Turbulent Business Cycles

Ding Dong
Hong Kong University of Science and Technology

Zheng Liu
Federal Reserve Bank of San Francisco

Pengfei Wang
HSBC School of Business
Peking University

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ABSTRACT. Firm-level evidence suggests that turbulence that reshuffles firms’ productivity rankings rises sharply in recessions. An increase in turbulence reallocates labor and capital from high- to low-productivity firms, reducing aggregate TFP and the stock market value of firms. A real business cycle model with heterogeneous firms and financial frictions can generate the observed macroeconomic and reallocation effects of turbulence. In the model, increased turbulence makes high-productivity firms less likely to remain productive, reducing their expected equity values and tightening their borrowing constraints relative to low-productivity firms. This leads to a reallocation that reduces aggregate TFP. Unlike uncertainty, turbulence changes both the conditional mean and the conditional variance of the firm productivity distribution, enabling a turbulence shock to generate a recession with synchronized declines in aggregate activities.

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Dong: Hong Kong University of Science and Technology. Email: ding.dong@connect.ust.hk. Liu: Federal Reserve Bank of San Francisco. Email: Zheng.Liu@sf.frb.org. Wang: Peking University HSBC Business School and PHBS Sargent Institute of Quantitative Economics and Finance. Email: pfwang@phbs.pku.edu.cn. We are grateful to Nick Bloom, Marc Dordal i Carreras, Larry Christiano, Steve Davis, Zhigang Ge, Bart Hobijn, Oscar Jorda, Matthias Kaldorf, Nobu Kiyotaki, Sylvain Leduc, Byoungchan Lee, Huiyu Li, Guido Lorenzoni, Erik Loualiche (discussant), Guangyu Pei, Franck Portier, Vincezo Quadrini, Petr Sedlacek (discussant), Mark Spiegel, Bo Sun, Stephen Terry, Mauricio Ulate, Dan Wilson, Yicheng Wang, Zebang Xu(discussant), Zhiwei Xu, Shengxing Zhang, Fuyang Zhao(discussant), Shangyao Zhou(discussant) and seminar participants at the FRBSF, PHBS, HKUST, ZJU AFR, Deutsche Bundesbank, 6th PKU-NUS Joint Conference, AMES 2022, 2022 CCER Summer Institute, SED 2022, SITE 2022, CFRC 2023, ESEM 2023, and CICF 2023 for helpful comments, to Yuan Wang for excellent research assistance, and to Anita Todd for editorial assistance. Pengfei Wang acknowledges financial support from National Science Foundation (project No.72150003, No.72125007). The views expressed herein are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of San Francisco or of the Federal Reserve System.
I. Introduction

Recessions are characterized by declines in aggregate economic activity. They are also characterized by a sharp rise in micro-level turbulence with increased churn in firm productivity rankings. This paper studies the macroeconomic and reallocation effects of turbulence shocks over business cycles.

We develop an empirical measure of turbulence using data from publicly traded U.S. firms listed in Compustat. We first construct a measure of firm-level total factor productivity (TFP) following the approach in the literature (Syverson, 2004; Foster et al., 2008; Bloom et al., 2018). We then sort the measured firm-level TFP in each year and estimate the Spearman rank correlations (denoted by $\rho_t$) between adjacent years. A low Spearman correlation indicates more churning of firm rankings in the productivity distribution: a high-productivity firm this year is less likely to maintain its productivity ranking next year; whereas a low-productivity firm now might become more productive in the future. Turbulence is inversely related to the Spearman correlation of firm-level TFP, and we measure it by $1 - \rho_t$.

Our measure of turbulence is countercyclical, rising sharply in recessions. An increase in turbulence reallocates labor and capital from high- to low-productivity firms, with the magnitude of the reallocation effects depending partly on financial frictions. Reflecting its reallocation effects, turbulence is negatively correlated with aggregate manufacturing TFP and the aggregate stock market value of firms. Turbulence is also associated with synchronized and persistent declines in real GDP, consumption, investment, and employment.

To understand the economic mechanism through which turbulence can drive the observed macroeconomic fluctuations and cross-sectional reallocation, we construct a real business cycle (RBC) model with heterogeneous firms and financial frictions. In the model, firms produce a homogeneous good using capital and labor, subject to idiosyncratic productivity shocks. Firms rely on external financing of working capital, with the borrowing capacity constrained by a fraction of the expected future equity value (Jermann and Quadrini, 2012; Liu and Wang, 2014; Lian and Ma, 2021). Firms also face idiosyncratic production distortions, reflecting differential policy interventions or government subsidies at the firm level (Hsieh and Klenow, 2009; Buera and Shin, 2013; Moll, 2014). At each given level of productivity, firms with sufficiently high levels of subsidies choose to operate, facing binding credit constraints while those with low levels of subsidies remain inactive. Given productivity, there is an endogenously determined threshold level of subsidy, at which a firm is indifferent between producing and staying inactive.

Under the stochastic process of the idiosyncratic productivity shocks, a firm can maintain its productivity from the current period to the next period with a time-varying probability $\rho_t$. With the complementary probability $1 - \rho_t$, the firm’s productivity will be an independent
and identically distributed (i.i.d.) random variable. A lower value of $\rho_t$ implies more frequent switching in firm productivity rankings between adjacent periods or, equivalently, greater turbulence.

The model predicts that a shock that increases turbulence leads to a recession. With greater turbulence, a high-productivity firm today would be less likely to remain productive in the future and a low-productivity firm today would be more likely to get a better productivity draw in the future. Thus, the expected equity value of a high-productivity firm falls relative to that of a low-productivity firm. Since firms’ borrowing capacity depends on the expected equity value, turbulence disproportionately tightens the current-period credit constraints for high-productivity firms and reallocates labor and capital from high- to low-productivity firms. This reallocation reduces aggregate TFP. The endogenous decline in TFP is quantitively important, enabling the model to generate a recession with synchronized declines in aggregate output, consumption, investment, and labor hours. These model predictions are in line with empirical evidence.

Financial frictions are crucial for amplifying the macroeconomic effects of turbulence shocks. Since labor and capital are perfectly mobile across firms, competition for input factors from high-productivity firms bids up wages and capital rents. Absent credit constraints and production subsidies, resources would be concentrated in the most productive firms, and the equilibrium allocation would be efficient. Credit constraints and idiosyncratic production distortions restrict the borrowing capacity of high-productivity firms, allowing some low-productivity firms to stay active in production. Such financial frictions lead to steady-state misallocation and they also create room for between-firm reallocation following a turbulence shock. Such reallocation leads to procyclical TFP, enabling the model to generate business cycle comovements.

The presence of financial frictions implies that competitive equilibrium allocations are inefficient. Appropriate policy interventions can potentially mitigate credit constraints and improve allocative efficiency. Since financial frictions are the key transmission channel for turbulence, policy interventions that alleviate credit constraints might mitigate its recessionary effect.

We use our model framework to evaluate the effectiveness of two alternative policy interventions for stabilizing turbulence-driven recessions. The first policy is a borrowing subsidy that reduces the effective costs of hiring capital and labor, therefore reducing the amount of working capital that firms need to finance. The second policy is credit easing, under which the government injects liquidity to enhance the borrowing capacity of active firms. Each policy is transitory and unexpected, and it is triggered by the realization of a turbulence shock, with the same persistence as that of the shock.
Under our calibration, both types of policies are effective for mitigating the recessionary effects of turbulence relative to the \textit{laissez-faire} benchmark economy with no policy interventions. However, the policies operate through different channels and therefore have different implications for reallocation.

Borrowing subsidies reduce the effective costs of hiring input factors for all firms, expanding the set of active firms at each level of productivity and boosting aggregate output. However, by enabling a larger fraction of low-productivity firms to stay active, the policy exacerbates misallocation, reducing aggregate TFP relative to the benchmark. The decline in TFP partly offsets the stimulus effects on aggregate output.

Credit easing expands the borrowing capacity for active firms. Competition for input factors from high-productivity firms pushes up equilibrium wages and capital rents, forcing some low-productivity firms to stay inactive. This reallocation improves aggregate TFP, contributing to increased output.\footnote{The two types of policy interventions—borrowing subsidies and credit easing—do not necessarily improve welfare relative to the benchmark, because they both incur a deadweight loss. We use these counterfactual policies to highlight the transmission mechanism of turbulence shocks. We do not study optimal policy here because welfare depends on the calibration of the sizes of the deadweight losses (see also Gertler and Karadi (2011)).}

\section*{II. Related literature}

Our work is closely related to the important contribution of Bloom et al. (2018), who study the macroeconomic implications of micro-level uncertainty. They show that, in a real business cycle model with capital and labor adjustment costs, an increase in micro-level uncertainty (i.e., an increase in the standard deviation of the firm-level TFP shocks) reduces net aggregate investment, net hiring, and aggregate output. However, in their model, aggregate consumption rises following an uncertainty shock. To generate a recession with aggregate comovements requires a simultaneous negative shock to the level of aggregate TFP.

Unlike uncertainty, which is a mean-preserving spread of the productivity distribution, turbulence changes not just the conditional variance but also the conditional mean of firm-level productivity. Following an increase in turbulence, firms with high productivity in the current period may not be as productive in the future. Thus, this turbulence-induced changes in conditional expectations of future firm productivity, together with credit constraints, lead to reallocation from high- to low-productivity firms, reducing aggregate TFP. The endogenous decline in TFP in turn leads to a recession with aggregate comovements in our model, without relying on simultaneous shocks to the level of TFP.
Our model illustrates the importance of financial frictions for propagating turbulence shocks. Existing studies show that financial frictions are also important for the transmission of uncertainty shocks (Gilchrist et al., 2014; Christiano et al., 2014; Alfaro et al., 2018; Arellano et al., 2019). In the model of Arellano et al. (2019), for example, hiring is risky because firms need to finance input costs before they receive revenues, and firms face idiosyncratic productivity shocks between paying for inputs and receiving revenues. An increase in the volatility of idiosyncratic shocks raises default risks, and firms respond by pulling back hiring and reducing production. Since firms are ex ante identical, they make identical hiring decisions. Thus, an increase in uncertainty in their model (i.e., firm-level volatility) does not lead to reallocation of capital and labor inputs. In our model, however, reallocation is the central mechanism for propagating turbulence shocks.

