# Optimal Monetary Policy in a Model of the Credit Channel *

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**Abstract**

We consider a simple extension of the basic new-Keynesian setup in which we relax the assumption of frictionless financial markets. In our economy, asymmetric information and default risk lead banks to optimally charge a lending rate above the risk-free rate. Our contribution is threefold. First, we derive analytically the loglinearised equations which characterise aggregate dynamics in our model and show that they nest those of the new-Keynesian model. A key difference is that marginal costs increase not only with the output gap, but also with the credit spread and the nominal interest rate. Second, we find that financial market imperfections imply that exogenous disturbances, including technology shocks, generate a trade-off between output and inflation stabilisation. Third, we show that, in our model, an aggressive easing of policy is optimal in response to adverse financial market shocks.

*Keyworks*: optimal monetary policy, financial markets, asymmetric information

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# 1 Introduction

Central banks devote much effort to the analysis of the financial positions of households, firms and financial institutions, and to monitor the evolution of credit aggregates and interest rate spreads. One reason is that financial market conditions are perceived to be factors which contribute to shape the performance of the economy and to affect its inflationary prospects.

In several historical episodes, central banks have also reacted sharply to changes in financial conditions. One example are the US developments during the late 1980s, when banks experienced large loan losses as a consequence of the bust in the real estate market. Due to weak financial conditions, banks could not raise new capital and, because of the requirement to comply with the Basel Accord, they were forced to cut back on loans. This led to a slowdown in credit growth and aggregate spending. According to Rudebusch (2006), this slowdown contributed to the FOMC decision to reduce the Federal funds rate well below what suggested by an estimated Taylor rule. A more recent example is provided by the financial market turmoil initiated in 2007 with the deterioration in the performance of nonprime mortgages in the US. Over 2007 and 2008, concerns about the ongoing deterioration of financial market conditions and tightening of credit conditions led to sharp cuts in policy interest rates in many countries.

Developments of the sort outlined above raise obvious questions on the appropriateness of these policy responses. Through which exact channels are these shocks transmitted to the real economy? Should financial market variables matter *per se* for monetary policy, or should they only be taken into account to the extent that they affect output and inflation? Can an increase in credit spreads generate a large enough economic reaction to justify interest rate cuts of the magnitude observed over 2007-2008?

The answer to these questions requires an analysis of the optimal monetary policy implications of models in which financial frictions play a causal role. It is also important to understand how exactly financial frictions interact with other distortions, notably nominal rigidities, to modify the scope for monetary policy actions.

To study whether and how financial market conditions ought to have a bearing on monetary policy decisions, we analyze the simplest possible extension of the basic new-Keynesian setup, in which results can be derived analytically. We assume that firms need to pay wages in advance of production and that informational frictions imply that they must borrow at a premium over the risk-free rate. As in Bernanke, Gertler and Gilchrist (1989) and Carlstrom and Fuerst

(1997, 1998), we rely on the costly state verification set-up in Townsend (1979) to characterise the optimal debt contract between firms and financial intermediaries. The advantage of relying on a micro-founded debt contract is that the model parameters will be policy invariant and our optimal policy analysis will not be subject to the Lucas critique.

We obtain two main sets of results.

First, we show that the loglinear approximation of the aggregate structural equations of our model is similar in structure to the one arising in the new-Keynesian setup with frictionless financial markets. As in the new-Keynesian case, private sector decisions can be characterized by an intertemporal IS equation and a Phillips curve. These relationships, however, include additional terms to reflect the existence of informational asymmetries. The main difference is that firms' marginal costs reflect, on top of the costs of labour input, also the credit spread and the nominal interest rate. The latter two variables matter because they determine the cost of credit for firms in the economy.

The loglinearized equilibrium equations also show that technology and financial market shocks operate as exogenous cost-push factors in the model. This is noticeable for technology shocks, which in the benchmark model with frictionless financial markets generate fully efficient fluctuations in output and consumption. In our model, however, these fluctuations produce variations in firms' exposure to external finance and leverage. The ensuing volatility in credit spreads and bankruptcy rates represents the inefficient implications of technology shocks in the presence of credit frictions.

Our second set of results concerns optimal policy. Using an analytic, second-order approximation of the welfare function, we demonstrate that welfare is directly affected not just by the volatility of inflation and the output gap, as in the benchmark case with frictionless financial markets, but also by the volatility of the nominal interest rate and of the credit spread. As a result, the target rule which would characterise optimal policy under discretion ought to include a reaction to credit spreads, even if with a small coefficient.

We also study whether optimal monetary policy should strive to bring equilibrium allocations back to a fully efficient level, or whether instead it should only attempt to implement a constrained optimum in which financial frictions are treated as given. The latter option may appear to be intuitively appealing, based on the observation that credit spreads ultimately stem from an information asymmetry which cannot be eliminated through policy interventions. In our model, however, financial market imperfections will interact with other frictions, such as nominal price rigidity. Consistently with general second-best results, it will turn out to be the case that monetary policy can undo some of the adverse implications on welfare of financial market imperfections.

We then characterise optimal policy under commitment from a numerical viewpoint. We show that the optimal policy reaction to technology shocks is not dramatically different from the case with frictionless financial markets and from the prescriptions of a simple policy rule of the Taylor type. More specifically, near complete inflation stabilization remains optimal.

In reaction to a financial market shock which increases the credit spread, however, optimal policy deviates markedly from the prescriptions of a Taylor rule. The main channel through which a persistent increase in credit spreads affects the economy has to do with the dynamics of the cost of credit. If this goes up after an exogenous shock, firms will incur a higher cost of servicing their debt and they will therefore try to increase their mark-ups. As a result, real wages will fall, persistently so if the original shock is also persistent. The expected persistent reduction in real wages will induce an immediate drop in households' consumption, which will be the main driver of the economic slowdown.

A Taylor rule would prescribe an interest rate tightening to meet the rise in inflation. Optimal monetary policy, however, is aggressively expansionary after the shock. While sustaining the inflationary pressure through the ensuing stimulus of aggregate demand, the interest rate cut directly contrasts the cost-push effect on inflation of the higher spread. The net effect on inflation is actually milder than under a Taylor rule.

Our paper is not the first attempt to analyze monetary policy in models with credit frictions. Ravenna and Walsh (2006) characterizes optimal monetary policy when firms need to borrow in advance to finance production. However, there is no default risk in that model and the cost of financing for firms is the risk-free rate. We show that our model nests that of Ravenna and Walsh (2006) in the special case in which the costs of asymmetric information disappear. Faia and Monacelli (2006) compares the welfare losses of various optimized simple interest rate rules in models with a structure similar to ours, but it does not characterize fully optimal (Ramsey) monetary policy. Similarly, Christiano, Motto and Rostagno (2006) argues that the monetary policy reaction to a stock market boom/bust cycle would be superior, in terms of welfare, if liquidity developments were taken into account.

Our paper is closest to recent work by Cúrdia and Woodford (2008), which also characterizes optimal monetary policy in a model where financial frictions matter, because of heterogeneity in the spending opportunities available to different households. Our work differs in the underlying source of financial frictions. Financial frictions are microfounded in our model and credit spreads arise from an explicit characterization of optimal debt contracts. Cúrdia and Woodford (2008) assume instead a flexible, reduced-form function linking the credit spread to macroeconomic conditions. Finally, Faia (2008) studies optimal monetary policy in a model with microfounded financial frictions similar to ours, but the focus of that paper is solely on technology shocks and the richer environment prevents an analytical characterization of the results.

The paper proceeds as follows. In section 2, we describe the environment and derive the conditions characterizing the equilibrium of the economy when financial contracts are written in nominal terms. In section 3, we discuss the log-linearized version of our model, in comparison to the new-Keynesian benchmark. This enables us to highlight the effect of financial market frictions on inflation and output dynamics. In section 4, we derive a simple quadratic approximation of the social welfare, which we compare to the one arising under frictionless financial markets. In section 5, we derive the first-order conditions of the social planner problem under discretion and we discuss the role of financial frictions for the optimal conduct of monetary policy. We then characterize numerically optimal monetary policy under commitment. Section 6 concludes.

## 2 The environment

The economy is inhabited by a representative infinitely-lived household and by a continuum of risk-neutral entrepreneurs. Households own firms producing differentiated goods in the retail sector, while entrepreneurs own firms producing a homogeneous good in the wholesale sector.

Financial market imperfections, in the form of asymmetric information and costly state verification, affect the activity of wholesale firms. These firms produce according to a technology that is linear in labor and subject to idiosyncratic productivity shocks. Entrepreneurs need to raise external finance to pay workers in advance of production but, due to the idiosyncratic shock, they face the risk of default on their debt. Lending occurs through perfectly competitive financial intermediaries ('banks'), which are able to ensure a safe return to households by providing funds to the continuum of firms. Firms and banks stipulate debt contracts, which are the optimal contractual arrangements between lenders and borrowers in this costly state verification environment.

The timing of events is as follows. At the beginning of the period, after the occurrence of aggregate shocks, the financial market opens. Households make their portfolio decisions. They decide how to allocate nominal wealth among existing assets, namely money, a portfolio of nominal state-contingent bonds, and deposits. Deposits are collected by a zero-profit bank and used to finance firms' production. Each wholesale firm stipulates a contract with a bank in order to raise external finance.

In the second part of the period, the goods market opens. Wholesale firms produce homogenous goods and sell them to the retail sector. If revenues are sufficient, they repay the debt and devote remaining profits to the financing of entrepreneurial consumption. Otherwise, they default and their production is sized by banks. Firms in the retail sector buy the homogeneous good from wholesale firms in a competitive market and use them to produce differentiated goods at no costs. Because of this product differentiation, retail firms acquire some market power and become price makers. However, they are not free to change their price at will, because prices are subject to Calvo contracts. Retail goods are then purchased by households and wholesale entrepreneurs for own consumption.

## 2.1 Households

At the beginning of period $t$, the financial market opens. First, the interest on nominal financial assets acquired at time $t-1$ is paid. The households, holding an amount $W_t$ of nominal wealth, choose to allocate it among existing nominal assets, namely money $M_t$, a portfolio of nominal state-contingent bonds $Z_{t+1}$ each paying a unit of currency in a particular state in period $t+1$, and one-period deposits denominated in units of currency $D_t$ paying back $R_t^d D_t$ at the end of the period.

In the second part of the period, the goods market opens. Households' money balances are increased by the nominal amount of their revenues and decreased by the value of their expenses. Taxes are also paid or transfers received. The amount of nominal balances brought into period $t+1$ is equal to

$$M_t + P_t w_t h_t + V_t - P_t c_t - T_t, \tag{1}$$

where $h_t$ is hours worked, $w_t$ is the real wage, $V_t$ are nominal profits transferred from retail producers to households, and $T_t$ are lump-sum nominal taxes collected by the government. $c_t$ denote a CES aggregator of a continuum $j \in (0,1)$ of differentiated consumption goods produced by retail firms,

$$c_t = \left[ \int_0^1 c_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

with $\varepsilon > 1$. $P_t(j)$ denotes the price of good $j$, and $P_t = \left[ \int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$ is the price of the CES aggregator.

Nominal wealth at the beginning of period $t+1$ is given by

$$W_{t+1} = Z_{t+1} + R_t^d D_t + R_t^m \{ M_t + P_t w_t h_t + V_t - P_t c_t - T_t \}, \tag{2}$$

where $R_t^m$ denotes the interest paid on money holdings.

