A Monetary Business Cycle Model With Labor Market Frictions*

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Abstract

We introduce search and matching frictions into a monetary DSGE model. When job separations are exogenous, we find that the fit of the model is poor by comparison with the current standard, the sticky wage formulation proposed by Erceg, Henderson and Levin (2000) (EHL). Model fit is comparable to EHL when separations are endogenized. The precise way in which separations are modeled matters. When separations are modeled as optimizing a total surplus criterion, endogeneity of separations does not improve fit. Fit is improved when the separation decision is made by the firm. In all cases, the treatment of separations is fully taken into account ex ante, when workers and firms solve their bargaining problem.

Keywords: DSGE, labor market frictions, endogenous separations, unemployment, Bayesian estimation
JEL codes: E2, E3, E5, J6

*The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Executive Board of the European Central Bank or of Sveriges Riksbank.

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1. Introduction

Recently, Hall (2005a,b,c) and Shimer (2005a,b), Gertler, Sala and Trigari (2006) (henceforth GST), Christiano, Ilut, Motto, and Rostagno (2007) (CIMR) and others have integrated the search and matching framework of Mortensen and Pissarides (1994) into business cycle models. This literature focuses entirely on a time varying job finding rate to explain the bulk of movements in unemployment and other aggregate variables. The job separation rate is assumed to be constant over the business cycle. As emphasized by den Haan, Ramey and Watson (2000) and Fujita and Ramey (2008), this assumption is counterfactual.[1] Figure (7.1) displays the Fujita and Ramey separation data, which exhibits considerable cyclicality. We argue that understanding business cycle fluctuations requires understanding why the separation rate fluctuates.

We start with a standard DSGE model with nominal and real rigidities as in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2007). In this model there is no unemployment, job search or separations and wages are modelled as being sticky in the manner proposed by Erceg, Henderson and Levin (2000) (henceforth EHL). We call this the *EHL model*. We integrate a standard search and matching framework with exogenous separations and sticky nominal wages into this model, using the version of the GST model developed in CIMR (the *exogenous separations model*). Then, we relax the assumption of exogenous separations by introducing idiosyncratic worker productivity and we consider two different ways to endogenize separations. In the first approach, separations are determined by maximizing firm surplus (the *employer surplus model*). In the second approach, separations are determined by maximizing total (worker plus firm) surplus (the *total surplus model*).

We estimate four models using Bayesian techniques: the EHL model; exogenous separations model; and the employer and total surplus models. We use the following 7 time series for estimation: GDP, Consumption, Investment, Hours, Real Wages, Inflation and the Federal Funds Rate.

According to the marginal data likelihood the EHL model performs best in explaining our 7 macro time series. The employer surplus model is a close second, followed by the total surplus model. The worst-fitting model is the exogenous separation model. That the EHL model fits somewhat better than the employer surplus model is perhaps not surprising, since the latter model imposes severe restrictions on the data.

In terms of impulse responses to a monetary policy shock EHL and the employer surplus specifications are virtually identical while this is not the case for the other two unemployment models.

[1] The job separation rate is defined as the number of people that move from employment to unemployment divided by the number of employed people.

According to the variance decompositions of the estimated model, the separation rate in the employer surplus specification is driven equally by shocks to neutral technology, labor preferences, price markup and the nominal interest rate.

Moreover, the employer surplus model correctly predicts the second moments of the separation rate, the unemployment rate and vacancies posted. All other models are either not able to address the second moments of these data or simply get it entirely wrong. Note that these results are interesting in so far as we do not use any of these data in the estimation so far.

Summing up, our results indicate by and large that EHL and the employer surplus specifications perform similarly in terms of standard macro variables and outperform the total surplus and exogenous separation specifications of the labor market.

The paper is organized as follows. Section 2 introduces the EHL model. The unemployment models are developed in section 3. In section 4 the estimation is discussed. Results are presented in section 5. Finally, section 6 concludes.

2. A Baseline Model

2.1. Firms

A homogeneous good, Y_t, is produced using

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_{d,t}}} di \right]^{\lambda_{d,t}}, \ 1 \leq \lambda_{d,t} < \infty. \tag{2.1}$$

The good is produced by a competitive, representative firm which takes the price of output, P_t, and the price of inputs, $P_{i,t}$, as given.

The i^{th} intermediate good producer has the following production function:

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^{+} \phi,$$

where $K_{i,t}$ denotes the labor services rented by the i^{th} intermediate good producer. Firms must borrow a fraction of the wage bill, so that one unit of labor costs is denoted by

$$W_t R_t^f,$$

with

$$R_t^f = \nu_t^f R_t + 1 - \nu_t^f, \tag{2.2}$$

where W_t is the aggregate wage rate, R_t is the interest rate on working capital loans, and ν_t^f corresponds to the fraction that must be financed in advance.

The firm's marginal cost, divided by the price of the homogeneous good is denoted by mc_t :

$$
\begin{aligned}
mc_t &= \frac{\tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k P_t\right)^{\alpha} \left(W_t R_t^f\right)^{1-\alpha} \frac{1}{\epsilon_t}}{z_t^{1-\alpha} P_t} \qquad (2.3) \\
&= \tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k\right)^{\alpha} \left(\bar{w}_t R_t^f\right)^{1-\alpha} \frac{1}{\epsilon_t},
\end{aligned}
$$

where r_t^k is the nominal rental rate of capital scaled by P_t. Also, τ_t^d is a tax-like shock, which affects marginal cost, but does not appear in a production function. In the linearization of a version of the model in which there are no price and wage distortions in the steady state, τ_t^d is isomorphic to a disturbance in λ_d, i.e., a markup shock.

Productive efficiency dictates that another expression for marginal cost must also be satisfied:

$$
\begin{aligned}
mc_t &= \tau_t^d \frac{1}{P_t} \frac{W_t R_t^f}{MP_{l,t}} \\
&= \tau_t^d \frac{1}{P_t} \frac{W_t R_t^f}{\epsilon_t (1-\alpha) z_t^{1-\alpha} \left(k_{i,t} z_{t-1}^{+} \Psi_{t-1} / H_{i,t}\right)^{\alpha}} \\
&= \tau_t^d \frac{\left(\mu_{\Psi,t}\right)^{\alpha} \bar{w}_t R_t^f}{\epsilon_t (1-\alpha) \left(\frac{k_{i,t}}{\mu_{z^+,t}} / H_{i,t}\right)^{\alpha}} \qquad (2.4)
\end{aligned}
$$

The i^{th} firm is a monopolist in the production of the i^{th} good and so it sets its price. Price setting is subject Calvo frictions. With probability ξ_d the intermediate good firm cannot reoptimize its price, in which case,

$$
P_{i,t} = \tilde{\pi}_{d,t} P_{i,t-1}, \; \tilde{\pi}_{d,t} \equiv \left(\pi_{t-1}\right)^{\kappa_d} \left(\bar{\pi}_t\right)^{1-\kappa_d-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d},
$$

where κ_d, $\varkappa_d$, $\kappa_d + \varkappa_d \in (0,1)$ are parameters, π_{t-1} is the lagged inflation rate and $\bar{\pi}_t$ is the central bank's target inflation rate. Also, $\breve{\pi}$ is a scalar which allows us to capture, among other things, the case in which non-optimizing firms either do not change price at all (i.e., $\breve{\pi} = \varkappa_d = 1$) or that they index only to the steady state inflation rate (i.e., $\breve{\pi} = \bar{\pi}$, $\varkappa_d = 1$).

With probability $1-\xi_d$ the firm can change its price. The problem of the i^{th} intermediate good producer which has the opportunity to change price is to maximize discounted profits:

$$
E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \{P_{i,t+j} Y_{i,t+j} - mc_{t+j} P_{t+j} Y_{i,t+j}\},
$$

subject to the requirement that production equal demand. In the above expression, υ_t is the multiplier on the household budget constraint. It measures the marginal value to the household of one unit of profits, in terms of currency. In the profit function, we replace the firm's output with the demand function:

$$\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_d}{\lambda_d-1}} Y_t = Y_{i,t},$$

to obtain, after rearranging,

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} P_{t+j} Y_{t+j} \{ \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{1-\frac{\lambda_d}{\lambda_d-1}} - mc_{t+j} \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{\frac{-\lambda_d}{\lambda_d-1}} \},$$

or,

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} P_{t+j} Y_{t+j} \{ (X_{t,j}\tilde{p}_t)^{1-\frac{\lambda_d}{\lambda_d-1}} - mc_{t+j} (X_{t,j}\tilde{p}_t)^{\frac{-\lambda_d}{\lambda_d-1}} \},$$

where

$$\frac{P_{i,t+j}}{P_{t+j}} = X_{t,j}\tilde{p}_t,\ X_{t,j} \equiv \begin{cases} \frac{\tilde{\pi}_{d,t+j}\cdots\tilde{\pi}_{d,t+1}}{\pi_{t+j}\cdots\pi_{t+1}}, & j > 0 \\ 1, & j = 0. \end{cases} .$$

The i^{th} firm maximizes profits by choice of $\tilde{p}_t$. The fact that this variable does not have an index, i, reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_d}{\lambda_d-1}+1}$, rearranging, and scaling we obtain[2]:

$$E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j} [\tilde{p}_t X_{t,j} - \lambda_d mc_{t+j}] = 0,$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j} = \psi_{z^+,t+j} \tilde{y}_{t+j} X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, we obtain,

$$\tilde{p}_t^d = \frac{E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j} \lambda_d mc_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j} X_{t,j}} = \frac{K_t^d}{F_t^d},$$

say, where

$$\begin{aligned} K_t^d &\equiv E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j} \lambda_d mc_{t+j} \\ F_t^d &= E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j} X_{t,j}. \end{aligned}$$

[2] See section (7.2) on how variables are scaled in the model.

These objects have the following convenient recursive representations:

$$E_t\left[\psi_{z^+,t}\tilde{y}_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_d}} \beta\xi_d F_{t+1}^d - F_t^d\right] = 0$$

$$E_t\left[\lambda_d\psi_{z^+,t}\tilde{y}_t mc_t + \beta\xi_d\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d\right] = 0.$$

Turning to the aggregate price index:

$$\begin{aligned} P_t &= \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_d}} di\right]^{(1-\lambda_d)} \\ &= \left[\left(1-\xi_p\right)\tilde{P}_t^{\frac{1}{1-\lambda_d}} + \xi_p\left(\tilde{\pi}_{d,t}P_{t-1}\right)^{\frac{1}{1-\lambda_d}}\right]^{(1-\lambda_d)} \end{aligned} \tag{2.5}$$

After dividing by P_t and rearranging:

$$\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d} = \left(\tilde{p}_t^d\right)^{\frac{1}{1-\lambda_d}}. \tag{2.6}$$

In sum, the equilibrium conditions associated with price setting are:[3]

$$E_t\left[\psi_{z^+,t}y_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_d}} \beta\xi_d F_{t+1}^d - F_t^d\right] = 0 \tag{2.7}$$

$$E_t\left[\lambda_d\psi_{z^+,t}y_t mc_t + \beta\xi_d\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d\right] = 0, \tag{2.8}$$

$$\mathring{p}_t = \left[(1-\xi_d)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right)^{\lambda_d} + \xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_d}{1-\lambda_d}}\right]^{\frac{1-\lambda_d}{\lambda_d}} \tag{2.9}$$

$$\left[\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right]^{(1-\lambda_d)} = \frac{K_t^d}{F_t^d} \tag{2.10}$$

[3] When we linearize about steady state and set $\varkappa_d = 0$, we obtain,

$$\begin{aligned} \hat{\pi}_t - \widehat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_d\beta}E_t\left(\hat{\pi}_{t+1} - \widehat{\bar{\pi}}_{t+1}^c\right) + \frac{\kappa_d}{1+\kappa_d\beta}\left(\hat{\pi}_{t-1} - \widehat{\bar{\pi}}_t^c\right) \\ &\quad -\frac{\kappa_d\beta\left(1-\rho_\pi\right)}{1+\kappa_d\beta}\widehat{\bar{\pi}}_t^c \\ &\quad +\frac{1}{1+\kappa_d\beta}\frac{\left(1-\beta\xi_d\right)\left(1-\xi_d\right)}{\xi_d}\widehat{mc}_t, \end{aligned}$$

where a hat indicates log-deviation from steady state.

$$\tilde{\pi}_{d,t} \equiv \left(\pi_{t-1}\right)^{\kappa_d} \left(\bar{\pi}_t\right)^{1-\kappa_d-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d} \tag{2.11}$$

The intermediate output good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t + \tilde{I}_t \tag{2.12}$$

Here, C_t denotes household consumption, G_t government consumption and $\tilde{I}_t$ is a homogenous investment good. Some of the latter good is used to add to the physical stock of capital, $\bar{K}_t$. The rest of the investment good is used in maintenance expenditures, which arise from the utilization of capital, $a\left(u_t\right)\bar{K}_t$. Here, u_t denotes the utilization rate of capital, with capital services being defined by:

$$K_t = u_t \bar{K}_t.$$

We adopt the following functional form for a :

$$a(u) = 0.5\sigma_b\sigma_a u^2 + \sigma_b\left(1-\sigma_a\right)u + \sigma_b\left(\left(\sigma_a/2\right)-1\right), \tag{2.13}$$

where σ_a and σ_b are the parameters of this function. Finally, the integral in (2.12) denotes domestic resources allocated to exports. The determination of consumption, investment and export demand is discussed below.

2.2. Households

Household preferences are given by:

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln\left(C_t - bC_{t-1}\right) - \zeta_t^h A_L \frac{\left(h_{j,t}\right)^{1+\sigma_L}}{1+\sigma_L}\right]. \tag{2.14}$$

The household owns the economy's stock of physical capital. It determines the rate at which the capital stock is accumulated and the rate at which it is utilized. The household owns the stock of net foreign assets and determines its rate of accumulation.

2.2.1. Technology for Capital Accumulation

The law of motion of the physical stock of capital is:

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \Upsilon_t F\left(I_t, I_{t-1}\right),$$

where

$$F\left(I_t, I_{t-1}\right) = \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right) I_t,$$

and

$$\begin{aligned} \tilde{S}(x) &= \frac{1}{2}\left\{\exp\left[\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right] + \exp\left[-\sqrt{\tilde{S}''}\left(x - \mu_{z^+}\mu_\Psi\right)\right] - 2\right\} \\ &= 0,\ x = \mu_{z^+}\mu_\Psi. \end{aligned} \tag{2.15}$$

Also,

$$
\begin{aligned}
\tilde{S}'(x) &= \frac{1}{2}\sqrt{\tilde{S}''}\left\{\exp\left[\sqrt{\tilde{S}''}\left(x-\mu_{z^+}\mu_\Psi\right)\right]-\exp\left[-\sqrt{\tilde{S}''}\left(x-\mu_{z^+}\mu_\Psi\right)\right]\right\} \\
&= 0,\ x=\mu_{z^+}\mu_\Psi.
\end{aligned}
\tag{2.16}
$$

and

$$
\begin{aligned}
\tilde{S}''(x) &= \frac{1}{2}\tilde{S}''\left\{\exp\left[\sqrt{\tilde{S}''}\left(x-\mu_{z^+}\mu_\Psi\right)\right]+\exp\left[-\sqrt{\tilde{S}''}\left(x-\mu_{z^+}\mu_\Psi\right)\right]\right\} \\
&= \tilde{S}'',\ x=\mu_{z^+}\mu_\Psi.
\end{aligned}
$$

Also,

$$
\begin{aligned}
F_1\left(I_t,I_{t-1}\right) &= \left(1-\tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)-\tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} \\
&= 1,\ \frac{I_t}{I_{t-1}}=\mu_{z^+}\mu_\Psi,
\end{aligned}
$$

and,

$$
\begin{aligned}
F_2\left(I_t,I_{t-1}\right) &= \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)^2 \\
&= 0,\ \frac{I_t}{I_{t-1}}=\mu_{z^+}\mu_\Psi.
\end{aligned}
$$

Scaling,

$$
\begin{aligned}
F\left(I_t,I_{t-1}\right) &= \left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)z_t^+\Psi_t i_t \\
F_1\left(I_t,I_{t-1}\right) &= \left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)-\tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}} \\
F_2\left(I_t,I_{t-1}\right) &= \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)^2
\end{aligned}
$$

In this notation, the law of motion of capital is written,

$$
\bar{k}_{t+1}z_t^+\Psi_t=\left(1-\delta\right)\bar{K}_t z_{t-1}^+\Psi_{t-1}+\Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)z_t^+\Psi_t i_t,
$$

or,

$$
\bar{k}_{t+1}=\frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t+\Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)i_t. \tag{2.17}
$$

2.2.2. Household Consumption and Investment Decisions

The first order condition for consumption is:

$$\frac{\zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \psi_{z^+,t} p_t^c \left(1 + \tau_t^c\right) = 0. \tag{2.18}$$

To define the intertemporal Euler equation associated with the household's capital accumulation decision, we need to define the rate of return on a period t investment in a unit of physical capital, R_{t+1}^k :

$$R_{t+1}^k = \frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - \frac{p_{t+1}^i}{\Psi_{t+1}}a(u_{t+1})\right]P_{t+1} + (1-\delta)P_{t+1}P_{k',t+1} + \tau_t^k \delta P_t P_{k',t}}{P_t P_{k',t}}, \tag{2.19}$$

where it is convenient to recall

$$\frac{p_t^i}{\Psi_t}P_t = P_t^i,$$

the date t price of the homogeneous investment good. Here, $P_{k',t}$ denotes the price of a unit of newly installed physical capital, which operates in period $t+1$. This price is expressed in units of the homogeneous good, so that $P_t P_{k',t}$ is the currency price of physical capital. The numerator in the expression for R_{t+1}^k represents the period $t+1$ payoff from a unit of additional physical capital. The timing of the capital tax rate reflects the assumption that the relevant tax rate is known at the time the investment decision is made. The expression in square brackets in (2.19) captures the idea that maintenance expenses associated with the operation of capital are deductible from taxes. The last expression in the numerator expresses the idea that physical depreciation is deductible at historical cost. It is convenient to express R_t^k in terms of scaled variables:

$$\begin{aligned} R_{t+1}^k &= \frac{P_{t+1}\Psi_{t+1}}{P_t\Psi_{t+1}} \frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - \frac{p_{t+1}^i}{\Psi_{t+1}}a(u_{t+1})\right] + (1-\delta)P_{k',t+1} + \tau_t^k\delta\frac{P_t}{P_{t+1}}P_{k',t}}{P_{k',t}} \\ &= \pi_{t+1}\frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - p_{t+1}^i a(u_{t+1})\right] + (1-\delta)\Psi_{t+1}P_{k',t+1} + \tau_t^k\delta\frac{P_t}{P_{t+1}}\Psi_{t+1}P_{k',t}}{\Psi_{t+1}P_{k',t}}. \end{aligned}$$

so that

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - p_{t+1}^i a(u_{t+1})\right] + (1-\delta)p_{k',t+1} + \tau_t^k\delta\frac{\mu_{\Psi,t+1}}{\pi_{t+1}}p_{k',t}}{p_{k',t}}. \tag{2.20}$$

Capital is a good hedge against inflation, except for the way depreciation is treated. A rise in inflation effectively raises the tax rate on capital because of the practice of valuing depreciation at historical cost. The first order condition for capital implies:

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R_{t+1}^k}{\pi_{t+1}\mu_{z^+,t+1}}. \tag{2.21}$$

We differentiate the Lagrangian representation of the household's problem with respect to I_t :

$$-\upsilon_t P_t^i + \omega_t \Upsilon_t F_1 \left(I_t, I_{t-1} \right) + \beta \omega_{t+1} \Upsilon_{t+1} F_2 \left(I_{t+1}, I_t \right) = 0,$$

where υ_t denotes the multiplier on the household's nominal budget constraint and ω_t denotes the multiplier on the capital accumulation technology. In addition, the price of capital is the ratio of these multipliers:

$$P_t P_{k',t} = \frac{\omega_t}{\upsilon_t}.$$

Expressing the investment first order condition in terms of scaled variables,

$$\begin{aligned} -\frac{\psi_{z^+,t}}{z_t^+} \frac{p_t^i}{\Psi_t} + \upsilon_t P_t P_{k',t} \Upsilon_t \left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right) \frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right] & \\ + \beta \upsilon_{t+1} P_{t+1} P_{k',t+1} \Upsilon_{t+1} \tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t} \right) \left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t} \right)^2 &= 0. \end{aligned}$$

Now multiply by $z_t^+ \Psi_t$

$$\begin{aligned} -\psi_{z^+,t} p_t^i + \psi_{z^+,t} p_{k',t} \Upsilon_t \left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right) \frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right] & \\ + \beta \psi_{z^+,t+1} p_{k',t+1} \Upsilon_{t+1} \tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t} \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \mu_{\Psi,t+1}\mu_{z^+,t+1} &= 0. \end{aligned} \tag{2.22}$$

The first order condition associated with capital utilization is:

$$\Psi_t r_t^k = a'\left(u_t \right),$$

or, in scaled terms,

$$\bar{r}_t^k = a'\left(u_t \right). \tag{2.23}$$

The tax rate on capital income does not enter here because of the deductibility of maintenance costs.

2.2.3. Financial Assets

The household does the economy's saving. Period t saving occurs by the acquisition an asset which is used to finance the working capital requirements of firms. This asset pays a nominally non-state contingent return from t to $t+1$, R_t. The first order condition associated with this asset is:

$$-\psi_{z^+,t} + \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}} \left[\frac{R_t - \tau_t^b \left(R_t - \pi_{t+1} \right)}{\pi_{t+1}} \right] = 0, \tag{2.24}$$

where τ_t^b is the tax rate on the real interest rate on bond income (for additional discussion of τ^b, see section 2.3.) A consequence of our treatment of the taxation on bonds is that the steady state real after tax return on bonds is invariant to π.

2.2.4. Wage Setting

Finally, we consider wage setting. We suppose that the specialized labor supplied by households is combined by labor contractors into a homogeneous labor service as follows:

$$H_t = \left[\int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \ 1 \leq \lambda_w < \infty,$$

where h_j denotes the j^{th} household supply of labor services. Households are subject to Calvo wage setting frictions as in Erceg, Henderson and Levin (2000) (EHL). With probability $1 - \xi_w$ the j^{th} household is able to reoptimize its wage and with probability ξ_w it sets its wage according to:

$$W_{j,t+1} = \tilde{\pi}_{w,t+1} W_{j,t} \tag{2.25}$$

$$\tilde{\pi}_{w,t+1} = (\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1})^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}, \tag{2.26}$$

where $\kappa_w, \varkappa_w, \vartheta_w, \kappa_w + \varkappa_w \in (0,1)$. The wage updating factor, $\tilde{\pi}_{w,t+1}$, is sufficiently flexible that we can adopt a variety of interesting schemes.