Our work is related to the economic development literature on capital misallocation under financial frictions (Midrigan and Xu, 2014; Moll, 2014; Buera and Shin, 2013; Gopinath et al., 2017; Liu et al., 2021). Indeed, our measure of turbulence is analogous to the persistence of idiosyncratic productivity in the continuous-time model of Moll (2014). In his model, more persistent idiosyncratic productivity shocks create an incentive for firms to save more in order to mitigate the impact of potentially binding credit constraints, resulting in relatively smaller steady-state productivity losses but also slower transitions to the steady-state. Other things being equal, the less persistent the idiosyncratic productivity shocks are, the greater the impact of financial frictions on aggregate productivity (Buera and Shin, 2013). We focus on the business cycle implications of turbulence. Thus, our work complements this development literature.

The countercyclical behavior of turbulence that we find is consistent with other empirical studies based on different data and measurements. For example, Aghion et al. (2021) construct a measure of turbulence based on the rate of new product additions and subtractions (i.e., product churn) using US Census of Manufactures data. They find that product churn

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3There is a large strand of literature on the macroeconomic effects of uncertainty shocks. Examples include Bloom (2009), Bachmann et al. (2013), Fernández-Villaverde et al. (2015), Jurado et al. (2015), Baker et al. (2016), Leduc and Liu (2016), Basu and Bundick (2017), Bansal et al. (2019), Berger et al. (2020), and many others. For recent surveys of the uncertainty literature, see Bloom (2014) and Fernández-Villaverde and Guerrón-Quintana (2020).
rises sharply during recessions. Bernard and Okubo (2016) and Dekle et al. (2021) also report evidence of countercyclical product churn based on Japanese manufacturing data. We add to this empirical literature by documenting the macroeconomic and reallocation effects of turbulence.\footnote{Similar reallocation effects can arise from labor market churns (Pratap and Quintin, 2011) or supply-chain disruptions (Meier, 2020).}

To our knowledge, our paper represents a first attempt to study the transmission mechanism of turbulence shocks over the business cycle using firm-level data and a quantitative business cycle model featuring firm heterogeneity and financial frictions.

III. EMPIRICAL METHODOLOGY

This section describes our empirical methods of measuring turbulence and the macroeconomic and reallocation effects of turbulence.

III.1. Defining turbulence. Consider the production function for firm \( j \) in period \( t \)

\[
Y_{jt} = z_{jt} F(K_{jt}, N_{jt}),
\]

where \( Y_{jt} \) denotes value-added output, \( K_{jt} \) and \( N_{jt} \) denote capital and labor inputs, respectively, and \( F(K, N) \) is the production function. The persistent idiosyncratic productivity, \( z_{jt} \), follows the stochastic process

\[
z_{j,t+1} = \begin{cases} 
  z_{jt} \text{ with prob } \rho_t, \\
  \bar{z} \text{ with prob } 1 - \rho_t,
\end{cases}
\]

where \( \bar{z} \) is an i.i.d. random variable with the cumulative density function \( \bar{G}(z) \).

Under the stochastic process in Eq. (2), a firm’s productivity level in period \( t + 1 \) can stay the same as that in period \( t \), in which case the ranking of firm productivity also stays the same. This occurs with the probability \( \rho_t \). With the complementary probability \( 1 - \rho_t \), the firm’s productivity in period \( t + 1 \) is an i.i.d. random variable \( \bar{z} \), which is independent of the period-\( t \) productivity, such that firm productivity in \( t + 1 \) would be uncorrelated with that in \( t \).

The term \( \rho_t \) measures the persistence of firm-level TFP. In the extreme case with \( \rho_t = 1 \) for all \( t \), a firm’s productivity level would be permanent: high-productivity firms would remain productive and low-productivity firms would remain unproductive. In the other extreme with \( \rho_t = 0 \), firm productivity would be an i.i.d. process, with no persistence. In the more general case with \( \rho_t \in (0, 1) \), firm productivity is persistent, and the persistence is time varying. A decline in \( \rho_t \) implies that high-productivity firms in period \( t \) would be less likely to remain productive in period \( t + 1 \) and low-productivity firms in period \( t \) would have a
chance to become more productive. Thus a decline in \( \rho_t \) increases the churn of firm rankings in the productivity distribution. We measure micro-level turbulence by \( 1 - \rho_t \).

Turbulence is related to but different from the micro-level uncertainty studied by Bloom et al. (2018). An increase in micro-level uncertainty corresponds to a mean-preserving spread of the cross-sectional productivity distribution—an increase in the variance or inter-quartile range (IQR) of productivity. An increase in turbulence also raises the conditional variance of the productivity distribution, as does uncertainty. Thus, turbulence is positively correlated with micro-level uncertainty.\(^5\)

However, unlike uncertainty, turbulence changes not only the conditional variance but also the conditional mean of the firm-level productivity distribution. Through its impact on the conditional mean of firm productivity, a turbulence shock generates between-firm reallocation, which is essential for generating procyclical aggregate productivity and business cycle comovements.

Furthermore, turbulence does not affect the \textit{ex ante} stationary distribution of productivity. That is, a turbulence shock is an \textit{ex ante} distribution-preserving shock, as shown in Proposition 1 below.\(^6\)

\textit{Proposition 1.} The cross-sectional stationary distribution of idiosyncratic productivity (denoted by \( G_t(z) \)) is invariant to the realization of \( \rho_t \).

\textit{Proof.} Under the stochastic process of idiosyncratic productivity specified in Eq. (2), the cumulative density function of productivity is given by

\[
G_{t+1}(z) = Pr(z_{t+1} \leq z) = Pr(z_t \leq z)\rho_t + Pr(\tilde{z} \leq z)(1 - \rho_t) = G_t(z)\rho_t + \tilde{G}(z)(1 - \rho_t).
\]

Under the stationarity of the distribution of \( z \), we have \( G_t(z) = \tilde{G}(z) \) for all \( t \). Thus, the stationary distribution is independent of the realization of \( \rho_t \). \( \square \)

\textbf{III.2. Measuring turbulence.} If idiosyncratic productivity is perfectly measured, then the Spearman rank correlation of firm productivity between adjacent periods would provide a correct measure of \( \rho_t \) and thus of turbulence (i.e., \( 1 - \rho_t \)).\(^7\)

\(^5\)For example, the correlation between our turbulence measure and the IQR of firm-level TFP from Compustat data is about 0.55.

\(^6\)The distribution-preserving turbulence that we study here can be viewed a discrete-time counterpart to the persistence of idiosyncratic productivity shocks in the continuous-time models of Moll (2014), which is also orthogonal to the stationary productivity distribution.

\(^7\)We formally show this in Proposition 3 in appendix A.1.
III.2.1. Measurement challenges. In general, however, firm-level productivity can be measured with errors. It is well-known in the productivity literature that revenue-based measures of firm-level TFP contain not only true productivity but also information about demand conditions (Syverson, 2004). Measurement errors in productivity can pose challenges for measuring turbulence.

To put the measurement challenges into context, consider the case with i.i.d. measurement errors in observed firm TFP, which is given by

\[ a_{j,t} = z_{j,t} + \tau_{j,t} \],

where \( a_{j,t} \) denotes observed TFP of firm \( j \) at time \( t \), \( z_{j,t} \) denotes true productivity that follows the process in Eq. (2), and \( \tau_{j,t} \) denotes a measurement error that is uncorrelated with \( z_{j,t} \) and is i.i.d. across firms and across time, drawn from the normal distribution \( N(0, \sigma_t^2) \). We allow the variance of the measurement error (denoted by \( \sigma_t \)) to be time-varying.

Given the stochastic process of true productivity in Eq. (2), the measured TFP follows the process

\[ a_{j,t+1} = \begin{cases} 
  z_{j,t} + \tau_{j,t+1} & \text{with prob } \rho_t, \\
  \tilde{z} + \tau_{j,t+1} & \text{with prob } 1 - \rho_t,
\end{cases} \]

\[ = \begin{cases} 
  a_{j,t} + \tau_{j,t+1} - \tau_j \equiv e_{j,t+1} & \text{with prob } \rho_t, \\
  \tilde{z} + \tau_{j,t+1} & \text{with prob } 1 - \rho_t.
\end{cases} \]

The presence of measurement errors in productivity gives rise to two challenges in estimating the true process of turbulence. The first challenge is heteroskedasticity. With a time-varying volatility of \( \tau_{j,t} \), the variance of the residual term \( e_{j,t+1} \) in Eq. (5) is also time-varying. Thus, the OLS estimator of the auto-correlation in \( a_{j,t} \) can be biased and inconsistent. We tackle this heteroskedasticity issue by estimating the rank correlations of the observed firm productivity between adjacent periods, because the rank distribution is time-invariant regardless of the functional forms of the underlying distribution of the observed productivity.

The second challenge is the standard endogeneity problem in a dynamic panel model. In the empirical specification (5), the residual term \( e_{j,t+1} \) is correlated with the independent variable \( a_{j,t} \) because both are functions of the measurement error \( \tau_{j,t} \). In the spirit of Arellano and Bond (1991), we address the endogeneity problem by an instrumental-variable (IV) estimation approach, using the rankings of the lagged productivity \( a_{j,t-1} \) and \( a_{j,t-2} \) as instruments for the ranking of \( a_{j,t} \). Since \( corr(a_{j,t-1}, e_{j,t+1}) = 0 \) and \( corr(a_{j,t}, a_{j,t-1}) > 0 \), these IVs satisfy both the exclusion restriction and the relevance condition.
III.2.2. The data. To implement the IV approach to measuring turbulence, we use firm-level data from Compustat Fundamentals Annual database. To obtain measures of industry-level employment, payroll, and price indices, we use information from the NBER-CES Manufacturing Industry Database.\footnote{The Compustat database is accessed through Wharton Research Data Service at: https://wrds-web.wharton.upenn.edu/wrds/. The NBER-CES database is accessed through: https://www.nber.org/research/data/nber-ces-manufacturing-industry-database.} By combining these two data sources, we obtain an unbalanced panel with 48,197 firm-year observations. This full sample (Sample 1) includes all listed firms in all manufacturing industries covered by NBER-CES in the years from 1958 to 2016.\footnote{We include firms incorporated in the US (Compustat fic='USA') that trade on major stock exchanges (NYSE, AMEX, and NASDAQ, Compustat exchg = 11, 12 or 14), for which the native currency is US dollars (Compustat curcd='USD'). We exclude firm-year observations with obvious errors: missing or nonpositive values in reported revenue, employment, and capital. We remove a firm if it was involved in a major merger or acquisition that affected its asset by more than 10 percent.} Table 1 presents the summary statistics of our samples.

