Monetary Policy Drivers of Bond and Equity Risks

John Y. Campbell, Carolin Pflueger, and Luis M. Viceira[1]

First draft: March 2012
This draft: March 2014

---

[1]Campbell: Department of Economics, Littauer Center, Harvard University, Cambridge MA 02138, USA, and NBER. Email john_campbell@harvard.edu. Pflueger: University of British Columbia, Vancouver BC V6T 1Z2, Canada. Email carolin.pflueger@sauder.ubc.ca. Viceira: Harvard Business School, Boston MA 02163 and NBER. Email lviceira@hbs.edu. We are grateful to Alberto Alesina, Robert Barro, Philip Bond, Mikhail Chernov, Paul Beaudry, Ian Dew-Becker, Alexander David, Adlai Fisher, Ben Friedman, Lorenzo Garlappi, Gita Gopinath, Robin Greenwood, Joshua Gottlieb, Howard Kung, Leonid Kogan, Deborah Lucas, Greg Mankiw, Michael Woodford, conference and seminar participants at the University of British Columbia, the Harvard Monetary Economics Seminar, the 2013 HBS Finance Research Retreat, the University of Calgary, the Vienna Graduate School of Finance, ECWFC 2013, PNWCF 2014, the Jackson Hole Finance Conference 2014, the ASU Sonoran Winter Finance Conference 2014, and especially our discussants Rossen Valkanov and Gregory Duffee for helpful comments and suggestions. This material is based upon work supported by Harvard Business School Research Funding and the PH&N Centre for Financial Research at UBC.

Abstract

The exposure of US Treasury bonds to the stock market has moved considerably over time. While it was slightly positive on average in the period 1960-2011, it was unusually high in the 1980s and negative in the 2000s, a period during which Treasury bonds enabled investors to hedge macroeconomic risks. This paper explores the effects of monetary policy parameters and macroeconomic shocks on nominal bond risks, using a New Keynesian model with habit formation and discrete regime shifts in 1979 and 1997. The increase in bond risks after 1979 is attributed primarily to a shift in monetary policy towards a more anti-inflationary stance, while the more recent decrease in bond risks after 1997 is attributed primarily to an increase in the persistence of monetary policy interacting with continued shocks to the central bank's inflation target. Endogenous responses of bond risk premia amplify these effects of monetary policy on bond risks.

1 Introduction

In different periods of history, long-term Treasury bonds have played very different roles in investors' portfolios. During the Great Depression of the 1930s, and once again in the first decade of the 21st Century, Treasury bonds served to hedge other risks that investors were exposed to: the risk of a stock market decline, and more generally the risk of a weak macroeconomy, with low output and high unemployment. Treasuries performed well both in the Great Depression and in the two recessions of the early and late 2000s. During the 1970s and particularly the 1980s, however, Treasury bonds added to investors' macroeconomic risk exposure by moving in the same direction as the stock market and the macroeconomy. A number of recent papers including Baele, Bekaert, and Inghelbrecht (2010), Campbell, Sunderam, and Viceira (2013), Christiansen and Ranaldo (2007), David and Veronesi (2013), Guidolin and Timmermann (2006), and Viceira (2012) have documented these developments.

Given the importance of Treasury bonds as an asset class, it is natural to ask what economic factors determine their risk properties. One way to do this is to use identities that link bond returns to movements in bond yields, and that link nominal bond yields to expectations of future short-term real interest rates, expectations of future inflation rates, and time-varying risk premia on longer-term bonds over short-term bonds. Barsky (1989), Shiller and Beltratti (1992), and Campbell and Ammer (1993) were early examples of this approach. A more recent literature has proceeded in a similar spirit, building on the no-arbitrage restrictions of affine term structure models (Duffie and Kan 1996, Dai and Singleton 2000, 2002, Duffee 2002) to estimate multifactor term structure models with both macroeconomic and latent factors (Ang and Piazzesi 2003, Ang, Dong, and Piazzesi

2007, Rudebusch and Wu 2007). Although these exercises can be informative, they are based on a reduced-form econometric representation of the stochastic discount factor and the process driving inflation. This limits the insights they can deliver about the economic determinants of bond risks.

A more ambitious approach is to build a general equilibrium model of bond pricing. Real business cycle models have an exogenous real economy, driven by shocks to either goods endowments or production, and an inflation process that is either exogenous or driven by monetary policy reactions to the real economy. Papers in the real business cycle tradition often assume a representative agent with Epstein-Zin preferences, and generate time-varying bond risk premia from stochastic volatility in the real economy and/or the inflation process (Bansal and Shaliastovich 2013, Buraschi and Jiltsov 2005, Burkhardt and Hasseltoft 2012, Gallmeyer et al 2007, Piazzesi and Schneider 2006). Some papers instead derive time-varying risk premia from habit formation in preferences (Bekaert, Engstrom, and Grenadier 2010, Bekaert, Engstrom, and Xing 2009, Buraschi and Jiltsov 2007, Dew-Becker 2013, Wachter 2006). Under either set of assumptions, this work allows only a limited role for monetary policy, which determines inflation (at least in the long run) but has no influence on the real economy.[2] Accordingly a recent literature has explored the asset pricing implications of New Keynesian models, in which price stickiness allows monetary policy to have real effects. Recent papers in this literature include Andreasen (2012), Bekaert, Cho, and Moreno (2010), Van Binsbergen et al (2012), Dew-Becker (2014), Kung (2013), Li and Palomino (2013), Palomino (2012), Rudebusch and Wu (2008), and Rudebusch and Swanson (2012).

[2]A qualification to this statement is that in some models, such as Buraschi and Jiltsov (2005), a nominal tax system allows monetary policy to affect fiscal policy and, through this indirect channel, the real economy.

We follow this second approach and quantitatively investigate two candidate explanations for the empirical instability in bonds risk properties: changes in monetary policy or changes in macroeconomic shocks. First, U.S. monetary policy differed substantially between the pre-Volcker period, the inflation fighting period of Volcker and Greenspan, and the recent period of increased central bank transparency. If the central bank affects the macroeconomy through nominal interest rates, it is natural to think that these changes should affect the risks of bonds and stocks.

Second, the nature of economic shocks has changed dramatically over time. While oil supply shocks were prominent during the 1970s and 1980s, more recent output fluctuations have been associated with the information technology revolution and financial sector shocks. It is intuitive that whenever supply shocks are dominant, bonds should be risky assets. Macroeconomic supply shocks, such as the oil supply shocks of the 1970s and 1980s, might generate high inflation recessions and therefore lead bonds to perform poorly at the same time as stocks.

Figure 1 helps motivate our analysis, showing a timeline of changing US bond risks, monetary policy regimes, and oil price shocks from Hamilton (2009). Figure 1 measures bond risks with rolling window CAPM betas and return volatilities of nominal bonds.[3] Before 1979, the beta of bonds was close to zero but slightly positive. The bond beta became strongly positive during the 1980s and 1990s and it finally flipped sign and became negative in the late 1990s. Figure 1 also shows two monetary policy dates, corresponding to major shifts in monetary policy. The first monetary policy break date corresponds to the appointment of Paul Volcker as chairman of the board of Governors, which arguably

[3]Figure 1 shows the CAPM beta and bond return volatility of ten-year nominal bond returns. We use daily returns over the past quarter. For a detailed data description, see Section 3.

marked a significant change from the previous more accommodative monetary policy regime (Clarida, Gali, and Gertler 1999). The second monetary policy date marks the first quarter of 1997, coinciding with Alan Greenspan's well-noted "Central Banking in a Democratic Society" speech (Greenspan 1996). At first glance, monetary policy changes in the late 1990s might not be as salient as Paul Volcker's appointment. However, this was a period of very significant monetary policy shifts towards transparency and gradualism, with the central bank taking more cautious interest rate decisions and implementing them over longer periods of time. Most tangibly, the Federal Reserve started publishing detailed transcripts of Federal Open Market Committee (FOMC) meetings in the mid 1990s, with plausibly significant implications for monetary policy decisions and bond risks. For example, a 2013 Bloomberg News article notes that the number of dissenting votes at FOMC meetings started to fall exactly when the Fed started to publish FOMC meeting minutes and it has been essentially zero since 1997 (our regime shift date).[4]

Figure 1 provides strong motivational evidence that changes in monetary policy are important for understanding changing bond risks. Figure 1, Panel A shows that changes in bond betas line up closely with monetary policy breaks. Bond return volatility also lines up with monetary policy break dates, with the exception of a short-lived spike at the beginning of the middle subperiod.

In contrast, Figure 1 shows that oil price shocks do not line up closely with nominal bond betas. This observation suggests that empirical changes in supply shock uncertainty may not be sufficient to explain changes in bond risks. Oil price shocks represent only a subset of macroeconomic supply shocks and therefore the evidence in Figure 1 is merely

[4]Joshua Zumbrun, 2013, Greenspans Bequest to Yellen Is Board Harmony Shown in Records, *Bloomberg News*, November 6.

suggestive. However, the main empirical analysis in this paper systematically examines the role of time-varying shock volatilities for nominal bond betas and corroborates the suggestive evidence in Figure 1.

This paper builds on the New Keynesian asset pricing literature and makes two contributions. First, we formulate a New Keynesian model in which bonds and stocks can both be priced from assumptions about their payoffs, and in which time-varying risk premia, driven by habit formation and stochastic volatility, generate realistic variances and covariances for these asset classes. Most previous New Keynesian asset pricing papers have concentrated on the term structure of interest rates, and have paid little attention to the implied pricing of equities. This contrasts with the integrated treatment of the bond and stock markets in several papers that use reduced-form affine or real business cycle models (Ang and Ulrich, 2012, Bansal and Shaliastovich 2013, Bekaert, Engstrom, and Grenadier 2010, Koijen, Lustig, and Van Nieuwerburgh, 2010, Campbell 1986, Campbell, Sunderam, and Viceira 2013, d'Addona and Kind 2006, Dew-Becker 2013, Eraker 2008, Hasseltoft 2008, Lettau and Wachter 2011, Wachter 2006).

Second, we use our model to relate changes in bond risks to periodic regime changes in the parameters of the central bank's monetary policy rule and the volatilities of macroeconomic shocks. In this way we contribute to the literature on monetary policy regime shifts (Andreasen 2012, Ang, Boivin, Dong, and Kung 2011, Bikbov and Chernov 2013, Boivin and Giannoni 2006, Chib, Kang, and Ramamurthy 2010, Clarida, Gali, and Gertler 1999, Palomino 2012, Rudebusch and Wu 2007, Smith and Taylor 2009). While this literature has begun to focus on the implications of monetary regime shifts for the term structure of interest rates, previous papers have not looked at the implications for the comovements of bonds and equities as we do here. Our structural analysis takes account of various channels by which the monetary policy regime affects the sensitivities of bond and stock returns to macroeconomic shocks, including endogenous responses of risk premia.

The organization of the paper is as follows. Section 2 lays out a basic New Keynesian model that explains interest rates, inflation, and medium-term deviations of output from trend (the "output gap") using three structural equations: an investment-saving curve (IS) that describes real equilibrium in the goods market based on the Euler equation of a representative consumer, a Phillips curve (PC) that describes the effects of nominal frictions on inflation, and a monetary policy reaction function (MP) embodying a Taylor rule as in Clarida, Gali, and Gertler (1999), Taylor (1993), and Woodford (2001). This section also solves for the stochastic discount factor (SDF) implied by the New Keynesian IS curve, and uses it to price bonds and stocks.

Section 3 describes our data sources and presents summary statistics for our full sample period, 1960Q1 through 2011Q4, and for three subperiods, 1960Q1–1979Q2, 1979Q3–1996Q4, and 1997Q1–2011Q4. These subperiods are chosen to match both shifts in monetary policy and changes in measured bond risks. This section also estimates the parameters of the monetary policy reaction function, over the full sample and the three subperiods, using reduced-form regression methodology.

Section 4 calibrates our model to fit both macroeconomic and asset pricing data over our three subperiods. Section 5 presents counterfactual analysis, asking how bond risks would have evolved over time if the monetary policy rule, or the volatilities of macroeconomic shocks, had been stable instead of time-varying. Section 6 explores the implications of our model for inflation-indexed bonds. Section 7 concludes, and an online appendix (Campbell, Pflueger, and Viceira 2013) presents additional details.

2 A New Keynesian Asset Pricing Model

We model the dynamics of macroeconomic time series with a standard New Keynesian framework consisting of a log-linearized Euler equation, a Phillips curve, and a monetary policy function. We integrate asset pricing into the framework by deriving the Euler equation from a stochastic discount factor (SDF) that also prices stocks and bonds in the model. The SDF links asset returns and macroeconomic and monetary variables in equilibrium in a standard no-arbitrage setup.

The Euler equation is a standard New Keynesian building block and provides an equivalent of the Investment and Savings (IS) curve. We provide a micro-founded log-linearized Euler equation relating current output to the lagged output gap, the expected future output gap, and the real interest rate. Euler equations with both backward-looking and forward-looking components are common in the dynamic stochastic general equilibrium (DSGE) literature (Christiano, Eichenbaum, and Evans 2005, Boivin and Giannoni 2006, Smets and Wouters 2007, Canova and Sala 2009).[5] Fuhrer (2000) argues that allowing for a backward-looking component is important for capturing the empirical hump-shaped output response to a monetary policy shock. The forward-looking component follows from standard household dynamic optimization.

[5]Christiano, Eichenbaum, and Evans (2005) and Boivin and Giannoni (2006) derive a backward- and forward-looking linearized Euler equation in a model where utility depends on the difference between consumption and an internal habit stock. A backward-looking component in the Euler equation can also be derived in a model with multiplicative external habit (Abel 1990, Fuhrer 2000).

We derive an Euler equation with both backward-looking and forward-looking components from a consumption-based SDF in which the marginal utility of consumption depends on the current and lagged values of the output gap, and its conditional volatility varies inversely with the output gap. This assumption about the volatility of marginal utility implies that real risk premia increase during recessions, consistent with the empirical evidence on stock and bond return predictability (Chen 1991, Cochrane 2007, Cochrane and Piazzesi 2005, Fama 1990, Fama and French 1989, Lamont 1998, Lettau and Ludvigson 2001). A parametric model that exhibits these properties and produces analytically tractable expressions for asset prices and expected returns after suitable log-linearization of the SDF is the habit-formation model of Campbell and Cochrane (1999), in which utility is a power function of the difference between consumption and habit–the consumption surplus. We therefore adopt this specification of utility for analytical convenience. Finally, shocks to marginal utility, or demand shocks, introduce shocks to the Euler equation.

The empirical evidence in Figure 2, Panel A justifies our assumption that the marginal utility of consumption is a function of the current and lagged output gaps. This figure plots the time series of stochastically detrended consumption—log real consumption of nondurables and services less a 24-quarter moving average—and the log output gap. The two series move very closely together, almost surprisingly so given the measurement issues in both series, with a correlation of 90%. We model surplus consumption to be linear in the current output gap and the lagged output gap, with a negative coefficient on the lag. We can therefore think of surplus consumption as a transformation of stochastically detrended consumption that accentuates higher-frequency movements.

The second building block of a New Keynesian model is the Phillips curve (PC) equation that links inflation and real output in equilibrium. We assume a PC with both forward- and backward-looking components to capture the price setting behavior of firms. While a Calvo (1983) model of monopolistically competitive firms and staggered price setting implies a forward-looking Phillips curve, a backward-looking Phillips curve can arise when price setters update their information infrequently (Mankiw and Reis 2002).

The third building block of the model is an equation describing the behavior of the central bank. We assume that the central bank's policy instrument is the short-term nominal interest rate. The central bank sets this interest rate according to a Taylor (1993) monetary policy (MP) rule, as a linear function of the "inflation gap" (the deviation of inflation from the central bank's target), the output gap, and the lagged nominal interest rate. Empirically, the Fed appears to smooth interest rates over time, and we capture this by modeling the nominal short rate as adjusting gradually to the target rate. This approach is fairly standard in the New Keynesian literature, although there is some debate over the relative importance of partial adjustment and serially correlated unobserved fundamentals in the MP rule (Rudebusch 2002, Coibion and Gorodnichenko 2012).

We allow for a time-varying central bank inflation target. Historical US inflation appears highly persistent (Ball and Cecchetti 1990, Stock and Watson 2007). We capture this empirical regularity by modeling the inflation target as a unit root process. Movements in our estimated inflation target may capture episodes where public expectations of central bank behavior are not well anchored, because the central bank lacks credibility, even if the central bank's true target is relatively stable (Orphanides and Williams 2004).

To close the model we need to make identification assumptions. DSGE models are often under-identified or only very weakly identified (Canova and Sala 2009, An and Schorfheide

2007) because the mapping between underlying parameters and model moments can be highly nonlinear. Restrictions on the form of the monetary policy shock may be necessary to identify monetary policy parameters (Backus, Zin, and Chernov, 2013). We adopt identification assumptions commonly used in the structural vector autoregression literature to help identify the central bank's monetary policy rule, using exclusion restrictions that allow us to estimate the monetary policy rule by Ordinary Least Squares (OLS).

2.1 Euler equation with habit formation

Standard no-arbitrage conditions in asset pricing imply that the gross one-period real return $(1 + R_{t+1})$ on any asset satisfies

$$1 = \mathrm{E}_t \left[M_{t+1} \left(1 + R_{t+1} \right) \right], \tag{1}$$

where M_{t+1} is the stochastic discount factor (SDF). Household optimization implies a SDF of the form

$$M_{t+1} = \frac{\beta U'_{t+1}}{U'_t}, \tag{2}$$

where U'_t is the marginal utility of consumption at time t and β is a time discount factor. Substitution of (2) into (1) produces the standard Euler equation.

The Euler equation for the return on a one-period real T-bill can be written in log form as:

$$\ln U'_t = r_t + \ln \beta + \ln \mathrm{E}_t U'_{t+1}, \tag{3}$$

where we write r_t for the log yield at time t—and return at time $t+1$—on a one-period real Treasury bill. Similarly, we write i_t to denote the log yield on a one-period nominal T-bill. We use the subscript t for short-term nominal and real interest rates to emphasize that they are known at time t. For simplicity, we assume that short-term nominal interest rates contain no risk premia or that $i_t = r_t + E_t\pi_{t+1}$, where π_{t+1} is inflation from time t to time $t+1$. This approximation is justified if uncertainty about inflation is small at the quarterly horizon, as appears to be the case empirically.

Equation (3) also describes the nominal interest rate given a model for expected inflation. Substituting $r_t = i_t - E_t\pi_{t+1}$ into (3), and dropping constants to reduce the notational burden, we have:

$$\ln U'_t = (i_t - \mathrm{E}_t\pi_{t+1}) + \ln \mathrm{E}_t U'_{t+1}. \tag{4}$$

We assume that $\ln U'_t$ is a linear function of the current and lagged log output gap x_t and that its conditional volatility is also an exponential affine function of x_t with a negative slope so that the volatility of marginal utility is higher when the output gap is low. These assumptions imply an Euler equation for the real riskfree rate that relates the real interest rate to the current output gap, its first-order lag, and its expected value.

To see this, consider a habit formation model of the sort proposed by Campbell and Cochrane (1999), where utility is a power function of the difference between consumption C and habit H:

$$U_t = \frac{(C_t - H_t)^{1-\alpha} - 1}{1-\alpha} = \frac{(S_t C_t)^{1-\alpha} - 1}{1-\alpha}. \tag{5}$$

Here $S_t = (C_t - H_t)/C_t$ is the surplus consumption ratio and α is a curvature parameter that controls risk aversion. Relative risk aversion varies over time as an inverse function of the surplus consumption ratio: $-U_{CC}C/U_C = \alpha/S_t$.

Marginal utility in this model is

$$U_t' = (C_t - H_t)^{-\alpha} = (S_t C_t)^{-\alpha}\,, \tag{6}$$

and log marginal utility is given by $\ln U_t' = -\alpha(s_t + c_t)$. Assuming lognormality, or taking a second-order Taylor approximation, the Euler equation (4) becomes

$$-\alpha(s_t + c_t) = (i_t - \mathrm{E}_t\pi_{t+1}) - \alpha\mathrm{E}_t(s_{t+1} + c_{t+1}) + \frac{\alpha^2}{2}\sigma_t^2, \tag{7}$$

where $\sigma_t^2 = \mathrm{Var}_t(s_{t+1} + c_{t+1})$.

Now suppose that

$$s_t + c_t = x_t - \theta x_{t-1} - v_t, \tag{8}$$

where the error term v_t is white noise uncorrelated with current or lagged x_t, or any other information variables known in advance. We have argued that the empirical output gap is closely related to stochastically detrended consumption, so the expression (8) can be interpreted as a simple transformation of stochastically detrended consumption

Furthermore, assume that the volatility of marginal utility is higher when the output gap is low. For some $0 < b < 1$:

$$\sigma_t^2 = \overline{\sigma}^2 \exp(-bx_t) \approx \overline{\sigma}^2(1 - bx_t). \tag{9}$$

Here $\overline{\sigma}$ is the conditional volatility of surplus consumption when the output gap is zero.

Substituting (8) and (9) into (7) yields the Euler equation:

$$x_t = \rho^{x-} x_{t-1} + \rho^{x+} \mathrm{E}_t x_{t+1} - \psi \left(i_t - \mathrm{E}_t \pi_{t+1} \right) + u_t^{IS}, \tag{10}$$

where $\rho^{x-} = \theta/(1+\theta^*)$, $\rho^{x+} = 1/(1+\theta^*)$, $\psi = 1/\alpha(1+\theta^*)$, $u_t^{IS} = v_t/(1+\theta^*)$, and $\theta^* = \theta - \alpha b \overline{\sigma}^2/2 < \theta$.

Several points are worth noting about the IS curve (10). First, because $\theta^* < \theta$, the coefficients on the lagged output gap and the expected future output gap sum to more than one. Second, the slope of the IS curve ψ does not equal the elasticity of intertemporal substitution (EIS) of the representative consumer. Third, shocks to the IS curve result from marginal utility or demand shocks in equation (8). Alternatively, we can interpret these shocks as incorporating any divergences between consumption surplus and the output gap that are uncorrelated with the other shocks in the model.

2.2 Macroeconomic dynamics

We complement the consumers' Euler equation with standard building blocks of New Keynesian macroeconomic models. We assume that consumers and firms do not incorporate contemporaneous monetary policy shocks into their time t decisions, similarly to Christiano, Eichenbaum and Evans (2005). Consumers and price-setting firms form their time t expectations based on monetary policy shocks up to time $t-1$ and IS, PC and inflation target shocks up to time t. We denote the expectation with respect to this information set by:

$$\mathrm{E}_{t-}(\cdot) = \mathrm{E}(\cdot | u_t^{IS}, u_{t-1}^{IS}, u_{t-2}^{IS}, ..., u_t^{PC}, u_{t-1}^{PC}, u_{t-2}^{PC}, ..., u_{t-1}^{MP}, u_{t-2}^{MP}, u_t^{*}, u_{t-1}^{*}, ...). \tag{11}$$

The assumption that consumers and firms make decisions based on E_{t-} expectations implies that monetary policy shocks do not affect macroeconomic aggregates contemporaneously, but only with a lag. This identification assumption is common in the structural VAR literature (Christiano, Eichenbaum, and Evans, 1999) and it is helpful for our empirical strategy in that we can estimate the monetary policy Taylor rule by OLS.

The dynamics of the output gap, inflation, and Fed Funds rate can then be summarized by the linearized system of equations:

$$\begin{aligned}
x_t &= \rho^{x-} x_{t-1} + \rho^{x+} \mathrm{E}_{t-} x_{t+1} - \psi(\mathrm{E}_{t-} i_t - \mathrm{E}_{t-} \pi_{t+1}) + u_t^{IS}, && (12)\\
\pi_t &= \rho^{\pi} \pi_{t-1} + (1-\rho^{\pi}) \mathrm{E}_{t-} \pi_{t+1} + \lambda x_t + u_t^{PC}, && (13)\\
i_t &= \rho^{i}(i_{t-1} - \pi^*_{t-1}) + (1-\rho^{i}) \left[\gamma^x x_t + \gamma^{\pi} (\pi_t - \pi^*_t)\right] + \pi^*_t + u_t^{MP}, && (14)\\
\pi^*_t &= \pi^*_{t-1} + u^*_t. && (15)
\end{aligned}$$

Equation (12) is the IS curve (10) with the expectational timing assumption (11). Equation (13) is a standard New Keynesian equation that determines inflation from the price-setting behavior of firms. It has parameters ρ^{π}, determining the relative weight on past inflation and expected future inflation, and λ, governing the sensitivity of inflation to the output gap.

Equation (14) is a central bank reaction function along the lines of Clarida, Gali, and Gertler (1999), Taylor (1993), and Woodford (2001). It determines the short-term nominal interest rate with parameters ρ^{i}, controlling the influence of past interest rates on current interest rates, γ^{x}, governing the reaction of the interest rate to the output gap, and γ^{π}, governing the response of the interest rate to inflation relative to its target level π^*_t.

Equation (15) specifies that the central bank's inflation target follows a random walk.

Our monetary policy specification does not explicitly depend on nominal long-term yields. However, a persistent inflation target shifts the term structure similarly to a level factor. In that sense, our model is similar to models where the level factor of the nominal term structure directly enters the central bank's monetary policy function (Rudebusch and Wu 2007, 2008).

Finally, we assume that the vector of shocks

$$u_t = [u_t^{IS}, u_t^{PC}, u_t^{MP}, u_t^{*}]' \tag{16}$$

is independently and conditionally normally distributed with mean zero and variance-covariance matrix:

$$\mathrm{E}_{t-1}\left[u_t u_t'\right] = \Sigma_u \times (1 - bx_{t-1}) = \begin{bmatrix} (\overline{\sigma}^{IS})^2 & 0 & 0 & 0 \\ 0 & (\overline{\sigma}^{PC})^2 & 0 & 0 \\ 0 & 0 & (\overline{\sigma}^{MP})^2 & 0 \\ 0 & 0 & 0 & (\overline{\sigma}^{*})^2 \end{bmatrix} \times (1 - bx_{t-1})\,. \tag{17}$$

Equation (17) has two important properties. First, the variances of all shocks in the model, not just the shock to the Euler equation, are proportional to $(1 - bx_{t-1})$, and thus linear in the output gap. This proportionality assumption makes the model relatively tractable and helps us fit the volatilities of bond and stock returns. Second, for parsimony we assume that all the shocks in the model are uncorrelated with each other. The assumption that monetary policy shocks u_t^{MP} and u_t^{*} are uncorrelated with the IS

and PC shocks captures the notion that all systematic variation in the short-term nominal interest rate is reflected in the monetary policy rule.