Consider the j^{th} household that has an opportunity to reoptimize its wage at time t. We denote this wage rate by $\tilde{W}_t$. This is not indexed by j because the situation of each household that optimizes its wage is the same. In choosing $\tilde{W}_t$, the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize:

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-\zeta_{t+i}^h A_L \frac{(h_{j,t+i})^{1+\sigma_L}}{1+\sigma_L} + \upsilon_{t+i} W_{j,t+i} h_{j,t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w} \right],$$

where τ_t^y is a tax on labor income and τ_t^w is a payroll tax. Also, υ_t is the multiplier on the household's period t budget constraint. The demand for the j^{th} household's labor services, conditional on it having optimized in period t and not again since, is:

$$h_{j,t+i} = \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}.$$

Here, it is understood that $\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \equiv 1$ when $i = 0$. Substituting this into the objective function,

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L}$$

$$+ \upsilon_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],$$

It is convenient to recall the scaling of variables:

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+,\ \bar{w}_t = \frac{W_t}{z_t^+ P_t},\ \tilde{y}_t = \frac{Y_t}{z_t^+},\ w_t = \tilde{W}_t / W_t,\ z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

Then,

$$\begin{aligned}
\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} &= \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\bar{w}_{t+i} z_{t+i}^+ P_{t+i}} = \frac{\tilde{W}_t}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} \\
&= \frac{W_t \left(\tilde{W}_t / W_t\right)}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} = \frac{\bar{w}_t \left(\tilde{W}_t / W_t\right)}{\bar{w}_{t+i}} X_{t,i} = \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i},
\end{aligned}$$

where

$$\begin{aligned}
X_{t,i} &= \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i}\pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}},\ i > 0 \\
&= 1,\ i = 0.
\end{aligned}$$

It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i \to \infty$. Thus,

$$X_{t,i} = \frac{\left(\pi_t^c \cdots \pi_{t+i-1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1} \cdots \bar{\pi}_{t+i}\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^i\right)^{\varkappa_w} \left(\mu_{z^+}^i\right)^{\vartheta_w}}{\pi_{t+i}\pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}$$

In steady state,

$$\begin{aligned}
X_{t,i} &= \frac{\left(\bar{\pi}^i\right)^{\kappa_w} \left(\bar{\pi}^i\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^i\right)^{\varkappa_w} \left(\mu_{z^+}^i\right)^{\vartheta_w}}{\bar{\pi}^i \mu_{z^+}^i} \\
&= \left(\frac{\breve{\pi}^i}{\bar{\pi}^i}\right)^{\varkappa_w} \left(\mu_{z^+}^i\right)^{\vartheta_w - 1} \\
&\to 0,
\end{aligned}$$

in the no-indexing case, when $\breve{\pi} = 1$, $\varkappa_w = 1$ and $\vartheta_w = 0$.

Simplifying using the scaling notation,

$$\begin{aligned}
&E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \\
&+ \upsilon_{t+i} W_{t+i} \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],
\end{aligned}$$

or,

$$\begin{aligned}
&E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \\
&+ \psi_{z^+,t+i} w_t \bar{w}_t X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],
\end{aligned}$$

or,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}$$
$$+\psi_{z^+,t+i} w_t^{1+\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],$$

Differentiating with respect to w_t,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \lambda_w (1+\sigma_L) w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)-1}$$
$$+\psi_{z^+,t+i} w_t^{\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}] = 0$$

Dividing and rearranging,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}$$
$$+\frac{\psi_{z^+,t+i}}{\lambda_w} w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}] = 0$$

Solving for the wage rate:

$$\begin{aligned} w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} &= \frac{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}} \\ &= \frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}} \end{aligned}$$

where

$$\begin{aligned} K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L} \\ F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}. \end{aligned}$$

Thus, the wage set by reoptimizing households is:

$$w_t = \left[\frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}\right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_L)}}.$$

We now express $K_{w,t}$ and $F_{w,t}$ in recursive form:

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w \zeta_{t+1}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1})^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \right)^{1+\sigma_L} \\
&\quad + (\beta\xi_w)^2 \zeta_{t+2}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+2}} \frac{\left(\pi_t^c \pi_{t+1}^c\right)^{\kappa_w} (\bar{\pi}_{t+1}\bar{\pi}_{t+2})^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^2\right)^{\varkappa_w} \left(\mu_{z^+}^2\right)^{\vartheta_w}}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\sigma_L} \\
&\quad + ...
\end{aligned}$$

or,

$$\begin{aligned}
K_{w,t} &= \zeta_t^h H_t^{1+\sigma_L} + E_t \beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1})^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \{ \zeta_{t+1}^h H_{t+1}^{1+\sigma_L} \\
&\quad + \beta\xi_w \left(\left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}} \frac{\left(\pi_{t+1}^c\right)^{\kappa_w} (\bar{\pi}_{t+2})^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+2}\mu_{z^+,t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\sigma_L} \zeta_{t+2}^h + ... \} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1})^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1},
\end{aligned}$$

using,

$$\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^+ P_{t+1}}{\bar{w}_t z_t^+ P_t} = \frac{\bar{w}_{t+1}\mu_{z^+,t+1}\pi_{t+1}}{\bar{w}_t} \tag{2.27}$$

Also,

$$
\begin{aligned}
F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w} \\
&= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} \\
&\quad + \beta\xi_w \frac{\psi_{z^+,t+1}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \frac{1-\tau_{t+1}^y}{1+\tau_{t+1}^w} \\
&\quad + (\beta\xi_w)^2 \frac{\psi_{z^+,t+2}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \\
&\quad \times \left(\frac{\left(\pi_t^c \pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}\bar{\pi}_{t+2}\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^2\right)^{\varkappa_w} \left(\mu_{z^+}^2\right)^{\vartheta_w}}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \frac{1-\tau_{t+2}^y}{1+\tau_{t+2}^w} \\
&\quad + \ldots
\end{aligned}
$$

or,

$$
\begin{aligned}
F_{w,t} &= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} \\
&\quad + \beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} \{ \frac{\psi_{z^+,t+1}}{\lambda_w} H_{t+1} \frac{1-\tau_{t+1}^y}{1+\tau_{t+1}^w} \\
&\quad + \beta\xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\left(\pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+2}\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+2}\mu_{z^+,t+2}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} \frac{\psi_{z^+,t+2}}{\lambda_w} H_{t+2} \frac{1-\tau_{t+2}^y}{1+\tau_{t+2}^w} \\
&\quad + \ldots \} \\
&= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} + \beta\xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1},
\end{aligned}
$$

so that

$$
F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1},
$$

We obtain a second restriction on w_t using the relation between the aggregate wage rate and the wage rates of individual households:

$$
W_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}.
$$

Dividing both sides by W_t and rearranging,

$$
w_t = \left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w}.
$$

Substituting, out for w_t from the household's first order condition for wage optimization:

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)}\bar{w}_t F_{w,t}=K_{w,t}.$$

We now derive the relationship between aggregate homogeneous hours worked, H_t, and aggregate household hours,

$$h_t \equiv \int_0^1 h_{j,t}dj.$$

Substituting the demand for $h_{j,t}$ into the latter expression, we obtain,

$$\begin{aligned} h_t &= \int_0^1 \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_t dj \\ &= \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}}\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \\ &= \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \end{aligned} \tag{2.28}$$

where

$$\mathring{w}_t \equiv \frac{\mathring{W}_t}{W_t},\ \mathring{W}_t = \left[\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj\right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

Also,

$$\mathring{W}_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\tilde{\pi}_{w,t}\mathring{W}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}},$$

so that,

$$\begin{aligned} \mathring{w}_t &= \left[(1-\xi_w)(w_t)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}} \\ &= \left[(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}. \end{aligned} \tag{2.29}$$

In addition to (2.29), we have following equilibrium conditions associated with sticky wages[4]:

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} \mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \frac{1-\tau_t^y}{1+\tau_t^w} + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} \tag{2.31}$$

$$K_{w,t} = \zeta_t^h \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t\right)^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \tag{2.32}$$

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}. \tag{2.33}$$

2.3. Fiscal and Monetary Authorities

We suppose that monetary policy follows a Taylor rule of the following form:

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[r_\pi \log\left(\frac{\pi_{t+1}}{\bar{\pi}_t}\right) + r_y \log\left(\frac{y_{t-1}}{y}\right)\right] + \varepsilon_{R,t}.$$

The parameters would be taken as unknowns to be estimated. In addition, $\bar{\pi}_t$ is an exogenous process that characterizes the central bank's consumer price index inflation target and its steady state value corresponds to the steady state of actual inflation.

[4]Log linearizing these equations about the nonstochastic steady state and under the assumption of $\varkappa_w = 0$, we obtain

$$E_t \left[\begin{array}{c} \eta_0 \widehat{\bar{w}}_{t-1} + \eta_1 \widehat{\bar{w}}_t + \eta_2 \widehat{\bar{w}}_{t+1} + \eta_3\left(\hat{\pi}_t - \widehat{\bar{\pi}}_t^c\right) + \eta_4\left(\hat{\pi}_{t+1} - \rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right) \\ +\eta_5\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) + \eta_6\left(\hat{\pi}_t^c - \rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right) \\ +\eta_7\hat{\psi}_{z^+,t} + \eta_8\hat{H}_t + \eta_9\hat{\tau}_t^y + \eta_{10}\hat{\tau}_t^w + \eta_{11}\hat{\zeta}_t^h \\ +\eta_{12}\hat{\mu}_{z^+,t} + \eta_{13}\hat{\mu}_{z^+,t+1} \end{array}\right] = 0, \tag{2.30}$$

where

$$b_w = \frac{[\lambda_w\sigma_L - (1-\lambda_w)]}{[(1-\beta\xi_w)(1-\xi_w)]}$$

and

$$\begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \\ \eta_{12} \\ \eta_{13} \end{pmatrix} = \begin{pmatrix} b_w\xi_w \\ \left(\sigma_L\lambda_w - b_w\left(1+\beta\xi_w^2\right)\right) \\ b_w\beta\xi_w \\ -b_w\xi_w \\ b_w\beta\xi_w \\ b_w\xi_w\kappa_w \\ -b_w\beta\xi_w\kappa_w \\ (1-\lambda_w) \\ -(1-\lambda_w)\sigma_L \\ -(1-\lambda_w)\frac{\tau^y}{(1-\tau^y)} \\ -(1-\lambda_w)\frac{\tau^w}{(1+\tau^w)} \\ -(1-\lambda_w) \\ -b_w\xi_w \\ b_w\beta\xi_w \end{pmatrix}.$$

We model government consumption expenditures as

$$G_t = g_t z_t^+.$$

where g_t is an exogenous stochastic process, orthogonal to the other shocks in the model. All distortionary taxes are assumed to follow exogenous stochastic processes. Lump-sum transfers are assumed to balance the government budget.

2.4. Resource Constraint

We begin by deriving a relationship between total output of the homogeneous good, Y_t, and aggregate factors of production. We first consider the production of the homogenous output good:

$$\begin{aligned} Y_t^{sum} &= \int_0^1 Y_{i,t} di \\ &= \int_0^1 \left[(z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^\alpha - z_t^+ \phi \right] di \\ &= \int_0^1 \left[z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{H_{it}} \right)^\alpha H_{it} - z_t^+ \phi \right] di \\ &= z_t^{1-\alpha} \epsilon_t \left(\frac{K_t}{H_t} \right)^\alpha \int_0^1 H_{it} di - z_t^+ \phi \end{aligned}$$

where K_t is the economy-wide average stock of capital services and H_t is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$Y_t^{sum} = z_t^{1-\alpha} \epsilon_t K_t^\alpha H_t^{1-\alpha} - z_t^+ \phi.$$

Recall that the demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\mathring{Y}_t \equiv \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} di = Y_t P_t^{\frac{\lambda_d}{\lambda_d - 1}} \left(\mathring{P}_t \right)^{\frac{\lambda_d}{1-\lambda_d}},$$

say, where

$$\mathring{P}_t = \left[\int_0^1 P_{i,t}^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{2.34}$$

Dividing by P_t,

$$\mathring{p}_t = \left[\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}},$$

or,

$$\mathring{p}_t = \left[(1-\xi_p) \left(\frac{1-\xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1-\xi_p} \right)^{\lambda_d} + \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \mathring{p}_{t-1} \right)^{\frac{\lambda_d}{1-\lambda_d}} \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{2.35}$$

The preceding discussion implies:

$$Y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \mathring{Y}_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[z_t^{1-\alpha} \epsilon_t K_t^\alpha H_t^{1-\alpha} - z_t^+ \phi \right],$$

or, after scaling by z_t^+,

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^\alpha H_t^{1-\alpha} - \phi \right],$$

where

$$k_t = \bar{k}_t u_t. \tag{2.36}$$

We replace aggregate homogeneous labor, H_t, with aggregate household labor, h_t, as follows:

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^\alpha \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1-\alpha} - \phi \right]. \tag{2.37}$$

It is convenient to also have an expression that exhibits the uses of the homogeneous output,

$$z_t^+ y_t = G_t + C_t + \tilde{I}_t,$$

or, after scaling by z_t^+:

$$y_t = g_t + c_t + \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t} \mu_{z^+,t}} \right).$$

3. Alternative Representation of the Labor Market

This section replaces the model of the labor market in our baseline model with the search and matching framework of Mortensen and Pissarides (1994) and, more recently, Hall (2005a,b,c) and Shimer (2005a,b). We integrate the framework into our environment - which includes capital and monetary factors - following the Gertler, Sala and Trigari (2006) (henceforth GST) strategy implemented in Christiano, Ilut, Motto, and Rostagno (2007). A key feature of the GST model is that there are wage-setting frictions, but they do not have a direct impact on on-going worker employer relations. However, wage-setting frictions have an impact on the effort of an employer in recruiting new employees. In this sense, the setup is not vulnerable to the Barro (1977) critique of sticky wages. The model is also attractive because of the richness of its labor market implications: the model differentiates between hours worked and the quantity of people employed, it has unemployment and vacancies.

The labor market in our alternative labor market model is a modified version of the GST model. GST assume wage-setting frictions of the Calvo type, while we instead work with Taylor-type frictions. In addition, we adopt a slightly different representation of the production sector in order to maximize comparability with our baseline model. A key difference is that we allow for endogenous separation of employees from their jobs, as in e.g. den Haan, Ramey and Watson (2000). In what follows, we first provide an overview and after that we present the detailed decision problems of agents in the labor market.

3.1. Sketch of the Model

As in the discussion of section 2.1, we adopt the Dixit-Stiglitz specification of homogeneous goods production. A representative, competitive retail firm aggregates differentiated intermediate goods into a homogeneous good. Intermediate goods are supplied by monopolists, who hire labor and capital services in competitive factor markets. The intermediate good firms are assumed to be subject to the same Calvo price setting frictions in the baseline model.

In the baseline model, the homogeneous labor services supplied to the competitive labor market by labor retailers (contractors) who combine the labor services supplied to them by households who monopolistically supply specialized labor services (see section 2.1). The modified model dispenses with the specialized labor services abstraction. Labor services are instead supplied to the homogeneous labor market by 'employment agencies'. The change leaves the equilibrium conditions associated with the production of the homogeneous good unaffected.[5]

Each employment agency retains a large number of workers. At the beginning of the period a fraction, $1-\rho$, of workers is randomly selected to separate from the firm and go into unemployment. Also, a number of new workers arrive from unemployment in proportion to the number of vacancies posted by the agency in the previous period. After separation and new arrivals occur, the nominal wage rate is set.

[5] An alternative (perhaps more natural) formulation would be for the intermediate good firms to do their own employment search. We instead separate the task of finding workers from production of intermediate goods in order to avoid adding a state variable to the intermediate good firm, which would complicate the solution of their price-setting problem.

The nominal wage paid to an individual worker is determined by Nash bargaining, which occurs once every N periods. The employees of an agency are represented by a union at negotiations. This assumption has no consequences except that it makes clear which wage (i.e. the collectively negotiated wage) will apply to workers arriving at the agency during the duration of the wage contract. Each employment agency is permanently allocated to one of N different cohorts. Cohorts are differentiated according to the period in which they renegotiate their wage. Since there is an equal number of agencies in each cohort, $1/N$ of the agencies bargain in each period. The wage in agencies that do not bargain in the current period is updated from the previous period according to the same rule used in our baseline model.

Next, each worker realizes an idiosyncratic productivity shock and workers with a shock below an endogenously determined cutoff separate into unemployment. The cutoff level of productivity is chosen relative to a particular surplus criterion, maximizing the surplus of the employment agency. The intensity of each worker's labor effort is then determined by an efficiency criterion. To explain how labor intensity is chosen, we discuss the implications of increased intensity for the worker and for the employment agency. The utility function of the household in the present labor market model is a modified version of (2.14):

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \{ \zeta_{t+l}^c \log(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l}^h A_L \left[\sum_{i=0}^{N-1} \frac{\left(\varsigma_{i,t+l}\right)^{1+\sigma_L}}{1+\sigma_L} \left[1 - \mathcal{F}\left(\bar{a}_{t+l}^i\right)\right] l_{t+l}^i \right] \}, \quad (3.1)$$

where $\left[1 - \mathcal{F}\left(\bar{a}_{t+l}^i\right)\right] l_{t+l}^i$ is the quantity of people working in cohort i and $\varsigma_{i,t}$ is the intensity with with each worker in cohort i works. As in GST, we follow the family household construct of Merz (1995) in supposing that each household has a large number of workers. Although the individual worker's labor market experience - whether employed or unemployed - is determined in part by idiosyncratic shocks, the household has sufficiently many workers that the total fraction of workers employed, L_t, as well as the fractions allocated among the different cohorts, $\left[1 - \mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i$, $i = 0, ..., N-1$, is the same for each household. We suppose that all the household's workers are supplied inelastically to the labor market (i.e., labor force participation is constant). Each worker passes randomly from employment with a particular agency to unemployment and back to employment according to the endogenous probabilities described below.

The household's currency receipts arising from the labor market are:

$$(1 - L_t) P_t^c b^u z_t^+ + \sum_{i=0}^{N-1} W_t^i \overbrace{\left[1 - \mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i}^{\text{quantity of people working in cohort } i} \varsigma_{i,t} \frac{1 - \tau_t^y}{1 + \tau_t^w} \quad (3.2)$$

where W_t^i is the nominal wage rate earned by workers in cohort $i = 0, ..., N-1$. The index, i, indicates the number of periods in the past when bargaining occurred most recently. As in our baseline model, there is a labor income tax τ_t^y and a payroll tax τ_t^w that affect the after-tax wage. Note that we implicitly assume that labor intensity, $\varsigma_{i,t}$, is cohort-specific. This is explained below. The presence of the term involving b^u indicates the assumption that unemployed workers receive a payment of $b^u z_t^+$ final consumption goods. The unemployment benefits are financed by lump sum taxes.

Let the price of labor services, W_t, denote the marginal gain to the employment agency that occurs when an individual worker raises labor intensity by one unit. Because the employment agency is competitive in the supply of labor services, W_t is taken as given and is the same for all agencies, regardless of which cohort it is in. Labor intensity equates the worker's marginal cost to the agency's marginal benefit:

$$W_t \frac{\mathcal{E}_t^i}{1-\mathcal{F}_t^i} = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L} \frac{1}{\upsilon_t \frac{1-\tau_t^y}{1+\tau_t^w}} \tag{3.3}$$

for $i = 0, ..., N-1$. Here,

$$\begin{aligned} \mathcal{E}_t^j &\equiv \mathcal{E}\left(\bar{a}_t^j; \sigma_{a,t}\right) \equiv \int_{\bar{a}_t^j}^{\infty} a d\mathcal{F}\left(a; \sigma_{a,t}\right) \\ \mathcal{F}_t^i &= \mathcal{F}\left(\bar{a}_t^j; \sigma_{a,t}\right) = \int_0^{\bar{a}_t^j} d\mathcal{F}\left(a; \sigma_{a,t}\right). \end{aligned}$$

Division by $1-\mathcal{F}_t^i$ is required in (3.3) so that the expectation is relative to the distribution of a conditional on $a \geq \bar{a}_t^j$. To understand the expression on the right of (3.3), note that the marginal cost, in utility terms, to an individual worker who increases labor intensity by one unit is $\zeta_t^h A_L \varsigma_{i,t}^{\sigma_L}$. This is converted to after-tax currency units by dividing by the multiplier, υ_t, on the household's nominal budget constraint as well as by the tax wedge due to labor income taxes and payroll taxes. Scaling (3.3) by $P_t z_t^+$yields:

$$\bar{w}_t \frac{\mathcal{E}_t^i}{1-\mathcal{F}_t^i} = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L} \frac{1}{\psi_{z^+,t} \frac{1-\tau_t^y}{1+\tau_t^w}} \tag{3.4}$$

Labor intensity will be different across cohorts because $\mathcal{E}_t^i / \left(1-\mathcal{F}_t^i\right)$ in (3.4) is indexed by cohort. When the wage rate is determined by Nash bargaining, it is taken into account that labor intensity is determined according to (3.4) and that some workers will endogenously separate. Note, that the ratio

$$\frac{\mathcal{E}_t^i}{\left(1-\mathcal{F}_t^i\right) \varsigma_{i,t}^{\sigma_L}}$$

will be the same for all cohorts since all other variables in (3.4) are not indexed by cohort.

Finally, the employment agency in the i^{th} cohort determines how many employees it will have in period $t+1$ by choosing vacancies, v_t^i. The vacancy posting costs associated with v_t^i

are:

$$\frac{\kappa z_t^+}{2}\left(\frac{Q_t^\iota v_t^i}{\left[1-\mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i}\right)^2\left[1-\mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i,$$

units of the domestic homogeneous good. Here, $\left[1-\mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i$ denotes the number of employees in the i^{th} cohort after endogenous separations have occurred and $\kappa z_t^+/2$ is a cost parameter which is assumed to grow at the same rate as the overall economic growth rate. Also, Q_t is the probability that a posted vacancy is filled. The functional form of our cost function nests GT and GST when $\iota = 1$. With this parameterization the cost function is in terms of the number of people hired, not the number of vacancies per se. We interpret this as reflecting that the GT and GST specifications emphasize internal costs (such as training and other) of adjusting the work force, and not search costs. In models used in the search literature (see, e.g., Shimer (2005a)), vacancy posting costs are independent of Q_t, i.e., $\iota = 0$. We also plan to investigate this latter case. We suspect that the model implies less amplification in response to expansionary shock in the case, $\iota = 0$. In a boom, Q_t can be expected to fall, so that with $\iota = 1$, costs of posting vacancies decrease in the GT specification.

3.2. Model Details

An employment agency in the i^{th} cohort which does not renegotiate its wage in period t sets the period t wage, $W_{i,t}$, as in (2.25):

$$W_{i,t} = \tilde{\pi}_{w,t} W_{i-1,t-1},\ \tilde{\pi}_{w,t} \equiv \left(\pi_{t-1}^c\right)^{\kappa_w}\left(\bar{\pi}_t\right)^{(1-\kappa_w-\varkappa_w)}\left(\breve{\pi}\right)^{\varkappa_w}\left(\mu_{z^+}\right)^{\vartheta_w}, \tag{3.5}$$

for $i = 1, ..., N-1$ (note that an agency that was in the i^{th} cohort in period t was in cohort $i-1$ in period $t-1$) where $\kappa_w, \varkappa_w, \vartheta_w, \kappa_w + \varkappa_w \in (0,1)$. After wages are set, employment agencies in cohort i decide on endogenous separation, post vacancies to attract new workers in the next period and supply labor services, $l_t^i \varsigma_{i,t}$, into competitive labor markets.

