Welfare Consequences of Sustainable Finance*

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Abstract

In lieu of carbon taxes to address global warming, sustainable investment mandates are proposed to incentivize firms to achieve net-zero emissions. With underspending on mitigation due to externalities, we model the welfare consequences of these investments in firms that qualify by spending enough on decarbonization technologies. Our dynamic stochastic general equilibrium model generates several testable predictions. A cost-of-capital wedge between qualified and unqualified firms equals firm mitigation divided by its Tobin's q. Given projections of global warming and cost of decarbonization technologies, we calculate the mandate size and cost-of-capital wedge needed to meet net-zero targets.

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1 Introduction

In light of the failure to implement carbon emissions taxes to address global warming, there is growing pressure on the financial and corporate sectors to provide alternative solutions. One widely-discussed solution is sustainable investment mandates to incentivize firms to meet net-zero emissions by either 2030 or 2050, consistent with the goals of the 2015 Paris Climate Agreement. These mandates are typically implemented as passive screens whereby a fraction of wealth is restricted to invest in companies that meet certain sustainability guidelines.[1] To meet net-zero targets and to qualify to be held in investors' sustainable portfolios, firms have to spend enough on decarbonization measures.

According to a recent Intergovernmental Panel on Climate Change (IPCC) special report (Rogelj et.al. (2018)), most mitigation pathways to net-zero emissions require a portfolio of decarbonization technologies, including renewables and negative emission technologies (NETs) such as afforestation and reforestation, soil carbon sequestration, bioenergy with carbon capture and storage (BECCs), and direct air capture (DAC).[2] The costs of these existing technologies—while currently high—are expected to fall over the next thirty years.

Support for such a solution comes from both private financial actors, including large asset managers such as Blackrock, and public ones, encompassing sovereign wealth funds and pension plans. Recently, central banks have indicated interest in similar mandates, as reflected in the Network for Greening the Financial System (NGFS). Despite their widely publicized pledges, there is still considerable skepticism due to the ambiguity of these mandates. Indeed, the Biden administration is pressing the financial community to "disambiguate" sustainable finance, i.e., to clarify these commitments.[3]

To address this issue, we seek to quantify the amount of capital that has to be dedicated to these mandates along with the costs to shareholders in order for the industrial

[1]According to US SIF Foundation in January 2019, around 38% of assets under management already undergo some type of sustainability screening and over 80% of these screens as implemented as passive portfolios.

[2]One reason is that for heavy industrial sectors like cement and steel, which generate nearly 20% of global CO2 emissions, switching fuel sources is not a viable option for achieving net-zero (de Pee (2018)).

[3]Zac Colman, 03/12/2021 on *Politico.com* "Kerry to Wall Street: Put your money behind your climate PR."

sector to meet net-zero emissions targets. Whereas integrated assessment models (Nordhaus (2017)) embed the global warming externality in emissions to analyze the social cost of carbon, we consider an economy where capital is the only input of production and there is an externality when it comes to corporate mitigation spending in order to evaluate the social benefits of decarbonization spending. Since rising global temperatures result in more frequent and destructive weather disasters that damage capital stock (National Academy of Sciences (2016)),[4] decarbonization spending, which comes at the expense of firm productivity, reduce the expected losses from these disasters by keeping global temperatures from rising.

Because the benefits of this mitigation only affect the aggregate risk and the market price of risk, which firms take as given, there is over-investment and over-accumulation of capital and underspending on decarbonization. While a carbon emissions tax is required to curtail emissions in integrated assessment models of emissions curtailment (Golosov, Hassler, Krusell and Tsyviski (2014)), a tax on capital is needed in our model to fund mitigation to address this market failure and achieve first-best (Hong, Wang and Yang (2020)). Like a carbon emissions tax in traditional integrated assessment models, a capital tax in our model can be quantitatively significant since weather disasters cause significant welfare losses (Rietz (1988), Barro (2006), Weitzman (2009), and Pindyck and Wang (2013)) for households with non-expected utility (Epstein and Zin (1989) and Weil (1990)).

Instead of a capital tax to achieve first best, we consider restrictions on the representative investor's portfolio, i.e., sustainable finance mandates. The representative investor, who has access to a complete set of financial securities (e.g., all contingencies including idiosyncratic shocks are dynamically spanned), is restricted to passively index a fixed fraction of total wealth to firms that meet sustainability guidelines. To be included in the representative investor's sustainable portfolio, otherwise ex-ante identical unsustainable firms have to spend

[4]For instance, climate models point to increased frequency and damage from hurricanes that make landfall (Grinsted, Ditlevsen, and Christensen (2019), Kossin et.al. (2020)). Similarly, the wildfires in the Western US states are also linked to climate change (Abatzoglou and Williams (2016)). See Bansal, Ochoa, and Kiku (2017) for the impact of higher temperature on growth stocks and Hong, Karolyi, and Scheinkman (2020) for a review of evidence on the damage of natural disasters for financial markets.

a minimum amount on decarbonization which they otherwise would not due to externalities.

The cost of capital and firm value for sustainable and unsustainable firms are endogenously determined so as to leave value-maximizing firms indifferent between being sustainable and not — the Tobin's q or stock price is the same for all firms in equilibrium. The risk-free rate, stock-market risk premium, Tobin's q for aggregate capital, and growth rates are jointly determined in equilibrium. Despite being a dynamic stochastic general equilibrium model, the solution is intuitive and offers several testable restrictions.

Since firms have the same Tobin's q, the investment and growth paths of both sustainable and unsustainable firms are identical (path by path) over time. A firm's Tobin's q is given by a Gordon Growth formula: compared to unsustainable firms, sustainable firms have lower cashflows to pay out (a lower numerator) due to mitigation spending but have a smaller denominator due to a lower cost of capital (the expected return required by the representative investor). The cash-flow effect and the discount-rate effect have to exactly offset each other so as to leave all firms indifferent between being a sustainable and an unsustainable firm.

Hence, the equilibrium cost-of-capital wedge between sustainable and unsustainable firms is equal to a sustainable firm's mitigation spending divided by its Tobin's q. The lower cost of capital for sustainable firms subsidizes their decarbonization, which they would have otherwise invested or distributed to shareholders. The benefits of this mitigation accrue to the entire economy. For a given fraction of wealth allocated to sustainability mandates, we calculate the welfare-maximizing qualification criterion. A higher fraction of wealth allocated to sustainable firms leads to more unsustainable firms becoming sustainable, higher aggregate mitigation spending, and higher welfare, therefore moving the competitive economy closer to the first-best.

An earlier generation of papers considered the real effects of ethical and socially responsible investing. Notably, Heinkel, Kraus, and Zechner (2001) examine the effects of green portfolio restrictions using a static constant absolute risk aversion (CARA) framework with no capital accumulation, an exogenous interest rate, and a given fixed cost that brown firms can pay to become green. Hong and Kacperczyk (2009) find that ethical investing mandates indeed generate cost-of-capital wedges.

But to conduct welfare calculations when it comes to firm mitigation of global warming, it is desirable to have a dynamic stochastic general equilibrium model where qualification criterion based on decarbonization spending, interest rates, and capital accumulation are endogenous. Hence, our paper combines integrated assessment models of climate change, typically done in a social planner setting, with richer competitive financial markets to evaluate the viability of sustainable finance mandates to avoid climate disasters.

In this vein, given projections of global warming and cost of decarbonization technologies, we use our model to calculate the fraction of wealth dedicated to sustainable firms, the firm-level qualification threshold based on decarbonization spending, and the cost-of-capital wedge that are needed to incentivize firms to reach net-zero targets. It is important to caveat that projections for global warming and cost of decarbonization technologies have considerable uncertainty, which are modeled in Barnett, Brock, and Hansen (2020). We do not take a stand on which projections are accurate. Our contribution is to show, for a given set of projections, how to calculate the financial commitment needed from sustainable finance to incentivize firms to reach net-zero targets.

Each year, the global industrial sector emits around 50 billion metric tons of greenhouse gases. The social cost of carbon (SSC) ranges from US $177-805 metric ton of emissions (Ricke, Drouet, Caldeira and Tavoni (2018)). As a baseline scenario, we suppose that the portfolio of decarbonization technology costs over the next thirty years reach either 144 dollars per metric ton (low cost) or 288 dollars per metric ton (high cost).[5] This would mean firms need to spend 3.67 (7.4) trillion dollars on mitigation or 0.6% (1.2%) of 600 trillion dollars of global capital stock.[6]

We then calibrate our model to a damage scenario absent mitigation from Burke, Hsiang, and Miguel (2015). We map one of their damage scenarios, which corresponds to -0.3% GDP expected growth rate per annum globally, over to our damage function that depends on two

[5] According to estimates from a McKinsey Sustainability report (de Pee (2018)), decarbonization of just the heavy industries that account for 20% of the global carbon emissions will cost around 20 trillion dollars up to 2050. See Gates (2021) for a discussion of costs of NETs. Estimates of carbon capture vary greatly depending on location and might be as low as 60-100 dollars per to (Schmelz, Hochman and Miller (2020)).

[6] Gadzinki, Schuller and Vacchino (2018) estimate global capital stock (including both traded and non-traded assets) in 2016 to be between 500 and 600 trillion dollars.

parameters: the arrival rate of weather disasters and the fat-tailed damage conditioned on an arrival (Barro and Jin (2011)). We then calibrate the impact of decarbonization spending on mitigating expected losses by assuming that annual decarbonization spending (either 0.6% or 1.2% of the global capital stock) will offset the damage from higher temperatures and yield a positive annual growth rate that can range anywhere from 0.5% (low effectiveness) to 1.5% (high effectiveness). Typically, welfare gains from mitigation increase with the effectiveness of decarbonization in insulating growth rates from global warming damage. Our calibration, like recent work on integrated assessment models (Cai and Lontzek (2019), Daniel, Litterman, and Wagner (2019)), uses parameters from the long-run risk framework (Bansal and Yaron (2004)).

As a baseline scenario, we consider a low cost (with mitigation spending being around 0.6% of capital stock) and moderate effectiveness of decarbonization (by calibrating to a 1% annual growth rate that will result from reaching net-zero emissions goals). In the first-best economy, aggregate mitigation spending is about 1.72% of the aggregate capital stock per annum, which comes at the expense of lower consumption and investment. Welfare is significantly higher in the first-best economy, by around 36% to reflect the urgency of the climate change problem.

The 0.6% net-zero emissions target mitigation spending increases welfare by about 25% compared with the competitive market economy (with no mitigation). Due to the force of aggregate risk reduction from mitigation, the risk-free rate is higher (2.1% compared to 0.82% per annum under competitive equilibrium with no mitigation) and Tobin's q is only slightly lower (1.77 compared to 1.79). This implies that the net-zero target capital tax rate is 0.36%=0.6%/1.77, i.e., the aggregate mitigation spending divided by Tobin's q.

The amount of sustainable capital and the welfare-maximizing qualification criterion needed to achieve a targeted decarbonization spending depend on firm policy requiring certain fraction of their cashflows to be paid out as dividends. With a restriction that a firm pays around 36% of its revenues as dividends (i.e., roughly the payout ratio for mature consumer or industrial companies), achieving the net-zero target aggregate spending of 0.6% of capital stock per annum requires that at least 38% of wealth be allocated to mandates and a qualification criterion requiring a firm to spend 1.6% of its capital stock each year on mitigation. The compensating cost-of-capital advantage for a sustainable form over an unsustainable one is 0.90% per annum. A higher dividend payout requirement implies that first-best can only be achieved with a greater fraction of wealth committed to mandates since these sustainable firms can spend less on mitigation. Large enough dividend payout requirements may mean mandates cannot achieve first-best since firms ultimately cannot spend much on mitigation.

As we vary the effectiveness of decarbonization technology in limiting damage to economic growth, we find fairly similar financial commitments. However, as we increase the cost of decarbonization technology, i.e., suppose that net-zero targets require spending 1.2% as opposed 0.6% of capital stock on decarbonization measures, then the amount of sustainable capital needed increases substantially from 38% of wealth to 60% of wealth and so does the cost-of-capital wedge, rising from 0.90% to 1.15%.

There are multiple ways to check the commitment of sustainable finance in our framework, whether it is disclosure of firm-level mitigation spending or using the cost-of-capital wedge as a proxy for the costs that shareholders are bearing to fund mitigation. One implication of our analysis is that for sustainable finance to confront the climate change problem it has to be different than what has been labeled sustainable finance in the past. Assets under management devoted to sustainable funds is on average 20% over the last twenty years. Over a long sample period, there is little evidence that there are significant differences in the cost of capital for sustainable versus unsustainable firms (see Matos (2020) for a review of the evidence). According to our model, this means that the qualification criterion to be a sustainable firm has historically been not stringent.[7]

Moreover, our assumption of a restriction on the representative investor's portfolio to passively index to sustainable firms generates realistic downward sloping demand curves (Shleifer (1986), Chang, Hong and Liskovich (2015), Kashyap, Kovrijnykh, Pavlova (2018),

[7]However, evidence based on return differences for higher versus low carbon emissions companies in the last few years (Bolton and Kacperczyk (2020)) suggest that qualification criterion might be getting more stringent.

