Technical Annexes for 'Monetary Policy Rules for an Open Economy' by Nicoletta Batini, Richard Harrison and Stephen Millard

Annex A: First order conditions

Following the discussion of the model in sections 2.1-2.3 of the main text, here we consider the problems facing agents in each of sector in turn.

Households

Household $j \in (0,1)$ solves the following problem:

$$\text{Max } E_0 \sum_{t=0}^{\infty} b^t \left(\exp(n_t) \ln(c_t(j) - x_c c_{t-1}(j)) + d \ln(1 - h_t(j)) + \frac{c}{1-e} \left(\frac{\Omega_t(j)}{P_t} \right)^{1-e} \right)$$

Subject to

$$M_t(j) + B_t(j) + \frac{B_{f,t}(j)}{e_t(j)} + P_t \int r_t(s) b_t(s,j) ds =$$
$$M_{t-1}(j) + (1+i_{t-1}) B_{t-1}(j) + (1+i_{f,t-1}) \frac{B_{f,t-1}(j)}{e_t} + P_t \int b_{t-1}(s,j) ds + W_t(j) h_t(j) + D_t + T_t - P_t c_t(j) \quad \text{(A1)}$$

$$\Omega_t(j) = M_{t-1}(j) + T_t + (1+i_{t-1}) B_{t-1}(j) + (1+i_{f,t-1}) \frac{B_{f,t-1}(j)}{e_t} - B_t(j) - \frac{B_{f,t}(j)}{e_t} \quad \text{(A2)}$$

$$c_t = c_{M,t}^{g} c_{N,t}^{1-g} \quad \text{(A3)}$$

$$P_t = \frac{P_{M,t}^{g} P_{N,t}^{1-g}}{g^g (1-g)^{1-g}} \quad \text{(A4)}$$

where the variables are defined as in the text. The household chooses c_M, c_N, W, M, B, $b(s)$ and B_f to solve the maximisation problem.

To solve this problem we substitute the definitions of aggregate consumption and the aggregate price level into the utility function, the budget constraint and the definition of 'money' (A2). We let the Lagrange multipliers on these two constraints be denoted λ_1 and λ_2, respectively. Suppressing the *j* index throughout, we differentiate to get:

$$\frac{g\,\exp(n_t)}{c_t - xc_{t-1}}\left(\frac{c_{N,t}}{c_{M,t}}\right)^{1-g} - l_{1,t}P_{M,t} = bgx\left(\frac{c_{N,t}}{c_{M,t}}\right)^{1-g} E_t\left(\frac{\exp(n_{t+1})}{c_{t+1} - xc_t}\right) \qquad \text{(A5)}$$

$$\frac{(1-g)\exp(n_t)}{c_t - xc_{t-1}}\left(\frac{c_{N,t}}{c_{M,t}}\right)^{g} - l_{1,t}P_{N,t} = b(1-g)x\left(\frac{c_{N,t}}{c_{M,t}}\right)^{g} E_t\left(\frac{\exp(n_{t+1})}{c_{t+1} - xc_t}\right) \qquad \text{(A6)}$$

$$l_{1,t} + l_{2,t} = b(1+i_t)E_t\left(l_{1,t+1} + l_{2,t+1}\right) \qquad \text{(A7)}$$

$$\frac{l_{1,t} + l_{2,t}}{e_t} = b(1+i_{f,t})E_t\left(\frac{l_{1,t+1} + l_{2,t+1}}{e_{t+1}}\right) \qquad \text{(A8)}$$

$$l_{2,t} = \frac{c}{P_t}\left(\frac{\Omega_t}{P_t}\right)^{-e} \qquad \text{(A9)}$$

$$l_{1,t} = bE_t\left(l_{1,t+1} + l_{2,t+1}\right) \qquad \text{(A10)}$$

The choice of the nominal wage discussed in section 2.3.2. The first order condition is:

$$E_t\sum_{s=0}^{\infty}(b\,f_W)^s\left[\frac{(1+p)^s W_t(j)}{P_{t+s}}\Lambda_{1,t+s} - (1+q_W)\frac{d}{1-h_{t+s}(j)}\right]h_{t+s}(j) = 0\,. \qquad \text{(A11)}$$

Equation (A12) features the real marginal utility of consumption, Λ_1, which is related to the marginal utility of nominal consumption in a simple manner: $\Lambda_1 = P\,l_1$. This is discussed in more detail below.

