26 [0.23,0.30]
σ_w	$\Gamma^{-1}(0.3, 1)$	0.46 [0.38,0.56]	0.96 [0.84,1.11]
σ_ε	$\Gamma^{-1}(0.3, 1)$	0.17 [0.15,0.19]	0.20 [0.18,0.23]
ρ_{α_c}	$B(0.5, 0.2)$		0.70 [0.57,0.83]
ρ_{α_i}	$B(0.5, 0.2)$		0.85 [0.77,0.93]
ρ_{α_π}	$B(0.5, 0.2)$		0.74 [0.62,0.86]
σ_{α_c}	$\Gamma^{-1}(0.3, 1)$		0.51 [0.45,0.59]
σ_{α_i}	$\Gamma^{-1}(1, 5)$		1.43 [1.26,1.63]
σ_{α_π}	$\Gamma^{-1}(0.3, 1)$		0.12 [0.10,0.13]
$\overline{\mathbf{g}}$	$B(0.025, 0.01)$		0.013 [0.01,0.017]

Table 1 - Prior distributions and Posterior estimates, baseline model.

Note: Γ denotes Gamma distribution, B denotes Beta distribution, N denotes Normal distribution, Γ^{-1} denotes Inverse Gamma distribution, and U denotes Uniform distribution. The prior distributions are expressed in terms of mean and standard deviation, except for the Uniform prior, for which lower and upper bounds are shown. Posterior means and 95% credible intervals have been calculated over 200,000 Metropolis-Hastings draws, discarding a burn-in of 40% draws. The sample is 1981:III-2011:I.

	ε_t^g	ε_t^b	ε_t^i	ε_t^a	ε_t^p	ε_t^w	ε_t^r	α_t^c	α_t^i	α_t^π	% Sentiment
$horizon =$ _Short-Run_ (0-4)											
y_t	0.279 [0.095]	0.104 [0.084]	0.256 [0.628]	0.019 [0.080]	0.050 [0.078]	0.010 [0.020]	0.063 [0.014]	0.051	0.155	0.007	**21.3%**
c_t	0.001 [0.001]	0.421 [0.378]	0.019 [0.265]	0.001 [0.141]	0.058 [0.144]	0.008 [0.034]	0.136 [0.037]	0.339	0.005	0.004	**34.8%**
i_t	0.001 [0.008]	0.007 [0.011]	0.571 [0.899]	0.001 [0.019]	0.037 [0.046]	0.009 [0.013]	0.022 [0.005]	0.002	0.334	0.009	**34.5%**
π_t	0.001 [0.010]	0.021 [0.005]	0.002 [0.022]	0.010 [0.066]	0.652 [0.769]	0.011 [0.121]	0.002 [0.006]	0.013	0.001	0.280	**29.4%**
$horizon =$ _Business Cycle_ (4-24)											
y_t	0.119 [0.017]	0.015 [0.008]	0.191 [0.387]	0.065 [0.404]	0.065 [0.119]	0.019 [0.057]	0.081 [0.008]	0.025	0.347	0.065	**43.7%**
c_t	0.023 [0.026]	0.043 [0.019]	0.112 [0.279]	0.052 [0.484]	0.112 [0.123]	0.025 [0.059]	0.227 [0.010]	0.126	0.165	0.107	**39.8%**
i_t	0.007 [0.073]	0.014 [0.002]	0.246 [0.660]	0.015 [0.139]	0.044 [0.081]	0.019 [0.040]	0.037 [0.004]	0.036	0.526	0.047	**60.9%**
π_t	0.009 [0.054]	0.064 [0.005]	0.077 [0.398]	0.045 [0.149]	0.062 [0.250]	0.097 [0.130]	0.062 [0.015]	0.118	0.134	0.325	**57.7%**

Table 2 - Forecast Error Variance Decomposition.   The main entries refer to the model estimation with observed expectations, learning and sentiment. The numbers below in brackets refer to the estimation under rational expectations.