Koijen and Yogo (2019)) even with complete financial spanning. Hence our model makes a sharp prediction linking the cost-of-capital wedge between sustainable and unsustainable firms to the amount of mitigation. But a cost-of-capital wedge also arises whenever there is demand for firms with sustainability attributes by a subset of investors who cannot perfectly hedge the idiosyncratic firm risks due to lack of full financial spanning.

In this vein, Pastor, Stambaugh, and Taylor (2020) model how some investors' non-pecuniary taste for green stocks generates three-fund separation in a static CAPM setting and differences in expected returns between green and brown stocks. But the cost-of-capital wedge in their model also depends on investor preferences and the nature of incomplete spanning. Pedersen, Fitzgibbons, and Pomorski (2020) also incorporate potential mispricing of fundamental information captured by sustainability measures. To the extent that demand curves are flat, it can be more efficient to engage activist strategies relying on voting as opposed to divestment to effect change (Gollier and Pouget (2014), Broccardo, Hart, and Zingales (2020) and Oehmke and Opp (2020)).

Our paper proceeds as follows. In Section 2, we describe our model. In Section 3, we analyze the first-best outcome or planner's solution. We then solve our sustainable finance mandate model in Section 4. We calibrate our model in Section 5 to business-as-usual global warming forecasts and calculate the main variables of interests. We conclude in Section 6.

2 Model

While mitigating climate disaster risk benefits the society, doing so is privately costly for the firm. We model sustainable finance mandates as portfolio restrictions on the representative agent's portfolio and examine the extent to which it encourages firms to provide risk mitigation and quantify its implications for social welfare. We use a representative-agent framework for expositional simplicity, where this agent can be interpreted as representing both public (e.g., sovereign wealth funds) and private investors.

On the demand side for financial assets, the representative agent holds and invests the entire wealth of the economy between sustainable (S) firms, unsustainable (U) firms, and the risk-free bonds. The agent has to invest an α fraction of the entire aggregate wealth in a sustainable type-S firm. The risk-averse representative agent is required to meet the sustainable investment mandate at all times when allocating assets.

On the supply side, a portfolio of S firms and a portfolio of U firms will arise endogenously in equilibrium, which we refer to as S-portfolio and U-portfolio, respectively. For a firm to qualify to be type-S, it has to spend at least a fraction m of its capital on mitigation via a portfolio of decarbonization technologies so as to reduce disaster risk. Otherwise, it is labeled a type-U for unsustainable.

2.1 Firm Production, Capital Accumulation, and Disaster Shocks

The firm's output at t, Y_t, is proportional to its capital stock, K_t, which is the only factor of production:

$$Y_t = AK_t \,, \tag{1}$$

where $A > 0$ is a constant that defines productivity for all firms. This is a version of widely-used AK models in macroeconomics and finance.[8] All firms start with the same level of initial capital stock K_0 and have the same production and capital accumulation technology. Additionally, they are subject to the same shocks (path by path).

That is, there is no idiosyncratic shock in our model. This simplifying assumption makes our model tractable and allows us to focus on the impact of the investment mandate on equilibrium asset pricing and resource allocation. Despite being identical in all aspects, some firms choose to be sustainable while others remain unsustainable in equilibrium.

Investment and Capital Accumulation. Let I_t and X_t denote the firm's investment and mitigation spending, respectively. As in Pindyck and Wang (2013), the firm's capital stock, K_t, evolves as:

$$dK_t = \Phi(I_{t-}, K_{t-})dt + \sigma K_{t-} d\mathcal{B}_t - (1 - Z)K_{t-} d\mathcal{J}_t \,. \tag{2}$$

[8] There are pros and cons of using an AK model for our climate-change analysis. For analyzing weather disasters such as hurricanes which have been shown to have permanent effects on capital and output, an AK model setup is natural. But an AK setup might miss important features of growth rate dynamics in other settings (Jones (1995)).

As in Lucas and Prescott (1971) and Jerrmann (1998), we assume that $\Phi(I, K)$, the first term in (2), is homogeneous of degree one in I and K, and thus can be written as

$$\Phi(I, K) = \phi(i)K \ , \tag{3}$$

where $i = I/K$ is the firm's investment-capital ratio and $\phi(\cdot)$ is increasing and concave. This specification captures the idea that changing capital stock rapidly is more costly than changing it slowly. As a result, installed capital earns rents in equilibrium so that Tobin's q, the ratio between the value and the replacement cost of capital exceeds one.

The second term captures continuous shocks to capital, where $\mathcal{B}_t$ is a standard Brownian motion and the parameter σ is the diffusion volatility (for the capital stock growth). This $\mathcal{B}_t$ is the source of shocks for the standard AK models in macroeconomics. This diffusion shock is common to all firms. Had we introduced an additional shock that is idiosyncratic across firms, our solution would remain unchanged as firms can perfectly hedge idiosyncratic shocks at no cost and our aggregation results remain valid.

The firm's capital stock is also subject to an aggregate jump shock. We capture this jump effect via the third term, where $\mathcal{J}_t$ is a (pure) jump process with a constant arrival rate, which we denote by $\lambda > 0$. To emphasize the timing of potential jumps, we use $t-$ to denote the pre-jump time so that a discrete jump may or may not arrive at t. Examples of jumps include hurricanes or wildfires that destroy physical and housing capital stock.

When a jump arrives ($d\mathcal{J}_t = 1$), it permanently destroys a stochastic fraction $(1 - Z)$ of the firm's capital stock K_{t-}, as Z is the recovery fraction where $Z \in (0, 1)$. (For example, if a shock destroyed 15 percent of capital stock, we would have $Z = .85$.) There is no limit to the number of these jump shocks.[9] If a jump does not arrive at t, i.e., $d\mathcal{J}_t = 0$, the third term disappears.

[9]Stochastic fluctuations in the capital stock have been widely used in the growth literature with an AK technology, but unlike the existing literature, we examine the economic effects of shocks to capital that involve discrete (disaster) jumps.

2.2 Mitigation and Externality

We use $\Xi_{t-}(Z)$ and $\xi_{t-}(Z)$ to denote the cumulative distribution function (cdf) and probability density function (pdf) at time $t-$ for the recovery fraction, Z, conditional on a jump arrival at t, respectively. We postulate that the cdf $\Xi_{t-}(Z)$ and pdf $\xi_{t-}(Z)$ depend on the pre-jump *aggregate* mitigation spending $\mathbf{X}_{t-}$ and the *aggregate* capital stock $\mathbf{K}_{t-}$ in the economy. Let $\mathbf{x}_{t-} = \mathbf{X}_{t-}/\mathbf{K}_{t-}$. We use boldfaced notations for aggregate variables.

To preserve our model's homogeneity property, we assume that the cdf $\Xi_{t-}(Z)$ and pdf $\xi_{t-}(Z)$ depend on mitigation spending purely via the pre-jump scaled aggregate mitigation spending ($\mathbf{x}_{t-}$). That is, if we simultaneously double the aggregate mitigation spending $\mathbf{X}_{t-}$ and aggregate capital stock $\mathbf{K}_{t-}$, the cumulative distribution $\Xi_{t-}(Z)$ is unchanged. It is sometimes useful to make the dependence of $\Xi_{t-}(Z)$ and $\xi_{t-}(Z)$ on scaled aggregate mitigation spending $\mathbf{x}_{t-}$ explicit: $\Xi_{t-}(Z) = \Xi(Z; \mathbf{x}_{t-})$ and $\xi_{t-}(Z) = \xi(Z; \mathbf{x}_{t-})$.

As disaster shocks are aggregate and disaster damages are only curtailed by *aggregate* mitigation spending $\mathbf{X}$, absent mandates or other incentive programs, firms have no incentives to mitigate on their own as the economy is competitive and their own mitigation spending have no impact on the *aggregate* mitigation spending (Hong, Wang and Yang (2020)).

Alternative specification: mitigation spending changes the disaster arrival rate. Since global warming is expected to magnify not just the damage but also the frequency of disasters, we can consider an alternative specification where by spending on mitigation, a private agent reduces the likelihood of a disaster arrival λ. Specifically, suppose that for a given pre-jump mitigation spending X_{t-} with an implied scaled mitigation $x_{t-} = X_{t-}/K_{t-}$, the jump arrival rate changes to $\lambda(\mathbf{x}_t)$ from the constant arrival rate λ absent mitigation. We assume that mitigation spending decreases the disaster arrival rate, $\lambda'(\mathbf{x}_t) < 0$. All the other parts of the model remain the same as in our baseline model. The core mechanism in our model remains the same. The welfare theorem again fails as no firms have incentives to mitigate to reduce the disaster arrival rate on their own as the economy is competitive and firms have incentives to free ride on others.

2.3 Sustainable Investment Mandates

Let $\mathbf{1}_t^S$ be an indicator function describing the status of a firm at t. To qualify as a sustainable (S) firm at t, the firm has to spend at least M_t at t on disaster risk mitigation, which contributes to the reduction of aggregate risk. That is, $\mathbf{1}_t^S = 1$ if and only if the firm's mitigation spending X_t satisfies:

$$X_t \geq M_t\,. \tag{4}$$

Otherwise, $\mathbf{1}_t^S = 0$ and the firm is unsustainable (U).

To preserve our model's homogeneity property, we assume that the mandated mitigation spending is proportional to firm size K_t:

$$M_t = m_t K_t\,, \tag{5}$$

where m_t is the minimal level of mitigation per unit of the firm's capital stock to qualify a firm to be sustainable. That is, it is cheaper for a firm (with smaller K_t) to qualify as a sustainable firm. Later, we endogenize the S-firm qualification threshold, m_t, to maximize the representative agent's utility.

The investment mandate α creates the inelastic demand for S firms. In equilibrium, the remaining $1-\alpha$ fraction is invested in the U-portfolio so that the agent has no investment in the risk-free bonds in equilibrium.

2.4 Optimal Firm Mitigation

Each firm can choose to be either a sustainable (S) or a unsustainable firm (U). We assume that a firm's mitigation is observable and contractible. While spending on aggregate risk mitigation yields no monetary payoff for the firm, doing so allows it to be included in the S-portfolio.

A value-maximizing firm chooses whether to be sustainable or unsustainable depending on which strategy yields a higher value. Let Q_t^n denote the the market value of a type-n firm at t, where $n = \{S, U\}$. By exploiting our model's homogeneity property, we conjecture and verify that the equilibrium value of a type-n firm at time t must satisfy:

$$Q_t^n = q^n K_t^n \,, \tag{6}$$

where q^n is Tobin's average q for a type n-firm.

In equilibrium, as mitigation spending has no direct benefit for the firm, if the firm chooses to be U, i.e., $\mathbf{1}_t^S = 0$, it will set $X_t = 0$. Moreover, even if a firm chooses to be a S firm, it has no incentive to spend more than M_t, i.e., (4) always binds for a type-S firm.

As we later verify, the equilibrium expected rate of return for a type-n firm, which we denote by r^n, is constant. A type-n firm maximizes its present value:

$$\max_{I^n, X^n} \mathbb{E}\left(\int_0^\infty e^{-r^n t} CF_t^n dt\right) \tag{7}$$

subject to the standard transversality condition specified in the Appendix A.1. In equation (7), CF_t^n is the firm's cash flow at t, which is given by

$$CF_t^S = AK_t^S - I_t^S - X_t^S \quad \text{and} \quad CF_t^U = AK_t^U - I_t^U \,, \tag{8}$$

as an unsustainable firm spends nothing on mitigation.

Since I_t and X_t are both proportional to K_t, spending on X_t effectively reduces the productivity of firms. Hence, X_t can be broadly interpreted as various mitigatory activities that reduce firm productivity including limiting carbon emissions or spending on other forms of mitigation.

In addition, we assume that there is a lower bound on payouts in the economy given by $\widehat{CF}_t$ for all firms at time t, i.e. $CF_t^n \geq \widehat{CF}_t^n$ with $n = \{S, U\}$ should be satisfied for all $t \geq 0$. For simplicity, we assume that $\widehat{CF}_t^n$ is always proportional to the firm's capital, i.e. $\widehat{CF}_t^n = \widehat{cf} K_t^n$ where $\widehat{cf} > 0$ is constant. This payout lower bound is meant to capture realistic features of corporate governance that indirectly limit firm investment and mitigation spending.[10]

[10] We can express this payout constraint in several alternative ways. For example, one way is to require a firm to pay out at least a fraction of its earnings, which equals revenue minus capital depreciation, $(A-\delta)K_t$. Another way is to require a firm to pay out a fraction of its free cash flow, which equals revenue minus investment costs, $(AK_t - I_t)$. These formulations are very similar in essence. For illustrative purposes, we choose the payout constraint as a fraction of the firm's capital stock.