2.3 Modeling bonds and stocks

We use the exact loglinear framework of Campbell and Ammer (1993) to express excess log returns on nominal and real bonds as a function of changes in expectations of future short-term interest rates, inflation, and risk premia. In our model, risk premia vary over time and the expectations hypothesis of the term structure of interest rates does not hold. We maintain our previous simplifying approximation that risk premia on one period nominal bonds equal zero, but risk premia on longer-term bonds are allowed to vary.

We write $r_{n-1,t+1}$ for the real one-period log return on a real n-period bond from time t to time t+1 and $xr_{n-1,t+1}$ for the corresponding return in excess of r_t. $r^{\$}_{n-1,t+1}$ denotes the nominal one-period return on a similar nominal bond and $xr^{\$}_{n-1,t+1}$ the corresponding excess return over i_t. We use the identities:

$$\begin{aligned} r^{\$}_{n-1,t+1} - \mathrm{E}_t r^{\$}_{n-1,t+1} &= -\left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{n-1} \left(\hat{\imath}_{t+j} + \pi^*_{t+j}\right) \\ &\quad - \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{n-1} xr^{\$}_{n-j-1,t+1+j} \qquad (18) \\ r_{n-1,t+1} - \mathrm{E}_t r_{n-1,t+1} &= -\left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{n-1} r_{t+j} \\ &\quad - \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{n-1} xr_{n-j-1,t+1+j}, \qquad (19) \end{aligned}$$

Nominal bond excess returns reflect shocks to the long-term inflation target, news about the nominal interest rate gap, and news about future nominal bond excess returns. Real long-term excess bond returns reflect news about the real interest rate gap and news about future real bond excess returns.

We model stocks as a levered claim on the log output gap x_t. We assume that log dividends are given by:

$$d_t = \delta x_t. \tag{20}$$

We interpret δ as capturing a broad concept of leverage, including operational leverage. The interpretation of dividends as a levered claim on the underlying fundamental process is common in the asset pricing literature (Abel 1990, Campbell 1986, 2003).

We write r^e_{t+1} for the log stock return and xr^e_{t+1} for the log stock return in excess of r_t. Following Campbell (1991) we use a loglinear approximation to decompose stock returns into dividend news, news about real interest rates, and news about future excess stock returns ignoring constants:

$$\begin{aligned} r^e_{t+1} - E_t r^e_{t+1} &= \delta \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=0}^{\infty} \rho^j \Delta x_{t+1+j} - \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{\infty} \rho^j r_{t+j} \\ &\quad - \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{\infty} \rho^j xr^e_{t+1+j}. \end{aligned} \tag{21}$$

Here ρ is a loglinearization constant close to 1.

2.4 Model solution and stability

We define the inflation and nominal interest rate gaps as:

$$\hat{\pi}_t = \pi_t - \pi_t^*, \tag{22}$$

$$\hat{\imath}_t = i_t - \pi_t^*. \tag{23}$$

We solve for the dynamics of the vector of state variables

$$\hat{Y}_t = [x_t, \hat{\pi}_t, \hat{\imath}_t]'. \tag{24}$$

The state variable dynamics have a solution of the form

$$\hat{Y}_t = P\hat{Y}_{t-1} + Qu_t. \tag{25}$$

We solve for $P \in \mathbb{R}^{3\times 3}$ and $Q \in \mathbb{R}^{3\times 4}$ using the method of generalized eigenvectors (see e.g. Uhlig 1999).

In principle, the model can have more than one solution. We only consider dynamically stable solutions with all eigenvalues of P less than one in absolute value, yielding non-explosive solutions for the output gap, inflation gap, and interest rate gap. Cochrane (2011) argues that there is no economic rationale for ruling out solutions on the basis of an explosive inflation path. In general, in our model an explosive solution for inflation is also explosive for the output gap and the real interest rate. We find it reasonable to rule out such solutions with explosive real dynamics.

The inclusion of backward-looking terms in the IS curve and Phillips curve means that there exist at most a finite number of dynamically stable equilibria of the form (25). This is true even when the monetary policy reaction to inflation (γ^{π}) is smaller than one, which usually leads to an indeterminate equilibrium in highly stylized Keynesian models with only forward-looking components (Cochrane, 2011).

We apply multiple equilibrium selection criteria proposed in the literature to rule out unreasonable solutions. Whenever there exist multiple dynamically stable solutions, these additional criteria allow us to pick a unique solution. We require the solution to be 'expectationally stable' (Evans 1985, 1986, McCallum 2003). Expectational stability requires that for small deviations from rational expectations, the system returns to the equilibrium. We also impose the forward criterion of Cho and Moreno (2011), which requires that expectations about shocks arbitrarily far in the future do not affect the current equilibrium. Finally, we also impose the solution selection criterion of Uhlig (1999), which is closely related to the minimum state variable solution proposed by McCallum (2004). We only consider solutions that are real-valued and have finite entries for Q. The Appendix provides full details on the model solution and solution criteria.

2.4.1 Stochastic discount factor

We can express innovations to log consumption plus habit as:

$$\begin{aligned} s_{t+1} + c_{t+1} - E_t\left(s_{t+1} + c_{t+1}\right) &= Q^M u_{t+1}, && (26) \\ Q^M &= e_1 Q - (1+\theta^*) e_1. && (27) \end{aligned}$$

In (9) we assumed that the conditional variance of the log SDF is linear in the output gap. Using equation (17) for the changing variance of the shock vector u_{t+1}, we can now verify this assumption with:

$$\sigma_t^2 = Q^M \Sigma_u Q^{M\prime}(1 - bx_t). \tag{28}$$

The variance of the SDF conditional on a zero output gap is $\overline{\sigma}^2 = Q^M \Sigma_u Q^{M\prime}$.

2.4.2 Solutions for bond and stock returns

We obtain solutions for unexpected nominal and real bond returns of the form:

$$r^{\$}_{n-1,t+1} - E_t r^{\$}_{n-1,t+1} = A^{\$,n} u_{t+1}, \tag{29}$$

$$r_{n-1,t+1} - E_t r_{n-1,t+1} = A^n u_{t+1}. \tag{30}$$

Up to constant terms, log yields of nominal and real zero coupon bonds equal:

$$y^{\$}_{n,t} = \pi^*_t + B^{\$,n} \hat{Y}_t, \tag{31}$$

$$y_{n,t} = B^n \hat{Y}_t. \tag{32}$$

The loglinear decompositions (18) and (19) for nominal and real bonds are exact, so the solutions for model bond returns and yields (29), (30), (31), and (32) are also exact, conditional on our loglinearization of the stochastic discount factor. The vectors $A^{\$,n} \in \mathbb{R}^{1\times 4}$, $A^n \in \mathbb{R}^{1\times 4}$, $B^{\$,n} \in \mathbb{R}^{1\times 3}$, and $B^n \in \mathbb{R}^{1\times 3}$ are defined recursively.

We obtain that unexpected and expected log excess stock returns take the following loglinear approximate forms:

$$r_{t+1}^{e} - E_t r_{t+1}^{e} = A^{e} u_{t+1}, \tag{33}$$

$$E_t x r_{t+1}^{e} = (1 - b x_t) b^{e}, \tag{34}$$

for some $A^e \in \mathbb{R}^{1\times 4}$ and some $b^e \in \mathbb{R}$. The online Appendix presents solution details, including bond betas, log dividend price ratios, and multi-period expected log equity returns in excess of short-term T-bills.

2.4.3 An estimable VAR

While standard empirical measures are available for the output gap, we do not observe the interest rate and inflation gaps. We therefore cannot directly estimate the recursive law of motion (25). However, for a long-term bond maturity n, we can estimate a VAR(1) in the vector:

$$Y_t = \left[x_t, \pi_t, i_t, y_{n,t}^{\$}\right]'. \tag{35}$$

The model implies that:

$$Y_{t+1} = P^{Y} Y_t + Q^{Y} u_{t+1}^{Y}. \tag{36}$$

Here, $u_t^Y = u_t$ and P^Y and Q^Y are determined by P, Q and the loadings for long-term nominal bond yields $B^{\$,n}$.

3 Preliminary Empirical Analysis

3.1 Monetary policy regimes

We break our sample into three subperiods corresponding to major shifts in monetary policy. The first subperiod, 1960.Q1–1979.Q2, covers the Fed chairmanships of William M. Martin, Arthur Burns, and G. William Miller. The second subperiod, 1979.Q3–1996.Q4, covers the Fed chairmanships of Paul Volcker and Alan Greenspan. The third subperiod, 1997.Q1-2011.Q4, covers the period after Alan Greenspan's "Central Banking in a Democratic Society" speech at the end of 1996.Q4 (Greenspan 1996). The third period contains the later part of Greenspan's chairmanship and the earlier part of Ben Bernanke's chairmanship.

Our first two subperiods are identical to those considered by Clarida, Gali, and Gertler (CGG, 1999), while our third covers data that has become available since. Following CGG, we assume that transitions from one regime to another are structural breaks, completely unanticipated by investors. This approach is motivated by the empirical observation that regimes in the nominal bond beta and monetary policy regimes are typically long-lasting on the order of one to two decades. While we recognize the importance of allowing agents to anticipate potential future changes in policy and to optimize according to such expectations, we think that a parsimonious model like ours still brings substantive insights which are likely to survive in a more sophisticated but less analytically tractable model.

Our choice of a third regime for monetary policy draws on several observations. First, in 1994 the Federal Reserve started to publish the transcripts of FOMC meetings shortly after each meeting. Observers of the Federal Reserve have noted that this increased transparency opened Fed deliberations to intense scrutiny by investors and the public. Consequently, increased transparency may have led to changes in the conduct of monetary policy towards more gradualism, with the central bank deciding on smaller interest rate changes, adopting more cautious policies, and implementing them over longer time horizons.

Second, Federal Reserve governors and chairmen have repeatedly noted a sense of increased uncertainty about the effects of monetary policy since the mid-1990s. Anecdotal evidence suggests that this led the Federal Reserve to adopt a more persistent monetary policy (Greenspan 1996, Bernanke 2004, Orphanides 2003).[6]

Third, observers of the Federal Reserve have interpreted Greenspan's 1996 speech as a signal of increased central bank concern with U.S. and international capital market conditions. In fact, this speech is popularly known as the "Irrational Exuberance" speech. These considerations might also have pushed towards transparency and gradualism to the extent that the Federal Reserve aims to mitigate short-term bond return volatility.

It is tempting to conclude that asset market considerations support a framework with both the output gap and stock returns—or some measure of asset valuations—in the monetary policy rule. In contrast, Greenspan (1996) argues that "central bankers do not need to be concerned if a collapsing asset bubble does not threaten to impair the real economy."Both anecdotal evidence and the empirical evidence in Rigobon and Sack (2003) are

[6]Greenspan (1996): "At different times in our history a varying set of simple indicators seemed successfully to summarize the state of monetary policy and its relationship to the economy. (...) Unfortunately, money supply trends veered off path several years ago as a useful summary of the overall economy." Bernanke (2004): "As a general rule, the Federal Reserve tends to adjust interest rates incrementally, in a series of small or moderate steps in the same direction. (...) Many central bankers and researchers have pointed to the pervasive uncertainty associated with analyzing and forecasting the economy as a reason for central bank caution in adjusting policy." See also the analysis in Stein (2013).

therefore consistent with our monetary policy rule that incorporates asset prices only to the extent that they reflect inflation and output fluctuations.

Finally, Figure 1 in the introduction shows that substantial changes in the sign and magnitude of nominal bond betas and bond return volatilities line up reasonably well with our proposed monetary policy regimes. Our estimates of the monetary policy rule shown below also provide robust empirical support for the existence of a third monetary policy regime.

3.2 Data and summary statistics

Our empirical analysis uses quarterly US data on output, inflation, interest rates, and aggregate bond and stock returns from of 1960.Q1 to 2011.Q4. GDP in 2005 chained dollars and the GDP deflator are from the Bureau of Economic Analysis via the Fred database at the St. Louis Federal Reserve. The end-of-quarter Federal Funds rate is from the Federal Reserve's H.15 publication. We use quarterly potential GDP in 2005 chained dollars from the Congressional Budget Office.[7] The end-of-quarter three-month T-bill is from the CRSP monthly Treasury Fama risk free rates files. We use log yields based on average of bid and ask quotes. The end-of-quarter five year bond yield is from the CRSP monthly Treasury Fama-Bliss discount bond yields. We use the value-weighted combined NYSE/AMEX/Nasdaq stock return including dividends from CRSP, and measure the dividend-price ratio using data for real dividends and the S&P 500 real price.[8] Interest rates, and inflation are in annualized percent, while the log output gap is in natural

[7]Table 2-3 of the CBO's August 2012 report "An Update to the Budget and Economic Outlook: Fiscal Years 2012 to 2022" (http://www.cbo.gov/publication/43541)

[8]The source is Robert Shiller's website at http://www.econ.yale.edu/ shiller/data.htm.

percent units. All yields are continuously compounded. We consider log returns in excess of the log T-bill rate.

Table 1 shows summary statistics for the log output gap, inflation, the Federal Funds rate, and the 5 year nominal bond yield for the US over the full sample period 1960-2011 and over each subperiod. The log real output gap has a first-order quarterly autocorrelation of 0.96 over the full sample period, implying a half life of 5 years. Realized inflation, the Fed Funds rate and the 5-year nominal bond yield are also highly persistent in the full sample and across subperiods. The average log output gap was positive in the earliest subperiod, and negative afterwards. Inflation and interest rates have been significantly lower in the latest subperiod compared to the early subperiods.

In our model, expected excess log stock returns vary negatively with the output gap. It is therefore important to verify empirically if this relation exists, and to examine the relation of the output gap with well known predictors of excess stock returns in the data such as the price-dividend ratio.

Figure 2, Panel B shows the log output gap and the log price-dividend ratio for the full sample period. The correlation between the two variables in Figure 3 is 0.18 for the full sample period, but it is 0.39 for 1960.Q1–1979.Q3, 0.41 for 1979.Q3–1996.Q4, and 0.74 for 1997.Q1–2011.Q4. The procyclicality of the price-dividend ratio is evident in Figure 2. Occasional long-lasting shifts in the relative levels of the two variables, particularly the secular increase in the price-dividend ratio during the bull market at the end of the 20th Century, decrease the full sample correlation relative to the subsample correlations.

Time-varying volatilities of shocks imply that the equity premium in our model varies inversely with the output gap. We wish to compare time-variation in empirical and model risk premia. Table 2 estimates the predictive relation between quarterly equity excess returns and the output gap:

$$r^e_{t+1} - i_t = a^0 + a^x x_t + \epsilon_{t+1}. \tag{37}$$

Table 2 shows that the full sample estimate of a^x is negative and significant, consistent with our model specification. Subsample estimates vary around the full sample estimate of $a^x = -0.49$.

We also consider the log dividend-price ratio (the negative of the log price-dividend ratio) as a predictor of equity excess returns. Table 3 reports regressions of one through five year log equity excess returns onto the lagged log dividend-price ratio. We report regressions over our full sample period 1960-2011 and an extended sample period 1947-2011. Table 3 shows that the log dividend-price ratio predicts equity excess returns with positive coefficients. The coefficients in Table 3 are comparable to those reported in Campbell and Cochrane (1999), but smaller in magnitude due to our inclusion of more recent data.

3.3 Estimating monetary policy rules

The central bank's Taylor rule parameters are key inputs for calibrating the model for each subperiod. The model incorporates exclusion restrictions, such that if we knew the output gap, the inflation gap, and the interest rate gap we could estimate the monetary policy function by OLS. Unfortunately, the inflation gap and interest rate gap are not directly observable. We therefore follow CGG in estimating the monetary policy rule in terms of the output gap, inflation, and the Fed Funds rate:

$$i_t = c^0 + c^x x_t + c^\pi \pi_t + c^i i_{t-1} + \epsilon_t.$$

We use the estimated values $\hat{c}^x$, $\hat{c}^\pi$, and $\hat{c}^i$ to pin down the calibrated values of the monetary policy parameters according to: $\hat{\rho}^i = \hat{c}^i$, $\hat{\gamma}^x = \hat{c}^x/(1-\hat{c}^i)$, and $\hat{\gamma}^\pi = \hat{c}^\pi/(1-\hat{c}^i)$.

The estimated monetary policy functions in Table 4 yield consistent estimates of the monetary policy parameters only if the inflation target is constant or contemporaneously uncorrelated with the output gap and inflation gaps. We will therefore need to verify that this bias is quantitatively small in the calibrated model.

The estimates in Table 4 suggest that monetary policy has varied substantially over time. During the earliest subperiod, 1960.Q1–1979.Q2, the central bank raised nominal interest rates less than one-for-one with inflation. In contrast, the central bank raised nominal interest rates more than one-for-one with inflation during the Volcker-Greenspan-Bernanke periods (1979.Q3–2011.Q4).[9] This finding is consistent with the empirical evidence reported by CGG and updates it to the most recent period.

The point estimates of $\hat{\gamma}^x$ in Table 4 also suggest that the central bank has put somewhat higher weight on output fluctuations in the earliest and latest subperiods than during the middle subperiod, although the estimates of neither $\hat{\gamma}^x$ nor $\hat{c}^x$ are statistically significant in the latest subperiod. This empirical finding is similar to Hamilton, Pruitt, and Borger (2011) who use the reaction of Fed Funds futures to macroeconomic announcements to

[9] $\hat{\gamma}^\pi$ appears to be less precisely estimated in the latest subperiod, while $\hat{c}^\pi$ is precisely estimated. This results from the nonlinear relation between the parameters. The coefficient $\hat{\gamma}^\pi$ is $\hat{c}^\pi$ divided by $(1-\hat{c}^i)$. Because $\hat{c}^i$ is very close to 1 in the latest subperiod, standard errors for $\hat{\gamma}^\pi$ based on the delta method tend to be very large.

estimate monetary policy rules before and after 2000.

Consistent with our informal observations about increased gradualism in monetary policy, during the most recent period the monetary policy rule explains 91% of the variation in the Federal Funds rate, implying that deviations from the monetary policy rule have been extremely small. Moreover, the loading of the lagged Fed Funds rate into the monetary policy function during this subperiod is very large at 0.89, almost twice as large as in the earlier subperiods. We will see that this increase in estimated policy persistence is important for understanding changing bond risks.

The increase in monetary policy persistence in the third subperiod is not driven by the onset of the global financial crisis or the subsequently binding zero lower bound for the nominal interest rate. The Appendix shows estimated monetary policy rules for two parts of the third subperiod, before and after the start of the financial crisis, which we take to be the third quarter of 2008. Not surprisingly, none of the estimated coefficients of the monetary policy rule are statistically different from zero during the post-crisis period. But more interestingly, the pre-crisis period still shows very strong persistence in monetary policy.

4 Model Calibration

We now calibrate our model to key empirical moments for the US over the three subsamples: 1960.Q1-1979.Q2, 1979.Q3-1996.Q4, and 1997.Q1-2011.Q4. Table 5 summarizes the calibration parameters, while Tables 6 and 7 compare key empirical and model moments.

We separate the parameters into two blocks. The first block of parameters corresponds to our main candidate explanations for changes in bond betas. It comprises the monetary policy rule parameters γ^x, γ^π, and ρ^i and the shock volatilities $\bar{\sigma}^{IS}$, $\bar{\sigma}^{PC}$, $\bar{\sigma}^{MP}$, and $\bar{\sigma}^*$. We allow these parameters to change over time. We use Table 4 to pin down the monetary policy parameters γ^x, γ^π, and ρ^i for each subperiod. The second block of time-invariant parameters comprises the preference parameters and the Phillips curve parameters (ρ, δ, α, ρ^π, ρ^{x+}, ρ^{x-}, and λ).

Previous evidence from Smets and Wouters (2007) supports our selection of parameter blocks. They estimate a structural New Keynesian model separately for the periods 1966-1979 and 1984-2004 and find that preference parameters are largely stable across subperiods. In their estimation, the most important parameter changes across those two subperiods are in the shock volatilities and the monetary policy parameters, which are exactly the parameters that we allow to vary across subperiods.[10]

We set the Phillips curve slope to $\lambda = 0.30$ following CGG. We set the backward-looking component of the Phillips curve to $\rho^\pi = 0.80$ following the empirical estimates of Fuhrer (1997). Thus we model the Phillips curve as strongly backward looking, consistent with estimates obtained using inflation and the output gap. Gali and Gertler (1999) find some empirical evidence in favor of a forward-looking curve using the labor share of income instead of the output gap. The Appendix shows that our results are not sensitive to the precise value of the backward-looking parameter in the Phillips curve as long as this parameter is not too close to zero, in which case there may not be a solution to our model.

[10] They also find some evidence that the Phillips curve flattens in their second subperiod. For simplicity we keep the parameters of the Phillips curve stable across subperiods, but we verify in the Appendix that in our model a flatter Phillips curve leads to a more positive bond beta in the second and third periods. Allowing for a flattening of the curve over time would therefore deepen rather than resolve the puzzle why the nominal bond beta has been negative since the late 1990s.

Over the full sample period, the standard deviations of four-quarter real dividend growth and four-quarter output gap growth are 5.35% and 2.20%, respectively. We use the ratio of these empirical standard deviations to pin down the model leverage parameter at $\delta = 2.43$, corresponding to a leverage ratio of 59%. We interpret δ as capturing a broad concept of leverage, including operational leverage. We choose the loglinearization parameter ρ as in Campbell and Ammer (1993), scaled to quarterly frequency. We set the preference parameter to $\alpha = 30$ to generate plausible equity return volatility.

We choose the remaining parameters to fit a large number of moments. Let superscript p denote subperiod parameters. The remaining parameters, ρ^{x+}, ρ^{x-}, $\sigma^{IS,p}$, $\sigma^{PC,p}$, $\sigma^{MP,p}$, and $\sigma^{*,p}$ for $p = 1, 2, 3$, are chosen to minimize the distance between model and empirical moments. We fit the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta, a total of 23 moments.[11]

The standard deviations of shocks, evaluated at a zero value of the output gap, change considerably across time periods. We estimate substantially larger volatilities of MP shocks and inflation target shocks for the period 1979–1996 than for the earliest and latest subperiods. The estimated volatility of PC shocks is largest in the earliest subperiod, and the estimated volatility of IS shocks is smallest in the most recent subperiod.

Table 5, Panel B shows the values of calibration parameters that are implied by the parameters shown in Panel A. The calibrated Euler equation has economically significant

[11] The objective function is the sum of squared differences between model and empirical moments summed over all three sub-periods. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes. The online Appendix describes details of the optimization procedure.

forward-looking and backward-looking components with a backward-looking component of 0.45 and a forward-looking component of 0.62. The forward- and backward-looking Euler equation components sum to more than one as a result of time-varying risk premia. The parameters imply $\theta = \rho^{x-}/\rho^{x+} = 0.73$, so investors' habit moves less than one for one with the lagged output gap.

The slope of the IS curve ψ with respect to the real interest rate is linked to the curvature parameter controlling risk aversion α and ρ^{x+} by $\psi = \rho^{x+}/\alpha$, as in Section 2.1. The implied slope of the IS curve is 0.02. This value is close to zero, but is in line with the empirical findings in Yogo (2004) and earlier work by Hall (1988).[12] Our calibration procedure requires a small value for ψ to match empirical moments, especially the substantial persistence in the empirical output gap. The parameters that control the volatility of the SDF, b and $\overline{\sigma}$, change little across subperiods.

Table 6 shows calibrated and empirical volatilities of VAR(1) residuals, volatilities of stock and bond returns, nominal bond betas, and Taylor rule regressions of the same form as in Table 4. Model moments are calculated from 2000 simulations of length 250, corresponding closely to our empirical sample size of 61 years of quarterly observations.

The calibrated model provides a close fit for the standard deviations of VAR(1) residuals and stock and bond return volatilities for each of the subperiods. Moreover, the calibrated model fits well the time variation in the nominal bond beta. Both empirical and model nominal bond betas were small but positive in the first subperiod, larger and positive in the second subperiod, and negative during the third subperiod. These changes in bond risks are the primary object of interest in our analysis.

[12] However the long-run risk literature, following Bansal and Yaron (2004), presents an opposing view.

The model's simulated Taylor rules also correspond closely to their empirical counterparts for each subperiod. This finding is reassuring in that it suggests that we can indeed identify time-varying monetary policy parameters from the regressions reported in Table 4 even without estimating the unobservable inflation target.

Our calibration generates additional model moments not used in the fitting procedure that can be used to evaluate the model's out-of-sample performance. Table 7 shows that many of these moments are comparable to their empirical counterparts. The calibration generates a positive and plausible correlation between the output gap and the log price-dividend ratio, especially given that empirical subperiod correlations between these two variables tend to exceed full sample correlations. The log dividend-price ratio is persistent, but less persistent and less volatile than in the data.

Table 7 shows that the model output gap and log dividend price ratio both predict stock excess returns with the right signs, even if the model generates stronger return predictability than one can estimate in the data. However, we would expect to find weaker evidence for predictability in the data to the extent that the observed output gap and the dividend-yield are noisy measures of the true variables. Finally, the model real short-term rate has a low average standard deviation of 1.90%. The coefficient from regressing $s_t + c_t$ onto the output gap x_t is positive, consistent with the intuition that marginal utility should be low when the output gap is high.

5 Counterfactual Analysis of Changing Bond Risks

We are now in a position to investigate the role of changing monetary policy and macroeconomic shocks for nominal bond betas. Our calibrated model replicates the shift in nominal bond betas over time. Since we allow both monetary policy and the volatilities of shocks to vary across subperiods, both may contribute to time-varying bond risks to varying degrees.