3.2.1. The Employment-Agency Problem

To understand how agencies bargain and how they make their employment decisions, it is useful to consider $F\left(l_t^0, \omega_t\right)$, the value function of the representative employment agency in the cohort that negotiates its wage in the current period. The arguments of F are the agency's workforce after beginning-of-period exogenous separations and new arrivals, l_t^0, and an arbitrary value for the nominal wage rate, ω_t. We are thus interested in the firm's problem after the wage rate has been set, when endogenous separations take place, followed by the setting of vacancies. To simplify notation, we leave out arguments of F that correspond to economy-wide variables. We find it convenient to adopt a change of variables. We suppose that the firm chooses a particular monotone transform of vacancy postings, which we denote by $\tilde{v}_t^i$:

$$\tilde{v}_t^i \equiv \frac{Q_t^\iota v_t^i}{\left(1-\mathcal{F}_t^j\right) l_t^i},$$

where $1-\mathcal{F}_t^j$ denotes the fraction of the beginning-of-period t workforce in cohort j which remains after endogenous separations. The agency's hiring rate is related to $\tilde{v}_t^i$ by:

$$\chi_t^i = Q_t^{1-\iota}\tilde{v}_t^i. \tag{3.6}$$

The timing in the endogenous separation model is that at the beginning of period t, exogenous separations occur, and new arrivals occur. Then, if this is a bargaining period, bargaining occurs. Then, idiosyncratic productivities are realized and a cutoff productivity, $\bar{a}_t^j$, is determined. Thus, the fraction of the current workforce in cohort j that is let go is $\mathcal{F}_t^j$ and the fraction that survives is $1-\mathcal{F}_t^j$. So, if l_t^j is the work force just after exogenous separations and new arrivals, then

$$\left(1-\mathcal{F}_t^j\right) l_t^j$$

is the size of the workforce after endogenous separations. The law of motion of the work force in each cohort is:

$$l_{t+1}^{j+1} = \left(\chi_t^j + \rho\right)\left(1-\mathcal{F}_t^j\right) l_t^j, \tag{3.7}$$

for $j = 0, 1, ..., N-1$, with the understanding here and throughout that $j = N$ is to be interpreted as $j = 0$ and where l_{t+1}^{j+1} is the workforce after new arrivals and exogenous separations in period $t+j$. Expression (3.7) is deterministic, reflecting the assumption that the agency employs a large number of workers.. After endogenous separations, agencies post vacancies.

The value function of the firm is:

$$\begin{aligned} F\left(l_t^0, \omega_t\right) &= \sum_{j=0}^{N-1} \beta^j E_t \frac{\upsilon_{t+j}}{\upsilon_t} \max_{\tilde{v}_{t+j}^j}\Big[\int_{\bar{a}_{t+j}^j}^{\infty} \left(W_{t+j}a - \Gamma_{t,j}\omega_t\right) \varsigma_{j,t+j} \overbrace{d\mathcal{F}(a)}^{\text{`fraction' of } l_{t+j}^j \text{ with productivity } a} \\ &- \overbrace{P_{t+j}\frac{\kappa z_{t+j}^+}{2}\left(\tilde{v}_t^j\right)^2\left(1-\mathcal{F}_{t+j}^j\right)}^{\text{costs are proportional to workforce after current period separations}} \Big] l_{t+j}^j + \beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right), \end{aligned}$$

where $\varsigma_{j,t}$ is assumed to satisfy (3.4). Simplifying,

$$\begin{aligned} F\left(l_t^0, \omega_t\right) &= \sum_{j=0}^{N-1} \beta^j E_t \frac{\upsilon_{t+j}}{\upsilon_t} \max_{\tilde{v}_{t+j}^j}\Big[\left(W_{t+j}\mathcal{E}_{t+j}^j - \Gamma_{t,j}\omega_t\left[1-\mathcal{F}_{t+j}^j\right]\right)\varsigma_{j,t+j} \\ &\quad - P_{t+j}\frac{\kappa z_{t+j}^+}{2}\left(\tilde{v}_t^j\right)^2\left(1-\mathcal{F}_{t+j}^j\right)\Big] l_{t+j}^j \\ &\quad + \beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right), \end{aligned} \tag{3.8}$$

Here,

$$\Gamma_{t,j} = \begin{cases} \tilde{\pi}_{w,t+j} \cdots \tilde{\pi}_{w,t+1}, & j > 0 \\ 1 & j = 0 \end{cases}. \tag{3.9}$$

Also, $\tilde{W}_{t+N}$ denotes the Nash bargaining wage rate that will be negotiated when the agency next has an opportunity to do so. At time t, the agency takes $\tilde{W}_{t+N}$ as given.

Writing out (3.8):

$$\begin{aligned} F\left(l_t^0, \omega_t\right) &= \max_{\left\{v_{t+j}^j\right\}_{j=0}^{N-1}} \{ \left[\left(W_t \mathcal{E}_t^0 - \omega_t \left(1 - \mathcal{F}_t^0\right)\right) \varsigma_t - P_t \frac{\kappa z_t^+}{2} \left(\tilde{v}_t^0\right)^2 \left(1 - \mathcal{F}_t^0\right) \right] l_t^0 \\ &+ \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t} \left[\left(W_{t+1} \mathcal{E}_{t+1}^1 - \Gamma_{t,1} \omega_t \left(1 - \mathcal{F}_{t+1}^1\right)\right) \varsigma_{t+1} - P_{t+1} \frac{\kappa z_{t+1}^+}{2} \left(\tilde{v}_{t+1}^1\right)^2 \left(1 - \mathcal{F}_{t+1}^1\right) \right] \\ &\times \left(\chi_t^0 + \rho\right) \left[1 - \mathcal{F}_t^0\right] l_t^0 \\ &+ \beta^2 E_t \frac{\upsilon_{t+2}}{\upsilon_t} \left[\left(W_{t+2} \mathcal{E}_{t+2}^2 - \Gamma_{t,2} \omega_t \left(1 - \mathcal{F}_{t+2}^2\right)\right) \varsigma_{t+2} - P_{t+2} \frac{\kappa z_{t+2}^+}{2} \left(\tilde{v}_{t+2}^2\right)^2 \left(1 - \mathcal{F}_{t+2}^2\right) \right] \\ &\times \left(\chi_{t+1}^1 + \rho\right) \left(\chi_t^0 + \rho\right) \left(1 - \mathcal{F}_{t+1}^1\right) \left(1 - \mathcal{F}_t^0\right) l_t^0 \\ &+ \beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right). \end{aligned}$$

The firm chooses vacancies to solve the problem in (3.8). We impose the following property:

$$F\left(l_t^0, \omega_t\right) = J\left(\omega_t\right) l_t^0, \tag{3.10}$$

where $J\left(\omega_t\right)$ is not a function of l_t^0. The function, $J\left(\omega_t\right)$, is the surplus that a firm bargaining in the current period enjoys from a match with an individual worker, when the current wage is ω_t. For convenience, we omit the expectation operator E_t below. Let

$$\begin{aligned} J\left(\omega_t\right) &= \max_{\left\{v_{t+j}^j\right\}_{j=0}^{N-1}} \{ \left(W_t \mathcal{E}_t^0 - \omega_t \left(1 - \mathcal{F}_t^0\right)\right) \varsigma_{0,t} - P_t z_t^+ \frac{\kappa}{2} \left(\tilde{v}_t^0\right)^2 \left[1 - \mathcal{F}_t^0\right] \\ &+ \beta \frac{\upsilon_{t+1}}{\upsilon_t} \left[\left(W_{t+1} \mathcal{E}_{t+1}^1 - \Gamma_{t,1} \omega_t \left(1 - \mathcal{F}_{t+1}^1\right)\right) \varsigma_{1,t+1} - P_{t+1} z_{t+1}^+ \frac{\kappa}{2} \left(\tilde{v}_{t+1}^1\right)^2 \left(1 - \mathcal{F}_{t+1}^1\right) \right] \times \\ &\left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(1 - \mathcal{F}_t^0\right) \\ &+ \beta^2 \frac{\upsilon_{t+2}}{\upsilon_t} \left[\left(W_{t+2} \mathcal{E}_{t+2}^2 - \Gamma_{t,2} \omega_t \left(1 - \mathcal{F}_{t+2}^2\right)\right) \varsigma_{2,t+2} - P_{t+2} z_{t+2}^+ \frac{\kappa}{2} \left(\tilde{v}_{t+2}^2\right)^2 \left(1 - \mathcal{F}_{t+2}^2\right) \right] \times \\ &\left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota} + \rho\right) \left(1 - \mathcal{F}_{t+1}^1\right) \left[1 - \mathcal{F}_t^0\right] \\ &+ \ldots + \\ &+ \beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J\left(\tilde{W}_{t+N}\right) \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_t^{1-\iota} + \rho\right) \cdots \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \times \\ &\left(1 - \mathcal{F}_{t+N-1}^{N-1}\right) \cdots \left(1 - \mathcal{F}_t^0\right) \}. \end{aligned}$$

Differentiate with respect to $\tilde{v}_t^0$ and multiply the result by $\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota}$, to obtain:

$$\begin{aligned}
0 &= -P_t z_t^+ \kappa \tilde{v}_t^0 \left[1-\mathcal{F}_t^0\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota} \\
&\quad +\beta \frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^1-\Gamma_{t,1}\omega_t\left[1-\mathcal{F}_{t+1}^1\right]\right)\varsigma_{1,t+1}-P_{t+1}z_{t+1}^+\frac{\kappa}{2}\left(\tilde{v}_{t+1}^1\right)^2\left(1-\mathcal{F}_{t+1}^1\right)\right]\times \\
&\quad \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left[1-\mathcal{F}_t^0\right] \\
&\quad +\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left[1-\mathcal{F}_{t+2}^2\right]\right)\varsigma_{2,t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\left(1-\mathcal{F}_{t+2}^2\right)\right]\times \\
&\quad \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\
&\quad +...+ \\
&\quad +\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\times \\
&\quad \left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\cdots\left[1-\mathcal{F}_t^0\right]\} \\
&= J\left(\omega_t\right)-\left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}+P_t z_t^+\frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2\left[1-\mathcal{F}_t^0\right] \\
&\quad -P_t z_t^+\kappa\tilde{v}_t^0\left[1-\mathcal{F}_t^0\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota}
\end{aligned}$$

Since the latter expression must be zero, we conclude:

$$\begin{aligned}
J\left(\omega_t\right) &= \left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}-P_t z_t^+\frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2\left[1-\mathcal{F}_t^0\right]+P_t z_t^+\kappa\tilde{v}_t^0\left[1-\mathcal{F}_t^0\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota} \\
&= \left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}+P_t z_t^+\kappa\left[\frac{1}{2}\left(\tilde{v}_t^0\right)^2+\tilde{v}_t^0\frac{\rho}{Q_t^{1-\iota}}\right]\left[1-\mathcal{F}_t^0\right]
\end{aligned}$$

Next, we obtain simple expressions for the vacancy decisions from their first order necessary conditions for optimality. Multiplying the first order condition for $\tilde{v}_{t+1}^1$ by

$$\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+1}^{1-\iota}},$$

we obtain:

$$\begin{aligned}
0 &= -\beta\frac{\upsilon_{t+1}}{\upsilon_t}P_{t+1}z_{t+1}^+\kappa\tilde{v}_{t+1}^1\left[1-\mathcal{F}_{t+1}^1\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+1}^{1-\iota}}\left[1-\mathcal{F}_t^0\right] \\
&\quad +\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{2,t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\left[1-\mathcal{F}_{t+2}^2\right]\right]\times \\
&\quad \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\
&\quad +...+ \\
&\quad +\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\times \\
&\quad \left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\cdots\left[1-\mathcal{F}_t^0\right]
\end{aligned}$$

Substitute out the period $t+2$ and higher terms in this expression using the first order condition for $\tilde{v}_t^0$. After rearranging, we obtain,

$$\frac{P_t z_t^+\kappa\tilde{v}_t^0}{Q_t^{1-\iota}}=\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\begin{array}{c}\left(W_{t+1}\mathcal{E}_{t+1}^1-\Gamma_{t,1}\omega_t\left(1-\mathcal{F}_{t+1}^1\right)\right)\varsigma_{1,t+1} \\ +P_{t+1}z_{t+1}^+\kappa\left(1-\mathcal{F}_{t+1}^1\right)\left(\frac{\left(\tilde{v}_{t+1}^1\right)^2}{2}+\frac{\tilde{v}_{t+1}^1\rho}{Q_{t+1}^{1-\iota}}\right)\end{array}\right].$$

Following the pattern set with $\tilde{v}_{t+1}^{1}$, multiply the first order condition for $\tilde{v}_{t+2}^{2}$ by

$$\left(\tilde{v}_{t+2}^{2}Q_{t+2}^{1-\iota}+\rho\right)\frac{1}{Q_{t+2}^{1-\iota}}.$$

Substitute the period $t+3$ and higher terms in the first order condition for $\tilde{v}_{t+2}^{2}$ using the first order condition for $\tilde{v}_{t+1}^{1}$ to obtain, after rearranging,

$$P_{t+1}z_{t+1}^{+}\kappa\tilde{v}_{t+1}^{1}\frac{1}{Q_{t+1}^{1-\iota}}=\beta\frac{\upsilon_{t+2}}{\upsilon_{t+1}}\left[\begin{array}{c}\left(W_{t+2}\mathcal{E}_{t+2}^{2}-\Gamma_{t,2}\omega_{t}\left(1-\mathcal{F}_{t+2}^{2}\right)\right)\varsigma_{2,t+2}\\+P_{t+2}z_{t+2}^{+}\kappa\left(1-\mathcal{F}_{t+2}^{2}\right)\left(\frac{\left(\tilde{v}_{t+2}^{2}\right)^{2}}{2}+\frac{\tilde{v}_{t+2}^{2}\rho}{Q_{t+2}^{1-\iota}}\right)\end{array}\right]$$

Continuing in this way, we obtain,

$$P_{t+j}z_{t+j}^{+}\kappa\tilde{v}_{t+j}^{j}\frac{1}{Q_{t+j}^{1-\iota}}=\beta\frac{\upsilon_{t+j+1}}{\upsilon_{t+j}}\left[\begin{array}{c}\left(W_{t+j+1}\mathcal{E}_{t+j+1}^{j+1}-\Gamma_{t,j+1}\omega_{t}\left(1-\mathcal{F}_{t+j+1}^{j+1}\right)\right)\varsigma_{t+j+1}\\+P_{t+j+1}z_{t+j+1}^{+}\kappa\left(1-\mathcal{F}_{t+j+1}^{j+1}\right)\left(\frac{\left(\tilde{v}_{t+j+1}^{j+1}\right)^{2}}{2}+\frac{\tilde{v}_{t+j+1}^{j+1}\rho}{Q_{t+j+1}^{1-\iota}}\right)\end{array}\right],$$

for $j=0,1,...,N-2$. Now consider the first order necessary condition for the optimality of $\tilde{v}_{t+N-1}^{N-1}$. After multiplying this first order condition by

$$\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+N-1}^{1-\iota}},$$

we obtain,

$$\begin{aligned}0\ =\ & -\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_{t}}P_{t+N-1}z_{t+N-1}^{+}\kappa\tilde{v}_{t+N-1}^{N-1}\left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\left(\tilde{v}_{t}^{0}Q_{t}^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^{1}Q_{t+1}^{1-\iota}+\rho\right)\cdots\\&\cdots\left(\tilde{v}_{t+N-2}^{N-2}Q_{t+N-2}^{1-\iota}+\rho\right)\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+N-1}^{1-\iota}}\left[1-\mathcal{F}_{t+N-2}^{N-2}\right]\cdots\left[1-\mathcal{F}_{t}^{0}\right]\\&+\beta^{N}\frac{\upsilon_{t+N}}{\upsilon_{t}}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_{t}^{0}Q_{t}^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^{1}Q_{t}^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\times\\&\left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\cdots\left[1-\mathcal{F}_{t}^{0}\right]\}\end{aligned}$$

or,

$$P_{t+N-1}z_{t+N-1}^{+}\kappa\tilde{v}_{t+N-1}^{N-1}\frac{1}{Q_{t+N-1}^{1-\iota}}=\beta\frac{\upsilon_{t+N}}{\upsilon_{t+N-1}}J\left(\tilde{W}_{t+N}\right).$$

Making use of our expression for J, we obtain:

$$P_{t+N-1}z_{t+N-1}^{+}\kappa\tilde{v}_{t+N-1}^{N-1}\frac{1}{Q_{t+N-1}^{1-\iota}}=\beta\frac{\upsilon_{t+N}}{\upsilon_{t+N-1}}\left[\begin{array}{c}\left(W_{t+N}\mathcal{E}_{t+N}^{0}-\tilde{W}_{t+N}\left(1-\mathcal{F}_{t+N}^{0}\right)\right)\varsigma_{0,t+N}\\+P_{t+N}z_{t+N}^{+}\kappa\left(\frac{\left(\tilde{v}_{t+N}^{0}\right)^{2}}{2}+\frac{\tilde{v}_{t+N}^{0}\rho}{Q_{t+N}^{1-\iota}}\right)\left[1-\mathcal{F}_{t+N}^{0}\right]\end{array}\right].$$

The above first order conditions apply over time to a group of agencies that bargain at date t. We now express the first order conditions for a fixed date and different cohorts:

$$\begin{aligned}P_{t}z_{t}^{+}\kappa\tilde{v}_{t}^{j}\frac{1}{Q_{t}^{1-\iota}}\ =\ & \beta\frac{\upsilon_{t+1}}{\upsilon_{t}}\left[\begin{array}{c}\left(W_{t+1}\mathcal{E}_{t+1}^{j+1}-\Gamma_{t-j,j+1}\tilde{W}_{t-j}\left(1-\mathcal{F}_{t+1}^{j+1}\right)\right)\varsigma_{j+1,t+1}\\+P_{t+1}z_{t+1}^{+}\kappa\left[1-\mathcal{F}_{t+1}^{j+1}\right]\left(\frac{\left(\tilde{v}_{t+1}^{j+1}\right)^{2}}{2}+\frac{\tilde{v}_{t+1}^{j+1}\rho}{Q_{t+1}^{1-\iota}}\right)\end{array}\right],\\&\text{for }j=0,...,N-2\end{aligned}$$

Divide both sides by $P_t z_t^+$ and express the result in terms of scaled variables:

$$\begin{aligned} \kappa\tilde{v}_t^j \frac{1}{Q_t^{1-\iota}} &= \beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\left[\begin{array}{c} \left(\bar{w}_{t+1}\mathcal{E}_{t+1}^{j+1} - G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}\left(1-\mathcal{F}_{t+1}^{j+1}\right)\right)\varsigma_{j+1,t+1} \\ +\kappa\left[1-\mathcal{F}_{t+1}^{j+1}\right]\left(\frac{\left(\tilde{v}_{t+1}^{j+1}\right)^2}{2} + \frac{\tilde{v}_{t+1}^{j+1}\rho}{Q_{t+1}^{1-\iota}}\right) \end{array}\right], \quad (3.11) \\ &\text{for } j = 0,...,N-2 \end{aligned}$$

where

$$\begin{aligned} G_{t-i,i+1} &= \frac{\tilde{\pi}_{w,t+1}\cdots\tilde{\pi}_{w,t-i+1}}{\pi_{t+1}\cdots\pi_{t-i+1}}\left(\frac{1}{\mu_{z^+,t-i+1}}\right)\cdots\left(\frac{1}{\mu_{z^+,t+1}}\right),\ i\geq 0, \quad (3.12) \\ w_t &= \frac{\tilde{W}_t}{W_t},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t}. \end{aligned}$$

Also,

$$G_{t,j} = \begin{cases} \frac{\tilde{\pi}_{w,t+j}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+j}\cdots\pi_{t+1}}\left(\frac{1}{\mu_{z^+,t+1}}\right)\cdots\left(\frac{1}{\mu_{z^+,t+j}}\right) & j>0 \\ 1 & j=0 \end{cases}. \quad (3.13)$$

The scaled vacancy first order condition of agencies that are in the last period of their contract is:

$$\kappa\tilde{v}_t^{N-1}\frac{1}{Q_t^{1-\iota}} = \beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\left[\begin{array}{c} \left(\bar{w}_{t+1}\mathcal{E}_{t+1}^{0} - w_{t+1}\bar{w}_{t+1}\left(1-\mathcal{F}_{t+1}^{0}\right)\right)\varsigma_{0,t+1} \\ +\kappa\left[1-\mathcal{F}_{t+1}^{0}\right]\left(\frac{\left(\tilde{v}_{t+1}^{0}\right)^2}{2} + \frac{\tilde{v}_{t+1}^{0}\rho}{Q_{t+1}^{1-\iota}}\right) \end{array}\right]. \quad (3.14)$$

We require the derivative of J with respect to ω_t. By the envelope condition, we can ignore the impact of a change in ω_t on endogenous separations and vacancy decisions, and only be concerned with the direct impact of ω_t on J. Taking the derivative of (**??**):

$$\begin{aligned} J_{w,t} &= -\left(1-\mathcal{F}_t^0\right)\varsigma_{0,t} \\ &-\beta\frac{\upsilon_{t+1}}{\upsilon_t}\Gamma_{t,1}\varsigma_{1,t+1}\left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right) \\ &-\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\Gamma_{t,2}\varsigma_{2,t+2}\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\left(1-\mathcal{F}_{t+2}^2\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\ &-...-\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\Gamma_{t,N-1}\varsigma_{N-1,t+N-1}\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\cdots\left(\chi_{t+1}^{N-2}+\rho\right)\times \\ &\left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left[1-\mathcal{F}_t^0\right]. \end{aligned}$$

Let,

$$\Omega_{t+j}^j = \begin{cases} \left(1-\mathcal{F}_{t+j}^j\right)\prod_{l=0}^{j-1}\left(\chi_{t+l}^l+\rho\right)\left(1-\mathcal{F}_{t+l}^l\right) & j>0 \\ 1-\mathcal{F}_t^0 & j=0 \end{cases}. \quad (3.15)$$

It is convenient to express this in recursive form:

$$\begin{aligned}
\Omega_t^0 &= 1-\mathcal{F}_t^0 \\
\Omega_{t+1}^1 &= \left(1-\mathcal{F}_{t+1}^1\right)\left(\chi_t^0+\rho\right)\overbrace{\left(1-\mathcal{F}_t^0\right)}^{\Omega_t^0} \\
\Omega_{t+2}^2 &= \left(1-\mathcal{F}_{t+2}^2\right)\left(\chi_{t+1}^1+\rho\right)\overbrace{\left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_t^0\right)\left(1-\mathcal{F}_{t+1}^1\right)}^{\Omega_{t+1}^1}
\end{aligned}$$

so that

$$\Omega_{t+j}^j=\left(1-\mathcal{F}_{t+j}^j\right)\left(\chi_{t+j-1}^{j-1}+\rho\right)\Omega_{t+j-1}^{j-1},$$

for $j=1,2,....$. It is convenient to define these objects at date t as a function of variables dated t and earlier for the purposes of implementing these equations in Dynare:

$$\begin{aligned}
\Omega_t^0 &= 1-\mathcal{F}_t^0 \\
\Omega_t^1 &= \left(1-\mathcal{F}_t^1\right)\left(\chi_{t-1}^0+\rho\right)\overbrace{\left(1-\mathcal{F}_{t-1}^0\right)}^{\Omega_{t-1}^0} \\
\Omega_t^2 &= \left(1-\mathcal{F}_t^2\right)\left(\chi_{t-1}^1+\rho\right)\overbrace{\left(\chi_{t-2}^0+\rho\right)\left(1-\mathcal{F}_{t-2}^0\right)\left(1-\mathcal{F}_{t-1}^1\right)}^{\Omega_{t-1}^1}
\end{aligned}$$

so that

$$\Omega_t^j=\left(1-\mathcal{F}_t^j\right)\left(\chi_{t-1}^{j-1}+\rho\right)\Omega_{t-1}^{j-1},$$

Then,

$$\begin{aligned}
J_{w,t} &= -\varsigma_{0,t}\Omega_t^0 \\
&\quad -\beta\frac{\upsilon_{t+1}}{\upsilon_t}\Gamma_{t,1}\varsigma_{1,t+1}\Omega_{t+1}^1 \\
&\quad -\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\Gamma_{t,2}\varsigma_{2,t+2}\Omega_{t+2}^2 \\
&\quad -...-\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\Gamma_{t,N-1}\varsigma_{N-1,t+N-1}\Omega_{t+N-1}^{N-1} \\
&= -\sum_{j=0}^{N-1}\beta^j\frac{\upsilon_{t+j}}{\upsilon_t}\Gamma_{t,j}\Omega_{t+j}^j\varsigma_{j,t+j}.
\end{aligned}$$