2.5 Dynamic Consumption and Asset Allocation

The representative agent makes all the consumption and asset allocation decisions. We thus use individual and aggregate variables for the agent interchangeably. For example, the aggregate wealth, $\mathbf{W}_t$, is equal to the representative agent's wealth, W_t. Similarly, the aggregate consumption, $\mathbf{C}_t$, is equal to the representative agent's consumption, C_t.

The representative agent has the following investment opportunities: (a) the S portfolio which includes all the sustainable firms; (b) the U portfolio which includes all other firms that are unsustainable; (c) the risk-free asset that pays interest at a constant risk-free interest rate r determined in equilibrium; and (d) actuarially fair insurance claims for disasters with every possible recovery fraction Z (and also for diffusion shocks.)

Type-S and type-U portfolios. The S and U portfolios include all the S and U firms, respectively. Let $\mathbf{Q}_t^S$ and $\mathbf{Q}_t^U$ denote the aggregate market value of the S portfolio firm and the U portfolio at t, respectively. Similarly, Let $\mathbf{D}_t^S$ and $\mathbf{D}_t^U$ denote the aggregate dividend of the S portfolio firm and the U portfolio at t, respectively.

We conjecture and then verify that the cum-dividend return for the type-n portfolio is given by

$$\frac{d\mathbf{Q}_t^n + \mathbf{D}_{t-}^n dt}{\mathbf{Q}_{t-}^n} = r^n dt + \sigma d\mathcal{B}_t - (1-Z)\left(d\mathcal{J}_t - \lambda dt\right), \tag{9}$$

where r^n is the endogenous constant expected cum-dividend return for a type-n firm in equilibrium. In equation (9), the diffusion volatility is equal to σ as in equation (2). The third term on the right side of equation (9) is a jump term capturing the effect of disasters on return dynamics. Both the diffusion volatility and jump terms are martingales (and this is why r^n is the expected return.) Note that the only difference between the S- and U-portfolio is the expected return. The diffusion and jump terms are the same as those in the capital evolution dynamics given in equation (2).

Disaster risk insurance (DIS). We define DIS as follows: a DIS for the survival fraction in the interval $(Z, Z+dZ)$ is a swap contract in which the buyer makes insurance payments $p(Z)dZ$, where $p(Z)$ is the equilibrium insurance premium payment, to the seller and in exchange receives a lump-sum payoff if and only if a shock with survival fraction in $(Z, Z+dZ)$ occurs. That is, the buyer stops paying the seller if and only if the defined disaster event occurs and then collects one unit of the consumption good as a payoff from the seller. The DIS contract is priced at actuarially fairly terms so that investors earn zero profits.

Preferences. We use the Duffie and Epstein (1992) continuous-time version of the recursive preferences developed by Epstein and Zin (1989) and Weil (1990), so that the representative agent has homothetic recursive preferences given by:

$$V_t = \mathbb{E}_t \left[\int_t^\infty f(C_s, V_s) ds \right] , \tag{10}$$

where $f(C, V)$ is known as the normalized aggregator given by

$$f(C, V) = \frac{\rho}{1-\psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega-1}} . \tag{11}$$

Here ρ is the rate of time preference, ψ the elasticity of intertemporal substitution (EIS), γ the coefficient of relative risk aversion, and we let $\omega = (1-\psi^{-1})/(1-\gamma)$. Unlike expected utility, recursive preferences as defined by (10) and (11) disentangle risk aversion from the EIS. An important feature of these preferences is that the marginal benefit of consumption is $f_C = \rho C^{-\psi^{-1}}/[(1-\gamma)V]^{\omega-1}$, which depends not only on current consumption but also (through V) on the expected trajectory of future consumption.

If $\gamma = \psi^{-1}$ so that $\omega = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C, V) = \frac{\rho\, C^{1-\gamma}}{1-\gamma} - \rho V. \tag{12}$$

This more flexible utility specification is widely used in asset pricing and macroeconomics for at least two important reasons: 1) conceptually, risk aversion is very distinct from the EIS, which this preference is able to capture; 2) quantitative and empirical fit with various asset pricing facts are infeasible with standard CRRA utility but attainable with this recursive utility, as shown by Bansal and Yaron (2004) and the large follow-up long-run risk literature.

Wealth dynamics. Let W_t denote the representative agent's wealth. Let H_t^S and H_t^U denote the dollar amount invested in the S and U portfolio, respectively. Let H_t denote the agent's wealth allocated to the market portfolio at t. That is, $H_t = H_t^S + H_t^U$. The dollar amount, $(W_t - H_t)$ is the dollar amount invested in the risk-free asset. For disasters with recovery fraction in $(Z, Z + dZ)$, $\delta_t(Z)W_t dt$ gives the total demand for the DIS over time period $(t, t + dt)$.

The agent accumulates wealth as:

$$\begin{aligned} dW_t &= r\left(W_{t-} - H_{t-}\right)dt + \left(r^S H_{t-}^S + r^U H_{t-}^U\right)dt + \sigma H_{t-} d\mathcal{B}_t - (1 - Z)\, H_{t-}(d\mathcal{J}_t - \lambda dt) \\ &\quad - C_{t-}dt - \left(\int_0^1 \delta_{t-}(Z)p(Z)dZ\right) W_{t-}dt + \delta_{t-}(Z)W_{t-}d\mathcal{J}_t\,. \end{aligned} \tag{13}$$

The first term in (13) is the interest income from savings in the risk-free asset, the second term is the expected return from investing in the S and U portfolios. Note that the expected returns are different: r^S and r^U for the S and U portfolios, respectively. The third and fourth terms are the diffusion and jump martingale terms for the stock market portfolio. Note that the stochastic (shock) components of the returns (diffusion and jumps) for the two portfolios are identical path by path. The fifth term is the standard consumption outflow term. The sixth term is the total DIS premium paid by the consumer before the arrival of disasters. Note that this term captures the financial hedging cost. The last term describes the DIS payments by the DIS seller to the household when a disaster occurs.

The total market capitalization of the economy, $\mathbf{Q}_t$, is given by

$$\mathbf{Q}_t = q^S \mathbf{K}_t^S + q^U \mathbf{K}_t^U\,. \tag{14}$$

Let π_t^S and π_t^U denote the fraction of total wealth W_t allocated to the S and U portfolio at time t, respectively. That is, $H_t^S = \pi_t^S H_t$, $H_t^U = \pi_t^U H_t$, and the remaining fraction $1 - \left(\pi_t^S + \pi_t^U\right)$ of W_t is allocated to the risk-free asset.

In equilibrium, the investment mandate requires that the total capital investment in the S portfolio has to be at least an α fraction of the total stock market capitalization $\mathbf{Q}_t$:

$$H_t^S \geq \alpha \mathbf{Q}_t\,. \tag{15}$$

In equilibrium, the total stock market capitalization $\mathbf{Q}_t$ depends on the mandate. We later derive a closed-form expression for the relation between $\mathbf{Q}_t$ and α.

Rewriting equation (13), we express the household's wealth dynamics as:

$$\begin{aligned} dW_t &= \left[rW_{t-} - C_{t-} + \left(\pi^S_{t-} \cdot (r^S - r) + \pi^U_{t-}(r^U - r)\right) W_{t-}\right] dt - \left(\int_0^1 \delta_{t-}(Z)p(Z)dZ\right) W_{t-}dt \\ &\quad + \left(\pi^S_{t-} + \pi^U_{t-}\right) W_{t-} \left[\sigma d\mathcal{B}_t - (1-Z)\left(d\mathcal{J}_t - \lambda dt\right)\right] + \delta_{t-}(Z)W_{t-}d\mathcal{J}_t \,. \end{aligned} \tag{16}$$

Let $\mathbf{Y}_t$, $\mathbf{C}_t$, $\mathbf{I}_t$, and $\mathbf{X}_t$ denote the aggregate output, consumption, investment, and mitigation spending, respectively. Adding across all type-S and U firms, we obtain the aggregate resource constraint:

$$\mathbf{Y}_t = \mathbf{C}_t + \mathbf{I}_t + \mathbf{X}_t \,. \tag{17}$$

2.6 Competitive Equilibrium

We define the competitive equilibrium subject to the investment mandate as follows: (1) the representative agent dynamically chooses consumption and asset allocation among the S portfolio, the U portfolio, and the risk-free asset subject to the investment mandate given in (15); (2) each firm chooses its status (S or U), and investment policy I to maximizes its market value; (3) all firms that choose sustainable investment policies are included in the S portfolio and all remaining firms are included in the U portfolio; and (4) all markets clear.

The market-clearing conditions include (i) the net supply of the risk-free asset is zero; (ii) the representative agent's demand for the S portfolio is equal to the total supply by firms choosing to be sustainable; (iii) the representative agent's demand for the U portfolio is equal to the total supply by firms choosing to be brown; (iv) the net demand for the DIS of each possible recovery fraction Z is zero; and (v) the goods market clears, i.e., the resource constraint given in (20) holds.

Because the risk-free asset and all DIS contracts are in zero net supply, the agent's entire wealth W_t is invested in the S and U portfolios.

2.7 Optimal Qualification Criterion

Finally, for a given level of α, we endogenize the criterion at the firm level characterized by the scaled mitigation threshold $M_t = m_t K_t$, for a firm to qualify as a sustainable firm. Specifically, at time 0, the planner announces $\{M_t; t \geq 0\}$ and commits to the announcement with the goal of maximizing the representative agent's utility given in equation (10) taking into account that the representative agent and firms take the mandate as given and optimize in competitive equilibrium.[11] Since no firm spends more than M_t to qualify as an S firm, the equilibrium aggregate mitigation spending satisfies:

$$\mathbf{X_t} = \alpha M_t. \tag{18}$$

Comment. In our model, the representative agent represents investors in the whole economy including both the private and public sectors. We may also interpret our representative-agent model as one with heterogeneous agents where an α fraction of them are sustainable investors, who have investment mandates (e.g., large asset managers and sovereign wealth funds), and the remaining $1-\alpha$ fraction do not. The sustainable investors group has inelastic demand for sustainable firms and moreover they do not lend their shares out for other investors to short sustainable firms.

3 First-Best: Capital Tax

Before solving the model for the ESG economy, we first report the first-best solution where the planner chooses aggregate $\mathbf{C}$, $\mathbf{I}$, and $\mathbf{X}$ to maximize the representative agent's utility defined earlier. (We drop the payout constraints $CF_t^n \geq \widehat{CF}_t^n$ for $n = \{S, U\}$.)

As our model features the homogeneity property, it is convenient to work with scaled variables at both aggregate and individual levels. We use lower-case variables to denote the corresponding upper-case variables divided by contemporaneous capital stock. For example, at the firm level, $i_t = I_t/K_t$, $\phi_t = \Phi_t/K_t$, and $x_t = X_t/K_t$. Similarly, at the aggregate level,

[11] Broadly speaking, our mandate choice is related to the optimal fiscal and monetary policy literature in macroeconomics. See Ljungqvist and Sargent (2018) for a textbook treatment.

$\mathbf{x}_t = \mathbf{X}_t/\mathbf{K}_t$. For consumers, $c_t = \mathbf{c_t} = \mathbf{C}_t/\mathbf{K}_t$.

Let $V(K)$ denote the representative agent's value function. As in Hong, Wang, and Yang (2020), the following Hamilton-Jacobi-Bellman (HJB) equation characterizes the planner's optimization problem:

$$0 = \max_{C,I,x} f(C,V) + \Phi(I,K)V'(K) + \frac{\sigma^2 K^2}{2}V''(K) + \lambda \int_0^1 \left[V(ZK) - V(K)\right]\xi(Z;x)dZ\,, \quad (19)$$

subject to the following resource constraint at all t:

$$AK_t = C_t + I_t + x_t K_t\,. \quad (20)$$

The first-order condition (FOC) for investment I is

$$f_C(C,V) = \Phi_I(I,K)V'(K)\,. \quad (21)$$

And the FOC with respect to mitigation spending is

$$f_C(C,V) = \frac{1}{K}\lambda \int_0^1 \left[\frac{\partial \xi(Z;x)}{\partial x} V(ZK)\right] dZ\,, \quad (22)$$

if the solution is strictly positive, $x > 0$. Otherwise, $x = 0$ as mitigation cannot be negative. The representative agent's value function takes the following homothetic form:

$$V(K) = \frac{1}{1-\gamma}(bK)^{1-\gamma}\,, \quad (23)$$

where b is a constant measuring the agent's certainty-equivalent wealth and is endogenously determined.

