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Examining the Sources of Excess Return Predictability: Stochastic Volatility or Market Inefficiency?*

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Abstract

We use a consumption based asset pricing model to show that the predictability of excess returns on risky assets can arise from only two sources: (1) stochastic volatility of fundamental variables, or (2) departures from rational expectations that give rise to predictable investor forecast errors and market inefficiency. While controlling for stochastic volatility, we find that a variable which measures non-fundamental noise in the Treasury yield curve helps to predict 1-month-ahead excess stock returns, but only during sample periods that include the Great Recession. For these sample periods, higher noise predicts lower excess stock returns, implying that a shortage of arbitrage capital in financial markets caused excess returns to drop below the levels justified by fundamentals. The statistical significance of the predictor variables that control for stochastic volatility are also typically sensitive to the sample period. Measures of implied and realized stock return variance cease to be significant when the COVID-influenced data from early 2020 onward is included.

Keywords: *Equity Premium, Excess Volatility, Return Predictability, Market Sentiment, Time Series Momentum, Yield Curve Noise.*
JEL Classification: E44, G12.

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1 Introduction

A vast literature, pioneered by Fama and French (1988), examines the so-called "predictability" of excess returns on risky assets. Predictability is typically measured by the size of a slope coefficient and the adjusted R-squared statistic in forecasting regressions over various time horizons. This paper examines the predictability question from both a theoretical and empirical perspective.

Our theoretical approach employs a standard consumption based asset pricing model. We show that the predictability of excess returns on risky assets can arise from only two sources: (1) stochastic volatility of fundamental variables, or (2) departures from rational expectations that give rise to predictable investor forecast errors and market inefficiency. Specifically, we show that excess returns on risky assets can be represented by an additive combination of conditional variance terms and investor forecast errors. This result holds for any stochastic discount factor, any consumption or dividend process, and any stream of bond coupon payments.

The conditional variance terms that appear in the expression for excess returns can be a source of predictability if one or more of the model's fundamental variables exhibit stochastic volatility. In the absence of stochastic volatility, rationally time-varying risk aversion, by itself, is not a source of predictable excess returns. But if we allow for a departure from rational expectations, then the resulting investor forecast errors will be predictable, serving as an alternative source of predictable excess returns. Under extrapolative expectations, excess returns are predictable, but stochastic volatility does not contribute to this predictability. This is because the investor's subjective forecasts do not take into account the laws of motion for the fundamental variables that give rise to stochastic volatility. We provide analytical examples to illustrate each of these cases.

Our empirical approach examines whether 1-month-ahead excess returns on stocks relative to the risk free rate can be predicted using data from 1990.M1 to 2020.M12. Motivated by our theoretical results, we look for evidence that market inefficiency (in the form of non-rational expectations) contributes to the predictability of excess returns while controlling for the presence of stochastic volatility. As an indirect control for stochastic volatility, we include the price-dividend ratio—a typical predictor variable in the literature. We show that under rational expectations, the equilibrium price-dividend ratio in the model will depend on any fundamental state variables that give rise to stochastic volatility. As direct controls for stochastic volatility, we include the implied variance and the realized variance of recent stock returns.

Numerous studies have found that measures of investor sentiment or return momentum can often help to predict excess stock returns.[1] It is tempting to conclude that such results provide evidence of market inefficiency. However, we cannot rule out that measures of sentiment or momentum are linked to movements in the underlying fundamental variables that give rise to stochastic volatility. Indeed, Audrino et al. (2020) find that measures of investor sentiment and investor attention for individual stocks have significant predictive power for a stock's future realized return volatility. Wang et al. (2006) find that the lagged stock market return (a measure of momentum) helps to predict both the implied volatility and the realized volatility of stock returns. The same study confirms the findings of Brown and Cliff (2004) that the lagged stock market return helps to predict future changes in sentiment.[2] Based on these results, we include measures of sentiment and momentum as indirect controls for stochastic volatility. Our sentiment variable is the 12-month change in the University of Michigan's consumer sentiment index. Our momentum variable is the trailing 1-month change in the excess stock return. As an additional control variable, we interact the 12-month sentiment change with our measure of return momentum because this variable turns out to be a more robust predictor than either sentiment or momentum in isolation.

The predictor variable that we use to detect market inefficiency is a measure of non-fundamental "noise" in the Treasury yield curve, as constructed by Hu et al. (2013). Specifically, the noise variable is the monthly average root mean squared deviation between the daily Treasury yield curve and a daily model-fitted, no-arbitrage yield curve. The noise variable captures the degree to which "limits to arbitrage" due to a lack of liquidity in financial markets may allow asset prices to deviate from fundamental values. According to Hu et al. (2013, p. 2344), "our noise measure does not simply capture the liquidity concerns specific to the Treasury market, but rather reflects how different liquidity crises might transmit through financial markets via the movements of arbitrage capital. In other words, rather than being a measure specific to the Treasury market, our noise measure is a reflection of overall market conditions."

Given that the noise variable in our predictive regressions is constructed using data from the Treasury yield curve, we include the monthly average Treasury term spread (the difference between the 10-year and 3-month constant maturity Treasury yields) as an additional control for stochastic volatility. The term spread captures expectations of future monetary policy actions that, in turn, could influence asset return volatility.

We find that the regression coefficient on the noise variable is statistically significant, but only during sample periods that include the Great Recession. For these sample periods, higher noise predicts lower excess stock returns, implying that a shortage of arbitrage capital

[1] For sentiment, see Huang, et al. (2014) and Adämmer and Schüssler (2020). For momentum, see Moskowitz et al. (2012), Neely, et al. (2014), and Cujean and Hasler (2017).

[2] Lansing and Tubbs (2019) report a similar finding.

in financial markets during the Great Recession caused excess returns to drop below the levels justified by fundamentals.

Our full-sample predictability regression for the period from 1990.M1 to 2020.M12 yields an adjusted R-squared statistic of 8.3%. If we omit the noise variable, then the adjusted R-squared statistic drops to 4.8%. In out-of-sample tests, including the noise variable markedly improves the out-of-sample R-squared statistic when the out-of-sample period includes the Great Recession, but not otherwise.

As for the seven predictor variables that control for stochastic volatility, we find that statistical significance is also typically sensitive to the sample period. The price-dividend ratio, the implied and realized variances of stock returns, and the sentiment-momentum interaction variable are not significant during recession periods. The implied stock return variance and the realized stock return variance both cease to be statistically significant when the COVID-influenced data from early 2020 onward is included. Using 120-month rolling regressions, we show that only three predictor variables (price-dividend ratio, implied stock return variance, and sentiment-momentum interaction variable) are significant across a diverse set of sample periods. The remaining predictor variables are either intermittently significant, rarely significant, or never significant. For the most recent 120-month sample period that runs from 2011.M1 to 2020.M12 and includes the COVID-influenced data, the only significant predictor variable is the price-dividend ratio.

Overall, our results reinforce the findings of Welch and Goyal (2008), Chen and Hong (2012), and others who demonstrate the difficulty of identifying any robust predictors of excess stock returns. Our results are also in line with the findings of Farmer et al. (2022) who identify short time intervals that exhibit significant out-of-sample predictability of excess stock returns. These time intervals, called "pockets of predictability," are interspersed with longer intervals that exhibit little or no evidence of predictable excess stock returns. They demonstrate that an asset pricing model with "sticky expectations" can generate such outcomes. Georges and Pereira (2021) develop a model with boundedly-rational, machine learning agents that delivers similar results. Interestingly, the lack of robustness in the predictability of excess stock returns could itself represent evidence in favor of market inefficiency.

1.1 Related literature

Numerous empirical studies find that measures of investor sentiment or investor attention can often help to predict *raw* stock returns, as opposed to excess stock returns. Examples include Brown and Cliff (2005), Lemmon and Portniaguina (2006), Tetlock (2007), Schmeling (2009), García (2013), Klemola et al (2016), and Fraiberger et al. (2018). Our theoretical results demonstrate that if some variable helps to predict raw stock returns, even after controlling for the presence of stochastic volatility, then this result need not imply market inefficiency.

Several studies link the predictability of excess returns to evidence of departures from rational expectations. Bacchetta, Mertens, and van Wincoop (2009) find that financial markets which exhibit predictable excess returns also exhibit predictable forecast errors of returns from surveys, arguing against full rationality of the survey forecasts. Piazzesi et al. (2015) find evidence of departures from rational expectations in expected excess bond returns from surveys. Cieslik (2018) shows that investors' real time forecast errors about the short-term real interest rate help to account for predictability in the bond risk premium.

The idea that excess stock returns tend to be more predictable around recessions has been documented by Henkel et. al. (2011), Dalig and Halling (2012), Neely, et al. (2014), Cujean and Hasler (2017), and Gómez-Cram (2022). Using data through December 2019, Gómez-Cram (2022) shows that an asset pricing model with sticky expectations about future cash flows can help account for the empirical evidence.

2 Excess returns in a consumption-based model

The framework for our theoretical analysis is a standard consumption-based asset pricing model. For any type of purchased asset and any specification of investor preferences, the first-order condition of the representative investor's optimal saving choice yields

$$1 = \widehat{E}_t \left[M_{t+1} R^i_{t+1} \right], \tag{1}$$

where M_{t+1} is the investor's stochastic discount factor and R^i_{t+1} is the gross holding period return on asset type i from period t to $t+1$. The symbol $\widehat{E}_t$ represents the investor's subjective expectation, conditional on information available at time t. Under rational expectations, $\widehat{E}_t$ corresponds to the mathematical expectation operator E_t evaluated using the objective distributions of all shocks, which are assumed known to the rational investor.

For a dividend-paying stock, we have $R^s_{t+1} = \left(d_{t+1} + p^s_{t+1}\right)/p^s_t$, where p^s_t is the ex-dividend stock price and d_{t+1} is the dividend received in period $t+1$. For a default-free bond that pays a stream of coupon payments (measured in consumption units) we have $R^b_t = \left(1 + \delta p^b_{t+1}\right)/p^b_t$, where p^b_t is the ex-coupon bond price and δ is a parameter that governs the decay rate of the coupon payments. A bond purchased in period t yields a coupon stream of 1, δ, δ^2... starting in period $t+1$. When $\delta = 1$, we have a consol bond that delivers a perpetual stream of coupon payments, each equal to one consumption unit. More generally, the value of δ can be calibrated to achieve a target value for the Macaulay duration of the bond, i.e., the present-value weighted average maturity of the bond's cash flows.[3] When $\delta = 0$, we have a one period discount bond that delivers a single coupon payment of one consumption unit in

[3] See, for example, Lansing (2015).

period $t+1$. In this case, $R^f_{t+1} \equiv 1/p^b_t$ is the risk-free rate of return which is known with certainty in period t.

With time-separable constant relative risk aversion (CRRA) preferences, we have $M_{t+1} = \beta\left(c_{t+1}/c_t\right)^{-\alpha}$, where β is the subjective time discount factor, c_t is the investor's real consumption, and α is the risk aversion coefficient. With recursive preferences along the lines of Epstein and Zin (1989), we have $M_{t+1} = \beta^{\omega}\left(c_{t+1}/c_t\right)^{-\omega/\psi}\left(R^c_{t+1}\right)^{\omega-1}$, where $R^c_{t+1} \equiv \left(c_{t+1}+p^c_{t+1}\right)/p^c_t$ is the gross return on an asset that delivers a claim to consumption c_{t+1} in period $t+1$, ψ is the elasticity of intertemporal substitution (EIS), and $\omega \equiv (1-\alpha)/\left(1-\psi^{-1}\right)$. In the special case when $\alpha = \psi^{-1}$, we have $\omega = 1$ such that Epstein-Zin preferences coincide with CRRA preferences. With external habit formation preferences along the lines of Campbell and Cochrane (1999), we have $M_{t+1} = \beta\left[s_{t+1}c_{t+1}/\left(s_t c_t\right)\right]^{-\alpha}$, where $s_t \equiv 1 - x_t/c_t$ is the surplus consumption ratio, x_t is the external habit level, and α is a curvature parameter that governs the steady state level of risk aversion.

For stocks, equation (1) can be rewritten as

$$p^s_t/d_t \;=\; \widehat{E}_t\left[M_{t+1}\frac{d_{t+1}}{d_t}\left(1+p^s_{t+1}/d_{t+1}\right)\right], \tag{2}$$

where p^s_t/d_t is the price-dividend ratio and d_{t+1}/d_t is the gross growth rate of dividends. At this point, it is convenient to define the following nonlinear change of variables:

$$z^s_t \;\equiv\; M_t\frac{d_t}{d_{t-1}}\left(1+p^s_t/d_t\right), \tag{3}$$

where z^s_t represents a composite variable that depends on the stochastic discount factor, the growth rate of dividends, and the price-dividend ratio.[4] The investor's first-order condition (2) becomes

$$p^s_t/d_t = \widehat{E}_t z^s_{t+1}, \tag{4}$$

which shows that the equilibrium price-dividend ratio is simply the investor's conditional forecast of the composite variable z^s_{t+1}. Substituting $p^s_t/d_t = \widehat{E}_t z^s_{t+1}$ into the definition (3) yields the following transformed version of the investor's first-order condition

$$z^s_t \;=\; M_t\frac{d_t}{d_{t-1}}(1+\widehat{E}_t z^s_{t+1}). \tag{5}$$

The gross stock return can now be written as

$$\begin{aligned} R^s_{t+1} \;&=\; \frac{d_{t+1}+p^s_{t+1}}{p^s_t} \;=\; \frac{\left(1+p^s_{t+1}/d_{t+1}\right)}{p^s_t/d_t}\frac{d_{t+1}}{d_t} \\ &=\; \frac{z^s_{t+1}}{\widehat{E}_t z^s_{t+1}}\frac{1}{M_{t+1}}, \end{aligned} \tag{6}$$

[4]This nonlinear change of variables technique is also employed by Lansing (2010, 2016) and Lansing and LeRoy (2014).

where we have eliminated p_t^s/d_t using equation (4) and eliminated $p_{t+1}^s/d_{t+1}+1$ using the definitional relationship (3) evaluated at time $t+1$.