The household's problem is to maximize preferences, defined as

$$E_o \left\{ \sum_0^\infty \beta^t \left[ u(c_t) + \kappa(m_t) - v(h_t) \right] \right\}, \tag{3}$$

where $u_c > 0$, $u_{cc} < 0$, $\kappa_m \geq 0, \kappa_{mm} < 0$ and $v_h > 0$, $v_{hh} > 0$, and $m_t \equiv M_t / P_t$ denotes real balances. The problem is subject to the budget constraint

$$M_t + D_t + E_t \left[ Q_{t,t+1} Z_{t+1} \right] \leq W_t, \tag{4}$$

Define $\pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\Delta_{m,t} \equiv \frac{R_t - R_t^m}{R_t}$. The optimality conditions can be written as

$$\frac{v_h(h_t)}{u_c(c_t)} = w_t \tag{5}$$

$$\frac{1}{R_t} = E_t \left[ Q_{t,t+1} \right] \tag{6}$$

$$R_t = R_t^d$$

$$u_c(c_t) + \kappa_m(m_t) = \beta R_t E_t \left\{ \frac{u_c(c_{t+1}) + \kappa_m(m_{t+1})}{\pi_{t+1}} \right\}$$

$$\frac{\kappa_m(m_t)}{u_c(c_t)} = \frac{\Delta_{m,t}}{1 - \Delta_{m,t}}. \tag{7}$$

Moreover, the optimal allocation of expenditure between the different types of goods leads to the demand functions

$$c_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} c_t, \tag{8}$$

where $P_t(j)$ is the price of good $j$.

### 2.2 Wholesale firms

The wholesale sector consists of a continuum of competitive firms, indexed by $i$, owned by infinitely lived entrepreneurs. Each firm produces the amount $y_{i,t}$ of a homogeneous good, using a linear technology

$$y_{i,t} = A_t \omega_{i,t} l_{i,t}. \tag{9}$$

Here $A_t$ is an aggregate, serially correlated productivity shock and $\omega_{i,t}$ is an idiosyncratic, iid productivity shock with distribution function $\Phi$ and density function $\phi$.

The production function (9) reflects our choice to abstract from capital accumulation. This is in contrast with most of the literature that introduces credit frictions in macro-models, where entrepreneurs are assumed to decide in period $t$ how to allocate their profits to consumption and investment expenditures (see e.g. Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999)). The value of the stock of capital available to firms in period $t+1$ provides the firm with a certain net worth (internal funds) that can be used in that period production. In that environment, aggregate shocks affect the evolution of firms' net worth, thus creating endogenous persistence. In our model, we assume instead that each firm receives a constant endowment $\tau$ at the beginning of each period, which can be used as internal funds. Since these funds are not sufficient to finance the firm's desired level of production, firms need to raise external finance. As a result, financial frictions have important effects also in our economy. For example, a spread arises endogenously between the loan rate charged by financial intermediaries to firms and the risk-free rate, to reflect the existence of default risk. At the same time, our simpler set-up enables us to provide an analytical characterization of economic dynamics and of optimal policy in the presence of credit constraints and information asymmetry.

#### 2.2.1 Labor demand

As in Christiano and Eichenbaum (1992) and Ravenna and Walsh (2006), we assume that firms need to pay factors of production before the proceeds from the sale of output are received.

Firms need to raise external finance to pay for wages. Before observing the idiosyncratic productivity shock, and after observing the aggregate shocks, they sign a contract with the financial intermediary to raise the amount $P_t\left(x_{i,t}-\tau\right)$, for total funds at hand $P_t x_{i,t}$, where[1]

$$x_{i,t} \geq w_t l_{i,t}. \tag{10}$$

We assume that entrepreneurs sell output only to retailers. Let $\overline{P}_t$ be the price of the wholesale homogenous good, and $\frac{\overline{P}_t}{P_t}=\chi_t^{-1}$ the relative price of wholesale goods to the aggregate price of retail goods. Each firm $i$'s demand for labor is derived by solving the problem

$$\max \quad \left[\frac{\overline{P}_t}{P_t}\mathcal{E}\left[A_t\omega_{i,t}l_{i,t}\right]-w_t l_{i,t}\right]$$

subject to the financing constraint (10), where the expectation $\mathcal{E}[\cdot]$ is taken with respect to the idiosyncratic shock unknown at the time of labor hiring decision, and $w_t$ denotes the payment of labor services measured in terms of the final consumption good. Denote the Lagrange multiplier on the financing constraint as $(q_{i,t}-1)$. Optimality requires that

$$q_{i,t} = q_t = \frac{A_t}{w_t\chi_t} \tag{11}$$

$$x_{i,t} = w_t l_{i,t} \tag{12}$$

implying that

$$\mathcal{E}\left[y_{i,t}\right]=\chi_t q_t x_{i,t}. \tag{13}$$

Equation (13) states that, as the production function is constant return to scale, wholesale firms must sell at a mark-up $\chi_t q_t$ over firms' production costs. This allows them to cover for the presence of monitoring costs and for the monopolistic distortion in the retail sector. This latter matters for firms in the wholesale sector because $P_t$ is the deflator of the nominal wage, and thus affects real marginal costs faced by wholesale producers.

Equation (12) states that the financing constraint is always binding. Given the contract stipulated by the firm with the financial intermediary (which sets the amount of funds $x_{i,t}$ and the repayment on these funds), the firm always finds it profitable to use the entire amount of

[1] We assume that the support of the aggregate productivity shock, $A_t$, is such that there is always a need for external finance. In the absence of this assumption, for sufficiently large negative shocks, $w_t l_{i,t}$ might be smaller than $\tau$, in which case firms could pay the wage bill using only their nominal internal funds.

funds and to produce, also when expected productivity is low. This way, it can minimize the probability of default.

### 2.2.2 The financial contract

Loans are stipulated in units of currency after all aggregate shocks have occurred, and repaid at the end of the same period. Lending occurs through the financial intermediary, which collects deposits from households and use them to finance loans to firms.

Firms face an idiosyncratic productivity shock, whose realization is observed at no costs only by the entrepreneur. The financial intermediary can monitor its realization but only at a cost, which is assumed to be a fraction of the value of the loan. If the realization of the idiosyncratic shock is sufficiently low, the value of the firm's production is not sufficient to repay the loan and the firm defaults. Households lend to firms through a financial intermediary, which is able to ensure a safe return. This is possible because by lending to the continuum of firms $i \in (0, 1)$ producing the wholesale good, the financial intermediary can differentiate the risk due to the presence of idiosyncratic shocks.

The informational structure corresponds to a costly state verification problem. The solution is a standard debt contract (see e.g. Gale and Hellwig, 1985) which is derived in the appendix. The terms of the contract are identical for all firms. The optimality conditions can be written as

$$q_t = \frac{R_t}{1 - \mu_t \Phi\left(\overline{\omega}_t\right) + \frac{\mu_t f(\overline{\omega}_t) \phi(\overline{\omega}_t)}{f_{\overline{\omega}}(\overline{\omega}_t)}} \tag{14}$$

$$x_t = \frac{R_t \tau}{R_t - q_t g\left(\overline{\omega}_t; \mu_t\right)}. \tag{15}$$

where $\overline{\omega}_t$ is a threshold for the distribution of the idiosyncratic productivity shock below which firms go bankrupt, and $f\left(\overline{\omega}_t\right)$ and $g\left(\overline{\omega}_t; \mu_t\right)$ are the expected shares of output accruing to the entrepreneur and the bank, respectively. $\mu_t$ denotes the share of value of the firm's input which is lost as a result of monitoring activities. Given the large time-variation in bankruptcy costs documented by Natalucci et al. (2004), it is assumed to be subject to serially correlated shocks.

Compared to the standard assumption of real debt contracts employed by Bernanke, Gertler and Gilchrist (1989) and Carlstrom and Fuerst (1997, 1998), our assumption of nominal contracts has two consequences.

The first is that monetary policy has real effects in our model – beyond those caused by the assumption of Calvo prices. The reason is not related to the impact of the higher nominal interest rate on the quantity of loans. Substituting equation (14) into equation (15), it can be noticed that a change in the nominal interest rate has no direct impact on the amount of real funds borrowed by entrepreneurs (the amount of funds is only modified in general equilibrium, to the extent that it induces changes in the threshold $\overline{\omega}_t$). The real effects of monetary policy arise entirely through the impact of the nominal interest rate on the financial mark-up $q_t$. An increase in the nominal interest rate increases the opportunity cost of lending funds for the financial intermediary and is therefore passed on to loan rates. The real effects of monetary policy in our model are therefore similar to those present in a cost-channel model. Loan rates, however, increase more than one-to-one with respect to the risk-free rate. The increase the latter variable makes it more difficult for firms to pay back their debt, and default probabilities must increase. As a result, credit spreads must also rise in equilibrium.

The second effect of the assumption of nominal contracts is that the fraction of the loan lost in monitoring activities is also in terms of currency, not in terms of physical goods – as is typically the case when contracts are in real terms. Intermediate firms sell their entire output to the retail sector at the end of the period and use the monetary proceedings from the sale to pay bank loans. To the extent that banks choose to monitor individual firms' productivity levels, some of the money will not be available to pay households' deposits. Thus financial frictions do not generate a loss of resources in our economy, but introduce an additional cost to be taken into account by banks when agreeing on an appropriate interest rate on loans.

An important implication of this assumption is that fluctuations in bankruptcy rates will only have an impact on utility (and welfare) indirectly, to the extent that they have undesirable implications on the mark-up $q_t$ or in the amount of loans. With real contracts, on the contrary, monitoring costs amount to a distruction of goods which would otherwise have been available for consumption: fluctuations in bankruptcy rates therefore have a direct utility cost.

The gross interest rate on loans can be backed out from the debt repayment, which requires $\overline{P}_t \overline{\omega}_t \chi_t q_t x_t = R_t^l P_t (x_t - \tau)$. This expression can be used to write the spread between the loan rate and the risk-free rate, $\Delta_t \equiv R_t^l / R_t^d$, as

$$\Delta_t = \frac{\overline{\omega}_t}{g(\overline{\omega}_t; \mu_t)}. \tag{16}$$

### 2.2.3 Entrepreneurs

Entrepreneurs have linear preferences over consumption and are infinitely lived. They consume a CES basket of differentiated goods similar to that of households.

At the end of each period, entrepreneurs sell their output to the retail sector and, if they do not default, repay the debt. Remaining profits are entirely allocated to final consumption goods

$$\int_0^1 P_t(j)\, e_{i,t}(j)\, dj = \overline{P}_t\left(\omega_{i,t} - \overline{\omega}_t\right)\chi_t q_t x_t,$$

where $e_{i,t}(j)$ is firm $i$'s consumption of good $j$. Notice that $\int_0^1 P_t(j)\, e_{i,t}(j) = P_t e_{i,t}$,where $e_{i,t}$ is the demand of the final consumption good of entrepreneur $i$. Aggregating across firms, we obtain $e_t = f(\overline{\omega}_t)\, q_t x_t$, where $e_t = \int_0^1 e_{i,t} di$ is the aggregate entrepreneurial consumption of the final consumption good. Using equations (14)-(15), we can rewrite aggregate entrepreneurial consumption as

$$e_t = \tau R_t \left(1 + \frac{\mu_t \phi(\overline{\omega}_t)}{f_{\overline{\omega}}(\overline{\omega}_t)}\right)^{-1} \tag{17}$$

Equation (17) shows that entrepreneurial consumption depends only on the nominal interest rate, on the bankruptcy threshold $\overline{\omega}_t$, and on the exogenous shock $\mu_t$.

As mentioned above, an increase in the nominal interest rate has no direct effect on loans and affects financial conditions mainly by inducing an increase in the mark-up $q_t$. This reflects into higher firms' profits so that, ceteris paribus, a higher $R_t$ leads to an increase in entrepreneurial consumption.

Changes in the threshold $\overline{\omega}_t$ act instead by modifying the output share $f(\overline{\omega}_t)$ (together with $g(\mu_t, \overline{\omega}_t)$). Since $f(\overline{\omega}_t)$ and entrepreneurs' profits are decreasing in the threshold, an increase in bankruptcy rates tends to depress entrepreneurial consumption.

Finally, a higher $\mu_t$ induces changes in the threshold $\overline{\omega}_t$. If total production changes little, firms have to pay a higher interest rate spread to cover for higher monitoring costs, and $\overline{\omega}_t$ tends to increase, leading to a reduction in entrepreneurial consumption. If however the shock is sufficiently contractionary, the demand for credit will fall and $\overline{\omega}_t$ will decrease.