Substituting (23) into the investment FOC (21) and the FOC (22) for mitigation spending, we obtain:

$$b = (A-i-x)^{1/(1-\psi)} \left[\frac{\rho}{\phi'(i)}\right]^{-\psi/(1-\psi)}\,, \quad (24)$$

$$\rho(A-i-x)^{-\psi^{-1}} b^{\psi^{-1}-1} = \frac{\lambda}{1-\gamma} \int_0^1 \left[\frac{\partial \xi(Z;x)}{\partial x} Z^{1-\gamma}\right] dZ\,. \quad (25)$$

And then by substituting (24) into (25), we obtain

$$1 = \frac{\lambda}{(1-\gamma)\phi'(i)} \int_0^1 \left[\frac{\partial \xi(Z;x)}{\partial x} Z^{1-\gamma}\right] dZ\,. \quad (26)$$

Finally, substituting (23) and (24) into (19) and simplifying the expression, we obtain

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\frac{(A-i-x)\phi'(i)}{\rho} - 1\right] + \phi(i) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\left[\int_0^1 \left[\xi(Z;x)Z^{1-\gamma}\right] dZ - 1\right]. \quad (27)$$

We now summarize the first-best solution. First, jointly solving (26) and (27), we obtain the first-best levels of $\mathbf{i}^{FB}$ and $\mathbf{x}^{FB}$. Second, we obtain the first-best level for $\mathbf{b}^{FB}$ using (24) and $\mathbf{c}^{FB} = A - \mathbf{i}^{FB} - \mathbf{x}^{FB}$. Third, we infer the equilibrium aggregate value of capital (average q), by using $\mathbf{q}^{FB} = 1/\phi'(\mathbf{i}^{FB})$. Finally, we obtain the first-best capital tax rate: $\mathbf{x}^{FB}/\mathbf{q}^{FB}$.

4 Solution

In this section, we solve for the equilibrium solution with the ESG mandate.

First, we consider the micro-level problem where firms take the sustainability mandate m as given and make decisions.

4.1 Firm Optimization

For a firm to be sustainable, it spends the minimal required m fraction of its capital stock:

$$x_t^S = \frac{X_t^S}{K_t^S} = m. \quad (28)$$

Any additional spending on mitigation is suboptimal as it yields no further benefit to the firm. All other firms spend nothing on mitigation and hence are unsustainable, i.e., $x_t^U = 0$.

Next, we solve for optimal investment policies for both types of firms. The firm's objective (7) implies that $\int_0^s e^{-r^n t} CF_t^n dt + e^{-r^n s} Q_s^n$ is a martingale under the physical measure. We obtain the following Hamilton-Jacobi-Bellman (HJB) equation by using Ito's Lemma:

$$r^n Q^n = \max_{I^n}\ CF^n + \left(\Phi(I^n, K^n)Q_K^n + \frac{1}{2}(\sigma K^n)^2 Q_{KK}^n\right) + \lambda \mathbb{E}\left[Q^n(ZK^n) - Q^n(K^n)\right], \quad (29)$$

where r^n is the cost of capital and CF^n is the cash flow for a type-n firm given by (8), and the choice of investment is subject to the firm payout constraint, $AK^n - X^n - I^n \geq \widehat{cf}\, K^n$. Here, $\mathbb{E}[\,\cdot\,]$ is the conditional expectation operator with respect to the distribution of recovery fraction Z. Recall that the last term only depends on the aggregate mitigation spending $\mathbf{X}_t$ and has the same effect on all firms.

By using our model's homogeneity property, $Q_t^n = q^n K_t$, we obtain the following

$$r^n q^n = \max_{i^n} \; cf^n + g(i^n) q^n \,, \tag{30}$$

subject to $i^n \leq A - x^n - \widehat{cf}$, where $g(i)$ is the expected firm growth rate:

$$g(i) = \phi(i) - \lambda(1 - \mathbb{E}(Z)) \tag{31}$$

and $cf^n = CF^n/K^n$ is the scaled cash flow for a type-n firm. As $x^S = m$ and $x^U = 0$, we have $cf^S = A - i^S - m \geq \widehat{cf}$ for a type-S firm and $cf^U = A - i^U \geq \widehat{cf}$ for a type-U firm.

The investment FOC for both types of firms implied by (30) is the following well known condition in the q-theory literature:

$$q^n = \frac{1}{\phi'(i^n)} \,. \tag{32}$$

A type-n firm's marginal benefit of investing is equal to its marginal q, q^n, multiplied by $\phi'(i^n)$. Equation (32) states that this marginal benefit, $q^n \phi'(i^n)$, is equal to one, the marginal cost of investing at optimality. The homogeneity property implies that a firm's marginal q is equal to its average q (Hayashi, 1982). Next, we determine aggregate variables.

4.2 Market Equilibrium

Since the financial investment and growth opportunities are time invariant, the equilibrium risk-free rate r, the expected returns (r^S and r^U) for the S and U portfolios, Tobin's average q for all firms (and also the aggregate capital stock) are all constant over time.

As our model features perpetual growth, we may also calculate q^S and q^U by using the Gordon growth model (with constant endogenous discount rate and growth rate) as follows:

$$q^S = \max_{i \leq A - m - \widehat{cf}} \frac{A - i - m}{r^S - g(i)} \quad \text{and} \quad q^U = \max_{i \leq A - \widehat{cf}} \frac{A - i}{r^U - g(i)} \,. \tag{33}$$

As a firm can choose being either sustainable or not, it must be indifferent between the two options at all time. That is, in equilibrium, all firms have the same Tobin's q, which in equilibrium is also Tobin's $\mathbf{q}$ for the aggregate economy:

$$q^S = q^U = \mathbf{q} \,. \tag{34}$$

Equations (32) and (34) imply that all firms also have the same equilibrium investment-capital ratio, which is also the aggregate $\mathbf{i}$:

$$i^S = i^U = \mathbf{i} \tag{35}$$

As the investment-capital ratio is the same for the two types of firms ($i^S = i^U$) and $x^S = m > x^U = 0$, the cash flows difference between a U and an S firm is exactly the mitigation spending: $cf^U - cf^S = m$ where $cf^U = A - i$.

Since each S firm spends mK_t^S units on mitigation and all firms are of the same size, we have the following relation between the scaled mitigation m at the firm level and scaled mitigation at the aggregate level $\mathbf{x} = \mathbf{X}/\mathbf{K}$:

$$m(\mathbf{x}) = \frac{\mathbf{x}}{\alpha} \geq \mathbf{x}\,. \tag{36}$$

The mitigation spending mandate for a firm, m, is larger than the aggregate scaled mitigation, $\mathbf{x}$, as only an α fraction of firms are sustainable.

By using (33), (34), and (35), we obtain $\mathbf{q}$, the average q for aggregate capital stock:

$$\mathbf{q} = \frac{A - \mathbf{i} - m(\mathbf{x})}{r^S - g(\mathbf{i})} = \frac{A - \mathbf{i}}{r^U - g(\mathbf{i})} \tag{37}$$

subject to the scaled aggregate investment constraint:

$$\mathbf{i} \leq A - \widehat{cf} - (\mathbf{x}/\alpha)\,. \tag{38}$$

In equilibrium, the aggregate consumption is equal to the aggregate dividend:

$$\mathbf{c} = \mathbf{cf} = A - \mathbf{i} - \mathbf{x}\,. \tag{39}$$

Using the second equality in (37), we obtain the following expression:

$$\mathbf{q} = \frac{\mathbf{c}}{r^M(\mathbf{x}) - \mathbf{g}(\mathbf{i})} = \frac{A - \mathbf{i} - \mathbf{x}}{r^M(\mathbf{x}) - \mathbf{g}(\mathbf{i})} = \frac{1}{\phi'(\mathbf{i})}\,. \tag{40}$$

Next, we report that the equilibrium risk-free rate r, and aggregate stock-market risk premium, $r^M(\mathbf{x}) - r(\mathbf{x})$, are the same as in a representative-firm economy (with no mitigation) but with two modifications: (1) productivity set at $A - \mathbf{x}$ and (2) the cdf for the recovery fraction Z given by $\Xi(Z; \mathbf{x})$ for a given $\mathbf{x}$.

Equilibrium risk-free rate r and expected market return $r^M(\mathbf{x})$ for a given $\mathbf{x}$. Using the results in Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), we calculate the aggregate stock-market risk premium, $r^M(\mathbf{x}) - r(\mathbf{x})$, by using

$$r^M(\mathbf{x}) - r(\mathbf{x}) = \gamma\sigma^2 + \lambda\mathbb{E}^{\mathbf{x}}\left[(1-Z)(Z^{-\gamma}-1)\right] . \tag{41}$$

The risk-free rate is

$$r(\mathbf{x}) = \rho + \psi^{-1}\phi(\mathbf{i}) - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \lambda\mathbb{E}^{\mathbf{x}}\left[\left(Z^{-\gamma}-1\right) + \left(\psi^{-1}-\gamma\right)\left(\frac{1-Z^{1-\gamma}}{1-\gamma}\right)\right] . \tag{42}$$

The first two terms in (42) capture the standard Ramsey channels via the discount rate (ρ) and the expected growth of capital ($\phi(\mathbf{i})$). The third term captures the precautionary savings effect and the last term is the jump-induced volatility and higher-order moments. Aggregate mitigation spending $\mathbf{X}_t = \mathbf{x}\mathbf{K}_t$ has a direct effect on the distribution of Z and also an indirect effect on r via its impact on $\mathbf{i}$. To make the distribution of Z on $\mathbf{x}$ explicit, we use $\mathbf{x}$ as the superscript for the expectation operator for jump distributions, e.g., in equations (41) and (42) for the stock market risk premium, $r^M(\mathbf{x}) - r$, and the risk-free rate, r.

Next, we calculate that the aggregate investment, average q, and consumption.

Aggregate investment, average q, and consumption: $\mathbf{i}$, $\mathbf{q}$, and $\mathbf{c}$ for a given $\mathbf{x}$. For a given level of $\mathbf{x}$, we solve for the aggregate scaled investment $\mathbf{i}$ by substituting (41) for $r^M(\mathbf{x})$ and $\mathbf{g} = \phi(\mathbf{i}) - \lambda\mathbb{E}^{\mathbf{x}}(1-Z)$, given in (31), into the last equality in (40) subject to $\mathbf{i} \leq A - m - \widehat{cf}$. It is a constrained optimization problem with the only unknown, $\mathbf{i}$.

Using the optimal investment policy as a function of mitigation, $\mathbf{i}(\mathbf{x})$, we then obtain the aggregate $\mathbf{q}$ by using $\mathbf{q}(\mathbf{x}) = 1/\phi'(\mathbf{i}((\mathbf{x}))$ (equivalently (40)), the aggregate scaled consumption/dividends $\mathbf{c}(\mathbf{x}) = \mathbf{cf}(\mathbf{x})$ by using (39), and the market risk premium $r^M(\mathbf{x}) - r$ by using (41). Having characterized all aggregate variables as functions of $\mathbf{x}$, we next turn to the cost-of-capital calculations for S and U firms.

Cost-of-capital wedge. It is helpful to use θ^n to denote the wedge between the expected return for a type-n firm, r^n, and the aggregate stock-market return, r^M, and write for

$n = \{S, U\}$,

$$r^n = r^M + \theta^n \,. \tag{43}$$

As an α fraction of the total stock market is the S portfolio and the remaining $1-\alpha$ fraction is the U portfolio, we have

$$r^M = \alpha \cdot r^S + (1-\alpha) \cdot r^U \,. \tag{44}$$

Using the last equality in (37) and (40), we obtain

$$\theta^U \;\;=\;\; \frac{\mathbf{x}}{\mathbf{q}} = \frac{\alpha m(\mathbf{x})}{q(\mathbf{x})} > 0 \,. \tag{45}$$

Equation (45) states that investors demand a higher rate of return to invest in U firms than in the aggregate stock market. The expected return wedge between the U-portfolio and the market portfolio is equal to θ^U, which is equal to the aggregate mitigation spending $\mathbf{X}$ divided by aggregate stock market value $\mathbf{Q}$. This ratio $\mathbf{x}/\mathbf{q}$ can be viewed as a "tax" on the unsustainable firms by investors in equilibrium.

Substituting (43) into (44) and using (45), we obtain:

$$\theta^S = -\frac{1-\alpha}{\alpha}\,\theta^U = -\frac{1-\alpha}{\alpha}\frac{\mathbf{x}}{\mathbf{q}} = -\,(1-\alpha)\,\frac{m(\mathbf{x})}{q(\mathbf{x})} < 0 \,. \tag{46}$$

The cost-of-capital difference between U and S firms is given by

$$r^U - r^S = \theta^U - \theta^S = \frac{1}{\alpha}\frac{\mathbf{x}}{\mathbf{q}} = \frac{m(\mathbf{x})}{\mathbf{q}} \,. \tag{47}$$

By being sustainable, a firm lowers its cost of capital from r^U to r^S by $r^U - r^S$. To enjoy this benefit, the firm spends $m(\mathbf{x})$ on mitigation. To make it indifferent between being sustainable and not, the cost-of-capital wedge is given by $r^U - r^S = m(\mathbf{x})/\mathbf{q}$, the ratio between the firm's mitigation spending, $m(\mathbf{x})K$, and its market value, $\mathbf{q}K$.