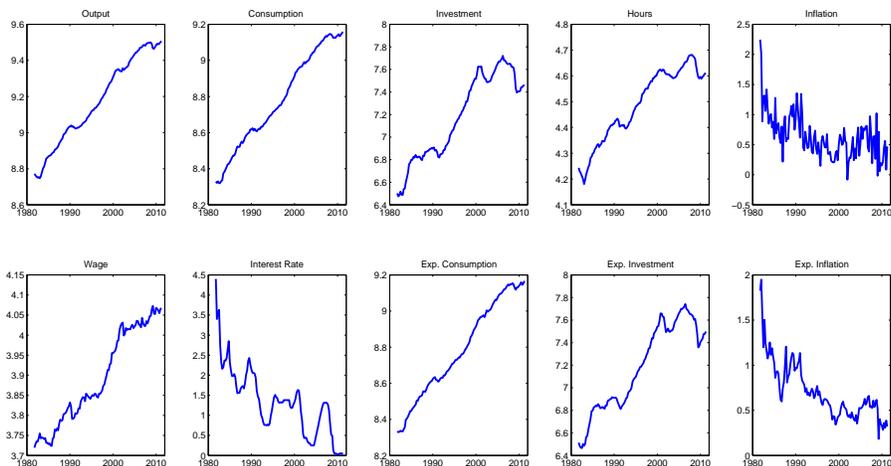
FIGURE 1. Raw Data Series. Output, consumption, investment, wage, consumption expectations, investment expectations are all expressed in real terms and are reported in log levels; hours are also in log levels. The interest rate is levels and converted to quarterly, while inflation and expected inflation are obtained as the log first difference of the price level. All data correspond to real-time vintages (the series were redefined to maintain the same base year across the sample, when necessary.
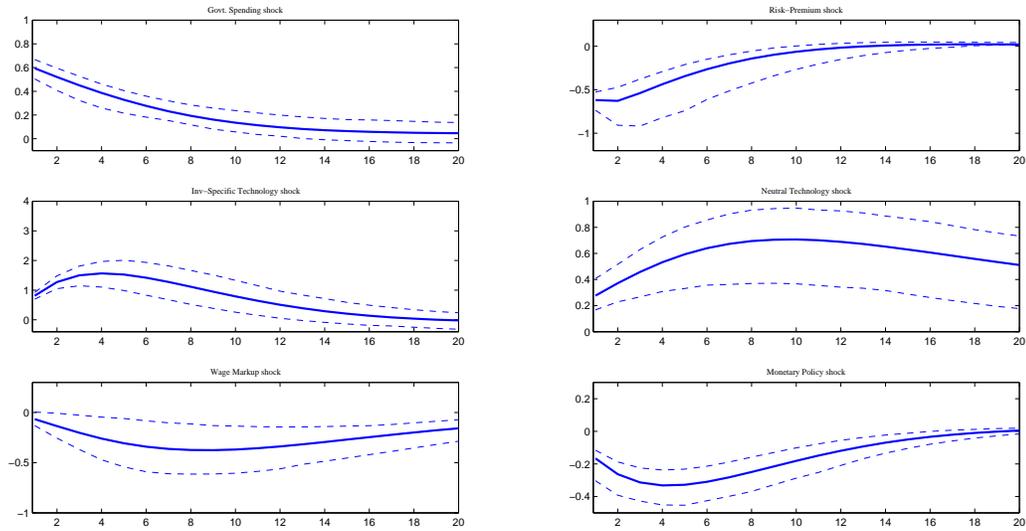
FIGURE 2. Impulse response functions of output y_t to government spending, risk-premium, investment-specific technology and neutral technology shocks, wage markup shocks, and monetary policy shocks, under rational expectations. The graphs show mean impulse responses across MCMC draws, along with 5% and 95% percentile error bands.
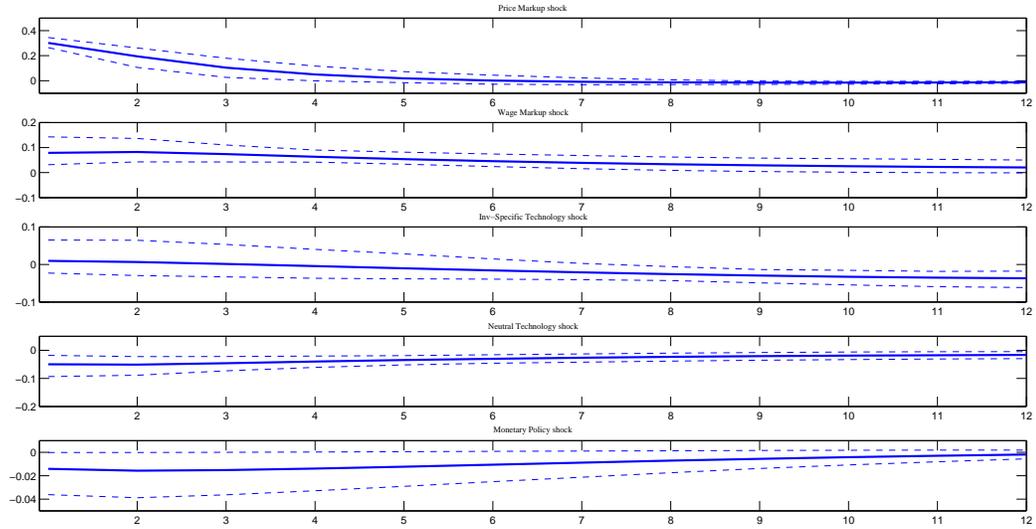
FIGURE 3. Impulse response function of π_t to price markup, wage markup, investment-specific, neutral technology, and monetary policy shocks, under rational expectations. The graphs show mean impulse responses across MCMC draws, along with 5% and 95% percentile error bands.