## 2.3 Retail firms

As in Bernanke, Gertler and Gilchrist (1999), monopolistic competition occurs at the "retail" level. More specifically, a continuum of monopolistically competitive retailers buy wholesale output from entrepreneurs in a competitive market and then differentiate it at no cost. Because of product differentiation, each retailer has some market power. Profits are distributed to the households, who own firms in the retail sector.

Let $Y_t(j)$ be the quantity of output sold by retailer $j$. This quantity can be used for households' consumption, $c_t(j)$, and for entrepreneurs' consumption, $e_t(j)$. Hence,

$$Y_t(j) = c_t(j) + e_t(j).$$

The final good $Y_t$ is a CES composite of individual retail goods

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{18}$$

with $\varepsilon > 1$.

### 2.3.1 Price setting

We assume that each retailer can change its price with probability $1-\theta$, following Calvo (1983). Let $P_t(j)$ denote the price for good $j$ set by retailers that can change the price at time $t$, and $Y_t(j)$ the demand faced given this price. Then each retailer chooses its price to maximize expected discounted profits, given by

$$E_t\left[\sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t(j) - \overline{P}_{t+k}}{P_{t+k}} Y_{t+k}(j)\right],$$

where $\overline{Q}_{t,t+k} = \beta \frac{u_c(c_{t+1})+\kappa_m(m_{t+1})}{u_c(c_t)+\kappa_m(m_t)}$.

Denote $P_t^*$ as the optimal price set by producers who can reset prices at time $t$. The first-order conditions of the firm's profit maximization problem imply that

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon-1} \frac{E_t\left\{\sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{\overline{P}_{t+k}}{P_{t+k}^{1-\varepsilon}} P_t^{-\varepsilon} Y_{t+k}\right\}}{E_t\left\{\sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}} Y_{t+k}\right\}}.$$

Now define

$$\begin{aligned}
\Theta_{1,t} &\equiv \frac{\overline{P}_t}{P_t} Y_t + E_t \left\{ \sum_{k=1}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{\overline{P}_{t+k}}{P_{t+k}^{1-\varepsilon}} P_t^{-\varepsilon} Y_{t+k} \right\} \\
\Theta_{2,t} &\equiv Y_t + E_t \left\{ \sum_{k=1}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}} Y_{t+k} \right\}
\end{aligned}$$

Using the expression for the aggregate price index, $P_t = \left[ \theta P_{t-1}^{1-\varepsilon} + (1-\theta) \left( P_t^* \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$, and substituting out $\frac{P_t^*}{P_t}$, we can recursify the first order condition as

$$\begin{aligned}
1 &= \theta \pi_t^{\varepsilon-1} + (1-\theta) \left( \frac{\varepsilon}{\varepsilon-1} \frac{\Theta_{1,t}}{\Theta_{2,t}} \right)^{1-\varepsilon} \\
\Theta_{1,t} &= \frac{1}{\chi_t} Y_t + \theta E_t \left[ \pi_{t+1}^{\varepsilon} \overline{Q}_{t,t+1} \Theta_{1,t+1} \right] \\
\Theta_{2,t} &= Y_t + \theta E_t \left[ \pi_{t+1}^{\varepsilon-1} \overline{Q}_{t,t+1} \Theta_{2,t+1} \right].
\end{aligned}$$

### 2.3.2 Price dispersion

Recall that the aggregate retail price level is given by $P_t = \left[ \int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$. Define the relative price of differentiated good $j$ as $p_t(j) \equiv \frac{P_t(j)}{P_t}$ and divide both sides by $P_t$ to express everything in terms of relative prices, $1 = \int_0^1 \left( p_t(j) \right)^{1-\varepsilon} dj$.

Define also the relative price dispersion term as

$$s_t \equiv \int_0^1 \left( p_t(j) \right)^{-\varepsilon} dj.$$

This equation can be written in recursive terms as

$$s_t = (1-\theta) \left( \frac{1 - \theta \pi_t^{\varepsilon-1}}{1-\theta} \right)^{-\frac{\varepsilon}{1-\varepsilon}} + \theta \pi_t^{\varepsilon} s_{t-1}.$$

## 2.4 Monetary policy

Monetary policy will be characterised either as an optimal Ramsey plan, or as a simple Taylor-type rule.

In addition, however, the central bank needs to specify a rule for either $R_t^m$ or $M_t^s$. It is convenient to express this rule in terms of $\Delta_{m,t}$. In order to facilitate the comparison of our model with the standard New-Keynesian setup, we assume that

$$\Delta_{m,t} = \Delta_m,$$

for all $i$. Then,

$$\kappa_m(m_t) = \frac{\Delta_m}{1-\Delta_m} u_c(c_t)$$

and we can define

$$U(c_t, \Delta_{m,t}) \equiv u_c(c_t)\left(1 + \frac{\Delta_m}{1-\Delta_m}\right).$$

Under a policy of constant $\Delta_{m,t}$, money demand becomes recursive and can therefore be neglected for the solution of the system.

We assume a functional form $U(c_t; \Delta_m) - v(h_t) = \frac{c_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \psi \frac{h_t^{1+\varphi}}{1+\varphi}$ and we define $\pi_{t+1} \equiv \log \pi_{t+1}$, $\widehat{p}_t(j) = \log p_t(j)$, $a_t = \log A_t$, and $\widehat{\mu}_t = \log \mu_t$.

### 2.5 Market clearing

Market clearing conditions are listed below.

Money:

$$M_t^s = M_t,$$

Bonds:

$$Z_t = 0$$

Labor:

$$h_t = l_t$$

Loans:

$$D_t = P_t(x_t - \tau)$$

Wholesale goods:

$$y_t = \int_0^1 Y_t(j)\,dj$$

Retail goods:

$$Y_t(j) = c_t(j) + e_t(j), \text{ for all } j.$$

# 3 The linearized equilibrium conditions

The appendix presents the system of equilibrium conditions linearized around a zero-inflation steady state.

In order to characterize the optimal response of monetary policy, it is convenient to rewrite the linearized system in deviation from the efficient equilibrium. This latter is an equilibrium where $\mu_t = 0$, $\tau \approx 0$, prices are flexible, the monopolistic distortion is eliminated with an appropriate subsidy, and $R_t$ reacts to technology shocks in such a way as to achieve zero inflation. In the presence of the cost channel, fluctuations in $R_t$ also introduce a distortion in the economy. We provide households with a subsidy that compensates for such distortion, as in De Fiore and Tristani (2008). We denote a variable with a hat and a superscript $e$ as the log-deviation of the variable from its steady state in the efficient equilibrium, which is characterized by

$$\begin{aligned} \widehat{Y}_t^e &= E_t \widehat{Y}_{t+1}^e - \sigma \widehat{r}_t^e \\ \left(\sigma^{-1} + \varphi\right) \widehat{Y}_t^e &= \left(1 + \varphi\right) a_t, \end{aligned}$$

and where $\widehat{r}_t^e$ denotes the real interest rate.

We find it useful to define the output gap, $\widetilde{Y}_t$, as actual output in deviation from efficient output, when both variables are linearized around the actual steady state $Y$. Note that under this definition the output gap will not be zero in steady state, but equal to the difference between the two steady states $y^* \equiv \log Y - \log Y^e$.

We can now rewrite the system as

$$\widehat{\Delta}_t = \frac{1 + \varphi + \sigma^{-1} \frac{Y}{c}}{\delta_1} \widetilde{Y}_t - \frac{\sigma^{-1} \frac{e}{c}}{\delta_1} \widehat{R}_t + \frac{1}{\delta_1} \widehat{\xi}_{2,t} \tag{19}$$

$$\begin{aligned} \widetilde{Y}_t \;\; = \;\; & E_t \widetilde{Y}_{t+1} - \sigma \frac{1 + \sigma^{-1} \frac{e}{c}}{1 - \varphi \frac{e}{c}} \left(\widehat{R}_t - E_t \pi_{t+1} - \widehat{r}_t^e\right) \\ & - \frac{\alpha_1 - \alpha_2 \frac{e}{c}}{1 - \varphi \frac{e}{c}} \left(\widehat{\Delta}_t - E_t \widehat{\Delta}_{t+1}\right) + \frac{\frac{e}{c}}{1 - \varphi \frac{e}{c}} \left(\widehat{R}_t - E_t \widehat{R}_{t+1}\right) + \upsilon_t \end{aligned} \tag{20}$$

$$\pi_t = \overline{\kappa} \left(\sigma^{-1} + \varphi\right) \widetilde{Y}_t + \overline{\kappa} \widehat{R}_t + \overline{\kappa} \left(\sigma^{-1} \alpha_1 + \alpha_2\right) \widehat{\Delta}_t + \beta E_t \pi_{t+1} - \overline{\kappa} \widehat{\xi}_{1,t} \tag{21}$$

for coefficients $\alpha_1, \alpha_2, \alpha_3, \delta_1,\ \delta_2$ defined in the appendix and $\overline{\kappa} \equiv (1-\theta)(1-\beta\theta)/\theta$. Notice that $\alpha_1 > 0$ and $\alpha_2 > 0$. The composite shocks $\widehat{\xi}_{1,t}$, $\widehat{\xi}_{2,t}$ and $\upsilon_t$ are defined as

$$\widehat{\xi}_{1,t} \equiv -\sigma^{-1}\frac{1+\varphi}{\sigma^{-1}+\varphi}\left(E_t a_{t+1} - a_t\right) + \left(\alpha_3 - \sigma^{-1}\alpha_1\frac{g_\mu \mu}{g}\right)\widehat{\mu}_t \tag{22}$$

$$\widehat{\xi}_{2,t} \equiv -\sigma^{-2}\frac{e}{c}\frac{1+\varphi}{\sigma^{-1}+\varphi}\left(E_t a_{t+1} - a_t\right) + \left(1+\sigma^{-1}\frac{e}{c}\right)\frac{1+\varphi}{\sigma^{-1}+\varphi}a_t - \delta_2\widehat{\mu}_t \tag{23}$$

$$\upsilon_t \equiv \frac{\frac{e}{c}}{1-\varphi\frac{e}{c}}E_t\left(\widehat{\xi}_{1,t+1} - \widehat{\xi}_{1,t}\right) + \alpha_1\frac{1+\sigma^{-1}\frac{e}{c}}{1-\varphi\frac{e}{c}}\frac{g_\mu \mu}{g}E_t\left(\widehat{\mu}_{t+1} - \widehat{\mu}_t\right) \tag{24}$$

where $g_\mu$ denotes the partial derivative of $g(\overline{\omega}_t; \mu_t)$ with respect to $\mu$.

Equation (19) shows that the spread between the loan rate and the policy rate increases with excess aggregate demand. An increase in the demand for retail (and thus also for wholesale) goods implies an implicit tightening of the credit constraint, since the exogenously given amount of internal funds must now be used to finance a higher level of debt. The increased default risk generates a larger spread. For the same reasons, the spread decreases with the nominal interest rate. An increase in the latter variable generates a reduction in the demand for final goods and thus in the demand for input in their production (wholesale goods). For a given amount of internal funds, leverage and the risk of default fall, reducing the spread.

Equation (20) is a forward-looking IS-curve describing the determinants of the gap between actual output and its efficient level. The first line of the expression shows that, as in the standard new-Keynesian model, the gap is affected by its expected future value and by the real interest rate. In our model, however, the output gap also depends on the expected change in the nominal interest rate and in the credit spread, as well as on the shock $\upsilon_t$. Note that this dependence is not present in a cost channel model: it would disappear in the absence of monitoring costs.