Equilibrium portfolios. In equilibrium the fraction of total wealth invested in the S portfolio is equal to α, the fraction of the market capitalization that is mandated to be sustainable: $\pi^S = \alpha$. Also, the fraction of total wealth invested in the U portfolio satisfies $\pi^U = (1-\alpha)$ and the risk-free asset holdings is zero. That is, $H_t^S = \alpha W_t = \mathbf{Q}_t^S = \alpha \mathbf{Q}_t$, $H_t^U = (1-\alpha)W_t = \mathbf{Q}_t^U = (1-\alpha)\mathbf{Q}_t$, and $W_t = \mathbf{Q}_t = \mathbf{Q}_t^S + \mathbf{Q}_t^U$. The disaster hedging position must be zero $\delta(Z) = 0$ for all Z.

4.3 Testable Restrictions

Equations (46) and (47) are testable restrictions of our model. At the firm level, Modigliani Miller financing irrelevance theorem holds. Thus, when interpreting our model and conducting empirical analysis, we un-lever the firm by computing its cost of capital with a weighted average cost of all types of capital, including debt, equity, and other financial claims.

In the traditional risk-return sense, an unsustainable firm is not "riskier" than a sustainable firm in our model as dividend payments for a sustainable firm are larger than for an unsustainable firm path by path creating a seeming "arbitrage" profits. However, the investors of the S-portfolio demand a lower rate of return due to the investment mandate.

Hence, it is the investment mandate that causes the cost of capital for sustainable and unsustainable firms to be different rather than the covariance between a firm's return and the stock-market return. Since in our laissez-faire competitive-market model private sectors provide no aggregate risk mitigation, the mandate increases welfare as it encourages private sectors to mitigate aggregate risk which in turn generates positive externality.

Finally, we note another testable restriction is that the cum-dividend return, $\frac{d\mathbf{Q}_t^n+\mathbf{D}_{t-}^n dt}{\mathbf{Q}_{t-}^n}$, given in (9) is higher for U than for S firms path by path, but the realized capital gain, $d\mathbf{Q}_t^n/\mathbf{Q}_{t-}^n$, is always the same for the two types.[12] That is, the return wedge for the U and S firms comes solely from the dividend-yield difference between them: $(cf^U/\mathbf{q})-cf^S/\mathbf{q} = m/\mathbf{q}$.

4.4 Welfare-Maximizing $\mathbf{x}$ and Firm Qualification Criterion $m(\mathbf{x})$

So far, we have taken a given level of scaled aggregate mitigation spending, $\mathbf{x}$, as given. Recall that the representative agent's welfare is measured by $\mathbf{b}$ given in (23)-(24):

$$\mathbf{b}(\mathbf{x}) \;=\; (A-\mathbf{i}-\mathbf{x})^{1/(1-\psi)}\left(\frac{\rho}{\phi'(\mathbf{i})}\right)^{-\psi/(1-\psi)} . \tag{48}$$

We may endogenize $\mathbf{x}$ by maximizing (48). Let $\mathbf{x}^{FB}$ denote the maximand of (48), which is the first-best level of $\mathbf{x}$.

[12] This result follows from the equilibrium properties that the two types have the same Tobin's average q, i.e., $Q_t^U/K_t^U = Q_t^S/K_t^S = \mathbf{q}$, which implies $i^S = i^U = \mathbf{i}$ via the investment optimality condition and also $g^S = g^U = \mathbf{g}$. As a result, the expected capital gains is equal to the expected growth rate, $\mathbb{E}_{t-}(dQ_t^U/Q_{t-}^U) = \mathbb{E}_{t-}(dQ_t^S/Q_{t-}^S) = \mathbf{g}dt$, which follows from (2).

First-best. The minimal amount of capital needed to attain the first-best, $\underline{\alpha}^{FB}$, is

$$\underline{\alpha}^{FB} = \frac{\mathbf{x}^{FB}}{A - \mathbf{i}^{FB} - \widehat{cf}} \leq 1 \,. \tag{49}$$

If there exists a level of $\underline{\alpha}^{FB} \leq 1$, the first-best is attainable. Otherwise, the first-best is attainable. The intuition for this condition is as follows. Provided that at least an $\underline{\alpha}^{FB}$ fraction of wealth supports sustainable investments under dividend constraints $cf^n \geq \widehat{cf}$, i.e., $\alpha > \underline{\alpha}^{FB}$, the first-best outcome is attainable by setting the mitigation spending mandate for an individual firm by setting $m^{FB} = \mathbf{x}^{FB}/\alpha$. When α is very close to $\underline{\alpha}^{FB}$, the sustainable firms spend their entire post-investment capital on mitigation and pays minimal dividends out $\widehat{cf}$. As α increases, more firms become sustainable as the cost of mitigation spending for each firm is reduced. This equilibrium adjustment also has implications for the cost-of-capital wedge and other equilibrium price and quantity variables.

Now consider the case where $\alpha < \underline{\alpha}^{FB}$, where $\underline{\alpha}^{FB}$ is given in (49). In this case, the first-best cannot be attained. The qualification threshold for being a sustainable firm is now set at

$$m(\mathbf{x}) = \mathbf{x}/\alpha \,. \tag{50}$$

The solution requires an α fraction of the firm in the economy to pay minimal dividends $(cf = \widehat{cf})$ and spend their entire free cash flows, $\left(A - \mathbf{i} - \widehat{cf}\right)$, on mitigation. In this case, the (scaled) aggregate mitigation spending is $\mathbf{x} = \alpha \left(A - \mathbf{i} - \widehat{cf}\right)$.

We choose $\mathbf{x}$ to maximize the representative consumer's utility, equivalently the welfare measure b given in (24) for the value function (23), subject to $\mathbf{x} = \alpha(A - \mathbf{i} - \widehat{cf})$ and

$$\frac{\alpha\,\widehat{cf} + (1-\alpha)(A - \mathbf{i})}{r^M - \mathbf{g}(\mathbf{i})} = \frac{1}{\phi'(\mathbf{i})} = \mathbf{q} \,. \tag{51}$$

5 Quantitative Analysis

In this section, we operationalize our model. First, we calibrate our model and choose parameter values based on business-as-usual projections of the damage of global warming to economic growth. Second, we describe our quantitative results and findings.

5.1 Calibration

Preferences parameters. We choose consensus values for the coefficient of relative risk aversion, $\gamma = 2$ and the time rate of preferences, $\rho = 5\%$ per annum. Estimates of the EIS ψ in the literature vary considerably, ranging from a low value near zero to values as high as two.[13] We choose $\psi = 1.1$ which is larger than one, as emphasized by Bansal and Yaron (2004) and the long-run risk literature for asset-pricing purposes.

Production parameters. As in Pindyck and Wang (2013), we specify the investment-efficiency function $\phi(i)$ as

$$\phi(i) = i - \frac{\eta i^2}{2} - \delta \,, \tag{52}$$

where δ is the depreciation rate and η measures the degree of adjustment costs. We set the productivity parameter: $A = 22\%$ per annum and the adjustment-cost parameter $\eta = 3.5$ as in Eberly, Rebelo, and Vincent (2012), the annual depreciation rate $\delta = 6\%$ as in Stokey and Rebelo (1995), and the annual diffusion volatility $\sigma = 14\%$ close to that in Pindyck and Wang (2013).

Disaster arrival rate λ and damage function. We calibrate the disaster arrival rate and damage function using business-as-usual projections from Burke, Hsiang, and Miguel (2015). Their projections are based on a set of panel regressions documenting the adverse effects of exogenous annual changes in temperature (i.e. weather shocks) for economic growth (Dell, Jones, and Olken (2014)).[14] They quantify the potential impact of warming on national and global incomes by combining these estimated response functions (which can also be modeled as non-linear as opposed to linear functions) with "business as usual" scenarios (Representative Concentration Pathway (RCP) 8.5) of future warming and different assumptions

[13] Attanasio and Vissing-Jørgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero. Guvenen (2006) reconciles the conflicting evidence on the elasticity of intertemporal substitution from a macro perspective.

[14] This panel regression approach initially focused on how weather affects crop yields (Schenkler and Roberts (2009)) by using location and time fixed effects. But it is now applied to many other contexts including economic growth and productivity. The main idea is that extreme annual temperature fluctuations are plausibly exogenous shocks that causally trace out the impact of higher temperatures on output.

regarding future baseline economic and population growth. This approach assumes future economies respond to temperature changes similarly to today's economies.

Their projection is that absent mitigation median global GDP per capita in 2100 will be 76.3% of what it is in 2010, i.e., 23.6% lower per capita in 2100 compared to 2010 due to global warming absent mitigation. A 23.6% lower GDP per capita in 2100 compared to 2010 maps into an annual GDP per capita growth rate of -0.3% absent mitigation, as $(1-1/100)^{90}=0.763$ where 90 is the number of years between 2010 and 2100.

Table 1: Parameter Values

Parameters	Symbol	Value
elasticity of intertemporal substitution	ψ	1.1
time rate of preference	ρ	0.05
coefficient of relative risk aversion	γ	2
productivity	A	22%
quadratic adjustment cost parameter	η	3.5
capital diffusion volatility	σ	14%
depreciation rate	δ	6%
dividend constraint	$\widehat{cf}$	8%
power-law exponent with no mitigation	β_0	100
jump arrival rate	λ	4.19
mitigation technology parameter	β_1	8.9×10^3

All parameter values, whenever applicable, are continuously compounded and annualized.

Since extreme annual temperatures are related weather disasters (Auffhammer, Hsiang, Schlenker, and Sobel (2013)), we map these business-as-usual projections into our disaster framework. As in Barro (2006) and Pindyck and Wang (2013), we assume that the cdf of the recovery fraction, Z, is given by a power-law function defined over $(0,1)$ (Gabaix (2009)):

$$\Xi(Z;\mathbf{x})=Z^{\beta(\mathbf{x})}\,, \tag{53}$$

where $\beta(\mathbf{x})$ depends on scaled aggregate mitigation $\mathbf{x}$. To ensure that our model is well defined (and economically relevant moments are finite), we require $\beta(\mathbf{x})>\gamma-1$. As in

Hong, Wang, and Yang (2020), we use the following linear specification for $\beta(\mathbf{x})$:

$$\beta(\mathbf{x}) = \beta_0 + \beta_1 \mathbf{x}\,, \tag{54}$$

with $\beta_0 \geq \max\{\gamma - 1, 0\}$ and $\beta_1 > 0$. The coefficient β_0 is the exponent for recovery Z in the absence of mitigation. The coefficient β_1 is a key parameter that measures the efficiency of the mitigation technology.

Conditional on a jump arrival, the expected fractional capital loss, $\ell(\mathbf{x})$, is given by

$$\ell(\mathbf{x}) = \mathbb{E}^{\mathbf{x}}(1 - Z) = \frac{1}{\beta(\mathbf{x}) + 1} = \frac{1}{\beta_0 + \beta_1 \mathbf{x} + 1}\,. \tag{55}$$

The larger the value of $\beta(\mathbf{x})$, the smaller the expected fractional loss $\mathbb{E}^{\mathbf{x}}(1 - Z)$. We set the power-law parameter in the absence of mitigation $\beta_0 = 100$. Absent mitigation ($\mathbf{x} = \mathbf{0}$), the implied expected fractional capital loss is $\ell(0) = 1/(\beta_0 + 1) = 1/101 \approx 1\%$ as $\beta_0 = 100$.

Recall that for a given $\mathbf{x}$, the expected aggregate growth rate, $\mathbf{g}$, is

$$\mathbf{g} = \phi(\mathbf{i}) - \lambda \mathbb{E}^{\mathbf{x}}(1 - Z) = \phi(\mathbf{i}) - \frac{\lambda}{\beta(\mathbf{x}) + 1} = \phi(\mathbf{i}) - \lambda \ell(\mathbf{x})\,. \tag{56}$$

Absent mitigation ($\mathbf{x} = 0$), applying our solution procedure we obtain $\mathbf{i} = 12.65\%$ per annum. The implied jump arrival rate is $\lambda = 4.19$ per annum in order to match the -0.3% growth rate per annum from Burke, Hsiang, and Miguel (2015). We report these parameters associated with the business-as-usual competitive equilibrium in Table 1.

Mitigation spending on decarbonization technology. According to estimates from a McKinsey Sustainability report (de Pee (2018)), decarbonization of just the heavy industries like cement that account for 20% of the global carbon emissions will cost around 20 trillion dollars up to 2050 (or 0.75 trillion dollars per year just for heavy sectors). Each year, the entire global industrial sector emits around 50 billion metric tons of greenhouse gases. As a baseline scenario, we suppose that the portfolio of decarbonization technology costs over the next thirty years reach either 144 dollars per metric ton (low cost) or 288 dollars per metric ton (high cost). These numbers are lower than widely quoted range for the social cost of carbon. This would mean firms need to spend 3.67 (7.4) trillion dollars annually on decarbonization or 0.6% (1.2%) of 600 trillion dollars of global capital stock.