In terms of scaled variables,

$$J_{w,t}=-\sum_{j=0}^{N-1}\beta^j\frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}G_{t,j}\Omega_{t+j}^j\varsigma_{j,t+j}. \tag{3.16}$$

The following is an expression for J_t evaluated at $\omega_t = \tilde{W}_t$, in terms of scaled variables. Dividing by $P_t z_t^+$:

$$
\begin{aligned}
J_{z^+,t} &= \frac{J\left(\tilde{W}_t\right)}{P_t z_t^+} = \frac{\left(W_t \mathcal{E}_t^0 - \tilde{W}_t\left(1-\mathcal{F}_t^0\right)\right)}{P_t z_t^+}\varsigma_{0,t} - \frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2\left(1-\mathcal{F}_t^0\right) \\
&+\beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\left[\frac{\left(W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\tilde{W}_t\left(1-\mathcal{F}_{t+1}^1\right)\right)}{P_{t+1}z_{t+1}^+}\varsigma_{1,t+1} - \frac{\kappa}{2}\left(\tilde{v}_{t+1}^1\right)^2\left(1-\mathcal{F}_{t+1}^1\right)\right] \\
&\times\left(\chi_t^0+\rho\right)\left[1-\mathcal{F}_t^0\right] \\
&+\beta^2\frac{\psi_{z^+,t+2}}{\psi_{z^+,t}}\left[\frac{\left(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\tilde{W}_t\left(1-\mathcal{F}_{t+2}^2\right)\right)}{P_{t+2}z_{t+2}^+}\varsigma_{2,t+2} - \frac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\left(1-\mathcal{F}_{t+2}^2\right)\right] \\
&\times\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right) \\
&+ ... + \\
&+\beta^N\frac{\psi_{z^+,t+N}}{\psi_{z^+,t}}J_{z^+,t+N}\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\cdots\left(\chi_{t+N-1}^{N-1}+\rho\right) \\
&\times\left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^0\right)
\end{aligned}
$$

or,

$$
\begin{aligned}
J_{z^+,t} &= \sum_{j=0}^{N-1}\beta^j\frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}\left[\left(\bar{w}_{t+j}\frac{\mathcal{E}_{t+j}^j}{1-\mathcal{F}_{t+j}^j} - G_{t,j}w_t\bar{w}_t\right)\varsigma_{j,t+j} - \frac{\kappa}{2}\left(\tilde{v}_{t+j}^j\right)^2\right]\Omega_{t+j}^j \\
&+\beta^N\frac{\psi_{z^+,t+N}}{\psi_{z^+,t}}J_{z^+,t+N}\frac{\Omega_{t+N}^N}{1-\mathcal{F}_{t+N}^0}. \qquad (3.17)
\end{aligned}
$$

We now turn to the firm's decision about which workers to cut. Denote:

$$
\mathcal{F}_{\bar{a},t}^j \equiv \frac{d\mathcal{F}_t^j}{d\bar{a}_t^j},\ \mathcal{E}_{a,t}^j \equiv \frac{d\mathcal{E}_t^j}{d\bar{a}_t^j}.
$$

Consider the decision about $\bar{a}_t^0$, the productivity cutoff for endogenous separation for the cohort which bargains today. The bargain is assumed to have already occurred. We continue to denote the outcome of the wage bargain by ω_t. Then,

$$
\begin{aligned}
&\frac{dJ\left(\omega_t\right)}{d\bar{a}_t^0} = \left(W_t\mathcal{E}_{a,t}^0 + \omega_t\mathcal{F}_{a,t}^0\right)\varsigma_{0,t} + P_t z_t^+\frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2\mathcal{F}_{a,t}^0 \\
&-\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\begin{array}{c}\left(W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\omega_t\left(1-\mathcal{F}_{t+1}^1\right)\right)\varsigma_{1,t+1} \\ -P_{t+1}z_{t+1}^+\frac{\kappa}{2}\left(\tilde{v}_{t+1}^1\right)^2\left[1-\mathcal{F}_{t+1}^1\right]\end{array}\right]\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\mathcal{F}_{a,t}^0 \\
&-\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\begin{array}{c}\left(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{2,t+2} \\ -P_{t+2}z_{t+2}^+\frac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\left[1-\mathcal{F}_{t+2}^2\right]\end{array}\right]\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1Q_{t+1}^{1-\iota}+\rho\right)\left[1-\mathcal{F}_{t+1}^1\right]\mathcal{F}_{a,t}^0 \\
&... \\
&-\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right) \\
\times\quad &\left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\cdots\left[1-\mathcal{F}_{t+1}^1\right]\mathcal{F}_{at}^0 = 0
\end{aligned}
$$

We can simplify this equation by multiplying by $[1-\mathcal{F}_t^0]/\mathcal{F}_{a,t}^0$:

$$\begin{aligned}&\left(W_t\mathcal{E}_{a,t}^0+\omega_t\mathcal{F}_{a,t}^0\right)\varsigma_{0,t}\left[1-\mathcal{F}_t^0\right]/\mathcal{F}_{a,t}^0+P_tz_t^+\tfrac{\kappa}{2}\left(\tilde{v}_t^0\right)^2\left[1-\mathcal{F}_t^0\right]\\ &-\beta\tfrac{\upsilon_{t+1}}{\upsilon_t}\left[\begin{array}{c}\left(W_{t+1}\mathcal{E}_{t+1}^1-\Gamma_{t,1}\omega_t\left(1-\mathcal{F}_{t+1}^1\right)\right)\varsigma_{1,t+1}\\ -P_{t+1}z_{t+1}^+\tfrac{\kappa}{2}\left(\tilde{v}_{t+1}^1\right)^2\left[1-\mathcal{F}_{t+1}^1\right]\end{array}\right]\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left[1-\mathcal{F}_t^0\right]\\ &-\beta^2\tfrac{\upsilon_{t+2}}{\upsilon_t}\left[\begin{array}{c}\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{2,t+2}\\ -P_{t+2}z_{t+2}^+\tfrac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\left[1-\mathcal{F}_{t+2}^2\right]\end{array}\right]\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1Q_{t+1}^{1-\iota}+\rho\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right]\\ &\qquad\qquad\qquad\qquad\ldots\\ &-\beta^N\tfrac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\\ &\times\left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\cdots\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right]=0\end{aligned}$$

So,

$$\begin{aligned}&\left(W_t\mathcal{E}_{a,t}^0+\omega_t\mathcal{F}_{a,t}^0\right)\varsigma_{0,t}\left[1-\mathcal{F}_t^0\right]/\mathcal{F}_{a,t}^0+P_tz_t^+\frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2\left[1-\mathcal{F}_t^0\right]\\ &-J\left(\omega_t\right)+\left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}-P_tz_t^+\frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2\left[1-\mathcal{F}_t^0\right]=0,\end{aligned}$$

or,

$$J\left(\tilde{W}_t\right)=\varsigma_{0,t}\left\{\left(W_t\mathcal{E}_t^0-\tilde{W}_t\left(1-\mathcal{F}_t^0\right)\right)+\left(W_t\mathcal{E}_{a,t}^0+\tilde{W}_t\mathcal{F}_{a,t}^0\right)\left[1-\mathcal{F}_t^0\right]/\mathcal{F}_{a,t}^0\right\}.$$

Now consider $\bar{a}_t^1$:

$$\begin{aligned}&\left(W_t\mathcal{E}_{a,t}^1+\Gamma_{t-1,1}\tilde{W}_{t-1}\mathcal{F}_{a,t}^1\right)\varsigma_{1,t}+P_tz_t^+\frac{\kappa}{2}\left(\tilde{v}_t^1\right)^2\mathcal{F}_{a,t}^1\\ =\;&\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^2-\Gamma_{t-1,2}\tilde{W}_{t-1}\left(1-\mathcal{F}_{t+1}^2\right)\right)\varsigma_{2,t+1}-P_{t+1}z_{t+1}^+\frac{\kappa}{2}\left(\tilde{v}_{t+1}^2\right)^2\left(1-\mathcal{F}_{t+1}^2\right)\right]\\ &\times\left(\tilde{v}_t^1Q_t^{1-\iota}+\rho\right)\mathcal{F}_{a,t}^1\\ &+\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^3-\Gamma_{t-1,3}\tilde{W}_{t-1}\left(1-\mathcal{F}_{t+2}^3\right)\right)\varsigma_{3,t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{2}\left(\tilde{v}_{t+2}^3\right)^2\left(1-\mathcal{F}_{t+2}^3\right)\right]\\ &\times\left(\tilde{v}_t^1Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^2Q_{t+1}^{1-\iota}+\rho\right)\left(1-\mathcal{F}_{t+1}^2\right)\mathcal{F}_{a,t}^1\\ &+\ldots+\\ &+\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}J\left(\tilde{W}_{t+N-1}\right)\left(\tilde{v}_t^1Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^2Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-2}^{N-1}Q_{t+N-2}^{1-\iota}+\rho\right)\\ &\times\left(1-\mathcal{F}_{t+N-2}^{N-1}\right)\cdots\left(1-\mathcal{F}_{t+1}^2\right)\mathcal{F}_{a,t}^1,\end{aligned}$$

or, after multiplying by $[1-\mathcal{F}_t^1]/\mathcal{F}_{a,t}^1$:

$$\begin{aligned}&\left(W_t\mathcal{E}_{a,t}^1+\Gamma_{t-1,1}\tilde{W}_{t-1}\mathcal{F}_{a,t}^1\right)\varsigma_{1,t}\left[1-\mathcal{F}_t^1\right]/\mathcal{F}_{a,t}^1+P_tz_t^+\frac{\kappa}{2}\left(\tilde{v}_t^1\right)^2\left(1-\mathcal{F}_t^1\right)\\ =\;&\sum_{j=1}^{N-2}\beta^j\frac{\upsilon_{t+j}}{\upsilon_t}\left[\begin{array}{c}\left(W_{t+j}\mathcal{E}_{t+j}^{j+1}-\Gamma_{t-1,j+1}\tilde{W}_{t-1}\left(1-\mathcal{F}_{t+j}^{j+1}\right)\right)\varsigma_{2,t+j}\\ -P_{t+j}z_{t+j}^+\tfrac{\kappa}{2}\left(\tilde{v}_{t+j}^{j+1}\right)^2\left(1-\mathcal{F}_{t+j}^{j+1}\right)\end{array}\right]\tilde{\Omega}_{t+j}^1\\ &+\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}J\left(\tilde{W}_{t+N-1}\right)\tilde{\Omega}_{t+N-1}^1,\end{aligned}$$

where:

$$\begin{aligned}
\tilde{\Omega}_t^1 &= \left(\chi_t^1+\rho\right)\left(1-\mathcal{F}_t^1\right) \\
\tilde{\Omega}_{t,2}^1 &= \left(\chi_t^1+\rho\right)\left(\chi_{t+1}^2+\rho\right)\left(1-\mathcal{F}_t^1\right)\left(1-\mathcal{F}_{t+1}^2\right) \\
\tilde{\Omega}_{t,3}^1 &= \left(\chi_t^1+\rho\right)\left(\chi_{t+1}^2+\rho\right)\left(\chi_{t+2}^3+\rho\right)\left(1-\mathcal{F}_t^1\right)\left(1-\mathcal{F}_{t+1}^2\right)\left(1-\mathcal{F}_{t+2}^3\right) \\
& ... \\
\tilde{\Omega}_{t,N-1}^1 &= \left(\chi_t^1+\rho\right)\cdots\left(\chi_{t+N-2}^{N-1}+\rho\right)\left(1-\mathcal{F}_{t+N-2}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^1\right)
\end{aligned}$$

The condition for $\bar{a}_t^q$ is:

$$\begin{aligned}
&\left(W_t\mathcal{E}_{a,t}^q+\Gamma_{t-q,q}\tilde{W}_{t-q}\mathcal{F}_{a,t}^q\right)\varsigma_{q,t}\left[1-\mathcal{F}_t^q\right]/\mathcal{F}_{a,t}^q+P_tz_t^+\frac{\kappa}{2}\left(\tilde{\upsilon}_t^q\right)^2\left[1-\mathcal{F}_t^q\right] \qquad (3.18)\\
= &\sum_{j=1}^{N-(q+1)}\beta^j\frac{\upsilon_{t+j}}{\upsilon_t}\left[\begin{array}{c}\left(W_{t+j}\mathcal{E}_{t+j}^{j+q}-\Gamma_{t-q,j+q}\tilde{W}_{t-1}\left(1-\mathcal{F}_{t+j}^{j+q}\right)\right)\varsigma_{q,t+j} \\ -P_{t+j}z_{t+j}^+\frac{\kappa}{2}\left(\tilde{\upsilon}_{t+j}^{j+q}\right)^2\left(1-\mathcal{F}_{t+j}^{j+q}\right)\end{array}\right]\tilde{\Omega}_{t,t+j-1}^q \\
&+\beta^{N-q}\frac{\upsilon_{t+N-q}}{\upsilon_t}J\left(\tilde{W}_{t+N-q}\right)\tilde{\Omega}_{t,t+N-q-1}^q,
\end{aligned}$$

where it is understood that the term involving the summation is deleted when $q=N-1$. The above expression is defined for $q=0,...,N-1$, where

$$\begin{aligned}
\tilde{\Omega}_{t,t}^q &= \left(\chi_t^q+\rho\right)\left(1-\mathcal{F}_t^q\right) \\
\tilde{\Omega}_{t,t+1}^q &= \left(\chi_{t+1}^{q+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{q+1}\right)\overbrace{\left(\chi_t^q+\rho\right)\left(1-\mathcal{F}_t^q\right)}^{\tilde{\Omega}_{t,t}^q} \\
\tilde{\Omega}_{t,t+2}^q &= \left(\chi_{t+2}^{q+2}+\rho\right)\left(1-\mathcal{F}_{t+2}^{q+2}\right)\overbrace{\left(\chi_t^q+\rho\right)\left(1-\mathcal{F}_t^q\right)\left(\chi_{t+1}^{q+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{q+1}\right)}^{\tilde{\Omega}_{t,t+1}^q}.
\end{aligned}$$

More generally,

$$\tilde{\Omega}_{t,t+j}^q=\left(\chi_{t+j}^{q+j}+\rho\right)\left(1-\mathcal{F}_{t+j}^{q+j}\right)\tilde{\Omega}_{t,t+j-1}^q,$$

for $j=0,1...,N-1-q$. This expression is defined for $q=0,...N-1$. The $\tilde{\Omega}$'s can be written in the following form, which is convenient for Dynare:

$$\begin{aligned}
\tilde{\Omega}_{t,t}^q &= \left(\chi_t^q+\rho\right)\left(1-\mathcal{F}_t^q\right) \\
\tilde{\Omega}_{t-1,t}^q &= \left(\chi_t^{q+1}+\rho\right)\left(1-\mathcal{F}_t^{q+1}\right)\left(\chi_{t-1}^q+\rho\right)\left(1-\mathcal{F}_{t-1}^q\right) \\
\tilde{\Omega}_{t-2,t}^q &= \left(\chi_t^{q+2}+\rho\right)\left(1-\mathcal{F}_t^{q+2}\right)\left(\chi_{t-2}^q+\rho\right)\left(1-\mathcal{F}_{t-2}^q\right)\left(\chi_t^{q+1}+\rho\right)\left(1-\mathcal{F}_t^{q+1}\right),
\end{aligned}$$

or, generally, (?the next equation needs checking?)

$$\tilde{\Omega}_{t-j,t}^q=\left(\chi_t^{q+j}+\rho\right)\left(1-\mathcal{F}_t^{q+j}\right)\tilde{\Omega}_{t-j+1,t}^q,$$

It is convenient to scale (3.18) by $P_t z_t^+$, $q = 0, ..., N-1$ (recall, the expression involving the summation sign is understood to be absent when $q = N-1$) :

$$
\begin{aligned}
&\left(\bar{w}_t \mathcal{E}_{a,t}^q + G_{t-q,q} w_{t-q} \bar{w}_{t-q} \mathcal{F}_{a,t}^q\right) \varsigma_{q,t} \left[1 - \mathcal{F}_t^q\right] / \mathcal{F}_{a,t}^q + \frac{\kappa}{2} \left(\tilde{v}_t^q\right)^2 \left[1 - \mathcal{F}_t^q\right] \\
= &\sum_{j=1}^{N-(q+1)} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} \left[\begin{array}{c} \left(\bar{w}_{t+j} \mathcal{E}_{t+j}^{j+q} - G_{t-q,j+q} w_{t-q} \bar{w}_{t-q} \left(1 - \mathcal{F}_{t+j}^{j+q}\right)\right) \varsigma_{q,t+j} \\ -\frac{\kappa}{2} \left(\tilde{v}_{t+j}^{j+q}\right)^2 \left(1 - \mathcal{F}_{t+j}^{j+q}\right) \end{array} \right] \tilde{\Omega}_{t,j}^q \\
&+\beta^{N-q} \frac{\psi_{z^+,t+N-q}}{\psi_{z^+,t}} J_{z^+,t+N-q} \tilde{\Omega}_{t,N-q}^q,
\end{aligned}
$$

where G and Γ are defined in (3.12) and (3.9) respectively.

3.2.2. The Worker Problem

We now turn to the worker. For the bargaining problem, we require the worker's value function before they know if they will survive the endogenous separation. It is convenient to begin by defining the worker's value function after they have survived the endogenous separation. We do so first. We then derive the value function of an unemployed worker, and finally we consider the value function of the employed worker before endogenous separations occur.

The period t value of being a worker in an agency in cohort i is V_t^i :

$$
\begin{aligned}
V_t^i &= \Gamma_{t-i,i} \tilde{W}_{t-i} \varsigma_{i,t} \frac{1-\tau_t^y}{1+\tau_t^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L) \upsilon_t} & (3.19) \\
&+\beta E_t \frac{\upsilon_{t+1}}{\upsilon_t} \left(\rho \left(1 - \mathcal{F}_{t+1}^{i+1}\right) V_{t+1}^{i+1} + \left[1 - \rho + \rho \mathcal{F}_{t+1}^{i+1}\right] U_{t+1}\right), & (3.20)
\end{aligned}
$$

for $i = 0, 1, ..., N-1$. Here, ρ is the exogenous probability of remaining with the agency in the next period and $\left(1 - \mathcal{F}_{t+1}^{i+1}\right)$ is the endogenous probability of remaining with the agency. Also, U_t is the value of being unemployed in period t. The values, V_t^i and U_t, pertain to the beginning of period t, after job separation and job finding has occurred. Scaling V_t^i by $P_t z_t^+$, we obtain:

$$
\begin{aligned}
V_{z^+,t}^i &= G_{t-i,i} w_{t-i} \bar{w}_{t-i} \varsigma_{i,t} \frac{1-\tau_t^y}{1+\tau_t^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L) \psi_{z^+,t}} & (3.21) \\
&+\beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left(\rho \left(1 - \mathcal{F}_{t+1}^{i+1}\right) V_{z^+,t+1}^{i+1} + \left[1 - \rho + \rho \mathcal{F}_{t+1}^{i+1}\right] U_{z^+,t+1}\right), & (3.22)
\end{aligned}
$$

for $i = 0, 1, ..., N-1$, where

$$
\frac{V_t^i}{P_t z_t^+} = V_{z^+,t}^i, \; U_{z^+,t+1} = \frac{U_{t+1}}{P_{t+1} z_{t+1}^+}.
$$

In our analysis of the Nash bargaining problem, we must have the derivative of V_t^0 with respect to the wage rate. To define this derivative, it is useful to have:

$$\mathcal{M}_{t+j} = \left(1 - \mathcal{F}_t^0\right) \cdots \left(1 - \mathcal{F}_{t+j}^j\right), \tag{3.23}$$

for $j = 0, ..., N-1$. Then, the derivative of V, which we denote by $V_w^0\left(\omega_t\right)$, is:

$$\begin{aligned} V_w^0\left(\omega_t\right) &= E_t \sum_{j=0}^{N-1} \left(\beta\rho\right)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w} \Gamma_{t,j} \frac{\upsilon_{t+j}}{\upsilon_t} \\ &= E_t \sum_{j=0}^{N-1} \left(\beta\rho\right)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w} \Gamma_{t,j} \frac{\psi_{z^+,t+j} P_t z_t^+}{\psi_{z^+,t} P_{t+j} z_{t+j}^+} \\ &= E_t \sum_{j=0}^{N-1} \left(\beta\rho\right)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w} G_{t,j} \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}. \end{aligned} \tag{3.24}$$

Note ω_t has no impact on the intensity of labor effort. This is determined by (3.4), independent of the wage rate paid to workers.

The value of being an unemployed worker is U_t :[6]

$$U_t = P_t z_t^+ b^u \left(1 - \tau_t^y\right) + \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t} [f_t V_{t+1}^x + \left(1 - f_t\right) U_{t+1}], \tag{3.25}$$

where f_t is the probability that an unemployed worker will land a job in period $t+1$. Also, V_t^x is the period $t+1$ value function of a worker who finds a job, before it is known which agency the job is found with:

$$V_{z^+,t}^x = \sum_{i=0}^{N-1} \frac{\chi_{t-1}^i \left(1 - \mathcal{F}_{t-1}^i\right) l_{t-1}^i}{m_{t-1}} V_{z^+,t}^{i+1}, \tag{3.26}$$

after scaling. Here, the total number of new matches is given by:

$$m_t = \sum_{j=0}^{N-1} \chi_t^j \left(1 - \mathcal{F}_t^j\right) l_t^j. \tag{3.27}$$

In (3.26),

$$\frac{\chi_{t-1}^i \left(1 - \mathcal{F}_{t-1}^i\right) l_{t-1}^i}{m_{t-1}}$$

is the probability of finding a job in an agency which was of type i in the previous period, conditional on being a worker who finds a job in t.

Scaling (3.25),

$$U_{z^+,t} = b^u \left(1 - \tau_t^y\right) + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} [f_t V_{z^+,t+1}^x + \left(1 - f_t\right) U_{z^+,t+1}] \tag{3.28}$$

[6] Note that in the model, as in swedish data, unemployment benefits are subject to a (labor) income tax.

This value function applies to any unemployed worker, whether they got that way because they were unemployed in the previous period and did not find a job, or they arrived into unemployment because of an exogenous separation, or because they arrived because of an endogenous separation.