A higher spread between loan and deposit rates is contractionary in our model, because it induces an increase in bankruptcy rates and a fall in entrepreneurial consumption. In our calibration, an expected increase in the spread between periods $t$ and $t+1$ tends instead to be expansionary, in spite of the fact that entrepreneurs are myopic in their consumption patterns. The transmission of this effect operates through households' consumption. Through the aggregate resource constraint, the reduction in $t+1$ entrepreneurial consumption, which is due to the higher expected spread, also tends to imply an increase in future households' consumption. Since households are forward looking, this effect will feed through to current households' consumption, thereby leading to an expansionary effect on output.

On top of the standard real interest rate effect, changes in the nominal interest rate have an impact on output which operates through similar, but opposite, channels to those of the spread. A higher nominal interest rate will in fact have a small expansionary effect, as it will increase the financial mark-up and entrepreneurial consumption. However, an expected increase in the nominal interest rate will be contractionary, as it will lead to an expected fall in households' future consumption.

Equation (21) represents an extended Phillips curve. The first determinant of inflation in this equation is an output gap term. This term is standard, even if it enters here with a different coefficient reflecting the presence of entrepreneurs in the economy. Ceteris paribus, a higher demand for retail goods, and correspondingly for intermediate goods to be used as production inputs, implies that wholesale firms need to pay a higher real wage to induce workers to supply the required labor services. As in the cost channel model, equation (21) also includes a nominal interest rate term, whose increase also pushes up marginal costs. Finally, the novel feature of our model is the presence of a credit spread in the equation. A higher credit spread implies a higher cost of external finance for wholesale firms and therefore exerts independent pressure on inflation.

The credit spread and the nominal interest rate act as endogenous "cost-push" terms in the economy. While pushing up marginal costs and inflation, an increase in either term also exerts downward pressure on economic activity. For the nominal interest rate, this happens through the ensuing increase in the real interest rate, which induces households to postpone their consumption to the future. For the credit spread, the main channel of transmission to aggregate demand is a fall in the real wage, through which firms try to offset the increase in financing costs.

All three equations (19), (20), (21) are also affected by all exogenous disturbances, which therefore act as exogenous "cost-push" factors in the Phillips curve. More specifically, technology shocks are also partly inefficient through their effect on the credit market. This is in contrast with the standard new-Keynesian model, in which they only generate efficient variations in output. The reason is that the output expansion which will typically follow a positive technology shock generates the need for an increase in external finance and in leverage, hence leading to an increase in the credit spread. In turn, the higher credit spread will affect output and inflation through the channels described above.

In the remainder of this section, we show that our model nests both the cost-channel model of Ravenna and Walsh (2006) and the standard new-Keynesian model.

We consider first the special case when monitoring costs are zero, i.e. $\mu_t = 0$, for all $t$ and $\tau \approx 0$. In this case, firms still need to borrow in advance of production. However, the information asymmetry concerning wholesale firms' productivity disappears because banks can monitor at no cost. Economic dynamics can then characterised as (see the Appendix)

$$\widetilde{Y}_t = E_t \widetilde{Y}_{t+1} - \sigma \left( \widehat{R}_t - E_t \pi_{t+1} - \widehat{r}_t^e \right)$$

$$\pi_t = \overline{\kappa} \left( \left( \sigma^{-1} + \varphi \right) \widetilde{Y}_t + \widehat{R}_t \right) + \beta E_t \pi_{t+1}$$

The equations above coincide with the reduced-form system of equilibrium conditions obtained by Ravenna and Walsh (2006) in their model of the "cost-channel," where firms borrow in advance of production but, since there is no asymmetric information nor default risk, they simply pay the risk-free rate on these funds.

Finally, the system would boil down to the new-Keynesian model in the absence of nominal debt contracts, in which case the nominal interest rate would not affect marginal costs.

## 3.1 Impulse responses

As a benchmark for comparison with the optimal policy case, we provide some evidence on the quantitative implications of the model through an impulse response analysis. For this purpose, we close the model with a simple monetary policy rule of the Taylor-type with interest rate smoothing

$$\widehat{R}_t = (1 - 0.8) \left( 2.0 \cdot \widehat{\pi}_t + 0.1 \cdot \widetilde{Y}_t \right) + 0.8 \cdot \widehat{R}_{t-1} + u_t^p$$

where $u_t^p$ is an i.i.d. monetary policy shock. The parameters of the rule are chosen in line with the values estimated in Smets and Wouters (2007) for the US.

The structural parameters are set in line with the literature. We set long-run monitoring costs at 15% of the firm's output, i.e. $\mu = 0.15$, a value consistent with the empirical estimates in Levin, Natalucci and Zakrajsek (2004). We then calibrate the standard deviations of idiosyncratic shocks ($\sigma_\omega$) and the subsidy $\tau$ so that that the annualized steady state spread $\Delta$ is equal to 2% and roughly 1% of firms go bankrupt each quarter. As to monopolistic competition and retail pricing, we assume $\varepsilon = 7$, leading to a steady-state mark-up of 17%, and a probability of not being able to re-optimize prices $\theta = 0.66$, implying that prices are changed on average every 3 quarters. Finally, we set the persistence of technology and monitoring cost shocks to 0.9.

Figure 1 displays impulse responses to a positive 1% technology shock under the Taylor rule[2] in our model – denoted as "credit channel model" – and in two well-known benchmarks: a model with the cost channel, which is obtained when $\mu_t = 0$ and $\tau = 0$; and a standard new-Keynesian model.

The most notable feature of Figure 1 is that the three models with nominal rigidities produce extremely similar impulse responses under the Taylor rule. As is typically the case, a technology shock exerts downward pressure on inflation (denoted as "inf") and on the interest rate on deposits ("i_dep"). The fall in inflation corresponds to almost the same negative output gap ("ygap") in our model and in the standard new-Keynesian model. It is slightly less pronounced, and turns positive after a few quarters, in the model with the cost channel. In the latter model, the fall in the policy interest rate has an expansionary effect through the ensuing reduction in marginal costs. In our model, the same effect is counteracted by an increase in the credit spread so that the output gap remains negative as in the new-Keynesian model. The responses of households' consumption ("cons_h") are equally very similar.

Our model also has implications for the stock of credit and the spread between loan and deposit rates. Credit expands almost one-to-one with production and households' consumption, but this also implies an increase in leverage, as firms' net worth is constant. As a result, the bankruptcy rate in the economy increases and so does the credit spread.

A pro-cyclical response of the credit spread to technology shocks is standard in models adopting the Carlstrom and Fuerst (1997) set-up, but the data show that spreads tend to increase during recessions – this is the case, for example, for the difference between lowest and highest rates on corporate bond yields in the US (see e.g. Figure 1 in Levin et al., 2004). This is a problem in terms of the ability of our model to replicate a key feature of the credit market data solely through fluctuations in technology shocks. Nevertheless, our model would indeed be capable of generating countercyclical credit spreads, if other shocks were allowed to

[2] Since the steady states of output $y$ and of the efficient level of output $y^e$ are different, the output gap term in the Taylor rule is written as $gap = \widehat{y}_t - \widehat{y}_t^e - y^*$.

drive business cycle fluctuations. For example, we show below that shocks to monitoring costs do give rise to a countercyclical response of the credit spread. A combination of technology shocks and monitoring cost shocks would easily generate a negative unconditional correlation between output and spreads, even if technology shocks would continue explaining the bulk of fluctuations in output and inflation.

The similarity between the impulse responses of the different models in Figure 1 is also likely to be related to our simplifying assumption which prevents firms from accumulating net worth during expansions. The quantitative implications of our model would probably change if we relaxed this assumption. For example, it would reduce the procyclical response of spreads to technology shocks, as firms would not need to finance the whole expansion in output through an increase in external funds. Their leverage would therefore not increase as much as it has to when internal funds are given. This would also generate more substantial differences between the impulse responses of models with and without financial frictions.

Figure 2 presents impulse responses to a policy shock. The similarity of three models is even more striking in this figure. The contraction in the output gap and the corresponding fall in inflation is virtually indistinguishable in the three models, and so is the monetary policy response. As in the case of technology shocks, the quantity of credit, leverage, and the spread between loan and deposit rates all move downwards with output, after a policy tightening.

It should be emphasized that the specific results in Figures 1 and 2 depend on the exact specification of the policy rule. With the original Taylor rule (with response coefficient of 1.5 on inflation and 0.5 to the output gap), for example, the responses of some variables – notably the output gap and inflation – would be more different across models. Other features which are often employed to increase the realism of models with nominal rigidities, e.g. habit formation, could generate further differences across models.

Nevertheless, Figures 1 and 2 suggest that it may be very difficult to discriminate empirically across models without looking also at financial variables, such as interest rate spreads or the stock of loans. They also suggest that the existence of credit frictions is not a sufficient ingredient for financial variables to play a quantitatively important role in shaping the monetary policy transmission mechanism. At least in our set-up, even if financial variables do react endogenously to economic developments and do play a direct role in the way shocks are transmitted through the economy, they modify little the reaction of output and inflation to "standard" macroeconomic shocks.

In spite of the results in Figures 1 and 2, however, credit frictions turn out to be important in two respects. First, they modify the objective of monetary policy compared to the case of frictionless financial markets. Second, they become relevant when shocks which affect the macroeconomy originate in financial markets. We analyze these two implications of credit frictions in the remainder of the paper.

# 4 Second order welfare approximation

Following Woodford (2003), we obtain a policy objective function by taking a second order approximation to the utility of the economy's representative agents. Since our economy is populated by households and entrepreneurs, the policy objective function will be a weighted average of the (approximate) utility functions of these two agents. The approximation to the objective function takes a form which nests the one in the benchmark new-Keynesian model (see Woordford, 2003) as a special case.

Under the functional form for household's utility defined above, the appendix shows that the present discounted value of social welfare can be approximated by

$$W_{t_0} \simeq \varsigma c^{1-\sigma^{-1}} \left[ \varkappa - \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} L_t \right] + t.i.p. \tag{25}$$

where $\varsigma$ is the weight assigned to households' utility, $t.i.p.$ denotes terms independent of policy and

$$L_t \equiv \kappa_\pi \pi_t^2 + \frac{1}{2}\frac{Y}{c}\varphi \left(\widetilde{Y}_t - y^*\right)^2 + \frac{1}{2}\sigma^{-1}\left(\widetilde{c}_t - y^*\right)^2 - \sigma^{-1}\frac{e}{c}\left(\widehat{Y}_t^e + y^*\right)\widehat{e}_t + \frac{e}{c}\left(1 - \frac{1-\varsigma}{\varsigma}c^{\sigma^{-1}}\right)\left(\widehat{e}_t + \frac{1}{2}\widehat{e}_t^2\right) \tag{26}$$

where $\widehat{e}_t$ is log-entrepreneurial consumption (in deviation from the steady state) and $\varkappa$ and $\kappa_\pi$ are parameters defined in the appendix.

The first three terms in equation (26) are common to the new-Keynesian model. Intuitively, social welfare decreases with variations of inflation around its target, and of the output gap around its (non-zero) steady state level. The first reason for disliking variations in the output gap is that households wish to smooth their labour supply. The second reason is that households also wish to smooth consumption over time. Unlike in the benchmark new-Keynesian model, the consumption smoothing motive only applies to households' consumption, $\widetilde{c}_t^2$, rather than to total output, because entrepreneurs are risk-neutral and thus indifferent about the timing of their consumption.

The main difference relative to the benchmark New-Keynesian model with frictionless financial markets is in the additional terms now appearing in the welfare approximation.

The term which is proportional to $\left(\widehat{Y}_t^e + y^*\right)\widehat{e}_t$ contributes positively to welfare. The presence of this term is again related to households' consumption smoothing motive (this term would disappear if households utility were linear, i.e. when $\sigma^{-1} = 0$). Under an output expansion induced by a technology shock, an increase in entrepreneurial consumption absorbs aggregate resources and thus contributes to smooth the path of households' consumption over time.

The last two terms in equation (26), which are proportional to $\widehat{e}_t$ and $\widehat{e}_t^2$, have an ambiguous impact on welfare, depending on whether the weight of households in social welfare is larger or smaller than a certain threshold $\varsigma = \left(1 + c^{-\sigma^{-1}}\right)^{-1}$.