Figure 3 plots nominal bond betas against the monetary policy reaction coefficients γ^x and γ^π. γ^π captures the long-run response of monetary policy to an increase in the inflation gap, while γ^x captures the response to an increase in the output gap. Each panel corresponds to one subperiod. The following parameters vary across panels and equal the calibration values for the respective subperiod: the persistence of monetary policy ρ^i, and the volatilities of shocks $\bar{\sigma}^{IS}$, $\bar{\sigma}^{PC}$, $\bar{\sigma}^{MP}$, and $\bar{\sigma}^*$. Red and orange colors indicate positive bond betas, while blue and green colors indicate negative bond betas. White areas indicate that no stable solution exists. We show the estimated combinations of monetary policy parameters for each subperiod.

The contour lines in Figure 3 show combinations of reaction coefficients γ^x and γ^π that keep bond betas fixed. These contours are upward-sloping in all three panels, showing that γ^π increases the bond beta while γ^x reduces it. However, the contour lines are relatively flat, which shows that the effect of γ^π on the bond beta is much larger than the effect of γ^x. In addition, the contours shift across Panels A through C, indicating that changes in parameters other than γ^x and γ^π are important for matching changes in bond betas.

Turning to specific results for our three subsamples, Figure 3, Panel A shows that nominal bond betas are positive for a wide range of monetary policy reaction coefficients

γ^x and γ^π in the presence of period 1 shock volatilities and period 1 monetary policy persistence. A monetary policy rule with a higher weight on stabilizing inflation γ^π, such as those estimated for our second and third subsamples, would have produced even more positive bond betas than were actually observed during 1960.Q1–1979.Q2.

Panel B shows that in the presence of period 2 shocks and monetary policy persistence, the sign of the nominal bond beta is highly sensitive to the central bank's weight on inflation stabilization γ^π. The central bank's strong emphasis on inflation stabilization during 1979.Q3–1996.Q4 is reflected in the strongly positive nominal bond beta during this period. This emphasis continued in period 3, so in the presence of period 2 shock volatilities and period 2 monetary policy persistence, period 3 monetary policy reaction coefficients would have produced a positive nominal beta, and not the negative beta actually observed.

Panel C indicates that the negative nominal bond beta since the late 1990s cannot be attributed to a change in the monetary policy reaction coefficients γ^π or γ^x. In the presence of period 3 shock volatilities and period 3 monetary policy persistence, the nominal bond beta would have been negative even if the weights on output and inflation stabilization, γ^x and γ^π, had remained constant between periods 2 and 3. It would have been even more negative than actually observed if monetary reaction coefficients in period 3 had been those measured for period 1.

Given the secondary role for the output reaction coefficient γ^x in Figure 3, we replace this parameter with the monetary persistence parameter in Figure 4, which plots nominal bond betas against the monetary policy parameters γ^π and ρ^i. The contour lines in this figure are upward-sloping and convex, becoming extremely steep as monetary persistence approaches its maximum value of one. This shows that γ^π increases the bond beta while ρ^i

reduces it, and ρ^i has a nonlinear effect that is stronger at high levels of persistence. This is important given the increase in persistence we have estimated for our final subperiod.

Figure 4 indicates that strong monetary policy persistence during the most recent subperiod has helped generate a negative nominal bond beta. Moreover, a move to third-period monetary policy persistence would have acted to decrease the nominal bond beta for all subperiod calibrations. However, a move to third-period monetary policy persistence and weight on inflation stabilization would not have been sufficient to make the bond beta negative given the other parameters estimated for the first subperiod.

We can also use Figures 3 and 4 to understand model implications for nominal bond betas when the nominal interest rate approaches the zero lower bound. The figures indicate that bond betas tend to be negative when the monetary policy coefficients approach zero, consistent with the empirical behavior of bond betas since the financial crisis.

5.1 Impulse responses

To develop further insight into the mechanisms of the model that produce the patterns illustrated in Figures 3 and 4, we now report impulse response functions implied by the model in each subperiod. Figure 5 shows dynamic responses for macroeconomic variables and asset prices following one-standard-deviation increases in each of the fundamental model shocks in period 1 starting from zero. Each panel shows three lines, each line corresponding to one subperiod calibration. We consider the responses of the output gap, the inflation gap, the nominal and real interest rates, the dividend-price ratio, and the nominal long-term yield to shocks.

An IS shock has qualitatively similar effects across all three calibrations. Higher expected inflation raises the long-term nominal yield, while slow real growth following an initial increase in the output gap pushes up the dividend-price ratio. Stock prices are inversely related to the dividend-price ratio and bond prices are inversely related to bond yields, so IS shocks tend to generate positive comovement between bond and stock returns.

A PC shock acts as an inflationary supply shock, lowering the output gap and raising inflation for all three subperiod calibrations. However, the responses of the short-term interest rate, the dividend-price ratio, and the nominal long-term yield vary across calibrations. The dividend-price ratio and the nominal long-term yield move in the same direction for periods 1 and 2, contributing to positive nominal bond betas for those periods. In contrast, for the period 3 calibration a PC shock moves the dividend-price ratio and the long-term nominal yield in opposite directions and contributes to the negative nominal bond beta for that period.

For subperiod 3, nominal bond yields fall in response to PC shocks for two reasons. First, the greater persistence of period 3 monetary policy decreases the immediate response in the Federal Funds rate, muting any immediate effect on longer-term nominal yields. Second, nominal bonds have a negative beta in period 3, implying that their risk premia are especially low in times of high aggregate risk. The recession following a PC shock increases the safety appeal of bonds, further lowering nominal bond yields and amplifying the negative bond beta. This is an example of an important amplification mechanism in our model.

A monetary policy (MP) shock has very small effects on both the dividend price ratio and the nominal yield. Intuitively, our calibration incorporates a small slope of the IS curve, so transitory monetary policy shocks have little effect

A shock to the central bank's inflation target has much more pronounced effects on inflation and nominal interest rates. An inflation target shock leads to permanent increases in inflation and nominal short-term and long-term yields. In the transition period, inflation is below the new target and this leads the Federal Reserve to lower real interest rates, boosting output. A positive inflation target shock acts similarly to a supply shock inducing firms to increase supply. Inflation target shocks therefore induce opposite movements in the dividend-price ratio and the long-term nominal yield, implying that inflation target shocks tend to generate a negative nominal bond beta.

Figure 5 illustrates why higher monetary policy persistence decreases the nominal bond beta. Intuitively, inflation target shocks contribute to negative bond betas and larger ρ^i amplifies the impact of inflation target shocks. When ρ^i is large, monetary policy operates more through expectations of future policy rates and less through contemporaneous policy rate adjustments. In the period 3 calibration, the central bank does not lower real interest rates immediately but only with a long lag after an inflation target shock. Therefore, an inflation target shock leads to an even stronger increase in nominal bond yields and a more negative bond beta in the period 3 calibration.

The effect of monetary policy persistence ρ^i on the nominal bond beta is compounded by the dynamic behavior of risk premia. If ρ^i is large, then bond returns are largely driven by inflation target shocks and are therefore countercyclical. Since nominal bond returns are then positively correlated with marginal utility, nominal bond yields incorporate low or even negative risk premia during recessions when volatility is high. This procyclicality of bond risk premia amplifies the countercyclicality of bond returns.

5.2 Marginal analysis

Table 8 analyzes the marginal effect of each parameter on the nominal bond beta, the volatility of nominal bond returns, and the volatility of equity returns across subperiods. We provide derivatives with respect to the MP parameters γ^x, γ^π and ρ^i, and also with respect to the log standard deviations of shocks—thus we can interpret the derivatives with respect to the standard deviations of shocks as semi-elasticities. All other parameters are held constant at their 1960.Q1-1979.Q2, 1979.Q3-1996.Q4, or 1997.Q1-2011.Q4 values.

We also decompose the same three asset pricing moments using partial semi-elasticities while holding constant the loadings of bond and stock returns onto the fundamental model shocks A^e and $A^{\$,n}$. These partial effects take account of bond and stock responses to each of the fundamental model shocks, similarly to the impulse responses shown in Figure 5, without taking account of alterations in responses caused by changing second moments (for example, the risk premium amplification mechanism discussed in the previous subsection).[13]

The top panel in Table 8 shows that the MP inflation coefficient γ^π tends to increase the nominal bond beta, while the MP output gap coefficient γ^x and MP persistence ρ^i tend to decrease the nominal bond beta. Monetary policy persistence has a strongly nonlinear effect on the nominal bond beta, and the magnitude of this effect is particularly large for the 1997.Q1–2011.Q4 calibration. The nominal bond beta increases in the PC shock volatility and decreases in the inflation target shock volatility. These signs are consistent

[13] The nominal bond beta partial semi-elasticity also holds constant the standard deviation of equity returns. Equivalently, this partial semi-elasticity captures the effect of shock volatilities on the covariance of bond and stock returns scaled by the inverse of a constant equity volatility. The nominal bond beta partial semi-elasticities sum to two times the calibrated nominal bond beta for each subperiod. The partial semi-elasticities for the standard deviations of asset returns sum to the calibrated standard deviation of asset returns for each subperiod. The online Appendix presents detailed formulas for semi-elasticities and partial semi-elasticities.

with the partial-semi elasticities but are substantially amplified by endogenous responses of risk premia. IS and MP shock volatilities have small and negative effects on the nominal bond beta.

The middle panel of Table 8 shows that the effect of parameters on bond return volatility is especially nonlinear, with total derivatives switching sign exactly when the nominal bond beta switches sign. But partial semi-elasticities with respect to shock volatilities, which ignore the endogenous responses of risk premia, are all positive. The partial semi-elasticity with respect to PC shock volatility is especially important in the middle subperiod, while the partial semi-elasticity with respect to inflation target shocks is especially important in the most recent subperiod.

The third panel of Table 8 looks at equity volatility. The volatility of the PC shock is clearly the single most important driver of equity volatility in the model, followed by the inflation target shock volatility.

Table 9 uses the semi-elasticities of Table 8 to decompose the changes in bond and equity risks. We report linear approximations of how much the change in each monetary policy parameter and in each shock volatility has contributed to changes in the nominal bond beta, nominal bond return volatility, and equity return volatility. Table 9 weights total derivatives as reported in Table 8 by the corresponding parameter change from one period to the next. Total derivatives can vary across subperiods and we therefore average total derivatives across lagged and led periods.

Table 9 also reports the total linearized change in bond and equity risks due to the combined change in monetary policy parameters γ^x, γ^π, ρ^i, $\bar{\sigma}^{MP}$, and $\bar{\sigma}^*$ and due to the combined changes in the volatilities of supply and demand shocks $\bar{\sigma}^{IS}$, and $\bar{\sigma}^{MP}$. Comparing the sum of linearized changes in bond and equity risks and the model-implied changes in bond and equity risks gives the change due to model nonlinearity. The linear effects of the individual parameters and the nonlinearity effect sum to the total model change reported at the top of Table 9.

Table 9 shows that the reduction in the central bank's response to output, γ^x, and the increase in the central bank's response to inflation, γ^π, were important contributors to the increases in the nominal bond beta and the volatility of bond returns that occurred in 1979. On the other hand, changes in the volatility of supply (PC) shocks and inflation target shocks acted to decrease the nominal bond beta and the volatility of bond returns at the 1979 regime change. Thus monetary policy changes and shock volatility changes offset each other to some degree in 1979.

In 1997, the increase in monetary policy persistence is most important for understanding the decline in the nominal bond beta, but the increase in the central bank's reaction to output also plays a role. Once again changes in the volatilities of macroeconomic shocks worked against these changes and offset them to some degree. The bottom of the table shows that nonlinear interaction effects, for example between the persistence of monetary policy and the volatility of inflation target shocks, are important at both regime changes but particularly so in 1997.

6 Implications for inflation-indexed bonds

Inflation-indexed bonds, or Treasury Inflation Protected Securities (TIPS), have been issued in the US since 1997. TIPS have since become a meaningful source of funding for the

U.S. Treasury and an important investment vehicle for institutional and retail investors (Campbell, Shiller, and Viceira 2009), which makes it interesting to understand monetary policy implications for the second moments of real bond returns. Since TIPS were not available during periods 1 and 2, we can only compare model and empirical real bond return moments for period 3.

Table 10 shows that in the third subperiod, when inflation-indexed bonds are available, the model-implied inflation-indexed bond beta is negative and quantitatively close to the empirical TIPS beta. We do not use TIPS data in our calibration procedure and TIPS moments therefore provide additional verification of the model's out-of-sample performance. The partial derivatives in Table 10 show that PC shocks are the main driver of the negative TIPS beta in this subperiod. Figure 5 shows that a PC shock increases inflation and depresses output, independently of the monetary policy regime. In the most recent monetary policy regime, the nominal rate is extremely sticky. An increase in inflation therefore leads to a drop in real interest rates and moves TIPS valuations and equity valuations in opposite directions. This mechanism would likely be even stronger for monetary policy at the zero lower bond, when nominal interest rates are constant.

Phillips curve shocks also explain why our model predicts that, had TIPS been issued in the second subperiod, they would have exhibited a positive beta. The strongly anti-inflationary monetary policy in the second subperiod implies that the central bank reacts to a Phillips curve shock by raising the real interest rate. This, in turn, leads to a fall in the price of TIPS at the same time that output and stock prices are low.

The 1960.Q1-1979.Q2 calibration of our model implies that if real bonds had existed during that period, they would have been extremely safe. We obtain extremely large values for the volatility of inflation-indexed bond returns, which is a result of real bond risk premia decreasing sharply and nonlinearly in the monetary policy parameter γ^{π}. However, we show in the Appendix that the real bond beta is negative for a wide region around the period 1 monetary policy parameters. The intuition why real bonds should have been safe during that period again hinges on Phillips curve shocks. In a regime, where the central bank raises the policy rate less than one-for-one with inflation, real bonds do well when the economy experiences a PC shock.[14]

7 Conclusion

Given the importance of nominal bonds in investment portfolios, and in the design and execution of fiscal and monetary policy, financial economists and macroeconomists need to understand the determinants of nominal bond risks. This is particularly challenging because the risk characteristics of nominal bonds are not stable over time.

This paper argues that understanding bond risks requires modeling the influence of monetary policy on the macroeconomy, particularly the relation between output and inflation, and understanding how macroeconomic supply and demand shocks and central bank responses to those shocks affect asset prices. We propose a model that integrates the building blocks of a New Keynesian model into an asset pricing framework in which risk and consequently risk premia can vary in response to macroeconomic conditions. We calibrate

[14] Given that the region around period 1 monetary policy parameters includes much more reasonable values for the real bond beta, we believe we could recalibrate our model relatively easily with a restriction that the implied volatility or beta of TIPS must fall within a certain range. Given that the model mechanism generally seems robust to small changes in parameters, we don't expect that this would change the model implications.

our model to US data between 1960 and 2011, a period in which macroeconomic conditions, monetary policy, and bond risks have experienced significant changes. We allow discrete regime changes just before the third quarter of 1979 and the first quarter of 1997.

Our model is sufficiently rich to allow for a detailed exploration of the monetary policy drivers of bond and equity risks. We find that two elements of monetary policy have been especially important drivers of bond risks during the last half century. First, a strong reaction of monetary policy to inflation shocks increases both the beta of nominal bonds and the volatility of nominal bond returns. Large increases in short-term nominal interest rates in response to inflation shocks tend to lower real output and stock prices, while causing bond prices to fall. Our model attributes the large positive beta and high volatility of nominal bonds after 1979 to a change in monetary policy towards a more anti-inflationary stance. Evidence of such a change has been reported by Clarida, Gali, and Gertler (1999) and other papers studying monetary policy regimes, but our model clarifies how this alters the behavior of the bond market.

Second, a monetary policy that smooths nominal interest rates over time implies that positive shocks to long-term target inflation cause real interest rates to fall, driving up output and equity prices, while increasing nominal long-term interest rates. This makes nominal bond returns countercyclical, implying a negative risk premium because nominal bonds hedge against deflationary recessions. Our model attributes the negative beta of nominal bonds since 1997 to a significant increase in the persistence of monetary policy—or a shift towards gradualism–together with continuing shocks to the central bank's inflation target. These inflation target shocks may be interpreted literally, as the result of shifting central bank preferences, or more broadly as the result of imperfect credibility in monetary policy (Orphanides and Williams 2004).

Our model implies that changes in the volatility of supply shocks, or shocks to the Phillips Curve, can also affect bond risks. Supply shocks move inflation and output in opposite directions, making bond returns procyclical. They also have a strong effect on the volatility of equity returns. However, we do not estimate large changes over time in the volatility of supply shocks and so our model does not attribute the historical changes in bond betas to this source.

We find that it is particularly important to take account of changing risk premia. Because macroeconomic volatility is countercyclical in our model, assets with positive betas have risk premia that increase in recessions, driving down their prices and further increasing their betas. Assets with negative betas, on the other hand, become even more desirable hedges during recessions; this increases their prices and makes their betas even more negative. Thus the dynamic responses of risk premia amplify sign changes in betas that originate in changes in monetary policy, and underline the importance of nonlinear effects in understanding the impact of changes in monetary policy and macroeconomic shocks on asset prices.

We show that the model generates empirically plausible inflation-indexed bond betas and volatilites for the most recent monetary policy regime, which is the only one of our regimes when the US Treasury issued inflation-indexed bonds. This is despite the fact that we did not use inflation-indexed bonds to fit the model.

Our analysis has several limitations that can be addressed in future research. First, since we use a New Keynesian model, the micro-foundations of our model are not as clear and detailed as is standard in the dynamic stochastic general equilibrium literature. We have little to say about the production side of the economy or the labor market. Second, our use of a habit-formation model shuts down the pricing of long-run risks that is the focus of a large literature following Bansal and Yaron (2004). Third, the regime shifts we consider are unanticipated, once-and-for-all events rather than stochastically recurring events whose probabilities are understood by market participants. Finally, we calibrate our model to US historical data but it will be valuable to extend this analysis to comparative international data on monetary policy in relation to bond and stock returns. Countries such as the UK, where inflation-indexed bonds have been issued for several decades, will provide particularly useful evidence on the comparative risks of real and nominal bonds, and their changes over time.

References

Abel, Andrew B., 1990, "Asset Prices under Habit Formation and Catching up with the Joneses", *American Economic Review Papers and Proceedings* 80, 38–42.

An, Sungbae, and Frank Schorfheide, 2007, "Bayesian Analysis of DSGE Models", *Econometric Reviews* 26, 113–172.

Andreasen, Martin M., 2012, "An Estimated DSGE Model: Explaining Variation in Nominal Term Premia, Real Term Premia, and Inflation Risk Premia", *European Economic Review* 56, 1656–1674.

Ang, Andrew, Jean Boivin, Sen Dong, and Rudy Loo-Kung, 2011, "Monetary Policy Shifts and the Term Structure", *Review of Economic Studies* 78, 429–457.

Ang, Andrew and Monika Piazzesi, 2003, "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables", *Journal of Monetary Economics* 50, 745–787.

Ang, Andrew, Sen Dong, and Monika Piazzesi, 2007, "No-Arbitrage Taylor Rules", unpublished paper, Columbia University and University of Chicago.

Ang, Andrew, and Maxim Ulrich, 2012, "Nominal Bonds, Real Bonds, and Equity", unpublished paper, Columbia University.

Backus, David, Mikhail Chernov, and Stanley E. Zin, 2013, "Identifying Taylor Rules in Macro-Finance Models", NBER Working Paper No. 19360.

Baele, Lieven, Geert Bekaert, and Koen Inghelbrecht, 2010, "The Determinants of Stock and Bond Return Comovements", *Review of Financial Studies* 23, 2374–2428.

Ball, Laurence and Stephen Cecchetti, 1990, "Inflation and Uncertainty at Long and Short Horizons", *Brookings Papers on Economic Activity* 1, 215-245.

Bansal, Ravi and Ivan Shaliastovich, 2013, "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets", *Review of Financial Studies* 26, 1–33.

Bansal, Ravi, and Amir Yaron, 2004, "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance* 59, 1481-1509.

Barsky, Robert B., 1989, "Why Don't the Prices of Stocks and Bonds Move Together?", *American Economic Review* 79, 1132–1145.

Bekaert, Geert, Seonghoon Cho, and Antonio Moreno, 2010, "New Keynesian Macroeconomics and the Term Structure", *Journal of Money, Credit, and Banking* 42, 33–62.

Bekaert, Geert, Eric Engstrom, and Steve Grenadier, 2010, "Stock and Bond Returns with Moody Investors", *Journal of Empirical Finance* 17, 867–894.

Bekaert, Geert, Eric Engstrom, and Yuhang Xing, 2009, "Risk, Uncertainty, and Asset Prices", *Journal of Financial Economics* 91, 59–82.

Bernanke, Ben S., 2004, "Gradualism," remarks by Governor Ben S. Bernanke at an economics luncheon co-sponsored by the Federal Reserve Bank of San Francisco (Seattle Branch) and the University of Washington, Seattle, Washington May 20, 2004. Available online at www.federalreserve.gov/boarddocs/speeches/2004/200405202/.

Bikbov, Ruslan and Mikhail Chernov, 2013, "Monetary Policy Regimes and the Term Structure of Interest Rates", *Journal of Econometrics* 174, 27–43.

Van Binsbergen, Jules H., Jesús Fernández-Villaverde, Ralph Koijen, and Juan Rubio-Ramírez, 2012, "The Term Structure of Interest Rates in a DSGE Model with Recursive Preferences", *Journal of Monetary Economics* 59, 634–648.

Burkhardt, Dominic and Henrik Hasseltoft, 2012, "Understanding Asset Correlations", unpublished paper, University of Zurich and Swiss Finance Institute.

Boivin, Jean, and Marc P. Giannoni, 2006, "Has Monetary Policy Become More Effective?", *Review of Economics and Statistics* 88, 445–462.

Buraschi, Andrea and Alexei Jiltsov, 2005, "Inflation Risk Premia and the Expectations Hypothesis", *Journal of Financial Economics* 75, 429–490.

Buraschi, Andrea and Alexei Jiltsov, 2007, "Habit Formation and Macroeconomic Models of the Term Structure of Interest Rates", *Journal of Finance* 62, 3009–3063.

Calvo, Guillermo, 1983, "Staggered Prices in a Utility Maximizing Framework", *Journal of Monetary Economics* 12, 383–98.

Campbell, John Y., 1986, "Bond and Stock Returns in a Simple Exchange Model", *Quarterly Journal of Economics* 101, 785–803.

Campbell, John Y., 1991, "A Variance Decomposition for Stock Returns", *Economic Journal* 101:157–179.

Campbell, John Y., 2003, "Consumption-Based Asset Pricing", Chapter 13 in George Constantinides, Milton Harris, and Rene Stulz eds. *Handbook of the Economics of Finance Vol. IB*, North-Holland, Amsterdam, 803–887.

Campbell, John Y. and John Ammer, 1993, "What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns", *Journal of Finance* 48, 3–37.

Campbell, John Y. and John H. Cochrane, 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior", *Journal of Political Economy* 107, 205–251.

Campbell, John Y., Carolin Pflueger, and Luis Viceira, 2013, "Appendix to Monetary Policy Drivers of Bond and Equity Risks", online at http://scholar.harvard.edu/campbell/publications.

Campbell, John Y., Robert J. Shiller, and Luis M. Viceira, 2009, "Understanding Inflation-Indexed Bond Markets", *Brookings Papers on Economic Activity* 79-120, Spring 2009.

Campbell, John Y., Adi Sunderam, and Luis M. Viceira, 2013, "Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds", unpublished paper, Harvard University.

Canova, Fabio, and Luca Sala, 2009, "Back to Square One: Identification Issues in DSGE Models" *Journal of Monetary Economics*, 56, 431–449.

Chen, Nai-fu, 1991, "Financial Investment Opportunities and the Macroeconomy", *Journal of Finance* 46, 529–554.

Chib, Siddhartha, Kyu Ho Kang, and Srikanth Ramamurthy, 2010, "Term Structure of Interest Rates in a DSGE Model with Regime Changes", unpublished paper, Washington University in St. Louis.

Cho, Seonghoon, and Antonio Moreno, 2011, "The Forward Method as a Solution Refinement in Rational Expectations Models", *Journal of Economic Dynamics and Control* 35, 257–272.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, 1999, "Monetary Policy Shocks: What Have We Learned and to What End?", in John B. Taylor and Michael Woodford (eds.) *Handbook of Macroeconomics 1999*, 65–148, Elsevier.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, 2005, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy* 113, 1–45.

Christiansen, Charlotte and Angelo Ranaldo, 2007, "Realized Bond-Stock Correlation: Macroeconomic Announcement Effects", *Journal of Futures Markets* 27, 439–469.

Clarida, Richard, Jordi Gali, and Mark Gertler, 1999, "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature* 37, 1661–1707.

Cochrane, John H., 2007, “Financial Markets and the Real Economy”, in Rajnish Mehra ed. *Handbook of the Equity Premium*, Elsevier, 237–325.

Cochrane, John H., 2011, “Determinacy and Identification with Taylor Rules,” *Journal of Political Economy* 119, 565–615.

Cochrane, John H. and Monika Piazzesi, 2005, “Bond Risk Premia”, *American Economic Review* 95, 138–160.

Coibion, Olivier and Yuriy Gorodnichenko, 2012, “Why Are Target Interest Rate Changes So Persistent?”, *American Economic Journal: Macroeconomics* 4, 126–162.

d’Addona, Stefano and Axel H. Kind, 2006, “International Stock-Bond Correlations in a Simple Affine Asset Pricing Model”, *Journal of Banking and Finance* 30, 2747–2765.

Dai, Qiang and Kenneth Singleton, 2000, “Specification Analysis of Affine Term Structure Models”, *Journal of Finance* 55, 1943–1978.

Dai, Qiang and Kenneth Singleton, 2002, “Expectations Puzzles, Time-Varying Risk Premia, and Affine Models of the Term Structure”, *Journal of Financial Economics* 63, 415–442.

David, Alexander and Pietro Veronesi, 2013, “What Ties Return Volatilities to Fundamentals and Price Valuations?”, *Journal of Political Economy* 121, 682–746.

Dew-Becker, Ian, 2013, “A Model of Time-Varying Risk Premia with Habits and Production”, unpublished paper, Duke University.

Dew-Becker, Ian, 2014, “Bond Pricing with a Time-Varying Price of Risk in an Estimated Medium-Scale New Keynesian Model”, forthcoming *Journal of Money, Credit, and Banking*.