Following Bloom et al. (2018), we focus on the subset of firms with 25+ years of observations in our sample. We use this pseudo-balanced panel as our baseline sample (Sample 2).
for estimating firm-level TFP. The baseline sample contains about 25,790 firm-year observations. Since firms in the baseline sample are older than those in the full sample, they are also larger on average in terms of assets, value added, capital, and employment, although their average growth rates of employment and capital are slower.\footnote{In a robustness check, we further narrow down the sample and focus on industries with more than 20 firms in each year. This sample (Sample 3) contains about 19,000 firm-year observations. Firms in Sample 3 have similar characteristics as those in Sample 2.}

III.2.3. \textit{Measured turbulence}. We measure firm-level TFP based on Solow residuals calculated from the constant-returns production function

$$tfp_{ijt} = y_{ijt} - \alpha_t k_{ijt} - (1 - \alpha_t)n_{ijt},$$

where $tfp_{ijt}$ denotes the TFP (in log units) of firm $j$ in industry $i$ and year $t$, and $y_{ijt}$, $k_{ijt}$ and $n_{ijt}$ denote the firm’s value added, capital input, and labor input, respectively, all in log units.\footnote{The cyclical properties and the macroeconomic and reallocation effects of turbulence are similar when we construct a measure of turbulence based on a production function with decreasing returns to scale (see Appendix B).}

Following Bloom et al. (2018), we assume that the cost share of capital input $\alpha_{it}$ is common for all firms within an industry $i$, although it can vary across time.\footnote{In our sample, the average value of the cost share of capital (weighted by the value of shipment) is about $0.34$. We provide some details of our approach to measuring value added, capital and labor inputs, and the capital share in Appendix A.}

After obtaining firm-level TFP, we construct a measure of turbulence using an IV estimation approach. Specifically, we rank firms within each industry (at the 3-digit level) by deciles of their productivity levels. We then estimate the rank correlation of firm TFP between year $t+1$ and $t$, using the rankings in years $t-1$ and $t-2$ as instrument variables for the ranking in year $t$. The time series of the estimated Spearman correlations corresponds to our measure of $\rho_t$.

Turbulence is measured by $1 - \rho_t$. Intuitively, a decline in $\rho_t$ implies that a high-productivity firm in year $t$ would be less likely to remain as productive in $t+1$, whereas a low-productivity firm in year $t$ might get a high productivity draw in year $t+1$. A reduction in $\rho_t$ reshuffles productivity rankings across firms between $t$ and $t+1$, increasing turbulence.\footnote{Bloom et al. (2018) estimate the Spearman correlations of plant rankings in the TFP distribution across adjacent years using Census Manufacturing (CM) data, although they do not directly confront the measurement challenges that we highlight here. Our IV estimate of $\rho_t$ using Compustat data is positively correlated with their OLS estimate using CM data (shown in Figure A1 of their online appendix at \url{https://nbloom.people.stanford.edu/sites/g/files/sbiybj4746/f/rubc_appendix_0.pdf}), with a correlation coefficient of 0.49.}
Figure 1. Decomposing churning in firm’s TFP ranking

This figure plots the times series of the estimated $1 - \rho_t$ using the OLS and the IV approaches. The black line shows the OLS estimates. The blue bars show the IV estimates, representing the true levels and variations in turbulence. The red bars show the OLS bias.

Our estimated turbulence series based on the IV approach is quite different from the OLS estimation, which can be biased because of the heteroskedasticity and endogeneity issues discussed earlier. The OLS bias is substantial, as shown in Figure 1. The black line shows the turbulence series estimated using the OLS approach. The blue bars show the turbulence series estimated using the IV approach. The red bars indicate the OLS bias. When we correct for the OLS bias, the average level of the estimated turbulence in our sample is about halved. In other words, OLS estimation would over-state the level of true turbulence. The bias is notably smaller than average during recessions, especially during the 2008-2009 global financial crisis.

III.3. Cyclical properties of turbulence. Figure 2 plots the time series of firm-level turbulence from 1965 to 2015 (corresponding to the blue bars in Figure 1). The mean, standard deviation, and autocorrelation of the estimated turbulence ($1 - \rho_t$) are 0.049, 0.021, and 0.69, respectively. The figure shows that turbulence is countercyclical, rising sharply in recessions.\footnote{In Appendix A, we show that the baseline estimate of $\rho_t$ is robust to alternative samples and alternative approaches to measuring TFP. Our measured turbulence displays an upward trend. Since an increase in turbulence raises firm-level volatility, the trend increase in turbulence is consistent with the empirical evidence}
Our measure of turbulence is negatively correlated with manufacturing TFP, as shown in panel A of figure 3. A rise in turbulence (blue line) is typically associated with a decline in TFP relative to trend (red line), and the correlation between the two series is about -0.28.

Turbulence is also negatively correlated with the stock market value of firms, as shown in panel B of figure 3. The market value of assets is calculated based on firms’ stock prices at the end of the fiscal year, multiplied by the shares outstanding and deflated by the consumer price index. The correlation between turbulence with the asset value is negative, at -0.30.\textsuperscript{15}

III.4. Reallocation effects of turbulence. To examine the reallocation effects of turbulence, we estimate the empirical specification

\[ x_{jt} = \beta_0 + \beta_1 \text{High}_T \text{TFP}_{jt} + \beta_2 \text{Turb}_t \ast \text{High}_T \text{TFP}_{jt} + \mu_j + \eta_t + \epsilon_{jt}, \]  

that the volatility for publicly traded firms has increased steadily over time, whereas the volatility of privately held firms has declined (Davis et al., 2006).

\textsuperscript{15}In Compustat, the firm stock price at the end of the fiscal year is the variable “PRCC\_F,” the shares outstanding is “CSHO,” the firm asset is “AT,” and the book equity is “CEQ.”
Figure 3. Correlation of turbulence with manufacturing TFP and stock market value

Note: The series of manufacturing TFP and firm value are computed as an average of firm-level TFP and stock market value in our benchmark sample. The series are detrended using the HP filter, with a smoothing parameter of 6.25. The gray shaded bars indicate NBER recession dates.

where the dependent variable $x_{jt}$ denotes the growth rate of employment, capital, sales, or market value of firm $j$ in year $t$ from $t - 1$, $Turb_t$ denotes measured turbulence, and $High_{TFP_{j}}$ is a dummy variable that equals one if firm $j$’s TFP level is above the median within its industry and zero otherwise. To mitigate potential biases associated with firm entries and exits, we focus on a pseudo balanced panel, with firms appearing at least 25 years in our sample from 1958 to 2016 (Sample 2), following Bloom et al. (2018). In estimating (7), we control for firm fixed effects ($\mu_j$) and year fixed effects ($\eta_t$). The term $\epsilon_{jt}$ denotes regression errors.

The relative sensitivity of the firm-level activity of high-productivity firms to turbulence is captured by the coefficient on the interaction term (i.e., $\beta_2$). The parameter $\beta_1$ measures the average effects of productivity on firm growth absent turbulence shocks. The parameter $\beta_0$ is a constant intercept. The estimation results are displayed in Table 2.

The table shows that the estimated values of $\beta_2$ are negative and statistically significant at the 99-percent confidence level for all measures of firm growth. Thus, an increase in
Table 2. Impact of turbulence on firms with different levels of productivity

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>$\Delta n_{jt}$</th>
<th>$\Delta k_{jt}$</th>
<th>$\Delta y_{jt}$</th>
<th>$\Delta v_{jt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>High_TFP$_{jt}$</td>
<td>0.006</td>
<td>0.018**</td>
<td>0.094***</td>
<td>0.053***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
<td>(0.006)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Turb$<em>t$ * High_TFP$</em>{jt}$</td>
<td>-0.945***</td>
<td>-0.969***</td>
<td>-1.220***</td>
<td>-0.940***</td>
</tr>
<tr>
<td></td>
<td>(0.095)</td>
<td>(0.144)</td>
<td>(0.086)</td>
<td>(0.139)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.052***</td>
<td>0.060***</td>
<td>0.034***</td>
<td>0.032***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

Firm Fixed Effect: Yes Yes Yes Yes
Year Fixed Effect: Yes Yes Yes Yes
Observations: 24,501 24,501 24,501 21,687

Note: This table shows the estimation results from the empirical specification that regresses firm-level variables (including the growth rates of employment, capital expenditure, sales, and firm value) on the measured turbulence (Turb) for firms with different levels of TFP. The dummy High_TFP$_{jt}$ equals one if firm $j$’s TFP is above the median within its industry and zero otherwise. All regressions use the pseudo panel of Compustat firms that appear for at least 25 years from 1965 to 2015. The standard errors shown in the parentheses are clustered by industry. The stars denote the p-values: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

Turbulence is associated with declines in the firm-level growth rates for high-productivity firms relative to those for low-productivity firms. The point estimates imply that a one-standard-deviation increase in turbulence reduces the relative growth rates of employment and capital for high-productivity firms by about 6 percent and 2.8 percent, respectively. It also reduces the relative growth rates of sales and the market value for high-productivity firms by about 1.7 and 1.6 percent, respectively.\(^{16}\) Thus, turbulence has important impact

\(^{16}\)The standard deviation of our measured turbulence is 2.1 percent. The impact of a one-standard-deviation increase in turbulence (from its mean level of 0.049) on the relative employment growth rate for high-productivity firms is thus $(-0.945) \times 0.049 \times 2.1 \approx -0.097$ percentage points, or about 6 percent drop from the average growth rate of 1.62% for high-productivity firms. In our sample, the average growth rates of capital, sales, and market value of the high-productivity firms are 2.8%, 1.7%, and 1.6%, respectively. Thus, the impact of a one standard deviation shock to turbulence on the relative growth rates of capital,
on firm-level activity, with greater adverse effects on high-productivity firms than on low-productivity firms. These findings suggest that turbulence reallocates capital and labor from high- to low-productivity firms, contributing to reducing aggregate TFP.