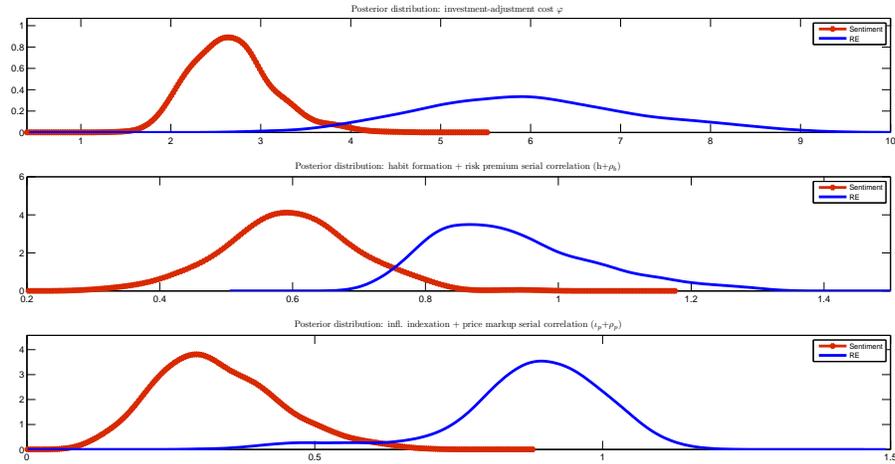
FIGURE 4. Posterior Distributions: Endogenous and Exogenous Sources of Persistence. Note: the top panel shows the posterior distributions for the investment-adjustment cost coefficient. The red line refers to the distribution obtained from the model with subjective expectations, learning, and sentiment, the blue line to the distribution obtained from the rational expectations estimation. The middle panel shows the posterior distribution for the sum of the habit formation in consumption coefficient and the serial correlation of the risk-premium shock. The bottom panel shows the posterior distribution for the sum of the inflation indexation coefficient and the serial correlation of the price markup shock.
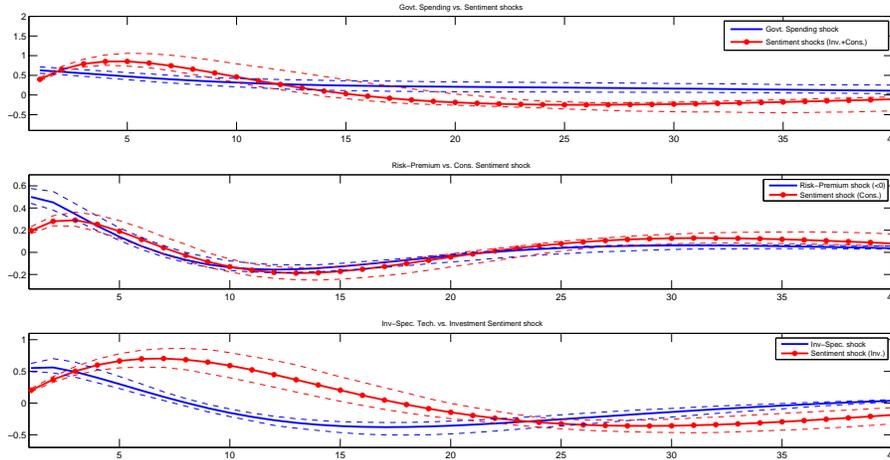
FIGURE 5. Impulse response function of y_t to structural and sentiment shocks, under observed expectations. The structural shocks considered are government spending, risk-premium, and investment-specific. The sentiment shocks are those related to consumption and investment expectations.