Duffee, Greg, 2002, “Term Premia and Interest Rate Forecasts in Affine Models”, *Journal of Finance* 57, 405–443.

Duffie, Darrell and Rui Kan, 1996, “A Yield-Factor Model of Interest Rates”, *Mathematical Finance* 6, 379–406.

Eraker, Bjørn, 2008, “Affine General Equilibrium Models”, *Management Science* 54, 2068–2080.

Evans, George, 1985, “Expectational Stability and the Multiple Equilibria Problem in Linear Rational Expectations”, *The Quarterly Journal of Economics* 100, 1217–1233.

Evans, George, 1986, "Selection Criteria for Models with Non-Uniqueness", *Journal of Monetary Economics* 18, 147–157.

Fama, Eugene F., 1990, "Stock Returns, Expected Returns, and Real Activity", *Journal of Finance* 45, 1089–1108.

Fama, Eugene F. and Kenneth R. French, 1989, "Business Conditions and Expected Returns on Stocks and Bonds", *Journal of Financial Economics* 25, 23–50.

Fuhrer, Jeffrey C., 1997, "The (Un)Importance of Forward-Looking Behavior in Price Specifications", *Journal of Money, Credit, and Banking* 29, 338–350.

Fuhrer, Jeffrey C., 2000, "Habit Formation in Consumption and Its Implications for Monetary-Policy Models", *American Economic Review* 90, 367–390.

Gali, Jordi, and Mark Gertler, 1999, "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics* 44, 195–222.

Greenspan, Alan, 1996, "The Challenge of Central Banking in a Democratic Society", remarks by Chairman Alan Greenspan at the Annual Dinner and Francis Boyer Lecture of The American Enterprise Institute for Public Policy Research, Washington, D.C., December 5. Available online at http://www.federalreserve.gov/newsevents/speech/1996speech.htm.

Gallmeyer, Michael, Burton Hollifield, Francisco Palomino, and Stanley Zin, 2008, "Term Premium Dynamics and the Taylor Rule", unpublished paper, Texas A&M University, Carnegie Mellon University, and University of Michigan.

Guidolin, Massimo, and Allan Timmermann, 2007, "Asset Allocation under Multivariate Regime Switching", *Journal of Economic Dynamics and Control* 31, 3503–3544.

Hall, Robert E., 1988, "Intertemporal Substitution in Consumption", *Journal of Political Economy* 96, 221–273.

Hamilton, James, 2009, "Causes and Consequences of the Oil Shock of 2007-08", *Brookings Papers on Economic Activity* 215–261, Spring 2009.

Hamilton, James, Seth Pruitt, and Scott Borger, 2011, "Estimating the Market-Perceived Monetary Policy Rule", *American Economic Journal: Macroeconomics* 3, 1–28.

Hasseltoft, Henrik, 2008, "The 'Fed-Model' and the Changing Correlation of Stock and Bond Returns: An Equilibrium Approach", unpublished paper, Stockholm School of Economics.

Koijen, Ralph, Hanno Lustig, and Stijn Van Nieuwerburgh, 2010, "The Cross-Section and Time-Series of Stock and Bond Returns", NBER working paper No. 15688.

Kung, Howard, 2013, "Macroeconomic Linkages Between Monetary Policy and the Term Structure of Interest Rates", *Journal of Financial Economics*, forthcoming.

Lamont, Owen, 1998, "Earnings and Expected Returns", *Journal of Finance* 53, 1563–1587.

Lettau, Martin and Sydney Ludvigson, 2001, "Consumption, Aggregate Wealth, and Expected Stock Returns", *Journal of Finance* 56, 815–849.

Lettau, Martin and Jessica Wachter, 2011, "The Term Structures of Equity and Interest Rates", *Journal of Financial Economics* 101, 90–113.

Li, Erica X.N. and Francisco Palomino, 2013, "Nominal Rigidities, Asset Returns, and Monetary Policy", unpublished paper, Cheung Kong Graduate School of Business and University of Michigan.

Mankiw, N. Gregory, and Ricardo Reis, 2002, "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve", *Quarterly Journal of Economics* 177, 1295–1328.

McCallum, Bennett T., 2003, "Multiple-Solution Indeterminacies in Monetary Policy Analysis", *Journal of Monetary Economics* 50, 1153–1175.

McCallum, Bennett T., 2004, "On the Relationship Between Determinate and MSV Solutions in Linear RE Models", *Economics Letters* 84, 55–60.

Orphanides, Athanasios (2003). "Monetary Policy Evaluation with Noisy Information," *Journal of Monetary Economics*, 50 (April), pp. 605-31.

Orphanides, Athanasios and John Williams, 2004, "Imperfect Knowledge, Inflation Expectations, and Monetary Policy", Chapter 5 in Ben S. Bernanke and Michael Woodford eds. *The Inflation-Targeting Debate*, University of Chicago Press.

Palomino, Francisco, 2012, "Bond Risk Premiums and Optimal Monetary Policy", *Review of Economic Dynamics* 15, 19–40.

Piazzesi, Monika and Martin Schneider, 2006, "Equilibrium Yield Curves", in D. Acemoglu, K. Rogoff, and M. Woodford (eds.) *NBER Macroeconomics Annual 2006*, 317-363, MIT Press, Cambridge, MA.

Rigobon, Roberto and Brian Sack, 2003, "Measuring the Reaction of Monetary Policy to the Stock Market", *Quarterly Journal of Economics* 639–669.

Rudebusch, Glenn D., 2002, "Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia", *Journal of Monetary Economics* 49, 1161–1187.

Rudebusch, Glenn D. and Eric T. Swanson, 2012, "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks", *American Economic Journal: Macroeconomics* 4, 105–143.

Rudebusch, Glenn D. and Tao Wu, 2007, "Accounting for a Shift in Term Structure Behavior with No-Arbitrage and Macro-Finance Models", *Journal of Money, Credit, and Banking* 39, 395–422.

Rudebusch, Glenn D. and Tao Wu, 2008, "A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy", *Economic Journal* 118, 906–926.

Shiller, Robert J. and Andrea Beltratti, 1992,"Stock Prices and Bond Yields: Can Their Comovements Be Explained in Terms of Present Value Models?", *Journal of Monetary Economics* 30, 25–46.

Smets, Frank, and Rafael Wouters, 2007, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach", *American Economic Review* 97, 586–606.

Smith, Josephine M. and John B. Taylor, 2009, "The Term Structure of Policy Rules", *Journal of Monetary Economics* 56, 907–917.

Stein, Jeremy C., 2013, "Gradualism in Monetary Policy: A Time-Consistency Problem?", unpublished notes.

Stock, James H. and Mark Watson, 2007, "Why Has US Inflation Become Harder to Forecast?", *Journal of Money Credit and Banking* 39, 3–33.

Taylor, John B., 1993, "Discretion versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy* 39, 195–214.

Uhlig, Harald, 1999, "A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily", in Ramon Marimon and Andrew Scott (eds.) *Computational Methods for the Study of Dynamic Economies*, 30–61, Oxford University Press.

Viceira, Luis M., 2012, "Bond Risk, Bond Return Volatility, and the Term Structure of Interest Rates", *International Journal of Forecasting* 28, 97–117.

Wachter, Jessica A., 2006, "A Consumption-Based Model of the Term Structure of Interest Rates", *Journal of Financial Economics* 79, 365–399.

Woodford, Michael, 2001, "The Taylor Rule and Optimal Monetary Policy", *American Economic Review Papers and Proceedings* 91, 232–237.

Yogo, Motohiro, 2004, "Estimating the Elasticity of Intertemporal Substitution When Instruments Are Weak", *Review of Economics and Statistics* 86, 797–810.

Tables and Figures

Table 1: **Summary Statistics**

1960.Q1-2011.Q4	Output Gap	Inflation	Fed Funds	Nom. Bond Yield
Mean	-0.77	3.50	5.78	6.24
Std.	2.91	2.38	3.77	2.75
AR(1) Coefficient	0.96	0.88	0.89	0.98
	(0.02)	(0.03)	(0.03)	(0.02)
AR(4) Coefficient	0.73	0.79	0.81	0.91
	(0.05)	(0.04)	(0.04)	(0.03)

1960.Q1-1979.Q2	Output Gap	Inflation	Fed Funds	Nom. Bond Yield
Mean	0.64	4.38	5.27	5.80
Std.	2.79	2.72	2.45	1.63
AR(1) Coefficient	0.93	0.88	0.86	0.96
	(0.04)	(0.06)	(0.06)	(0.35)
AR(4) Coefficient	0.61	0.78	0.60	0.92
	(0.09)	(0.07)	(0.10)	(0.06)

1979.Q3-1996.Q4	Output Gap	Inflation	Fed Funds	Nom. Bond Yield
Mean	-1.92	3.76	8.67	8.73
Std.	2.14	2.24	4.04	2.55
AR(1) Coefficient	0.93	0.87	0.78	0.94
	(0.04)	(0.05)	(0.08)	(0.04)
AR(4) Coefficient	0.50	0.78	0.74	0.77
	(0.10)	(0.06)	(0.08)	(0.08)

1997.Q1-2011.Q4	Output Gap	Inflation	Fed Funds	Nom. Bond Yield
Mean	-1.27	2.06	3.08	3.90
Std.	3.13	1.01	2.32	1.51
AR(1) Coefficient	1.00	0.51	0.95	0.94
	(0.03)	(0.12)	(0.04)	(0.05)
AR(4) Coefficient	0.89	0.28	0.78	0.80
	(0.09)	(0.13)	(0.09)	(0.08)

Full sample and sub period summary statistics. US quarterly log output gap (%), GDP deflator inflation (%, Annualized), Fed Funds rate (%, Annualized), and 5 year nominal yield (%, Annualized). Yields and inflation continuously compounded. Standard errors in parentheses.

Table 2: **Predicting Stock Returns with Output Gap**

Log Exc. Stock Ret. xr^e_{t+1}	60.Q1-11.Q4	60.Q1-79.Q2	79.Q2-96.Q4	97.Q1-11.Q4
Output Gap x_t	-0.49*	-0.61*	-0.32	-0.47
	(0.20)	(0.30)	(0.48)	(0.40)
Constant	0.59	0.81	1.12	0.18
	(0.61)	(0.95)	(1.14)	(1.36)
R^2	0.03	0.04	0.01	0.02

Quarterly realized log excess stock returns (%, Quarterly) from quarter t to quarter $t+1$ onto the output gap (%) in quarter t. Newey-West standard errors with 2 lags in parentheses. * and ** denote significance at the 1% and 5% levels.

Table 3: **Predicting Stock Returns with Dividend Price Ratio**

Log Exc. Stock Ret. $xr^e_{t\to t+k}$	k=4	k=8	k=12	k=16	k=20
Log Dividend Price Ratio $d_t - p_t$	0.08	0.15	0.19*	0.21**	0.25**
	(0.05)	(0.09)	(0.09)	(0.08)	(0.06)
Constant	0.34	0.62*	0.79*	0.91**	1.10**
	(0.18)	(0.29)	(0.31)	(0.27)	(0.23)
R^2	0.04	0.07	0.09	0.10	0.12
Sample			1960.Q1-2011.Q1		

$xr^e_{t\to t+k}$	k=4	k=8	k=12	k=16	k=20
Log Dividend Price Ratio $d_t - p_t$	0.11*	0.21**	0.28**	0.34**	0.43**
	(0.04)	(0.07)	(0.08)	(0.09)	(0.10)
Constant	0.44**	0.82**	1.13**	1.40**	1.76**
	(0.15)	(0.24)	(0.27)	(0.29)	(0.33)
R^2	0.08	0.15	0.22	0.27	0.33
Sample			1947.Q1-2011.Q4		

k quarter log excess stock returns onto lagged log dividend price ratio, both in natural units. Newey-West standard errors with $k+4$ lags in parentheses. * and ** denote significance at the 1% and 5% levels.

Table 4: **Estimating the Monetary Policy Function**

Fed Funds i_t	60.Q1-11.Q4	60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4
Output Gap x_t	0.06	0.18**	-0.04	0.05
	(0.04)	(0.06)	(0.13)	(0.04)
Inflation π_t	0.21	0.30**	0.83**	0.21**
	(0.11)	(0.07)	(0.21)	(0.07)
Lagged Fed Funds i_{t-1}	0.81**	0.56**	0.43*	0.89**
	(0.05)	(0.10)	(0.17)	(0.06)
Constant	0.42	0.91*	1.75	-0.12
	(0.26)	(0.38)	(0.92)	(0.29)
R^2	0.79	0.75	0.69	0.91
Implied $\hat{\gamma}^x$	0.32	0.42**	-0.07	0.44
	(0.21)	(0.13)	(0.22)	(0.21)
Implied $\hat{\gamma}^\pi$	1.08**	0.69**	1.44**	1.92*
	(0.43)	(0.16)	(0.19)	(1.26)
Implied $\hat{\rho}^i$	0.81**	0.56**	0.43**	0.89**
	(0.05)	(0.10)	(0.17)	(0.06)

We estimate $i_t = c^0 + c^x x_t + c^\pi \pi_t + c^i i_{t-1} + \epsilon_t$. All variables are described in Table 1. Since the inflation target is not directly observable it is omitted. Implied parameters are calculated according to $\hat{\rho}^i = \hat{c}^i$, $\hat{\gamma}^x = \hat{c}^x/(1-\hat{c}^i)$, and $\hat{\gamma}^\pi = \hat{c}^\pi/(1-\hat{c}^i)$. Newey-West standard errors with 6 lags in parentheses. Standard errors for $\hat{\gamma}^x$ and $\hat{\gamma}^\pi$ are calculated by the delta method. * and ** denote significance at the 5% and 1% levels. Significance levels for implied parameters are based on an ordinary least squares likelihood ratio test.

Table 5: **Parameter Choices**

Panel A: Calibration Parameters

Time-Invariant Parameters				
Log-Linearization Constant	ρ		0.99	
Leverage	δ		2.43	
Preference Parameter	α		30	
Backward-Looking Comp. PC	ρ^{π}		0.80	
Slope PC	λ		0.30	
Forward-Looking Comp. IS	ρ^{x+}		0.62	
Backward-Looking Comp. IS	ρ^{x-}		0.45	

Monetary Policy Rule		60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4
MP Coefficient Output	γ^{x}	0.42	-0.07	0.44
MP Coefficient Infl.	γ^{π}	0.69	1.44	1.92
Backward-Looking Comp. MP	ρ^{i}	0.56	0.43	0.89

Std. Shocks				
Std. IS	$\bar{\sigma}^{IS}$	0.45	0.43	0.26
Std. PC shock	$\bar{\sigma}^{PC}$	1.08	0.80	0.93
Std. MP shock	$\bar{\sigma}^{MP}$	1.04	2.03	0.26
Std. infl. target shock	$\bar{\sigma}^{*}$	0.37	0.70	0.53

Panel B: Implied Parameters

Time-Invariant Implied Parameters				
SDF Lag Output Gap	θ		0.72	
SDF Lag with Varying Risk Premia	θ^{*}		0.61	
Slope IS	ψ		0.02	

Time-Varying Implied Parameters				
Heteroskedasticity Parameter	b	7.18	8.27	7.89
Volatility SDF	σ	0.32	0.30	0.31

Table 6: **Model and Empirical Moments**

	60.Q1-79.Q2		79.Q3-96.Q4		97.Q1-11.Q4	
Std. VAR(1) Residuals	Empirical	Model	Empirical	Model	Empirical	Model
Output Gap	0.92	0.81	0.75	0.77	0.65	0.51
Inflation	1.12	1.22	0.89	0.92	0.80	1.02
Fed Funds Rate	1.22	1.15	2.07	2.17	0.66	0.55
Log Nominal Yield	0.48	0.41	0.85	0.72	0.55	0.70
Std. Asset Returns						
Std. Eq. Ret.	17.62	18.82	15.34	17.50	20.08	17.95
Std. Nom. Bond Ret.	4.85	3.91	9.11	7.09	5.55	5.83
Nominal Bond Beta	0.06*	0.07	0.20*	0.22	-0.17**	-0.15
Taylor Rule: Fed Funds onto Output, Infl. and Lag. Fed Funds						
Output	0.18**	0.21	-0.04	-0.12	0.05	0.01
Inflation	0.30**	0.33	0.83**	0.65	0.21**	0.19
Lagged Fed Funds	0.56**	0.61	0.43*	0.39	0.89**	0.80

This table reports model second moments, conditional on the output gap x_t being at its unconditional mean of zero. The law of total variance implies that unconditional second moments are equal to the conditional second moments reported in this table. For details of the derivation, see the Appendix. * and ** denote significance at the 5% and 1% levels. We use Newey-West standard erros with 2 lags for the nominal bond beta and Newey-West standard errors with 6 lags for the empirical Taylor rule estimation in the bottom panel.

Table 7: **Additional Empirical and Model Moments**

	Empirical	Model
AR(1) Coefficient Output Gap	0.96	0.83
AR(4) Coefficient Output Gap	0.73	0.18
Correlation(x, p-d)	0.18	0.47
Std(d-p)	0.40	0.15
AR(4) Coefficient d-p	0.92	0.11
Slope 1 year exc. Stock ret. wrt d-p	0.08	0.84
Slope 5 year exc. Stock ret. wrt d-p	0.25	1.03
Slope quarterly stock ret. wrt x	-0.49	-3.30
Std(real rate) (% Ann.)		1.90
Regression s+c onto x		0.23

Additional model and empirical moments are not explicitly fitted by the calibration procedure. Model moments show averages across three sub sample calibrations weighted by sub sample length. Wold's theorem for vector processes implies that the unconditional second moments of the state variables are identical for conditionally homoskedastic and heteroskedastic VAR(1) processes with identical unconditional variance-covariance matrix of innovations. We therefore use a VAR(1) with matrix of slope coefficients P and conditionally homoskedastic, iid vector of innovations $\epsilon_t \sim N(0, Q\Sigma_u Q')$ to simulate all model moments. For details of the derivation, see the Appendix.

Table 8: **Marginal Effects of Parameters**

		Total Derivative			Partial Derivative		
Nominal Bond Beta		60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4	60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4
MP Coefficient Output	γ^x	-3.92	-1.50	-1.37			
MP Coefficient Inflation	γ^π	5.01	1.80	1.86			
MP Persistence	ρ^i	-1.85	-1.91	-20.90			
IS Shock Std.	$\bar{\sigma}^{IS}$	-0.56	-0.11	-0.09	-0.01	-0.01	-0.01
PC Shock Std.	$\bar{\sigma}^{PC}$	3.43	3.87	5.25	0.22	0.58	-0.05
MP Shock Std.	$\bar{\sigma}^{MP}$	-0.28	-0.33	-0.06	0.01	0.02	0.00
Infl. Target Shock Std.	$\bar{\sigma}^{*}$	-2.59	-3.42	-5.09	-0.08	-0.15	-0.24
Std. Nominal Bond Returns		60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4	60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4
MP Coefficient Output	γ^x	-16.06	-11.72	14.20			
MP Coefficient Inflation	γ^π	20.94	14.35	-19.04			
MP Persistence	ρ^i	-6.98	-14.48	215.46			
IS Shock Std.	$\bar{\sigma}^{IS}$	-1.99	-0.28	1.81	0.22	0.42	0.89
PC Shock Std.	$\bar{\sigma}^{PC}$	14.49	32.35	-52.90	1.24	5.55	0.05
MP Shock Std.	$\bar{\sigma}^{MP}$	-0.83	-2.30	0.78	0.35	0.42	0.10
Infl. Target Shock Std.	$\bar{\sigma}^{*}$	-7.76	-22.68	56.14	2.11	0.71	4.80
Std. Equity Returns		60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4	60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4
MP Coefficient Output	γ^x	-1.59	-1.29	-1.12			
MP Coefficient Inflation	γ^π	0.77	0.75	0.66			
MP Persistence	ρ^i	-0.20	-0.32	-1.70			
IS Shock Std.	$\bar{\sigma}^{IS}$	0.23	0.05	0.00	0.24	0.07	0.01
PC Shock Std.	$\bar{\sigma}^{PC}$	17.41	11.87	14.89	17.24	11.63	14.78
MP Shock Std.	$\bar{\sigma}^{MP}$	-0.08	-0.16	-0.06	0.08	0.18	0.06
Infl. Target Shock Std.	$\bar{\sigma}^{*}$	1.27	5.74	3.11	1.26	5.63	3.09

Derivatives with respect to monetary policy rule parameters and log standard deviations of shocks (semi-elasticities). Partial derivatives hold constant the loadings of bond and stock returns. Nominal bond beta partial derivatives also hold equity volatility constant.

Table 9: **Decomposing Changes in Bond and Equity Risks**

		Nominal Bond Beta		Std. Nom. Bond Returns		Std. Equity Returns	
Change Date		79.Q3	97.Q1	79.Q3	97.Q1	79.Q3	97.Q1
Empirical Change		0.13	-0.36	4.26	-3.56	-2.28	4.74
Model Change		0.15	-0.37	3.18	-1.27	-1.32	0.45
Total Derivative x Parameter Change							
MP Coefficient Output	γ^x	1.35	-0.74	6.83	0.12	0.71	-0.62
MP Coefficient Inflation	γ^π	2.63	0.86	13.43	-0.50	0.57	0.34
MP Persistence	ρ^i	0.26	-4.97	1.44	42.64	0.04	-0.44
MP Shock Std.	$\bar{\sigma}^{MP}$	-0.20	0.42	-1.01	1.80	-0.08	0.23
Infl. Target Shock Std.	$\bar{\sigma}^*$	-1.92	1.14	-9.55	-3.72	2.18	-1.23
IS Shock Std.	$\bar{\sigma}^{IS}$	0.01	0.05	0.04	-0.36	0.00	-0.01
PC Shock Std.	$\bar{\sigma}^{PC}$	-1.10	0.67	-6.93	-1.04	-4.47	1.97
Combined Effects							
MP Sub-Total		2.11	-3.28	11.14	40.34	3.42	-1.73
IS&PC Shocks Sub-Total		-1.09	0.72	-6.90	-1.40	-4.47	1.95
Total Linear Changes		1.03	-2.56	4.24	38.94	-1.06	0.22
Nonlinearity Effect		-0.87	2.19	-1.06	-40.21	-0.26	0.23

We average total derivatives across subsequent periods weighting sub periods according to their sample size. We multiply average weighted total derivatives by the parameter change between subsequent periods. Total derivatives for each sub period are reported in Table 8 and parameter values for each sub period are reported in Table 5. The "MP Sub-Total" row sums the linear effects of γ^x, γ^π, ρ^i, $\bar{\sigma}^{MP}$, and $\bar{\sigma}^*$. The "IS&PC Shocks Sub-Total" row sums the linear effects of $\bar{\sigma}^{IS}$ and $\bar{\sigma}^{PC}$. The row "Total Linear Changes" reports the sum of all linear effects reported in the panel above. The row "Nonlinearity Effect" shows the difference between the model change and the total of linear changes.

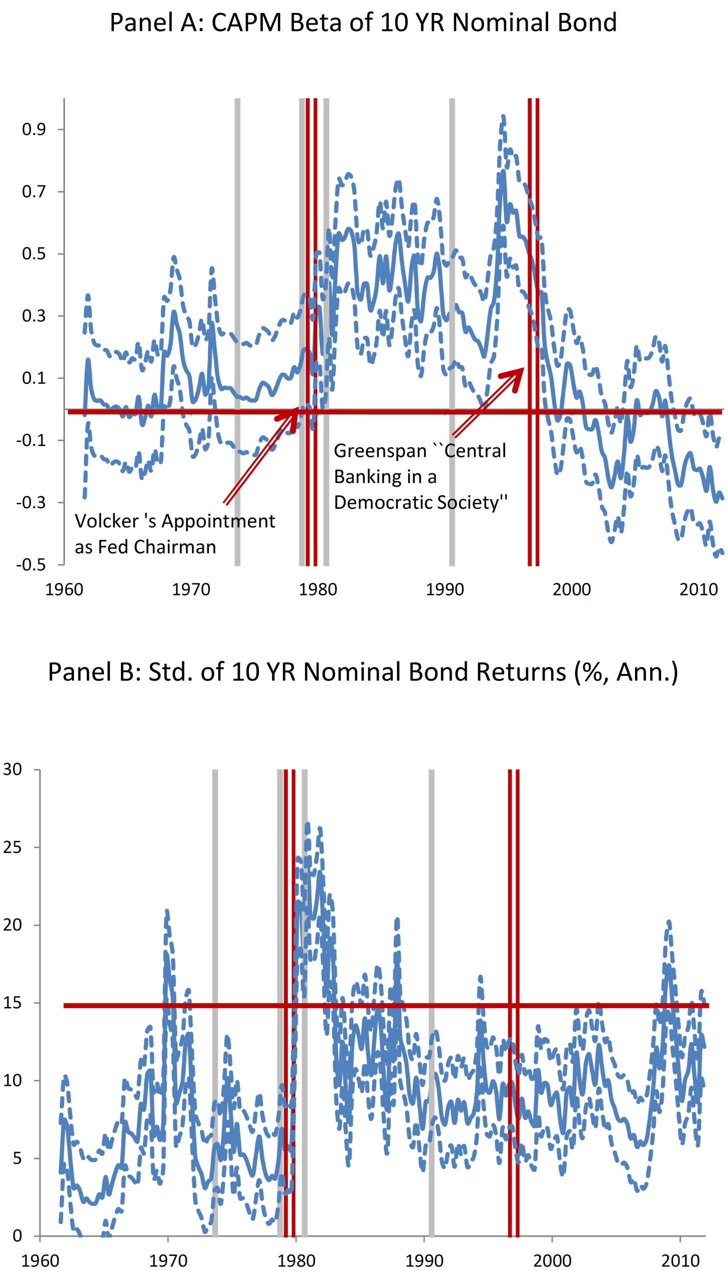
Table 10: **Risks of Inflation-Indexed Bonds**

Panel A: Moments	60.Q1-79.Q2		79.Q3-96.Q4		97.Q1-11.Q4	
	Empirical	Model	Empirical	Model	Empirical	Model
Std. Infl.-Indexed Bond Ret.	N/A	7632.40	N/A	18.09	4.27	3.24
Infl.-Indexed Bond Beta	N/A	-215.55	N/A	1.03	-0.08	-0.16

Panel B: Marginal Effects of Parameters		Total Derivative			Partial Derivative		
Infl.-Indexed Bond Beta		60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4	60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4
MP Coefficient Output	γ^x	-5584.50	-0.19	-2.62			
MP Coefficient Inflation	γ^π	6372.20	0.17	2.86			
MP Persistence	ρ^i	-2294.10	-0.17	-27.97			
IS Shock Std.	σ^{IS}	-2396.60	-0.05	-0.04	-83.36	0.02	0.00
PC Shock Std.	σ^{PC}	2506.40	0.06	0.11	-324.06	1.37	-0.26
MP Shock Std.	σ^{MP}	-292.40	-0.04	-0.09	-0.07	0.02	0.01
Infl. Target Shock Std.	σ^*	182.60	0.03	0.02	-23.61	0.66	-0.05
Std. Infl.-Indexed Bond Ret.		60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4	60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4
MP Coefficient Output	γ^x	204920.00	-4.47	46.28			
MP Coefficient Inflation	γ^π	-233180.00	3.63	-50.25			
MP Persistence	ρ^i	84310.00	-3.15	495.45			
IS Shock Std.	σ^{IS}	91810.00	-0.71	1.10	6205.30	0.28	0.52
PC Shock Std.	σ^{PC}	-88470.00	13.24	0.09	1330.10	11.87	2.02
MP Shock Std.	σ^{MP}	10740.00	-0.85	2.03	0.00	0.19	0.27
Infl. Target Shock Std.	σ^*	-6450.00	6.41	0.02	96.90	5.75	0.42

This table shows the model-implied beta and standard deviation of returns on inflation-indexed bonds with five years to maturity. We approximate inflation-indexed bond log excess returns according to $\widehat{xr}_{5YR,t+1} = -19y^{TIPS}_{5YR,t+1} + 20y^{TIPS}_{5YR,t} - i_t + \pi_{t+1}$, where $y^{TIPS}_{5YR,t}$ is the Treasury Inflation Protected (TIPS) end-of-quarter yield with five years to maturity from Bloomberg (USGGT05Y Index). Quarterly TIPS returns are available starting 1997.Q4. We use Newey-West standard errors with 2 lags to assess statistical significance of the empirical bond beta. * and ** denote significance at the 5% and 1% levels.