Non-Traded Sector

As described in section 2.3.1 producer $k \in (0,1)$ in the non-traded sector choose prices to solve the following problem.

$$\max E_t\sum_{j=0}^{\infty}(b\,f_N)^j\,\Lambda_{1,t+j}\left(\frac{(1+p)^j P_{N,t}(k)}{P_{t+j}} - V_{t+j}\right)y_{N,t+j}(k)$$

subject to $y_{N,t+j}(k)=\left(\frac{(1+p)^{j}P_{N,t+j}(k)}{P_{N,t+j}}\right)^{-\frac{(1+q_N)}{q_N}} y_{N,t+j}$.

The first order condition is:

$$E_t\sum_{j=0}^{\infty}(bf_N)^j\Lambda_{1,t+j}\left(\frac{-q_N(1+p)^jP_{N,t}(k)}{P_{t+j}}+(1+q_N)V_{t+j}\right)y_{N,t+j}(k)=0\,. \tag{A12}$$

The real unit cost, V, in units of final consumption is given by:

$$V_{t+s}=\min\left\{\frac{W_{t+s}}{P_{t+s}}h_{N,t+s}(k)+\frac{P_{I,t+s}}{P_{t+s}}I_{N,t+s}(k)\right\}\text{ subject to }A_{N,t+s}h_{N,t+s}(k)^{a_N}I_{N,t+s}(k)^{(1-a_N)}=1$$

The first order conditions to this problem imply that:

$$\frac{W_t}{P_{I,t}}=\frac{a_N}{1-a_N}\frac{I_{N,t}(k)}{h_{N,t}(k)},$$

for all $k\in(0,1)$ at all dates t. Because non-traded producers are price takers in the factor market, the equilibrium ratio of intermediates to labour is constant across firm in this sector:

$$\frac{W_t}{P_{I,t}}=\frac{a_N}{1-a_N}\frac{I_{N,t}}{h_{N,t}}. \tag{A13}$$

The constancy of the intermediate:labour ratio implies that the aggregate output in the non-traded producers is given by:

$$y_{N,t}=\left[\int_0^1 y_{N,t}(k)^{1/(1+q_N)}dk\right]^{1+q_N}$$

$$y_{N,t}=\left[\int_0^1 y_{N,t}(k)^{1/(1+q_N)}dk\right]^{1+q_N}$$

$$=A_{N,t}\left[\int_0^1[h_{N,t}(k)^{a_N}I_{N,t}(k)^{(1-a_N)}]^{1/(1+q_N)}dk\right]^{1+q_N}=A_{N,t}\left(\frac{h_{N,t}}{I_{N,t}}\right)^{a_N}\left[\int_0^1 I_{N,t}(k)^{1/(1+q_N)}dk\right]^{1+q_N}$$

So,

$$y_{N,t} = A_{N,t} h_{N,t}^{\alpha_N} I_{N,t}^{1-\alpha_N}. \tag{A14}$$

The minimised unit cost for all firms in the non-traded sector is found to be:

$$V_t = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} \frac{W_t^{\alpha_N}(P_{I,t})^{1-\alpha_N}}{A_{N,t} P_t}. \tag{A15}$$

Export sector

As described in section 2.2.2 exports are produced using a Cobb-Douglas technology:

$$y_{X,t} = A_{X,t} h_{X,t}^{\alpha_X} I_{X,t}^{1-\alpha_X} \tag{A16}$$

Efficient production implies that factor demands are given by:

$$\frac{W_t}{P_{X,t}} = \alpha_X A_{X,t} \left(\frac{I_{X,t}}{h_{X,t}} \right)^{1-\alpha_X} \tag{A17}$$

$$\frac{P_{I,t}}{P_{X,t}} = (1-\alpha_X) A_{X,t} \left(\frac{h_{X,t}}{I_{X,t}} \right)^{\alpha_X} \tag{A18}$$

Export demand is:

$$X_t = \left(\frac{e_t P_{X,t}}{P_t^*} \right)^{-\eta} y_{f,t}^{b}. \tag{A19}$$

Intermediate goods sector

Producers in both the non-traded and export sectors purchase imported intermediates from retailers who solve a pricing problem described in section 2.3.1. The first order condition is:

$$E_{t-1}\sum_{s=0}^{\infty}(b\,f_I)^s\Lambda_{1,t+s}\left(\frac{-q_I(1+p)^s P_{I,t}(k)}{P_{t+s}}+(1+q_I)\frac{P^*_{I,t+s}}{e_{t+s}P_{t+s}}\right)y_{I,t+s}(k)=0 \qquad \text{(A20)}$$