Starting again from equation (1) and proceeding in a similar fashion yields the following transformed first-order condition for bonds:

$$z_t^b = M_t(1+\delta \widehat{E}_t z_{t+1}^b), \tag{7}$$

where $z_t^b \equiv M_t(1+\delta p_t^b)$ and $p_t^b = \widehat{E}_t z_{t+1}^b$. The gross bond return can now be written as

$$\begin{aligned} R_{t+1}^b &= \frac{1+\delta p_{t+1}^b}{p_t^b} \\ &= \frac{z_{t+1}^b}{\widehat{E}_t z_{t+1}^b}\frac{1}{M_{t+1}}. \end{aligned} \tag{8}$$

When $\delta = 0$ we have $z_{t+1}^b = M_{t+1}$ and the above expression simplifies to $R_{t+1}^b = R_{t+1}^f = 1/(\widehat{E}_t M_{t+1})$.

Combining equations (6) and (8) yields the following ratio of the gross stock return to the gross bond return:

$$\frac{R_{t+1}^s}{R_{t+1}^b} = \frac{z_{t+1}^s}{\widehat{E}_t z_{t+1}^s}\frac{\widehat{E}_t z_{t+1}^b}{z_{t+1}^b}. \tag{9}$$

Taking logs of both sides of equation (9) yields the following compact expression for the excess stock return, i.e., the realized equity premium:

$$\log(R_{t+1}^s/R_{t+1}^b) = \log\left[z_{t+1}^s/(\widehat{E}_t z_{t+1}^s)\right] - \log\left[z_{t+1}^b/(\widehat{E}_t z_{t+1}^b)\right], \tag{10}$$

where the second term on the right side simplifies to $\log[M_{t+1}/(\widehat{E}_t M_{t+1})]$ when $\delta = 0$.

Similarly, we can compute the excess bond return which compares the return on a longer-term bond $(\delta > 0)$ to the risk free rate $(\delta = 0)$. In this case, we have

$$\log(R_{t+1}^b/R_{t+1}^f) = \log\left[z_{t+1}^b/(\widehat{E}_t z_{t+1}^b)\right] - \log\left[M_{t+1}/(\widehat{E}_t M_{t+1})\right]. \tag{11}$$

Equations (10) and (11) are striking. If we apply the approximation $\log(A/B) \simeq (A-B)/B$ to the terms that appear on the right sides of equations (10) and (11), then $A-B$ would represent the investor's forecast error. Imposing rational expectations such that $\widehat{E}_t = E_t$ might therefore seem to imply that $\log(A/B)$ should be wholly unpredictable. However, as we show below, predictability can arise under rational expectations if the model exhibits stochastic volatility. Nonetheless, the intuition of $\log(A/B) \simeq (A-B)/B$ helps to explain why is it very difficult for consumption-based asset pricing models to generate significant predictability of excess returns under rational expectations. The same intuition also helps to explain why these same models struggle to produce a sizeable mean equity premium, except in cases where there is a high degree of curvature in investor preferences. The high degree of curvature serves to invalidate the approximation $\log(A/B) \simeq (A-B)/B$.

3 Predictability from stochastic volatility

In the special case of CRRA utility, normally and independently distributed consumption growth, and $c_t = d_t$, the equilibrium price-dividend ratio is constant. The realized equity premium relative to the risk free rate is $\log(R^s_{t+1}/R^f_{t+1}) = \varepsilon_{t+1} + (\alpha - 0.5)\,\sigma^2_\varepsilon$, where ε_{t+1} is the innovation to consumption growth and σ^2_ε is the associated variance which is not stochastic.[5] In this special case, excess returns at time $t+1$ are not predictable using variables dated time t or earlier. But as we show below, models that exhibit stochastic volatility can generate predictability of excess returns under rational expectations.

When solving consumption-based asset pricing models, it is common to employ approximation methods that deliver conditional log-normality of the relevant variables. If a random variable q_t is conditionally log-normal, then

$$\log\left(E_t q_{t+1}\right) \;=\; E_t\left[\log\left(q_{t+1}\right)\right] \;+\; \tfrac{1}{2} Var_t\left[\log\left(q_{t+1}\right)\right], \tag{12}$$

where Var_t is the mathematical variance operator conditional on information available to the investor at time t.

Starting from equation (10) and imposing rational expectations such that $\widehat{E}_t = E_t$, we make the assumption that the composite variables z^s_{t+1} and z^b_{t+1} are both conditionally log-normal. Making use of equation (12) to eliminate $\log\left(E_t z^s_{t+1}\right)$ and $\log\left(E_t z^b_{t+1}\right)$ yields the following alternate expression for the excess stock return

$$\begin{aligned} \log(R^s_{t+1}/R^b_{t+1}) \;&=\; \left[\log\left(z^s_{t+1}\right) - E_t \log\left(z^s_{t+1}\right)\right] \;-\; \left[\log(z^b_{t+1}) - E_t \log(z^b_{t+1})\right] \\ &\quad -\tfrac{1}{2} Var_t\left[\log\left(z^s_{t+1}\right)\right] + \tfrac{1}{2} Var_t[\log(z^b_{t+1})] \end{aligned} \tag{13}$$

where $z^b_{t+1} = M_{t+1}$ for a 1-period discount bond with $\delta = 0$. Notice that the first two terms in equation (13) are the investor's forecast errors for $\log\left(z^s_{t+1}\right)$ and $\log(z^b_{t+1})$, respectively. These forecast errors cannot be a source of predictability under rational expectations. However, the last two terms in equation (13) show that predictability can arise under rational expectations if the laws of motion for the endogenous variables $\log\left(z^s_{t+1}\right)$ and $\log(z^b_{t+1})$ exhibit stochastic volatility. This is because the conditional variance terms at time t would partly determine the realized excess return at time $t+1$.

Specializing equation (13) to the case where $\delta = 0$ such that $R^b_{t+1} = R^f_{t+1}$ and $z^b_{t+1} = M_{t+1}$,

[5] For the derivation, see Lansing and LeRoy (2014), Appendix B. Note that in the risk neutral case with $\alpha = 0$, we have the result that $E[R^s_{t+1}/R^f_{t+1}] = E[\exp(\varepsilon_{t+1} - 0.5\sigma^2_\varepsilon)] = 1$.

we have

$$\log(R^s_{t+1}/R^f_{t+1}) = \left[\log\left(z^s_{t+1}\right) - E_t\log\left(z^s_{t+1}\right)\right] - \left[\log(M_{t+1}) - E_t\log(M_{t+1})\right]$$

$$-\tfrac{1}{2}Var_t[\log\underbrace{(M_{t+1}R^s_{t+1}\,p^s_t/d_t)}_{=z^s_{t+1}}] + \tfrac{1}{2}Var_t\left[\log(M_{t+1})\right], \tag{14}$$

where the last line exploits the definition of z^s_{t+1}. Equation (14) implies that the rational expected excess return on stocks is given by

$$E_t\log(R^s_{t+1}/R^f_{t+1}) = -\tfrac{1}{2}Var_t[\log(M_{t+1}R^s_{t+1}\,p^s_t/d_t)] + \tfrac{1}{2}Var_t\left[\log(M_{t+1})\right], \tag{15}$$

where R^f_{t+1} is known at time t.

Following Campbell (2014), an alternative expression for the rational expected excess return on stocks can be derived by decomposing the conditional rational expectation in equation (1) as follows

$$\underbrace{E_t\left[M_{t+1}R^s_{t+1}\right]}_{=1} = \underbrace{E_tM_{t+1}}_{=1/R^f_{t+1}}E_tR^s_{t+1} + Cov_t\left[M_{t+1},\ R^s_{t+1}\right]. \tag{16}$$

Solving the above expression for $E_t\left(R^s_{t+1}\right)/R^f_{t+1}$ and then taking logs yields

$$\log(E_tR^s_{t+1}/R^f_{t+1}) = \log\left\{1 - Cov_t\left[M_{t+1},\ R^s_{t+1}\right]\right\}, \tag{17}$$

$$E_t\log(R^s_{t+1}/R^f_{t+1}) = \log\left\{1 - Cov_t\left[M_{t+1},\ R^s_{t+1}\right]\right\} - \tfrac{1}{2}Var_t\left[\log R^s_{t+1}\right], \tag{18}$$

where, in going from equation (17) to (18), we have assumed conditional log-normality of the gross stock return R^s_{t+1}. The above expression shows that the rational expected excess return on stocks will be predictable if $Cov_t\left[M_{t+1},\ R^s_{t+1}\right]$ or $Var_t\left[\log R^s_{t+1}\right]$ are time-varying. Attanasio (1991) undertakes a derivation similar to equation (18) and concludes (p. 481): "predictability of excess returns constitutes direct evidence against the joint hypothesis that markets are efficient and second moments are constant." While our derivation of equation (14) delivers a similar conclusion, it helps to focus attention on investor forecast errors as an alternative source of predictable excess returns when expectations are not fully rational.

3.1 Analytical example: Exogenous stochastic volatility

Here we provide an analytical example to show how stochastic volatility in the law of motion for consumption growth can generate predictable excess returns under rational expectations. Suppose the investor's stochastic discount factor is given by

$$M_{t+1} = \beta\exp(c_{t+1}/c_t)^{-\alpha}\left[\frac{1-\kappa_{t+1}}{1-\kappa_t}\right]^{-\alpha}, \tag{19}$$

where κ_t is a stochastic habit formation parameter that allows for time-varying risk aversion.[6] If we define $\exp(\eta_t) \equiv [1-\kappa_t]^{-1}$, then the investor's time-varying coefficient of relative risk aversion is given by $\alpha \exp(\eta_t)$.[7] The time-series process for the stochastic discount factor is governed by the following equations:

$$M_{t+1} = \beta \exp(-\alpha x_{t+1} + \alpha \eta_{t+1} - \alpha \eta_t), \tag{20}$$

$$x_{t+1} = \overline{x} + \rho_x (x_t - \overline{x}) + \sigma_t \varepsilon_{t+1}, \qquad |\rho_x| < 1, \qquad \varepsilon_t \sim NID(0,1), \tag{21}$$

$$\sigma^2_{t+1} = \overline{\sigma}^2 + \rho_\sigma \left(\sigma^2_t - \overline{\sigma}^2\right) + u_{t+1}, \qquad |\rho_\sigma| < 1, \qquad u_t \sim NID\left(0, \sigma^2_u\right), \tag{22}$$

$$\eta_{t+1} = \overline{\eta} + \rho_\eta (\eta_t - \overline{\eta}) + \omega_{t+1}, \qquad \left|\rho_\eta\right| < 1, \qquad \omega_t \sim NID\left(0, \sigma^2_\omega\right), \tag{23}$$

where $x_{t+1} \equiv \log(c_{t+1}/c_t)$ is real consumption growth that evolves as an AR(1) process with mean $\overline{x}$ and persistence parameter ρ_x. The innovation ε_{t+1} is normally and independently distributed (NID) with mean zero and variance of one. We allow for stochastic volatility along the lines of Bansal and Yaron (2004), where ρ_σ governs the persistence of volatility and u_{t+1} is the innovation to volatility.[8] The habit formation variable η_t evolves as an AR(1) process with mean $\overline{\eta}$ and persistence parameter ρ_η. Real dividend growth $x^d_{t+1} \equiv \log(d_{t+1}/d_t)$ is given by

$$x^d_{t+1} = x_{t+1} + v_{t+1}, \qquad v_t \sim NID\left(0, \sigma^2_v\right), \tag{24}$$

where v_{t+1} is an innovation with mean zero and variance σ^2_v.

Under rational expectations, we have

$$R^f_{t+1} = \beta^{-1} \exp\left[\alpha\overline{x} + \alpha\rho_x (x_t - \overline{x}) + \alpha(1-\rho_\eta)(\eta_t - \overline{\eta}) - \tfrac{1}{2}\alpha^2\sigma^2_t - \tfrac{1}{2}\alpha^2\sigma^2_\omega\right], \tag{25}$$

$$\log\left[M_{t+1}/(E_t M_{t+1})\right] = -\alpha\sigma_t\,\varepsilon_{t+1} + \alpha\omega_{t+1} - \tfrac{1}{2}\alpha^2\sigma^2_t - \tfrac{1}{2}\alpha^2\sigma^2_\omega. \tag{26}$$

The left side of equation (26) will be predictable only when σ^2_t is time-varying, i.e., when $\sigma^2_u > 0$.

Appendix A provides an approximate analytical solution for the composite variable z^s_{t+1} that appears in the excess stock return equation (10).[9] Under rational expectations, the

[6] The investor's external habit formation utility function is given by $U = (c_t - \kappa_t C_t)^{1-\alpha}/(1-\alpha)$, where C_t is aggregate consumption per person which the investor views as exogenous. In equilbrium, we have $c_t = C_t$.

[7] The risk aversion coefficient is defined as $-c_t U_{cc}/U_c$. We have $U_c = (c_t - \kappa_t C_t)^{-\alpha}$ and $U_{cc} = -\alpha(c_t - \kappa_t C_t)^{-\alpha-1}$. Imposing the equilibrium condition $c_t = C_t$ yields $-c_t U_{cc}/U_c = \alpha/(1-\kappa_t)$.

[8] When simulating their model, Bansal and Yaron (2004) ensure that σ^2_{t+1} remains positive by replacing any negative realizations with a very small number, which happens in about 5% of the realizations. We would obtain similar results if equation (22) was replaced by a GARCH(1,1) model in which σ^2_{t+1} depends on x^2_t and σ^2_t but not on u_{t+1}.