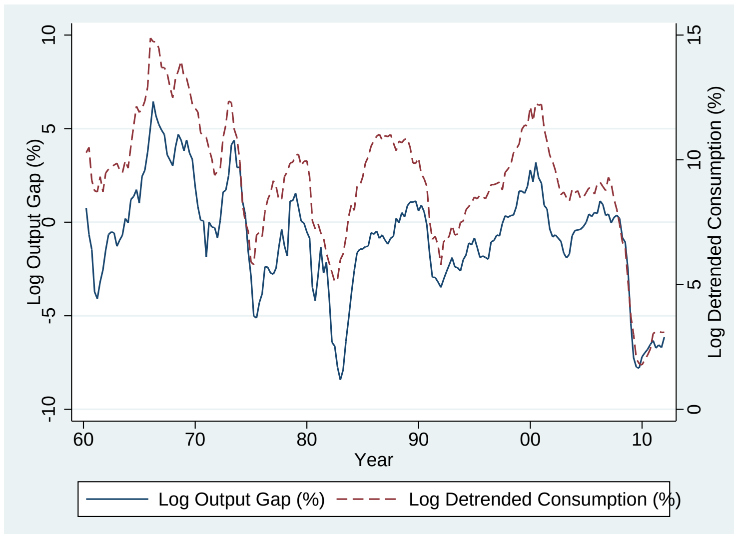
Figure 1: US Nominal Bond Beta, Bond Volatility, and Policy Dates



Nominal bond beta and standard deviation of nominal bond returns from daily bond and stock returns over past three months as in Campbell, Sunderam, and Viceira (2013). We model time-varying second moments as an unobserved trend AR(1) component plus white measurement noise. We show trend second moments estimated using the Kalman filter. 95% confidence intervals, which do not take into account parameter uncertainty, are shown in dashed. Gray vertical lines depict Hamilton (2009) oil price shocks.

Figure 2: US Output Gap

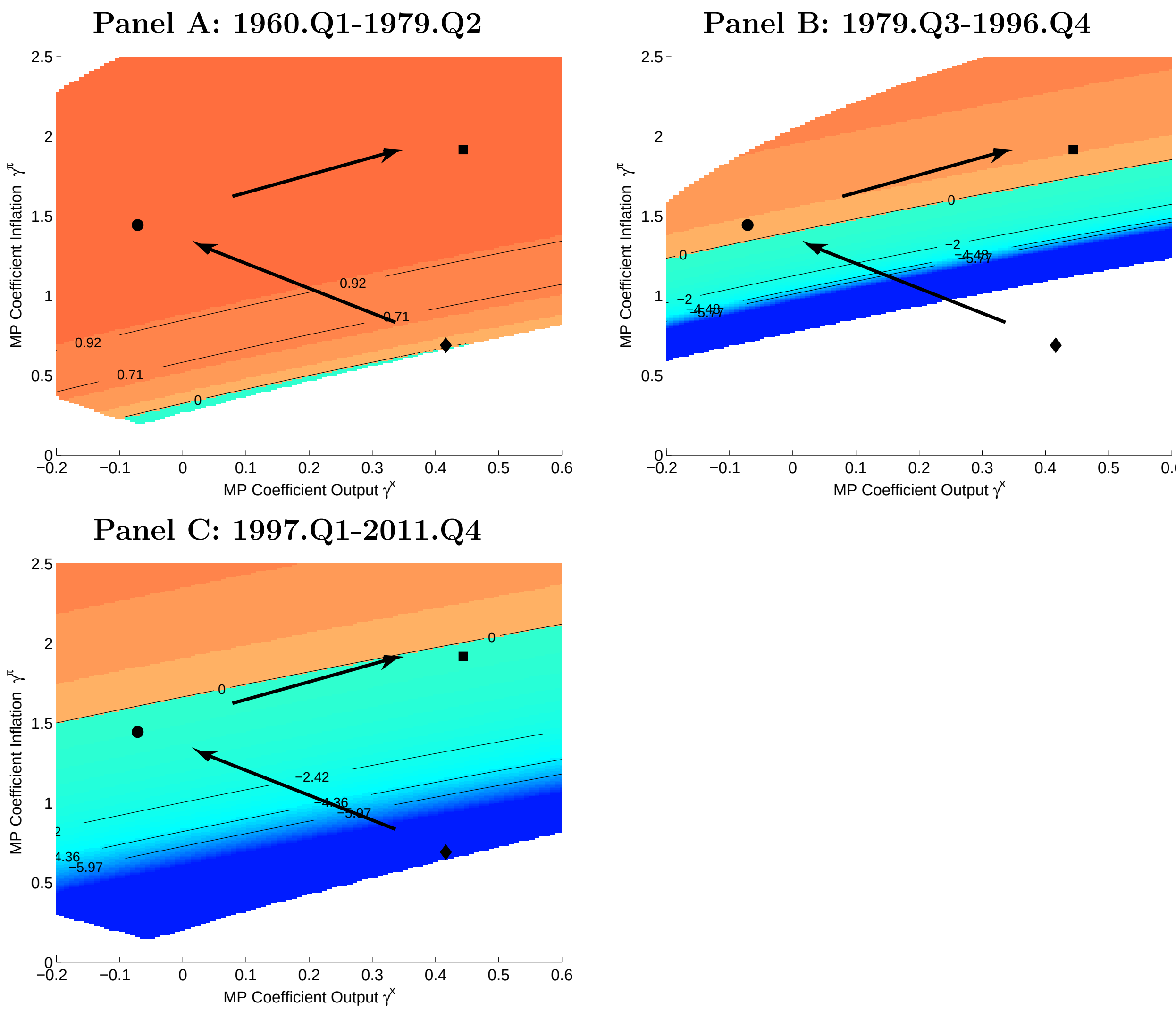
Panel A: Output Gap and Detrended Consumption



Panel B: Output Gap and Price-Dividend Ratio

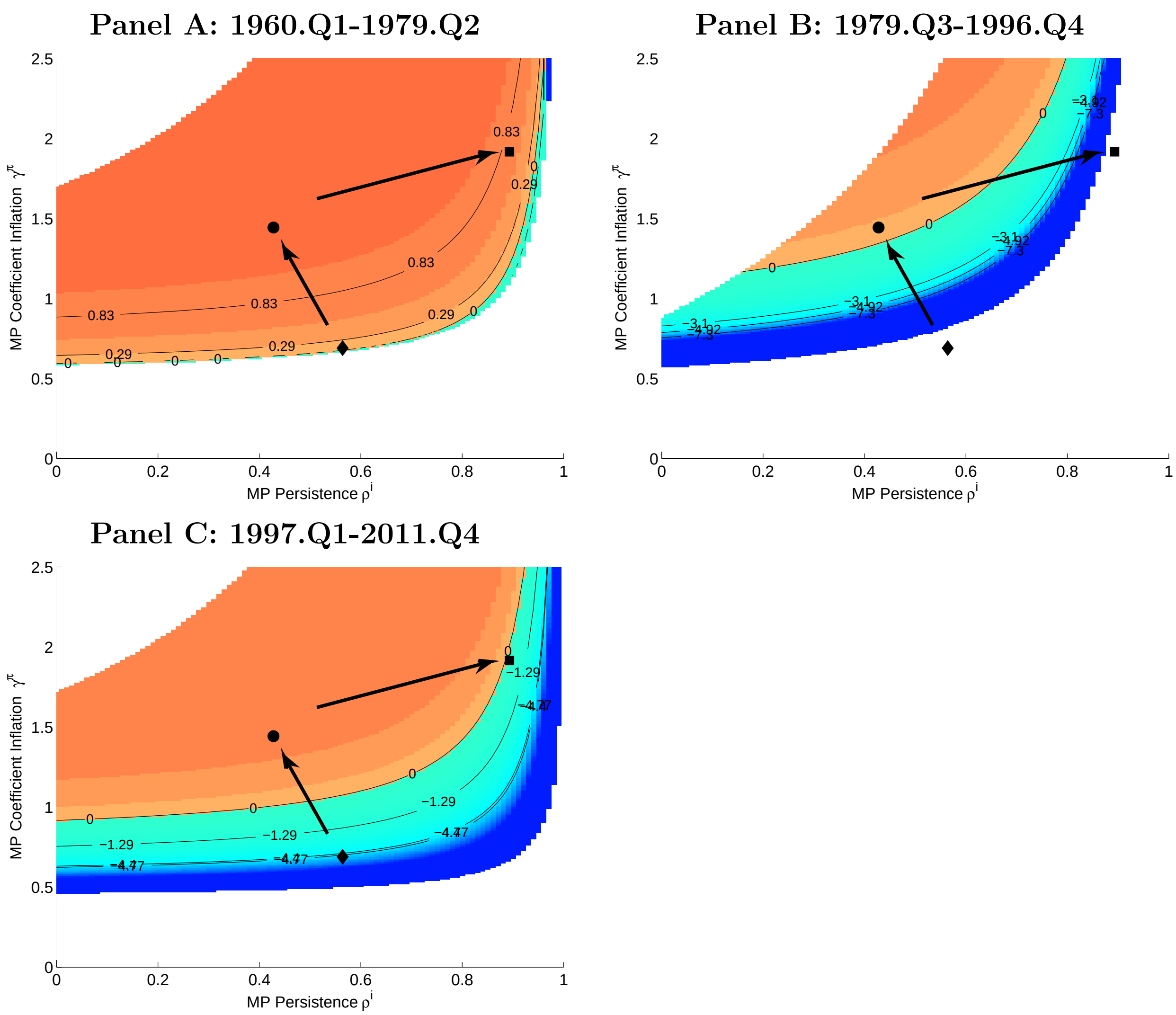
Panel A plots the time series of the US log real output gap together with log real consumption in excess of its 24 quarter moving average. We use real consumption expenditures data for nondurables and services from the Bureau of Economic Analysis National Income and Product Accounts Tables. The US log output gap (%) is described in Table 1. The end-of-quarter price dividend ratio is computed as the S&P 500 real price divided by real dividends averaged over the past 10 years.

Figure 3: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and γ^{x}



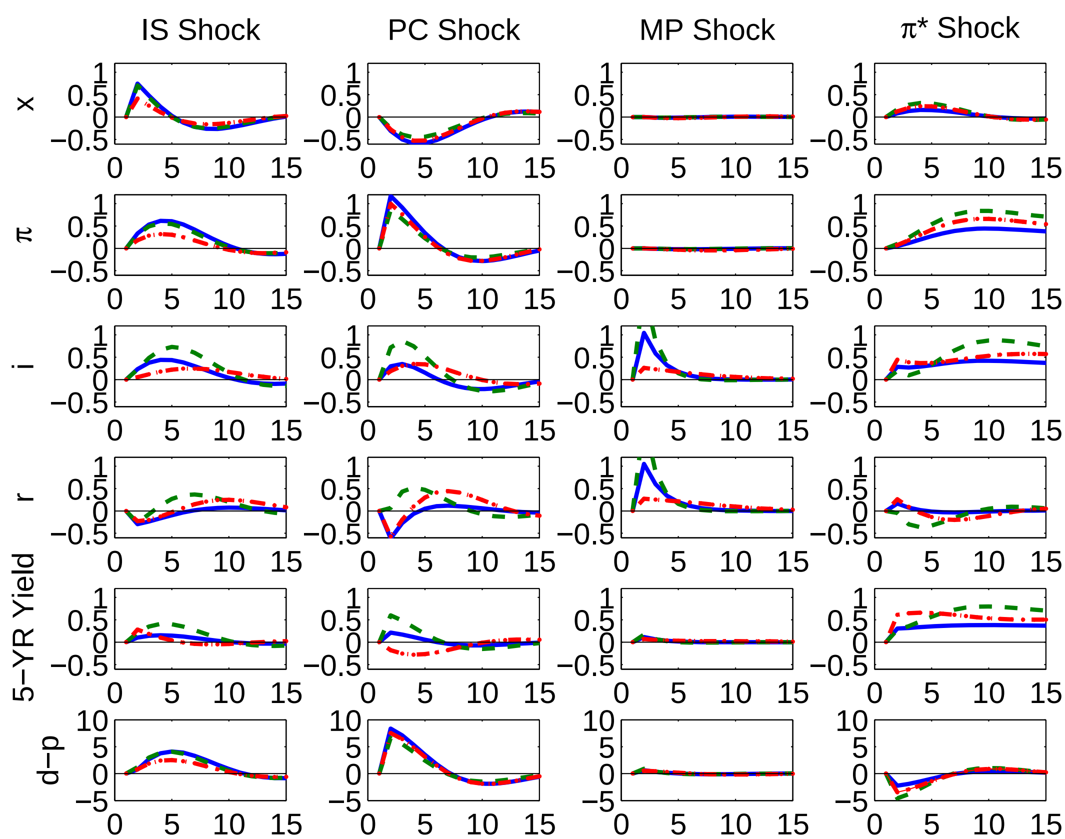
The volatilities of shocks and the monetary policy persistence parameter ρ^{i} are fixed at their calibrated values for 1960.Q1-1979.Q2 (Panel A), 1979.Q3-1996.Q4 (Panel B) or 1997.Q1-2011.Q4 (Panel C). We indicate calibrated monetary policy parameters as follows: 1960.Q1-1979.Q2 = diamond, 1979.Q3-1996.Q4 = circle, 1997.Q1-2011.Q4 = square. Arrows connect the sub period monetary policy rules in chronological order.

Figure 4: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and ρ^{i}



The volatilities of shocks and the monetary policy coefficient γ^{x} are fixed at their calibrated values for 1960.Q1-1979.Q2 (Panel A), 1979.Q3-1996.Q4 (Panel B) or 1997.Q1-2011.Q4 (Panel C). We indicate calibrated monetary policy parameters as follows: 1960.Q1-1979.Q2 = diamond, 1979.Q3-1996.Q4 = circle, 1997.Q1-2011.Q4 = square. Arrows connect the sub period monetary policy rules in chronological order.

Figure 5: Impulse Response Functions



We show impulses for the output gap, inflation, the nominal and real Federal Funds rates, the 5 year nominal yield, and the log dividend price ratio following one standard deviation shocks in period 1. We show impulse responses for the sub periods 1960.Q1-1979.Q2 (blue solid line), 1979.Q3-1996.Q4 (green dashed line), and 1997.Q1-2011.Q4 (red dash-dot line). Note that standard deviations of shocks vary across sub periods. The output gap and the dividend price ratios are in percent deviations from the steady state. All other variables are in annualized percent units.

Appendix: Monetary Policy Drivers of Bond and Equity Risks

John Y. Campbell, Carolin Pflueger, and Luis M. Viceira[1]

First draft: March 2012
This draft: March 2014

[1]Campbell: Department of Economics, Littauer Center, Harvard University, Cambridge MA 02138, USA, and NBER. Email john_campbell@harvard.edu. Pflueger: University of British Columbia, Vancouver BC V6T 1Z2, Canada. Email carolin.pflueger@sauder.ubc.ca. Viceira: Harvard Business School, Boston MA 02163 and NBER. Email lviceira@hbs.edu. We are grateful to Alberto Alesina, Robert Barro, Philip Bond, Mikhail Chernov, Paul Beaudry, Ian Dew-Becker, Alexander David, Adlai Fisher, Ben Friedman, Lorenzo Garlappi, Gita Gopinath, Robin Greenwood, Joshua Gottlieb, Howard Kung, Leonid Kogan, Deborah Lucas, Greg Mankiw, Michael Woodford, conference and seminar participants at the University of British Columbia, the Harvard Monetary Economics Seminar, the 2013 HBS Finance Research Retreat, the University of Calgary, the Vienna Graduate School of Finance, ECWFC 2013, PNWCF 2014, the Jackson Hole Finance Conference 2014, the ASU Sonoran Winter Finance Conference 2014, and especially our discussants Rossen Valkanov and Gregory Duffee for helpful comments and suggestions. This material is based upon work supported by Harvard Business School Research Funding and the PH&N Centre for Financial Research at UBC.

A Model Solution

Let π_t^* denote the central bank's inflation target at time t. We solve the model in terms of the output gap x_t and inflation and nominal interest rate gaps:

$$\begin{aligned}\hat{\pi}_t &= \pi_t - \pi_t^*, \quad (1)\\ \hat{i}_t &= i_t - \pi_t^*. \quad (2)\end{aligned}$$

Denote the vector of state variables by:

$$\hat{Y}_t = [x_t, \hat{\pi}_t, \hat{i}_t]'. \quad (3)$$

We can re-write the model dynamics in terms of the state variables as:

$$\begin{aligned}x_t &= \rho^{x-} x_{t-1} + \rho^{x+} E_{t-} x_{t+1} - \psi \left(E_{t-} \hat{i}_t - E_{t-} \hat{\pi}_{t+1} \right) + u_t^{IS}, \quad (4)\\ \hat{\pi}_t &= \rho^{\pi} \hat{\pi}_{t-1} + (1 - \rho^{\pi}) E_{t-} \hat{\pi}_{t+1} + \lambda x_t - \rho^{\pi} u_t^* + u_t^{PC}, \quad (5)\\ \hat{i}_t &= \rho^i \hat{i}_{t-1} + (1 - \rho^i) \left[\gamma^x x_t + \gamma^{\pi} \hat{\pi}_t \right] + u_t^{MP}, \quad (6)\\ \pi_t^* - \pi_{t-1}^* &= u_t^*. \quad (7)\end{aligned}$$

Using $E_{t-} \hat{i}_t = \hat{i}_t - u_t^{MP}$, we can write the model as:

$$0 = F E_{t-} \hat{Y}_{t+1} + G \hat{Y}_t + H \hat{Y}_{t-1} + M u_t. \quad (8)$$

where

$$\begin{aligned}F &= \begin{bmatrix} \rho^{x+} & \psi & 0 \\ 0 & (1-\rho^{\pi}) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (9)\\ G &= \begin{bmatrix} -1 & 0 & -\psi \\ \lambda & -1 & 0 \\ (1-\rho^i)\gamma^x & (1-\rho^i)\gamma^{\pi} & -1 \end{bmatrix}, \quad (10)\\ H &= \begin{bmatrix} \rho^{x-} & 0 & 0 \\ 0 & \rho^{\pi} & 0 \\ 0 & 0 & \rho^i \end{bmatrix}, \quad (11)\\ M &= \begin{bmatrix} 1 & 0 & \psi & 0 \\ 0 & 1 & 0 & -\rho^{\pi} \\ 0 & 0 & 1 & 0 \end{bmatrix}. \quad (12)\end{aligned}$$

We focus on solutions of the form:

$$\hat{Y}_t = P \hat{Y}_{t-1} + Q u_t. \quad (13)$$

Additional solutions, such as solutions depending on two lags of state variables, may exist, see e.g. Evans and McGough (2005). P has to satisfy:

$$F P^2 + G P + H = 0. \quad (14)$$

Following Uhlig (1999), we first solve for the generalized eigenvectors and eigenvalues of Ξ with respect to Δ, where:

$$\Xi = \begin{bmatrix} -G & -H \\ I_3 & 0_3 \end{bmatrix}, \tag{15}$$

$$\Delta = \begin{bmatrix} F & 0_3 \\ 0_3 & I_3 \end{bmatrix}. \tag{16}$$

For three generalized eigenvalues $\lambda_1, \lambda_2, \lambda_3$ with generalized eigenvectors $[\lambda_1 z_1', z_1']'$, $[\lambda_2 z_2', z_2']'$, $[\lambda_3 z_3', z_3']'$, a solution is given by

$$P = \Omega\Lambda\Omega^{-1}, \tag{17}$$

where $\Lambda = diag(\lambda_1, \lambda_2, \lambda_3)$ and $\Omega = [z_1, z_2, z_3]$. Generalized eigenvalues are stable if their absolute value is < 1.

Let e_k denotes the row vector with a 1 in position k and zeros otherwise. Q has to satisfy

$$Qe_k' = -[FP+G]^{-1}Me_k' \quad k = 1, 2, 4 \tag{18}$$

$$Qe_3' = -G^{-1}Me_3' \tag{19}$$

Provided that G is nonsingular, $G \times Q \times e_3' = -Me_3' = -[\psi, 0, 1]'$ implies that $Q \times e_3' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, i.e. the monetary policy shock has no contemporaneous effect on x_t or $\hat{\pi}_t$.

As long as we focus on solutions of the form (13) and the matrix of lagged terms H is non-singular, the solution cannot contain arbitrary random variables, or 'sunspots'. If we were to allow for more complicated solution forms, where $\hat{Y}_t$ can depend on two lags of itself as well as current and lagged shocks, sunspot solutions may be possible (Evans and McGough, 2005).

To see that solutions of the form (13) do not allow for sunspots, suppose the contrary. Assume that for some vector of random variables ϵ_t uncorrelated with $\hat{Y}_{t-1}$ and u_t:

$$\hat{Y}_t = P\hat{Y}_{t-1} + Qu_t + \epsilon_t. \tag{20}$$

The expression (20) corresponds to the definition of sunspot equilibria, see e.g. Cho and Moreno (2011). Then substituting (20) into (8) gives the same conditions for P and M as before and:

$$(FP+G)\epsilon_t \equiv 0. \tag{21}$$

But from (14), $(FP+G) \times P = -H$ is non-singular. Therefore, $FP+G$ is non-singular and $\epsilon_t \equiv 0$. This completes the proof that there are no sunspot solutions.

A.1 Equilibrium Selection and Properties

We are essentially solving a quadratic matrix equation, so picking a solution amounts to picking three out of six generalized eigenvalues. We only consider dynamically stable solutions with all eigenvalues less than 1 in absolute value, yielding non-explosive solutions for the output gap, inflation gap and interest rate gap. When there are only three generalized eigenvalues with absolute values less than 1, there exists a unique dynamically stable solution. For the period 1 calibration, we have $\gamma^{\pi} < 1$ and there exist multiple real-valued, dynamically stable solutions. The period 2 and 3 calibrations have unique dynamically stable solutions.

We only consider solutions that are real-valued, and have finite entries for Q. We also require the diagonal entries of Q to be positive. This requirement means that the immediate impact of a positive IS shock on the output gap is positive rather than negative.

We apply multiple equilibrium selection criteria, which have been proposed in the literature, to rule out "bubble" or unreasonable solutions. These different equilibrium refinements are not identical, but coincide in many cases. Therefore, there exists a unique solution satisfying all criteria for a large part of our parameter space.

McCallum (1983) proposes to pick the minimum state variable solution. This solution has a minimum set of state variables and satisfies a continuity criterion. Unfortunately, Uhlig (1999) points out that implementing this criterion directly can be computationally demanding. We therefore follow Uhlig (1999) in picking the solution with the minimum absolute eigenvalues, which under certain conditions coincides with the minimum state variable solution (McCallum 2004).

We also require that our solution is locally E-stable (Evans 1985, 1986, Evans and Honkapohja 1994) as a plausible necessary, but not sufficient, condition. Local E-stability intuitively requires that the solution is learnable. If agents expectations deviate slightly from equilibrium dynamics, the system will return to an E-stable equilibrium under a simple revision rule.

Finally, we ensure uniqueness of our solution by requiring that it equals the forward solution of Cho and Moreno (2011). The forward solution is obtained by imposing a zero terminal condition. Expectations about shocks far in the future do not affect the current equilibrium. Viewed differently, if we assume that all state variables are constant from time t+T onwards, we can solve for the time t output gap, inflation gap, and interest rate gap recursively. The forward solution obtains by letting T go to infinity.

Let *vec* denote vectorization. Applying Proposition 1.3 of Fudenberg and Levine (1998, p.25) the E-stability condition translates into the requirement that the eigenvalues of the derivative

$$\frac{\partial vec([FP+G]^{-1}H)}{\partial vec(P)} \tag{22}$$

have eigenvalues with absolute values less than 1.