Final imports sector

The first order condition for the pricing problem of retailers of final imported goods is given by:

$$E_{t-1}\sum_{s=0}^{\infty}(b\,f_M)^s\Lambda_{1,t+s}\left(\frac{-q_M(1+p)^s P_{M,t}(k)}{P_{t+s}}+(1+q_M)\frac{P^*_{t+s}}{e_{t+s}P_{t+s}}\right)y_{M,t+s}(k)=0\,. \qquad \text{(A21)}$$

Government

The government operates monetary policy by setting nominal interest rates according to a rule (described below) and prints as much money as is demanded at this level of nominal interest rates. Any seignorage revenue is distributed as a lump-sum transfer to consumers. For simplicity, we assume a zero supply of domestic bonds. Hence:

$$M_t - M_{t-1} = T_t - t_t\,. \qquad \text{(A22)}$$

Market Clearing

We have the following market clearing conditions in factor markets, goods markets and asset markets:

$$h_t = h_{X,t} + h_{N,t} \qquad \text{(A23)}$$

$$c_{N,t} = y_{N,t} \qquad \text{(A24)}$$

$$X_t = y_{X,t} \qquad \text{(A25)}$$

$$\int\int b_t(s,j)dsdj = 0 \qquad \text{(A26)}$$

Net foreign assets

The evolution of net foreign assets can be found by evaluating the household's budget constraint (A2) at market equilibrium and then aggregating across households. As discussed in section 2.4, the net foreign asset position (under our assumptions this is equal to the domestic holdings of foreign bonds) is non-stationary. To deal with this problem we do not include this equation in our system. Instead we use the equation to substitute foreign bond holdings out of the definition of 'money' (A3).

Annex B: Flexible-price steady state

We use the following notation. Variables without time subscripts are the steady state values. Lower case letters represent nominal variables expressed relative to the CPI (we also define the real value of foreign bond holdings as $b_f = B_f / eP$). We express nominal variables relative to the general price level in order to solve for steady state variables that are not trended (in steady state all nominal variables will follow the same trend path). In addition, the Lagrange multipliers l_1 and l_2 are homogenous of degree -1 so we scale them by the CPI, to give stationary multipliers $\Lambda_1 = P l_1$ and $\Lambda_2 = P l_2$. Throughout we use the real exchange rate definition, $q_t = \dfrac{e_t P_t}{P_t^*}$.

To construct a steady state, we first assume that all domestic nominal variables are growing at an annual rate of 2.5%. This means that, in steady state, the government is meeting an inflation target of 2.5%. For simplicity, we also assume that the steady state growth of foreign nominal variables is 2.5%. The implied steady state value of nominal interest rates at home and abroad will be given by:

$$i = i_f = \frac{1+p}{b} - 1.$$

In what follows, we use equations (A23) and (A27) before evaluating the steady state. We assume that steady state taxes are set to exactly offset steady state dividends. Finally, we choose a flexible price equilibrium so that, although price setters retain some monopoly power, they simply set prices as a mark up over unit costs.

Then, the first order conditions imply the following equations defining steady state values of the variables:

$$l_1 p_M = \frac{(1-bx)g}{(1-x)c}\left(\frac{c_N}{c_M}\right)^{1-g} \tag{A27}$$

$$l_1 p_N = \frac{(1-bx)(1-g)}{(1-x)c}\left(\frac{c_M}{c_N}\right)^{g} \tag{A28}$$