[9] Appendix A also outlines how the various asset pricing equations would change in the case of Epstein-Zin preferences.

approximate solution implies the following expression:

$$\begin{aligned}\log\left[z_{t+1}^{s}/(E_t z_{t+1}^{s})\right] &= a_1\sigma_t\,\varepsilon_{t+1}+a_2u_{t+1}+a_3v_{t+1}+a_4\omega_{t+1}\\ &\quad -\tfrac{1}{2}\left(a_1\right)^2\sigma_t^2-\tfrac{1}{2}\left(a_2\right)^2\sigma_u^2-\tfrac{1}{2}\left(a_3\right)^2\sigma_v^2-\tfrac{1}{2}\left(a_4\right)^2\sigma_\omega^2, \end{aligned}\tag{27}$$

where a_1 through a_4 are Taylor series coefficients that depend on the model parameters. Substituting equations (26) and (27) into the excess stock return equation (10) and imposing $\delta = 0$ such that $R_{t+1}^{b} = R_{t+1}^{f}$ yields

$$\begin{aligned}\log(R_{t+1}^{s}/R_{t+1}^{f}) &= (a_1+\alpha)\,\sigma_t\,\varepsilon_{t+1}\;+\;a_2u_{t+1}\;+\;a_3v_{t+1}\;+\;(a_4-\alpha)\,\omega_{t+1}\\ &\quad +\;\tfrac{1}{2}\left[\alpha^2-(a_1)^2\right]\sigma_t^2\;-\;\tfrac{1}{2}\left(a_2\right)^2\sigma_u^2\;-\;\tfrac{1}{2}\left(a_3\right)^2\sigma_v^2\;+\;\tfrac{1}{2}\left[\alpha^2-(a_4)^2\right]\sigma_\omega^2, \end{aligned}\tag{28}$$

which shows that excess stock returns will be predictable only when σ_t^2 is time-varying, provided that $\alpha^2 - (a_1)^2 \neq 0$. In the special case when $\rho_x = 0$, the first Taylor series coefficient becomes $a_1 = 1 - \alpha$ and the coefficient on σ_t^2 in equation (28) becomes $\alpha - 0.5$.

Equation (28) implies

$$E[R_{t+1}^{s}/R_{t+1}^{f}] = \exp[\alpha(\alpha + a_1)\sigma_t^2 + \alpha(\alpha - a_4)\sigma_\omega^2)]. \tag{29}$$

In the risk neutral case with $\alpha = 0$, investors do not view stocks as risky assets and we have $E[R_{t+1}^{s}/R_{t+1}^{f}] = 1$. Notice also that when $\sigma_\omega^2 > 0$, the model exhibits rationally time-varying risk aversion but this feature does not introduce predictability of excess stock returns. This is because the innovation variance σ_ω^2 for the habit formation variable η_t is not time-varying. This example shows that in the absence of stochastic volatility, rationally time-varying risk aversion, by itself, is not a source of predictable excess returns.

3.2 Predictability of raw stock returns

Many studies examine the predictability of raw stock returns as opposed to *excess* stock returns. Starting from equation (6) and making use of equations (20) and (27) yields the following expression for the raw stock return

$$\begin{aligned}\log\left(R_{t+1}^{s}\right) &= (a_1+\alpha)\,\sigma_t\,\varepsilon_{t+1}\;+\;a_2u_{t+1}\;+\;a_3v_{t+1}\;+\;(a_4-\alpha)\,\omega_{t+1}\\ &\quad -\;\tfrac{1}{2}\left(a_1\right)^2\sigma_t^2\;-\;\tfrac{1}{2}\left(a_2\right)^2\sigma_u^2\;-\;\tfrac{1}{2}\left(a_3\right)^2\sigma_v^2\;-\;\tfrac{1}{2}\left(a_4\right)^2\sigma_\omega^2\\ &\quad -\log\left(\beta\right)+\alpha\overline{x}+\;\alpha\rho_x\left(x_t-\overline{x}\right)\;+\;\alpha(1-\rho_\eta)\left(\eta_t-\overline{\eta}\right). \end{aligned}\tag{30}$$

Equation (30) shows that $\log\left(R_{t+1}^{s}\right)$ will be predictable due to the terms involving $x_t - \overline{x}$ and $\eta_t - \overline{\eta}$ even when volatility is not stochastic, i.e., when $\sigma_t^2 = \overline{\sigma}^2$ for all t. Hence, if some variable helps to predict raw stock returns, even after controlling for the presence of stochastic volatility, then this result need not imply market inefficiency.

3.3 Discussion

Using a log-linear approximation of the stock return identity, Cochrane (2005) shows that the variance of the log price-dividend ratio must equal the sum of the ratio's covariances with: (1) future dividend growth rates, (2) future risk-free rates, and (3) future excess stock returns. The magnitude of each covariance term is a measure of the predictability of each component when the price-dividend ratio is employed as the sole regressor in a forecasting equation. But under rational expectations, our theoretical results show that the price-dividend ratio (or any other variable) will predict future excess stock returns only in the presence of stochastic volatility.

It is important to note that the mere presence of the state variable σ_t^2 in equation (28) does not guarantee that the observed amount of excess return predictability will be statistically significant. Depending on the model calibration, the fundamental shock innovations ε_{t+1}, u_{t+1}, v_{t+1}, and ω_{t+1} may end up being the main drivers of fluctuations in *realized* excess returns, thus washing out the influence of the fundamental state variable σ_t^2 which is the sole driver of fluctuations in *expected* excess returns. This washing out effect appears to be present in most of the leading consumption based asset pricing models.

In the rational long-run risks model of Bansal and Yaron (2004), *exogenous* stochastic volatility is achieved by assuming an AR(1) law of motion for the volatility of innovations to consumption growth and dividend growth, along the lines of equation (22). In the rational external habit model of Campbell and Cochrane (1999), *endogenous* stochastic volatility is achieved via a nonlinear sensitivity function that determines how innovations to consumption growth influence the logarithm of the surplus consumption ratio.[10] Despite these features, subsequent analysis has shown that these fully-rational models fail to deliver significant predictability of excess stock returns.

Kirby (1998) had previously shown that the rational habit model of Abel (1990) and the rational recursive preferences model of Epstein and Zin (1989) both fail to generate significant predictability of excess stock returns. Chen and Hwang (2018) extend Kirby's analysis to the rational models of Campbell Cochrane (1999) and Bansal and Yaron (2004) and find that neither model can generate any significant predictable excess stock returns. Using simulated data, Beeler and Campbell (2012) show that the rational long-run risk models of Bansal and Yaron (2004) and Bansal et al. (2012) both fail to match predictability patterns observed in the data.

[10] Appendix B provides an analytical example of predictability that arises from endogenous stochastic volatility.

4 Predictability from market inefficiency

We now provide a analytical example to illustrate the second possible source of predictable excess returns, namely, departures from rational expectations that give rise to predictable investor forecast errors and market inefficiency.

4.1 Analytical example: Extrapolative expectations

Studies by Vissing-Jorgenson (2004), Amromin and Sharpe (2014), Frydman and Stillwagon (2018), and Da et al. (2021) all find evidence of extrapolative or procyclical expected returns among stock investors. Greenwood and Shleifer (2014) and Adam et al. (2017) show that measures of investor optimism about future stock returns are strongly correlated with past stock returns and the price-dividend ratio.[11] Interestingly, even though a higher price-dividend ratio in the data empirically predicts lower *realized* stock returns (Cochrane 2008), the survey evidence shows that investors fail to take this relationship into account; instead they continue to forecast high future returns on stocks following a sustained run-up in the price-dividend ratio. Using survey data, Casella and Gulen (2018) show that the ability of the dividend yield (inverse of the price-dividend ratio) to forecast 12-month ahead excess returns is contingent on a variable that measures the degree to which investors extrapolate past stock returns.

Along the lines of Lansing (2006), we model extrapolative expectations as $\widehat{E}_t M_{t+1} = A^f M_t$ and $E_t z^s_{t+1} = A^s z^s_t$, where $A^f > 0$ and $A^s > 0$ are extrapolation parameters. The value of A^i for $i = f,\ s$ governs the nature of the extrapolation, where $A^i = 1$ corresponds to a random walk forecast. For stocks, $A^s > 1$ can be viewed as "optimistic" about the future stock price while $A^s < 1$ can be viewed as "pessimistic." A more complex scheme could allow the extrapolation parameters to be time-varying and linked to past price movements.

The stochastic discount factor continues to be defined by equations (20) through (22). In this case, we have

$$\begin{aligned}
\log[M_{t+1}/(\widehat{E}_t M_{t+1})] &= \log\left[\frac{\beta \exp\left(-\alpha x_{t+1} + \alpha\eta_{t+1} - \alpha\eta_t\right)}{A^f \beta \exp\left(-\alpha x_t + \alpha\eta_t - \alpha\eta_{t-1}\right)}\right], \\
&= -\log(A^f) - \alpha\sigma_t\,\varepsilon_{t+1} \;+\; \alpha\omega_{t+1} \;+\; \alpha\,(1-\rho_x)\,(x_t - \overline{x}) \\
&\quad -\alpha\left(2-\rho_\eta\right)(\eta_t - \overline{\eta}) \;+\; \alpha(\eta_{t-1} - \overline{\eta}),
\end{aligned} \tag{31}$$

which shows that $\log[M_{t+1}/(\widehat{E}_t M_{t+1})]$ will be predictable due to the terms involving $x_t - \overline{x}$, $\eta_t - \overline{\eta}$, and $\eta_{t-1} - \overline{\eta}$.

[11] We confirm this finding in Figure 5 using survey data about investors' perceived probability of an increase in stock prices over the next year.

Appendix C provides an approximate analytical solution for the expression $z^s_{t+1}/\,(\widehat{E}_t z^s_{t+1})$. The approximate solution implies

$$\begin{aligned}\log[z^s_{t+1}/(\widehat{E}_t z^s_{t+1})] \;\;=\;\; & -\log(A^s)\;+(1-\alpha-b_1)\,\sigma_t\,\varepsilon_{t+1}\;+\;(1-b_2)v_{t+1}\;+\;(\alpha-b_3)\omega_{t+1} \\ & -\,(1-\alpha-b_1)\,(1-\rho_x)\,(x_t-\overline{x})\;-\;(1-b_2)v_t \\ & -\,[(\alpha-b_3)(1-\rho_\eta)+\alpha+b_4](\eta_t-\overline{\eta})\;+\;(\alpha+b_4)(\eta_{t-1}-\overline{\eta}), \end{aligned} \tag{32}$$

where b_1 through b_4 are Taylor series coefficients that depend on the model parameters. Substituting equations (31) and (32) into the excess stock return equation (10) and imposing $\delta = 0$ such that $R^b_{t+1} = R^f_{t+1}$ yields

$$\begin{aligned}\log(R^s_{t+1}/R^f_{t+1}) \;\;=\;\; & \log(A^f/A^s)\;+(1-b_1)\;\sigma_t\,\varepsilon_{t+1}\;+\;(1-b_2)v_{t+1}\;-\;b_3\omega_{t+1} \\ & -\,(1-b_1)(1-\rho_x)\,(x_t-\overline{x})\;-\;(1-b_2)\,v_t \\ & -\,[(\alpha+b_3)\rho_\eta-b_3+b_4](\eta_t-\overline{\eta})\;+\;b_4(\eta_{t-1}-\overline{\eta}), \end{aligned} \tag{33}$$

which shows that the four terms involving $x_t - \overline{x}$, v_t, $\eta_t - \overline{\eta}$, and $\eta_{t-1} - \overline{\eta}$ represent sources of predictable excess returns that arise from market inefficiency. Notice that in this example, the presence of stochastic volatility does not contribute to predictable excess returns. This is because the investor's subjective forecasts $\widehat{E}_t M_{t+1}$ and $\widehat{E}_t z^s_{t+1}$ do not take into account the fundamental law of motion (22) that governs the evolution of σ_t^2.

5 Predictability regressions

Our empirical approach examines whether 1-month-ahead excess returns on stocks relative to the risk free rate can be predicted using data from 1990.M1 to 2020.M12. Motivated by our theoretical results, we look for evidence that market inefficiency contributes to the predictability of excess returns while controlling for the presence of stochastic volatility. In this section we describe: (1) our motivation for the choice of predictor variables, (2) properties of the data, and (3) the results of our predictability regressions.

5.1 Choice of predictor variables

Our predictability regressions take the following form:

$$\begin{aligned}\mathbf{ersf}_{t+1} \;\;=\;\; & c_0 + c_1\,\mathbf{pd} + c_2\,\mathbf{iv} + c_3\,\mathbf{rv} + c_4\,\mathbf{term} + c_5\,\boldsymbol{\Delta}\mathbf{sent12} \\ & + c_6\,\boldsymbol{\Delta}\mathbf{ersf} + c_7\,\boldsymbol{\Delta}\mathbf{sent12} \times \boldsymbol{\Delta}\mathbf{ersf} + c_8\,\mathbf{noise}, \end{aligned} \tag{34}$$

where $\mathbf{ersf}_{t+1} \equiv \log(R^s_{t+1}/R^f_{t+1})$ is the realized excess return on stocks relative to the risk free rate in month $t+1$. The gross return on stocks R^s_{t+1} is measured by the 1-month nominal return on the S&P 500 stock index, including dividends. The gross risk free rate R^f_{t+1} is measured by the 1-month nominal return on a 3-month Treasury Bill. The predictor variables on the right side of equation (34) are all dated month t. We do not perform long-horizon predictability regressions because the empirical reliability of such results have been called into question by Boudoukh et al. (2008) and Bauer and Hamilton (2017).