We implement the Cho and Moreno (2011) criterion by requiring that the following sequence $P_n, n = 0, 1, ...$ converges to P

$$\begin{aligned} P_0 &= 0_{3\times 3} && (23) \\ P_{n+1} &= -[FP_n + G]^{-1} \times H && (24) \end{aligned}$$

This sequence P_n has at most one limit and therefore this selection criterion yields a unique solution.

A.2 Solving for SDF and Model Dynamics Simultaneously

We can solve for the matrices P and Q in terms of the model coefficients ρ^{x-}, ρ^{x+}, ψ, ρ^{π}, λ, ρ^{i}, γ^{x}, γ^{π}.

We now want to solve the model for a given slope coefficient of volatility with respect to the output gap b and the variance-covariance matrix Σ_u. We therefore solve for the slope coefficients ρ^{x-} and ρ^{x+} in terms of the preference parameters and volatilities.

We have that

$$\begin{aligned} \rho^{x-} &= \frac{\theta}{1+\theta^*} && (25) \\ \rho^{x+} &= \frac{1}{1+\theta^*} && (26) \\ \psi &= \frac{1}{\alpha(1+\theta^*)} && (27) \\ \theta^* &= \theta - \alpha b \bar{\sigma}^2/2 && (28) \\ \bar{\sigma}^2 &= Q^M \Sigma_u Q^{M\prime} && (29) \\ Q^M &= e_1 Q - (1+\theta^*)e_1 && (30) \end{aligned}$$

θ^* is therefore a fixed point:

$$\theta^* = \theta - \frac{1}{2}\alpha b \left(e_1 Q - (1+\theta^*)e_1\right) \Sigma_u \left(e_1 Q - (1+\theta^*)e_1\right)' \tag{31}$$

This fixed point therefore depends on the matrix Q, which depends on the solution for state variable dynamics. It would therefore substantially complicate the solution if we wanted to hold b constant across sub periods.

A.3 Bond Returns

We solve for nominal and real bond log return surprises in terms of the fundamental vector of shocks u_t. We use the loglinear framework of Campbell and Ammer (1993) and do not impose the Expectations Hypothesis. We maintain our previous simplifying approximation that risk premia on one period nominal bonds equal zero. Risk premia on longer-term bonds are allowed to vary.

We write $r_{n-1,t+1}$ for the real one-period return on a real n-period bond from time t to time t+1 and $xr_{n-1,t+1}$ for the corresponding return in excess of r_t. $r^{\$}_{n-1,t+1}$ denotes the nominal one-period return on a similar nominal bond and $xr^{\$}_{n-1,t+1}$ the corresponding excess return over i_t. We use the identities:

$$
\begin{aligned}
r^{\$}_{n-1,t+1} - E_t r^{\$}_{n-1,t+1} &= -(E_{t+1} - E_t) \sum_{j=1}^{n-1} \left(\hat{i}_{t+j} + \pi^*_{t+j} \right) && (32) \\
&\quad - (E_{t+1} - E_t) \sum_{j=1}^{n-1} xr^{\$}_{n-j-1,t+1+j} && (33) \\
r_{n-1,t+1} - E_t r_{n-1,t+1} &= -(E_{t+1} - E_t) \sum_{j=1}^{n-1} r_{t+j} && (34) \\
&\quad - (E_{t+1} - E_t) \sum_{j=1}^{n-1} xr_{n-j-1,t+1+j} && (35)
\end{aligned}
$$

We now derive recursive expressions for unexpected nominal and real bond returns. We guess the functional forms:

$$
\begin{aligned}
E_t xr^{\$}_{n-1,t+1} &= (1 - bx_t) b^{\$,n} && (36) \\
E_t xr_{n-1,t+1} &= (1 - bx_t) b^{n} && (37)
\end{aligned}
$$

The functional forms (36) and (37) hold for $n = 1$ with $b^{\$,1} = b^1 = 0$. Assuming (36) and (37) for maturities less than n, we can express (33) and (35) as:

$$
\begin{aligned}
-(E_{t+1} - E_t) \sum_{j=1}^{n-1} xr^{\$}_{n-j-1,t+1+j} &= b \sum_{j=1}^{n-1} b^{\$,n-j} e_1 P^{j-1} Q u_{t+1} && (38) \\
-(E_{t+1} - E_t) \sum_{j=1}^{n-1} xr_{n-j-1,t+1+j} &= b \sum_{j=1}^{n-1} b^{n-j} e_1 P^{j-1} Q u_{t+1} && (39)
\end{aligned}
$$

We can express (32) and (34) as:

$$
\begin{aligned}
-\left(E_{t+1}-E_t\right)\sum_{j=1}^{n-1}(\hat{i}_{t+j}+\pi^*_{t+j}) &= -e_3\left[I-P\right]^{-1}\left[I-P^{n-1}\right]Qu_{t+1} \\
&\quad -(n-1)u^*_{t+1} \qquad (40)\\
-\left(E_{t+1}-E_t\right)\sum_{j=1}^{n-1} r_{t+j} &= -(e_3-e_2P)\left[I-P\right]^{-1}\left[I-P^{n-1}\right]Qu_{t+1} \qquad (41)
\end{aligned}
$$

Denoting

$$
\begin{aligned}
S^{\$,n} &= -(n-1)e_4 - e_3\left[I-P\right]^{-1}\left[I-P^{n-1}\right]Q, \qquad (42)\\
S^{n} &= -(e_3-e_2P)\left[I-P\right]^{-1}\left[I-P^{n-1}\right]Q, \qquad (43)
\end{aligned}
$$

we obtain:

$$
\begin{aligned}
r^{\$}_{n-1,t+1}-E_t r^{\$}_{n-1,t+1} &= \underbrace{\left[S^{\$,n}+b\sum_{j=1}^{n-1} b^{\$,n-j}e_1P^{j-1}Q\right]}_{A^{\$,n}}u_{t+1}, \qquad (44)\\
r_{n-1,t+1}-E_t r_{n-1,t+1} &= \underbrace{\left[S^{n}+b\sum_{j=1}^{n-1} b^{n-j}e_1P^{j-1}Q\right]}_{A^{n}}u_{t+1}. \qquad (45)
\end{aligned}
$$

The conditional expected return adjusted for Jensen's inequality equals the conditional covariance between bond excess returns and marginal utility. It hence follows that:

$$
\begin{aligned}
b^{\$,n} &= \alpha\left[S^{\$,n}+b\sum_{j=1}^{n-1} b^{\$,n-j}e_1P^{j-1}Q\right]\Sigma_u Q^{M\prime} \qquad (46)\\
&\quad -\frac{1}{2}\left[S^{\$,n}+b\sum_{j=1}^{n-1} b^{\$,n-j}e_1P^{j-1}Q\right]\Sigma_u\left[S^{\$,n}+b\sum_{j=1}^{n-1} b^{\$,n-j}e_1P^{j-1}Q\right]' \qquad (47)
\end{aligned}
$$

Similarly, we obtain the recursive expression:

$$
\begin{aligned}
b^{n} &= \alpha\left[S^{n}+b\sum_{j=1}^{n-1} b^{n-j}e_1P^{j-1}Q\right]\Sigma_u Q^{M\prime} \qquad (48)\\
&\quad -\frac{1}{2}\left[S^{n}+b\sum_{j=1}^{n-1} b^{n-j}e_1P^{j-1}Q\right]\Sigma_u\left[S^{n}+b\sum_{j=1}^{n-1} b^{n-j}e_1P^{j-1}Q\right]'. \qquad (49)
\end{aligned}
$$

Up to a constant, log yields of nominal and real zero coupon bonds then equal:

$$\begin{aligned} y^{\$}_{n,t} &= \frac{1}{n}E_t\sum_{j=0}^{n-1} r^{\$}_{n-j-1,t+1+j} && (50)\\ &= \frac{1}{n}E_t\sum_{j=0}^{n-1} i_{t+j} - \frac{1}{n}E_t\sum_{j=0}^{n-1} bb^{\$,n-j}x_{t+j} && (51)\\ &= \pi^*_t + \underbrace{\left[\frac{1}{n}e_3[I-P]^{-1}[I-P^n] - \frac{b}{n}\sum_{j=0}^{n-1} b^{\$,n-j}e_1P^j\right]}_{\Gamma^{\$,n}}\hat{Y}_t && (52)\\ y_{n,t} &= \underbrace{\left[\frac{1}{n}(e_3-e_2P)[I-P]^{-1}[I-P^n] - \frac{b}{n}\sum_{j=0}^{n-1} b^{n-j}e_1P^j\right]}_{\Gamma^{n}}\hat{Y}_t && (53)\end{aligned}$$

We can then calculate the conditional slope of the term structure as follows:

$$\begin{aligned} y^{\$}_{n,t} - i_t &= \frac{1}{n}E_t\sum_{j=0}^{n-1} i_{t+j} - i_t + \frac{1}{n}E_t\sum_{j=0}^{n-1} b^{\$,n-j}(1-bx_{t+j}) && (54)\\ &= \frac{1}{n}E_t\sum_{j=0}^{n-1} \hat{i}_{t+j} - \hat{i}_t + \frac{1}{n}E_t\sum_{j=0}^{n-1} b^{\$,n-j}(1-bx_{t+j}) && (55)\\ &= \left(\Gamma^{\$,n} - e_3\right)\hat{Y}_t + \frac{1}{n}\sum_{j=0}^{n-1} b^{\$,n-j} && (56)\end{aligned}$$

With $\hat{Y}_t$ mean zero, the average slope of the term structure and the average conditional expected bond excess return are:

$$\begin{aligned} E\left(y^{\$}_{n,t} - i_t\right) &= \frac{1}{n}\sum_{j=0}^{n-1} b^{\$,n-j} && (57)\\ E\left(E_t xr^{\$}_{n-1,t+1} + \frac{1}{2}Var_t(xr^{\$}_{n-1,t+1})\right) &= \alpha A^{\$,n}\Sigma_u Q^{M\prime} && (58)\end{aligned}$$

A.4 Stock Returns

Modeling stocks as a levered claim on the output gap x_t, we assume that dividends are given by:

$$d_t = \delta x_t. \tag{59}$$

We interpret δ as capturing a broad concept of leverage, including operational leverage.

We write r^e_{t+1} for the log stock return and xr^e_{t+1} for the log stock return in excess of r_t. Following Campbell (1991) we decompose stock returns into dividend news, news about real interest rates, and news about future excess stock returns ignoring constants:

$$\begin{aligned} r^e_{t+1} - E_t r^e_{t+1} &= \delta\left(E_{t+1} - E_t\right)\sum_{j=0}^{\infty}\rho^j \Delta x_{t+1+j} - \left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{t+j} \\ &\quad - \left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j xr^e_{t+1+j} \end{aligned} \tag{60}$$

ρ is a loglinearization constant close to 1. Now guess the functional form:

$$E_t xr^e_{t+1} = (1 - bx_t)b^e. \tag{61}$$

Then:

$$r^e_{t+1} - E_t r^e_{t+1} = \left(\kappa A^x + A^r\right)u_{t+1}, \tag{62}$$

where

$$A^x = e_1[I - \rho P]^{-1}Q, \tag{63}$$

$$A^r = -\rho(e_3 - e_2 P)[I - \rho P]^{-1}Q, \tag{64}$$

$$\kappa = \delta(1 - \rho) + \rho \times b \times b^e. \tag{65}$$

We also write:

$$A^e = \left(\kappa A^x + A^r\right). \tag{66}$$

κA^x captures the stock returns' exposure to long-term news about the output gap. A^r captures the exposure of stock returns to real interest rate news.

The conditional equity premium adjusted for Jensen's inequality equals the conditional covariance of excess stock returns and marginal utility:

$$\begin{aligned} E_t xr^e_{t+1} + \frac{1}{2}Var_t\left(xr^e_{t+1}\right) &= \alpha Cov_t(r^e_{t+1}, s_{t+1} + c_{t+1}) && (67) \\ &= \alpha A^e \Sigma_u Q^{M\prime}(1 - bx_t) && (68) \end{aligned}$$

The average conditional equity premium is then given by:

$$E\left(E_t xr^e_{t+1} + \frac{1}{2}Var_t\left(xr^e_{t+1}\right)\right) = \alpha A^e \Sigma_u Q^{M\prime} \tag{69}$$

It then follows that expected stock returns indeed take the hypothesized form, where κ is the positive root of the quadratic equation:

$$\begin{aligned} 0 &= \kappa^2 + \kappa \times 2\frac{(\rho b)^{-1} - \alpha A^x \Sigma_u Q^{M\prime} + A^x \Sigma A^{r\prime}}{A^x \Sigma_u A^{x\prime}} \\ &\quad + \frac{-2\delta(1-\rho)(\rho b)^{-1} + A^r \Sigma_u A^{r\prime} - 2\alpha A^r \Sigma_u Q^{M\prime}}{A^x \Sigma_u A^{x\prime}} \end{aligned} \tag{70}$$

Applying the Campbell and Shiller (1988) approximate loglinear present value model to equity prices (ignoring constants), we obtain log dividend price ratio:

$$
\begin{aligned}
d_t - p_t &= -\delta E_t \sum_{j=0}^{\infty} \rho^j \Delta x_{t+1+j} + E_t \sum_{j=0}^{\infty} \rho^j (r^e_{t+1+j} - r_{t+j}) + E_t \sum_{j=0}^{\infty} \rho^j r_{t+j} \qquad (71) \\
&= \left[\delta e_1 (I - P) - (b \times b^e) e_1 + e_3 - e_2 P\right] [I - \rho P]^{-1} \hat{Y}_t. \qquad (72)
\end{aligned}
$$

The model has implications for the relation between the log dividend price ratio and expected long-term excess stock returns. Denoting the k-period log equity return in excess of short-term real T-bills by $xr^e_{t \to t+k}$:

$$
E_t xr^e_{t \to t+k} = -(b \times b^e) e_1 [I - P]^{-1} [I - P^k] \hat{Y}_t. \qquad (73)
$$

A.4.1 Bond-Stock Covariances

The conditional nominal and real bond-stock return covariances equal:

$$
\begin{aligned}
Cov_t(r^e_{t+1}, r^{\$}_{n-1,t+1}) &= A^{\$,n} \Sigma_u A^{e\prime} (1 - b x_t) \qquad (74) \\
Cov_t(r^e_{t+1}, r_{n-1,t+1}) &= A^{n} \Sigma_u A^{e\prime} (1 - b x_t) \qquad (75)
\end{aligned}
$$

The nominal bond return loadings $A^{\$,n}$, as defined in in (44), contain a term $-(n-1) \times [0, 0, 0, 1]$ increasing linearly in bond duration and for long-term nominal bonds this is the dominant term. If a positive shock to the inflation target increases stock returns, this term contributes negatively to the nominal bond-stock covariance. If a positive shock to the inflation target decreases stock returns, this term contributes positively.

The variances of equity excess returns, nominal and real bond excess returns are:

$$
\begin{aligned}
Var_t(r^e_{t+1}) &= A^e \Sigma_u A^{e\prime} (1 - b x_t), \qquad (76) \\
Var_t(r^{\$}_{n-1,t+1}) &= A^{\$,n} \Sigma_u A^{\$,n\prime} (1 - b x_t), \qquad (77) \\
Var_t(r_{n-1,t+1}) &= A^{n} \Sigma_u A^{n\prime} (1 - b x_t). \qquad (78)
\end{aligned}
$$

The conditional stock market betas of nominal and real bonds are independent of x_t and given by:

$$
\beta_t(r^{\$}_{n-1,t+1}) = \frac{A^{\$,n} \Sigma_u A^{e\prime}}{A^e \Sigma_u A^{e\prime}}, \qquad (79)
$$

$$
\beta_t(r_{n-1,t+1}) = \frac{A^{n} \Sigma_u A^{e\prime}}{A^e \Sigma_u A^{e\prime}}. \qquad (80)
$$

A.5 Estimable VAR(1) in Output, Inflation, and Nominal Yields

While standard empirical measures are available for the output gap, we do not observe the interest rate and inflation gaps. We therefore cannot directly estimate the recursive law of motion (13). However, for a long-term bond maturity n, we can estimate a VAR(1) in the vector:

$$Y_t = \begin{bmatrix} x_t \\ \pi_t \\ i_t \\ y^{\$}_{n,t} \end{bmatrix} \tag{81}$$

$$= A\left[\hat{Y}_t, \pi^*_t\right]'. \tag{82}$$

The model implies that:

$$Y_{t+1} = P^Y Y_t + Q^Y u^Y_{t+1}. \tag{83}$$

$$\tag{84}$$

Here:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ & \Gamma^{\$,n} & & 1 \end{bmatrix}, \tag{85}$$

$$P^Y = A \begin{bmatrix} P & 0 \\ 0 & 1 \end{bmatrix} A^{-1}, \tag{86}$$

$$Q^Y = A \begin{bmatrix} & Q & & \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{87}$$

$$u^Y_t = u_t \tag{88}$$

provided that the inverse of A exists.

A.6 Unconditional Second Moments

Expressions (74) through (80) allow us to calculate conditional covariances, variances, and betas, conditional on the output gap being at zero. This section shows that unconditional second moments of bond and stock returns are equal to the conditional second moments, evaluated at $x_t = 0$. The law of total variance says that for any random variables X_1 and X_2:

$$Var(X_1) = E(Var(X_1|X_2)) + Var(E(X_1|X_2). \tag{89}$$

We now apply the law of total variance to the unexpected equity return $r^e_{t+1} - E_t r^e_{t+1}$ and the output gap x_t. The unconditional variance of $r^e_{t+1} - E_t r^e_{t+1}$ is given by:

$$
\begin{aligned}
Var(r^e_{t+1} - E_t r^e_{t+1}) &= E\left[Var\left(r^e_{t+1} - E_t r^e_{t+1} \middle| x_t\right)\right] && (90)\\
&\quad + Var\left(E\left(r^e_{t+1} - E_t r^e_{t+1} \middle| x_t\right)\right) && (91)
\end{aligned}
$$

But $E\left(r^e_{t+1} - E_t r^e_{t+1} \middle| x_t\right) = 0$ for any value of x_t and therefore

$$
\begin{aligned}
Var(r^e_{t+1} - E_t r^e_{t+1}) &= E\left[Var\left(r^e_{t+1} - E_t r^e_{t+1} \middle| x_t\right)\right] && (92)\\
&= E\left[A^e \Sigma_u A^{e\prime}(1 - b x_t)\right] && (93)\\
&= A^e \Sigma_u A^{e\prime}. && (94)
\end{aligned}
$$

The expression (94) shows that the unconditional variance of equity returns equals the conditional variance of equity returns, evaluated at $x_t = 0$. It similarly follows that:

$$
\begin{aligned}
Var(r^{\$}_{n-1,t+1}) &= A^{\$,n} \Sigma_u A^{\$,n} && (95)\\
Var(r_{n-1,t+1}) &= A^n \Sigma_u A^n && (96)\\
Cov\left(r^e_{t+1}, r^{\$}_{n-1,t+1}\right) &= A^{\$,n} \Sigma_u A^{e\prime} && (97)\\
Cov\left(r^e_{t+1}, r_{n-1,t+1}\right) &= A^n \Sigma_u A^{e\prime} && (98)\\
& && (99)
\end{aligned}
$$

The unconditional betas of nominal and real bonds are therefore also equal to the conditional betas (79) and (80).

It is useful to be able to simulate unconditional second moments of model real interest rates, dividend-price ratios etc. We show that we can simulate those moments by simulating a conditionally homoskedastic VAR(1) with matrix of slope coefficients P and a conditionally homoskedastic, independently and identically distributed vector of innovations.

First, we show that the unconditional second moments of the state variables $\hat{Y}_t$ are the same as those for a conditionally homoskedastic VAR(1) with matrix of slope coefficients P and conditionally homoskedastic, independently and identically distributed vector of innovations $\epsilon_t \sim N(0, Q\Sigma_u Q')$. We denote such a conditionally homoskedastic VAR(1) process by $\tilde{Y}$. The fundamental errors of $\hat{Y}_t$ are given by:

$$
\hat{Y}_t - E(\hat{Y}_t | \hat{Y}_{t-1}, \hat{Y}_{t-2}, ...) = Q u_t. \tag{100}
$$

The vector of fundamental errors is uncorrelated across time and it therefore is vector white noise (Chapter 10, Hamilton 1994). Applying again the law of total variance, we obtain the unconditional variance-covariance matrix of the fundamental errors:

$$
\begin{aligned}
Var(Q u_t) &= E\left(Var\left(Q u_t | \hat{Y}_{t-1}\right)\right) + Var\left(E\left(Q u_t | \hat{Y}_{t-1}\right)\right) && (101)\\
&= Q \Sigma_u Q'. && (102)
\end{aligned}
$$

We can then apply Wold's theorem for vector processes (Chapter 10, Hamilton 1994) and write $\hat{Y}_t$ as a vector $MA(\infty)$ representation:

$$\hat{Y}_t = [I - PL]^{-1} Q u_t, \tag{103}$$

where L denotes the lag operator. By Hamilton (1994) Proposition 10.2:

$$Cov(\hat{Y}_t, \hat{Y}_{t-k}) = \sum_{j=0}^{\infty} P^{j+k} Q \Sigma_u Q' P^{j\prime}. \tag{104}$$

The process $\tilde{Y}_t$ has the same variance-covariance matrix of fundamental errors as $\hat{Y}_t$. The unconditional variances and covariances of $\tilde{Y}_t$ are hence also given by (104).

Second, we can simulate unconditional second moments of the log dividend price ratio, expected stock returns, and the real short-term interest rate by first simulating $\tilde{Y}$ and then computing the short-term real interest rate, the dividend-price ratio, and expected equity excess returns according to (72), (73), and (53) with $\tilde{Y}_t$ replacing $\hat{Y}_t$. This follows from the observation that if $\hat{Y}_t$ and $\tilde{Y}_t$ have identical variances and covariances at all leads and lags, then so do any linear combinations of $\hat{Y}_t$ and $\tilde{Y}_t$. The second moments of other quantities that are linear combinations of the state variables can be simulated similarly.

B A Note on Units

Our empirical yields and returns are in annualized percent units. Log real dividends and the log output gap are in natural percent units. Our empirical units are analogous to those used by CGG. Our empirical coefficients in Table 4 in the main paper can therefore be compared directly to those in CGG.

However, the Campbell and Shiller (1988) loglinearizations, the expression for the equity premium (67) and expected bond returns (47) are expressed in natural units. We therefore solve the model in natural units and subsequently report scaled parameters and model moments reflecting our choice of empirical units. Let a superscript ^{c} denote natural units used for solving the calibrated model. Values with no superscript denote the parameters and variables corresponding to empirical units.

Our quantities in empirical units are related to quantities in calibration units according to: $x_t = 100x_t^c$, $i_t = 400i_t^c$, $\pi_t = 400\pi_t^c$, and $y_t^{\$,n} = 400y_t^{\$,n}$ and $\pi_t^* = 400\pi_t^*$. We can therefore write the model as:

$$
\begin{aligned}
x_t &= \rho^{x-,c} x_{t-1} + \rho^{x+,c} E_{t-} x_{t+1} - \frac{\psi^c}{4}\left(E_{t-} i_t - E_{t-}\pi_{t+1}\right) + 100 \times u_t^{IS,c} && (105)\\
\pi_t &= \rho^{\pi,c}\pi_{t-1} + (1-\rho^{\pi,c}) E_{t-}\pi_{t+1} + 4\lambda^c x_t + 400 \times u_t^{PC,c} && (106)\\
i_t &= \rho^{i,c}\left(i_{t-1} - \pi^*_{t-1}\right) + (1-\rho^{i,c})\left[4\gamma^{x,c} x_t + \gamma^{\pi,c}(\pi_t - \pi^*_t)\right] + \pi^*_t + 400 u_t^{MP,c} && (107)\\
\pi^*_t &= \pi^*_{t-1} + 400 u^*_t && (108)
\end{aligned}
$$

Equations (105) through (108) imply relations between the empirical and calibration parameters:

$$
\begin{aligned}
\rho^{x-} &= \rho^{x-,c}, \rho^{x+} = \rho^{x+,c}, \psi = \frac{\psi^c}{4} && (109)\\
\rho^{\pi} &= \rho^{\pi,c}, \lambda = 4\lambda^c && (110)\\
\rho^{i} &= \rho^{i,c}, \gamma^x = 4\gamma^{x,c}, \gamma^{\pi} = \gamma^{\pi,c} && (111)\\
\bar{\sigma}^{IS} &= 100\bar{\sigma}^{IS,c}, \bar{\sigma}^{PC} = 400\bar{\sigma}^{PC,c}, \bar{\sigma}^{MP} = 400\bar{\sigma}^{MP,c}, \bar{\sigma}^* = 400\bar{\sigma}^* && (112)
\end{aligned}
$$

Fuhrer (1997) estimates a Phillips curve with both backward-looking and forward-looking components. Using inflation in annualized percent, and the log output gap in natural units, he find a backward-looking component of 0.8, a forward-looking component of 0.2, and a weight on the output gap of 0.12. We can therefore compare the parameter λ in empirical units directly to the magnitudes in CGG, Fuhrer (1997), and Roberts (1995).

Yogo (2004) scales interest rates and inflation to quarterly units. Our calibrated values for ψ^c in natural units can therefore be compared directly to the estimated values in Yogo (2004). We therefore report the value ψ^c corresponding to natural units rather than ψ corresponding to empirical units throughout the paper.

We choose the leverage parameter δ to match the relative volatilities of log real dividend growth and log output gap growth. We use four quarter growth rates to smooth out some of the more seasonal fluctuations. We consider four quarter log output growth $\Delta x_t = x_t - x_{t-4}$. The standard deviation of this growth rate over the period 1960.Q1-2011.Q4 is 2.20%. Let d_t denote the sum of log S&P 500 real dividends. Monthly real S&P 500 dividends are from Robert Shiller's web site. These real dividends are deflated by the not seasonally adjusted CPI-U with a basis of 1982-84=100. We obtain quarterly dividends by summing the level real dividends within the quarter. The standard deviation 1960.Q1-2011.Q4 of the four quarter log dividend growth rate $\Delta d_t = d_t - d_{t-4}$ equals 5.35%. Our model specifies dividends as a levered claim on the output gap with $d_t = \delta x_t$. We therefore set the leverage parameter δ to match the relative standard deviations of output and dividend growth. This gives $\delta = 2.43$.