The quadratic term in entrepreneurial consumption is due to two reasons. On the one hand, fluctuations in entrepreneurial consumption must be accompanied by changes in households' consumption through the aggregate resource constraint. Hence, households dislike fluctuations in $\widehat{e}_t$. On the other hand, entrepreneurs enjoy consumption volatility, because of their risk neutrality. The sign of the overall term proportional to $\widehat{e}_t^2$ in social welfare depends on which one of these two effects prevails, which is in turn determined by the relative importance of households in social utility.

The linear term in entrepreneurial consumption highlights the potential redistributional effects of policy in our model. A higher value of entrepreneurial consumption is obviously beneficial for entrepreneurial welfare. At the same time, any entrepreneurial consumption is detrimental for households' welfare, as it subtracts from the economy resources which could be consumed by households. The net effect of this term on welfare is again determined by the relative weight of households in social utility.

This linear term actually tends to dominate all second order terms. Depending on the exact weight of households in welfare, our model can therefore generate very different welfare implications and, as a result, different optimal policy responses to shocks. Given the simplicity of our framework, we prefer not to take a stance on the weight which would be most realistic given the relative size of entrepreneurs in actual economies. Instead, we select the weight so as to neutralise the redistributional incentives of optimal policy and, as a result, to maximise the comparability of our results with those of the standard new-Keynesian model.

The particular weight which achieves this objective is $\varsigma = \left(1 + c^{-\sigma^{-1}}\right)^{-1}$. As a result, first order terms disappear entirely from social welfare.

Under this special weight $\varsigma$, the loss function simplifies to

$$\begin{aligned} L_t \equiv\ & \kappa_\pi \pi_t^2 + \frac{1}{2}\frac{Y}{c}\varphi\left(\widetilde{Y}_t - y^*\right)^2 + \frac{1}{2}\sigma^{-1}\left[\frac{Y}{c}\left(\widetilde{Y}_t - y^*\right) - \frac{e}{c}\left(\widehat{R}_t + \delta_3\widehat{\Delta}_t - \delta_4\widehat{\mu}_t\right)\right]^2 \\ & + \sigma^{-1}\frac{e}{c}\frac{Y}{c}\left(\widehat{Y}_t^e + y^*\right)\left(\widetilde{Y}_t - \widehat{R}_t - \delta_3\widehat{\Delta}_t\right) \end{aligned} \tag{27}$$

where equations (16) and (17) were used to write entrepreneurial consumption in terms of the nominal interest rate $\widehat{R}_t$, the credit spread $\widehat{\Delta}_t$ and the exogenous shock $\widehat{\mu}_t$. This expression allows us to perform a complete derivation of optimal policy using the linearized policy equations (19)-(21).

Compared to the case of the standard new-Keynesian model, the novel terms in expression (27) are those with a coefficient proportional to $e/c$ (notice that these terms vanish when entrepreneurs disappear from the economy and $Y = c$).

These novel terms include first, within square brackets, elements proportional to the squared nominal interest rate and the squared loan-deposit rate spread. Hence, the presence of asymmetric information in the economy introduces directly both an interest rate smoothing and a "spread smoothing" motive for optimal policy. At the same time, these terms are relatively small in our calibration, where households' consumption takes up the lion share of output. Under normal circumstances, therefore, the interest rate smoothing concern is unlikely to be predominant compared to the objective of maintaining price stability.

The term within square brackets in equation (27) also includes a number of cross products between endogenous variables. More specifically, a planner would be averse to a positive covariance between the nominal interest rate and the loan-deposit rates spread. A high covariance would increase the volatility of entrepreneurial consumption, with negative spillovers on households' consumption-smoothing motive. The planner would however not be averse to a positive covariance between, on the one side, the output gap, on the other side, either the interest rate or the loan-deposit rates spread. A negative output gap, for example, would be welfare improving if accompanied by a fall in the policy interest rate, such that only entrepreneurial consumption would suffer from the reduction in output, while households' consumption would remain unchanged.

Finally, the last term in equation (27) shows that increases in the nominal interest rate and in the credit spread have a positive effect on welfare, if they are accompanied by an increase in the efficient level of output – i.e. an increase in productivity. The reason is that households are willing to reap the benefits of the higher productivity on real wages, but wish to smooth their consumption pattern over time. Higher firms' profits and entrepreneurial consumption at a time of high productivity helps to achieve the latter objective.

Our derivations above write welfare in terms of deviations from the efficient equilibrium. An alternative possibility would be to write all variables in deviation from the values which they would take in a "natural" equilibrium, in which prices are flexible and financial contracts are denominated in real terms. In a more general context, De Fiore and Tristani (2008) demonstrates that such an equilibium can be defined independently of monetary policy; it also demonstrates that price stability would be maintained at all times in the economy with nominal rigidities, if policy interest rates were set so as to "track" real rate of interest prevailing in the natural equilibrium. One may therefore conjecture that the natural equilibrium should coincide with the best implementable allocation in our economy. More specifically, one may think that a fraction of the variation in credit spreads due to efficient shocks – notably technology shocks – should also be efficient, and that this fraction should coincide with the behaviour of spreads in the natural equilibrium.

It turns out, however, that in our economy monetary policy can do better than implementing the natural equilibrium. Our numerical results below show that optimal policy would choose not to maintain price stability at all times, even if this equilibrium would be implementable by the aforementioned policy of tracking the natural rate. Consistently with general second-best results, monetary policy can undo some of the adverse implications on welfare of financial market imperfections.

# 5 Optimal policy

## 5.1 Discretion

When the welfare function can be approximated as in (25) and (27), the problem of the central bank is to maximize that objective, subject to the system of equilibrium conditions (19)-(21).

The appendix shows that, in the special case in which $\varphi = 0$ and $\sigma^{-1} = 1$, the target rule which characterizes the discretionary equilibrium takes the simple form

$$\pi_t = \nu_e \left(\widehat{Y}_t^e + y^*\right) - \nu_\pi \left[\left(\widetilde{Y}_t - y^*\right) - \frac{e}{Y}\left(\widehat{R}_t + \delta_3 \widehat{\Delta}_t\right)\right] - \nu_\pi \frac{e}{Y} \delta_4 \widehat{\mu}_t \tag{28}$$

for parameters $\nu_\pi$ and $\nu_e$ defined in the appendix.

Note that, in the frictionless case, $\nu_\pi = \frac{1}{\overline{\kappa}} \frac{1}{\kappa_\pi}$ and $\nu_e = 0$ so that the optimality condition becomes

$$\pi_t = -\frac{1}{\overline{\kappa}} \frac{1}{\kappa_\pi} \left(\widetilde{Y}_t - y^*\right)$$

which corresponds to standard results in the new-Keynesian case – see, for instance, Woodford (2003, chapter 7, p. 471). This target criterion implies that the central bank will choose to engineer a constant, positive inflation rate, given the output-inflation trade-off implicit in the Phillips curve. Any rise in inflation above that level would be met by a policy response such as to produce a negative output gap.

In our model, the target criterion which would be followed by a central bank under discretion is affected by the existence of financial frictions. While the output gap remains important, both other endogenous variables and shocks limit the ability of the central bank to use the output gap to achieve the desired level of inflation. A surge in inflation could also be countered through actions which affect the spread $\widehat{\Delta}_t$ and the nominal interest rate.

In addition exogenous shocks, including both the financial shock $\widehat{\mu}_t$ and technology shocks (through the efficient level of output $\widehat{Y}_t^e$), affect the target criterion. This implies that the optimal inflation rate varies in the face of these shocks. Differently from the benchmark new-Keynesian case, some temporary deviations from the central bank's objective may occasionally be desirable.

### 5.2 Optimal monetary policy under commitment

We characterize numerically the optimal monetary policy under commitment in the special case characterized above in which linear terms disappear from the quadratic approximation of the welfare function. Under this assumption, steady state inflation is zero and the linear system in equations (19)-(21) is a correct approximation.[3] In all cases, we concentrate on optimal policy under a timeless perspective, as in Woodford (2003).

Figure 3 displays impulse responses to a technology shock when monetary policy is set optimally. Once again, for the technology shock we contrast optimal policy in the credit channel model with the optimal policies which would arise in a model with the cost-channel and in the standard new-Keynesian model.[4]

Compared to the results in Figure 1, optimal policy leads to more significant differences in the three models considered here. As is well-known, optimal policy would ensure complete price stability and full stabilization of the output gap in the new-Keynesian model. The policy interest rate would fall on impact and then return slowly to the baseline.

Under the credit channel, near-full inflation stabilization remains optimal in response to technology shocks. However, the path of the policy interest rate which achieves this outcome would be somewhat different. The policy rate is kept constant for one period, before reaching levels roughly consistent with those in the new-Keynesian model. Partly as a result of the slowed monetary easing, output increases less than in the efficient equilibrium and a negative output gap ensues.

The impulse response of the output gap highlights the differences between our model and the cost channel model. In the latter case, the impact reduction in the nominal interest rate is more marked – even if not as aggressive as in the new-Keynesian benchmark – and strongly expansionary, so that the output gap increases after the technology shock. This is not the case in our model because of the increase in the spread, which remains procyclical as in the simple rule benchmark.

[3] Our numerical results are based on a full second order approximation of the policy equations, which we perform using Dynare. We compute the first order conditions of the welfare maximisation problem of the policy maker using Giovanni Lombardo's lq_solution routine available at http://home.arcor.de/calomba/symbsolve4_lnx.zip.

[4] In all cases, we assume the existence of a steady state subsidy which eliminates first-order terms in output from the second-order expansion of individuals' utility. The subsidy is slightly different in the three cases: it is equal to $\chi/(\chi-1)$ in the new-Keynesian model, $R\chi/(\chi-1)$ in the cost-channel model, and $q\chi/(\chi-1)$ in our model.

Figure 4 displays impulse responses to a positive shock to $\widehat{\mu}_t$. This shock is representative of a broader array of "financial shocks" which could be defined in our model, notably shocks to the $\tau$ subsidy, or to the standard deviation of idiosyncratic shocks.

The increase in $\widehat{\mu}_t$, which is assumed to be persistent, acts like a classical cost-push shock: it depresses households' consumption and output while creating inflationary pressures. On the one hand, it generates an immediate increase in the loan-deposit rate spread – the shock is normalized to produce a 1 percentage point increase in the spread. The larger spread pushes up firms' marginal costs and thus generates inflationary pressure. On the other hand, the increase in marginal costs also generates an increase in the mark-up $q_t$, which exerts downward pressure on wages. In general, households react to the lower wage rate with a reduction in both their labour supply and in their demand for consumption goods. In our particular calibration – utility is logarithmic in consumption and linear in leisure – labour supply is fully elastic and the adjustment takes place entirely through consumption, which falls one-to-one with the real wage.

In our model, therefore, the depressionary effects on output of an adverse financial shock do not arise directly through the higher cost of working capital induced by the higher loan rates. Since we abstract from investment, aggregate demand is actually very little sensitive to the increase in loan rates. The reason is that households' consumption, which represents the bulk of aggregate demand, is only affected by the risk-free nominal interest rate. The main channel through which the adverse financial shock is transmitted to the real economy has to do with the fact that surviving firms, i.e. firms which do not go bankrupt, need to make higher profits to finance the higher cost of finance. It is the ensuing reduction in real wages which produces the main squeeze on aggregate demand.

The policy response under a Taylor rule is to increase interest rates to meet the inflationary pressure. In spite of this policy response, inflation rises by 1 percentage point, partly due to the cost-channel effect of the nominal interest rate. The increase in $\widehat{\mu}_t$ also leads to an increase in bankruptcy rates, while the amount of credit falls.

Hence, contrary to the case of the technology shock, the spread moves anti-cyclically in response to a financial shock.