The variable **pd** is the price-dividend ratio for the S&P 500 stock index defined as the end-of-month nominal closing value of the index divided by cumulative nominal dividends over the past 12 months. Any consumption-based asset pricing model with rational expectations implies that the price-dividend ratio will depend on the model's fundamental state variables, including any that would give rise to the conditional variance terms in equation (13). We illustrate this idea in Appendix A using the rational asset pricing model of Section 3.1. Cochrane (2017) shows that the price-dividend ratio in U.S. data exhibits strong co-movement with a measure of "surplus consumption" constructed from the data using the parameters of Campbell and Cochrane (1999) habit formation model. Hence, including **pd** as a regressor is a way to control indirectly for the presence of stochastic volatility when the state variables that drive stochastic volatility are not directly observable.

To control directly for the presence of stochastic volatility, we include the variables **iv** and **rv** which are, respectively, the implied variance and the realized variance of returns on the S&P 500 index. Implied variance is measured by the end-of-month VIX-squared, de-annualized (i.e., $\text{VIX}^2/12$). Realized variance is measured by the sum of squared 5-minute log returns on the S&P 500 stock index over the month. Studies by Attanasio (1991), Guo (2006), and Welch and Goyal (2008) employ measures of realized stock return volatility as predictor variables. The difference between **iv** and **rv** is the "variance risk premium," as originally defined by Bollerslev et al. (2009). Numerous studies find that the variance risk premium can be a useful predictor of excess stock returns.[12] We include **iv** and **rv** as separate predictor variables rather than imposing the restriction that the regression coefficients on **iv** and **rv** must be of equal magnitude but opposite sign. Imposing such a restriction does not qualitatively affect our results.

The variable **term** is the monthly average yield spread between the 10-year and 3-month constant maturity Treasury securities. Studies by Welch and Goyal (2008) and Faria and Verona (2020) employ versions of the Treasury term spread as predictor variables. A study by Miranda-Agrippino and Rey (2020) finds that a single global factor, partly driven by U.S. monetary policy, helps to explain a significant share of the variance of equity and bond returns around the world. Given that the Treasury yield curve reflects expectations of future U.S. monetary policy, we view the inclusion of **term** as a way to control indirectly for the presence of stochastic volatility.

[12] See, for example, Drechsler and Yaron (2011), Bollerslev et al. (2014), Zhou (2018), and Pyun (2019).

As reviewed in the introduction, measures of sentiment and momentum have been shown to predict future stock return volatility (Audrino et al. 2020, Wang et al. 2006). We therefore include measures of sentiment and momentum as indirect controls for stochastic volatility. The variable **Δsent12** is the 12-month change in the University of Michigan's consumer sentiment index. We experimented with higher frequency changes in the sentiment index, but the resulting fit was not improved. The momentum variable **Δersf** is the 1-month change in the excess stock return. In a recent comprehensive study of excess return predictability, Gu et al. (2020) find that "allowing for (potentially complex) interactions among the baseline predictors" can substantially improve forecasting performance. Motivated by this finding, we interact the sentiment and momentum variables to obtain **Δsent12×Δersf** as an additional predictor variable. This interaction variable turns out to be a more robust predictor than either **Δsent12** or **Δersf** in isolation.

We experimented with including additional controls for stochastic volatility in the form of volatility measures for consumption growth or dividend growth, each computed using rolling data windows of various lengths. None of these measures were found to be statistically significant.

According to Shleifer and Vishney (1997, p. 35): "Arbitrage plays a critical role in the analysis of securities markets, because its effect is to bring prices to fundamental values and to keep markets efficient." They describe how forces such as performance-based access to arbitrage capital, or noise trader risk, can create "limits to arbitrage," thus allowing mispricing to persist. When investor expectations are fully-rational, the first-order condition (1) in our model represents a no-arbitrage condition for the equilibrium stock or bond price. But if investor expectations are not fully-rational, then the no-arbitrage condition will be violated, giving rise to mispricing that would influence the excess return on stocks relative to bonds. Motivated by these ideas, we use the variable **noise** to detect market inefficiency because it captures the degree of mispricing in U.S. Treasury bonds that make up the yield curve. According to Hu et al. (2013, p. 2342) "abnormal noise in Treasury prices is a symptom of a market in severe shortage of arbitrage capital. More importantly, to the extent that capital is allocated across markets for major marginal players in the market, this symptom applies not only to the Treasury market, but also more broadly to the overall financial market." Hu et al. (2013) construct their noise measure by fitting a theoretical no-arbitrage yield curve to the daily Treasury yield curve and then compute the root mean squared deviation between the two daily curves. The variable **noise** is the monthly average root mean squared deviation between the two daily yield curves. We obtain similar results using the end-of-month root mean squared deviation between the two daily yield curves.

We must acknowledge that the variables we use to control for stochastic volatility are imperfect. For example, departures from rational expectations could affect the price-dividend ratio, the variance of stock returns, or measures of sentiment and momentum. Numerous empirical studies starting with Shiller (1981) and LeRoy and Porter (1981) have shown that stock prices appear to exhibit excess volatility when compared to fundamentals, as measured by the discounted stream of ex post realized dividends.[13] A study by Greenwood et al. (2019) using stock returns for various U.S. industries finds that stock valuation ratios and stock return volatility both increase substantially during the 24 months preceding what they define as "bubble peaks." Inefficient movements in stock prices could influence the variable **term** if Federal Reserve monetary policy reacts to the stock market, as appears to be the case (Rigobon and Sack 2003, Hayford and Malliaris 2004, Lansing 2003, 2008, Cieslak and Vissing-Jorgensen 2021). As discussed in detail by Brav and Heaton (2002), it is extremely difficult to distinguish between rational and behavioral explanations of financial market phenomena. Nevertheless, in our empirical analysis, we treat all variables except **noise** as controls for stochastic volatility.

Figures 1 and 2 provide time series plots of our eight predictor variables. Notice that all of the predictor variables tend to exhibit extreme upward or downward movements during recessions. In our predictive regressions, we examine how recession periods influence the statistical significance of each predictor variable.

5.2 Data

We use monthly data for the period from 1990.M1 to 2020.M12. The starting date for the sample is governed by the availability of data for **iv** which makes use of the VIX index. The sources and methods used to construct the data are described in Appendix D.

Table 1 reports summary statistics for excess stock returns and our eight predictor variables. The mean excess return on stocks relative to the risk free rate is 0.59% per month. The summary statistics show that excess stock returns exhibit excess kurtosis. Five out of the eight predictor variables also exhibit excess kurtosis, namely, **iv, rv**, **Δersf, Δsent12×Δersf,** and **noise**.

Five of the eight predictor variables are highly persistent, namely, **pd, iv, term**, **Δsent12,** and **noise.** In Appendix E, we use a bootstrap procedure to gauge the quantitative impact of persistent regressors on the critical values of the standard t-statistic. The bootstrapped critical values are not substantially different from the asymptotic ones, but there are some noticeable shifts in either direction for some of the persistent predictor variables, particularly **pd**. Use of the bootstrapped t-statistics does not change any of our conclusions regarding the statistical significance of the **noise** variable which is the focus of our interest for detecting market inefficiency.

Table 1 shows that **noise** exhibits reasonably strong cross-correlations with **iv** and **rv.** All

[13] Lansing and LeRoy (2014) provide a recent update on this literature.

three of these variables tend to spike upwards during significant stock market declines, such as occurred during the financial crisis from late 2008 through early 2009 and during the onset of the COVID pandemic in March 2020. The cross-correlations between **noise** and the other five predictor variables are rather weak.

5.3 Predictive regressions

The results of our predictability regressions are summarized in Tables 2 through 5. Figures 3 and 4 plot estimated coefficients from a series of 120-month rolling regressions using all eight predictor variables. The t-statistics for the estimated coefficients in the tables and figures are computed using Newey-West HAC corrected standard errors. Bold entries in the tables indicate that the predictor variable is significant at the 5% level using the asymptotic critical values. Adjusted R-squared values are shown at the bottom of each regression specification. The rolling regressions show that the statistical significance of the eight predictor variables is often sensitive to the sample period.

Table 2 shows the full-sample regression results. Specification 1 includes **pd** only—a typical univariate specification in the literature. According to the theory, **pd** encodes any fundamental state variables that would give rise to stochastic volatility. Specification 2 adds **iv, rv,** and **term** as additional controls for stochastic volatility. Specification 3 goes further by including **Δsent12, Δersf,** and **Δsent12×Δersf.** As noted earlier, we interpret these three variables as additional controls for stochastic volatility because there is evidence in the literature that sentiment and momentum measures can help to predict future stock return volatility. Recall that stochastic volatility is the only source of predictability under rational expectations.

The estimated coefficient on **pd** in Table 2 is always negative and statistically significant in the full sample. This result is consistent with numerous previous studies which find that a higher price-dividend ratio predicts a lower excess stock return. The estimated coefficient on **iv** is positive while the estimated coefficient on **rv** is negative, but neither coefficient is statistically significant in the full sample. However, if we drop the COVID-influenced data from 2020.M3 onward, then both **iv** and **rv** become strongly significant with coefficients that are approximately equal in magnitude but opposite in sign. This result is consistent with previous findings in the literature that the variance risk premium (**iv** minus **rv**) helps to predict excess stock returns with a positive regression coefficient. The literature has interpreted the variance risk premium as a proxy for macroeconomic uncertainty. A positive regression coefficient on the variance risk premium implies that higher uncertainty in month t induces investors to demand a higher excess stock return in month $t+1$.

The rolling regression results in Figure 3 confirm that both **iv** and **rv** cease to be statistically significant when the COVID-influenced data enters the 120-month moving window.

Three COVID-influenced data points contribute to the breakdown of **iv** and **rv** (and by extension the breakdown of the variance risk premium) as predictors of 1-month-ahead excess stock returns. The variance risk premium turns strongly negative in 2020.M3, but this observation is followed by a strongly positive excess stock return in 2020.M4. The pre-COVID regression relationship predicts a *negative* excess stock return in 2020.M4. Subsequently, the variance risk premium turns positive in 2020.M8 and 2020.M9, but these two observations are followed by a strongly negative excess stock returns in 2020.M9 and 2020.M10, respectively. The pre-COVID regression relationship predicts *positive* excess stock returns in 2020.M9 and 2020.M10. This example shows that predictor variables that previously have been considered robust can lose their statistical significance in short order.

The estimated coefficients on $\mathbf{\Delta sent12}$ and $\mathbf{\Delta ersf}$ in Table 2 are not statistically significant in the full sample. A finding of non-significance for these two variables is a typical result across all of our regression specifications. However, the estimated coefficient on the interaction variable $\mathbf{\Delta sent12}\times\mathbf{\Delta ersf}$ is negative and strongly significant in both Specifications 3 and 4. Specification 3 delivers an adjusted R-squared statistic of 4.8% versus 0.90% for Specification 2. Further investigation of the interaction variable reveals that its statistical significance derives mainly from periods of declining sentiment and negative return momentum, forecasting a further decline in the excess stock return.[14]

The estimated coefficient on the variable **noise** in Table 2 is negative and strongly significant with a t-statistic of -3.164. As shown in Appendix E, the 2.5% percentile bootstrapped critical value for **noise** is -2.331. Higher values of **noise** predict lower 1-month-ahead excess stock returns, implying that a shortage of arbitrage capital in financial markets causes excess returns to drop below the levels justified by fundamentals. From Figure 2, we see that **noise** is highest during the Great Recession that runs from December 2007 to June 2009. But there are some smaller upward spikes in 1998.M10 (following the collapse of the hedge fund Long-Term Capital Management), 1999.M10 (heading into the dotcom bust), 2016.M12 (following the surprise U.S. presidential victory of Donald Trump), and 2020.M3 (coinciding with the onset of the COVID pandemic in the U.S.).

Table 3 shows split-sample regression results. The first split sample runs from 1990.M1 to 2005.M12 while the second runs from 2006.M1 to 2020.M12. The regression results for the first split sample are similar to the full-sample results, with the exception that **noise** is not significant in the first split sample which does not include the Great Recession. As discussed further below, **noise** is only significant in sample periods that include the Great Recession. The results for the second split sample show that **pd** is only significant in Specification 4 that includes **noise.** The contemporaneous correlation between **pd** and **noise** in the second

[14] For details, see Lansing and Tubbs (2019).

split sample is -0.76 whereas the two variables are essentially uncorrelated in the first split sample. Notice also that the regression coefficient on **pd** is much larger in magnitude in the second split sample. This is because **pd** exhibits a lower average value from 2006 onward. In Specification 3, the variable Δ**sent12**$\times\Delta$**ersf** is statistically significant in both split samples, but the significance is stronger in the first split sample. In going from Specification 3 to Specification 4 which includes **noise,** the adjusted R-squared statistic for the second split sample increases from 2.62% to 13.6%.

Tables 4 and 5 show how recession periods influence the statistical significance of each predictor variable. In Table 4, the dummy variable **GR**, dated month-t, is equal to 1 during the Great Recession from 2007.M12 to 2009.M6 and equal to 0 otherwise. In Table 5, the dummy variable **R**, dated month-t, is equal to 1 during recessions and equal to 0 otherwise. Unlike the values of the predictor variables themselves, the values of the dummy variables would not have been available in real time at the end of month-t. Nevertheless, these regressions provide a way of examining the sensitivity of the results to selected sample periods that are identified ex post.[15]

Table 4 shows that **noise** is only significant when **GR** $= 1$, confirming that clear evidence of market inefficiency is confined to a narrow sub-sample of data that coincides with the global financial crisis. Table 5 shows that the variables **pd**, **iv**, **rv,** and **Δsent12×Δersf** are not significant when **R** $= 1$, which serves to include the COVID recession data from 2020.M3 to 2020.M5. Setting **R** $= 0$ excludes the COVID recession data, thereby helping to improve the statistical significance of all four of these variables.