Due to our choice of empirical units, we use a slightly different transformation from the transition matrix P^c of the state variables in natural unit to the transition matrix P^Y of the estimable VAR(1). We have the relation:

$$\begin{aligned} Y_t &= \begin{bmatrix} x_t \\ \pi_t \\ i_t \\ y^{\$}_{n,t} \end{bmatrix} && (113) \\ &= A^c \left[\hat{x}^c_t, \hat{\pi}^c_t, \hat{i}^c_t, \pi^{c,*}_t \right]', && (114) \end{aligned}$$

where:

$$\begin{aligned} A^c &= diag(100, 400, 400, 400) \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ & \Gamma^{\$,n} & & 1 \end{bmatrix}, && (115) \\ P^Y &= A^c \begin{bmatrix} P & 0 \\ 0 & 1 \end{bmatrix} A^{c-1}, && (116) \\ Q^Y &= A^c \begin{bmatrix} & Q & & \\ 0 & 0 & 0 & 1 \end{bmatrix}, && (117) \\ u^{c,Y}_t &= \left[u^{c,IS}_t,\ u^{c,PC}_t,\ u^{c,MP}_t,\ u^{c,*}_t \right]', && (118) \\ Y_t &= P^Y Y_{t-1} + Q^Y u^{c,Y}_{t+1}. && (119) \end{aligned}$$

We report annualized percent standard deviations of equity and bond returns at the average output gap $x_t = 0$. We calculate annualized standard deviations of equity and bond returns in percent at $x_t = 0$:

$$\begin{aligned} Std_t(r^e_{t+1}) &= 200\sqrt{A^e \Sigma_u A^{e\prime}}, && (120) \\ Std_t(r^{\$}_{n-1,t+1}) &= 200\sqrt{A^{\$,n} \Sigma_u A^{\$,n\prime}}, && (121) \\ Std_t(r_{n-1,t+1}) &= 200\sqrt{A^n \Sigma_u A^{n\prime}}. && (122) \end{aligned}$$

We back out empirical shocks for each sub period separately. From the empirical series $Y^{emp}_t = \left[x^{emp}_t, \pi^{emp}_t, i^{emp}_t, y^{emp,\$,n}_t \right]$, we back out fundamental shocks in empirical units:

$$\begin{aligned} \left[u^{IS}_t, u^{PC}_t, u^{MP}_t, u^*_t \right]' &= \left[100 u^{c,IS}_t,\ 400 u^{c,PC}_t,\ 400 u^{c,MP}_t,\ 400 u^{c,*}_t \right]', && (123) \\ &= diag(100, 400, 400, 400) \times Q^{Y-1} \left(Y^{emp}_t - P^Y Y^{emp}_{t-1} \right). && (124) \end{aligned}$$

We transform the parameter b into empirical units according to $b = b^c/100$. Then $(1 - b^c x^c_t) = (1 - b x_t)$. We calculate the standard deviation of the volatility factor $(1 - b x_t)$ at $x_t = 0$ according to $b\sqrt{e_1 Q \Sigma_u Q' e_1'}$.

B.1 Partial Derivatives

We compute the partial derivative of the nominal bond beta with respect to $ln(\bar{\sigma}_u^k)$ as follows:

$$\frac{\partial \beta^{\$}}{\partial ln \bar{\sigma}_u^k}| = \frac{1}{A^e \Sigma_u A^{e\prime}} 2A^{\$,n} e_k^{\prime} e_k A^{e\prime} \bar{\sigma}_u^{k2} \tag{125}$$

$$\frac{\partial Std_t(r_{t+1}^e)}{\partial ln \bar{\sigma}_u^k} = \frac{200 A^e e_k^{\prime} e_k A^{e\prime} \bar{\sigma}_u^{k2}}{\sqrt{A^e \Sigma_u A^{e\prime}}} \tag{126}$$

$$\frac{\partial Std(r_{n-1,t+1}^{\$})}{\partial ln \bar{\sigma}_u^k} = \frac{200 A^{\$,n} e_k^{\prime} e_k A^{\$,n\prime} \bar{\sigma}_u^{k2}}{\sqrt{A^{\$,n} \Sigma_u A^{\$,n\prime}}} \tag{127}$$

The partial derivatives for the nominal bond beta sum to two times the calibrated nominal bond beta for each sub period. The partial derivatives for the standard deviations of asset returns sum to the calibrated standard deviation of asset returns for each sub period.

C Details of Moment Fitting Procedure

We minimize the distance between model and empirical moments summed over all three sub-periods. We use a superscript p to denote period p moments and a hat to denote empirically estimated moments. Our objective function is:

$$Obj = \sum_{p=1}^{3} \left[\left\| P^{Y,p} - \hat{P}^{Y,p} \right\|^2 + \left\| diag(Q^{Y,p} \Sigma_u Q^{Y,p}) - diag(\widehat{Q^{Y,p} \Sigma_u Q}^{Y,p}) \right\|^2 \right. \tag{128}$$

$$+ (\frac{1}{10}(Std^p(r_{n-1,t+1}^{\$}) - \widehat{Std^p(r_{n-1,t+1}^{\$}})))^2 \tag{129}$$

$$+ (\frac{1}{10}(Std^p(r_{t+1}^e) - \widehat{Std^p(r_{nt+1}^e}))))^2 \tag{130}$$

$$\left. + (10 \times (\beta^p(r_{n-1,t+1}^{\$}) - \widehat{\beta^p(r_{n-1,t+1}^{\$}})))^2 \right] \tag{131}$$

We optimize Obj over the following parameters: ρ^{x-}, ρ^{x+}, $\bar{\sigma}^{IS,p}$, $\bar{\sigma}^{PC,p}$, $\bar{\sigma}^{MP,p}$, and $\bar{\sigma}^{*,p}$, $p = 1, 2, 3$. We hold all other parameters constant at the values shown in Table 5 in the main paper.

In order to reduce the dimensionality of the minimization problem, we minimize Obj iteratively. First, we minimize with respect to the standard deviations of shocks while holding the Euler equation parameters ρ^{x+} and ρ^{x-} constant at initial guesses. Second, we minimize with respect to ρ^{x+} and ρ^{x-} while holding constant the standard deviations of shocks at their optimal values from the first step. Third, we minimize again with respect to the standard deviations of shocks holding constant ρ^{x+} and ρ^{x-} at their optimal values from the second step.

Step 1: Starting from an initial guess of $\rho^{x-} = 0.4503$ and $\rho^{x+} = 0.6161$, we first minimize with respect to $\bar{\sigma}^{IS,p}$, $\bar{\sigma}^{PC,p}$, $\bar{\sigma}^{MP,p}$, and $\bar{\sigma}^{*,p}$ holding ρ^{x-} and ρ^{x+} constant. Given ρ^{x-} and ρ^{x+}, we can minimize with respect to $\bar{\sigma}^{IS,p}$, $\bar{\sigma}^{PC,p}$, $\bar{\sigma}^{MP,p}$, and $\bar{\sigma}^{*,p}$ independently for each period p.

We use a simple and robust minimization procedure. We randomly draw 50000 parameter vectors. We draw $\bar{\sigma}^{IS,p}, \bar{\sigma}^{PC,p}, \bar{\sigma}^{MP,p}, \bar{\sigma}^{*,p}$ from independent uniform distributions. Our support intervals for 1960.Q1-1979.Q2 are such that $\left[(\bar{\sigma}^{IS,1}), (\bar{\sigma}^{PC,1}), (\bar{\sigma}^{MP,1}), (\bar{\sigma}^{*,1})\right] \in [0,0,0,0] \times [0.8256, 2.2069, 2.2768, 0.7715]$. Our support intervals for 1979.Q3-1996.Q4 are such that $\left[(\bar{\sigma}^{IS,2}), (\bar{\sigma}^{PC,2}), (\bar{\sigma}^{MP,2}), (\bar{\sigma}^{*,2})\right] \in [0,0,0,0] \times [0.7800, 1.3338, 4.3310, 1.1228]$. Our support intervals for 1997.Q1-2011.Q4 are such that $\left[(\bar{\sigma}^{IS,3}), (\bar{\sigma}^{PC,3}), (\bar{\sigma}^{MP,3}), (\bar{\sigma}^{*,3})\right] \in [0,0,0,0] \times [0.6153, 1.8542, 0.8119, 1.0595]$. Minimizing with respect to the volatilities of shocks for each sub sample yields:

$$\begin{pmatrix} \bar{\sigma}^{IS,1} & \bar{\sigma}^{IS,2} & \bar{\sigma}^{IS,3} \\ \bar{\sigma}^{PC,1} & \bar{\sigma}^{PC,2} & \bar{\sigma}^{PC,3} \\ \bar{\sigma}^{MP,1} & \bar{\sigma}^{MP,2} & \bar{\sigma}^{MP,3} \\ \bar{\sigma}^{*,1} & \bar{\sigma}^{*,2} & \bar{\sigma}^{*,3} \end{pmatrix} = \begin{pmatrix} 0.38 & 0.54 & 0.34 \\ 1.09 & 0.83 & 0.90 \\ 1.23 & 1.93 & 0.38 \\ 0.37 & 0.72 & 0.51 \end{pmatrix} \tag{132}$$

Step 2: In the second step, we minimize with respect to ρ^{x+} and ρ^{x-} while holding the volatilities of shocks constant at the values shown in (132). We randomly draw 10000 draws from two independent uniform distributions $U_1 \in [0,1]$ and $U_2 \in [0,1]$ and set $\rho^{x-} = 0.4253 + 0.05U_1$ and $\rho^{x+} = (1 - \rho^{x-}) + 0.2 \times \rho^{x-}U_2$, thereby ensuring that ρ^{x+} and ρ^{x-} sum to more than 1. We obtain minimizing parameter values $\rho^{x-} = 0.4466$ and $\rho^{x+} = 0.6224$ agreeing with the initial guesses up to two significant digits. Figure A.1 shows the objective function Obj against U_1 and U_2. Each dot corresponds to one combination of parameter values. Figure A.1 shows that the optimizing parameter values are in the middle of the ranges considered. We therefore are not at a boundary solution. Moreover, the optimal parameter values occur at a clear minimum, indicating that the parameters ρ^{x+} and ρ^{x-} are well identified.

Step 3: The third step is exactly the same as the first step, except that we hold ρ^{x-} and ρ^{x+} constant at their new values. Figure A.2 shows the objective function against the standard deviations of shocks for each sub sample period. If the volatilities of shocks are well identified, the lower envelopes of the scatter plots in Figure A.2 should have clear minima. It appears that the objective function exhibits clear minima with respect to each of the shock volatilities. Figure A.2 shows that all volatilities are in the interior of the intervals that we are optimizing over. This finding is reassuring in that it suggests that we are considering sufficiently large ranges.

The optimal volatilities of shocks are shown in Table 5 in the main paper. These optimal volatilities are close to the preliminary values (132). Moreover, the final values for ρ^{x+} and ρ^{x-} are very close to the initial guesses, indicating convergence of our algorithm.

D Additional Calibration Features and Robustness

Table A.1 shows the matrix of slope coefficients for the quarterly VAR(1) in the log output gap, inflation, the Federal Funds rate, and the 5 year nominal yield both in the model and in the data. Table A.1 shows that the calibrated model can generate substantial persistence in the output gap, inflation, Fed Funds rate, and the long-term nominal yield, even though the output gap is somewhat less persistent than in the data. The off-diagonal elements are generally small and often close to zero.

Figure A.3 shows a time series of smoothed shocks backed out from our sub period calibrations. For each sub period, we back out the fundamental model shocks by inverting the relation $Y_{t+1} = P^Y Y_t + Q^Y u^Y_{t+1}$ and plugging in the empirical time series for the vector Y_t and the model implied matrices P^Y and Q^Y.

Tables A.2 and A.3 present an alternative calibration and are analogous to Tables 5 and 6 in the main paper. The alternative calibration fits the volatility of VAR(1) residual volatilities, and the volatilities of bond and stock returns, but not the nominal bond beta. Table A.2 shows that in the alternative calibration we obtain a lower volatility of PC shocks in the middle sub period. Consequently, the alternative calibration obtains a negative nominal bond beta in the second sub period instead of a positive nominal bond beta.

Table A.4 shows additional moments from the calibration. The equity premium is close to 4% on an annualized basis.

The model assumes that shocks are uncorrelated for parsimony. We would therefore expect that model-implied shocks should be uncorrelated. Table A.5 reports the univariate correlations between IS, PC, MP, and inflation target shocks for each subperiod calibration. The average correlations are generally close to zero. However, the correlation between the inflation target shock and monetary policy shock stands out and is negative in all three subperiods. This implied negative correlation can quite plausibly be a result of assuming a monetary policy rule, which smoothes the interest rate gap, rather than the interest rate. We therefore consider robustness to an alternative monetary policy rule in the next section.

D.1 Robustness to Alternative MP Rule

In our main formulation, the central bank smoothes the difference between the Fed Funds rate and the inflation target, following an interest rate rule of the form:

$$i_t = \rho^i (i_{t-1} - \pi^*_{t-1}) + (1 - \rho^i) \left[\gamma^x x_t + \gamma^\pi (\pi_t - \pi^*_t)\right] + \pi^*_t + u^{MP}_t. \qquad (133)$$

In this section, we consider a similar, alternative formulation, which instead smoothes the level of the Fed Funds rate:

$$\begin{aligned} i_t &= \rho^i i_{t-1} + (1-\rho^i)\left[\gamma^x x_t + \gamma^\pi(\pi_t - \pi_t^*) + \pi_t^*\right] + \tilde{u}_t^{MP} & (134) \\ &= \rho^i(i_{t-1} - \pi_{t-1}^*) + (1-\rho^i)\left[\gamma^x x_t + \gamma^\pi(\pi_t - \pi_t^*)\right] + \pi_t^* - \rho^i \tilde{u}_t^* + \tilde{u}_t^{MP}. & (135) \end{aligned}$$

Expression (135) shows that the two monetary policy rules are equivalent if $u_t^{MP} = -\rho^i \tilde{u}_t^* + \tilde{u}_t^{MP}$, so the monetary policy shock in (133) will be more negatively correlated with the inflation target shock u_t^* than the monetary policy shock in (134). Considering a model calibration with $\tilde{u}_t^*$ and $\tilde{u}_t^{MP}$ independent, the alternative monetary policy rule can therefore address the negative correlation between implied MP and inflation target shocks in Table A.5.

The model solution takes exactly the same form as before, the only difference being that now:

$$M = \begin{bmatrix} 1 & 0 & \psi & 0 \\ 0 & 1 & 0 & -\rho^\pi \\ 0 & 0 & 1 & -\rho^i \end{bmatrix}. \quad (136)$$

We re-create Figures 3 and 4 with the monetary policy rule (135) using the parameter values shown in Table 5. Since we do not re-calibrate the model to match macroeconomic and asset pricing moments, we do not expect the modified model to match the empirical betas.

Figures A.4 and A.5 look very similar to Figures 3 and 4 in the main text, indicating that the identified monetary policy drivers act similarly and similarly strongly in the alternative model. Figures A.3 and A.4 differ from Figures 3 and 4 in that the read and blue regions have shifted slightly relative to the dots indicating the estimated monetary policy coefficients for each regime.

Importantly, the nominal bond beta is still strongly increasing in the inflation weight γ^π, weakly increasing in the output weight γ^x, and increasing and nonlinear in the monetary policy persistence coefficient ρ^i. Panel B of Figure A.5 indicates that even in this alternative, not fully calibrated model, the increase in monetary policy persistence would have switched the nominal bond beta from positive to negative if all other parameters had remained constant.

D.2 Robustness to Phillips Curve Parameter ρ^π

In our calibration, we use a Phillips curve with a backward looking component of $\rho^\pi = 0.8$, which is consistent with the empirical estimates of Fuhrer (1997). However, a wide range of empirical estimates are available for ρ^π are available in the literature. For instance, Gali and Gertler (1999) use the labor share of income instead of the output gap and estimate a smaller backward-looking component in the Phillips curve.

Choosing a high value of ρ^π ensures that our model has a unique solution for a large range of monetary policy functions.

We verify that the main mechanism identified in this paper is robust to choosing a lower value of ρ^π. Unfortunately, the range of parameter values with a stable and unique model solution shrinks rapidly when we choose smaller values of ρ^π. We therefore consider an alternative value for ρ^π of 0.7. Considering even smaller values would leave us with no solution for a large range of monetary policy functions.

Figures A.6 and A.7 are analogous to Figures 3 and 4 in the main paper, but they use a lower value for ρ^π. All other parameters are as in Table 5 in the main paper. We do not re-calibrate the model and we therefore do not expect the model to match empirical asset pricing moments exactly. The main difference between Figures A.6 and A.7 and Figures 3 and 4 in the main text is that in Figures A.6 and A.7 the regions where no solution exists (shown in white) are much larger. Whenever a solution exists, it is very similar to the one in the main calibration. Importantly, the dependence of the nominal bond beta on the monetary policy parameters γ^π, γ^x, and ρ^i looks very similar.

D.3 Effect of Phillips Curve Slope

There is some empirical evidence that in addition to the shock volatilities and the monetary policy rule parameters (which are allowed to vary across our subperiod calibrations), the slope of the Phillips curve. λ may also have changed over time. Smets and Wouters (2007) report some evidence that the slope of the Phillips curve may have decreased over time. We investigate whether this might explain the sign switch in the empirical nominal bond beta in the late 1990s.

Figure A.8 plots the nominal bond beta as a function of λ for all three subperiod calibrations. Figure A.8 shows that the nominal bond beta decreases in the slope of the Phillips curve λ if all other parameters are held constant at their period 2 or period 3 values. If the slope of the Phillips curve has decreased over time, our model indicates that this should have led to an increase in the nominal bond beta and not to the decrease observed in the late 1990s.

D.4 Robustness to Different Leverage Parameter

Our baseline model assumes that the output gap and dividends are perfectly correlated, while in the data the correlation is much lower. We could follow a similar route as in Campbell and Cochrane (1999) and address this issue by modeling dividends as proportional to the output gap plus an idiosyncratic shock. In order to maintain a realistic ratio of the dividend growth volatility and output gap growth volatility, we would need to reduce the sensitivity of dividends to the output gap.

Campbell and Cochrane (1999) remark that their habit formation model has very similar implications no matter whether they model dividend growth as perfectly or imperfectly correlated with consumption growth.

We study the sensitivity of our model implications with respect to the leverage parameter δ in order to understand how important the assumption of perfectly correlated dividends and output gap is for our findings. Adding an idiosyncratic, unpriced, shock to dividends is unlikely to change substantially any asset pricing dynamics. Modeling dividends as imperfectly correlated with the output gap should therefore yield implications that are very similar to reducing the leverage parameter δ.

Figure A.9 is equivalent to Figure 4 in the main text, but it sets $\delta = 1$, corresponding to an extremely low firm leverage ratio of 0%. We can see that the implications for nominal bond betas are qualitatively and quantitatively extremely similar to the baseline calibration. In fact, Figure A.9 and Figure 4 in the main paper are visually indistinguishable (except for the fact that Figure A.9 uses fewer pixels). We interpret Figure A.9 as indicating that our results are not sensitive to assuming a strong correlation between dividends and the output gap.

Figure A.10 explores the real bond beta as a function of monetary policy parameters. The figure is analogous to Figure 4, Panel A in the main paper, except that this figure plots the beta of real bonds instead of nominal bonds. We can see that the real bond beta is negative for a wide region around the period 1 monetary policy parameters, which are indicated with a diamond. Moreover, the real bond beta decreases sharply in the MP coefficient γ^{π}. Table 10 in the main paper implies that, had TIPS existed, they would have been extremely safe with a beta of -215.55. Given that the region around period 1 monetary policy parameters includes much more reasonable values for the real bond beta, we believe we could recalibrate our model relatively easily with a restriction that the implied volatility or beta of TIPS must fall within a certain range. Given that the model mechanism generally seems robust to small changes in parameters, we don't expect that this would change the model implications.

E Additional Empirical Results

Table A.6 reports Taylor rule estimates for superiod 3 (1997.Q-2011.Q4) and splits it into a pre-Lehman subsample (1997.Q1-2008.Q2) and a post-Lehman subsample (2008.Q3-2011.Q4). Interestingly, the estimates for the pre-Lehman subsample are virtually identical to the estimates for the full subperiod 3. The estimates reported in Table 4 in the main paper therefore do not appear to be mainly driven by the financial crisis, just as the negative nominal bond beta does not appear mainly driven by the financial crisis. For the post-Lehman subsample, the estimates for all monetary policy coefficients (γ^x, γ^{π}, and ρ^i) are all close to zero with small standard errors.

This is intuitive, since the most salient feature of monetary policy during the crisis was perhaps that the Federal Funds rate has been stuck at the zero lower bound. The empirical results in Table A.6 suggest that we can model post-Lehman monetary policy by setting all monetary policy parameters to zero.

Table A.7 tests for statistical significance of the changes in monetary policy parameters. It shows that the major changes (increase in γ^{π} from period 1 to period 2; increase in ρ^{i} from period 2 to period 3) are indeed statistically significant.

If changes in bond risks are driven by macroeconomic factors, then changes in bond risks should be reflected in changing macroeconomic correlations. Lower than expected inflation raises nominal bond prices, all else equal, so the inflation-output correlation should typically take the opposite sign from the bond-stock correlation.

Table A.8 compares sub-sample correlations of asset prices and macroeconomic variables. The empirical output gap is highly persistent and it is therefore unsurprising that three year equity excess returns are more strongly correlated with the output gap than highly volatile quarterly stock returns. We therefore use quarterly overlapping three year bond and stock excess returns for our comparison of asset return correlations and macroeconomic correlations. Table A.8 confirms our intuition that bond excess returns should at least partly reflect news about inflation and that equity excess returns should reflect the business cycle. In each sub period, empirical bond excess returns are negatively correlation with inflation and equity excess returns are positively correlated with the output gap.

Table A.8 confirms that the changes in the bond-stock comovement documented in Figure 1 and in Table 6 are robust to using three year returns instead of daily or quarterly returns. The correlation between three year stock returns and three year bond returns was positive and significant in the first sub-period, increased in the second sub period, and became negative and significant in the last sub period.

The bond-output gap, inflation-stock, and inflation-output gap correlations confirm our intuition that changing bond risks are related to the prevalence of inflationary recessions versus deflationary recessions during different regimes. The bond-output gap correlation typically has the same sign as the bond-stock correlation, while the inflation-stock return correlation and the inflation-output correlation has the opposite sign. The only exception to this pattern is the first sub period bond-output gap correlation, which takes a negative, but small and insignificant, value.

References

Bikbov, Ruslan, and Mikhail Chernov, 2013, "Monetary Policy Regime Changes and the Term Structure of Interest Rates ", *Journal of Econometrics* 174, 27-43.

Campbell, John Y., 1991, "A Variance Decomposition for Stock Returns, the H.G. Johnson Lecture to the Royal Economic Society ", *Economic Journal* 101, 157–179.

Campbell, John Y. and John Ammer, 1993, "What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns", *Journal of Finance* 48, 3–37.

Campbell, John Y., and Robert Shiller, 1988, "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors ", *Review of Financial Studies* 1, 195–228.

Cho, Seonghoon, and Antonio Moreno, 2011, "The Forward Method as a Solution Refinement in Rational Expectations Models ", *Journal of Economic Dynamics and Control* 35, 257–272.

Evans, George, 1985, "Expectational Stability and the Multiple Equilibria Problem in Linear Rational Expectations ", *The Quarterly Journal of Economics* 100(4), 1217–1233.

Evans, George, 1986, "Selection Criteria for Models with Non-Uniqueness ", *Journal of Monetary Economics* 18, 147–157.

Evans, George, and Seppo Honkapohja, 1994, "Learning, Convergence, and Stability with Multiple Rational Expectations Equilibria ", *European Economic Review* 38, 1071–1098.

Evans, George, and Bruce McGough, 2005, "Stable Sunspot Solutions in Models with Predetermined Variables ", *Journal of Economic Dynamics and Control* 19, 601–625.

Fudenberg, Drew, and David K. Levine, 1998, *The Theory of Learning in Games*, MIT Press, Cambridge, MA.

Fuhrer, Jeffrey C., 1997, "The (Un)Importance of Forward-Looking Behavior in Price Specifications ", *Journal of Money, Credit, and Banking* 29(3), 338–350.

McCallum, Bennett T., 1983, "On Non-Uniqueness in Rational Expectations Models An Attempt at Perspective ", *Journal of Monetary Economics* 11, 139–168.

McCallum, Bennett T., 2004, "On the Relationship Between Determinate and MSV Solutions in Linear RE Models ", *Economics Letters* 84, 55–60.

Roberts, John M., 1995, “New Keynesian Economics and the Phillips Curve”, *Journal of Money Credit and Banking*, 4(1) 975–984.

Smets, Frank, and Rafael Wouters, 2007, “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach”, *American Economic Review* 97, 586–606.

Stock, James and Mark Watson, 2010, “Modeling Inflation After the Crisis ”, NBER Working Paper 16488.

Uhlig, Harald, 1999, “A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily ”, in Ramon Marimon and Andrew Scott (eds.) *Computational Methods for the Study of Dynamic Economies*, 30–61, Oxford University Press.

Yogo, Moto (2004), “Estimating the Elasticity of Intertemporal Substitution When Instruments Are Weak”, *Review of Economics and Statistics* 86(3), 797–810.