We then calibrate the parameter β_1 as follows. Suppose that the aggregate mitigation spending is $\mathbf{x} = 0.6\%$ is able to stop the rise of global temperature and that the expected growth rate is not as severely damaged as it otherwise would be absent mitigation. There is in general uncertainty about how the abatement of temperatures will translate to the mitigation of damages. We set $\mathbf{g} = 1\%$ per annum as a baseline scenario and consider some alternative targets ranging from 0.5% to 1.5% in the comparative statics. For our baseline, solving (56) yields $\beta_1 = 8.9 \times 10^3$.

Lower bound on dividend payouts. Additionally, we impose a restriction that a firm pays about 36 percent of its revenues (AK) as dividends, i.e. $\widehat{cf} = 8\%$, which is roughly the payout ratio for mature consumer or industrial companies. We vary this restriction $\widehat{cf}$ in a comparative statics exercise.

Table 2: COMPARING COMPETITIVE MARKET SOLUTION WITH FIRST-BEST

A. Competitive Market Outcomes		
mitigation level	$\mathbf{x}$	0
aggregate investment	$\mathbf{i}$	12.65%
aggregate consumption/dividends	$\mathbf{c}$	9.35%
expected GDP growth rate	$\mathbf{g}$	-0.3%
(real) risk-free rate	r	0.82%
stock market risk premium	$r^M - r$	4.09%
stock market return volatility		14.30%
Tobin's q	$\mathbf{q}$	1.79
B. First-Best Outcomes		
first-best mitigation level	$\mathbf{x}^{FB}$	1.72%
aggregate investment	$\mathbf{i}^{FB}$	11.75%
aggregate consumption/dividends	$\mathbf{c}^{FB}$	8.53%
expected GDP growth rate	$\mathbf{g}^{FB}$	1.68%
(real) risk-free rate	r	2.76%
stock market risk premium	$r^M - r$	3.95%
stock market return volatility		14.05%
Tobin's q	$\mathbf{q}^{FB}$	1.70

All parameter values, whenever applicable, are continuously compounded and annualized.

In Table 2, we report equilibrium outcomes for key variables of interest for both the competitive equilibrium and first-best solutions. Panel A reports the competitive equilibrium predictions. There is no aggregate risk mitigation, $\mathbf{x} = \mathbf{0}$. The market risk premium is 4.09% per annum and the real risk-free rate is 0.82% per annum. Additionally, the implied Tobin's average $\mathbf{q}$ is 1.79 and the annual stock market volatility is 14.30%.

Panel B of Table 2 reports the first-best results. With $\beta_1 = 8.9 \times 10^3$, the first-best level of mitigation is $\mathbf{x}^{FB} = 1.72\%$ per annum. Mitigation spending makes the economy more sustainable turning the aggregate (expected) growth rate positive (1.68% per annum) from -0.3% per annum. Compared with the competitive equilibrium results in Panel A, in the first-best planner's economy, the real risk-free rate is higher (2.76% per annum) and the equity risk premium is slightly lower (3.95% per annum). While aggregate risk mitigation costs roughly 1.72% of the capital stock each year, causing both consumption and investment to be lower than in the competitive market economy, optimally mitigating aggregate risk nonetheless enhances welfare and generates sustainable growth.

5.2 Findings

Relation between x and outcome variables of interest. Figure 1 shows that as aggregate mitigation spending $\mathbf{x}$ increases, both the aggregate investment $\mathbf{i}$ and consumption $\mathbf{c}$ decrease (see Panels A and C). This is because fewer resources are left (after mitigation) to allocate between investment and consumption. The black dots in these panels correspond to the net-zero emissions target spending of $\mathbf{x} = 0.6\%$. At $\mathbf{x} = 0.6\%$, investment is 12.4% of capital stock and consumption is 9% of capital stock. Investment is only a bit lower compared to the competitive equilibrium, i.e. $\mathbf{x} = 0$ but consumption is significantly lower by 50 bps per annum.

Since the investment FOC implies that Tobin's $\mathbf{q}$ is monotonically increasing in $\mathbf{i}$, the market value of capital, $\mathbf{q}$, hence also decreases with aggregate $\mathbf{x}$ (Panel B). Panel D of Figure 1 plots the society's willingness to pay (WTP) to move away from a competitive

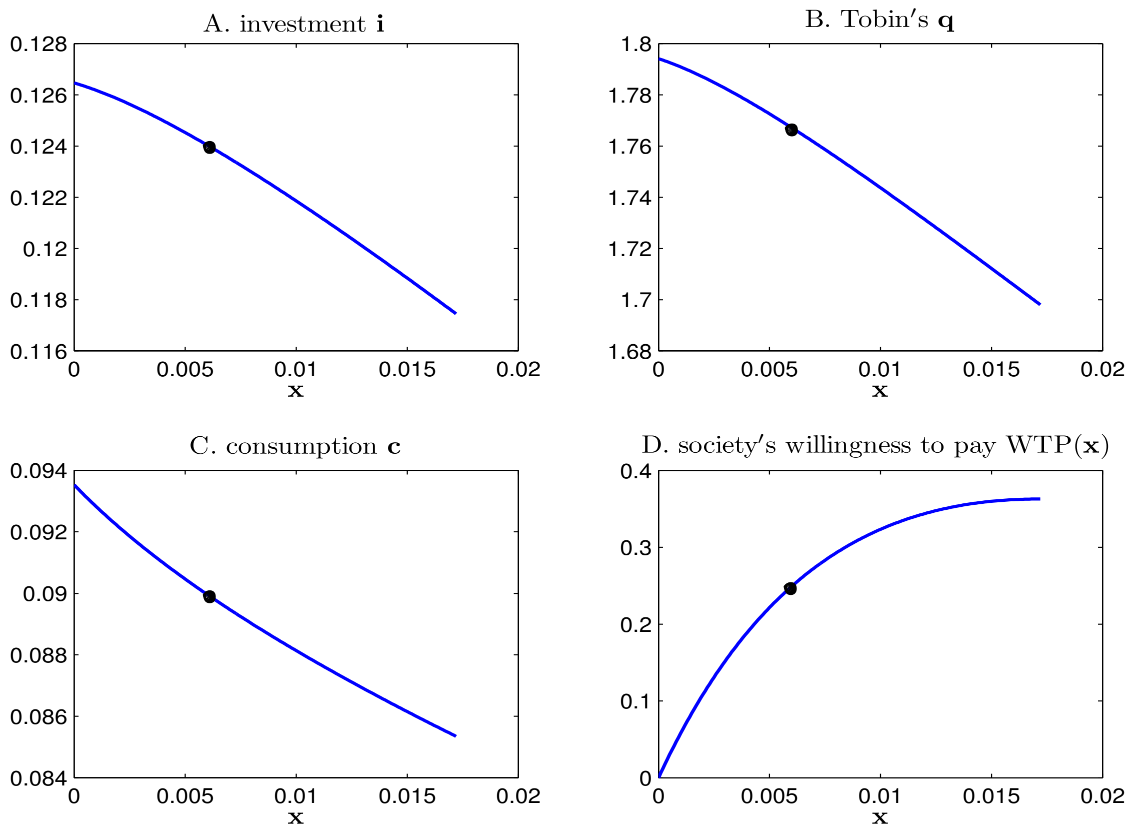

Figure 1: This figure plots the effects of aggregate mitigation $\mathbf{x}$ on aggregate investment $\mathbf{i}$, aggregate Tobin's $\mathbf{q}$, consumption $\mathbf{c}$ and society′s willingness to pay WTP($\mathbf{x}$). The parameters values are reported in Table 1.

equilibrium where $\mathbf{x} = 0$ to one with an aggregate mitigation spending level $\mathbf{x}$:

$$\text{WTP}(\mathbf{x}) = \frac{\mathbf{b}(\mathbf{x})}{\mathbf{b}^{CE}} - 1\,, \tag{57}$$

where $\mathbf{b}^{CE} = \mathbf{b}(\mathbf{x}^{CE}) = \mathbf{b}(0)$, as $\mathbf{x}^{CE} = 0$ in competitive equilibrium. Naturally, WTP($\mathbf{x}$) increases with $\mathbf{x}$.

The black dots in these panels again mark the net-zero emissions target corresponding to $\mathbf{x} = 0.6\%$. With the aggregate risk mitigation of $\mathbf{x} = 0.6$, the WTP is about 25% percent higher than in the competitive equilibrium. As aggregate risk mitigation $\mathbf{x}$ approaches 1.72% of the aggregate capital stock ($\mathbf{x} = 1.72\%$), welfare is maximized and welfare is 36% higher than in the competitive equilibrium. And Tobin's q when $\mathbf{x} = 0.6\%$ is only slightly lower than in the competitive equilibrium with ($\mathbf{x} = 0$) (1.77 compared to 1.79).

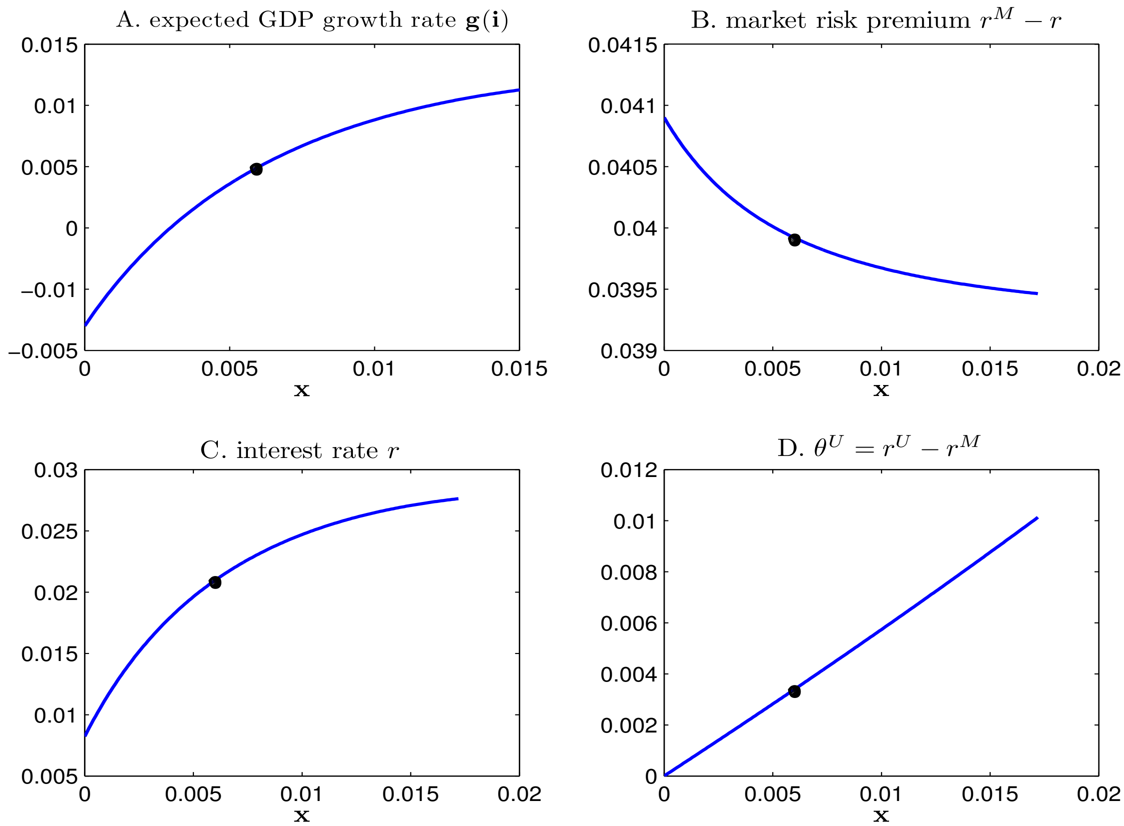

Figure 2: This figure plots the effects of (scaled) aggregate mitigation $\mathbf{x}$ on the expected growth rate $\mathbf{g}$, the market risk premium $r^M - r$, the interest rate, and the wedge between the sustainable firm's cost of capital and the market return, $\theta^U = r^U - r^M$. The parameters values are reported in Table 1.

Panel A of Figure 2 shows that the expected growth rate $\mathbf{g}$ increases with (scaled) aggregate mitigation spending $\mathbf{x}$. Two forces drive this relationship. On the one hand, since investment $\mathbf{i}$ decreases with $\mathbf{x}$ as we see from Figure 1 and growth increases with investment, we expect $\mathbf{g}$ to fall with $\mathbf{x}$. This is a standard investment crowding out effect from mitigation spending. On the other hand, increasing $\mathbf{x}$ also makes the economy less risky and hence lowers conditional damages, which increases $\mathbf{g}$. The latter force in our calibration dominates the former investment crowding out effect. That is, aggregate risk mitigation makes growth more sustainable. At the black dot indicating $\mathbf{x} = 0.6$, the expected growth rate $\mathbf{g}$ is by construction 1%.