Finally, we consider the value function of a worker before they know whether they will survive the endogenous separation cut. We denote this value function by $\tilde{V}_t^j$:

$$\tilde{V}_t^j = \mathcal{F}_t^j U_t + \left(1 - \mathcal{F}_t^j\right) V_t^j.$$

Total job matches must also satisfy the following matching function:

$$m_t = \sigma_m \left(1 - L_t\right)^{\sigma} v_t^{1-\sigma}, \tag{3.29}$$

where

$$L_t = \sum_{j=0}^{N-1} \left(1 - \mathcal{F}_t^j\right) l_t^j. \tag{3.30}$$

and σ_m is the productivity of the matching technology.[7]

In our environment, there is a distinction between effective hours and measured hours. Effective hours is the hours of each person, adjusted by their productivity, a. The average productivity of a worker in working in cohort j (i.e., who has survived the endogenous productivity cut) is $\mathcal{E}_t^j / \left(1 - \mathcal{F}_t^j\right)$. The number of workers who survive the productivity cut in cohort j is $\left(1 - \mathcal{F}_t^j\right) l_t^j$, so that our measure of total effective hours is:

$$\begin{aligned} H_t &= \sum_{j=0}^{N-1} \varsigma_{j,t} \mathcal{E}_t^j l_t^j, & (3.31) \\ \mathcal{E}\left(\bar{a}_t^j; \sigma_{a,t}\right) &= \int_{\bar{a}_t^j}^{\infty} a d\mathcal{F}\left(a; \sigma_{a,t}\right) = 1 - prob\left[v < \frac{\log\left(\bar{a}_t^j\right) + \frac{1}{2}\sigma_{a,t}^2}{\sigma_{a,t}} - \sigma_{a,t}\right], & (3.32) \end{aligned}$$

where $prob$ refers to the standard normal distribution. We also need:

$$\begin{aligned} \mathcal{F}\left(\bar{a}^j; \sigma_a\right) &= \int_0^{\bar{a}^j} d\mathcal{F}(a; \sigma_a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log\left(\bar{a}^j\right) + \frac{1}{2}\sigma_a^2}{\sigma}} \exp^{\frac{-v^2}{2}} dv & (3.33) \\ &= prob\left[v < \frac{\log\left(\bar{a}^j\right) + \frac{1}{2}\sigma_a^2}{\sigma_a}\right], \end{aligned}$$

[7] One could allow for time-variation of σ_m. This is necessary if we want to include vacancies among the observed variables, otherwise we have stochastic singularity as unemployment tomorrow, in that case, is fully determined by vacancies and unemployment today.

so that,

$$\begin{aligned}\mathcal{F}_{\bar{a}^j}\left(\bar{a}^j;\sigma_a\right) &= \frac{1}{\bar{a}^j\sigma_a}\frac{1}{\sqrt{2\pi}}\exp^{\frac{-\left[\frac{\log\left(\bar{a}^j\right)+\frac{1}{2}\sigma_a^2}{\sigma_a}\right]^2}{2}} \\ &= \frac{1}{\bar{a}^j\sigma_a}\text{Standard Normal pdf }\left(\frac{\log\left(\bar{a}^j\right)+\frac{1}{2}\sigma_a^2}{\sigma_a}\right).\end{aligned}$$

Total measured hours is:

$$H_t^{meas}=\sum_{j=0}^{N-1}\varsigma_{j,t}\left(1-\mathcal{F}_t^j\right)l_t^j.$$

The job finding rate is:

$$f_t=\frac{m_t}{1-L_t}. \tag{3.34}$$

The probability of filling a vacancy is:

$$Q_t=\frac{m_t}{v_t}. \tag{3.35}$$

The $i=0$ cohort of agencies in period t solve the following Nash bargaining problem:

$$\max_{\omega_t}\left(\tilde{V}_t^0-U_t\right)^{\eta_t}J\left(\omega_t\right)^{(1-\eta_t)}=\max_{\omega_t}\left(\left(1-\mathcal{F}_t^0\right)\left(V_t^0-U_t\right)\right)^{\eta_t}J\left(\omega_t\right)^{(1-\eta_t)} \tag{3.36}$$

where $V^0\left(\omega_t\right)-U_t$ is the match surplus enjoyed by a worker and η_t is the bargaining power of workers which we allow to follow an exogenous time-varying process. We denote the wage that solves this problem by $\tilde{W}_t$. Note that $\tilde{W}_t$ takes into account that intensity will be chosen according to (3.4) as well as (3.5). The first order condition associated with this problem is:

$$\eta_t V_{w,t}J_{z^+,t}+\left(1-\eta_t\right)\left[V_{z^+,t}^0-U_{z^+,t}\right]J_{w,t}=0, \tag{3.37}$$

after division by $z_t^+P_t$.

We assume that the posting of vacancies uses the homogeneous domestic good. We leave the production technology equation, (2.37), unchanged, and we alter the resource constraint:

$$\begin{aligned}y_t &= g_t+c_t^d+i_t^d \qquad (3.38)\\ &+\left(R_t^x\right)^{\eta_x}\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x}+\left(1-\omega_x\right)\right]^{\frac{\eta_x}{1-\eta_x}}\left(1-\omega_x\right)\left(p_t^x\right)^{-\eta_f}y_t^*+\frac{\kappa}{2}\sum_{j=0}^{N-1}\left(\tilde{v}_t^j\right)^2\left[1-\mathcal{F}_t^j\right]l_t^j.\end{aligned}$$

Total vacancies v_t are related to vacancies posted by the individual cohorts as follows:

$$v_t=\frac{1}{Q_t^\iota}\sum_{j=0}^{N-1}\tilde{v}_t^j\left(1-\mathcal{F}_t^j\right)l_t^j.$$

Note however, that this equation does not add a constraint to the model equilibrium. In fact, it can be derived from the equilibrium equations (3.35), (3.27) and (3.6).

3.3. An Alternative Bargaining Problem

Our baseline interpretation of the Nash bargaining problem is that the bargain is between the employment agency and a union which represents the 'average worker'. The worker's interests are summarized by $\tilde{V}_t$ and take into account that with some probability the worker will separate at some time during the contract. The worker's outside option is unemployment, and so its surplus is $\tilde{V}_t - U_t$. The firm's surplus corresponds to J_t and this takes into account that workers who arrive in the future, while the contract remains in force, will be paid the wage rate that solves the bargaining problem, (3.36). In addition, if bargaining with the firm breaks down and the union takes all the workers, l_t^0, into unemployment, then the value of the firm drops to zero. This is because not only are current revenues from l_t^0 set to zero, but the agency's ability to ever hire in the future is eliminated when l_t^0 is set to zero.

We now consider an alternative formulation of the bargaining problem, in which there is no union. In the alternative formulation, we imagine that bargaining occurs among a continuum of worker-agency representative pairs. Each bargaining session takes the outcomes of all other bargaining sessions as given. Because each bargaining session is atomistic, each session ignores its impact on the wage earned by workers arriving in the future during the contract. We assume that those future workers are simply paid the average of the outcome of all bargaining sessions. Since each bargaining problem is identical, the wage that solves each problem is the same and so the average wage coincides with the wage that solves the bargaining problem. There is an important distinction between the atomistic and the union approach. When the Nash bargaining problem is optimized with respect to the wage, the impact on the wage earned by future arriving workers is ignored. The outside option of the worker in the alternative scenario is the same as before, it is the unemployment state, U_t. The outside option of the agency is also the same as before, namely zero. To see this, note that the agency's present discounted value of profits, $F\left(l_t^0, \omega_t\right)$, still has the following form, which is linear in l_t^0 :

$$F\left(l_t^0, \omega_t\right) = J\left(\omega_t\right) l_t^0.$$

Suppose that each worker in l_t^0 is identified with a point, i, on the interval, $i \in [0, l_t^0]$. Then, profits can be written in terms of each individual worker as follows:

$$F\left(l_t^0, \omega_t\right) = \int_0^{l_t^0} J\left(\omega_t^i, i\right) di,$$

where ω_t^i denotes the wage negotiated by worker i. We adopt the Riemann interpretation of this integral:

$$F\left(l_t^0, \omega_t\right) = \lim_{M \to \infty} \sum_{j=1}^{M} J\left(\omega_t^{i_j}, i_j\right)\left(i_j - i_{j-1}\right),$$

where $i_0 = 0$, $i_0 < i_1 < ... < i_M = l_t^0$. Thus, in the finite-but-large-value of M case, we interpret $i_1, ..., i_M$ as the M workers. We suppose that if the bargaining session involving the i_j^{th} worker breaks down, then $J\left(\omega_t^{i_j}, i_j\right) = 0$. For this reason, in the atomistic version of the Nash bargaining problem, we set the outside option of the firm to zero.

We now turn to the computation of J_w for our alternative formulation. We consider the surplus associated with a single worker and denote the wage received by that worker by ω_t. We denote the average across the wages received by all workers by $\tilde{\omega}_t$. Then,

$$\begin{aligned} J(\omega_t) &= \max_{\{v_{t+j}^j\}_{j=0}^{N-1}} \{ \left(W_t \mathcal{E}_t^0 - \omega_t \left(1 - \mathcal{F}_t^0\right)\right) \varsigma_{0,t} - P_t z_t^+ \frac{\kappa}{2} \left(\tilde{v}_t^0\right)^2 \left(1 - \mathcal{F}_t^0\right) \\ &+\beta \frac{\upsilon_{t+1}}{\upsilon_t} [\left(W_{t+1} \mathcal{E}_{t+1}^1 \varsigma_{1,t+1} - P_{t+1} z_{t+1}^+ \frac{\kappa}{2} \left(\tilde{v}_{t+1}^1\right)^2 \left(1 - \mathcal{F}_{t+1}^1\right)\right) \left(\chi_t^0 + \rho\right) \left(1 - \mathcal{F}_t^0\right) \\ &-\Gamma_{t,1} \omega_t \left(1 - \mathcal{F}_{t+1}^1\right) \varsigma_{1,t+1} \rho \left(1 - \mathcal{F}_t^0\right) - \Gamma_{t,1} \tilde{\omega}_t \left(1 - \mathcal{F}_{t+1}^1\right) \varsigma_{1,t+1} \chi_t^0 \left(1 - \mathcal{F}_t^0\right)] \\ &... \end{aligned}$$

To simplify the notation and given that we only want this expression for the purpose of computing J_w, we drop all terms that do not involve ω_t:

$$\begin{aligned} J(\omega_t) &= -\omega_t \left(1 - \mathcal{F}_t^0\right) \varsigma_{0,t} \\ &+\beta \frac{\upsilon_{t+1}}{\upsilon_t} [-\Gamma_{t,1} \omega_t \varsigma_{1,t+1} \rho \left(1 - \mathcal{F}_{t+1}^1\right) \left(1 - \mathcal{F}_t^0\right)] \\ &+\beta^2 \frac{\upsilon_{t+2}}{\upsilon_t} \left[-\Gamma_{t,2} \omega_t \varsigma_{2,t+2}\right] \rho^2 \left(1 - \mathcal{F}_{t+2}^2\right) \left(1 - \mathcal{F}_{t+1}^1\right) \left(1 - \mathcal{F}_t^0\right) \\ &+... + \\ &+\beta^{N-1} \frac{\upsilon_{t+N-1}}{\upsilon_t} \left[-\Gamma_{t,N-1} \omega_t \varsigma_{N-1,t+N-1}\right] \rho^{N-1} \left(1 - \mathcal{F}_{t+N-1}^{N-1}\right) \cdots \left(1 - \mathcal{F}_t^0\right) \end{aligned}$$

So that,

$$\begin{aligned} J_{w,t} &= -\left(1 - \mathcal{F}_t^0\right) \varsigma_{0,t} \\ &+\beta \frac{\upsilon_{t+1}}{\upsilon_t} [-\Gamma_{t,1} \varsigma_{1,t+1} \rho \left(1 - \mathcal{F}_{t+1}^1\right) \left(1 - \mathcal{F}_t^0\right)] \\ &+\beta^2 \frac{\upsilon_{t+2}}{\upsilon_t} \left[-\Gamma_{t,2} \varsigma_{2,t+2}\right] \rho^2 \left(1 - \mathcal{F}_{t+2}^2\right) \left(1 - \mathcal{F}_{t+1}^1\right) \left(1 - \mathcal{F}_t^0\right) \\ &+... + \\ &+\beta^{N-1} \frac{\upsilon_{t+N-1}}{\upsilon_t} \left[-\Gamma_{t,N-1} \varsigma_{N-1,t+N-1}\right] \rho^{N-1} \left(1 - \mathcal{F}_{t+N-1}^{N-1}\right) \cdots \left(1 - \mathcal{F}_t^0\right), \end{aligned}$$

which (after scaling) is identical to (3.16) with the understanding that in the definition of Ω_{t+j}^j, $\chi_{t+l}^l = 0$. To implement this alternative version of the model, we simply use this definition of $J_{w,t}$ together with the previous definition of J_t in the equation that characterizes the solution to the Nash bargaining problem. This is the only change required to implement this alternative version of the model. An alternative representation of J_w is convenient, and highlights how firms discount future wages now in the same way as the household does in V_w :

$$
\begin{aligned}
J_{w,t} &= -\mathcal{M}_{t,0}\varsigma_{0,t} \\
&- (\beta\rho)\,\mathcal{M}_{t,1}\frac{\upsilon_{t+1}}{\upsilon_t}\Gamma_{t,1}\varsigma_{1,t+1} \\
&- (\beta\rho)^2\,\mathcal{M}_{t,2}\frac{\upsilon_{t+2}}{\upsilon_t}\Gamma_{t,2}\varsigma_{2,t+2} \\
&- ... \\
&- (\beta\rho)^{N-1}\,\mathcal{M}_{t,N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\Gamma_{t,N-1}\varsigma_{N-1,t+N-1},
\end{aligned}
$$

It is interesting to compare $J_{w,t}$ and $V_{w,t}$:

$$
V_{w,t} = \sum_{j=0}^{N-1} (\beta\rho)^j\,\mathcal{M}_{t,j}\varsigma_{j,t+j}\frac{1-\tau^y_{t+j}}{1+\tau^w_{t+j}}\Gamma_{t,j}\frac{\upsilon_{t+j}}{\upsilon_t}.
$$

Note that one is just the minus of the other, if we ignore the tax wedge. That is, absent the tax wedge a change in the wage simply reallocates resources between the firm and the worker. In our baseline case, this is not true because the firm and the worker discount the future differently. This implies that if there were not restrictions on the intertemporal pattern of wage payments in the baseline model, then it would be desirable to shift wages into the present. When we take into account the tax wedge, increases in the wage take resources away from the firm and only incompletely transfer them to households. As a result, we conjecture that the presence of the tax wedge causes the equilibrium pre-tax wage to be smaller.

3.4. Alternative Model of Endogenous Separations

In this section we consider a total surplus criterion for determining the a cutoff in period t. We begin by discussing the cutoff for cohort, $j = 0$. We identify each worker with a productivity level, a, in the current period and this allows us to define a total surplus for the firm and worker for each a. Because of the linearity in our environment, it must be that when we integrate over the surplus of all the individual a's, we arrive at the aggregate surplus implicit in our construction of the Nash bargaining problem:

$$
\left(1-\mathcal{F}_t^0\right)(V_t - U_t) + J_t = \int_{\bar{a}}^{\infty} s_t(a)dF(a), \tag{3.39}
$$

where $s(a_t)$ denotes the surplus of the match associated with a worker with productivity, a. Here, $(1-\mathcal{F}_t^0)(V_t - U_t)$ is the average surplus of workers in l_t^0, over all values of $a \geq 0$. We arrive at this expression from the fact that each worker with $a \geq \bar{a}$ receive the same surplus, $V_t - U_t$, and workers with $a < \bar{a}$ receive zero surplus. Similarly, J_t denotes the average surplus of a worker across all $a \geq \bar{a}$. Let $s_t(a)$ be defined as follows:

$$s_t(a) = V_t^0 - U_t + (W_t a - \omega_t)\varsigma_{0,t} - P_t z_t^+ \frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2 + \frac{D_t^0}{1-\mathcal{F}_t^0},$$

where D_t^0 denotes:

$$\begin{aligned}
D_t^0 &= \beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\omega_t\left(1-\mathcal{F}_{t+1}^1\right)\right)\varsigma_{1,t+1} - P_{t+1}z_{t+1}^+\frac{\kappa}{2}\left(\tilde{v}_{t+1}^1\right)^2\left[1-\mathcal{F}_{t+1}^1\right]\right] \\
&\quad \times\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left[1-\mathcal{F}_t^0\right] \\
&\quad +\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{2,t+2} - P_{t+2}z_{t+2}^+\frac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\left[1-\mathcal{F}_{t+2}^2\right]\right] \\
&\quad \times\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\
&\quad +\ldots+ \\
&\quad +\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right) \\
&\quad \times\left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\cdots\left[1-\mathcal{F}_t^0\right]\} \\
&= J_t - \left[\left(W_t\mathcal{E}_t^0 - \omega_t\left[1-\mathcal{F}_t^0\right]\right)\varsigma_{0,t} - P_t z_t^+\frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2\left(1-\mathcal{F}_t^0\right)\right],
\end{aligned}$$

so that

$$J_t = W_t\left[\mathcal{E}_t^0 - \bar{a}_t^0\left(1-\mathcal{F}_t^0\right)\right]\varsigma_{0,t} - \left(1-\mathcal{F}_t^0\right)\left(V_t^0 - U_t\right)$$

An alternative representation which may aid intuition is the following:

$$\left(1-\mathcal{F}_t^0\right)\left(V_t^0 - U_t\right) + J_t = \int_{\bar{a}_t^0}^{\infty} W_t\left[a - \bar{a}_t^0\right]\varsigma_{0,t}\,dF(a)$$

Dividing by $P_t z_t^+$, we obtain:

$$J_{z^+,t} = \bar{w}_t\left[\mathcal{E}_t^0 - \bar{a}_t^0\left(1-\mathcal{F}_t^0\right)\right]\varsigma_{0,t} - \left(1-\mathcal{F}_t^0\right)\left(V_{z^+,t}^0 - U_{z^+,t}\right). \tag{3.40}$$

Now, consider $\bar{a}_t^1$. We require a representation of the surplus of the match associated with productivity, a. The surplus of the firm is:

$$\begin{aligned}
\frac{F_t^1(\omega_{t-1})}{l_t^1} &\equiv J_t^1(\omega_{t-1}) = \left(W_t\mathcal{E}_t^1 - \Gamma_{t-1,1}\omega_{t-1}\left(1-\mathcal{F}_t^1\right)\right)\varsigma_{1,t} - P_t z_t^+\frac{\kappa}{2}\left(\tilde{v}_t^1\right)^2\left(1-\mathcal{F}_t^1\right) \\
&\quad +\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^2 - \Gamma_{t-1,2}\omega_{t-1}\left(1-\mathcal{F}_{t+1}^2\right)\right)\varsigma_{2,t+1} - P_{t+1}z_{t+1}^+\frac{\kappa}{2}\left(\tilde{v}_{t+1}^2\right)^2\left[1-\mathcal{F}_{t+1}^2\right]\right] \\
&\quad \times\left(\chi_t^1+\rho\right)\left[1-\mathcal{F}_t^1\right] \\
&\quad +\ldots+ \\
&\quad +\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}J\left(\tilde{W}_{t+N-1}\right)\left(\chi_t^1+\rho\right)\cdots\left(\chi_{t+N-2}^{N-1}+\rho\right)\left[1-\mathcal{F}_{t+N-2}^{N-1}\right]\cdots\left[1-\mathcal{F}_t^1\right]\}.
\end{aligned}$$

As noted before, $J_t^1(\omega_{t-1})$ is the average, over $a \geq \bar{a}_t^1$ of workers who are employed at the agency. It is convenient to express $J_t^1(\omega_{t-1})$ as a function of $J_{t-1}(\omega_{t-1})$:

$$\begin{aligned} J_{t-1}(\omega_{t-1}) &= \left(W_{t-1}\mathcal{E}_{t-1}^0 - \bar{a}_{t-1}^0\left(1-\mathcal{F}_{t-1}^0\right)\right)\varsigma_{0,t-1} - P_{t-1}z_{t-1}^+\frac{\kappa}{2}\left(\tilde{v}_{t-1}^0\right)^2 \\ &\quad + \beta\frac{\upsilon_t}{\upsilon_{t-1}}\left[\left(W_t\mathcal{E}_t^1 - \Gamma_{t-1,1}\omega_{t-1}\left(1-\mathcal{F}_t^1\right)\right)\varsigma_{1,t} - P_tz_t^+\frac{\kappa}{2}\left(\tilde{v}_t^1\right)^2\left(1-\mathcal{F}_t^1\right) + D_t^1\right] \\ &= \left(W_{t-1}\mathcal{E}_{t-1}^0 - \bar{a}_{t-1}^0\left(1-\mathcal{F}_{t-1}^0\right)\right)\varsigma_{0,t-1} - P_{t-1}z_{t-1}^+\frac{\kappa}{2}\left(\tilde{v}_{t-1}^0\right)^2 + \beta\frac{\upsilon_t}{\upsilon_{t-1}}J_t^1(\omega_{t-1}), \end{aligned}$$

where

$$\begin{aligned} &D_t^1 = \beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^2 - \Gamma_{t-1,2}\omega_{t-1}\left(1-\mathcal{F}_{t+1}^2\right)\right)\varsigma_{2,t+1} - P_{t+1}z_{t+1}^+\frac{\kappa}{2}\left(\tilde{v}_{t+1}^2\right)^2\left(1-\mathcal{F}_{t+1}^2\right)\right] \\ &\times\left(\chi_t^1+\rho\right)\left(1-\mathcal{F}_t^1\right) \\ &+\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^3 - \Gamma_{t-1,3}\omega_{t-1}\left(1-\mathcal{F}_{t+2}^3\right)\right)\varsigma_{3,t+2} - P_{t+2}z_{t+2}^+\frac{\kappa}{2}\left(\tilde{v}_{t+2}^3\right)^2\left(1-\mathcal{F}_{t+2}^3\right)\right] \\ &\times\left(\chi_t^1+\rho\right)\left[1-\mathcal{F}_t^1\right]\left(\chi_{t+1}^2+\rho\right)\left(1-\mathcal{F}_{t+1}^2\right) \\ &+ ... + \\ &+\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}J\left(\tilde{W}_{t+N-1}\right)\left(\chi_t^1+\rho\right)\cdots\left(\chi_{t+N-2}^{N-1}+\rho\right)\left(1-\mathcal{F}_{t+N-2}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^1\right)\}. \end{aligned}$$

If we define the total surplus associated with an $a-$productivity worker as follows:

$$s_t^1(a) = V_t^1 - U_t + \left(W_ta - \Gamma_{t-1,1}\omega_t\right)\varsigma_{1,t} - P_tz_t^+\frac{\kappa}{2}\left(\tilde{v}_t^1\right)^2 + \frac{D_t^1}{1-\mathcal{F}_t^1},$$

then,

$$\begin{aligned} \int_{\bar{a}_t^1} s_t^1(a)\,dF(a) &= \left(1-\mathcal{F}_t^1\right)\left(V_t^1-U_t\right) + \left(W_t\mathcal{E}_t^1 - \Gamma_{t-1,1}\omega_{t-1}\left(1-\mathcal{F}_t^1\right)\right)\varsigma_{1,t} \\ &\quad - P_tz_t^+\frac{\kappa}{2}\left(\tilde{v}_t^1\right)^2\left(1-\mathcal{F}_t^1\right) + D_t^1 \\ &= \left(1-\mathcal{F}_t^1\right)\left(V_t^1-U_t\right) + J_t^1(\omega_{t-1}), \end{aligned}$$

which is aggregate surplus in period t, over cohort $j=1$. Thus, the equation defining $\bar{a}_t^1$ is:

$$V_t^1 - U_t + \left(W_t\bar{a}_t^1 - \Gamma_{t-1,1}\omega_{t-1}\right)\varsigma_{1,t} - P_tz_t^+\frac{\kappa}{2}\left(\tilde{v}_t^1\right)^2 + \frac{D_t^1}{1-\mathcal{F}_t^1} = 0.$$

We scale by dividing by $P_tz_t^+$:

$$V_{z^+,t}^1 - U_{z^+,t} + \left(\bar{w}_t\bar{a}_t^1 - G_{t-1,1}\bar{w}_{t-1}w_{t-1}\right)\varsigma_{1,t} - \frac{\kappa}{2}\left(\tilde{v}_t^1\right)^2 + \frac{D_{z^+,t}^1}{1-\mathcal{F}_t^1} = 0.$$

We conclude that the efficiency conditions for cohorts, $j=1,..,N-1$ are:

$$V_{z^+,t}^j - U_{z^+,t} + \left(\bar{w}_t\bar{a}_t^j - G_{t-j,j}\bar{w}_{t-j}w_{t-j}\right)\varsigma_{j,t} - \frac{\kappa}{2}\left(\tilde{v}_t^j\right)^2 + \frac{D_{z^+,t}^j}{1-\mathcal{F}_t^j} = 0. \quad (3.41)$$

Thus, the equilibrium conditions determining $\bar{a}_t^j$, for $j = 0, ..., N-1$ are (3.40) and (3.41).