Figures 3 and 4 show the results of 120-month rolling regressions using Specification 4. Only three predictor variables, namely, **pd**, **iv**, and Δ**sent12**$\times\Delta$**ersf,** are significant across a diverse set of sample periods. The variables **rv**, **term**, and **noise** are intermittently significant. The variable Δ**sent12** is rarely significant while the variable Δ**ersf** is never significant. For the most recent 120-month sample period that runs from 2011.M1 to 2020.M12 and includes the COVID-influenced data, the only significant predictor variable is **pd**. The results of the rolling regressions highlight the difficulty of identifying any robust predictors of excess stock returns. The intermittent or short-lived significance of some of our predictor variables is consistent with the findings of "pockets of predictability" by Farmer et al. (2022). Using model simulations, they demonstrate that a departure from rational expectations about future cash growth, in the form of "sticky expectations," can produce such outcomes.

[15] An alternate way of showing how recessions influence the results would be to add recession interaction variables (e.g., **pd**×**R**) to the regression specification in Table 2. The R-squared statistic for such a regression would be the same as in Table 4 or 5, but the estimated coefficient on the interaction variable would now represent the difference between the regression coefficients for each of the two regimes. Our interest here is not whether the regression coefficients are statistically different across the two regimes, but instead whether the statistical significance of a given predictor variable derives from one regime or the other.

Table 6 compares goodness-of-fit statistics for predictive regressions that include the variable **noise** versus otherwise similar regressions that omit this variable. An asterisk (*) indicates the superior goodness-of-fit statistic for the two regressions being compared. The goodness of fit statistics are: (1) the root mean squared forecast error ($RMSFE$), (2) the mean absolute forecast error ($MAFE$), (3) the correlation coefficient between the forecasted excess return and the realized excess return ($Corr$), (4) the percentage of forecasted excess returns with the same sign as the realized excess return ($Sign$), and (5) either the adjusted R-squared statistic (for in-sample forecasts) or the out-of-sample R-squared statistic (for out-of-sample forecasts). The out-of-sample R-squared statistic compares the performance of the predictive regression to a benchmark forecast model that assumes constant excess stock returns. The statistic is defined as one minus the ratio of summed squared residuals from the predictive regression to summed squared deviations of realized excess returns from the mean excess return of the estimation sample. We consider two out-of-sample forecasting exercises. In the first exercise, the out-of-sample period runs from 2006.M1 to 2020.M12, which includes the Great Recession. In the second exercise, the out-of-sample period runs from 2010.M1 to 2020.M12, which excludes the Great Recession.

The top panel of Table 6 shows the results for the in-sample predictive regressions. The bottom two panels shows the results for the out-of-sample forecasting exercises. In all cases in Table 6, including **noise** in the predictive regression serves to improve forecast performance as measured by the goodness-of-fit statistic. But the improved forecast performance in the second out-of sample exercise is relatively minor. This is because the out-of-sample period excludes the Great Recession which accounts for the statistical significance of the **noise** variable.

Our predictive regressions identify clear evidence of market inefficiency only during the Great Recession. But other evidence suggests that investors' forecasts of future stock returns are not fully-rational. Figure 5, adapted from Lansing (2020), shows that the degree of investor optimism or pessimism about the stock market is strongly linked to recent movements in stock prices. The figure plots the results of a University of Michigan survey that asks people to assign a probability that stock prices will increase over the next year.[16] Movements in the mean probability response from the survey are strongly correlated with movements in the predictor variable **pd**.

In our regressions, higher values of **pd** predict lower excess stock returns. But the survey respondents fail to take this empirical relationship into account. Instead, the survey respondents assign a higher probability of a price increase (implying a higher expected excess stock

[16] The data is available from June 2002 onward from https://data.sca.isr.umich.edu/tables.php. The survey question reads: "Suppose that tomorrow someone were to invest one thousand dollars in a type of mutual fund known as a diversified stock fund. What do you think is the percent chance that this one thousand dollar investment will increase in value in the year ahead, so that it is worth more than one thousand dollars one year from now?"

return) when **pd** is higher. During both the 2008-2009 financial crisis and the onset of the COVID-19 pandemic in early 2020, the value of **pd** was very low. Rational investors with time-varying risk premiums should expect higher excess stock returns when **pd** is very low. But in contrast, the survey respondents were very pessimistic about future stock prices when **pd** was very low. The pattern in Figure 5 is suggestive of extrapolative expectations rather than rationally time-varying risk premiums. Our theoretical results in Section 4.1 demonstrate that investors' use of extrapolative expectations will give rise to predictable excess stock returns.

6 Conclusion

This paper shows that realized excess returns on risky assets can be represented by an additive combination of conditional variance terms and investor forecast errors. As a result, the predictability of realized excess returns can arise from only two sources: (1) stochastic volatility of fundamental variables, or (2) departures from rational expectations that give rise to predictable investor forecast errors.

Motivated by our theoretical results, we run predictability regressions for 1-month-ahead excess stock returns using data from 1990.M1 to 2020.M12. We look for evidence of market inefficiency while controlling for the presence of stochastic volatility. The predictor variable that we use to detect market inefficiency is a measure of non-fundamental noise in the Treasury yield curve. We acknowledge, however, that disentangling the two sources of predictability is difficult because departures from rational expectations could influence the predictor variables that we use to control for stochastic volatility.

We find that the statistical significance of the stochastic volatility control variables is typically sensitive to the sample period. For example, measures of implied and realized stock return variance cease to be significant when the COVID-influenced data from early 2020 onward is included. The Treasury yield curve noise variable is statistically significant only during sample periods that include the Great Recession. Overall, we interpret our empirical results as providing evidence that the predictability of excess stock returns, when present, can come from both of the two sources identified by the theory.

A Appendix: Rational solution with stochastic volatility

This appendix derives an approximate analytical solution to the rational model with exogenous stochastic volatility described in Section 3.1. Gelain and Lansing (2014) employ similar methods to derive an approximate analytical solution to a rational asset pricing model for housing that exhibits exogenous stochastic volatility in fundamental rent growth.[17] Substituting the functional forms for M_t and d_t/d_{t-1} into the transformed first-order condition for stocks (5) yields

$$z_t^s \;=\; \beta \exp\left[(1-\alpha)\, x_t + v_t + \alpha\eta_t - \alpha\eta_{t-1}\right]\left(1 + E_t z_{t+1}^s\right), \tag{A.1}$$

where $x_t \equiv \log\left(c_t/c_{t-1}\right)$. A conjectured solution to (A.1) takes the form

$$z_t^s \;=\; a_0 \exp\left[a_1\left(x_t - \overline{x}\right) + a_2\left(\sigma_t^2 - \overline{\sigma}^2\right) + a_3 v_t + a_4\left(\eta_t - \overline{\eta}\right) + a_5\left(\eta_{t-1} - \overline{\eta}\right)\right]. \tag{A.2}$$

Iterating ahead the conjectured solution (A.2) and then taking the conditional expectation yields

$$\begin{aligned}\underbrace{E_t z_{t+1}^s}_{=p_t^s/d_t} \;&=\; a_0 \exp\left[a_1\rho_x\left(x_t - \overline{x}\right) + \tfrac{1}{2}\left(a_1\right)^2\sigma_t^2 + a_2\rho_\sigma\left(\sigma_t^2 - \overline{\sigma}^2\right) + \tfrac{1}{2}\left(a_2\right)^2\sigma_u^2 + \tfrac{1}{2}\left(a_3\right)^2\sigma_v^2\right] \\ &\quad \times \exp[\left(a_4\rho_\eta + a_5\right)\left(\eta_t - \overline{\eta}\right) + \tfrac{1}{2}\left(a_4\right)^2\sigma_\omega^2],\end{aligned} \tag{A.3}$$

where $p_t^s/d_t = E_t z_{t+1}^s$ from equation (4). The above expression shows that p_t^s/d_t is a function of the fundamental state variable σ_t^2 that drives the stochastic volatility of consumption and dividend growth. This analytical result motivates the inclusion of the price-dividend ratio as an indirect control for stochastic volatility in the predictability regressions of Section 5.

The conditional forecast (A.3) is substituted into the transformed first order condition (A.1) which is then log-linearized to obtain

$$\begin{aligned}z_t^s \;&=\; F\left(x_t,\, \sigma_t^2,\, v_t,\, \eta_t,\, \eta_{t-1}\right), \\ &\simeq\; a_0 \exp\left[a_1\left(x_t - \overline{x}\right) + a_2\left(\sigma_t^2 - \overline{\sigma}^2\right) + a_3 v_t + a_4\left(\eta_t - \overline{\eta}\right) + a_5\left(\eta_{t-1} - \overline{\eta}\right)\right],\end{aligned} \tag{A.4}$$

where a_0 through a_5 are Taylor-series coefficients with $a_0 \equiv \exp\left\{E\left[\log\left(z_t^s\right)\right]\right\}$. After some manipulation, it can be shown that the Taylor series coefficients must satisfy the following

[17] Lansing (2010) demonstrates the accuracy of this solution method for the level of the price-dividend ratio by comparing the approximate analytical solution to the exact theoretical solution for the model version without stochastic volatility ($\sigma_u^2 = 0$) and without time-varying risk aversion ($\sigma_\omega^2 = 0$).

system of nonlinear equations

$$a_0 = \frac{\beta \exp[(1-\alpha)\overline{x}]}{1-\beta \exp\left[(1-\alpha)\overline{x}+(a_1)^2\,\overline{\sigma}^2/2+(a_2)^2\sigma_u^2/2+(a_3)^2\sigma_v^2/2+(a_4)^2\sigma_\omega^2/2\right]}, \tag{A.5}$$

$$a_1 = \frac{(1-\alpha)}{1-\rho_x\beta \exp\left[(1-\alpha)\overline{x}+(a_1)^2\,\overline{\sigma}^2/2+(a_2)^2\sigma_u^2/2+(a_3)^2\sigma_v^2/2+(a_4)^2\sigma_\omega^2/2\right]}, \tag{A.6}$$

$$a_2 = \frac{\left[(a_1)^2/2\right]\beta \exp\left[(1-\alpha)\overline{x}+(a_1)^2\,\overline{\sigma}^2/2+(a_2)^2\sigma_u^2/2+(a_3)^2\sigma_v^2/2+(a_4)^2\sigma_\omega^2/2\right]}{1-\rho_\sigma\beta \exp\left[(1-\alpha)\overline{x}+(a_1)^2\,\overline{\sigma}^2/2+(a_2)^2\sigma_u^2/2+(a_3)^2\sigma_v^2/2+(a_4)^2\sigma_\omega^2/2\right]}, \tag{A.7}$$

$$a_3 = 1, \tag{A.8}$$

$$a_4 = \frac{\alpha\left\{1-\beta \exp\left[(1-\alpha)\overline{x}+(a_1)^2\,\overline{\sigma}^2/2+(a_2)^2\sigma_u^2/2+(a_3)^2\sigma_v^2/2+(a_4)^2\sigma_\omega^2/2\right]\right\}}{1-\rho_\eta\beta \exp\left[(1-\alpha)\overline{x}+(a_1)^2\,\overline{\sigma}^2/2+(a_2)^2\sigma_u^2/2+(a_3)^2\sigma_v^2/2+(a_4)^2\sigma_\omega^2/2\right]} \tag{A.9}$$

$$a_5 = -\alpha, \tag{A.10}$$

provided that $\beta \exp\left[(1-\alpha)\,\overline{x} + (a_1)^2\,\overline{\sigma}^2/2 + (a_2)^2\,\sigma_u^2/2 + (a_3)^2\,\sigma_v^2/2 + (a_4)^2\,\sigma_\omega^2/2\right] < 1$. From equations (A.2) and (A.3), we can compute $\log[z_{t+1}^s/(E_t z_{t+1}^s)]$, yielding equation (27) in the text.

In the case of Epstein-Zin preferences, we have $M_{t+1} = \beta^\omega \exp\left[-\omega\, x_t/\psi\right]\left(R_{t+1}^c\right)^{\omega-1}$, where $R_{t+1}^c = \exp(x_t)\left(1+p_{t+1}^c/c_{t+1}\right)/(p_t^c/c_t)$ is the gross return on a consumption claim, ψ is the elasticity of intertemporal substitution, and $\omega \equiv (1-\alpha)/\left(1-\psi^{-1}\right)$. When $v_t = 0$ (such that $c_t = d_t$) and $\eta_t = \eta_{t-1}$ (such that risk aversion is constant), the first-order condition (A.1) becomes

$$z_t^c = \beta^\omega \exp\left[(1-\alpha)\,x_t\right]\left[1+(E_t z_{t+1}^c)^{1/\omega}\right]^\omega, \tag{A.11}$$

where the composite variable z_t^c is defined as $z_t^c \equiv \beta^\omega \exp\left[(1-\alpha)\,x_t\right](1+p_t^c/c_t)^\omega$. In this case, we have $p_t^c/c_t = (E_t z_{t+1}^s)^{1/\omega}$.

The investors's stochastic discount factor for the Epstein-Zin case can be rewritten as

$$M_{t+1} = \beta \exp[(1-\alpha-\omega)\,x_{t+1}/\omega]\left[z_{t+1}^c/(E_t z_{t+1}^c)\right]^{(\omega-1)/\omega}. \tag{A.12}$$

An approximate analytical solution to equation (A.11) can be obtained using methods similar to those employed in solving equation (A.1) above. Using the analytical solution for z_t^c, an approximate analytical solution for the risk free rate of return can be obtained by computing $R_{t+1}^f = 1/(E_t M_{t+1})$ where M_{t+1} is given by equation (A.12).

Substituting $p_t^c/c_t = (E_t z_{t+1}^s)^{1/\omega}$ (from the first-order condition) and $1 + p_{t+1}^c/c_{t+1} = \beta^{-1}(z_{t+1}^c)^{1/\omega}\exp[-(1-\alpha)\,x_{t+1}/\omega]$ (from the definition of z_{t+1}^c) into the expression for R_{t+1}^c confirms that

$$R_{t+1}^c = \frac{z_{t+1}^c}{E_t z_{t+1}^c}\frac{1}{M_{t+1}}. \tag{A.13}$$

The excess return on the consumption claim is given by

$$\log(R^c_{t+1}/R^f_{t+1}) \;=\; \log\left[z^c_{t+1}/(E_t z^c_{t+1})\right] \;-\; \log\left[M_{t+1}/(E_t M_{t+1})\right], \tag{A.14}$$

which is a special case of equation (10) in the text.