The reallocation effects of turbulence are robust to alternative high-productivity indicators (lagged high-TFP indicators or finer grouping of TFP rankings), alternative samples (with large industries or excluding top firms), and adding controls for potential reallocation effects of business cycle recessions, as we show in the Appendix (see Appendix A.3). In the Appendix, we also estimate the dynamic reallocation effects of turbulence using the local projections approach of Jordà (2005). The reallocation effects are persistent, lasting more than 5 years after the impact.

III.5. Financing constraints and the reallocation effects of turbulence. To examine the extent to which the reallocation effects of turbulence might depend on financial frictions, we estimate the empirical specification

\[ x_{it} = \beta_0 + \beta_1 \text{High_FF}_{it} + \beta_2 \text{Turb}_t \times \text{High_FF}_{it} + \mu_i + \eta_t + \epsilon_{it}, \]  

(8)

where the dependent variable \(x_{it}\) denotes interquartile range (IQR) of labor (or capital) of firms in industry \(i\) in year \(t\), \(\text{Turb}_t\) denotes measured turbulence, and \(\text{High_FF}_{it}\) is a dummy variable that equals one if industry \(i\)’s financial constraint is above the median level among all NAICS 4-digit industries in year \(t\). We measure a firm’s financial constraint using the Kaplan-Zingales (KZ) index following Kaplan and Zingales (1997) and Lamont et al. (2001).\(^{17}\) We obtain an industry-level measure of financial constraint by taking the within-industry sales-weighted average of the firm-level KZ indices. To mitigate potential biases associated with firm entries and exits, we focus on the sample with firms appearing 25+ years from 1958 to 2016 (Sample 2). We estimate the empirical specification (8) using the sample with firms appearing 25+ years (Sample 2), controlling for industry fixed effects (\(\mu_i\)) and year fixed effects (\(\eta_t\)). The term \(\epsilon_{it}\) denotes regression errors.

Changes in the IQR of employment (or capital) capture reallocations within an industry. For example, a decline in the IQR of employment following an increase in turbulence would indicate reallocation of labor from firms with high levels of employment to those with low levels of employment. We are interested in how financial frictions could affect the reallocation effects of turbulence. This effect is captured by the parameter \(\beta_2\) in Eq. (8), which measures sales, and market values are \((-0.969) \times 0.049 \times 2.1/3.6 \approx -2.8\%\), \((-1.220) \times 0.049 \times 2.1/7.3 \approx -1.7\%\), and \((-0.940) \times 0.049 \times 2.1/6.1 \approx -1.6\%\), respectively.

\(^{17}\)We follow Lamont et al. (2001) to construct an index of financial constraint according to Kaplan and Zingales (1997) classification to five readily available accounting variables: cash flow, market-to-book, leverage, dividends, and cash holdings. The coefficients associated with flow variables are adjusted to annual frequency. A higher index value suggests a firm is more constrained.
Table 3. Reallocation effect of turbulence: sensitivity to financial frictions

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>IQR of Employment (1)</th>
<th>(2)</th>
<th>IQR of Capital (3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High_FF_{it}</td>
<td>0.232***</td>
<td>0.271***</td>
<td>0.335***</td>
<td>0.385***</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.085)</td>
<td>(0.097)</td>
<td>(0.098)</td>
</tr>
<tr>
<td>Turb_{t} * High_FF_{it}</td>
<td>-4.791***</td>
<td>-5.448***</td>
<td>-6.866***</td>
<td>-7.741***</td>
</tr>
<tr>
<td></td>
<td>(1.548)</td>
<td>(1.566)</td>
<td>(1.792)</td>
<td>(1.810)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.869***</td>
<td>1.895***</td>
<td>2.090***</td>
<td>2.122***</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.025)</td>
<td>(0.028)</td>
<td>(0.028)</td>
</tr>
</tbody>
</table>

industry Fixed Effect | Yes | Yes | Yes | Yes |
Year Fixed Effect | Yes | Yes | Yes | Yes |
Observations | 3,647 | 3,552 | 3,647 | 3,552 |

Note: This table shows the regression of interquartile range of employment (or capital) on the measured turbulence (Turb) for industries with different levels of external financing dependence. In the baseline specification (Columns (1) and (3)), the dummy High_FF_{it} equals one if industry i's external financing dependence is above the median. In the alternative specification (Columns (2) and (4)), we use lagged indicator of external financing dependence (i.e., High_FF_{i,t-1} instead of High_FF_{it}). All regressions use the pseudo panel of Compustat firms that appear for at least 25 years in the sample from 1958 to 2015. The standard errors shown in the parentheses are clustered by industries. The stars denote the p-values: * p < 0.1; ** p < 0.05; *** p < 0.01.

the relative sensitivity of industry-level employment IQR (or capital IQR) to changes in turbulence for industries with high levels of financial friction. The parameter β₁ measures the average effect of financial constraint on the IQR of employment (or capital). The estimation results are displayed in Table 3.

In the baseline specifications with a contemporaneous indicator of financial constraint (Columns (1) and (3)), the estimated values of β₂ are negative and statistically significant at the 99 percent confidence level. Thus, an increase in turbulence is associated with greater declines the IQRs of both employment and capital in industries facing tighter financial constraints. The positive estimates of β₁ indicate that, absent turbulence, an industry with
tighter financial constraints has also greater within-industry dispersion of employment and capital.

The results are robust when we use a lagged indicator of financial constraints (i.e., $High_{FF_{i,t-1}}$ instead of $High_{FF_{i,t}}$), as reported in Columns (2) and (4) of the table. Overall, our estimation suggests that tighter financial constraints are associated with larger declines in the cross-sectional dispersion of employment and capital when turbulence rises.\footnote{In the Appendix, we show that our results are also robust to finer grouping of industries based on their financial constraint (see Table A.5).}

III.6. Macroeconomic effects of turbulence. We now examine the macroeconomic effects of turbulence. For this purpose, we estimate the impulse responses of several key

---

**Figure 4.** Estimated impulse response of macroeconomic variables to a turbulence shock

*Note:* This figure shows the impulse responses of macroeconomic variables to a one-standard-deviation (34.57\%) increase in the log-level of turbulence estimated from the local projections model (9). The solid lines show the point estimates of the impulse responses. The blue dashed lines show the 68\% confidence intervals.

*Source:* BEA, Compustat, NBER-CES, and authors’ calculations.
macroeconomic variables to a turbulence shock using the local projections approach of Jordà (2005).\footnote{As shown by Plagborg-Møller and Wolf (2021), linear local projections and vector autoregressions (VARs) estimate the same impulse responses when the lag structures are unrestricted, without imposing any parametric assumptions on the data generating process.}

We consider the empirical specification

\[ x_{t+h} - x_{t-1} = \beta_0^h + \beta_1^h \text{turb}_t + \beta_2^h \text{turb}_{t-1} + \beta_3^h (x_{t-1} - x_{t-2}) + \epsilon_{t+h} \quad h = 0, 1, 2, 3, 4. \tag{9} \]

The dependent variable \( x_{t+h} - x_{t-1} \) denotes the cumulative changes in the log-level of the variable of interest from year \( t - 1 \) to year \( t + h \), where \( h \) denotes the projection horizons (number of years). The list of dependent variables includes the macroeconomic times series of per capita real consumption, investment, private output (i.e., the sum of consumption and investment), and hours worked, and also the firm value and the manufacture TFP constructed from the firm-level and industry-level data. The independent variable \( \text{turb}_t \equiv \log(1 - \rho_t) \) denotes the log-level of turbulence in year \( t \). In estimating the local projections, we control for lagged turbulence \( \text{turb}_{t-1} \) and the lagged growth rate of the dependent variable \( x_{t-1} - x_{t-2} \). The term \( \epsilon_{t+h} \) is the regression residual. The parameter \( \beta_1^h \) measures the impulse responses of the macroeconomic variables to a turbulence shock at horizon \( h \).

Figure 4 plots the estimated impulse responses of the macroeconomic variables to a one-standard-deviation turbulence shock (i.e., an increase in the log-level of turbulence of 0.3457) for horizons up to five years.\footnote{Our measured annual series of (logged) turbulence has a first-order autocorrelation of 0.648 and a standard deviation of 0.4541, implying a standard deviation of the innovation of 0.3457.} The shock leads to a recession with synchronized and persistent declines in aggregate output, consumption, investment, hours worked, and firm value. It also leads to a decline in manufacturing TFP. These macroeconomic effects of turbulence are quantitatively important. For example, a one-standard-deviation increase in turbulence reduces per capita output by about 0.5 percent on impact, and by more than one percent within three years after the shock.

### IV. A REAL BUSINESS CYCLE MODEL WITH TURBULENCE SHOCKS

We now construct a real business cycle model to examine the economic mechanism through which turbulence can drive macroeconomic fluctuations and cross-sectional reallocation. In light of the empirical evidence presented in Section III, we incorporate into the model two key ingredients—firm heterogeneity and financial frictions. We show that these ingredients are both important for the transmission of turbulence shocks.
IV.1. The model. The model economy is populated by a continuum of infinitely lived households with measure one. The representative household has the utility function

\[ E \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \psi N_t^{1+\gamma} \right\}, \tag{10} \]

where \( C_t \) denotes consumption, \( N_t \) denotes labor hours, and \( E \) is an expectation operator. The parameter \( \beta \in (0, 1) \) is a subjective discount factor, \( \psi > 0 \) measures the relative weight on the disutility of working, and \( \gamma \geq 0 \) is the inverse Frisch elasticity of labor supply.

All markets are perfectly competitive. The household takes prices as given and maximizes the utility in Eq. (10) subject to the sequence of budget constraints

\[ C_t + K_{t+1} = (R_t + 1 - \delta)K_t + W_t N_t + D_t - T_t, \tag{11} \]

where \( K_{t+1} \) denotes the end-of-period capital stock, \( R_t \) denotes the capital rental rate, \( W_t \) denotes the real wage rate, \( D_t \) denotes the dividend income from firms, and \( T_t \) denotes a lump-sum tax paid to the government.