We now turn to the expression for $D^j_{z^+,t}$, $= 1, .., N-1$. With $j = 1$:

$$
\begin{aligned}
&\frac{D_t^1}{P_t z_t^+} = \beta \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \frac{1}{P_{t+1} z_{t+1}^+} \left[\begin{array}{c} \left(W_{t+1}\mathcal{E}_{t+1}^2 - \Gamma_{t-1,2}\omega_{t-1}\left(1-\mathcal{F}_{t+1}^2\right)\right)\varsigma_{2,t+1} \\ -P_{t+1}z_{t+1}^+\frac{\kappa}{2}\left(\tilde{v}_{t+1}^2\right)^2\left(1-\mathcal{F}_{t+1}^2\right) \end{array} \right] \\
&\times \left(\chi_t^1 + \rho\right)\left(1-\mathcal{F}_t^1\right) \\
&+\beta^2 \frac{\psi_{z^+,t+2}}{\psi_{z^+,t}} \frac{1}{P_{t+2} z_{t+2}^+} \left[\begin{array}{c} \left(W_{t+2}\mathcal{E}_{t+2}^3 - \Gamma_{t-1,3}\omega_{t-1}\left(1-\mathcal{F}_{t+2}^3\right)\right)\varsigma_{3,t+1} \\ -P_{t+2}z_{t+2}^+\frac{\kappa}{2}\left(\tilde{v}_{t+2}^3\right)^2\left(1-\mathcal{F}_{t+2}^3\right) \end{array} \right] \\
&\times \left(\chi_t^1 + \rho\right)\left(1-\mathcal{F}_t^1\right)\left(\chi_{t+1}^2 + \rho\right)\left(1-\mathcal{F}_{t+1}^2\right) \\
&+ ... + \\
&+\beta^{N-1} \frac{\psi_{z^+,t+N-1}}{\psi_{z^+,t}} \frac{J\left(\tilde{W}_{t+N-1}\right)}{P_{t+N-1} z_{t+N-1}^+} \left(\chi_t^1 + \rho\right)\cdots\left(\chi_{t+N-2}^{N-1} + \rho\right) \\
&\times \left(1-\mathcal{F}_{t+N-2}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^1\right)\},
\end{aligned}
$$

or, generalizing to arbitrary $j \in (1, N-1)$:

$$
\begin{aligned}
D^j_{z^+,t} \equiv \frac{D_t^j}{P_t z_t^+} = &\overbrace{\beta\tfrac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[\begin{array}{c} \left(\bar{w}_{t+1}\mathcal{E}_{t+1}^{j+1} - G_{t-j,j+1}\bar{w}_{t-j}w_{t-j}\left(1-\mathcal{F}_{t+1}^{j+1}\right)\right)\varsigma_{j+1,t+1} \\ -\frac{\kappa}{2}\left(\tilde{v}_{t+1}^{j+1}\right)^2\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\ \times\left(\chi_t^j+\rho\right)\left(1-\mathcal{F}_t^j\right) \end{array} \right]}^{\equiv 0, \text{ for } j=N-1} \qquad (3.42) \\
&+\beta^2 \frac{\psi_{z^+,t+2}}{\psi_{z^+,t}} \left[\left(\bar{w}_{t+2}\mathcal{E}_{t+2}^{j+2} - G_{t-j,j+2}\bar{w}_{t-j}w_{t-j}\left(1-\mathcal{F}_{t+2}^{j+2}\right)\right)\varsigma_{j+2,t+2} - \frac{\kappa}{2}\left(\tilde{v}_{t+2}^{j+2}\right)^2\left(1-\mathcal{F}_{t+2}^{j+2}\right)\right] \\
&\times\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right)\times\left(\chi_t^j+\rho\right)\left(1-\mathcal{F}_t^j\right) \\
&+ ... + \\
&+\beta^{N-j}\frac{\psi_{z^+,t+N-j}}{\psi_{z^+,t}} J_{z^+,t+N-j}\left(\chi_t^j+\rho\right)\cdots\left(\chi_{t+N-(j+1)}^{N-1}+\rho\right)\left(1-\mathcal{F}_{t+N-(j+1)}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^j\right)\}
\end{aligned}
$$

To implement the version of the model discussed in this subsection, we replace the N formulas in (3.18) by (3.40) and (3.41).

4. Estimation

We estimate the model using Bayesian techniques. In this section, we discuss the calibration parameters that we do not estimate, the data, the choice of priors, the specification of shocks and the measurement equations.

4.1. Calibration

We calibrate the models to US data. The time unit is a quarter. Parameters that are related to "great ratios" and other observable quantities in the data are calibrated. These include the discount factor β and the tax rate on bonds τ_b which are calibrated to yield a real interest of rate of roughly 2.5%. We calibrate the capital share α to 0.3, consistent with a capital-output ratio of roughly 10 on a quarterly basis. We set the depreciation rate δ to 0.02 to match the ratio of investment over output of 0.25 in the data. In line with output data, steady state balanced growth is calibrated to 0.44% on a quarterly basis. To keep things simple, we assume that all balanced growth is due to neutral unit root technology growth. The following table provides a list of all parameters that we calibrate and keep fixed in the estimation.

Parameter	Value	Description
α	0.3	Capital share in production
β	0.998	Discount factor
η_g	0.2	Government consumption share on GDP
τ_k	0.32	Capital tax rate
τ_w	0.0	Payroll tax rate
τ_c	0.05	Consumption tax rate
τ_y	0.24	Labor income tax rate
τ_b	0.0	Bond tax rate
μ_z	1.0044	Steady state growth rate of neutral technology
μ_ψ	1	Steady state growth rate of investment technology
$\bar{\pi}$	1.0061	Steady state gross inflation target
$\breve{\pi}$	1.0061	Third indexing base
ϑ_w	1	Wage indexing wrt.steady state technology
N	2	Number of agency cohorts/length of wage contracts
ι	1	Parameter in vacancy posting cost function
$bshare$	0.7	Replacement ratio taking utility of leisure into account
$unemp$	0.06	Steady state unemployment rate
$totalseparation$	0.05	Total separation rate
σ	0.5	Unemployment share in matching technology
σ_m	0.8	Level parameter in matching function

In the model, some further parameters are calculated endogenously in the steady state. These parameters are the bargaining power η to ensure a given unemployment rate, the disutility of labor scaling parameter A_L so that employed individuals spend a fraction of 0.25 of their time working and the exogenous separation rate ρ such that the unemployment models match total separations. Furthermore, the standard deviation of idiosyncratic worker productivity σ_a is set such that the to be estimated steady state endogenous separation rate $\digamma$ is obtained and we set the vacancy posting costs parameter κ to ensure a to be estimated steady state recruiting cost to output ratio.

4.2. Choice of priors

Table (7.11) shows our choice of priors which is in line with the literature. 2 parameters are novel: the steady state probability of endogenous separation F, which we assign a beta prior with a mean of 0.6% and a standard deviation of 0.05 and the steady state recruitment costs to output ratio for which we choose a prior mean of 0.05% and an inverse gamma distribution with 2 degrees of freedom.

4.3. Data and Measurement Equations

We estimate all 4 models using US data. Our sample period is 1985Q1-2008Q3. We use the following 7 data time series for all 4 models: real GDP, real private consumption, real private investment, hours worked, real wages, GDP deflator inflation and the Federal Funds rate. All real quantities are in per capita terms. Except for inflation and the nominal interest rate, we take logs and first differences. In addition, we demean each first-differenced time series for the estimation. Thus, we have the following data set:

$$\Delta \ln Y_t^{data}, \Delta \ln C_t^{data}, \Delta \ln I_t^{data}, \Delta \ln H_t^{data}, \Delta \ln (W_t/P_t)^{data},\ \pi_t^{data} R_t^{data}$$

We demean the first-differenced time series because in our sample from 1985Q1-2008Q3 variables such as output, consumption, real wages, investment grow on average at substantially different rates. The model, however, allows for two different real long-run growth rates only. In order to match these different trends in the data the estimation is likely to result in a series of negative or positive shocks for some stationary exogenous process. We want to avoid this and therefore demean the data. After the estimation we compare the growth rates of the data with those implied by the model. See figure (7.3) in the Appendix for plots of the above data used in the estimation. We do not use measurement errors in the estimation.

Below we report the measurement equations we use to link the model to the demeaned data. First differences are written in annualized percentages so model variables are multiplied by 400 accordingly. Furthermore our data series for inflation and interest rates are annualized, so we make the same transformation for the model variables i.e. multiplying by

400:[8]

$$
\begin{aligned}
\Delta \ln Y_t^{data} &= 400(\ln \mu_{z^+,t} + \Delta \ln y_t) - 400(\ln \mu_{z^+}) \\
\Delta \ln C_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln c_t) - 400(\ln \mu_{z^+}) \\
\Delta \ln I_t^{data} &= 400\left[\ln \mu_{z^+,t} + \ln \mu_{\psi,t} + \Delta\left(\ln i_t + a\left(u_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)\right] - 400(\ln \mu_{z^+} + \ln \mu_\psi) \\
\pi_t^{data} &= 400 \log \pi_t - 400 \log \pi \\
R_t^{data} &= 400(R_t - 1) - 400(R - 1)
\end{aligned}
$$

In the unemployment models, hours worked are measured as follows:

$$\Delta \ln H_t^{data} = 400\Delta \ln H_t^{meas},$$

whereas in the EHL model we have

$$\Delta \ln H_t^{data} = 400\Delta \ln h_t.$$

Furthermore, in the unemployment models, the demeaned real wage is measured by the demeaned employment-weighted average Nash bargaining wage in the model:

$$w_t^{avg} = \frac{1}{L}\sum_{j=0}^{N-1} l_t^j G_{t-j,j} w_{t-j} \bar{w}_{t-j}.$$

Given this definition the measurement equation for wages is:

$$\Delta \ln(W_t/P_t)^{data} = 400\Delta \ln \frac{\tilde{W}_t}{z_t^+ P_t} = 400(\ln \mu_{z^+,t} + \Delta \ln w_t^{avg}) - 400(\ln \mu_{z^+})$$

and for the EHL case it follows accordingly that

$$\Delta \ln(W_t/P_t)^{data} = 400\Delta \ln \frac{W_t}{z_t^+ P_t} = 400(\ln \mu_{z^+,t} + \Delta \ln \bar{w}_t) - 400(\ln \mu_{z^+})$$

4.4. Shocks

We estimate the model using 7 shocks in total. The following stochastic variables in the model evolve according to AR(1) processes, i.e.

[8] Note that in the data we measure $\pi_t^{data} = 400(\log P_t^{data} - \log P_{t-1}^{data})$. In the model, we have defined $\pi_t = \frac{P_t}{P_{t-1}}$. Matching data with the model results in the above measurement equations for inflation.

Stationary neutral technology:	$\epsilon_t = (1-\rho_\epsilon)\log\epsilon + \rho_\epsilon \log\epsilon_{t-1} + \varepsilon_{\epsilon,t}$
Stationary investment specific technology:	$\Upsilon_t = (1-\rho_\Upsilon)\log\Upsilon + \rho_\Upsilon \log\Upsilon_{t-1} + \varepsilon_{\Upsilon,t}$
Labor preference:	$\zeta_t^h = (1-\rho_{\zeta^h})\log\zeta^h + \rho_{\zeta^h} \log\zeta_{t-1}^h + \varepsilon_{\zeta^h,t}$
Government consumption:	$g_t = (1-\rho_g)\log g + \rho_g \log g_{t-1} + \varepsilon_{g,t}$
Price markup:	$\tau_t^d = (1-\rho_{\tau^d})\log\tau^d + \rho_{\tau^d} \log\tau_{t-1}^d + \varepsilon_{\tau^d,t}$
Inflation target:	$\bar{\pi}_t = (1-\rho_{\bar{\pi}})\log\bar{\pi} + \rho_{\bar{\pi}} \log\bar{\pi}_{t-1} + \varepsilon_{\bar{\pi},t}$

Finally, we have the shock to the nominal interest rate $\varepsilon_{R,t}$ which we assume to be i.i.d. All other exogenous processes in the model are set to their steady states in the estimation.

4.5. DSGE-VAR

We estimate all models using a standard DSGE-VAR Bayesian estimation procedure. This way, not only the marginal data likelihood provides information about the fit of each respective model but also the hyperparameter λ gives us an idea about the strength of the cross equation restrictions of the respective DSGE models. In particular, we seek to obtain information whether the added complexity in the unemployment models and the implied restrictions are supported or rejected by the data.

We obtain estimation results using random walk Metropolis-Hasting chain with 250000 draws after a burn-in of 50000 draws. The first 20 observations are used to initialize the estimation. Substantial analysis has been spent on ensuring that the Hessian used for the Metropolis-Hasting algorithm approximates the curvature well.

5. Results

In this section, we discuss the estimated parameters, the implied variance decompositions, first and second moments of data and models as well as impulse responses of shocks, all at the estimated posterior mean.

5.1. Posterior Mean Parameter Values

We start by commenting briefly on the parameter estimates at the posterior mean. See the prior-posterior table (7.11)in the appendix.

According to the marginal data likelihood the EHL model performs best in explaining our 7 macro time series. The employer surplus specification of the labor market comes next followed by the total surplus and the exogenous separation specifications. The differences in the fit between the EHL and the unemployment models may not be surprising as the latter impose much more cross equation restrictions on the data. What is interesting, however, is that the endogenous separations employer surplus specification outperforms the endogenous separations total surplus and the exogenous separation specifications.

The DSGE-VAR hyperparameter λ posterior mean estimate is 1.1 for the EHL model whereas it is 0.9 for the employer surplus model and 0.8 for the total surplus model and exogenous separations model. These values indicate that there are more cross equation restrictions imposed in the unemployment models relative to the EHL model. However, the difference is not striking.

The estimated posterior mean values for the standard parameters are in line with the existing literature. 2 parameters deserve special attention. The steady state endogenous separation probability F takes a mean of 0.63 and 0.62 in the employer and total surplus models respectively. Relative to the prior, the posterior has not moved much, which is also partly due to the relatively tight prior. By contrast, the other parameter, the steady state recruitment cost to GDP ratio has moved relatively much relative to the prior. The posterior is roughly 0.01%. At a first glance this number might appear very low. The following back-on-the-envelope calculation shows that the implied recruiting cost are reasonable. The annual job finding rate in Fujita and Ramey (2006) is roughly 1. The number of unemployed people in the US in 2008 is 8.924 millions. Given this information, the number of people that moved from unemployment to employment is in 2008 is 8.924 mullions. Suppose the 0.01% recruiting cost to GDP ratio as obtained from the estimation is true. Then, annual US nominal GDP in 2008 which equals 14 281 bn \$.times 0.01% amounts to 1.4281 bn \$ for total recruiting costs in the US in 2008. Now, dividing this number by the number of people that got recruited (e.g. moved from unemployment to employment) results in 160 \$ recruiting costs per person employed. This number does not appear to be unreasonable. However, it needs to be checked against the literature, of course.

5.2. Variance Decompositions and Moments

Table (7.12) presents asymptotic variance decompositions at the estimated posterior mean. Interestingly, the stationary neutral technology shock is much more important for GDP growth in the unemployment models compared to the EHL while the opposite is true for the stationary investment specific technology shock.

According to the results, the separation rate in the employer surplus specification is driven equally by shocks to neutral technology, labor preferences, price markup and the nominal interest rate.

Moreover, the table shows that especially labor market related variables are driven by different shocks depending on which unemployment model is considered. Note, however, that all four models have been estimated with the same data set which does not include data for separation and finding rates, unemployment and vacancies. A next step forward will be to use these additional data in the estimation of the respective unemployment models.

First and second moments of the data and the models are presented in table (7.13). Except for the growth rate for investment, hours and wages the first moments of the models are roughly in line with the data. As regards of the second moments, all models tend to overpredict the standard deviations of output and inflation and the unemployment models appear to underpredict the variability of hours and overpredict the variability of wages.

More importantly, however, the employer surplus model correctly predicts the second moments of the separation rate, the unemployment rate and vacancies posted. All other models are either not able to address the second moments of these data or simply get it entirely wrong. Note again, that these results are interesting in so far as we do not use any of these data in the estimation so far.

5.3. Impulse Responses

Finally, figures (7.4) to (7.10) show the impulse responses at the posterior mean for all shocks considered in the estimation.

In terms of impulse responses to a monetary policy shock the EHL and the employer surplus specifications are virtually identical with respect to the macro variables of the mode while this is not the case for the other two unemployment models.

Interestingly, the employer surplus model produces much more fluctuations in the unemployment rate compared to the total surplus model. Most of these fluctuations are due to large changes in the separation rate in response to a contractionary monetary policy. As apparent from the figure, cohort 1 employers , i.e. the ones that cannot renegotiate their wages, increase their separation heavily. By contrast, cohort 0 employers even decrease their separations. The reason being that cohort 0 employers are able to negotiate wages with their employees at the time when the monetary policy shock hits the economy.

An interesting feature of the employer surplus model is the positive response of hours per employee after a monetary tightening. It seems as if utilizing the intensive margin and at the same time laying people off is the optimal strategy for an employment agency that determines the separation rate according to its own surplus.

A perhaps counterfactual result, however, is the fall in the unemployment rate in the exogenous separations model after a monetary policy shock. We have experimented with the recruitment cost to GDP ratio. Had we imposed a prior mean of 0.25% instead of 0.05% for the exogenous separations model, the posterior would have been 0.23% while the marginal data likelihood would have worsened by 5 log points. However, given this alternative parameterization, the fall in the unemployment rate would disappear and turn into a rise in the unemployment rate.

Interestingly, according to the total surplus model, vacancies and unemployment rise in response to a monetary tightening. According to the data, there appears to be a strong negative correlation of vacancies and unemployment (see figure (7.2). On the one hand, our results from the variance decompositions confirm, that monetary policy shocks are not the main driving forces of business cycle fluctuations including vacancies and unemployment. On the other hand, this issue needs to the checked thoroughly. As a next step we are planning to include further data in the estimation for the unemployment models i.e. data on job separation and finding rates, unemployment and vacancies. Imposing the dynamics of these data on the models will be an important next step in order to build confidence in the results obtained so far as well as enables better identification of crucial parameters related to the respective labor market specifications.

6. Conclusion

In US data, job separation rates vary over the business cycle. In this paper we analyze the determinants of time varying job separation rates and their consequences for the macroeconomy. To do so, we have developed a monetary model with nominal rigidities, idiosyncratic worker productivity and endogenize separations along the following two dimensions: maximization of either the employer surplus or total employer-employee surplus determines how many employees are laid off in response to shocks. We contrast the implications of these alternative assumptions to the cases of exogenous separations and the Erceg, Henderson and Levin (2000) (EHL henceforth) approach to model the labor market without unemployment. We have estimated all four models on 7 key macro time series using Bayesian techniques. Our results indicate by and large that EHL and the employer surplus specifications perform very similar in terms of standard macro variables and outperform the total surplus and exogenous separation specifications of the labor market.

As a next step we are planning to include further data in the estimation for the unemployment models i.e. data on job separation and finding rates, unemployment and vacancies. Imposing the dynamics of these data on the models will be an important next step in order to build confidence in the results obtained so far as well as enables better identification of crucial parameters related to the respective labor market specifications.

