

Comments on Lars E. O. Svensson and Michael Woodford,

“Indicator Variables for Optimal Policy”

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This paper makes a remarkable achievement: simultaneous calculation of the rational expectations equilibrium and optimal control for a dynamic programming problem in which there is imperfect information about the state of the economy.

In the paper’s framework,  $X_t$  denotes a vector summarizing the true state of the economy at time  $t$ , which could include current and lagged values of variables of interest. The vector  $x_t$  contains forward-looking variables, which are presumed to be determined by a structural equation of the form

$$x_t = \rho_1 x_{t+1|t} + \rho_2 X_t + \rho_3 X_{t|t} + \rho_4 x_{t|t} + \rho_5 i_t. \quad (1)$$

This is another way of writing the second row of (2.1), with, for example,  $\rho_2 = -(A_{22}^1)^{-1} A_{21}^1$ .

The forward-looking variable may depend on the current real value of the state ( $X_t$ ), the inferred value based on information at time  $t$  ( $X_{t|t}$ ), the current inferred or expected future values of the forward-looking variables ( $x_{t|t}$  or  $x_{t+t|t}$ ), and values of a vector  $i_t$  that is controlled by the central bank.

The evolution of  $X_t$  is presumed to be governed by the first row of (2.1),

$$X_{t+1} = A_{11}^1 X_t + A_{12}^1 x_t + A_{11}^2 X_{t|t} + A_{12}^2 x_{t|t} + B_1 i_t + u_{t+1}. \quad (2)$$

The information set  $I_t$  on the basis of which the inferences  $X_{t|t}$  and forecasts  $x_{t+1|t}$  are formed consists of current and lagged values of a vector  $Z_t$  which is related to the macro and forward-looking variables according to

$$Z_t = D^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + D^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + v_t \quad (3)$$

where  $(u'_t, v'_t)'$  is vector white noise with  $u_t$  uncorrelated with  $v_t$ .

The parameter matrices  $\rho_j, A_{ij}^k, B, D^k$ , and variances of  $u$  and  $v$  are taken as given structural parameters, while  $i_t$  is chosen by the central bank so as to minimize (2.6). A limited-information rational-expectations equilibrium consists of (a) a process for  $x_t$  as a function of  $\{X_{t-j}, Z_{t-j}\}_{j=0}^{\infty}$ , and (b) processes for  $i_t, X_{t|t}, x_{t|t}, x_{t+1|t}$  as functions of  $\{Z_{t-j}\}_{j=0}^{\infty}$ , such that (1)-(3) hold with  $i_t$  minimizing (2.6) and  $X_{t|t}, x_{t|t}, x_{t+1|t}$  the optimal inferences of  $X_t, x_t, x_{t+1}$  based on  $\{Z_{t-j}\}_{j=0}^{\infty}$ . One solution turns out to be given by

$$x_t = G^1 X_t + G^2 X_{t|t}$$

$$i_t = F X_{t|t}$$

$$x_{t|t} = (G^1 + G^2) X_{t|t}$$

$$x_{t+1|t} = (G^1 + G^2)(H + J) X_{t|t}$$

where the inference  $X_{t|t}$  is the outcome of a Kalman filter (B.13),<sup>1</sup>

$$X_{t|t} = (H + J) X_{t-1|t-1} + K^* [Z_t - (L + M)(H + J) X_{t-1|t-1}].$$

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<sup>1</sup> In Svensson and Woodford's notation,  $K^* = K(I + MK)^{-1}$ .

The paper gives constructive algorithms for calculating  $(G^k, F, H, J)$  in terms of the structural parameters of equations (1) and (2) but not depending on (3), the quality of information available to the central bank and private sector; this is the essential certainty-equivalence result.

In the specific example of Section 5, the basic signal-extraction problem for the central bank is distinguishing between a shock to the Phillips curve and a shock to potential output. It is perhaps worth commenting on why there is any conceptual difference between these two objects.

Equation (5.1) can be written

$$\pi_t = \delta\pi_{t+1|t} + \kappa y_t + q_t \tag{4}$$

where  $q_t = -\kappa\bar{y}_t + \nu_t$ . The components  $\bar{y}_t$  and  $\nu_t$  follow independent AR(1) processes. However, if (4) were all that mattered or were observed, there would be no conceptual distinction between the two shocks, and no signal extraction problem. Since  $\pi_t$ ,  $\pi_{t+1|t}$ , and  $y_t$  are all known at time  $t$ ,  $q_t$  is observed without error and optimal inference would be equivalent to simply forecasting  $q_t$  from its known ARMA(2,1) structure.

The reason there is a signal extraction problem here is that the central bank cares about  $\bar{y}_t$  in and of itself, independently of its effects on inflation;  $\bar{y}_t$  is thus both a shock to inflation and a shock to the central bank's preferences. The marginal cost of letting inflation rise by one unit is  $\pi_t$ , while the marginal cost of letting output rise by one unit is  $\lambda(y_t - \bar{y}_t)$ . As a result, along an optimal path, the central bank's optimal response to a perceived shock to  $\bar{y}_t$  is to change  $y_t$  by the identical amount, with no change in  $\pi_t$ . By contrast, the optimal

response to a perceived shock to  $\nu_t$  is to tolerate higher inflation and lower output. This is the case with or without an ability to commit; either framework implies a unit coefficient relating  $y_t$  to  $\bar{y}_{t|t}$  and a negative correlation between  $y_t - \bar{y}_{t|t}$  and  $\pi_t$ .

This example is an excellent vehicle for showcasing the paper's key results on certainty equivalence with forward-looking variables and policy inertia arising from either inference problems or commitment. However, one would want to be cautious about extrapolating these results into practical guidelines for monetary policy. First, the rule of choosing low levels of inflation when output is high is clearly a result of having assumed that there are only supply shocks  $\nu_t$  and no shocks to aggregate demand. Second, the paper assumes that the central bank can hit a level of output dead-on at any value it likes, but can control inflation only imperfectly. I imagine there are some whose ideological persuasion would have led them to favor the opposite formulation. Third, the certainty equivalence results are very much dependent on the linear-quadratic framework. This is probably a reasonable way to think about things during calm periods such as the present, where the choice is clearly one of tweaking inflation or output a little bit one direction or another. But it is also important to remember that another, perhaps even more important, goal of monetary policy is arresting financial crises before they become full-blown. Capitalist economies are periodically subject to events that produce an extremely rapid flight from risky capital. In October 1987 or October 1929, for example, the Fed's number one job was to provide immediate liquidity and largely ignore the inference issues studied in this paper.