Compared to the responses under the Taylor rule, those obtained under optimal policy are striking because the policy interest rate moves in the opposite direction. Interest rates are immediately cut very aggressively and stay low for approximately one year, in spite of the inflationary pressure. The main reason for this policy response is that the financial shock is inefficient, hence the fall in households' consumption is entirely undesirable. The marked expansion in monetary policy is aimed at smoothing households' consumption path after the shock. At the same time, the interest rate cut counters inflation through the cost channel, even if it tends to fuel inflation through the aggregate demand stimulus.

All in all, compared to the Taylor rule case, households' consumption moves very little on impact and only reaches levels consistent with those attained under the Taylor rule after 3 quarters. At the same time, the increase in inflation is less pronounced, and less persistent than in the Taylor rule case.

## 6 Conclusion

Using a small, microfounded model with nominal rigidities and credit frictions, we have analyzed the implications of financial market conditions on macroeconomic dynamics and on optimal monetary policy.

In our simple set-up, it is possible to characterize analytically the linearized aggregate relations of the model and to obtain an approximate welfare criterion consistent with the microfoundations of the model. Our results show that, in general, monetary policy ought to pay attention to the evolution of financial market conditions, as captured for example by changes in credit spreads. On the one hand, these changes matter because they affect firms' marginal costs and have therefore an impact on output and inflation. On the other hand, they matter because of their impact on entrepreneurial consumption.

In our numerical analysis, we find that despite the presence of cost-push factors introduced by financial market frictions, optimal monetary policy does not produce substantial deviations from price stability. Nevertheless, our results suggest that there might be good reasons for a central bank to react with an aggressive easing to an adverse financial shock. Those types of shocks create an inefficient recession, whose negative consequences on consumption can be reduced.

Our results should help improve our understanding of the determinants of optimal policy decisions in models with credit frictions. The numerical findings presented in the figures should however be interpreted as an illustrative example. Their precise quantitative features should be cross-checked against those derived from more complex models with more realistic features.

# Appendix

## A The financial contract

The informational structure corresponds to a costly state verification problem. The solution is a standard debt contract (see e.g. Gale and Hellwig, 1985) such that: i) the repayment to the financial intermediary is constant in states when monitoring does not occur; ii) the firm is declared bankrupt when the fixed repayment cannot be honoured; iii) in case of bankruptcy, the financial intermediary monitors and completely seizes the firm's output.

Recall that the presence of agency costs implies that $y_{i,t} = \omega_{i,t}\chi_t q_t x_{i,t}$. Define

$$f(\overline{\omega}) \equiv \int_{\overline{\omega}}^{\infty} \omega\Phi(d\omega) - \overline{\omega}\left[1 - \Phi(\overline{\omega})\right]$$

$$g(\overline{\omega};\mu) \equiv \int_{0}^{\overline{\omega}} \omega\Phi(d\omega) - \mu\Phi(\overline{\omega}) + \overline{\omega}\left[1 - \Phi(\overline{\omega})\right]$$

as the expected shares of output accruing respectively to an entrepreneur and to a lender, after stipulating a contract that sets the fixed repayment at $\overline{P}_t\chi_t q_t\overline{\omega}_{it}x_{i,t}$ units of money. In case of default, a stochastic fraction $\mu_t$ of the input costs $x_{i,t}$, measured in units of money, is used in monitoring. We assume that $\mu_t$ follows a AR1 process. At the individual firm level, total output is split between the entrepreneur, the lender, and monitoring costs so that

$$f(\overline{\omega}_t) + g(\overline{\omega}_t;\mu_t) = 1 - \mu_t\Phi(\overline{\omega}_t).$$

The optimal contract is the pair $(x_{i,t}, \overline{\omega}_{i,t})$ that solves the following costly state verification problem:

$$\max\ \overline{P}_t\chi_t q_t f(\overline{\omega}_{i,t})x_{i,t}$$

subject to

$$\overline{P}_t\chi_t q_t g(\overline{\omega}_{i,t})x_{i,t} \geq R_t^d P_t\left(x_{i,t} - \tau\right) \tag{29}$$

$$\overline{P}_t\left[f(\overline{\omega}_{i,t}) + g(\overline{\omega}_{i,t};\mu_t) - 1 + \mu_t\Phi(\overline{\omega})\right] \leq 0 \tag{30}$$

$$\overline{P}_t\chi_t q_t f(\overline{\omega}_{i,t})x_{i,t} \geq P_t\tau \tag{31}$$

The optimal contract maximizes the entrepreneur's expected profits subject to the lender being willing to lend out funds, (29), the feasibility condition, (30), and the entrepreneur being willing to sign the contract, (31). Notice that the intermediary needs to pay back to the household a gross return equal to the safe interest on deposits, $R_t^d$. Since in equilibrium $R_t = R_t^d$, the financial intermediary's expected return on each unit of loans cannot be lower than $R_t$.

The optimality conditions can be written as

$$q_t = \frac{R_t}{1-\mu_t \Phi\left(\overline{\omega}_{i,t}\right)+\frac{\mu_t f\left(\overline{\omega}_{i,t}\right)\phi\left(\overline{\omega}_{i,t}\right)}{f_{\overline{\omega}}\left(\overline{\omega}_{i,t}\right)}}, \tag{32}$$

$$x_{i,t} = \left\{\frac{R_t}{R_t - q_t g\left(\overline{\omega}_{i,t};\mu_t\right)}\right\}\tau. \tag{33}$$

From equation (32), it follows that the terms of the contract depend on the state of the economy only through the aggregate mark-ups $\chi_t$ and $q_t$ and the return $R_t$. Hence, they are the same for all firms, $\overline{\omega}_{i,t} = \overline{\omega}_t$. Since initial wealth is also the same across firms, it follows from equation (33) that the size of the project is the same across firms.

# B The system in reduced form

The system of equilibrium conditions that characterizes the evolution of the aggregate variables (once a monetary policy rule is specified) can be linearized around a zero-inflation steady state as

$$\begin{aligned}
\left(1+\sigma^{-1}\frac{e}{c}\right)\widehat{\chi}_t &= (1+\varphi)\ a_t - \left(\sigma^{-1}\alpha_1+\alpha_2\right)\widehat{\Delta}_t - \left(\sigma^{-1}+\varphi\right)\widehat{Y}_t - \widehat{R}_t \qquad (34)\\
&\quad + \left(\alpha_3 - \sigma^{-1}\alpha_1\frac{g_\mu \mu}{g}\right)\widehat{\mu}_t \\
\delta_1\widehat{\Delta}_t &= \left(1+\varphi+\sigma^{-1}\frac{Y}{c}\right)\widehat{Y}_t - \sigma^{-1}\frac{e}{c}\widehat{R}_t - (1+\varphi)\ a_t - \delta_2\widehat{\mu}_t \qquad (35)\\
\widehat{Y}_t &= E_t\widehat{Y}_{t+1} - \frac{1}{\sigma^{-1}}\left(\widehat{R}_t - E_t\pi_{t+1}\right) + \alpha_1 E_t\left(\widehat{\Delta}_{t+1}-\widehat{\Delta}_t\right) \qquad (36)\\
&\quad + \frac{e}{c}E_t\left(\widehat{\chi}_{t+1}-\widehat{\chi}_t\right) + \alpha_1\frac{g_\mu \mu}{g}E_t\left(\widehat{\mu}_{t+1}-\widehat{\mu}_t\right) \\
\pi_t &= -\overline{\kappa}\left(1+\sigma^{-1}\frac{e}{c}\right)\widehat{\chi}_t + \beta E_t\pi_{t+1} \qquad (37)
\end{aligned}$$

where hats denote log-deviation of a variable from the steady state.

The coefficients of the system (34)-(37) are given by

$$
\begin{aligned}
\alpha_1 &= -\frac{\frac{f_{\overline{\omega}}\overline{\omega}}{\chi}\frac{y}{c}}{1-g_{\overline{\omega}}\Delta} > 0 \\
\alpha_2 &= -\frac{\mu\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)\frac{q}{R}}{(1-g_{\overline{\omega}}\Delta)} > 0 \\
\alpha_3 &= \left[\frac{\frac{g_\mu\mu}{g}\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)}{(1-g_{\overline{\omega}}\Delta)}+\frac{f\phi}{f_{\overline{\omega}}}-\Phi\right]\mu\frac{q}{R} \\
\alpha_4 &= -\frac{\mu\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)+\overline{\omega}\left(f_{\overline{\omega}}+\mu\phi\right)}{\left(f+\frac{\mu f\phi}{f_{\overline{\omega}}}\right)(1-g_{\overline{\omega}}\Delta)} > 0 \\
\alpha_5 &= \left(\frac{\alpha_4 g_\mu}{g}-\frac{\phi}{f_{\overline{\omega}}+\mu\phi}\right)\mu \\
\delta_1 &\equiv \left(1+\frac{\sigma^{-1}e}{c}\right)\alpha_4-\sigma^{-1}\alpha_1-\alpha_2 \\
\delta_2 &\equiv \alpha_3-\sigma^{-1}\alpha_1\frac{g_\mu\mu}{g}.
\end{aligned}
$$

When monitoring costs are zero, i.e. $\mu_t = 0$, for all $t$ and $\tau \approx 0$, we obtain

$$\widehat{\chi}_t = -\left(\sigma^{-1}+\varphi\right)\widehat{Y}_t-\widehat{R}_t+(1+\varphi)\,a_t$$

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1}-\sigma\left(\widehat{R}_t-E_t\pi_{t+1}\right)$$

$$\pi_t = -\overline{\kappa}\widehat{\chi}_t+\beta E_t\pi_{t+1}$$

# C Case with frictionless financial markets

When $\mu_t = 0$, for all $t$, $f(\overline{\omega}_t)+g(\overline{\omega}_t;\mu_t) = 1$. Also, since there are no monitoring costs, banks set $\overline{\omega}_t$ as high as possible subject to the constraint that the firm is willing to sign the contract, i.e.

$$f(\overline{\omega}_t) = \frac{\tau}{q_t x_t}$$

This maximizes banks' profits, as they can size the production of all defaulting firms at no cost. In such equilibrium,

$$
\begin{aligned}
g\left(\overline{\omega}_t;\mu_t\right) &= 1-\frac{\tau\chi_t}{y_t}\\
e_t &= \tau\\
Y_t &= c_t+\tau.
\end{aligned}
$$

Moreover, from the bank's zero profit condition, we have

$$
x_t=\frac{R_t\tau}{R_t-q_t\left(1-\frac{\tau\chi_t}{y_t}\right)}.
$$

The log-linearized system can then be written as

$$
\widehat{\chi}_t=-\left[\frac{(R-1)\,\chi\tau}{y}\left(1+\sigma^{-1}+\varphi\right)+\left(\sigma^{-1}+\varphi\right)\right]\widehat{Y}_t+\left(\frac{\chi\tau}{y}-\frac{1}{q}\right)R\widehat{R}_t+\left[\frac{(R-1)\,\chi\tau}{y}+1\right](1+\varphi)\,a_t
$$

$$
\widehat{Y}_t=E_t\widehat{Y}_{t+1}-\sigma\left(\widehat{R}_t-E_t\pi_{t+1}\right)
$$

$$
\pi_t=-\overline{\kappa}\left(1+\sigma^{-1}\frac{e}{c}\right)\widehat{\chi}_t+\beta E_t\pi_{t+1}
$$

In the limiting case where $\tau\approx 0$, $q_t=R_t$ and the system boils down to the equations reported in the text.