There is a continuum of firms, each endowed with a constant-returns technology that produces the final consumption good using capital and labor as inputs.\(^{21}\) Firms face idiosyncratic productivity shocks drawn at the beginning of each period, before hiring inputs. The production function for an individual firm is given by

\[ y_{jt} = A_t z_{jt} k_{jt}^{\alpha} n_{jt}^{1-\alpha}, \tag{12} \]

where \( y_{jt} \) denotes the output produced by firm \( j \) in period \( t \), and \( k_{jt} \) and \( n_{jt} \) denote the capital and labor inputs, respectively.

The term \( A_t \) denotes an aggregate productivity shock that follows the AR(1) process

\[ \ln(A_t) = \rho_A \ln(A_{t-1}) + \sigma_A \varepsilon_t^A, \tag{13} \]

where the innovation term \( \varepsilon_t^A \) follows the standard normal process. The parameter \( \rho_A \) and \( \sigma_A \) measure the persistence and the volatility, respectively, of the aggregate productivity shock.

The idiosyncratic productivity shock \( z_{jt} \) follows the stochastic process described in Eq. (2), which we rewrite here for convenience of referencing:

\[ z_{jt, t+1} = \begin{cases} z_{jt} & \text{with prob } \rho_t, \\ \tilde{z} & \text{with prob } 1 - \rho_t. \end{cases} \tag{14} \]

Here, the term \( \tilde{z} \) is an i.i.d. random variable with a finite number of states. Specifically, we assume that \( \tilde{z} = z_j \) with probability \( \pi_j \), for \( j = 1, 2, ..., J \). Without loss of generality, we}

\(^{21}\)In Appendix B, we show that the main model mechanism that drives the macroeconomic and reallocation effects of turbulence shocks is similar if the production technology exhibits decreasing returns of scale.
further assume that $z_1 < z_2 < ... < z_J$. The process features time-invariant cross-sectional distribution of firm productivity such that, regardless of the realization of $\rho_t \in (0, 1)$, there is always a fraction $\pi_j$ of firms with $z_{jt} = z_j$ in each period. Thus, in a stationary equilibrium, $\pi_j$ is the measure of firms with productivity $z_j$.

We measure turbulence by $1 - \rho_t$. If $\rho_t = 1$, then the idiosyncratic productivity $z_{jt}$ would be permanent. If $\rho_t = 0$, on the other hand, then each firm would face i.i.d. shocks to productivity with no persistence. A lower value of $\rho_t$ implies that a high-productivity firm in the current period may not maintain its productivity in the next period, whereas a low-productivity firm in the current period might be able to draw a better productivity in the next period. Thus, a decline in $\rho_t$ reshuffles firms’ productivity ranking across time, increasing turbulence.

We assume that the turbulence shock follows the stochastic process

$$
\ln(1 - \rho_t) = (1 - \rho_\bar{\rho}) \ln(1 - \bar{\rho}) + \rho_\rho \ln(1 - \rho_{t-1}) + \sigma_\rho \varepsilon_t^\rho, \tag{15}
$$

where $\bar{\rho}$ denotes the average level of $\rho_t$ and the innovation term $\varepsilon_t^\rho$ follows a standard normal process. The parameter $\rho_\rho$ and $\sigma_\rho$ measure the persistence and the volatility of the turbulence shock, respectively.

Firms rely on external financing of their working capital. In the beginning of each period, firms need to borrow from a competitive financial intermediary to cover payments for input factors, and these working capital loans are repaid within the period, after firms receive revenues. Following Jermann and Quadrini (2012) and Liu and Wang (2014), we assume that a firm’s borrowing capacity is constrained by a fraction $\theta$ of its expected equity value in the next period, in line with the empirical evidence of Lian and Ma (2021).

Firms at each level of productivity face idiosyncratic production distortions (denoted by $\tau_{jt}$), reflecting differential policy interventions or government subsidies at the firm level (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Buera and Shin, 2013; Moll, 2014). These production distortions drive a wedge between firms’ private and social marginal revenue products. We assume that $\tau_{jt}$ is drawn from a continuous i.i.d. distribution $F(\tau_{jt})$. Under credit constraints, the presence of idiosyncratic production distortions allows a fraction of firms at each level of productivity to stay active, enabling turbulence shocks to generate reallocation and endogenous fluctuations in aggregate TFP.\footnote{Including idiosyncratic distortions also serves a technical purpose in our model with a discrete distribution of idiosyncratic productivity. The continuity of the distribution function $F(\tau_{jt})$ implies a well-defined cutoff point $\tau_{jt}^*$ that determines the subset of active firms at each level of productivity $z_{jt}$.}

The firms’ optimizing problem is characterized by the Bellman equation

$$
V_t(z_{jt}, \tau_{jt}) = \max_{k_{jt}, n_{jt}} \tau_{jt} A_t z_{jt} k_{jt}^{\alpha} n_{jt}^{1-\alpha} - R_t k_{jt} - W_t n_{jt} + E_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}), \tag{16}
$$
subject to the working capital constraint

\[ R_t k_{jt} + W_t n_{jt} \leq \theta E_t M_{t+1} V_{t+1}(z_{jt+1}, \tau_{jt+1}) \equiv \theta B_{jt}. \]  

(17)

Here, the term \( V_t(z_{jt}, \tau_{jt}) \) denotes the value function of firm \( j \) that depends on the firm-level state variables \( z_{jt} \) and \( \tau_{jt} \). The value function \( V_t(z_{jt}, \tau_{jt}) \) also depends on aggregate shocks, which are summarized by the time subscript \( t \). The term \( M_{t+1} = \beta \frac{C_t}{C_{t+1}} \) denotes the stochastic discount factor determined by the marginal utilities of the representative household who owns all firms. The term \( B_{jt} \) denotes the expected present value of a firm with current productivity \( z_{jt} \).

Profit maximizing implies the conditional factor demand functions

\[
\frac{\tau_{jt} y_{jt}}{k_{jt}} = (1 + \mu_{jt}) R_t, 
\]  

(18)

and

\[
(1 - \alpha) \frac{\tau_{jt} y_{jt}}{n_{jt}} = (1 + \mu_{jt}) W_t, 
\]  

(19)

where \( \mu_{jt} \) denotes the Lagrangian multiplier associated with the credit constraint (17). Using the factor demand functions, we can write the firm’s flow profit as

\[
d_{jt} \equiv \left[ \tau_{jt} A_t z_{jt} \left( \frac{\alpha W_t}{(1 - \alpha) R_t} \right)^\alpha - \frac{W_t}{1 - \alpha} \right] n_{jt}. 
\]  

(20)

Since production subsidies follow an i.i.d. process, a firm would choose to be active in production if and only if its subsidy \( \tau_{jt} \) is sufficiently high such that \( d_{jt} \geq 0 \). It follows that there exists a threshold level of production subsidy \( \tau_{jt}^* \) such that, if \( \tau_{jt} \geq \tau_{jt}^* \), then a firm would be active in production, facing binding credit constraints. Otherwise, the firm would remain inactive. At the threshold level of subsidy, a firm earns zero profit and thus it would be indifferent between producing and staying inactive. The indifference condition determines the threshold level of subsidy

\[
\tau_{jt}^* = \frac{R_t^\alpha W_t^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha} A_t z_{jt}}. 
\]  

(21)

The threshold \( \tau_{jt}^* \) increases with the factor prices \( R_t \) and \( W_t \) and decreases with the productivity level \( z_{jt} \). Thus, given the factor prices, the fraction of active firms is larger for firms with higher productivity.

The presence of credit constraints and production distortions creates misallocation of resources. Absent those distortions, all resources would be allocated to the most productive firm (with productivity \( z_J \)). However, under those distortions, some low-productivity firms are able to produce because not all high-productivity firms are active. Specifically, at each level of productivity, there is a non-degenerate fraction of firms that are active, with the
share of active firms measured by $1 - F(\tau_{jt})$ for all $j \in 1, \ldots, J$. Such misallocation opens up a reallocation channel for turbulence shocks, as we show below.

Since active firms face binding credit constraints and inactive firms do not use any input factors, we obtain the conditional demand functions for labor and capital inputs

$$n_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{(1-\alpha)\theta B_{jt}}{W_t}, & \text{if } \tau_{jt} \geq \tau_{jt}^* \\ 0, & \text{otherwise.} \end{cases} \quad (22)$$

and

$$k_t(z_{jt}, \tau_{jt}) = \begin{cases} \frac{\alpha \theta B_{jt}}{R_t}, & \text{if } \tau_{jt} \geq \tau_{jt}^* \\ 0, & \text{otherwise.} \end{cases} \quad (23)$$

Given the factor demand functions, firm $j$’s value function can be written as

$$V_t(z_{jt}, \tau_{jt}) = \max \left\{ \frac{\tau_{jt}}{\tau_{jt}^*} - 1, 0 \right\} \theta B_{jt} + B_{jt}. \quad (24)$$

Since production subsidies are i.i.d. across time, the average value of a firm with productivity $z_{jt}$ is given by

$$\bar{V}_t(z_{jt}) = \int V_t(z_{jt}, \tau)dF(\tau) = \left[ 1 + \theta \int_{\tau_{jt}^*}^{\tau} \frac{(\tau - 1)dF(\tau)}{\tau_{jt}^*} \right] B_{jt} \equiv \Phi(\tau_{jt}^*)B_{jt}, \quad (25)$$

where the term $\Phi(\tau_{jt}^*) \equiv 1 + \theta \int_{\tau_{jt}^*}^{\tau} \frac{(\tau - 1)dF(\tau)}{\tau_{jt}^*}$ is a decreasing function of the threshold subsidy level $\tau_{jt}^*$.