Panel B of Figure 2 shows that the aggregate market risk premium, $r^M - r$, decreases with (scaled) aggregate mitigation spending $\mathbf{x}$. This is because the equilibrium jump risk premium is proportional to $\mathbb{E}^{\mathbf{x}}[(1-Z)(Z^{-\gamma}-1)]$, which decreases with $\mathbf{x}$ as Z becomes less fat tailed. At the black dot indicating $\mathbf{x}=0.6$, the market risk premium is a bit less than 4%, not much different from the level at the competitive equilibrium of no mitigation spending.

Panel C of Figure 2 shows that the equilibrium risk-free rate r increases with $\mathbf{x}$. Since increasing $\mathbf{x}$ makes the distribution for Z less fat-tailed, mitigation lowers the representative agent's precautionary savings demand and hence increases r as we see from (42). The risk-premium and risk-free rate results again reflect a key force in our model: mitigation spending lowers aggregate risk, which in turn reduces the agent's precautionary savings demand, supports sustainable growth, and enhances welfare. Due to the force of aggregate risk reduction from mitigation, the risk-free rate is higher (2.1% at the net-zero emissions target of $\mathbf{x}=0.6\%$ compared to 0.82% per annum under competitive equilibrium with no mitigation)

The cost of being an unsustainable firm. As we have shown, an unsustainable firm has a higher cost of capital than a sustainable one. Panel D of Figure 2 shows that the wedge between the cost of capital for an unsustainable firm and the market portfolio, $\theta^U=r^U-r^M$, increases with the scaled mitigation $\mathbf{x}$. Because $\mathbf{q}$ decreases with $\mathbf{x}$ (see Panel B of Figure 1) and $\theta^U=\mathbf{x}/\mathbf{q}$, θ^U unambiguously increases with $\mathbf{x}$. The intuition for this result is as follows. As $\mathbf{x}$ increases, the unsustainable firm's cost of capital relative to the market increases as more mitigation has to be done by sustainable firms. Note that this wedge θ^U does not depend on the mandate α. The black dot in Panel D shows that the expected return of unsustainable companies is close to 0.4% higher relative to the expected return of the market is higher when $\mathbf{x}=0.6$

The effect of α on equilibrium outcomes. Next, we analyze how aggregate mitigation $\mathbf{x}$, qualification criterion m for a firm to be sustainable, Tobin's q, and the cost-of-capital wedge, r^U-r^S between S and U firms, vary as we increase the fraction of capital to support sustainable investing, α. Panel A of Figure 3 shows that as α increases, aggregate mitigation

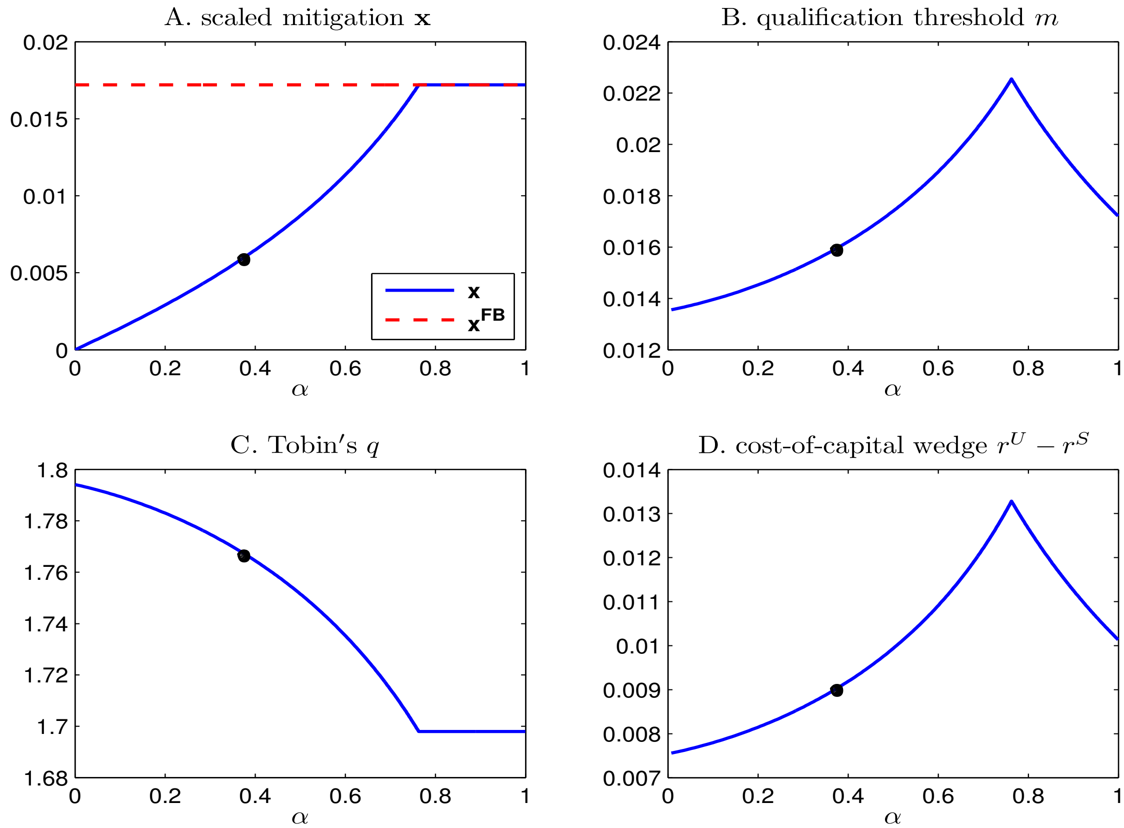

Figure 3: This figure plots the effects of (scaled) aggregate mitigation $\mathbf{x}$ on the expected growth rate $\mathbf{g}$, the market risk premium $r^M - r$, the interest rate, and the wedge between the sustainable firm's cost of capital and the market return, $\theta^U = r^U - r^M$. The parameters values are reported in Table 1.

$\mathbf{x}$ increases, eventually reaching the first-best level $\mathbf{x}^{FB}$. The reason is that the economy can support more mitigation spending with a higher α.

Panel B of Figure 3 shows that the optimal mandate m is increasing slightly with α for $\alpha \leq \underline{\alpha}^{FB}$ and then decreasing significantly in α for $\alpha > \underline{\alpha}^{FB}$. To see why, recall that for $\alpha < \underline{\alpha}^{FB}$ the economy does not attain the first-best and the sustainable firm spends their entire free cash flows on mitigation, i.e., $m = \mathbf{x}/\alpha = A - \mathbf{i} - \widehat{cf}$. Since investment $\mathbf{i}$ is decreasing in $\mathbf{x}$ (see Panel A of Figure 1) and $\mathbf{x}$ is increasing in α (Panel A), investment $\mathbf{i}$ must also decrease in α and hence the qualification threshold $m = A - \mathbf{i} - \widehat{cf}$ must increase in α in the region where $\alpha < \underline{\alpha}^{FB}$. When the first-best is attained, i.e. $\alpha > \underline{\alpha}^{FB}$, each S firm chooses $m = \mathbf{x}^{FB}/\alpha$, which is decreasing in α as more S firms share a fixed level of aggregate mitigation $\mathbf{x}^{FB}$.

Panel C of Figure 3 shows that Tobin's q decreases with α in the region where $\alpha < \underline{\alpha}^{FB}$, as a higher α implies a higher $\mathbf{x}$ which gives rise to a lower average q as shown in Panel B of Figure 1. When $\alpha > \underline{\alpha}^{FB}$, the economy attains first-best and therefore $q = \mathbf{q}^{FB}$.

Finally, Panel D shows that the cost-of-capital wedge between sustainable and unsustainable firms, $r^U - r^S$, increases with α until reaching $\underline{\alpha}^{FB}$. Recall that (47) shows the cost-of-capital wedge, $r^U - r^S$, is equal to $m/\mathbf{q}$. As m increases and Tobin's q decreases with α in the region where $\alpha < \underline{\alpha}^{FB}$ (Panels B and C), the equilibrium cost-of-capital wedge, $r^U - r^S$, increases with α. That is, an increase in α leads to an increase in the firm's qualification threshold m and a decrease in Tobin's q. These two forces reinforce each other causing $r^U - r^S = m/\mathbf{q}$ to increase with α.

However, the wedge $r^U - r^S$ decreases with α for $\alpha > \underline{\alpha}^{FB}$. This decreasing result follows from $r^U - r^S = m(\mathbf{x}^{FB})/q^{FB} = \mathbf{x}^{FB}/(\alpha q^{FB})$, as more firms are available to support first-best. As a result, each firm spends less on mitigation and its required compensation in terms of cheaper cost of capital falls.

Amount of capital and cost-of-capital wedge needed to support net-zero. From Figure 3, we can calculate the amount of capital, firm-level mitigation mandate, and cost-of-capital wedge needed to support the net-zero target. To support $\mathbf{x} = 0.6\%$, we need $\alpha = 37.6\%$. Each firm's mitigation spending is equal to $m = \mathbf{x}/\alpha = 0.6\%/37.6\% = 1.6\%$ of its own capital stock (captured by the black dots in these panels). Each firm pays $\widehat{cf} = 8\%$ as dividends to shareholders and uses the remaining (scaled) cash flow $A - m - \widehat{cf} = 22\% - 1.6\% - 8\% = 12.4\%$ for investment.

In contrast, the minimum fraction of wealth needed for sustainable finance to reach the first-best is $\underline{\alpha}^{FB} = 76.3\%$. That is, to attain the first-best planner's outcome, we need 76.3% of the firms in the economy to be sustainable. Each firm's mitigation spending is equal to $m = \mathbf{x}^{FB}/\underline{\alpha}^{FB} = 1.76\%/76.3\% = 2.31\%$ of its own capital stock. To achieve first-best, the wedge $r^U - r^S$ reaches the maximum of 1.33% per annum when $\alpha = \underline{\alpha}^{FB} = 76.3\%$.

5.3 Comparative Statics: Payout Constraints $cf_t \geq \widehat{cf}$

Figure 4 shows that when the payout constraint parameter $\widehat{cf}$ increases from 8% (for our baseline case) to 8.8%, first-best is unattainable because there does not exist a level of $\underline{\alpha}^{FB} \leq 1$ that satisfies equation (49). The intuition is that when the required payouts are too high, even if all firms in the economy ($\alpha = 1$) spend their entire free cash flows, $x = A - \mathbf{i} - \widehat{cf}$, on mitigation, the aggregate mitigation level is still below the first-best level: $\mathbf{x} < \mathbf{x}^{FB}$ as shown in Panel A. Panels B, C, and D show that qualitatively the effects of α on qualification threshold m, Tobin's q, and the cost-of-capital wedge, $r^U - r^S$ are the same as those in the baseline model for the region where $\alpha < \underline{\alpha}^{FB}$ (see Figure 3).

The quantitative effects become less significant for a given level of α since mitigation decreases as required minimal payouts increase. As a result, the amount of capital needed to achieve net-zero mitigation spending of $\mathbf{x} = 0.6\%$ is now around 75%. The qualification threshold m is now 80 bps and the cost-of-capital wedge is now 45bps since each sustainable firm is doing less mitigation.

Table 3: EFFECTS OF AGGREGATE MITIGATION LEVEL

$\mathbf{x}$	$\mathbf{g}$	β_1	α	m	$r^U - r^S$
0.6%	0.5%	5100	36.38%	1.65%	0.94%
0.6%	0.75%	6900	36.99%	1.62%	0.92%
0.6%	1%	8900	37.60%	1.60%	0.90%
0.6%	1.25%	11400	38.23%	1.57%	0.89%
0.6%	1.5%	14300	38.90%	1.54%	0.87%
1.2%	0.5%	3380	58.92%	2.04%	1.18%
1.2%	0.75%	4380	59.67%	2.01%	1.17%
1.2%	1%	5620	60.48%	1.98%	1.15%
1.2%	1.25%	7060	61.28%	1.96%	1.13%
1.2%	1.5%	8860	62.11%	1.93%	1.12%

All parameter values, whenever applicable, are continuously compounded and annualized.