B Appendix: Endogenous stochastic volatility

Endogenous stochastic volatility can arise from the nonlinear nature of the model's functional forms. Consider the time-separable exponential utility function $U = 1 - \exp(-\alpha c_t)$ which exhibits constant absolute risk aversion such that $-U_{cc}/U_c = \alpha$. The investor's stochastic discount factor is given by

$$\begin{aligned} M_{t+1} &= \beta \exp\left[-\alpha\left(c_{t+1} - c_t\right)\right] = \beta \exp\left(-\alpha c_t\, x_{t+1}\right), & \text{(B.1)} \\ x_{t+1} &= \overline{x} + \rho_x\left(x_t - \overline{x}\right) + \varepsilon_{t+1}, \qquad |\rho_x| < 1, \qquad \varepsilon_t \sim NID\left(0, \sigma^2_\varepsilon\right), & \text{(B.2)} \end{aligned}$$

where $x_{t+1} \equiv \left(c_{t+1} - c_t\right)/c_t$ is real consumption growth that evolves as an AR(1) process with constant innovation variance σ^2_ε.

Under rational expectations, we have

$$R^f_{t+1} = 1/(E_t M_{t+1}) \;=\; \beta^{-1} \exp\left\{c_t\left[\alpha\overline{x} + \alpha\rho_x\left(x_t - \overline{x}\right)\right] - \tfrac{1}{2}\alpha^2\sigma^2_\varepsilon c^2_t\right\}, \tag{B.3}$$

$$\log\left[M_{t+1}/(E_t M_{t+1})\right] \;=\; -\alpha c_t\, \varepsilon_{t+1} \;-\; \tfrac{1}{2}\alpha^2\sigma^2_\varepsilon c^2_t, \tag{B.4}$$

which shows that the left side of equation (B.4) will be predictable because c^2_t is time-varying and helps to partly determine the realized excess stock return at time $t+1$. Similarly, the term $\log\left[z^s_{t+1}/(E_t z^s_{t+1})\right]$ that appears in the excess stock return equation (10) will also be predictable.

C Appendix: Solution with extrapolative expectations

This appendix derives an approximate analytical solution for $z^s_{t+1}/(\widehat{E}_t z^s_{t+1})$ under extrapolative expectations. Substituting the extrapolative forecast $\widehat{E}_t z^s_{t+1} = A^s z_t$ together with the functional forms for M_t and d_t/d_{t-1} into the transformed first-order condition for stocks (5), and then solving for z^s_t yields

$$z^s_t \;=\; \frac{\beta \exp\left[\left(1-\alpha\right)x_t + v_t + \alpha\eta_t - \alpha\eta_{t-1}\right]}{1 - A^s\beta \exp\left[\left(1-\alpha\right)x_t + v_t + \alpha\eta_t - \alpha\eta_{t-1}\right]}, \tag{C.1}$$

where $x_t \equiv \log\left(c_t/c_{t-1}\right)$. The denominator of equation (B.1) can be approximated as

$$\begin{aligned} &1 - A^s\beta \exp\left[\left(1-\alpha\right)x_t + v_t + \alpha\eta_t - \alpha\eta_{t-1}\right] \;\simeq \\ &\qquad b_0 \exp[b_1\left(x_t - \overline{x}\right) + b_2 v_t + b_3\left(\eta_t - \overline{\eta}\right) + b_4\left(\eta_{t-1} - \overline{\eta}\right)], \end{aligned} \tag{C.2}$$

where b_0 through b_4 are Taylor-series coefficients. The Taylor series coefficients are given by

$$b_0 = 1 - A^s \beta \exp\left[(1-\alpha)\,\overline{x}\right], \tag{C.3}$$

$$b_1 = \frac{-A^s \beta (1-\alpha) \exp\left[(1-\alpha)\,\overline{x}\right]}{1 - A^s \beta \exp\left[(1-\alpha)\,\overline{x}\right]}, \tag{C.4}$$

$$b_2 = \frac{-A^s \beta \exp\left[(1-\alpha)\,\overline{x}\right]}{1 - A^s \beta \exp\left[(1-\alpha)\,\overline{x}\right]}, \tag{C.5}$$

$$b_3 = \frac{-A^s \beta \alpha \exp\left[(1-\alpha)\,\overline{x}\right]}{1 - A^s \beta \exp\left[(1-\alpha)\,\overline{x}\right]} \tag{C.6}$$

$$b_4 = \frac{A^s \beta \alpha \exp\left[(1-\alpha)\,\overline{x}\right]}{1 - A^s \beta \exp\left[(1-\alpha)\,\overline{x}\right]} \tag{C.7}$$

provided that $A^s \beta \exp\left[(1-\alpha)\,\overline{x}\right] < 1$.

Using equations (C.1) and (C.2), we have

$$\begin{aligned} \log[z^s_{t+1}/(A^s z^s_t)] \;\; = \;\; & -\log A^s \; + \; (1-\alpha-b_1)(x_{t+1}-x_t) \; + \; (1-b_2)(v_{t+1}-v_t) \\ & + \; (\alpha-b_3)(\eta_{t+1}-\eta_t) \; - \; (\alpha+b_4)(\eta_t-\eta_{t-1}), \end{aligned} \tag{C.8}$$

which can be transformed to obtain equation (32) in the text.

D Appendix: Data sources

Monthly data on the end-of-month nominal S&P 500 stock index, nominal dividends, and the nominal risk free rate of return are from Welch and Goyal (2008). Updated data through the end of 2020 are available from Amit Goyal's website.[18] The gross nominal return on the S&P 500 stock index in month t is defined as $(P_t + D_t/12)/P_{t-1}$, where P_t is the end-of-month nominal closing value of the index and D_t is cumulative nominal dividends over the past 12 months. The price-dividend ratio in month t is defined as P_t/D_t. Data on the implied variance and realized variance of the S&P 500 stock index are from Zhou (2018). Updated monthly data through the end of 2020 are available from Hao Zhou's website.[19] The implied variance is measured by the end-of-month VIX-squared, de-annualized (i.e., $\text{VIX}^2/12$). Realized variance is measured by the sum of squared 5-minute log returns of the S&P 500 stock index over the month. Both variance measures are expressed in percentage-squared terms and are available in real time at the end of the observation month. The monthly average spread in percent between the 10-year and 3-month constant maturity Treasury yields is from the FRED database of the

[18] www.hec.unil.ch/agoyal/.

[19] https://sites.google.com/site/haozhouspersonalhomepage/.

Federal Reserve Bank of St. Louis. The University of Michigan consumer sentiment index is from www.sca.isr.umich.edu/tables.html. The Treasury yield curve noise measure is from Hu, Pan, and Wang (2013). Updated daily data through the end of 2020 are available from Jun Pan's website.[20] The noise measure employed in the regressions is the monthly average of the daily noise measures. Similar results are obtained using the end-of-month noise measure.

E Appendix: Bootstrapped critical values

Stambaugh (1999) and Mankiw and Shapiro (1986) show that the estimated slope coefficient and its associated t-statistic exhibit finite-sample bias when one regresses stock returns (or excess stock returns) on the lagged price-dividend ratio, which is highly persistent. They further illustrate that the bias in the estimated slope coefficient depends on the contemporaneous correlation between innovations to excess stock returns and innovations to the price-dividend ratio. Upward movements in the stock price tend to drive up the excess stock return and the price-dividend ratio simultaneously, implying a positive correlation between the two innovations. The bias in the estimated slope coefficient is proportional to the bias in the estimate of the AR coefficient for the price-dividend ratio. Kendall (1954) shows that there is a large downward finite-sample bias in the estimate of the AR coefficient when the variable in question is highly persistent, as with the price-dividend ratio. The upshot is that the least squares estimate of the slope coefficient and its associated t-statistic in predictive regressions for excess stock returns can have non-trivial biases. Use of the standard asymptotic critical values for t-statistics can lead investigators to reject the null hypothesis more often than they should.

Table 1 shows that the predictor variables **pd**, **iv**, **term,** **Δsent12,** and **noise** are highly persistent. We wish to gauge the magnitude of potential size distortions of the standard t-statistic in our specific application where we regress excess stock returns in month $t+1$ on a constant and all eight predictor variables in month t. We address this issue using a slightly modified bootstrap procedure as laid out in Nelson and Kim (1993), Mark (1995), and Rapach and Wohar (2006). We postulate that the data under the null hypothesis are generated by the following system:

$$\log(R^s_{t+1}/R^f_{t+1}) = a_0 + \varepsilon_{1t+1}, \tag{E.1}$$

$$x^i_{t+1} = b_0 + b_1 x^i_t + ... + b_j x^i_{t-j+1} + \varepsilon^i_{2t+1}, \tag{E.2}$$

where x^i_t denotes one of the eight predictor variables. The innovation to excess stock returns ε_{1t+1} and the innovation to each of the eight predictor variables ε^i_{2t+1} are allowed to be contemporaneously correlated.

[20]http://en.saif.sjtu.edu.cn/junpan/

To obtain the bootstrapping parameters, we first estimate equation (E.1) using excess stock returns. We then estimate equation (E.2) for each of the eight predictor variables. The number of lags in equation (E.2) for each predictor variable is determined using the AIC, with a maximum of four lags. Given the parameter estimates from equations (E.1) and (E.2), we compute and store the residuals. Next, we take random draws (with replacement) of these OLS residuals in tandem, preserving the contemporaneous correlations between these residuals as in the original sample. For each simulation, we obtain a bootstrapped data of sample size $N \times 1.25$, where $N = 372$ is the sample length of monthly U.S. data from 1990.M1 to 2020.M12. We drop the first 25% of the bootstrapped data to remove any potential impact of the initial values, thus keeping the length of the pseudo-sample equal to the length of the U.S. data sample. Following Shaman and Stine (1988), we also implement a bias correction procedure for the estimated AR coefficients in equation (E.2). We use the bias-corrected parameter values and the randomly-drawn correlated residuals to generate bootstrapped data from equations (E.1) and (E.2).

We carry out the bootstrap procedure by simulating excess stock returns using equation (E.1) and simulating the evolution of the eight predictor variables using the eight versions of equation (E.2). We then use this simulated pseudo-sample to regress excess stock returns in month $t+1$ on a constant and all eight predictor variables in month t. For each bootstrapped sample, we compute and store the t-statistics for the eight slope coefficients. The t-statistics are computed using Newey-West HAC corrected standard errors. We repeat the process 1000 times and obtain an empirical distribution of the bootstrapped t-statistics. We report the 2.5% and 97.5% percentiles of the empirical distribution as the 5% empirical critical values. The bootstrapping results are reported in Table E.1.

The two-sided 5% asymptotic critical values of a t-statistic that adheres to a standard normal distribution are -1.96 and $+1.96$. The bootstrapped critical values in Table E.1 are not substantially different from the asymptotic ones, but there are some noticeable shifts in either direction for the persistent predictor variables, depending upon the direction of the underlying correlation between the simulated innovations.

For example, the 2.5% percentile of the bootstrapped t-statistic for **pd** is -2.936. This value is larger in absolute value than the asymptotic value of -1.96, thus raising the bar for one to reject the null hypothesis in favor of a negative coefficient. At the same time, the 97.5% percentile of the bootstrapped t-statistic for **pd** is 1.394, less than the asymptotic value of 1.96. This left-skewed distribution of the test statistics results from the positive correlation between innovations to excess stock returns and innovations to **pd,** which gives rise to downward bias in the slope coefficient and the associated t-statistic. On the other hand, the 2.5% and 97.5% percentiles of the bootstrapped t-statistic for **iv** are -1.783 and 2.478, respectively, indicating a slightly right-skewed distribution. The right-skewed distribution is consistent with a negative correlation between innovations to excess stock returns and innovations to **iv.**

The distributions of the t-statistics for the remaining six predictor variables in Table E.1 appear less skewed and closer to the standard normal distribution. The 2.5% and 97.5% percentiles of the bootstrapped t-statistic for **noise,** our key variable of interest, are both larger in magnitude than 1.96. But the bootstrapped distribution is quite symmetric around zero.

Table E.1: Bootstrapped critical t-statistics

Predictor variable	2.5% percentile	97.5% percentile
pd	−2.936	1.394
iv	−1.783	2.478
rv	−2.461	2.751
term	−2.269	2.340
Δsent12	−2.144	1.986
Δersf	−1.937	2.038
Δsent12×Δersf	−2.090	2.170
noise	−2.331	2.258

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Table 1: Summary statistics: 1990.M1 to 2020.M12

Variable	Mean	Std. Dev.	Skewness	Kurtosis	Autocorr.	Corr. with **noise**
ersf	0.59	4.28	−0.74	4.59	0.03	−0.20
pd	52.1	13.6	0.53	3.32	0.98	−0.18
iv	36.7	34.8	3.46	19.9	0.77	0.61
rv	21.9	48.1	9.24	106.9	0.44	0.42
term	1.69	1.14	−0.05	1.93	0.98	0.27
Δ**sent12**	−0.36	10.1	−0.63	3.46	0.85	−0.29
Δ**ersf**	0.01	5.95	0.58	4.45	−0.47	0.05
Δ**sent12** $\times$ Δ**ersf**	−5.07	68.1	−2.31	26.7	−0.26	−0.05
noise	2.88	2.03	4.08	26.8	0.95	1.00

Notes: **ersf** = excess return on S&P 500 stock index relative to the risk free rate in percent as measured by the return on a 3-month Treasury bill, **pd** = price-dividend ratio for the S&P 500 index defined as the end-of-month nominal closing value of the index divided by cumulative nominal dividends over the past 12 months, **iv** = implied variance in percent-squared from options on the S&P 500 stock index, **rv** = realized variance of the S&P 500 stock index in percent-squared using 5-minute return intervals over the month, **term** = monthly average spread in percent between the 10-year and 3-month constant maturity Treasury yields, **Δsent12** = 12-month change in the University of Michigan consumer sentiment index, **Δersf** = excess return momentum defined as the 1-month change in **ersf, noise** = monthly average root mean squared deviation between the daily Treasury yield curve and a daily model-fitted, no-arbitrage yield curve.