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7. Appendix

7.1. Tables and Figures

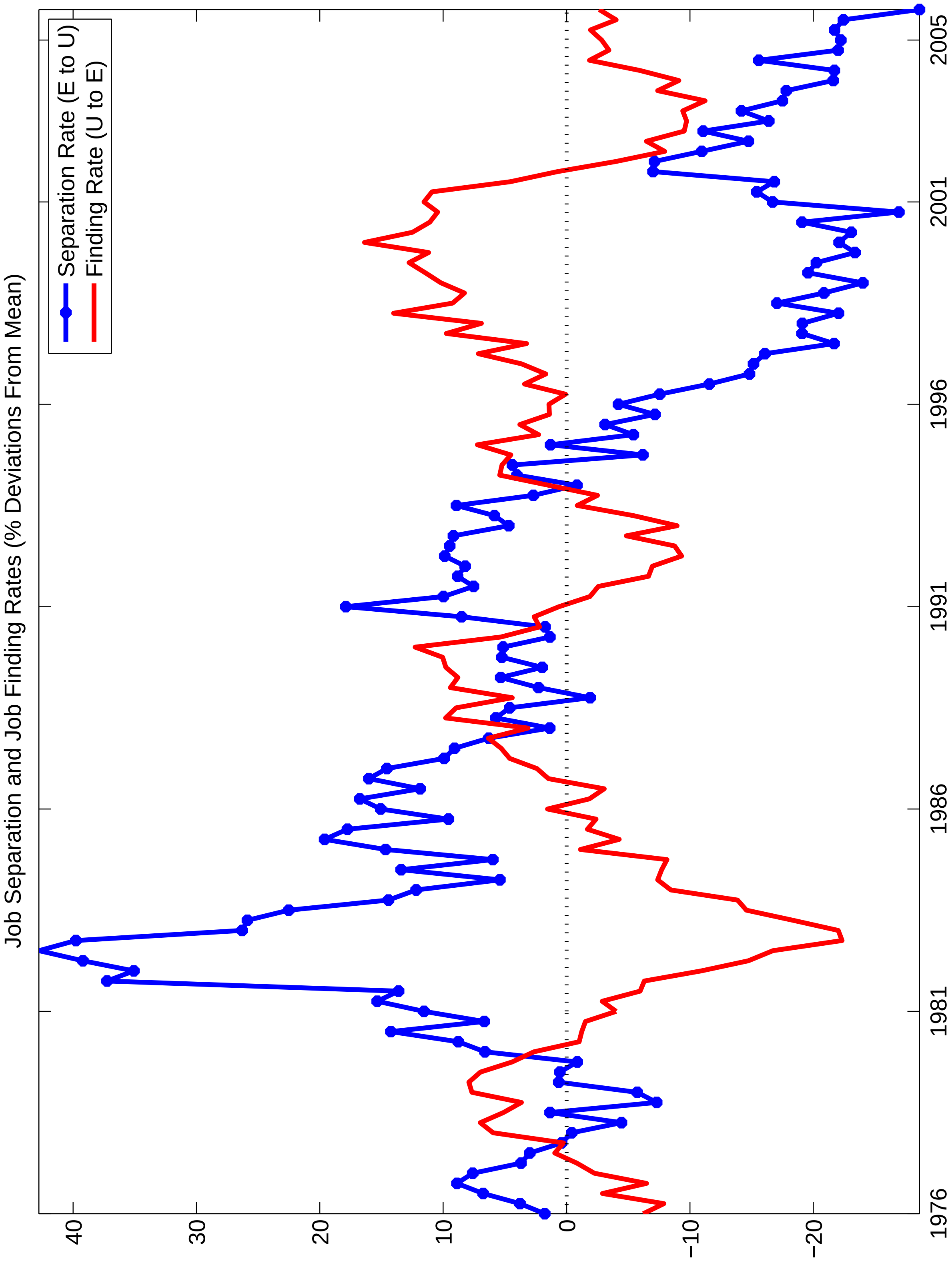

Figure 7.1: US Job Separations and Job Findings

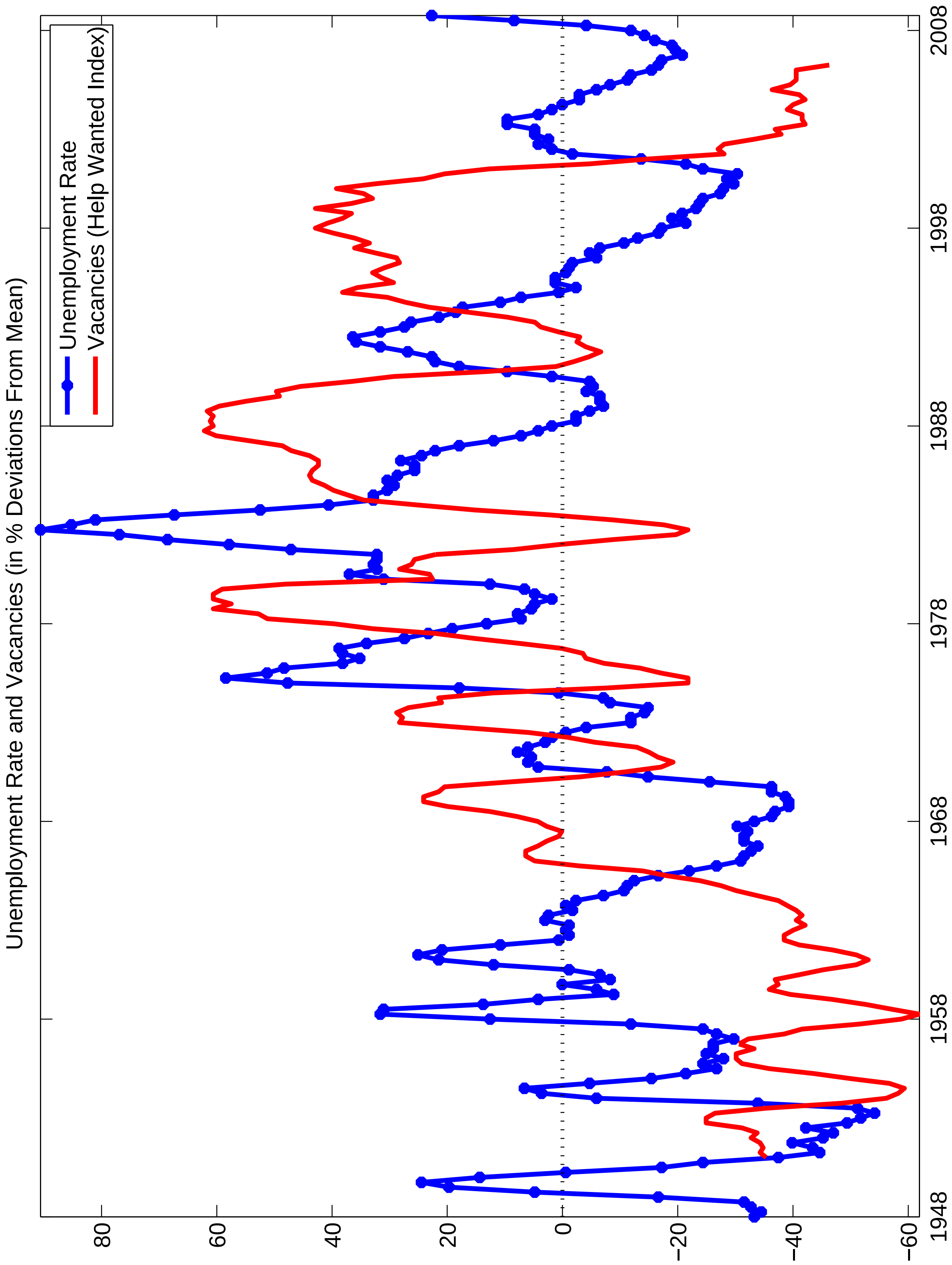

Figure 7.2: US Unemployment Rate and Vacancies (Help Wanted Index of the Conference Board)

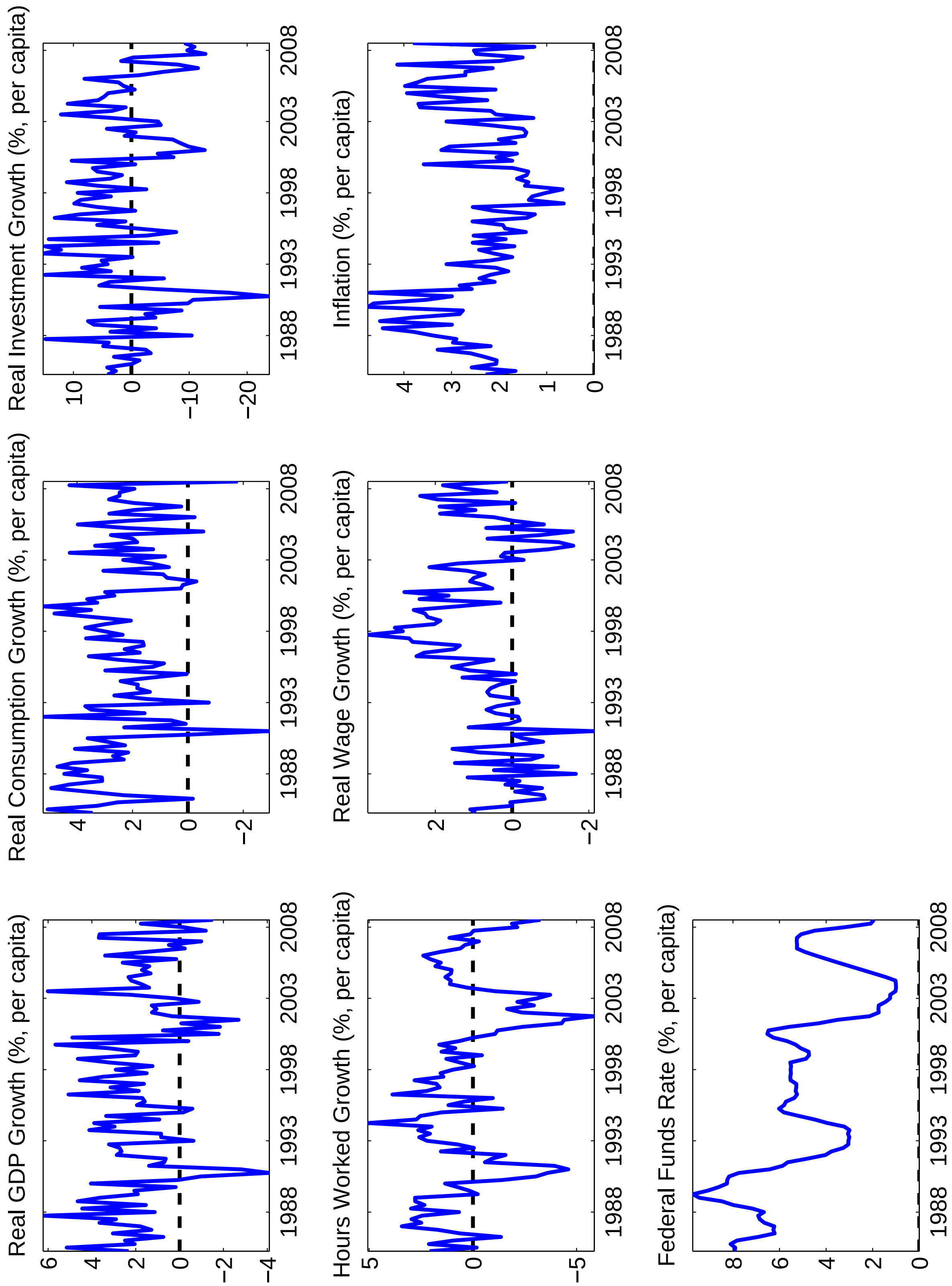

Figure 7.3: US Data used for Estimation

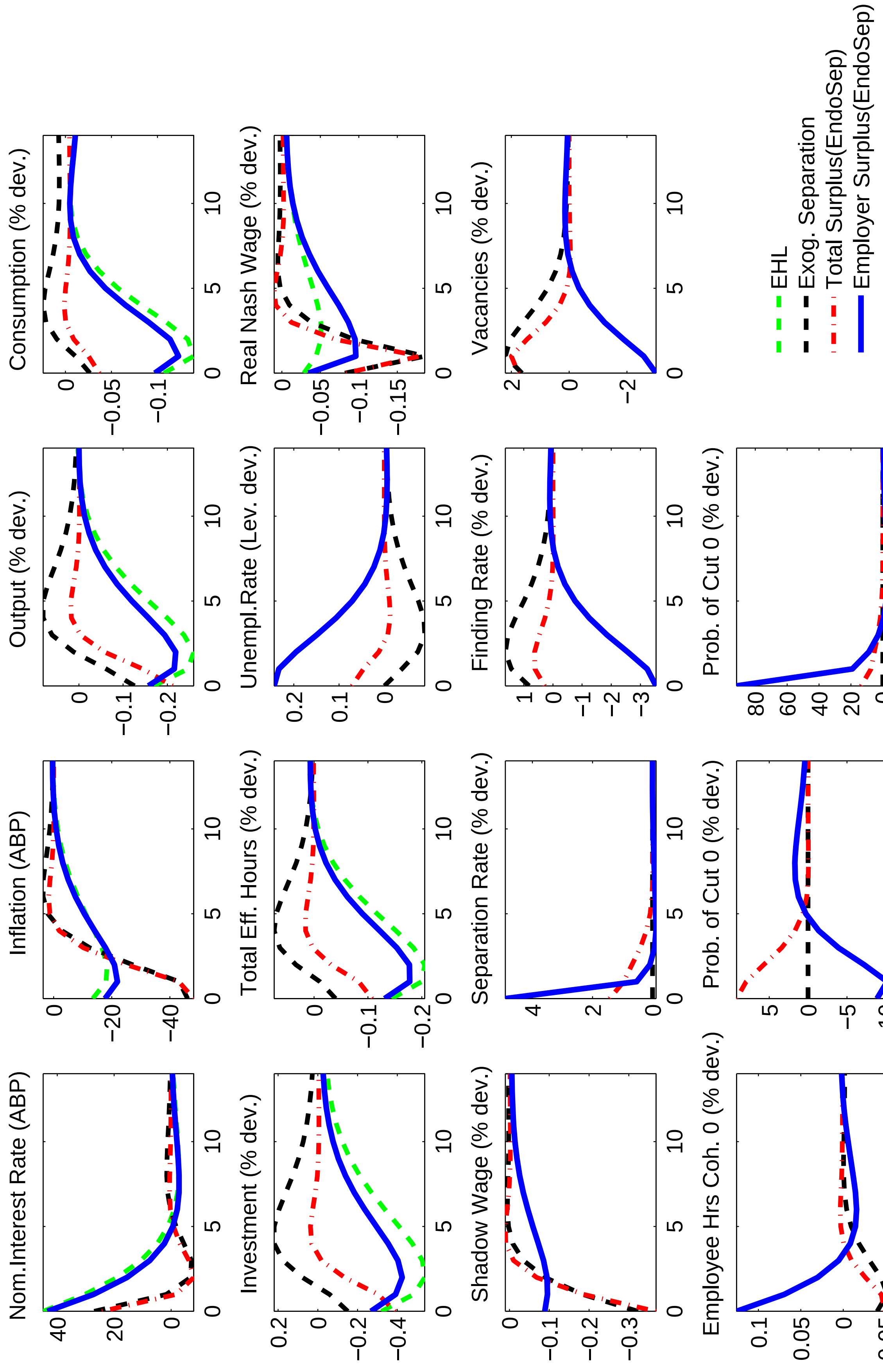

Figure 7.4: Impulse Responses to a Nominal Interest Rate Shock at the Estimated Posterior Mean

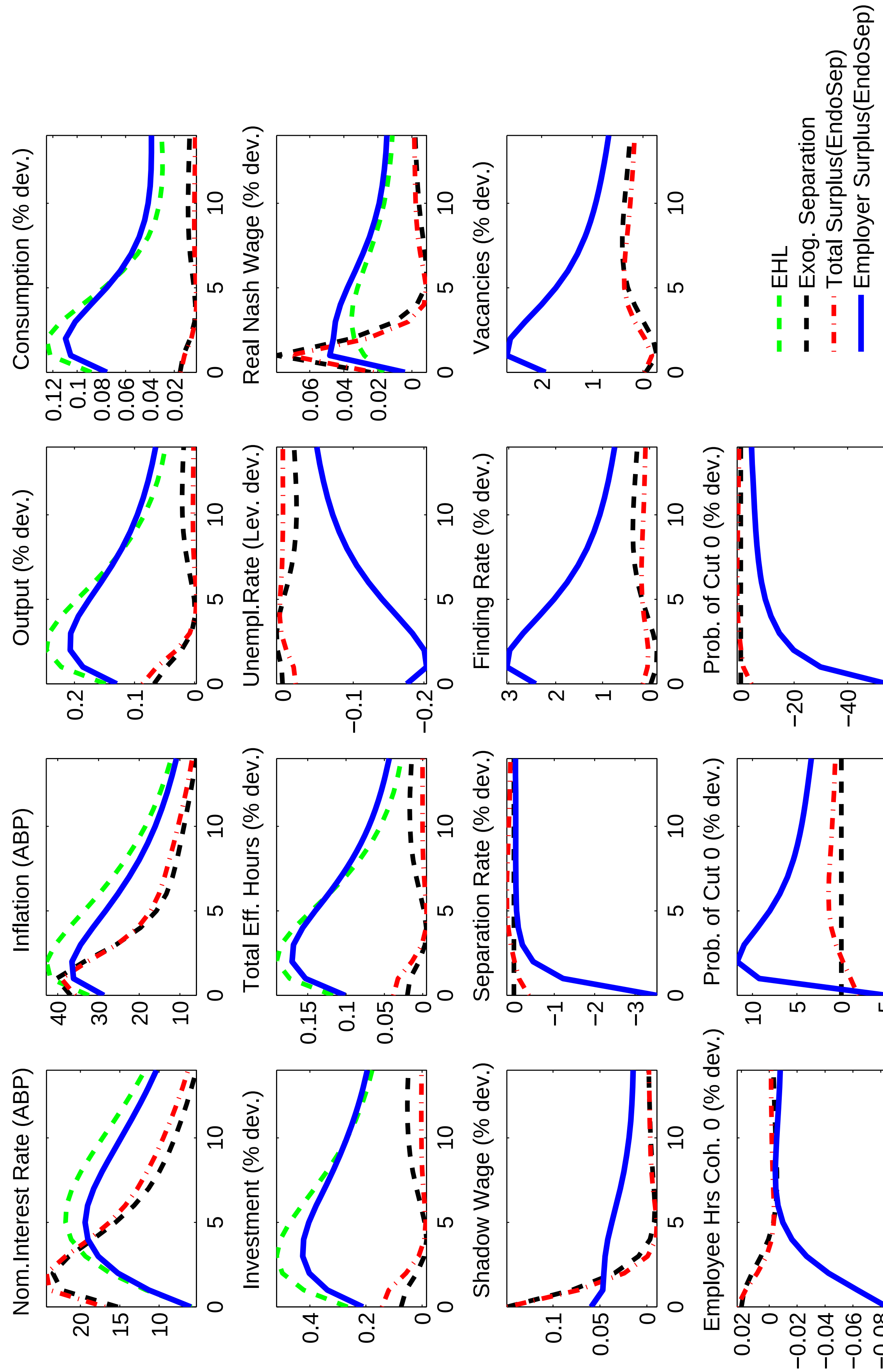

Figure 7.5: Impulse Responses to an Inflation Target Shock at the Estimated Posterior Mean

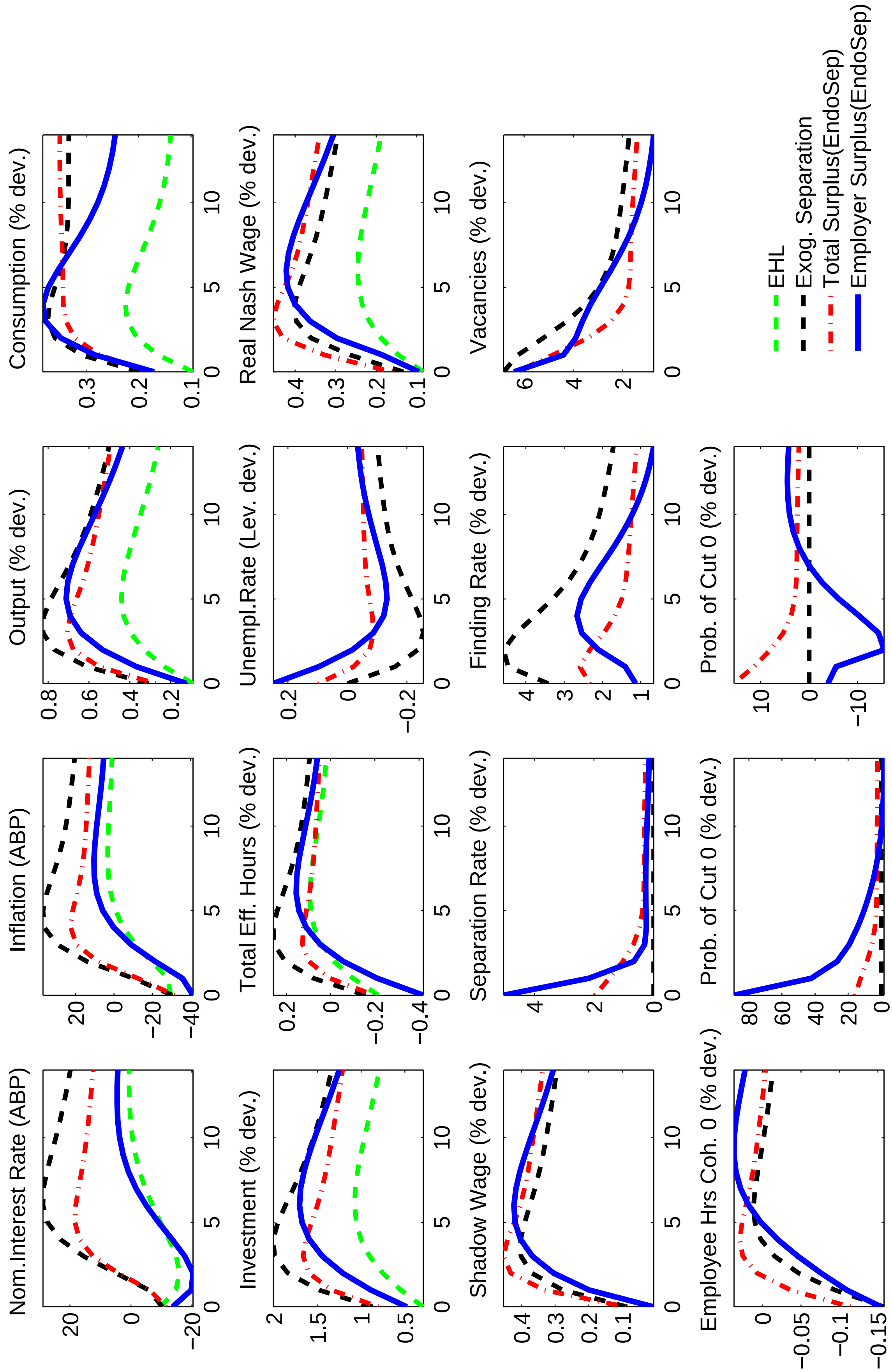

Figure 7.6: Impulse Responses to a Stationary Neutral Technology Shock at the Estimated Posterior Mean

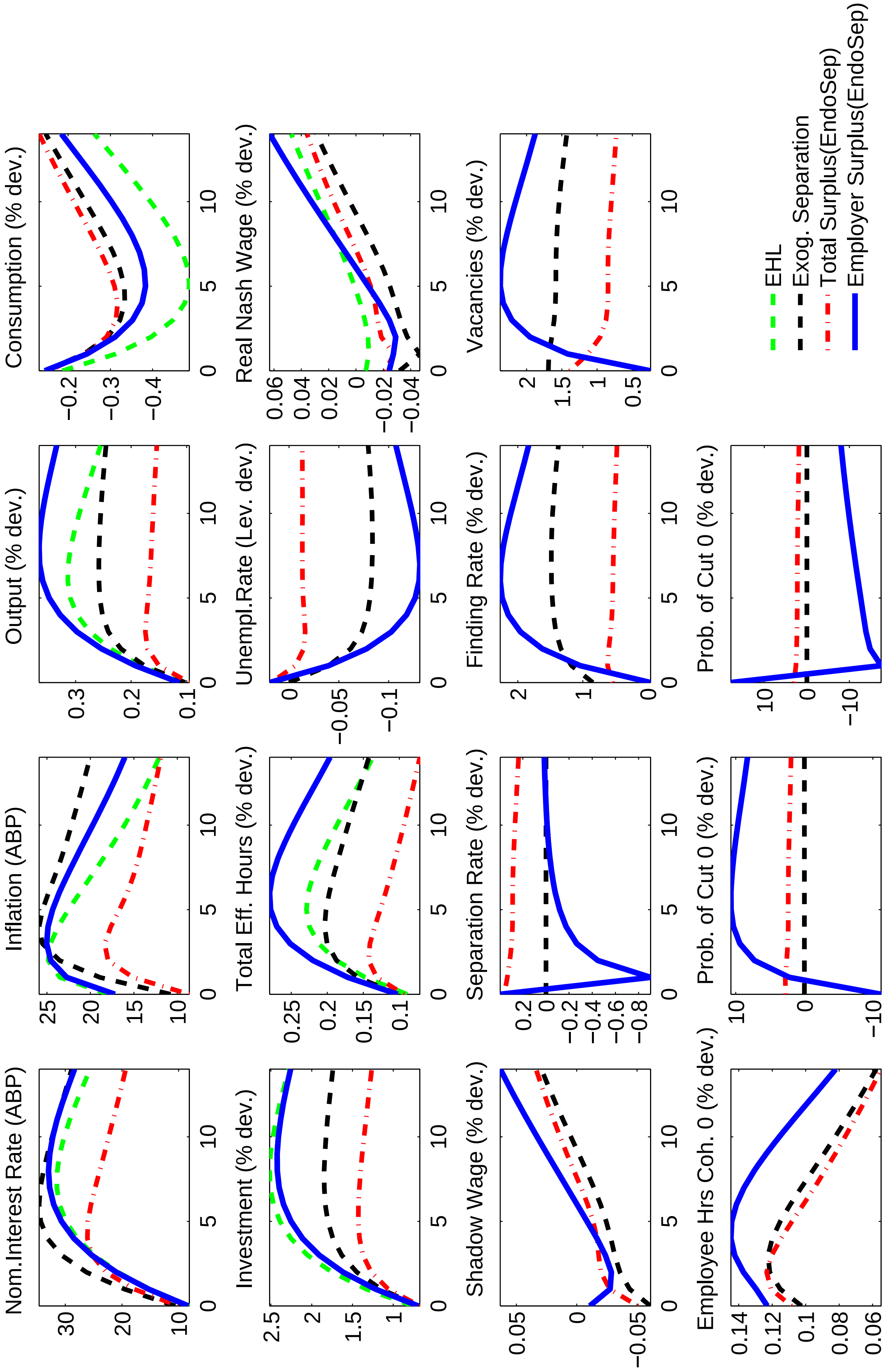

Figure 7.7: Impulse Responses to a Stationary Investment Specific Technology Shock at the Estimated Posterior Mean