Given the stochastic process of $z_{j,t+1}$ and the definition of $B_{jt}$ in Eq. (17), we have

$$B_{jt} = \beta \mathbb{E}_t \frac{C_t}{C_{t+1}} \left[ \rho_t \bar{V}_{jt+1} + (1 - \rho_t) \sum_{i=1}^{J} \pi_i \bar{V}_{it+1} \right]. \quad (26)$$

In a competitive equilibrium, markets for labor, capital, and final consumptions goods all clear. Labor market clearing implies that

$$N_t = \sum_j \pi_j N_{jt} \equiv \sum_j \pi_j \frac{(1-\alpha)\theta B_{jt}}{W_t} \left[ 1 - F(\tau_{jt}^*) \right]. \quad (27)$$

Capital market clearing implies that

$$K_t = \sum_j \pi_j K_{jt} \equiv \sum_j \pi_j \frac{\alpha \theta B_{jt}}{R_t} \left[ 1 - F(\tau_{jt}^*) \right]. \quad (28)$$

Goods market clearing implies that

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t, \quad (29)$$

where aggregate output $Y_t$ is given by

$$Y_t \equiv \sum_j \pi_j Y_{jt} = \sum_j \pi_j A_t z_{jt} K_{jt}^{\alpha} N_{jt}^{1-\alpha}. \quad (30)$$
Given aggregate output, aggregate capital and labor inputs, we define aggregate TFP as

\[ Z_t \equiv \frac{Y_t}{K_t^{\alpha}N_t^{1-\alpha}} = \frac{\sum_j \pi_j A_t z_{jt} K_{jt}^{\alpha} N_{jt}^{1-\alpha}}{K_t^{\alpha} N_t^{1-\alpha}}. \]  

(31)

**Definition.** A competitive equilibrium consists of the sequence of allocations \( \{C_t, Y_t, N_t, K_t\} \) and the sequence of prices \( \{W_t, R_t\} \) such that (i) taking all prices as given, the allocations solve the household’s utility maximizing problem and the firms’ profit maximizing problem; and (ii) markets for labor, capital, and goods all clear.

### IV.2. Steady-state allocations.

We now provide some analytical characterization of the steady-state equilibrium and show how the steady-state allocations vary with the average level of turbulence \((1 - \bar{\rho})\).

In Section IV.1, we have shown that a firm with productivity \(z_j\) chooses to produce (i.e., become active) if and only if its subsidy exceeds the threshold \(\tau^*_j\). The threshold level of subsidy is given by Eq. (21), and it is a decreasing function of firm productivity \(z_j\).

Figure 5 illustrates the production decisions for firms with different levels of productivity \((z)\) and subsidies \((\tau)\). The downward-sloping curve indicates the threshold function \(\tau^*(z)\). At each \(z\), a firm with a subsidy \(\tau \geq \tau^*(z)\) chooses to produce. Otherwise, it stays inactive. Thus, the region of active firms are those with \((\tau, z)\) lying above the threshold curve.

In an economy with a higher average level of turbulence (i.e., with a lower value of \(\bar{\rho}\)), a high-productivity firm is less likely to remain productive, reallocating resources to low-productivity firms. Such reallocations reduce aggregate TFP, lowering the factor prices. For any given \(z\), the declines in wages and capital rents reduce the threshold level of subsidy \(\tau^*\) (see Eq. (21)). Therefore, the threshold curve for production decisions shifts downward, with a flatter slope (red line), indicating that the increase in turbulence expands the active regions for low-productivity firms more than it does for high-productivity firms.

The reallocation effects of turbulence illustrated in Figure 5 can be formalized by the following proposition.

**Proposition 2.** Given the steady-state factor prices \(R\) and \(W\), an increase in average turbulence reduces the share of labor hours allocated to high-productivity firms. Specifically, define the relative share of labor hours as \(\eta_{ji} \equiv \frac{N_j}{N_i}\), where \(N_j\) and \(N_i\) denote labor hours allocated to active firms with productivity \(z_j\) and \(z_i\), respectively. Without loss of generality, we assume that \(z_j > z_i\). Then, we have

\[ \frac{\partial \eta_{ji}}{\partial \bar{\rho}} > 0. \]

(32)
Figure 5. Production decisions in the steady-state equilibrium

Note: This figure shows the steady-state relation between the threshold level of subsidy $\tau^*$ and firm productivity $z$ (blue line). Firms with $(\tau, z)$ lying above the threshold line are active in production and those below the line are inactive. The figure also shows the threshold line (red line) for production decisions in an economy with a higher average level of turbulence (i.e., a lower value of $\bar{\rho}$).

Proof. In the steady-state equilibrium, the relative share of labor is given by

$$\eta_{ij} = \frac{\pi_j}{\pi_i} \frac{1 - \beta \bar{\rho} \Phi(\tau^*_j)}{1 - \beta \bar{\rho} \Phi(\tau^*_i)} \left[1 - F(\tau^*_j)\right] \left[1 - F(\tau^*_i)\right]^{-1},$$

where we have used aggregated labor demand based on Eq. (22), with the term $B_j$ substituted out using the steady-state version of Eq. (26).

At given values of $W$ and $R$, the threshold $\tau^*_j$ is a function of $z_j$ only. Since $z_j > z_i$, it is easy to show that $\Phi(\tau^*_j) > \Phi(\tau^*_i)$ and that $1 - F(\tau^*_j) > 1 - F(\tau^*_i) > 0$. Thus, we have

$$\frac{\partial \eta_{ji}}{\partial \bar{\rho}} = \frac{\beta [\Phi(\tau^*_j) - \Phi(\tau^*_i)]}{(1 - \beta \bar{\rho} \Phi(\tau^*_j))^2} \frac{1 - F(\tau^*_j)}{1 - F(\tau^*_i)} > 0$$

(33)

□

When turbulence rises ($\bar{\rho}$ declines), current productivity is less predictive for future productivity, such that productive firms are less likely to stay productive. Thus, an increase in
turbulence lowers expected value of productive firms, reducing their borrowing capacity for financing working capital. As a consequence, labor is reallocated to less productive firms.

The analytical results in Proposition 2 are partial equilibrium in nature, because we have assumed that the factor prices \( W \) and \( R \) are independent of turbulence. However, since the production thresholds for firms with lower productivity are more sensitive to changes in the factor prices, an increase in turbulence that reduces the factor prices would disproportionately expand the active regions for low-productivity firms, reinforcing the misallocation effects of turbulence. We quantify the general equilibrium effects of a turbulence shock in the next section.

IV.3. \textbf{The calibration.} A period in our model corresponds to a year. We set the subject discount factor to \( \beta = 0.96 \), implying an annualized risk-free interest rate of 4 percent. Based on our estimated firm-level production function parameters using the Compustat and NBER-CES data, we calibrate the cost share of capital to \( \alpha = 0.34 \). We set the capital depreciation rate to \( \delta = 0.10 \) to match the average annual investment rate of 10 percent in the U.S. data (Eisfeldt and Rampini, 2006; Clementi and Palazzo, 2016). We assume that labor is indivisible in the sense of Hansen (1985) and Rogerson (1988), implying that \( \gamma = 0 \). We calibrate the relative utility weight on leisure \( \psi \) such that the steady-state labor hours are one-third of the time endowment. We set the parameter \( \theta \) to 0.35 to match the average ratio of working capital to market equity in the Compustat data. We set the annual persistence of aggregate TFP shocks to \( \rho_A = 0.95 \), following the real business cycle literature (Cooley and Prescott, 1995; Bloom et al., 2018). We normalize the standard deviation of the aggregate TFP shock to 1 percent (0.01).

The presence of the production distortions \( \tau_{jt} \) can potentially complicate the calibration of the turbulence shock. Our revenue-based measure of firm-level TFP contains not only true productivity shocks \( z_{jt} \) but also information about demand conditions summarized in the production distortion term \( \tau_{jt} \). Specifically, the model implies that

\[
\text{tfp}_{jt} = \log(z_{jt}) + \log(\tau_{jt}),
\]

where \( \text{tfp}_{jt} \) is the firm-level TFP (in log units) that we construct based on the production function using data from Compustat and NBER-CES. For tractability, we assume that \( \tau_{jt} \) is an i.i.d. process with a constant variance \( \sigma_\tau \) and that \( \tau_{jt} \) is uncorrelated with \( z_{jt} \).

If \( \sigma_\tau > 0 \), then the estimated average value of the Spearman rank correlation of the observed TFP would understate the true value of \( \tilde{\rho} \).

\(^{23}\)To see this, consider the extreme case with \( \tilde{\rho} = 1 \) (i.e., no changes in firm productivity ranking). Given the noise in observed productivity stemming from \( \tau_{jt} \), the estimated Spearman correlation using the observed TFP would be less than one.
Table 4. Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ Subjective discount factor</td>
<td>0.96</td>
<td>Average real interest rate of 4% per year</td>
</tr>
<tr>
<td>$\alpha$ Capital share</td>
<td>0.34</td>
<td>Average cost share of capital (NBER-CES)</td>
</tr>
<tr>
<td>$\delta$ Capital depreciation rate</td>
<td>0.10</td>
<td>Capital depreciation rate of 10% per year</td>
</tr>
<tr>
<td>$\gamma$ Inverse Frisch elasticity</td>
<td>0</td>
<td>Indivisible labor</td>
</tr>
<tr>
<td>$\psi$ Utility weight on leisure</td>
<td>2.15</td>
<td>Average hours of 1/3 of time endowment</td>
</tr>
<tr>
<td>$\theta$ Loan to value ratio</td>
<td>0.35</td>
<td>Working capital to equity ratio (Compustat)</td>
</tr>
<tr>
<td>$\bar{\rho}$ Firm-level TFP persistence</td>
<td>0.96</td>
<td>Estimated (Compustat and NBER-CES)</td>
</tr>
<tr>
<td>$\sigma_z$ Volatility of firm-level TFP shock</td>
<td>0.11</td>
<td>Estimated (Compustat and NBER-CES)</td>
</tr>
<tr>
<td>$\sigma_\tau$ Volatility of production distortion (log)</td>
<td>0.35</td>
<td>Estimated (Compustat and NBER-CES)</td>
</tr>
<tr>
<td>$\mu_\tau$ Average production distortion (log)</td>
<td>-0.045</td>
<td>Normalized</td>
</tr>
<tr>
<td>$\rho_p$ Persistence of turbulence shock</td>
<td>0.605</td>
<td>Estimated to match Bloom et al. (2018)</td>
</tr>
<tr>
<td>$\sigma_p$ Volatility of turbulence shock</td>
<td>0.58</td>
<td>Estimated to match Bloom et al. (2018)</td>
</tr>
<tr>
<td>$\rho_A$ Persistence of productivity shock</td>
<td>0.95</td>
<td>Cooley and Prescott (1995)</td>
</tr>
<tr>
<td>$\sigma_A$ Volatility of aggregate TFP shock</td>
<td>0.01</td>
<td>Normalized</td>
</tr>
</tbody>
</table>

would also distort the estimated value of $\sigma_z$. Thus, we need to jointly calibrate the values of $\sigma_\tau$, $\bar{\rho}$, and $\sigma_z$.