Table 2: Predicting excess returns on stocks: Full sample results

1990.M1 to 2020.M12	1	2	3	4
pd	$\mathbf{-0.036}$	$\mathbf{-0.045}$	$\mathbf{-0.046}$	$\mathbf{-0.054}$
	(-1.990)	(-2.120)	(-2.326)	(-2.709)
iv		0.010	0.010	0.031
		(0.480)	(0.419)	(1.448)
rv		-0.005	-0.002	-0.006
		(-0.238)	(-0.108)	(-0.360)
term		-0.261	-0.326	-0.102
		(-1.207)	(-1.403)	(-0.437)
Δsent12			0.042	0.026
			(1.570)	(1.135)
Δersf			0.001	0.016
			(0.016)	(0.408)
Δsent12 $\times$ Δersf			$\mathbf{-0.013}$	$\mathbf{-0.012}$
			(-3.371)	(-2.975)
noise				$\mathbf{-0.551}$
				(-3.164)
Adj. R^2	1.06%	0.90%	4.81%	8.30%

Notes: Dependent variable is **ersf** for month $t+1$. All regressions include a constant term with regressors dated month t. Newey-West HAC corrected t-statistics in parentheses. Boldface indicates significant at the 5% level using asymptotic critical values. See Table 1 for variable definitions.

Table 3: Predicting excess returns on stocks: Split sample results

1990.M1 to 2005.M12	1	2	3	4
pd	**−0.034**	**−0.053**	**−0.051**	**−0.051**
	(−2.191)	(−2.298)	(−2.308)	(−2.337)
iv		**0.033**	0.030	**0.032**
		(1.989)	(1.910)	(2.071)
rv		−0.008	−0.013	−0.013
		(−0.240)	(−0.432)	(−0.437)
term		−0.218	−0.301	−0.275
		(−0.645)	(−0.915)	(−0.826)
Δsent12			0.033	0.029
			(1.248)	(1.102)
Δersf			−0.028	−0.028
			(−0.701)	(−0.687)
Δsent12 × Δersf			**−0.017**	**−0.017**
			(−4.043)	(−4.001)
noise				−0.246
				(−0.832)
Adj. R^2	2.26%	3.50%	7.02%	6.82%
2006.M1 to 2020.M12	1	2	3	4
pd	−0.053	−0.126	−0.115	**−0.339**
	(−0.465)	(−1.146)	(−1.048)	(−3.658)
iv		−0.155	−0.013	0.019
		(−0.449)	(−0.333)	(0.497)
rv		0.005	0.007	−0.002
		(0.200)	(0.302)	(−0.109)
term		−0.240	−0.308	−0.059
		(−0.789)	(−0.854)	(−0.149)
Δsent12			0.051	−0.001
			(1.105)	(−0.036)
Δersf			0.052	0.087
			(0.806)	(1.238)
Δsent12 × Δersf			**−0.009**	−0.008
			(−2.027)	(−1.375)
noise				**−1.019**
				(−5.484)
Adj. R^2	0.00%	−0.97%	2.62%	13.6%

Notes: See Table 2.

Table 4: Predicting excess returns on stocks: Great Recession results

1990.M1 to 2020.M12	1	2	3	4
pd × **GR**	−0.220	−0.187	0.101	−0.603
	(−1.390)	(−0.689)	(−0.327)	(−1.933)
pd × (1 − **GR**)	**−0.043**	**−0.060**	**−0.059**	**−0.057**
	(−2.748)	(−2.814)	(−2.689)	(−2.706)
iv × **GR**		0.007	0.016	0.051
		(0.245)	(0.324)	(1.816)
iv × (1 − **GR**)		0.023	0.020	0.023
		(1.383)	(1.061)	(1.193)
rv × **GR**		**−0.028**	−0.024	**−0.031**
		(2.221)	(−0.936)	(2.109)
rv × (1 − **GR**)		0.006	0.007	0.005
		(0.623)	(0.684)	(0.510)
term × **GR**		2.976	3.731	2.257
		(1.607)	(1.940)	(1.336)
term × (1 − **GR**)		−0.220	−0.226	−0.153
		(−0.938)	(−0.877)	(−0.573)
Δsent12 × **GR**			**0.196**	−0.033
			(2.544)	(−0.341)
Δsent12 × (1 − **GR**)			0.009	0.006
			(0.335)	(0.226)
Δersf × **GR**			**0.773**	0.604
			(3.304)	(1.375)
Δersf × (1 − **GR**)			−0.007	−0.006
			(−0.212)	(−0.166)
Δsent12 × **Δersf** × **GR**			**0.021**	0.006
			(1.963)	(0.304)
Δsent12 × **Δersf** × (1 − **GR**)			**−0.008**	**−0.008**
			(−2.161)	(−2.121)
noise × **GR**				**−1.508**
				(−4.417)
noise × (1 − **GR**)				−0.231
				(−1.025)
Adj. R^2	3.92%	7.97%	11.0%	12.8%

Notes: See Table 2. The dummy variable **GR,** dated month-t, is equal to 1 during the Great Recession from 2007.M12 to 2009.M6 and equal to 0 otherwise. All regressions include constant × **GR** and constant × (1−**GR**).

Table 5: Predicting excess returns on stocks: All recession results

1990.M1 to 2020.M12	1	2	3	4
pd $\times$ **R**	−0.031	−0.038	−0.048	−0.080
	(−0.626)	(−0.710)	(−0.950)	(−1.700)
pd $\times$ (1 − **R**)	**−0.046**	**−0.076**	**−0.075**	**−0.071**
	(−2.790)	(−3.991)	(−3.886)	(−3.950)
iv $\times$ **R**		−0.032	−0.043	0.021
		−0.900)	(−1.499)	(0.685)
iv $\times$ (1 − **R**)		**0.060**	**0.057**	**0.064**
		(4.470)	(4.146)	(4.324)
rv $\times$ **R**		0.019	0.025	0.008
		(0.850)	(1.350)	(0.380)
rv $\times$ (1 − **R**)		**−0.044**	**−0.043**	**−0.048**
		(−2.083)	(−2.003)	(2.152)
term $\times$ **R**		0.783	0.925	**2.995**
		(0.531)	(0.666)	(2.047)
term $\times$ (1 − **R**)		−0.389	−0.395	−0.296
		(−1.916)	(−1.849)	(−1.315)
Δsent12 $\times$ **R**			0.062	0.077
			(0.483)	(0.797)
Δsent12 $\times$ (1 − **R**)			0.003	0.002
			(0.106)	(0.067)
Δersf $\times$ **R**			0.514	**0.913**
			(1.664)	(2.907)
Δersf $\times$ (1 − **R**)			−0.013	−0.010
			(−0.358)	(−0.287)
Δsent12 $\times$ **Δersf** $\times$ **R**			0.005	0.024
			(0.331)	(1.797)
Δsent12 $\times$ **Δersf** $\times$ (1 − **R**)			**−0.010**	**−0.009**
			(−2.139)	(−2.048)
noise $\times$ **R**				**−1.124**
				(−5.802)
noise $\times$ (1 − **R**)				−0.323
				(−1.485)
Adj. R^2	2.48%	6.86%	12.7%	15.9%

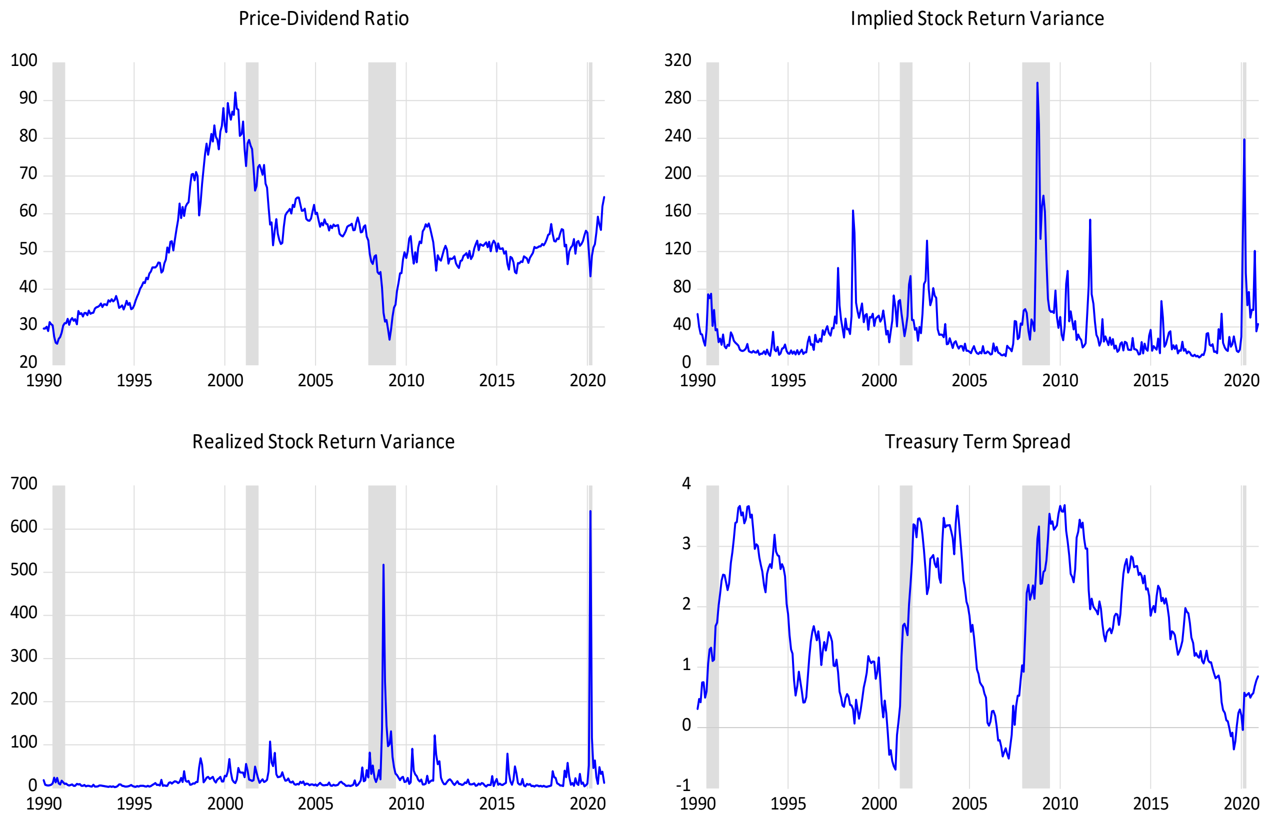
Notes: See Table 2. The dummy variable **R**, dated month-t, is equal to 1 during recessions and equal to 0 otherwise. All regressions include constant $\times$ **R** and constant $\times$ (1−**R**).

Table 6: Goodness-of Fit Statistics

1-month ahead forecast	$RMSFE$	$MAFE$	$Corr$	$Sign$	Adj. R^2	OOS R^2
In-sample with **noise**	4.03%*	2.99%*	0.32*	65.0%*	8.30%*	
In-sample without **noise**	4.11%	3.09%	0.26	63.1%	4.81%	
Split out-of-sample I with **noise**	4.30%*	3.13%*	0.25*	68.9%*	–	5.30%*
Split out-of-sample I without **noise**	4.44%	3.24%	0.17	65.6%	–	−0.01%
Split out-of-sample II with **noise**	4.46%*	3.11%*	0.00*	65.0%*	–	−0.17%*
Split out-of-sample II without **noise**	4.69%	3.30%	−0.09	63.1%	–	−0.29%