5.4 Other Scenarios

Finally, in Table 3, we consider how the amount of sustainable capital, mandate and cost-

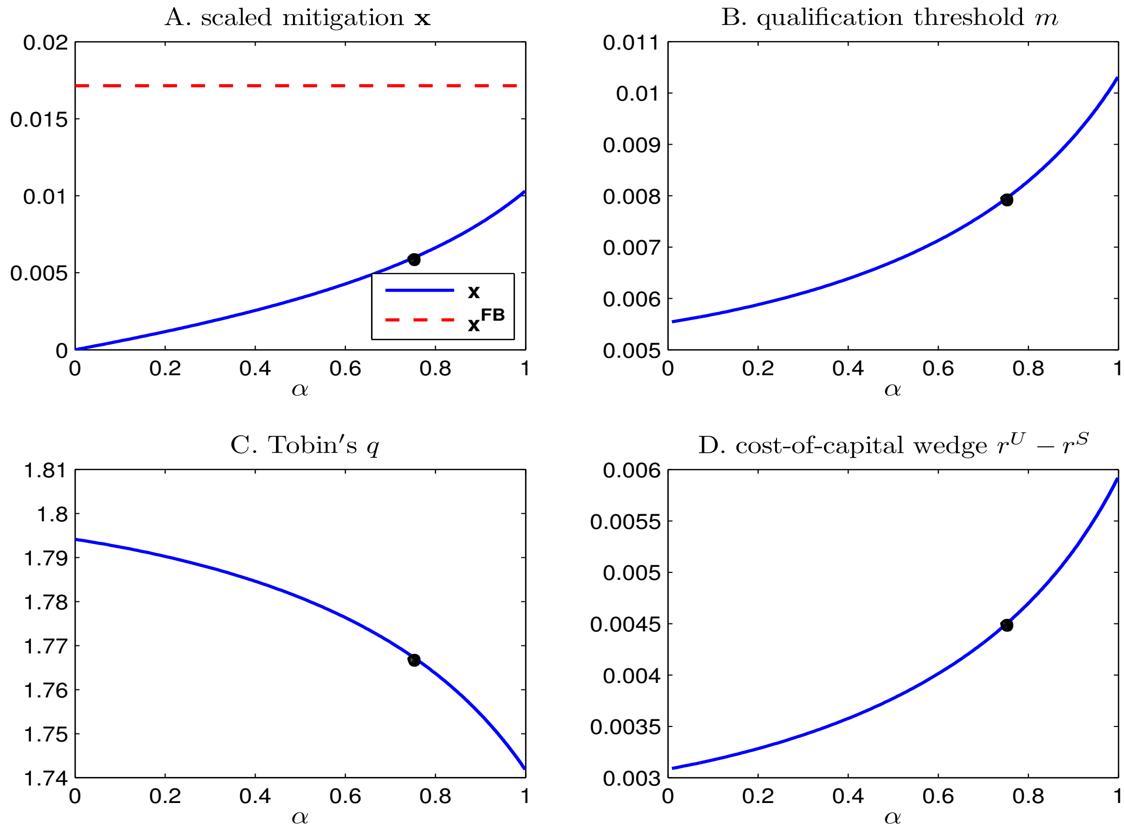

Figure 4: This figure plots the effects of α on optimal (scaled) aggregate mitigation $\mathbf{x}$, qualification threshold m, Tobin's average q, and the cost-of-capital wedge $r^U - r^S$. Dividend-capital lower bound is $\widehat{cf} = 8.8\%$ and all other parameters values are reported in Table 1.

of-capital wedge vary with two key comparative statics regarding the effectiveness of decarbonization technology and the cost of decarbonization technology.

In the first five rows, we fix the cost of decarbonization technology as in our baseline, i.e. $\mathbf{x} = 0.6\%$, and vary β_1 around our baseline of $\beta_1 = 8,900$ by targeting different growth rates g that are achieved by reaching net-zero. We consider a range from 0.5% growth to 1.5% growth. As the decarbonization becomes more effective, the amount of sustainable capital α increases slightly, the firm-level mandate m decreases slightly, and so too does the cost-of-capital wedge. But overall, there are not large differences in sustainable finance mandate outcomes as we vary the effectiveness of decarbonization technology.

In the next five rows, we increase the cost of decarbonization technology, i.e., suppose that net-zero targets require spending 1.2% as opposed 0.6% of capital stock on decarbonization measures. And then again vary the effectiveness of the decarbonization technology. Since we need to spend more to achieve the same set of growth rates, β_1's are lower in these five rows than the first five rows. Comparing $g = 1\%$ scenario across the to cost scenarios, the amount of sustainable capital needed increases substantially from 37.6% of wealth to 60.48% of wealth and so does the cost-of-capital wedge, rising from 0.90% to 1.15%. In other words, the cost of decarbonization technology is critical in determining the size of sustainable finance mandates.

6 Conclusion

Sustainable finance mandates have grown significantly in the last decade in lieu of government failures to address climate disaster externalities. Firms that spend enough resources on mitigation of these externalities qualify for sustainable finance mandates. These mandates incentivize otherwise ex-ante identical unsustainable firms to become sustainable for a lower cost of capital. We present and solve a dynamic stochastic general equilibrium model to address the welfare consequences. The model is highly tractable, including a simple formula that characterizes the cost-of-capital wedge between sustainable and unsustainable firms as the tax rate on firm value to subsidize mitigation. There are a number of testable implications that can be taken to the data and potential implications for the design of optimal sustainable finance mandates.

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Appendices

A Details for Model Solution

A.1 Firm Value Maximization

Using the standard dynamic programming, we obtain the following HJB equation for Q^n:

$$r^n Q^n = \max_{I^n, X^n} \; AK^n - I^n - X^n + \left(\Phi(I^n, K^n) Q_K^n + \frac{1}{2}(\sigma K^n)^2 Q_{KK}^n \right) + \lambda \mathbb{E}\left[Q^n(ZK^n) - Q^n(K^n) \right] , \tag{A.58}$$

subject to the dividend yield constraint $AK^n - X^n - I^n \geq \widehat{cf} K^n$. And then substituting $Q^n(K) = q^n K^n$ into (A.58), we obtain

$$r^n q^n = \max_{i^n, x^n} \; A - i^n - x^n + \phi(i^n) q^n + \lambda \left[\mathbb{E}(Z) - 1 \right] q^n , \tag{A.59}$$

subject to the dividend yield constraint $A - x^n - i^n \geq \widehat{cf}$.

The FOC for investment implied by (A.59) is

$$q^n = \frac{1}{\phi'(i^n)} , \tag{A.60}$$

which is the standard Tobin's q formula (e.g., Hayashi, 1982). As $x^U \geq 0$ and $x^S \geq m$, the optimal mitigation spending is $x^U = 0$ for a type-U firm and $x^S = m$ for a type-S firm. as no firm wants to spend more than it has to on mitigation.

We may rewrite (A.59) as

$$q^n = \max_{i^n \leq A - x^n - \widehat{cf}} \frac{A - i^n - x^n}{r^n - g(i^n)} , \tag{A.61}$$

where $g(i^n) = \phi(i^n) - \lambda(1 - \mathbb{E}(Z))$. Equation (A.61) implies (33).

As all firms have the same Tobin's q in equilibrium, we have $i^S = i^U = \mathbf{i} \leq A - m - \widehat{cf}$ and

$$\mathbf{q} = \frac{A - \mathbf{i} - m}{r^S - g(\mathbf{i})} = \frac{A - \mathbf{i}}{r^U - g(\mathbf{i})} . \tag{A.62}$$

A.2 Household's Optimization Problem

Using the same procedure as in Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), we can show that both the optimal risk-free asset holding and the jump hedging demand for all levels of Z are zero in equilibrium. Therefore, then may rewrite the household's wealth dynamics given by (13) as follows

$$dW_t = W_{t-} \left[\left[r + (r^S - r)\pi_{t-}^S + (r^U - r)(1 - \pi_{t-}^S) \right] dt + \sigma d\mathcal{B}_t - (1 - Z)\left(d\mathcal{J}_t - \lambda dt \right) \right] - C_{t-} dt , \tag{A.63}$$

where $\pi^S = H^S/(H^S + H^U) = H^S/W$.

The post-jump wealth is $W^{\mathcal{J}} = W - (1-Z)\,W = ZW$. The following HJB equation characterizes the value function $J(W)$:

$$0 = \max_{C,\pi^S} f(C,J) + \left[rW + \left((r^S - r)\pi^S + (r^U - r)(1-\pi^S) + \lambda(1-\mathbb{E}(Z))\right)W - C\right] J'(W)$$
$$+ \frac{\sigma^2 W^2 J''(W)}{2} + \lambda \int_0^1 \left[J\left(ZW\right) - J(W)\right]\xi(Z)dZ\,, \tag{A.64}$$

subject to $\pi^S \geq \alpha$. The FOC for consumption C is the standard condition:

$$f_C(C,J) = J'(W)\,. \tag{A.65}$$

Because the S- and the U-portfolio have exactly the same (diffusion and jump) risk exposures with probability one, the optimality for π^S is positive infinity if $r^S > r^U$ as we can see from (A.64). This is not an equilibrium. In equilibrium, $r^S \leq r^U$ and $\pi^S = \alpha$ holds. We later pin down the equilibrium relation between r^S and r^U.

Let $J_t = J(W_t)$ denote the household's value function. We show that

$$J(W) = \frac{1}{1-\gamma}(uW)^{1-\gamma}\,, \tag{A.66}$$

where u is a constant determined endogenously. Substituting (A.66) into the FOC (A.65) yields the following linear consumption rule:

$$C(W) = \rho^{\psi} u^{1-\psi} W\,. \tag{A.67}$$

A.3 Market Equilibrium

First, a sustainable firm spends minimally on mitigation: $x^S = \frac{X^S}{K^S}$. Second, in equilibrium, the household invests all wealth in the stock market and holds no risk-free asset, $H = W$ and $W = Q^S + Q^U$, and has zero disaster hedging position, $\delta(Z) = 0$ for all Z. Third, the representative agent's (dollar amount) investment in the S portfolio is equal to the total market value of sustainable firms, $\pi^S = \alpha$ and (dollar amount) investment for the U portfolio is equal to the total market value of unsustainable firms, $\pi^U = 1-\alpha$. Finally, goods market clears.

By using the preceding equilibrium conditions together with $H = W = Q^S + Q^U = q^S K^S + q^U K^U = q(K^S + K^U) = qK$, $W^{\mathcal{J}} = ZW$ and $\pi^S = \alpha$, we obtain

$$\alpha r^S + (1-\alpha) r^U = r + \gamma\sigma^2 + \lambda\mathbb{E}\left[(1-Z)(Z^{-\gamma} - 1)\right] = r^M\,, \tag{A.68}$$
$$p(Z) = \lambda Z^{-\gamma}\xi(Z)\,. \tag{A.69}$$

Using $\alpha r^S + (1-\alpha)r^U = r^M$, $\mathbf{x} = \alpha m$, and (A.62), we obtain

$$\begin{aligned}\frac{A - \mathbf{i} - \mathbf{x}}{r^M - g(\mathbf{i})} &= \frac{\alpha(A - \mathbf{i} - m) + (1-\alpha)(A - \mathbf{i})}{\alpha r^S + (1-\alpha) r^U - g(\mathbf{i})} \\ &= \frac{\alpha \mathbf{q}(r^S - g(\mathbf{i})) + (1-\alpha)\mathbf{q}(r^U - g(\mathbf{i}))}{\alpha(r^S - g(\mathbf{i})) + (1-\alpha)(r^U - g(\mathbf{i}))} = \mathbf{q}\,,\end{aligned} \tag{A.70}$$

which implies (37). And solving

$$\mathbf{q} = \frac{\mathbf{A}-\mathbf{i}}{\mathbf{r^M}+\theta^{\mathbf{U}}-\mathbf{g(i)}} = \frac{\mathbf{A}-\mathbf{i}}{\mathbf{r^U}-\mathbf{g(i)}} = \frac{\mathbf{A}-\mathbf{i}-\mathbf{x}}{\mathbf{r^M}-\mathbf{g(i)}}\,, \tag{A.71}$$

subject to $\mathbf{i} \leq A - m - \widehat{cf}$. we obtain $(A-\mathbf{i})\theta^U = \mathbf{x}(r^U - g(\mathbf{i}))$ and $\theta^U = \mathbf{x}/\mathbf{q} = \alpha m/q$ as shown in (45).

In addition, the optimal consumption rule given in (A.67) implies

$$c = \frac{C}{K} = \frac{C}{W}q = \rho^{\psi}u^{1-\psi}q\,. \tag{A.72}$$

And then substituting c given by (A.72) and the value function given in (A.66) into the HJB equation (A.64), and using $\pi = \alpha$, $\delta = 0$, and $H = W$, we obtain

$$\begin{aligned} 0 &= \frac{1}{1-\psi^{-1}}\left(\frac{c}{q}-\rho\right) + \left(\alpha r^S + (1-\alpha)r^U - \frac{c}{q} + \lambda(1-\mathbb{E}(Z))\right) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] \\ &= \frac{1}{1-\psi^{-1}}\left(\frac{c}{q}-\rho\right) + \left(r^M - \frac{c}{q} + \lambda(1-\mathbb{E}(Z))\right) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]. \end{aligned} \tag{A.73}$$

The goods market clear condition implies $c = \alpha(A - i^S - x^S) + (1-\alpha)(A - i^U) = \alpha q^S(r^S - g(i^S)) + (1-\alpha)q^U(r^U - g(i^U)) = q(r^M - \alpha g(i^S) - (1-\alpha)g(i^U)) = q(r^M - g(i))$. By using (A.62), we obtain

$$\frac{c}{q} = r^M - g(i)\,, \tag{A.74}$$

which implies (40). And then by substituting it into (A.73) and combining $r^M = r + \gamma\sigma^2 + \lambda\mathbb{E}\left[(1-Z)(Z^{-\gamma}-1)\right]$, we obtain (42) for the equilibrium interest rate.