

**Discussion of**  
**“Regime Shifts in a Dynamic Term Structure Model  
of U.S. Treasury Bond Yields”**  
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## **Main Questions:**

- Are regime shifts important for our understanding of interest rates and the term structure?
- Is regime shift risk priced?
- Are the transition probabilities state dependent?

## **Approach:**

- Arbitrage free model of the term structure. Standard multifactor model of the term structure. Affine structure.
- Regime shifts: in prices of risk of the standard factors, transition probabilities for the regimes.

## Why this setup:

- *No Arbitrage Model:*
  - Tractable implications for prices.
  - Once fit to one set of bond prices can price other bonds with confidence: won't induce implied prices that exhibit arbitrage opportunities.
  - Could price derivatives.
  - Seems to recover some important factors in a robust manner.

- *Regime switching:*

- There are regimes in interest rates: variation in fed policy, ...
- Standard affine models of the term structure don't fit conditional volatility well. Conditional volatility clusters. This approach may give a parsimonious and tractable model of volatility.
- Nonstationarity in the world? Shocks to operating procedure or financial structure.

## Some of the details:

- **Affine Term Structure model**

- Matches existing models
- Restrictions imposed to induce identification.

- **Regime switching model:**

- *State is known:*
  - \* Tractable.
  - \* But does this capture what we think of as regime shifts?
  - \* In models with hidden states, agents infer a conditional probability of the state. Exposure to regime risk varies with conditional view of the state.

– *Transition probabilities constant under risk-neutral probabilities*

\* Makes pricing tractable.

\* Given parameterization of true probabilities, this implies a model of the market price of regime-shift risk:

$$\Gamma_t^{jk} = \log \left( \frac{\pi_t^{jk}}{\pi^{*jk}} \right)$$

\* This is somewhat unusual. What type of model would result in this especially *given* the model for the transition probabilities:

$$\pi_t^{jk} = \frac{1}{1 + \exp(\eta_0^{jk} + \eta_Y^{jk} \cdot Y_t)}$$

## Some of the Results:

### *“Standard Part”*

- Standard factors: curvature and two slope factors.
- Factor loadings are constant, but intercepts move with the state.
- Slopes of the yield curves vary with the state.

## *“Markov Switching”*

- Regime H is associated with a recession. (Figure 4).
- Factors exhibit less mean reversion in state L.
- Regime shift risk is priced
- Yield curve gives information about transition probabilities.
  - Flatter curve implies higher probability of switching from L to H.
  - Probability of moving from H to L increases as short-term rates decline or butterfly spread declines.
- During recession the probability of moving from H to L is lower.

- Persistence in the state H during period 1983 to 1985. Credibility of the fed?
- Asymmetry in the transition probabilities. Unconditionally low probability of H to H. Under risk-neutral measure the probability of staying in HH is *much* larger.
- Humps in conditional volatility measures: figure 6. Is this a sign of macro effects on volatility? Is this in options?

- Factor Risk premia:
  - Market Price of Factor Risk depends on state: figure 7
  - Market Price of Regime Shift risk: bigger from H to L, than for L to H.
  - Time varying risk aversion? This feature appears in many models of equity markets. Is the variation implied here consistent with observations from the equity markets?
- Figure 8: MPRS risk H to L is affected by persistence of the underlying states by construction. Can this be interpreted through a utility function?
- No regime conditional volatility movement. Figure 10. Captures volatility during gulf war, but over stated later?

## Other Comments:

- What about other securities like derivatives? If this model is capturing conditional volatility, does the model price derivatives well?
- How much depends on including 1980 to 1983? Is this a period of structural change or is it an event in a stationary Markov chain? Hard problem!