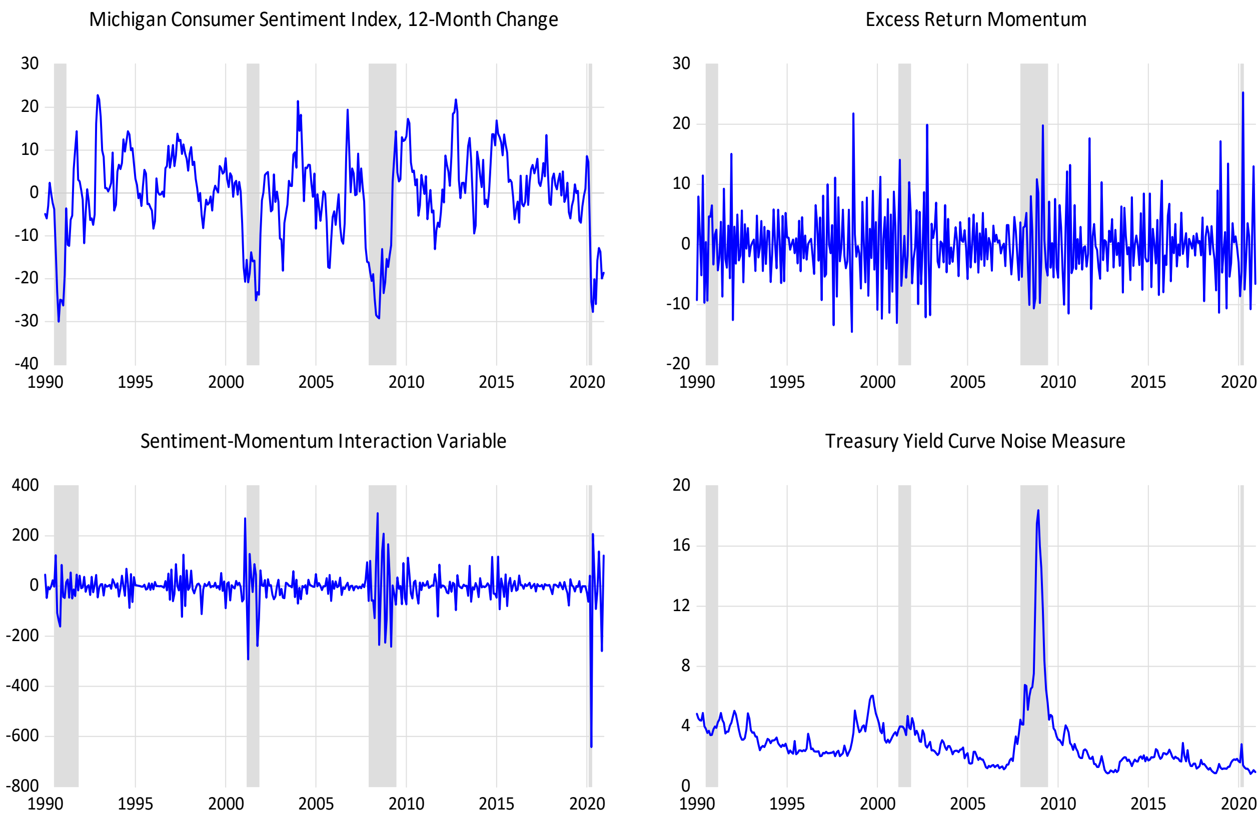
Notes: $RMSFE$ = Root mean squared forecast error, $MAFE$ = Mean absolute forecast error, $Corr$ = correlation coefficient between forecasted excess return and realized excess return, $Sign$ = percentage of forecasted excess returns with same sign as realized excess return, Adj. R^2 = Adjusted R-squared statistic for in-sample regressions, OOS R^2 = Out-of-sample R-squared statistic defined as $1-SSR/SST$, where SSR is the sum of the squared residuals from the predictive regression and SST is the sum of the squared deviations of realized excess returns from the mean excess return of the estimation sample. The in-sample regression equations correspond to columns 3 and 4 in Table 2 and cover the period from 1990.M1 to 2020.M12. For the split out-of-sample I regressions, the same equations are estimated for the period from 1990.M1 to 2005.M12 and then used to forecast excess stock returns for the period from 2006.M1 to 2020.M12. For the split out-of-sample II regressions, the same equations are estimated for the period from 1990.M1 to 2009.M12 and then used to forecast excess stock returns for the period from 2010.M1 to 2020.M12. An asterisk * indicates the superior goodness-of-fit statistic for the two regressions being compared.

Figure 1: *Predictor Variables*



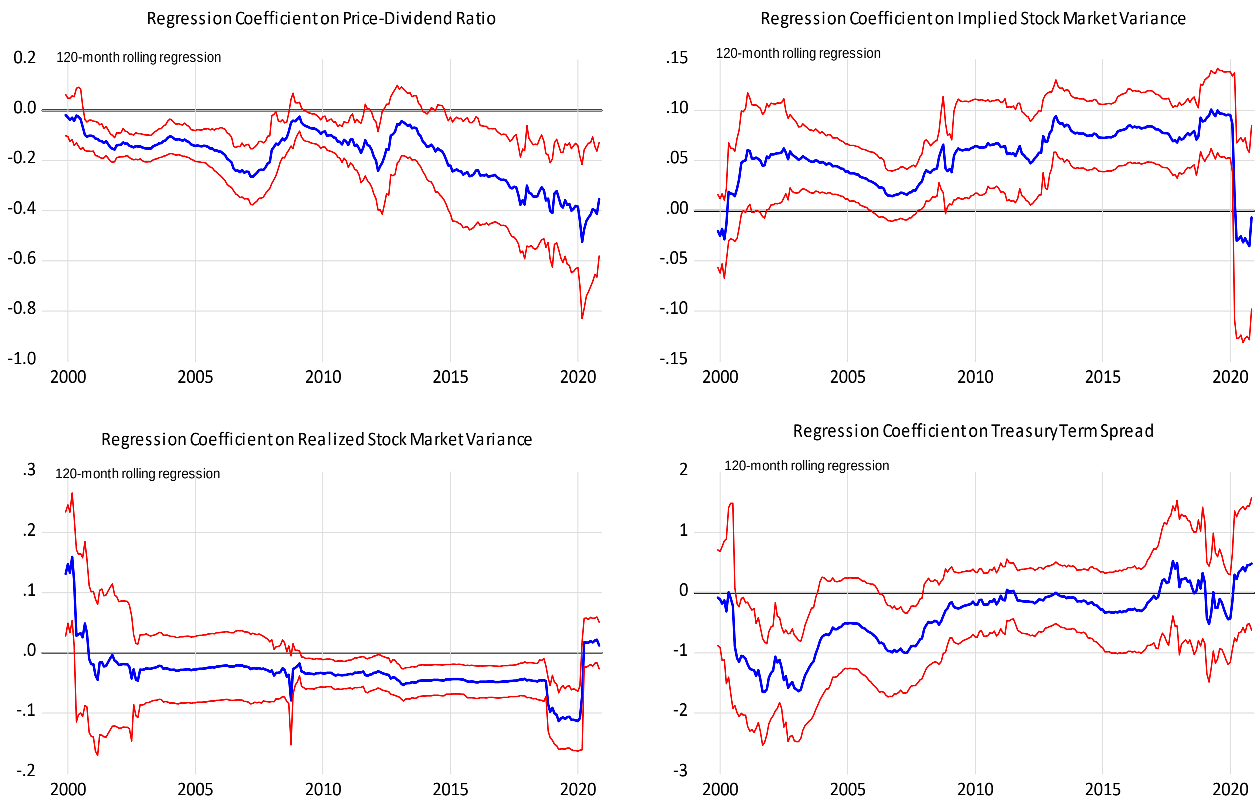
Notes: The predictor variables for one-month-ahead excess stock returns include the price-dividend ratio (**pd**), the implied stock return variance (**iv**), the realized stock return variance (**rv**), and the treasury term spread (**term**). These variables are included to control for the presence of stochastic volatility.

Figure 2: *Predictor Variables*



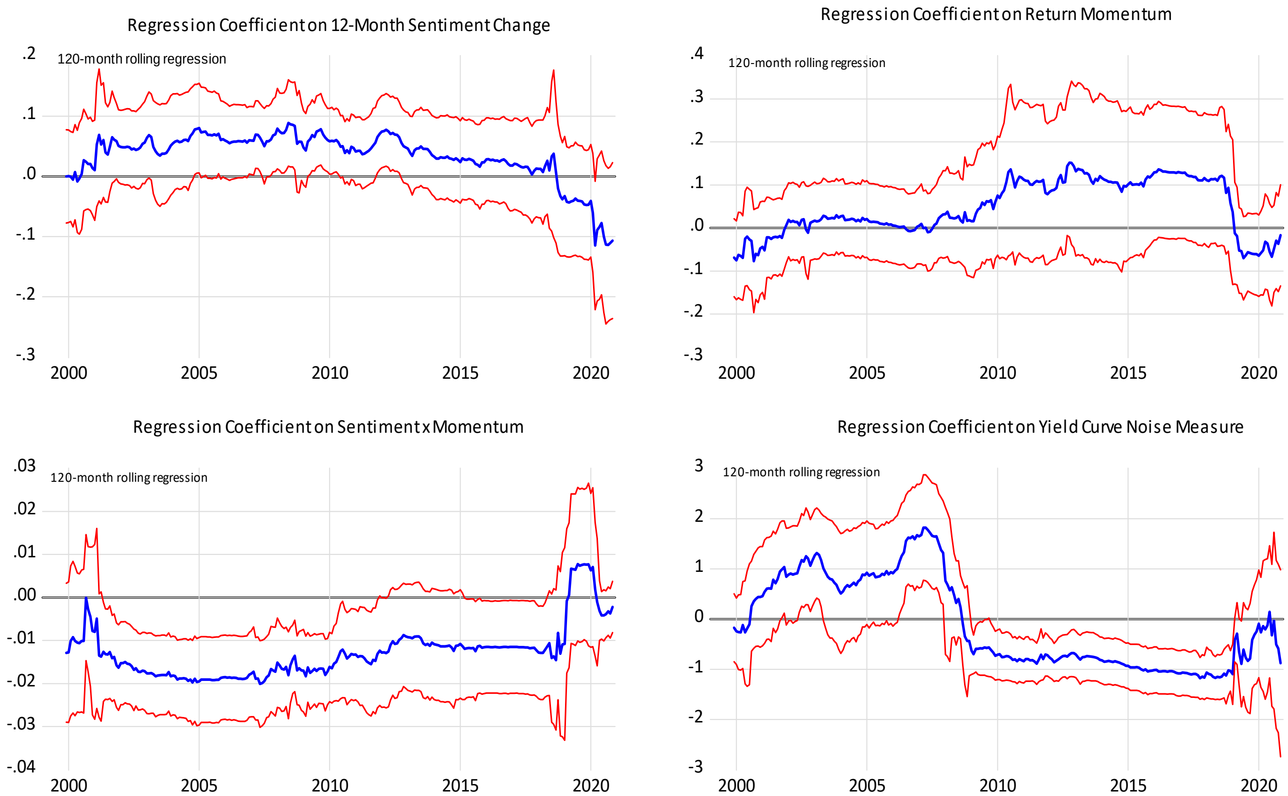
Notes: The predictor variables for one-month-ahead excess stock returns include a measure of shifts in consumer sentiment (**Δsent12**), a measure of excess return momentum (**Δersf**), a sentiment-momentum interaction variable (**Δsent12**$\times$**Δersf**), and a measure of non-fundamental noise in the treasury yield curve (**noise**). The first three of these variables are included to control for the presence of stochastic volatility whereas the **noise** variable is included to detect market efficiency.

Figure 3: *Rolling Regression Coeffficients*



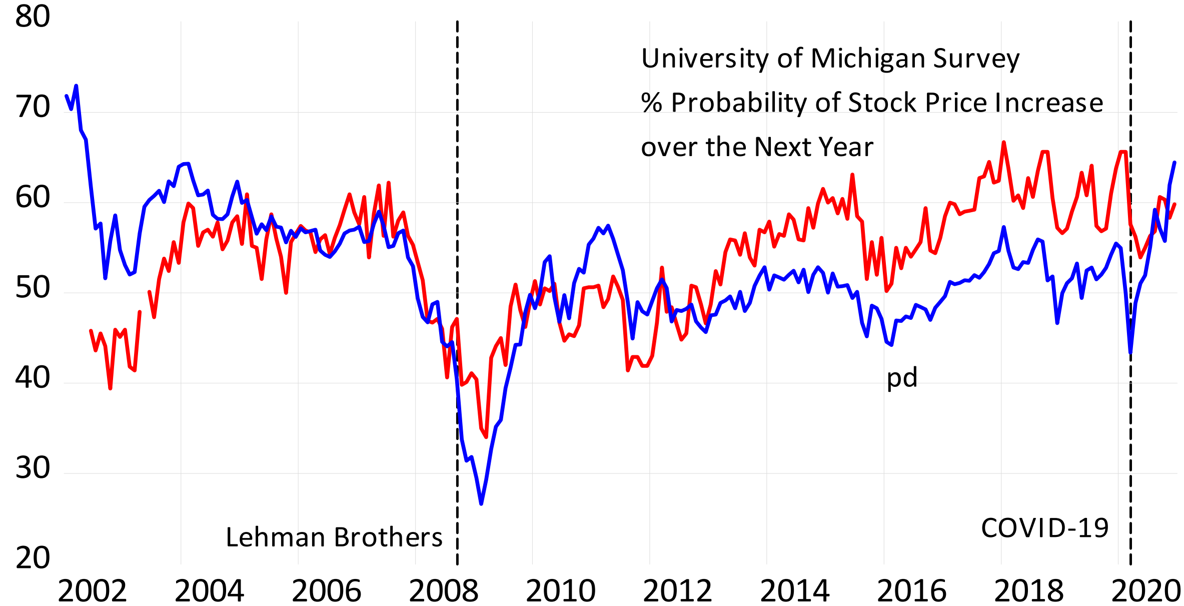
Notes: Thin red lines represent 95% confidence intervals using the asymptotic critical values. The rolling regression coefficient on **pd** exhibits a consistent negative sign and is mostly significant from the early 2000s onwards. The rolling regression coefficients on **iv** and **rv** exhibit mostly positive and negative signs, respectively. The rolling regression coefficient on **term** is rarely significant.

Figure 4: *Rolling Regression Coeffficients*



Notes: Thin red lines represent 95% confidence intervals using the asymptotic critical values. The rolling regression coefficient on Δ**sent12** is rarely significant while the rolling regression coefficient on Δ**ersf** is never significant. The rolling regression coefficient on Δ**sent12**$\times\Delta$**ersf** exhibits a mostly negative sign that is often significant. The rolling regression coefficient on **noise** is negative and significant for sample periods that include the Great Recession than runs from 2007.M12 to 2009.M6.

Figure 5: *Optimism or Pessimsim About Stocks is Strongly Linked to Recent Price Movements*



Notes: The predictor variable **pd** is plotted together with a gauge of investors' expectations about future stock returns from a University of Michigan survey. The survey records investors' perceived probability of an increase in stock prices over the next year. In predictive regressions, a higher value of **pd** forecasts lower excess stock returns. But the survey respondents fail to take this empirical relationship into account. Instead, the survey respondents assign a higher probability of a price increase (implying higher expected excess returns) when **pd** is higher. The pattern in Figure 5 is suggestive of extrapolative expectations rather than rationally time-varying risk premiums.