We implement this calibration by targeting three moments in the model to their counterparts in the firm-level data. Those three moments in the data include (1) the average value of the Spearman rank correlations of establishment-level TFP (0.72, estimated by Bloom et al. (2018)), (2) the standard deviation of the firm-level TFP shock (0.22, based on firm-level TFP constructed using the Compustat/NBER-CES data), and (3) the average IQR of equity values across firms (1.53, also from the Compustat data). This calibration implies that $\bar{\rho} = 0.96$, $\sigma_z = 0.11$, and $\sigma_\tau = 0.35$.

The presence of $\tau_{jt}$ can also affect the calibration of the turbulence shock process (i.e., $\rho_p$ and $\sigma_p$). Given our calibration of $\sigma_\tau$, $\bar{\rho}$, and $\sigma_z$, we use Eq. (34) to simulate the true productivity process $z_{jt}$ and calibrate the two parameters $\rho_p$ and $\sigma_p$ to target the persistence and the standard deviation of the turbulence measure based on establishment-level TFP ($t_{jtp_{jt}}$) constructed by Bloom et al. (2018). This process leads to our calibration of $\rho_p = 0.605$ and $\sigma_p = 0.58$.

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The calculations of Bloom et al. (2018) are based on establishment-level data from the Census of Manufactures (CM) and the Annual Survey of Manufactures (ASM), which provide much broader and more granular coverage of U.S. businesses than the firm-level data in the Compustat.
IV.4. **Impulse responses to a turbulence shock.** To examine the macroeconomic effects of turbulence shocks, we solve our model based on calibrated parameters. We simulate the model using third-order approximations of the equilibrium conditions around the deterministic steady-state. We then compute impulse responses of several key macroeconomic and distributional variables as deviations of those variables driven by the turbulence shock from their stochastic steady-state levels without the shock.\(^{25}\)

Figure 6 displays the impulse responses to a one-standard-deviation shock to turbulence. An increase in turbulence reduces the chance for a current high-productivity firm to remain as productive in the future, and it also increases the chance for a current low-productivity firm to get a higher productivity draw in the future. Thus, the equity values of high-productivity firms declines relative to those of low-productivity firms, leading to a decline in the IQR of firm value and disproportionately tightening the borrowing capacity of high-productivity firms in the current period. Facing tightened credit constraints, high-productivity firms pull back hiring, reallocating labor and capital to low-productivity firms. Since high-productivity firms use more capital and labor in the steady-state than low-productivity firms, the increase in turbulence reduces the IQR of labor and capital across firms, and it also reduces the IQR of sales, in line with the empirical evidence presented in Section III.4.

Through reallocation, a turbulence shock reduces aggregate TFP and leads to a recession with synchronized declines in aggregate output, consumption, investment, and labor hours, as in the data. The recessionary effects of turbulence are sizable and persistent. For example, a one-standard-deviation turbulence shock leads to a drop in aggregate output of about 2 percent on impact, and output stays below its steady-state level for more than five years after the shock.

IV.5. **The role of financial frictions.** Our empirical evidence suggests that financial frictions are important for the reallocation effects of turbulence (Section III.5). We now illustrate the quantitative importance of financial frictions for propagating turbulence shocks to driven macroeconomic fluctuations. For this purpose, we consider a counterfactual version of our model, in which firms’ borrowing capacity does not vary with the expected equity value.

\(^{25}\)We follow the approach in Fernández-Villaverde et al. (2011) and Leduc and Liu (2016) to compute the impulse responses. In particular, the model is first simulated for a large number of periods to compute the ergodic mean of each variable. It is then simulated using the ergodic means as a starting point. Finally, impulse responses to a turbulence shock are computed as the differences between the simulated path with the turbulence shock and the path with no shocks. This solution approach helps capture potential non-linear effects of the shock. Since turbulence shocks in our model have first-moment impact, the impulse responses generated from the third-order approximations are essentially the same as those from first-order approximations (and we have verified this).
Figure 6. Impulse response to a turbulence shock in the benchmark model

Note: This figure shows the impulse responses to a one-standard-deviation shock to turbulence in the calibrated benchmark model. The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

Specifically, we replace the working capital constraint with

$$R_t k_{jt} + W_t n_{jt} \leq \theta \beta E_t \frac{C_t}{C_{t+1}} \left[ \rho_t \bar{V}^{ss}_{jt} + (1 - \rho_t) \sum_{i=1}^{J} \pi_i \bar{V}^{ss}_{i} \right] \equiv \theta \bar{B}_{jt}, \quad (35)$$

where $\bar{V}^{ss}_{jt}$ denotes the steady-state equity value for firms with productivity $z_{jt}$. In this counterfactual, a turbulence shock can still influence firms’ borrowing capacity by changing the transition probability ($\rho_t$) of the future productivity distribution, but changes in firms’ expected equity value following a turbulence shock would have no effect on the borrowing capacity.

Figure 7 shows the impulse responses in the benchmark model (blue solid lines) and those in the counterfactual under this “quasi-fixed” borrowing capacity (red dash-dotted lines). The figure shows that the recession effects and the reallocation effects of a turbulence shock would be substantially dampened if firms’ borrowing capacity could not vary with the
**Figure 7.** Impulse response to a turbulence shock: Benchmark model vs. counterfactual with quasi-fixed borrowing capacity

*Note:* This figure shows the impulse responses to a one-standard-deviation shock to turbulence in the benchmark model (blue lines) and in the counterfactual with quasi-fixed borrowing capacity (red dash-dotted lines). The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

expected equity value. This counterfactual illustrates the importance of financial frictions—and in particular, the endogenous variations of the borrowing capacity with expected firm values—for propagating turbulence shocks.

**IV.6. Impulse responses to a micro-level uncertainty shock.** Turbulence is related to but different from micro-level uncertainty such as that studied by Bloom et al. (2018). Uncertainty is a mean-preserving spread of the firm-level productivity distribution. Similar to uncertainty, turbulence reshuffles firms’ rankings in future productivity distributions, which also raises the *conditional* variance of the idiosyncratic shocks. Different from uncertainty, turbulence also changes the conditional mean of the productivity distribution. By changing the conditional mean, a turbulence shock generates between-firm reallocation, which is amplified through financial frictions and leads to procyclical aggregate TFP and business cycle comovements.
To illustrate the relation between turbulence and uncertainty, we consider a micro-level uncertainty shock. Specifically, we assume that the i.i.d. production distortion $\tau_{jt}$ follows a log-normal distribution with a time-varying volatility $\sigma_{\tau,t}$ that follows the stationary stochastic process

$$\ln(\sigma_{\tau,t}) = (1 - \rho_{\sigma}) \ln(\sigma_{\tau}) + \rho_{\sigma} \ln(\sigma_{\tau,t-1}) + \sigma_{\sigma} \varepsilon_{t}^\sigma, \quad (36)$$

where the innovation term $\varepsilon_{t}^\sigma$ follows the standard normal process. The parameters $\rho_{\sigma}$ and $\sigma_{\sigma}$ measure the persistence and the standard deviation of the volatility shock, respectively. We normalize the mean value of $\log(\tau_{jt})$ to $\mu_{\tau,t} = -0.5\sigma_{\tau,t}^2$ such that the unconditional mean of $\tau_{jt}$ is always one (i.e., $E(\tau_{jt}) = 1$). Our uncertainty measure $\ln(\sigma_{\tau,t})$ here parallels the micro-level uncertainty in Bloom et al. (2018). We set the persistence of the micro-level uncertainty to $\rho_{\sigma} = 0.75$, consistent with Leduc and Liu (2016) and Bloom et al. (2018). We normalize the standard deviations of the uncertainty shock to 1 percent (0.01).
Figure 8 shows the impulse responses of the key macroeconomic and distributional variables following an increase in micro-level uncertainty. Similar to turbulence, uncertainty also leads to misallocation and reduces aggregate productivity, but through a different channel. An increase in uncertainty expands the right tails of the subsidy distribution, raising the average subsidies for active firms at each productivity level (i.e., those with \( \tau_j \geq \tau^*_j \)).\(^{26}\) Since the threshold level of subsidy (\( \tau^*_j \)) decreases with firm-level productivity (\( z_j \)), the expansion of the right tail of the subsidy distribution increases the mass of active low-productivity firms by more than that of high-productivity firms. Thus, uncertainty reallocates capital and labor towards low-productivity firms, reducing aggregate TFP. The recessionary effect of uncertainty shock is amplified through general equilibrium channels due to lower wage and rent. The decline in aggregate TFP can generate synchronized declines in output, consumption, investment and labor hours, which is in line with empirical estimates of the impulse response using local projection method (Figure A3 in Appendix).

An uncertainty shock disproportionately raises the average subsidy received by high-productivity firms, and thus, it raises the IQR of firm value and sales (inclusive of subsidies), consistent with empirical evidence (see Figure A3 in the Appendix). In contrast, a turbulence shock reduces the IQR of both firm value and sales.\(^{27}\)

IV.7. Impulse responses to an aggregate productivity shock. The macroeconomic and reallocation effects of turbulence are also different from those associated with other aggregate shocks such as a TFP shock.

Figure 9 displays the impulse responses to a negative TFP shock. Similar to a turbulence shock, a negative TFP shock generates a recession with synchronized declines in aggregate output, consumption, investment, and labor hours. The shock also reduces average firm value.

However, the TFP shock has different reallocation effects than turbulence. A decline in aggregate TFP raises the threshold level of subsidy for active firms at each level of productivity, shrinking the set of active firms. Since labor and capital are perfectly mobile across firms, all firms face the same wages and capital rents. A decline in aggregate TFP would therefore force more low-productivity firms into the inactive regions than high-productivity firms.

\(^{26}\)Uncertainty also expands the left tail of the distribution, but that does affect production because firms with subsidies in the left tails are inactive.