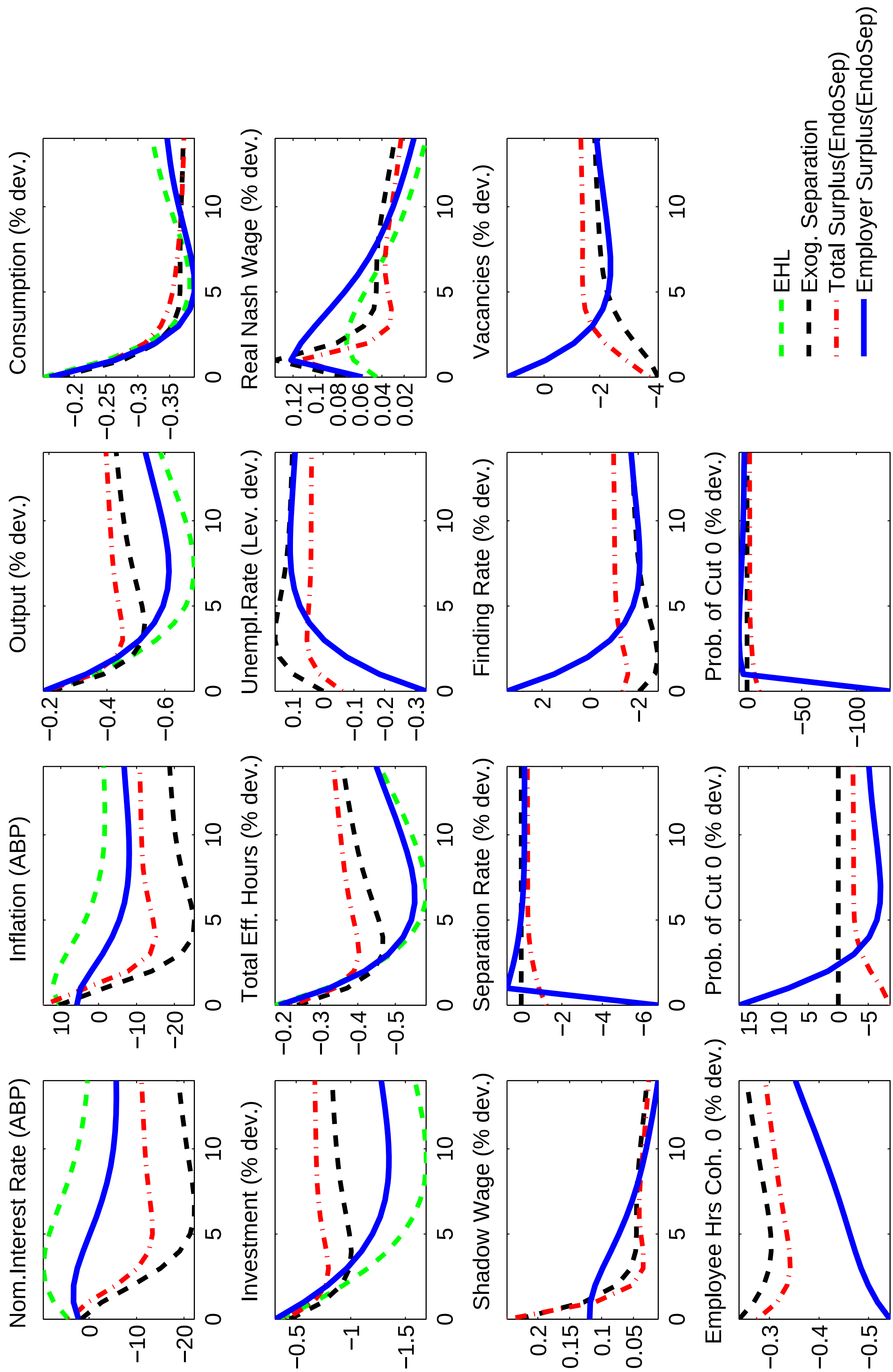

Figure 7.8: Impulse Responses to a Labor Preference Shock at the Estimated Posterior Mean

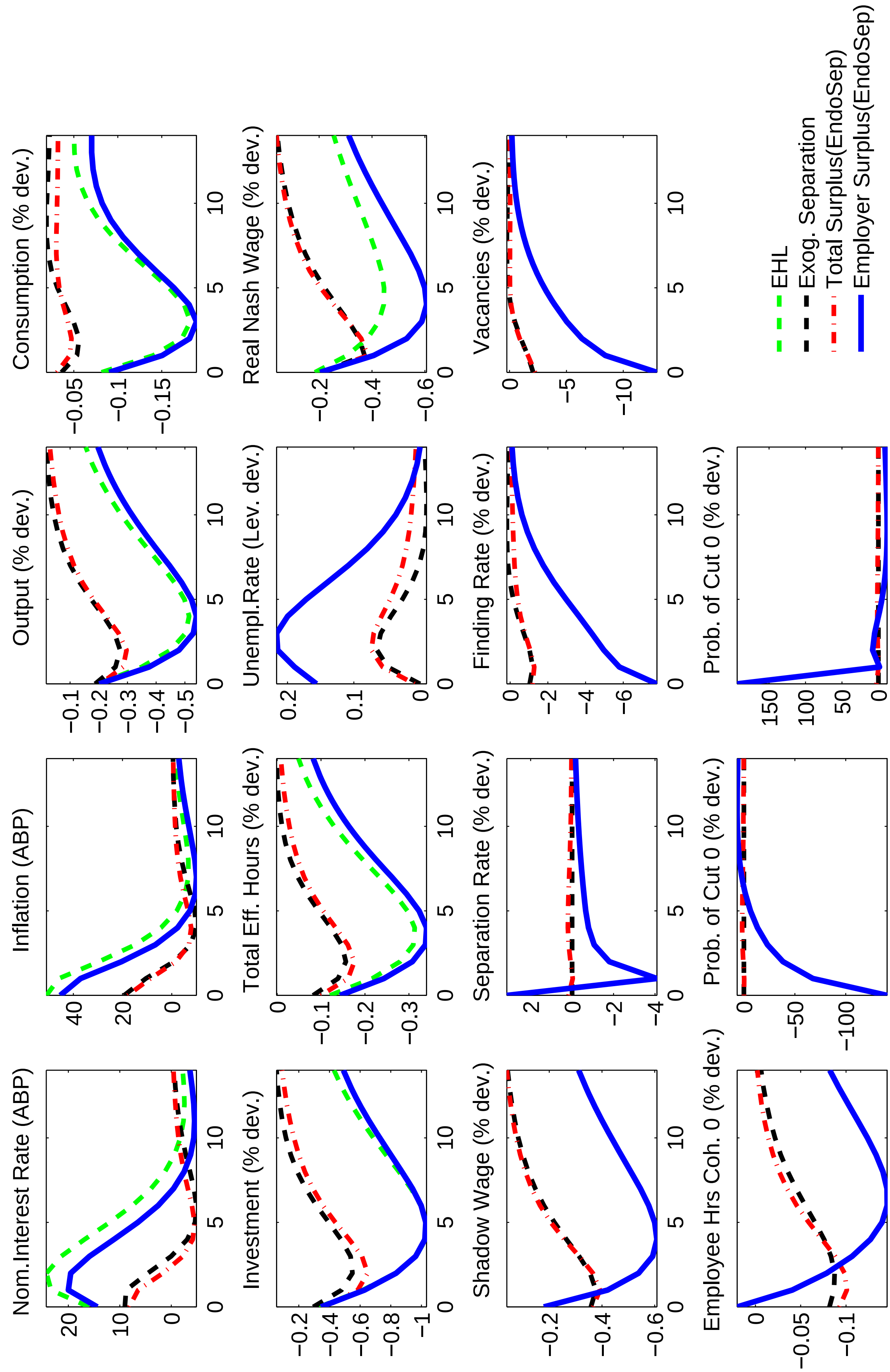

Figure 7.9: Impulse Responses to a Price Markup Shock at the Estimated Posterior Mean

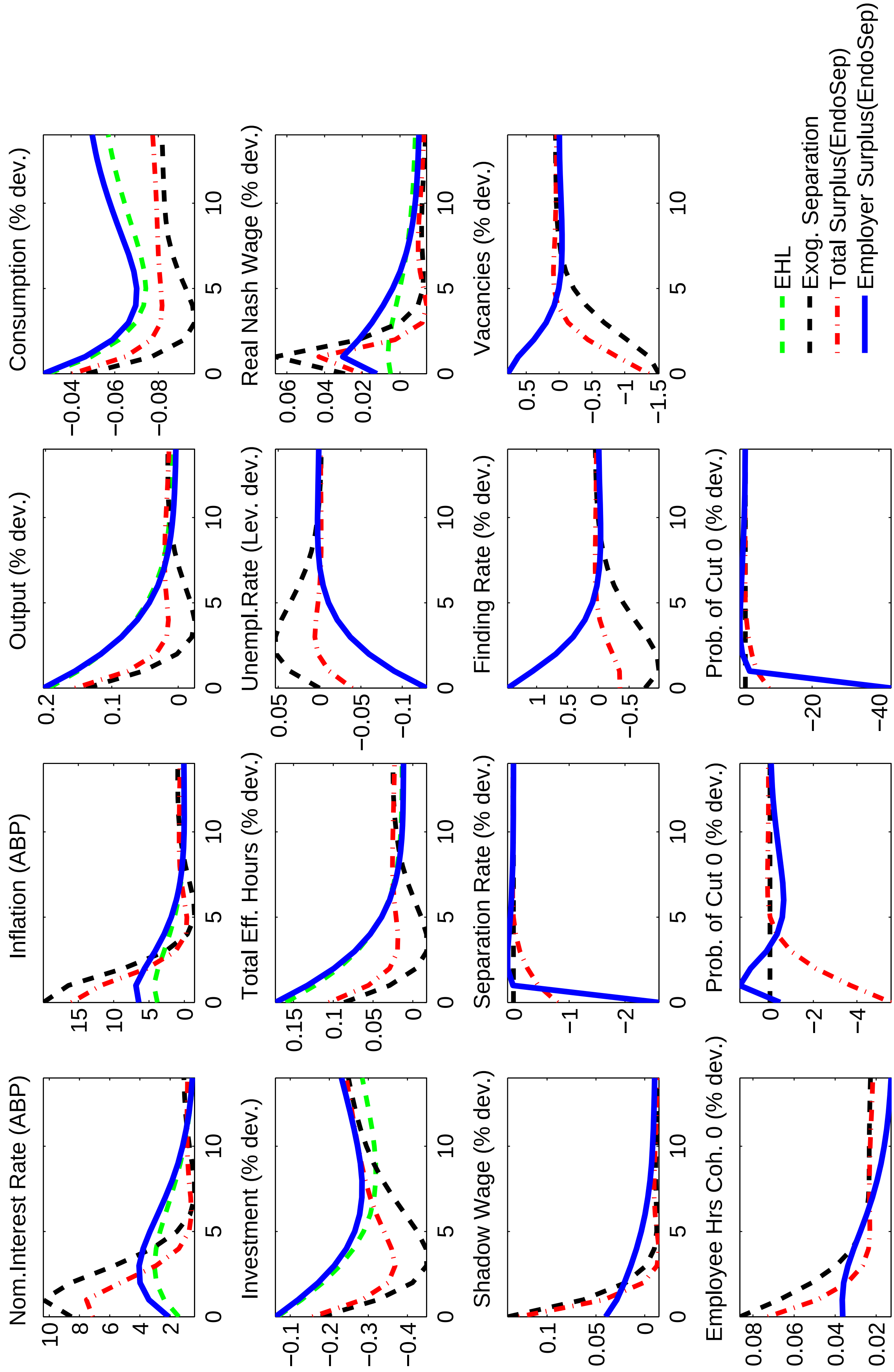

Figure 7.10: Impulse Responses to a Government Consumption Shock at the Estimated Posterior Mean

					EHL			Endogenous Separation: Employer Surplus			Endogenous Separation: Total Surplus			Exog. Separation		
		prior	prior mean	pstdev	mean	5%	95%	mean	5%	95%	mean	5%	95%	mean	5%	95%
Parameters																
ξ_d	Price stickiness	B	0.5	0.15	**0.74**	0.64	0.84	**0.76**	0.69	0.83	**0.64**	0.57	0.71	**0.63**	0.55	0.70
κ_d	Price indexation	B	0.5	0.1	**0.37**	0.21	0.52	**0.36**	0.21	0.49	**0.51**	0.35	0.67	**0.50**	0.33	0.65
λ_d	Price markup	N	1.2	0.05	**1.28**	1.21	1.36	**1.23**	1.15	1.31	**1.24**	1.16	1.32	**1.24**	1.16	1.33
b	Conumption habit	B	0.75	0.05	**0.64**	0.57	0.71	**0.62**	0.55	0.69	**0.59**	0.52	0.67	**0.61**	0.54	0.69
S''	Invest. adj. cost curv.	N	0.4	0.1	**0.39**	0.21	0.56	**0.38**	0.22	0.55	**0.10**	0.02	0.19	**0.14**	0.02	0.25
σ_a	Capac. adj. cost curv.	B	0.2	0.1	**0.29**	0.10	0.47	**0.43**	0.24	0.62	**0.40**	0.22	0.59	**0.37**	0.18	0.55
ν_f	Working capital share	B	0.5	0.15	**0.42**	0.17	0.65	**0.44**	0.20	0.68	**0.44**	0.20	0.69	**0.44**	0.19	0.67
ρ_R	Interest rate smoothing	B	0.75	0.1	**0.81**	0.76	0.87	**0.80**	0.74	0.86	**0.67**	0.58	0.76	**0.67**	0.58	0.77
r_π	Inflation weight	N	1.5	0.2	**1.88**	1.61	2.14	**1.90**	1.63	2.16	**1.91**	1.63	2.17	**1.91**	1.66	2.17
r_y	Output weight	N	0.5	0.1	**0.03**	-0.05	0.11	**0.02**	-0.07	0.10	**-0.08**	-0.14	-0.03	**-0.10**	-0.14	-0.06
κ_w	Wage indexation	B	0.5	0.1	**0.37**	0.24	0.50	**0.54**	0.40	0.69	**0.51**	0.36	0.67	**0.52**	0.36	0.69
ξ_w	Wage stickiness	B	0.5	0.15	**0.72**	0.61	0.84	-	-	-				-	-	-
λ_w	Wage markup	N	1.5	0.1	**1.48**	1.31	1.65	-	-	-	-	-	-	-	-	-
F	SS endog. separation (%)	G	0.6	0.05	-	-	-	**0.63**	0.56	0.72	**0.62**	0.54	0.71	-	-	-
s_R	SS recruit. costs/GDP (%)	IG	0.05	inf	-	-	-	**0.01**	0.01	0.01	**0.01**	0.01	0.02	**0.01**	0.01	0.02
AR(1) Parameters																
ρ_ε	Stat. neutral tech. shock	B	0.8	0.1	**0.92**	0.86	0.98	**0.79**	0.60	0.97	**0.93**	0.89	0.98	**0.94**	0.91	0.98
ρ_ψ	Stat. invest. tech. shock	B	0.8	0.1	**0.86**	0.71	0.99	**0.84**	0.65	0.99	**0.95**	0.90	0.99	**0.96**	0.92	0.99
$\rho_{\zeta h}$	Labor preference shock	B	0.8	0.1	**0.93**	0.84	1.00	**0.96**	0.92	1.00	**0.98**	0.96	1.00	**0.97**	0.95	1.00
ρ_g	Gov. spending shock	B	0.8	0.1	**0.90**	0.80	0.99	**0.87**	0.77	0.98	**0.91**	0.81	0.99	**0.91**	0.83	1.00
$\rho_{\tau d}$	Price markup shock	B	0.8	0.1	**0.87**	0.76	0.98	**0.86**	0.74	0.98	**0.75**	0.61	0.89	**0.79**	0.68	0.91
ρ_π	Inflation target shock	B	0.8	0.1	**0.89**	0.82	0.96	**0.89**	0.81	0.97	**0.86**	0.76	0.97	**0.86**	0.75	0.97
Standard Deviations																
σ_ε	Stat. neutral tech. shock	IG	1	Inf	**0.26**	0.21	0.31	**0.48**	0.38	0.59	**0.40**	0.31	0.49	**0.39**	0.30	0.47
σ_ψ	Stat. invest. tech. shock	IG	1	Inf	**1.74**	0.93	2.58	**1.63**	0.82	2.43	**0.84**	0.51	1.15	**0.92**	0.56	1.28
$\sigma_{\zeta h}$	Labor preference shock	IG	1	Inf	**3.97**	1.62	6.66	**2.14**	1.56	2.69	**1.53**	1.16	1.89	**1.45**	1.09	1.77
σ_g	Gov. spending shock	IG	1	Inf	**0.95**	0.74	1.15	**0.97**	0.75	1.19	**0.83**	0.63	1.02	**0.79**	0.60	0.99
$\sigma_{\tau d}$	Price markup shock	IG	1	Inf	**0.79**	0.36	1.25	**0.90**	0.48	1.30	**0.56**	0.36	0.75	**0.51**	0.34	0.67
σ_π	Inflation target shock	IG	0.1	Inf	**0.09**	0.05	0.14	**0.08**	0.04	0.11	**0.05**	0.03	0.06	**0.05**	0.03	0.06
σ_R	Nominal interest shock	IG	1	Inf	**0.13**	0.12	0.15	**0.13**	0.12	0.15	**0.14**	0.12	0.15	**0.14**	0.12	0.15
DSGE-VAR																
λ	DSGE-VAR hyperparam.	IG	1	Inf	**1.09**	0.86	1.32	**0.85**	0.71	0.97	**0.77**	0.66	0.87	**0.75**	0.65	0.85
Log Data Density						**-983**			**-1007**			**-1047**			**-1053**	

Notes: MCMC 250 000 draws; 50 000 burn in; acceptance ratio roughly 0.25

Figure 7.11: Estimated Parameters

		Δ GDP	Δ Consumption	Δ Investment	Δ Hours	Δ Wage	Inflation	Nom. Interest	Separation Rate	Finding Rate	Unemployment Rate	Vacancies
Stationary neutral technology shock	EHL	16	18	8	31	23	13	13	-	-	-	-
	Endo. Sep. - Empl. Surplus	30	27	25	55	23	14	6	20	10	11	18
	Endo. Sep. - Total Surplus	42	33	41	44	33	22	17	39	42	42	44
	Exog. Separation	51	36	47	44	26	26	20	-	40	39	41
Stationary investment technology shock	EHL	18	24	75	16	0	11	27	-	-	-	-
	Endo. Sep. - Empl. Surplus	6	30	42	5	1	30	64	1	19	19	14
	Endo. Sep. - Total Surplus	3	32	26	4	1	16	44	8	7	4	7
	Exog. Separation	4	29	29	7	1	24	45	-	16	16	15
Labor preference shock	EHL	20	22	9	21	3	8	14	-	-	-	-
	Endo. Sep. - Empl. Surplus	18	24	12	16	5	3	2	30	19	23	15
	Endo. Sep. - Total Surplus	17	31	10	27	5	22	22	32	46	41	42
	Exog. Separation	20	30	12	35	7	29	27	-	40	40	38
Government spending shock	EHL	13	2	1	12	0	0	1	-	-	-	-
	Endo. Sep. - Empl. Surplus	11	1	1	7	0	1	0	4	1	2	0
	Endo. Sep. - Total Surplus	8	2	2	7	1	1	1	4	0	1	1
	Exog. Separation	7	2	2	5	3	1	1	-	1	1	1
Price markup shock	EHL	22	16	6	11	72	20	13	-	-	-	-
	Endo. Sep. - Empl. Surplus	22	8	14	9	66	14	5	21	34	17	45
	Endo. Sep. - Total Surplus	14	1	10	7	42	2	1	1	4	9	3
	Exog. Separation	11	1	6	6	44	1	1	-	1	1	2
Inflation target shock	EHL	8	12	1	6	1	45	28	-	-	-	-
	Endo. Sep. - Empl. Surplus	5	4	3	3	1	32	13	9	11	16	6
	Endo. Sep. - Total Surplus	2	0	1	1	3	21	13	2	0	0	0
	Exog. Separation	1	0	0	0	3	10	6	-	0	0	0
Nominal interest rate shock	EHL	4	6	1	4	1	2	5	-	-	-	-
	Endo. Sep. - Empl. Surplus	8	7	4	5	3	6	10	15	7	12	3
	Endo. Sep. - Total Surplus	14	1	10	9	15	15	3	14	1	3	3
	Exog. Separation	6	1	3	3	16	8	2	-	3	3	3

Notes: Asymptotic variance decomposition; separation rate, finding rate, unemployment and vacancies not matched with data in the estimation

Figure 7.12: Asymptotic Variance Decomposition at the Estimated Posterior Mean

	Means (%)		Standard Deviations (%)				
	Data	All Models	Data	EHL	Endog. Sep. Empl. Surplus	Endog. Sep. Total Surplus	Exog. Separation
Δ GDP	1.75	1.75	2.00	2.48	2.66	2.59	2.69
Δ Consumption	2.30	1.75	1.58	1.68	1.71	1.45	1.56
Δ Investment	0.84	1.75	7.80	6.63	6.63	6.51	6.93
Δ Hours	0.34	0.00	2.15	2.12	0.69	0.48	0.48
Δ Wage	0.70	1.75	1.20	1.16	1.67	1.82	1.72
Inflation	2.45	2.44	0.95	1.75	1.76	1.78	2.48
Nom. Interest	4.98	4.80	2.18	1.71	1.75	1.65	2.39
Job Separation Rate	5.50	5.04	13.7	-	12.6	5.02	-
Job Finding Rate	72.3	79.5	6.88	-	22.5	12.1	21.1
Unemployment Rate	5.64	6.00	17.7	-	21.1	8.38	19.7
Vacancies	*	*	33.8	-	27.5	18.1	23.5

Notes: Job separation, job finding rates and unemployment in level for data mean and in percent deviations from steady state for standard deviations. Separation rate, finding rate, unemployment and vacancies are not matched with data in the estimation. Sample: 1985.1 to 2008.3 (if available).* Vacancies in the data are the help wanted index provided by the conference board. Since it is an index we do not report the mean value in the data and do not compare it with steady state vacancies of the models.

Figure 7.13: Comparison of Means and Standard Deviations of US Data and Models

7.2. Scaling of Variables

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, Ψ_t, is an embodied shock to technology and it is convenient to define the following combination of embodied and neutral technology:

$$\begin{aligned} z_t^+ &= \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &= \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{7.1}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+ \Psi_t$. Consumption goods C_t, government consumption G_t and the real wage $\frac{W_t}{P_t}$ are scaled by z_t^+. Also, υ_t is the shadow value in utility terms to the household of and $\upsilon_t P_t$ is the shadow value of one consumption good (i.e., the marginal utility of consumption). The latter must be multiplied by z_t^+ to induce stationarity. Thus,

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+ \Psi_t},\ i_t = \frac{I_t}{z_t^+ \Psi_t},\ c_t = \frac{C_t}{z_t^+},\ g_t = \frac{G_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t} \\ \psi_{z^+,t} &= \upsilon_t P_t z_t^+,\ \tilde{y}_t = \frac{Y_t}{z_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ w_t = \frac{\tilde{W}_t}{W_t},\ D_{z^+,t}^j \equiv \frac{D_t^j}{P_t z_t^+}. \end{aligned}$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_t P_{k',t},\ \bar{r}_t^k = \Psi_t r_t^k.$$

where $P_{k',t}$ is in units of the homogeneous good. We define the following inflation rates:

$$\pi_t = \frac{P_t}{P_{t-1}},\ \pi_t^i = \frac{P_t^i}{P_{t-1}^i}.$$

Here, P_t is the price of the homogeneous output good and P_t^i is the price of the domestic final investment good.