$$c w^{-e} = l_2 \tag{A29}$$

$$l_1 = \frac{b(l_1 + l_2)}{1+p} \tag{A30}$$

$$l_1 w = \frac{(1+q_W)d}{(1-h)} \tag{A31}$$

$$\frac{1-b}{b} b_f = p_X X - p_M c_M - p_I (I_X + I_N) \tag{A32}$$

$$w = m + \frac{1-b}{b} b_f \tag{A33}$$

$$c = c_M^{g} c_N^{1-g} \tag{A34}$$

$$p_N^{1-g} p_M^{g} = g^{g} (1-g)^{1-g} \tag{A35}$$

$$p_N = (1+q_N) v \tag{A36}$$

$$\frac{w}{p_I} = \frac{a_N}{1-a_N}\frac{I_N}{h_N} \tag{A37}$$

$$y_N = A_N h_N^{a_N} I_N^{1-a_N} \tag{A38}$$

$$v = \frac{w^{a_N} p_I^{1-a_N}}{a_N^{a_N}(1-a_N)^{1-a_N}} \tag{A39}$$

$$y_X = A_X h_X^{a_X} I_X^{1-a_X} \tag{A40}$$

$$\frac{w}{p_X} = a_X A_X \left(\frac{I_X}{h_X}\right)^{1-a_X} \tag{A41}$$

$$\frac{p_I}{p_X} = (1-a_X)A_X\left(\frac{h_X}{I_X}\right)^{a_X} \tag{A42}$$

$$X = \left(\frac{q}{p_X}\right)^{h} y_f^{b} \tag{A43}$$

$$p_I = (1+q_I)\tfrac{p_I^*}{q} \tag{A44}$$

$$p_M = (1+q_M)\tfrac{1}{q} \tag{A45}$$

$$c_N = y_N \tag{A46}$$

$$h = h_N + h_X \tag{A47}$$

$$X = y_X \tag{A48}$$

Annex C: A log-linear representation of the model

To solve the model we log-linearise the first order conditions of the model around the non-stochastic steady state defined by equations (A27) to (A48). As described in the main text we use (A2) evaluated at market equilibrium to substitute foreign bond holdings out of the model. As in Annex 2, we also substitute out for taxes, transfers and dividends. Log-linearising the consumers' first order conditions (equations (A5) to (A10)) gives us:

$$E_t\left(\frac{bx}{(1-bx)(1-x)}\hat{c}_{t+1} - \frac{bx}{1-bx}n_{t+1}\right) = \hat{\Lambda}_{1,t} + \hat{p}_{M,t} - \frac{1}{1-bx}n_t + \hat{c}_{M,t} - \left(1-\frac{1+bx^2}{(1-bx)(1-x)}\right)\hat{c}_t - \frac{x}{(1-bx)(1-x)}\hat{c}_{t-1} \tag{A49}$$

$$E_t\left(\frac{bx}{(1-bx)(1-x)}\hat{c}_{t+1} - \frac{bx}{1-bx}n_{t+1}\right) = \hat{\Lambda}_{1,t} + \hat{p}_{N,t} - \frac{1}{1-bx}n_t + \hat{c}_{N,t} - \left(1-\frac{1+bx^2}{(1-bx)(1-x)}\right)\hat{c}_t - \frac{x}{(1-bx)(1-x)}\hat{c}_{t-1} \tag{A50}$$

$$E_t\left(\hat{p}_{t+1}-\frac{\Lambda_1}{\Lambda_1+\Lambda_2}\hat{\Lambda}_{1,t+1}-\frac{\Lambda_2}{\Lambda_1+\Lambda_2}\hat{\Lambda}_{2,t+1}\right)=(i_t-i)-\frac{\Lambda_1}{\Lambda_1+\Lambda_2}\hat{\Lambda}_{1,t} -\frac{\Lambda_2}{\Lambda_1+\Lambda_2}\hat{\Lambda}_{2,t} \tag{A51}$$

$$E_t\left(\hat{p}^*_{t+1}+\hat{q}_{t+1}-\frac{\Lambda_1}{\Lambda_1+\Lambda_2}\hat{\Lambda}_{1,t+1}-\frac{\Lambda_2}{\Lambda_1+\Lambda_2}\hat{\Lambda}_{2,t+1}\right)=(i_{f,t}-i_f)+\hat{q}_t-\frac{\Lambda_1}{\Lambda_1+\Lambda_2}\hat{\Lambda}_{1,t} -\frac{\Lambda_2}{\Lambda_1+\Lambda_2}\hat{\Lambda}_{2,t}+z_t \tag{A52}$$

$$\hat{\Lambda}_{2,t}+e\hat{w}_t=0 \tag{A53}$$

$$E_t\left(\hat{p}_{t+1}-\frac{\Lambda_1}{\Lambda_1+\Lambda_2}\hat{\Lambda}_{1,t+1}-\frac{\Lambda_2}{\Lambda_1+\Lambda_2}\hat{\Lambda}_{2,t+1}\right)=\hat{\Lambda}_{1,t} \tag{A54}$$

where for any variable x, $\hat{x}=\ln\left(\frac{x_t}{x}\right)$ where x is its steady state value and z is an exogenous 'foreign exchange risk premium' shock.