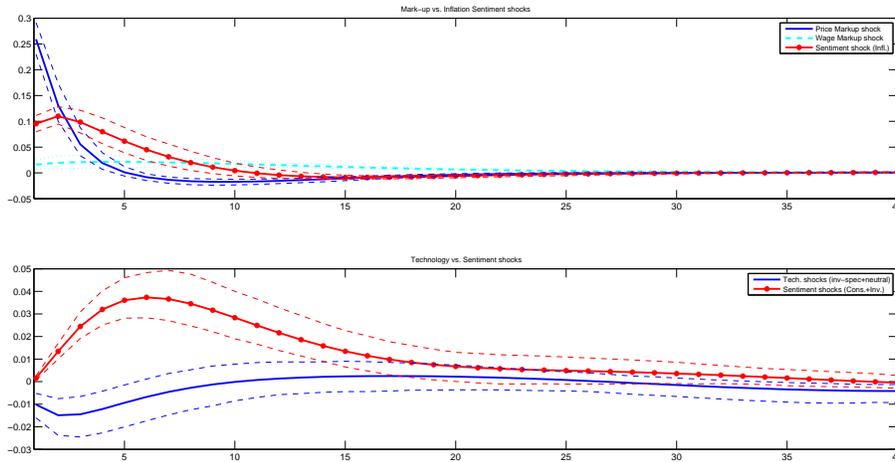
FIGURE 6. Impulse response function of inflation to structural and sentiment shocks, under observed expectations. The top panel shows the responses to price markup, wage markup, and inflation sentiment shocks. The bottom panel shows the sum of the responses to the investment-specific and technology shocks, together with the sum of the responses to the consumption and investment sentiment shocks.
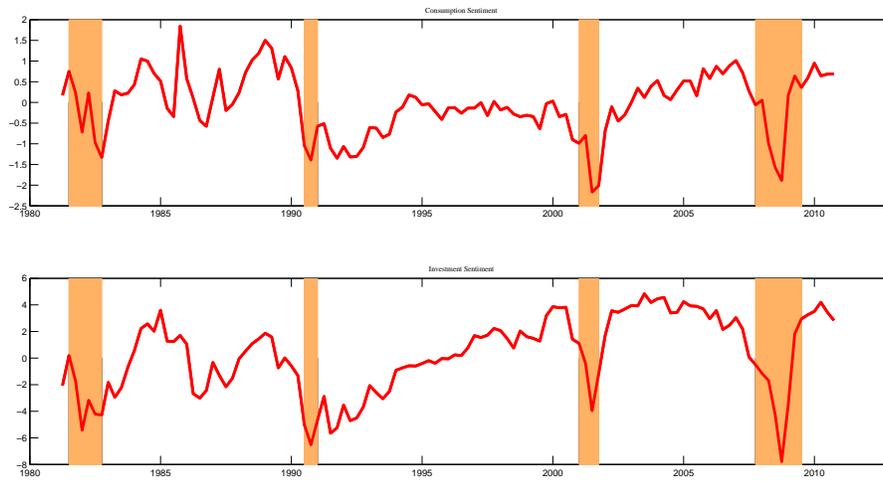
FIGURE 7. Sentiment series. The figure shows filtered series, average across MCMC draws. The top panel shows the sentiment related to consumption expectations; the bottom panel the sentiment related to investment expectations. Shaded yellow areas denote recessions as dated by the NBER recession dating committee.
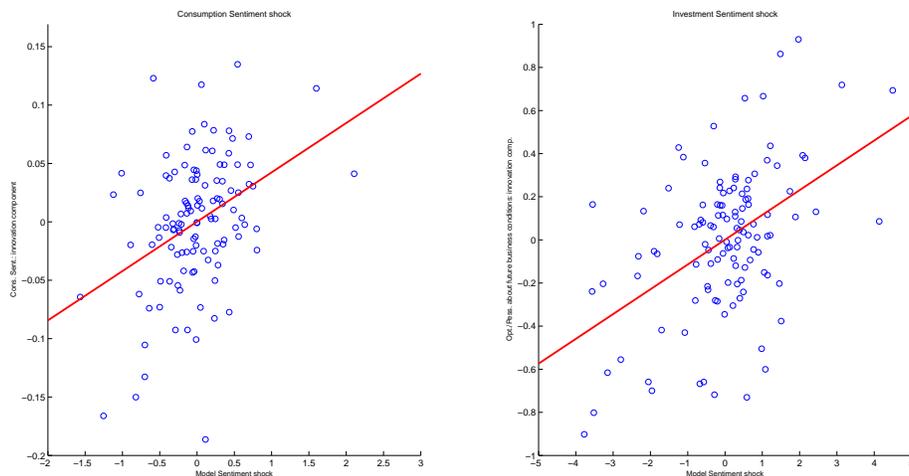
FIGURE 8. Scatter plots: sentiment shocks from DSGE model estimation versus innovations from actual survey sentiment indicators. The left scatter plot shows the relation (with regression line) between the DSGE-model-based consumption sentiment innovation and the innovation component in the University of Michigan Consumer Sentiment series (obtained by regressing the sentiment index on current economic indicators, as output, inflation, interest rates). The right scatter plot shows the relation between the DSGE-model-based investment sentiment innovation and the innovation component in the Conference Board percentage of respondents expecting better business conditions in the next six months and the percentage expecting worse business conditions in the next six months (obtained by regressing the dependent variable on the same vector of economic indicators).