## D Welfare approximation

Our monetary policy objective is derived as the second order approximation to a weighted average of the utilities of the household and of the entrepreneur, i.e.

$$
E_o\left\{\sum_0^\infty\beta^t\left[\varsigma U_t+(1-\varsigma)\,U_t^e\right]\right\}
$$

where $\varsigma$ is the weight of the utility of households in the policy objective. Households' temporary utility can then be approximated as

$$
U_t\simeq U+u_c c\left(\widehat{c}_t+\frac{1}{2}\left(1+\frac{u_{cc}c}{u_c}\right)\widehat{c}_t^2\right)-v_h h\left(\widehat{h}_t+\frac{1}{2}\left(1+\frac{v_{hh}h}{v_h}\right)\widehat{h}_t^2\right)
$$

where hats denote log-deviations from the deterministic steady state and $c$ and $h$ denote steady state levels. Similarly, entrepreneurial temporary utility $U_t^e$ can be expanded as

$$U_t^e \simeq e\left(1+\widehat{e}_t+\frac{1}{2}\widehat{e}_t^2\right)$$

where $e$ is the steady state level of entrepreneurial consumption.

Under the functional form $U_t = \frac{c_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \psi\frac{h_t^{1+\varphi}}{1+\varphi}$, households' temporary utility can be rewritten as

$$U_t \simeq \frac{c^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \psi\frac{h^{1+\varphi}}{1+\varphi} + c^{1-\sigma^{-1}}\widehat{c}_t - \psi h^{1+\varphi}\widehat{h}_t + \frac{1}{2}\left(c^{1-\sigma^{-1}}\left(1-\sigma^{-1}\right)\widehat{c}_t^2 - \psi h^{1+\varphi}\left(1+\varphi\right)\widehat{h}_t^2\right)$$

We can express hours, households' consumption and entrepreneurial consumption as

$$h_t = \frac{s_t Y_t}{A_t}$$

$$c_t = Y_t - e_t$$

$$e_t = \tau R_t\left[1+\frac{\mu_t\phi\left(\overline{\omega}_t\right)}{f_{\overline{\omega}}\left(\overline{\omega}_t\right)}\right]^{-1}$$

The period aggregate utility can be approximated as

$$\begin{aligned}\varsigma U_t + (1-\varsigma)U_t^e \;\simeq\;& \varsigma c^{1-\sigma^{-1}}\left(\frac{1}{1-\sigma^{-1}} - \frac{\psi}{1+\varphi}\frac{h^{1+\varphi}}{c^{1-\sigma^{-1}}}\right) + (1-\varsigma)e \\ &+\varsigma c^{1-\sigma^{-1}}\widehat{c}_t + (1-\varsigma)e\widehat{e}_t - \varsigma c^{1-\sigma^{-1}}\frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}}\widehat{h}_t \\ &+\frac{1}{2}\varsigma c^{1-\sigma^{-1}}\left(1-\sigma^{-1}\right)\widehat{c}_t^2 - \frac{1}{2}\varsigma c^{1-\sigma^{-1}}\frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}}(1+\varphi)\widehat{h}_t^2 + \frac{1}{2}(1-\varsigma)e\widehat{e}_t^2\end{aligned}$$

Now note that the resource constraint $c_t = Y_t - e_t$ can be approximated to second order as

$$\widehat{c}_t = \frac{Y}{c}\widehat{Y}_t - \frac{e}{c}\widehat{e}_t + \frac{1}{2}\frac{Y}{c}\widehat{Y}_t^2 - \frac{1}{2}\frac{e}{c}\widehat{e}_t^2 - \frac{1}{2}\widehat{c}_t^2$$

while the production function implies simply

$$\widehat{h}_t = -a_t + \widehat{s}_t + \widehat{Y}_t.$$

It follows that utility can be rewritten as

$$
\begin{aligned}
\frac{\varsigma U_t + (1-\varsigma) U_t^e - \varkappa}{c^{1-\sigma^{-1}}} \simeq\; & -\varsigma \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} \widehat{s}_t - \varsigma \left( \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} - \frac{Y}{c} \right) \widehat{Y}_t \\
& - \frac{e}{c} \left( \varsigma - (1-\varsigma) c^{\sigma^{-1}} \right) \left( \widehat{e}_t + \frac{1}{2} \widehat{e}_t^2 \right) \\
& - \frac{1}{2} \varsigma \sigma^{-1} \widehat{c}_t^2 - \frac{1}{2} \varsigma \left( \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} (1+\varphi) - \frac{Y}{c} \right) \widehat{Y}_t^2 \\
& + \varsigma \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} (1+\varphi) a_t \widehat{Y}_t + tips
\end{aligned}
$$

where

$$
\varkappa \equiv \varsigma c^{1-\sigma^{-1}} \left( \frac{1}{1-\sigma^{-1}} - \frac{1}{1+\varphi} \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} \right) + (1-\varsigma) e
$$

Now consider the FOC

$$
\frac{\psi h^{\varphi}}{c_t^{-\sigma^{-1}}} = \frac{A_t}{q_t \chi_t}
$$

Under perfect competition and frictionless credit markets, firms set the real wage at the marginal product of labor, $w_t = A_t$. In our model, equation (11) implies that $w_t = \frac{A_t}{q_t \chi_t}$. We provide households with a subsidy $\Omega_1$ such that $\frac{\psi h^{\varphi}}{c^{-\sigma^{-1}}} = w_t (1-\Omega_1)$, and

$$
1 - \Omega_1 = q\chi.
$$

It follows that in such a steady state

$$
\frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} = \frac{Ah}{c} = \frac{Y}{c}.
$$

This subsidy will allow us to ignore first order terms in output from welfare. Note, however, that the same subsidy will not bring the steady state of the economy back to the efficient (steady state) level. The actual steady state can in fact be written as

$$
Y = \psi^{-\left(\varphi+\sigma^{-1}\right)^{-1}} \left[ \frac{1-\Omega_1}{q\chi \left(1 - \frac{f(\varpi)}{\chi}\right)^{\sigma^{-1}}} \right]^{\left(\varphi+\sigma^{-1}\right)^{-1}}
$$

while the steady state level of efficient output is $Y^e = \psi^{-\left(\varphi+\sigma^{-1}\right)^{-1}}$. The subsidy which would make the actual steady state efficient is $1 - \Omega_1 = q\chi \left(1 - \frac{f(\varpi)}{\chi}\right)^{\sigma^{-1}} > q\chi$. Following Woodford

(2003), for small values of the distortions, $\Omega_1$ can be treated as an expansion parameter. The steady state gap $y^* \equiv \log Y - \log Y^e$ can then be loglinearized to yield $y^* \simeq -\left(\varphi + \sigma^{-1}\right)^{-1} \Omega_1$.

In addition, we focus on the case of a special Pareto weight $\varsigma = \frac{c^{\sigma-1}}{1+c^{\sigma-1}}$, which allows us to ignore first order terms in entrepreneurial consumption. It follows that the loss can be written as

$$\frac{\varsigma U_t + (1-\varsigma) U_t^e - \varkappa}{\varsigma c} \simeq -\frac{Y}{c}\widehat{s}_t - \frac{1}{2}\frac{Y}{c}\varphi \widehat{Y}_t^2 - \frac{1}{2}\sigma^{-1}\left(\frac{Y}{c}\widehat{Y}_t - \frac{e}{c}\widehat{e}_t\right)^2 + \frac{Y}{c}(1+\varphi) a_t \widehat{Y}_t + t.i.s.p.$$

Now note that in the fully-efficient steady state we would have

$$(1+\varphi) a_t = \sigma^{-1}\widehat{c}_t^e + \varphi \widehat{Y}_t^e + \left(\varphi + \sigma^{-1}\right)(y - y^e)$$

which can be used to substitute out the technology shock $a_t$ from the loss function.

In addition, a first order approximation to the equation for price dispersion, of first-order in $\widehat{s}_t$ and second-order in $\pi_t$ takes the form

$$\widehat{s}_t \simeq \frac{\theta}{1-\theta}\varepsilon \frac{\pi_t^2}{2} + \theta \widehat{s}_{t-1}.$$

This latter can be integrated forward to obtain

$$\widehat{s}_t \simeq \frac{\theta}{1-\theta}\varepsilon \sum_{s=t_0}^{t} \theta^{t-s}\frac{\pi_s^2}{2} + \theta^{t-t_0+1}\widehat{s}_{t_0-1}.$$

Multiplying this by $\beta^{t-t_0}$ and realizing that multiples of $\widehat{s}_{t_0-1}$ are independent of policy, we obtain

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0}\widehat{s}_t \simeq \frac{\theta}{(1-\theta)(1-\beta\theta)}\varepsilon \sum_{t=t_0}^{\infty} \beta^{t-t_0}\frac{\pi_t^2}{2} + t.i.p.$$

Finally, note that entrepreneurial consumption can be written as

$$\widehat{e}_t = \widehat{R}_t + \delta_3 \widehat{\Delta}_t - \delta_4 \widehat{\mu}_t$$

where

$$\begin{aligned}
\delta_3 &= -\frac{\mu\overline{\omega}\left(\phi_{\overline{\omega}} - \frac{\phi^2}{f_{\overline{\omega}}}\right)}{\left(f_{\overline{\omega}} + \mu\phi\right)\left(1 - \Delta g_{\overline{\omega}}\right)} > 0 \\
\delta_4 &= \left(\frac{\phi}{\mu\phi + f_{\overline{\omega}}} + \frac{\delta_3 g_\mu}{g}\right)\mu
\end{aligned}$$

It follows that the approximated welfare function can be written as in the main text, for

$$\kappa_\pi \equiv \frac{1}{2}\frac{Y}{c}\frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)} > 0$$

# E Optimal policy under discretion

Under discretion, the central bank tries to minimize the loss function (27) subject to the three-equation system (19)-(21). Denote as $\eta_{\Delta,t}$, $\eta_{Y,t}$ and $\eta_{\pi,t}$ the Lagrangean multipliers associated to the constraints. The first order conditions can be written as

$$\eta_{\pi,t} = 2\kappa_\pi\pi_t$$

and

$$\begin{aligned}
\eta_{Y,t} &= \frac{Y}{c}\varphi\left(\widetilde{Y}_t - y^*\right) + \sigma^{-1}\frac{Y}{c}z_t + \sigma^{-1}\frac{Y-c}{c}\frac{Y}{c}\left(\widehat{Y}_t^e + y^*\right) + \frac{1+\varphi+\sigma^{-1}\frac{Y}{c}}{\delta_1}\eta_{\Delta,t} + 2\overline{\kappa}\kappa_\pi\left(\sigma^{-1}+\varphi\right)\pi_t \\
\eta_{\Delta,t} &= -\frac{Y-c}{c}\delta_3\sigma^{-1}z_t - \sigma^{-1}\frac{Y-c}{c}\frac{Y}{c}\delta_3\left(\widehat{Y}_t^e + y^*\right) - \frac{\alpha_1 - \alpha_2\frac{e}{c}}{1-\varphi\frac{e}{c}}\eta_{Y,t} + 2\overline{\kappa}\kappa_\pi\left(\sigma^{-1}\alpha_1 + \alpha_2\right)\pi_t \\
0 &= -\frac{Y-c}{c}\sigma^{-1}z_t - \sigma^{-1}\frac{Y-c}{c}\frac{Y}{c}\left(\widehat{Y}_t^e + y^*\right) - \frac{\sigma^{-1}\frac{e}{c}}{\delta_1}\eta_{\Delta,t} - \sigma\frac{1}{1-\varphi\frac{e}{c}}\eta_{Y,t} + 2\overline{\kappa}\kappa_\pi\pi_t
\end{aligned}$$

where

$$z_t \equiv \frac{Y}{c}\left(\widetilde{Y}_t - y^*\right) - \frac{Y-c}{c}\left(\widehat{R}_t + \delta_3\widehat{\Delta}_t - \delta_4\widehat{\mu}_t\right)$$