The definition of W (equation (A3)) becomes:

$$\hat{w}_t-\frac{m}{w}\hat{m}_t+\frac{wh}{w}\hat{h}_t+\frac{wh}{w}\hat{w}_t-+\frac{c}{w}\hat{c}_t=0\,. \tag{A55}$$

The definitions of consumption and the price indices are:

$$\hat{c}_t=g\hat{c}_{M,t}+(1-g)\hat{c}_{N,t}\,, \tag{A56}$$

$$0=g\hat{p}_{M,t}+(1-g)\hat{p}_{N,t}\,. \tag{A57}$$

Wage setting is given by the following two equations (the derivation follows Erceg *et al* (1999, p25)).

$$\Delta\hat{W}_t=bE_t\Delta\hat{W}_{t+1}+\frac{(1-f_W)(1-bf_W)h}{f_W(1-h)[1+\frac{h(1+q_W)}{q_W(1-h)}]}\hat{h}_t-\frac{(1-f_W)(1-bf_W)}{f_W[1+\frac{h(1+q_W)}{q_W(1-h)}]}\hat{\Lambda}_{1,t} -\frac{(1-f_W)(1-bf_W)}{f_W[1+\frac{h(1+q_W)}{q_W(1-h)}]}\hat{w}_t \tag{A58}$$

$$\hat{w}_t=\hat{w}_{t-1}+\Delta\hat{W}_t-\hat{p}_t\,. \tag{A59}$$

Pricing decisions by non-traded goods producers are described by:

$$\Delta\hat{P}_{N,t} = bE_t\Delta\hat{P}_{N,t+1} + \frac{(1-f_N)(1-bf_N)}{f_N}\hat{v}_t - \frac{(1-f_N)(1-bf_N)}{f_N}\hat{p}_{N,t}, \tag{A60}$$

$$\hat{p}_{N,t} = \hat{p}_{N,t-1} + \Delta\hat{P}_{N,t} - \hat{p}_t, \tag{A61}$$

$$\hat{v}_t = a_N\hat{w}_t + (1-a_N)\hat{p}_{I,t} - \hat{A}_{N,t}. \tag{A62}$$

Efficient production by non-traded producers implies that:

$$\hat{w}_t - \hat{p}_{I,t} = \hat{I}_{N,t} - \hat{h}_{N,t}, \tag{A63}$$

$$\hat{y}_{N,t} = \hat{A}_{N,t} + a_N\hat{h}_{N,t} + (1-a_N)\hat{I}_{N,t}. \tag{A64}$$

The first order conditions for export producers become:

$$\hat{w}_t - \hat{p}_{X,t} - \hat{A}_{X,t} - (1-a_X)\hat{I}_{X,t} + (1-a_X)\hat{h}_{X,t} = 0, \tag{A65}$$

$$\hat{p}_{I,t} - \hat{p}_{X,t} - \hat{A}_{X,t} + a_X\hat{I}_{X,t} + a_X\hat{h}_{X,t} = 0. \tag{A66}$$

Export production is given by:

$$\hat{y}_{X,t} - \hat{A}_{X,t} - a_X\hat{h}_{X,t} - (1-a_X)\hat{I}_{X,t} = 0. \tag{A67}$$

Export demand can be written as:

$$\hat{X}_t + h\hat{q}_t + h\hat{p}_{X,t} - b\hat{y}_{F,t} = 0. \tag{A68}$$

Pricing of intermediates is described by:

$$\begin{aligned}\Delta\hat{P}_{I,t} = bE_{t-1}\Delta\hat{P}_{I,t+1} &+ \frac{(1-f_I)(1-bf_I)}{f_I}E_{t-1}\hat{p}^*_{I,t} \\ &- \frac{(1-f_I)(1-bf_I)}{f_I}E_{t-1}\hat{q}_t - \frac{(1-f_I)(1-bf_I)}{f_I}E_{t-1}\hat{p}_{I,t}\end{aligned} \tag{A69}$$

$$\hat{p}_{I,t} = \hat{p}_{I,t-1} + \Delta\hat{P}_{I,t} - \hat{p}_t. \tag{A70}$$

Pricing of final imports is described by:

$$\Delta\hat{P}_{M,t} = bE_{t-1}\Delta\hat{P}_{M,t+1} - \frac{(1-f_I)(1-bf_I)}{f_I}E_{t-1}\hat{q}_t\,, \tag{A71}$$

$$\hat{p}_{M,t} = \hat{p}_{M,t-1} + \Delta\hat{P}_{M,t} - \hat{p}_t\ . \tag{A72}$$

The relevant market-clearing conditions can be written as:

$$\frac{h_X}{h}\hat{h}_{X,t} + \frac{h_N}{h}\hat{h}_{N,t} - \hat{h}_t = 0\,, \tag{A73}$$

$$\hat{c}_{N,t} - \hat{y}_{N,t} = 0\,, \tag{A74}$$

$$\hat{X}_t - \hat{y}_{X,t} = 0\,. \tag{A75}$$

Together with some obvious lag identities and log-linearised definitions (for example the GDP identity) the model can be cast in the form of equations (22) and (23) in the main text. The calibration of the forcing processes is described in section 3.2 of the main text.