\(^{27}\)Here, the IQR of labor is computed based on the share of labor allocated to high-productivity firms (the top quartile) times the mass of high-productivity firms relative to that allocated to low-productivity firms (the bottom quartile). The IQRs of firm value and sales are measured by the ratio of the average value of high-productivity firms to that of low-productivity firms (excluding the effects from the mass of producers at each level of productivity), which is comparable to the empirical measures.
firms. This “cleansing effect” reallocates resources to more productive firms, raising the endogenous component of aggregate TFP, mitigating the recession. The negative TFP shock also reduces firm value dispersion because, among active firms, lower-productivity firms need to be compensated by higher average subsidies for them to remain active. Thus, the average subsidy for low-productivity firms increases relative to that for high-productivity firms, reducing the dispersion in firm value.\footnote{For distributional impacts of a TFP shock, see Figure A7 in Appendix A.6.3.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{impulse_response.png}
\caption{Impulse response to a negative aggregate TFP shock in the benchmark model}
\end{figure}

\textit{Note}: This figure shows the impulse responses to a one-standard-deviation negative shock to aggregate TFP. The horizontal axis shows the periods (years) since the impact of the shock. The vertical axis shows the percent deviations of each variable from its stochastic steady-state level.

IV.8. \textbf{Quantitative importance of turbulence shocks}. Turbulence has quantitatively important recessionary effects, both in the model and in the data. Figure 10 compares the model-implied impulse responses of aggregate output (blue solid line) with the empirical estimates of the impulse response (black solid line). A one-standard-deviation turbulence shock reduces aggregate output by around 1 percent after one year, both in the data and
Figure 10. Impulse response to turbulence shock: Model vs. data

Note: This figure shows the impulse responses of private aggregate output to a one-standard-deviation shock to turbulence in the data (red solid line) and in the calibrated annual version of the model (blue solid line). The dashed lines show the 68% confidence band around the empirical estimates of the impulse responses. The horizontal axis shows the years after the impact of the shock. The vertical axis shows the percent deviations of output in the model from its steady-state level and the percentage changes in output in the data relative to its pre-shock level.

in the model. The shock has persistent recessionary effects on aggregate output both in the model and in the data, although the theoretical impulse responses miss the hump estimated from the data. These findings suggest that turbulence plays an important role in driving business cycles.\footnote{We have also compared the theoretical impulse responses of investment, consumption, and labor hours with those in the data (see Appendix A.6.4). We find that, similar to the case of output, the model-implied responses of investment and consumption are both in line with those estimated from the data, although the calibrated model fails to generate the turbulence-driven hump in the data.}
V. Policy interventions

Appropriate policy interventions can potentially undo the financial frictions, stabilizing aggregate output and improving allocative efficiency. To illustrate this point, we consider two alternative policy interventions in response to a recession driven by a turbulence shock. One policy is a borrowing subsidy that reduces firms’ borrowing costs. The other is a credit-easing policy that expands firms’ borrowing capacity.

Under the borrowing subsidy policy (Policy I), the government subsidizes wages and rents for active firms at an exogenous rate \( \omega_{1t} \), such that firms with productivity \( z_{jt} \) face the effective credit constraint

\[
\tilde{R}_t k_{jt} + \tilde{W}_t n_{jt} \leq \theta B_{jt},
\]

where \( \tilde{R}_t = (1 - \omega_{1t})R_t \) and \( \tilde{W}_t = (1 - \omega_{1t})W_t \) denote the after-subsidy capital rental rate and real wage rate, respectively. The subsidies thus reduce the amount of working capital loans that firms need to borrow. The government finances the borrowing subsidies \( \Omega_{1t} = \omega_{1t}(W_tN_t + R_tK_t) \) by imposing lump-sum taxes on the representative household. We assume that operating this policy incurs a resource cost of \( \lambda_1 \Omega_{1t} \), where \( \lambda_1 \geq 0 \) reflects potential deadweight losses associated with the government program.

Under the credit easing policy (Policy II), the government injects liquidity into active firms, such that firms with productivity \( z_{jt} \) face the effective credit constraint

\[
R_t k_{jt} + W_t n_{jt} \leq \theta(1 + \omega_{2t})B_{jt},
\]

where \( \omega_{2t}B_{jt} \) is the amount of government transfers to active firms with expected equity value \( B_{jt} \). The total cost of the credit-easing policy is given by \( \Omega_{2t} = \omega_{2t} \sum_j \pi_j \theta B_{jt} [1 - F(\tau^*_j)] \), which is financed by lump-sum taxes on the household. Similar to the borrowing subsidy policy, we assume that credit easing also incurs a resource cost of \( \lambda_2 \Omega_{2t} \), where \( \lambda_2 \geq 0 \) reflects potential deadweight losses in operating the policy.

Following Bloom et al. (2018), we consider transitory and unanticipated policy interventions. A policy would be implemented only if a turbulence shock hits the economy, and the policy intervention has the same persistence as the shock. We evaluate the effectiveness of each of the two alternative policies—borrowing subsidies and credit easing—for mitigating the macroeconomic and reallocation effects of turbulence. For this purpose, we compare the impulse responses of aggregate output and aggregate TFP to a turbulence shock under borrowing subsidies (Policy I) or credit easing (Policy II) to those in the laissez-faire benchmark economy without policy intervention (No policy).

In each policy regime, we simulate the model economy for 1000 periods based on third-order approximations of the equilibrium system around the deterministic steady-state. A turbulence shock and a simultaneous policy intervention (if any) are implemented in period
Figure 11. Impact effects of a turbulence shock with alternative policy interventions

Note: This figure plots the responses of aggregate output and aggregate TFP in the impact period of a one-standard-deviation turbulence shock. Black bars show the responses in the benchmark economy with no policy interventions. Red bars represent the responses under an unanticipated and temporary borrowing subsidy policy (Policy I). Blue bars represent the responses under an unanticipated and temporary credit easing policy (Policy II).

The unanticipated policy stimulus has a size of 1 percent of steady-state output, with the same persistence as the turbulence shock. After a policy intervention is implemented, we allow the economy to evolve naturally for the remaining 20 years. We calculate the responses of each endogenous variable to the turbulence shock (with or without a policy intervention) as percent deviations from the stochastic steady-state.

Figure 11 shows the stabilizing effects of the two alternative policies relative to the benchmark economy, conditional on a one-standard-deviation turbulence shock. Compared to the benchmark economy without policy interventions (black bar), Policy I (red bar) is effective for stabilizing the declines in aggregate output (upper panel). By providing borrowing...
subsidies, the policy stimulates demand for labor and capital, and thus mitigating the declines in equilibrium hours, investment, and output. However, by reducing the effective factor prices, Policy I enables a larger fraction of low-productivity firms to produce, exacerbating misallocation and intensifying the decline in aggregate TFP following a turbulence shock (lower panel).

The credit easing policy is also effective for stabilizing turbulence-driven output declines relative to the benchmark economy (blue bar, upper panel). The policy expands the borrowing capacity for all active firms, enabling a larger share of high-productivity firms to finance working capital and produce. The increase in the share of active high-productivity firms pushes up equilibrium wages and rents, shrinking the set of active low-productivity firms. Thus, Policy II reallocates labor and capital to high-productivity firms, improving aggregate TFP relative to the benchmark (blue bar, lower panel).

These policy experiments suggest that temporary borrowing subsidies or credit easing policies are effective for stabilizing turbulence-driven output fluctuations. However, the reallocation consequences of the two policies are different. While borrowing subsidies exacerbate misallocation, credit easing alleviates it.

VI. Conclusion

Macroeconomic fluctuations often mask underlying cross currents with important cross-sectional reallocations. We study the implications of turbulence—a form of reallocation shocks—for business cycles. An increase in turbulence changes the conditional distribution of firms’ future productivity, leading to reallocations across firms. We measure turbulence based on firm-level TFP data and document evidence that turbulence is countercyclical, rising sharply in recessions. Turbulence has cross-sectional reallocation effects, the magnitude of which depends on financial frictions. Turbulence is negatively correlated with average firm equity values and aggregate TFP. An increase in turbulence is associated with synchronized declines in aggregate output, consumption, investment, and labor hours.

Using a real business cycle model augmented with firm heterogeneity and financial frictions, we have highlighted a quantitatively important reallocation channel, through which a turbulence shock drives macroeconomic fluctuations. An increase in turbulence reduces the likelihood for the current high-productivity firms to maintain their productivity rankings in the future, lowering their expected equity values relative to those of the current low-productivity firms. Facing tightened working capital constraints, high-productivity firms pull back hiring of capital and labor relative to low-productivity firms, leading to reallocation from high- to low-productivity firms and reducing aggregate TFP. Such declines in TFP
generate a recession with synchronized declines in aggregate output, consumption, investment, and labor hours, as in the data. A one-standard-deviation shock to turbulence leads to a drop in aggregate output of about 0.5 percent, with the recessionary effects persisting for more than five years, suggesting that turbulence plays an important role in driving business cycles.

Financial frictions are crucial for propagating turbulence shocks in our model. The presence of financial frictions also leads to misallocation. Policy interventions designed to alleviate credit constraints can potentially dampen the impact of turbulence and improve allocative efficiency. However, the particular approach to implementing such policy interventions can produce very different outcomes. For example, borrowing subsidies that reduce the amount of working capital loans that firms need to borrow can effectively boost aggregate output, mitigating the recessionary effects of turbulence. An alternative credit easing policy that expands firms’ borrowing capacity can also stimulate aggregate output. However, these two alternative policies have different implications for allocative efficiency. A borrowing subsidy enables more low-productivity firms to stay active, exacerbating misallocation, whereas credit easing allows high-productivity firms to expand production, improving aggregate productivity.

To illustrate the key transmission mechanism of turbulence, we have intentionally kept the model stylized. For example, the model abstracts from firm entries and exits. To the extent that firms rely on external financing and entering (exiting) firms have higher (lower) productivity than incumbent firms, we conjecture that introducing entry and exit decisions could potentially amplify the recessionary effects of turbulence through reallocation. Another direction of generalizing our study is to enrich the model by incorporating other real and nominal frictions such as habit formation, investment adjustment costs, and sticky prices and wages. With these additional frictions, the model can better fit time-series data and it can also be used to examine the role of monetary policy in stabilizing macroeconomic fluctuations driven by turbulence. We leave these important subjects for future research.