These equations can be solved for the three Lagrange multipliers and yield an additional optimality condition. In the case $\varphi = 0$ and $\sigma^{-1} = 1$, the latter condition can be written as equation (28) in the text, where

$$\nu_e \equiv \frac{1}{\overline{\kappa}}\frac{1}{\kappa_\pi}\frac{Y}{c}\frac{e}{c}\frac{2\,\frac{Y}{c}\,\delta_3+\frac{e}{c}\,\alpha_2\,\left(1+\frac{Y}{c}-\frac{e}{c}\,\delta_3\right)-\delta_1\,\left(1+\delta_3\right)-\alpha_1\,\left(1+\frac{Y}{c}-\frac{e}{c}\,\delta_3\right)}{\left(1+\frac{Y}{c}\right)\,\alpha_1+\left(2+3\,\frac{e}{c}+2\,\frac{e^2}{c^2}+2\,\frac{Y}{c}+\frac{Y-c}{c}\frac{Y}{c}\right)\,\alpha_2+\delta_1}$$

$$\nu_\pi \equiv \frac{1}{\overline{\kappa}}\frac{1}{\kappa_\pi}\frac{\frac{e}{Y}\,\alpha_1-\frac{e}{Y}\frac{e}{c}\,\alpha_2+\left(1+\frac{e}{Y}\right)\,\delta_1-2\,\frac{e}{c}\,\delta_3}{\left(1+\frac{y}{c}\right)\,\alpha_1+\left(2+3\,\frac{e}{c}+2\,\frac{e^2}{c^2}+2\,\frac{Y}{c}+\frac{e}{c}\frac{Y}{c}\right)\,\alpha_2+\delta_1}$$

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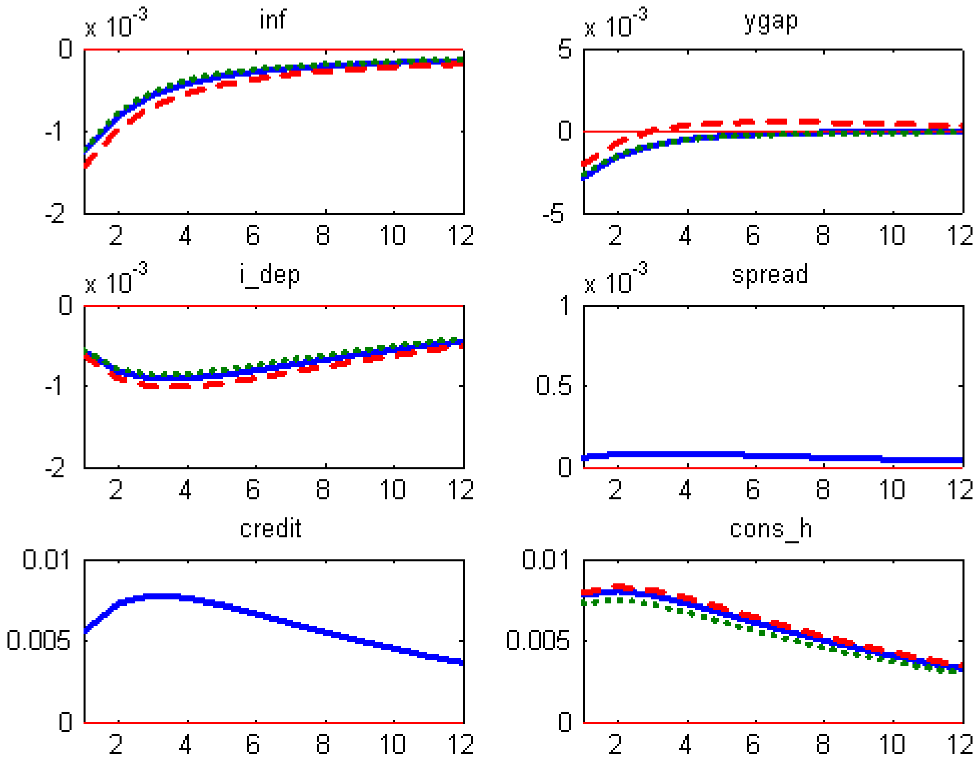
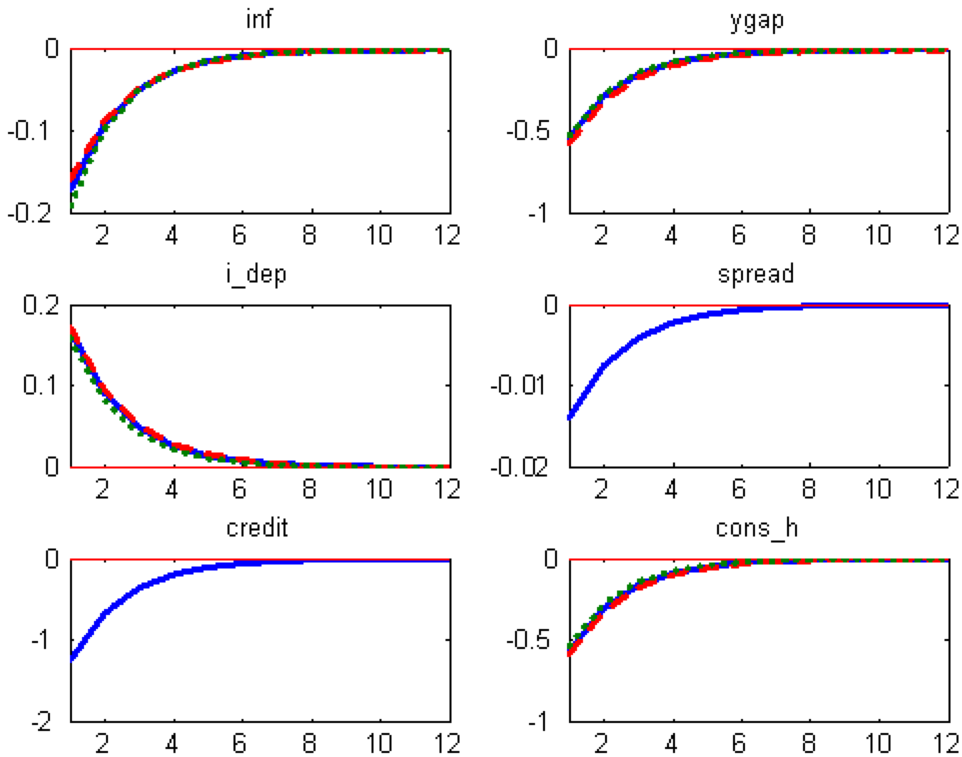
Figure 1: Impulse responses to a technology shock under a Taylor rule within different models



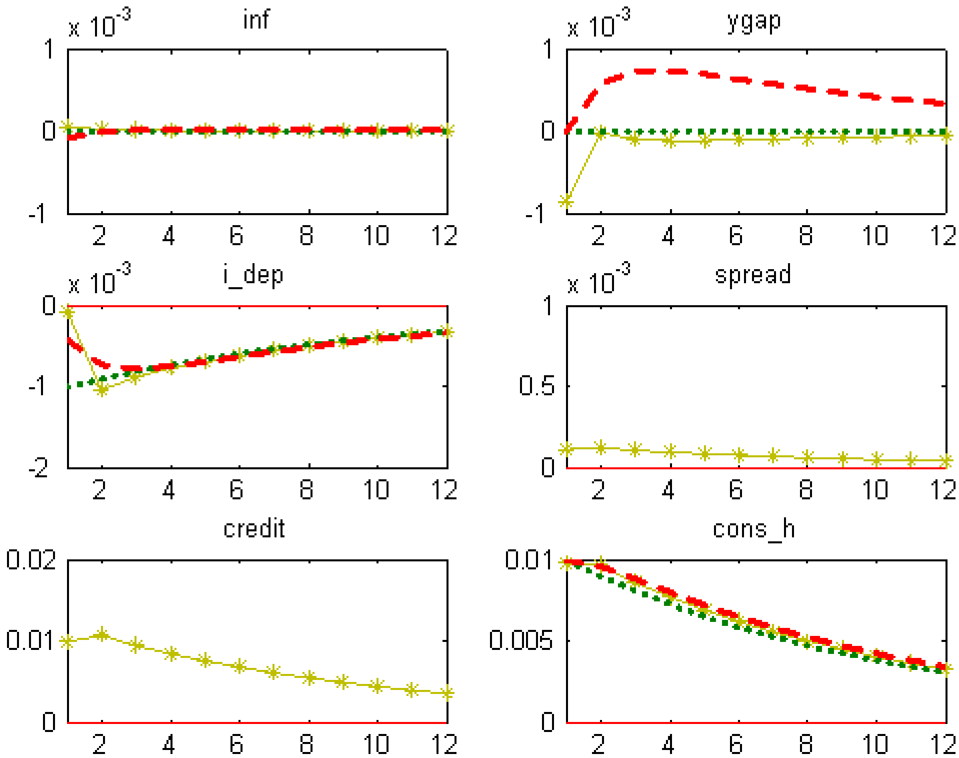

Legend: blue solid line: credit channel model; dashed red line: cost channel model; dotted green line: new-Keynesian model.

Figure 2: Impulse responses to a policy shock under a Taylor rule within different models



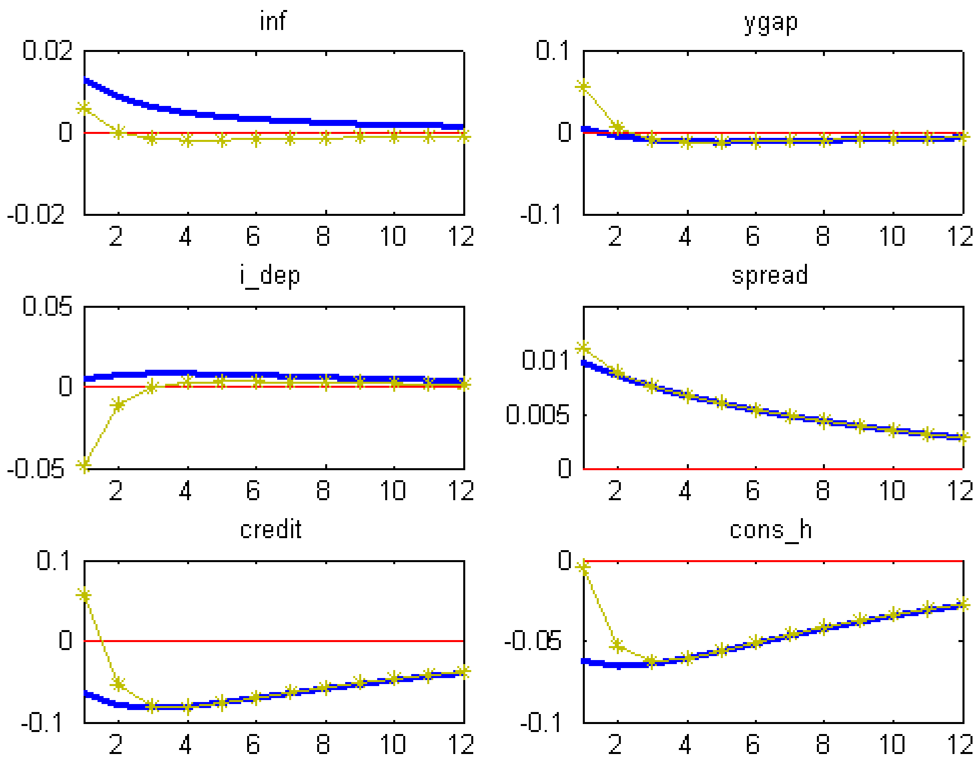

Legend: blue solid line: credit channel model; dashed red line: cost channel model; dotted green line: new-Keynesian model.

Figure 3: Impulse responses to a technology shock under optimal policy within different models




Legend: brown stars: optimal policy in the credit channel model; dashed red line: optimal policy in the cost channel model; dotted green line: optimal policy in the new-Keynesian model.
Note: in all cases, linear terms in the second order expansion of utility are set to zero through an appropriate steady state subsidy and, for the credit channel model, through a particular Pareto weight.

Figure 4: Impulse responses to a $\widehat{\mu}_t$ shock in the credit channel model: Taylor rule vs optimal policy




Legend: brown stars: optimal policy; blue solid line: simple rule.
Note: linear terms in the second order expansion of utility are set to zero through an appropriate steady state subsidy and a particular Pareto weight.