Tables and Figures

Table A.1: Empirical and Model VAR(1) Matrices for Sub Periods

	1960.Q1-1979.Q2 Model				1960.Q1-1979.Q2 Empirical			
	Coeff. on Lagged Variables				Coeff. on Lagged Variables			
Output Gap	0.74	-0.28	-0.05	0.33	0.97	-0.05	-0.20	0.19
Inflation	0.32	0.84	-0.03	0.19	-0.04	0.42	0.37	0.42
Fed Funds Rate	0.23	0.17	0.53	0.30	0.17	0.04	0.54	0.47
Log Nom. Yield	0.07	-0.02	-0.06	1.08	0.03	0.08	0.02	0.83
	1979.Q3-1996.Q4 Model				1979.Q3-1996.Q4 Empirical			
	Coeff. on Lagged Variables				Coeff. on Lagged Variables			
Output Gap	0.73	-0.79	-0.08	0.87	0.90	-0.08	0.02	-0.07
Inflation	0.32	0.54	-0.05	0.51	0.03	0.78	0.10	-0.04
Fed Funds Rate	0.23	1.03	0.47	-0.50	0.07	0.76	0.11	0.65
Log Nom. Yield	0.22	-0.20	-0.07	1.27	-0.05	0.22	-0.02	0.83
	1997.Q1-2011.Q4 Model				1997.Q1-2011.Q4 Empirical			
	Coeff. on Lagged Variables				Coeff. on Lagged Variables			
Output Gap	0.47	-0.22	-0.16	0.39	0.93	0.04	0.03	0.13
Inflation	0.17	0.87	-0.09	0.22	0.20	0.37	-0.17	-0.06
Fed Funds Rate	0.14	0.16	0.90	-0.06	0.00	0.14	0.68	0.47
Log Nom. Yield	-0.32	-0.10	-0.13	1.23	0.07	-0.06	0.06	0.75

P^Y is the matrix of slope coefficients of a quarterly VAR(1) in the log output gap, inflation, Fed Funds rate, and 5 Year Nominal Yield.

Table A.2: Alternative Calibration Parameter Choices

Time-Invariant Parameters				
Log-Linearization Constant	ρ		0.99	
Leverage	δ		2.43	
Preference Parameter	α		30	
Backward-Looking Comp. PC	ρ^{π}		0.80	
Slope PC	λ		0.30	
Forward-Looking Comp. IS	ρ^{x+}		0.62	
Backward-Looking Comp. IS	ρ^{x-}		0.45	
Monetary Policy Rule		60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4
MP Coefficient Output	γ^{x}	0.42	-0.07	0.44
MP Coefficient Infl.	γ^{π}	0.69	1.44	1.92
Backward-Looking Comp. MP	ρ^{i}	0.56	0.43	0.89
Std. Shocks				
Std. IS	$\bar{\sigma}^{IS}$	0.38	0.39	0.32
Std. PC shock	$\bar{\sigma}^{PC}$	1.02	0.73	0.98
Std. MP shock	$\bar{\sigma}^{MP}$	1.21	1.93	0.47
Std. infl. target shock	$\bar{\sigma}^{*}$	0.33	0.68	0.54

The alternative calibration puts no weight on the nominal bond beta in fitting the standard deviations of fundamental shocks. The time-invariant parameters and monetary policy rule parameters are identical to those in Table 5 in the main paper. The standard deviations of shocks differ from the calibration in Table 5 in the main paper.

Table A.3: Alternative Calibration Model and Empirical Moments

	60.Q1-79.Q2		79.Q3-96.Q4		97.Q1-11.Q4	
Std. VAR(1) Residuals	Empirical	Model	Empirical	Model	Empirical	Model
Output Gap	0.92	0.70	0.75	0.69	0.65	0.62
Inflation	1.12	1.14	0.89	0.84	0.80	1.08
Fed Funds Rate	1.22	1.28	2.07	2.05	0.66	0.68
Log Nominal Yield	0.48	0.42	0.85	0.77	0.55	0.60
Std. Asset Returns						
Std. Eq. Ret.	17.62	17.89	15.34	16.49	20.08	19.01
Std. Nom. Bond Ret.	4.85	3.98	9.11	6.55	5.55	5.42
Nominal Bond Beta	0.06	0.12	0.20	-0.05	-0.17	-0.04
Taylor Rule: Fed Funds onto Output, Infl. and Lag. Fed Funds						
Output	0.18	0.22	-0.04	-0.13	0.05	0.02
Inflation	0.30	0.34	0.83	0.65	0.21	0.19
Lagged Fed Funds	0.56	0.59	0.43	0.39	0.89	0.81

We compare model and empirical moments for the alternative calibration. Alternative calibration parameters are specified in Table A.2. The alternative calibration puts no weight on the nominal bond beta in fitting the standard deviations of fundamental shocks.

Table A.4: Appendix Model Moments

	Model				Empirical			
	60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4	Avg.	60.Q1-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4	60.Q1-11.Q4
Equity Premium.	3.61	3.12	3.29	3.35	3.23	8.12	4.94	5.36
Nom. Bond Exc. Ret.	0.25	0.68	-0.50	0.18	0.01	2.31	2.97	1.64
$E\left(y_{5,t}^{\$}-i_t\right)$	0.14	0.39	-0.03	0.18	0.74	1.32	1.14	1.05
$Corr(x_t, y_{5,t}^{\$}-i_t)$	0.05	0.31	0.93	0.39	-0.62	-0.21	-0.53	-0.46
$Corr(x_t, y_{5,t}^{\$})$	0.01	-0.02	0.31	0.09	-0.08	-0.34	0.82	-0.07
$Corr(x_t, i_t)$	-0.02	-0.16	-0.20	-0.12	0.22	-0.21	0.80	0.05
$xr_{5,t+1}^{\$}$ onto $(y_{5,t}^{\$}-i_t)$	-0.08	-0.71	4.12	0.92	2.24	3.09	2.37	2.84*
$xr_{5,t+1}^{\$}$ onto x_t	-1.23	-3.56	5.27	-0.14	-0.72*	-0.17	-0.16	-0.47
$xr_{5,t+1}^{\$}$ onto dp_t	0.06	0.14	-0.30	-0.02	0.06	0.02	-0.03	-0.01

The equity premium and the average nominal bond excess return show average returns in excess of a short-term bond adjusted for Jensen's inequality. The last three rows show regression coefficients of log 5 year bond excess returns (Annualized, %) onto the slope of the yield curve (Annualized, %), the output gap (%), and the log dividend price ratio (%), respectively. The last three rows show $*$ when the coefficient is significant at the 5% level with Newey-West standard errors with two lags.

Table A.5: Correlations of Implied Shocks

1960.Q1-1979.Q2

	IS	PC	MP	Infl. Target
IS	1.00	-0.24	0.11	-0.14
PC		1.00	0.04	-0.34
MP			1.00	-0.40
Infl. Target				1.00

1979.Q3-1996.Q4

	IS	PC	MP	Infl. Target
IS	1.00	0.05	-0.58	-0.07
PC		1.00	0.07	-0.49
MP			1.00	-0.31
Infl. Target				1.00

1997.Q1-2011.Q4

	IS	PC	MP	Infl. Target
IS	1.00	-0.68	0.00	-0.24
PC		1.00	0.07	0.11
MP			1.00	-0.75
Infl. Target				1.00

For each sub period, we back out the fundamental model shocks by inverting the relation $Y_{t+1} = P^Y Y_t + Q^Y u^Y_{t+1}$ and plugging in the empirical time series for the vector Y_t and the model implied matrices P^Y and Q^Y.

Table A.6: Empirical Monetary Policy Function Crisis Sample

Fed Funds i_t	97.Q1-11.Q4	97.Q1-08.Q2	08.Q3-11.Q4
Output Gap	0.05	0.17	-0.02
	(0.04)	(0.12)	(0.04)
Inflation	0.21**	0.27**	-0.01
	(0.07)	(0.09)	(0.02)
Lagged Fed Funds	0.89**	0.88**	0.03
	(0.06)	(0.09)	(0.04)
Constant	-0.12	-0.27	-0.04
	(0.29)	(0.27)	(0.33)
R^2	0.91	0.86	0.22
Implied $\hat{\gamma}^x$	0.44	1.43	-0.02
	(0.21)	(0.40)	(0.05)
Implied $\hat{\gamma}^\pi$	1.92*	2.29*	-0.01
	(1.26)	(1.55)	(0.02)
Implied $\hat{\rho}^i$	0.89**	0.88**	0.03
	(0.06)	(0.09)	(0.04)

This table estimates the monetary policy rule before and after the Lehman brothers bankruptcy in 2008.Q3. All variables and test specifications are described in Table 4 in the main text.

Table A.7: Estimating Changes in the Monetary Policy Rule

Fed Funds i_t	60.Q1-96.Q4	79.Q3-11.Q4	97.Q1-11.Q4
Dummy Period $\mathcal{T}$	79.Q3-96.Q4	97.Q1-11.Q4	08.Q3-11.Q4
Output Gap x_t	0.18**	-0.04	0.17
	(0.06)	(0.13)	(0.13)
Inflation π_t	0.30**	0.83**	0.27**
	(0.07)	(0.21)	(0.09)
Lagged Fed Funds i_{t-1}	0.56**	0.43*	0.88**
	(0.10)	(0.17)	(0.09)
Output Gap×Dummy $x_t I_{t\in\mathcal{T}}$	-0.22	0.09	-0.19
	(0.14)	(0.14)	(0.13)
Inflation×Dummy $\pi_t I_{t\in\mathcal{T}}$	0.53*	-0.62**	-0.28**
	(0.22)	(0.22)	(0.09)
Lagged Fed Funds×Dummy $i_{t-1} I_{t\in\mathcal{T}}$	-0.14	0.47*	-0.85**
	(0.20)	(0.18)	(0.10)
Dummy $I_{t\in\mathcal{T}}$	0.84	-1.87	0.23
	(1.00)	(0.96)	(0.40)
Constant	0.91*	1.75	-0.27
	(0.38)	(0.92)	(0.27)
R^2	0.77	0.85	0.93
Implied $\Delta\hat{\gamma}^x$	-0.49	0.52	-1.45
	(0.25)	(0.30)	(0.41)
Implied $\Delta\hat{\gamma}^\pi$	0.75*	0.47	-2.30
	(0.25)	(1.28)	(1.59)
Implied $\Delta\hat{\rho}^i$	-0.14	0.47*	-0.85
	(0.20)	(0.18)	(0.10)

Variables and tests are described in Table 4 in the main text. We estimate $i_t = c^0 + c^x x_t + c^\pi \pi_t + c^i i_{t-1} + d^0 I_{t\in\mathcal{T}} + d^x x_t I_{t\in\mathcal{T}} + d^\pi \pi_t I_{t\in\mathcal{T}} + d^0 i_{t-1} I_{t\in\mathcal{T}} + \epsilon_t$. Changes in monetary policy parameters, such as $\Delta\hat{\gamma}^x$, show estimated changes from the pre-$\mathcal{T}$ sub-sample to the $\mathcal{T}$ sub-sample. Standard errors for $\Delta\hat{\gamma}^x$ and $\Delta\hat{\gamma}^\pi$ are calculated by the delta method. Significance levels for changes in monetary policy parameters are based on a likelihood ratio test.

Table A.8: Sub-Period Correlations of Bond Returns, Stock Returns, Output Gap, and Inflation

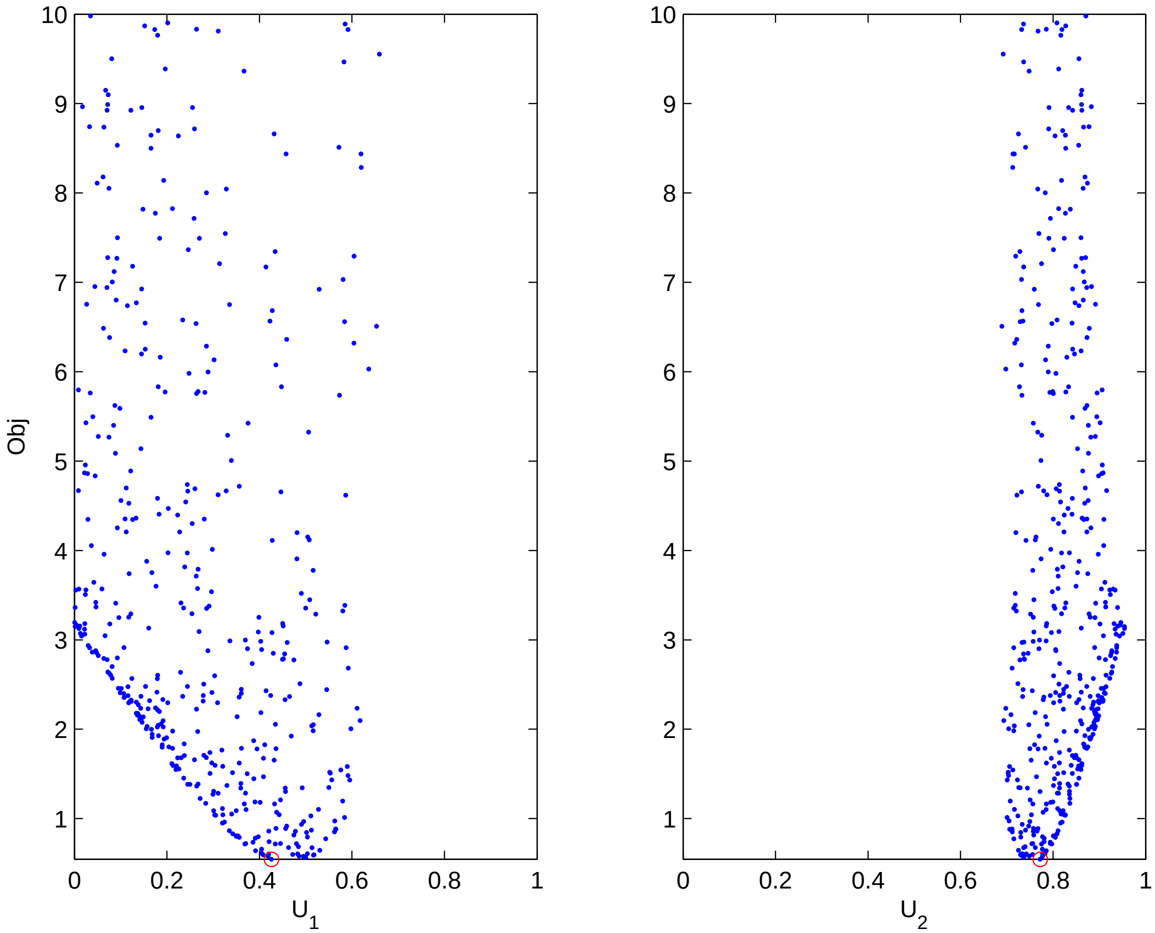
60.Q1-79.Q2	Bond Excess Returns	Stock Excess Returns	Output Gap	Inflation
Bond Excess Returns	1			
Stock Excess Returns	0.32*	1		
Output Gap	-0.17	0.36*	1	
Inflation	-0.25*	-0.60*	-0.14	1

79.Q3-96.Q4	Bond Excess Returns	Stock Excess Returns	Output Gap	Inflation
Bond Excess Returns	1			
Stock Excess Returns	0.46*	1		
Output Gap	0.30*	0.23	1	
Inflation	-0.74*	-0.27*	-0.16	1

97.Q1-11.Q4	Bond Excess Returns	Stock Excess Returns	Output Gap	Inflation
Bond Excess Returns	1			
Stock Excess Returns	-0.63*	1		
Output Gap	-0.55*	0.55*	1	
Inflation	-0.32*	0.12	0.34*	1

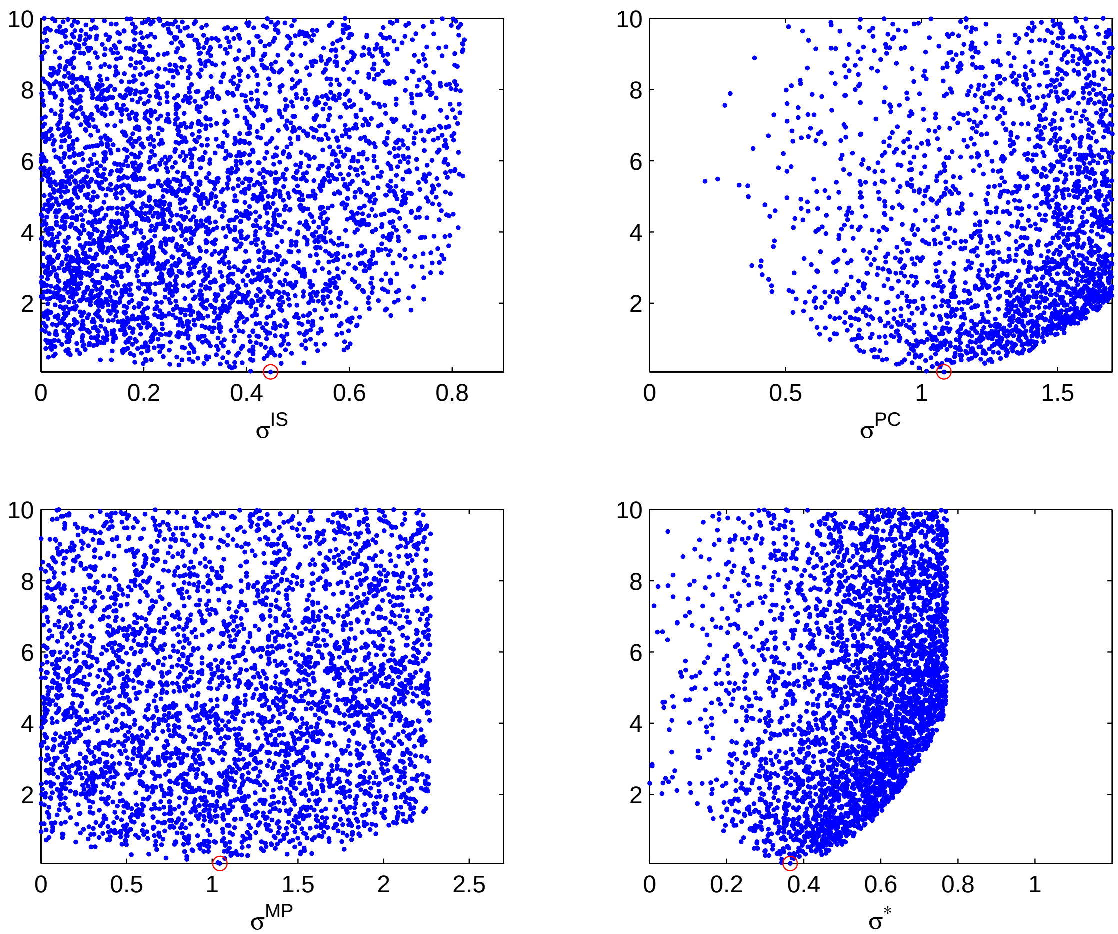
Quarterly overlapping 3 year log equity returns in excess of log three month T-bill, 3 year log excess return on 5 year nominal bond in excess of three month log T-bill. Quarterly inflation and output as in Table 1. We report correlations of log excess returns from time $t-12$ to t and macroeconomic variables as of quarter t. * and ** denote significance at the 5% and 1% level. Significance levels not adjusted for time series dependence.

Figure A.1: Minimizing with Respect to ρ^{x-} and ρ^{x+}



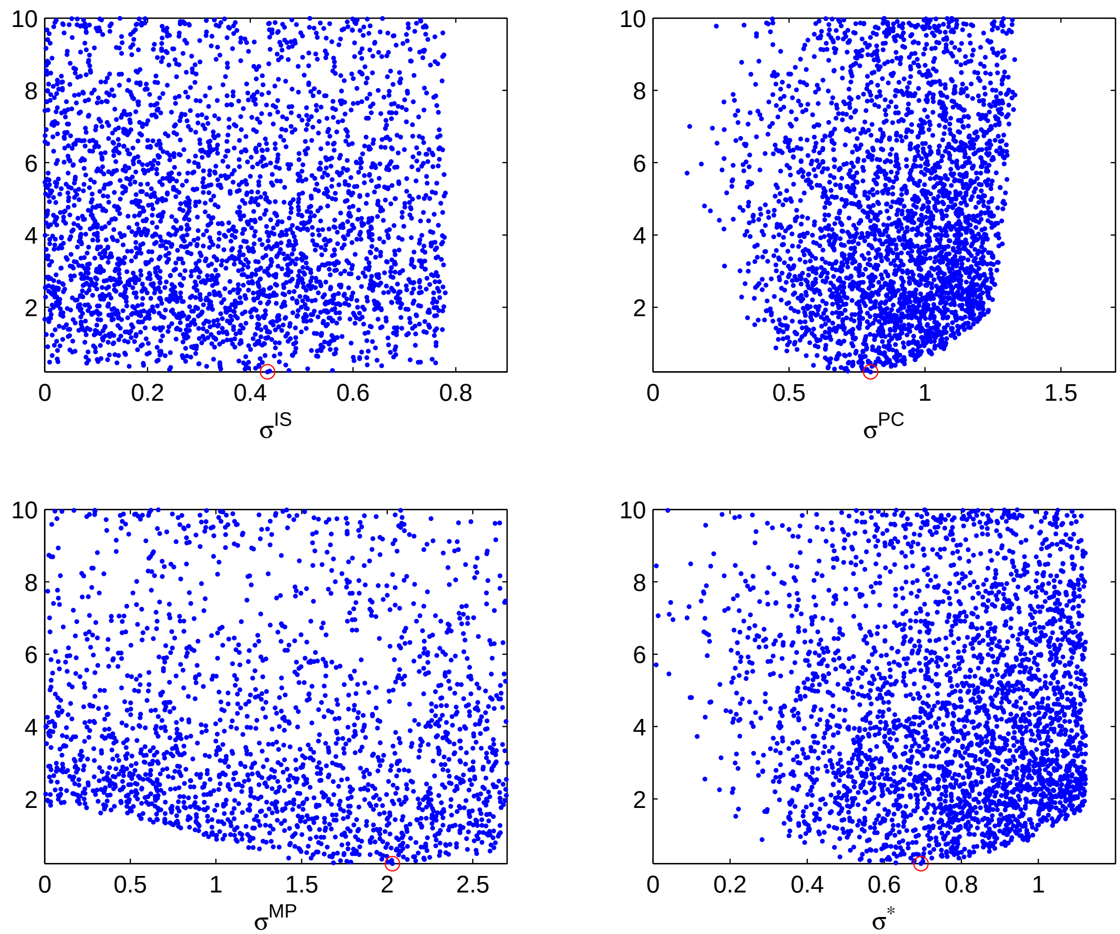
We minimize the objection function with respect to ρ^{x-} and ρ^{x+} while holding constant the volatilities of shocks at the values shown in (132). We randomly draw 10000 draws from two independent uniform distributions $U_1 \in [0,1]$ and $U_2 \in [0,1]$ and set $\rho^{x-} = 0.4253 + 0.05U_1$ and $\rho^{x+} = (1 - \rho^{x-}) + 0.2 \times \rho^{x-}U_2$. The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.

Figure A.2: (Panel A) Minimizing with Respect to Shock Volatilities: 1960.Q1-1979.Q2



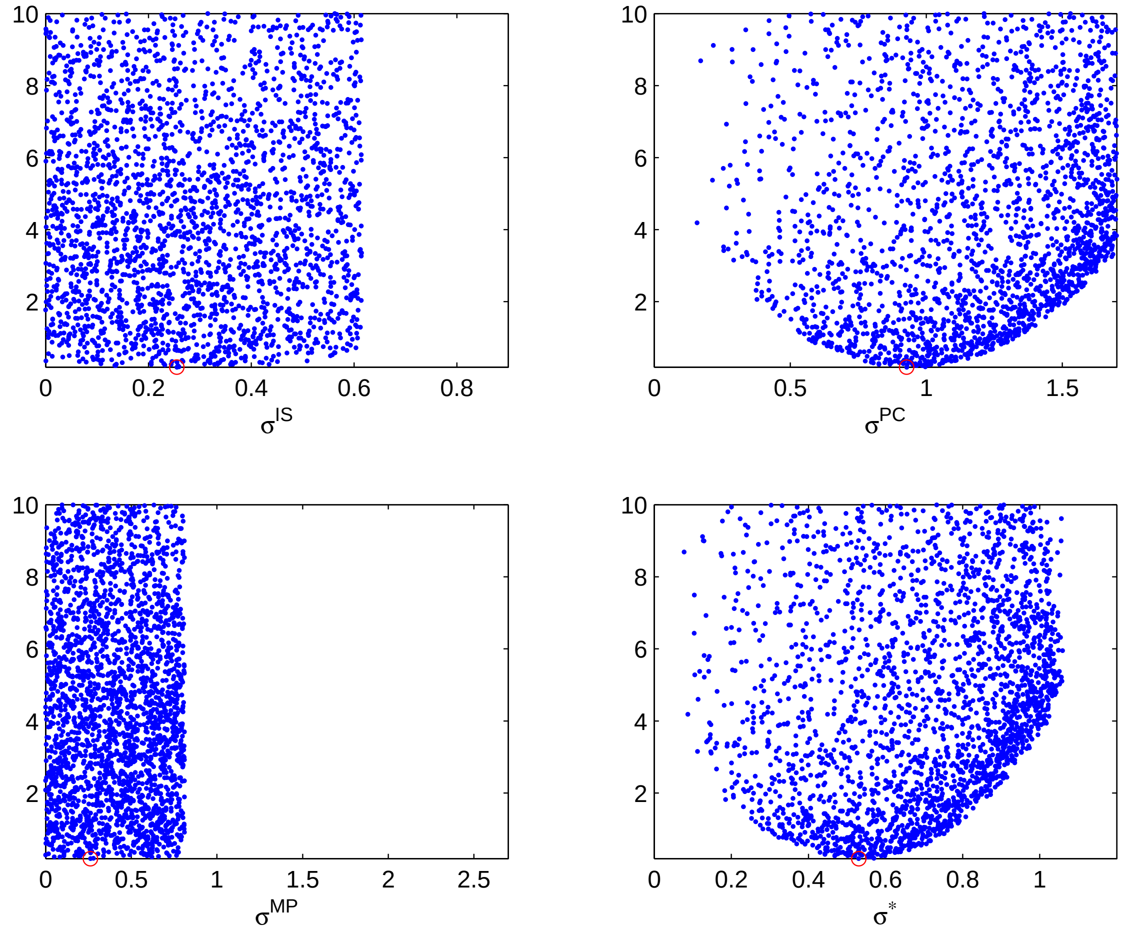
We minimize the objection function with respect to $\bar{\sigma}^{IS,1}$, $\bar{\sigma}^{PC,1}$, $\bar{\sigma}^{MP,1}$, and $\bar{\sigma}^{*,1}$ while holding constant all time-invariant parameters at the values shown in Table 5 in the main paper. The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments for that sub-period. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.

Figure A.2: (Panel B) Minimizing with Respect to Shock Volatilities: 1979.Q3-1996.Q4



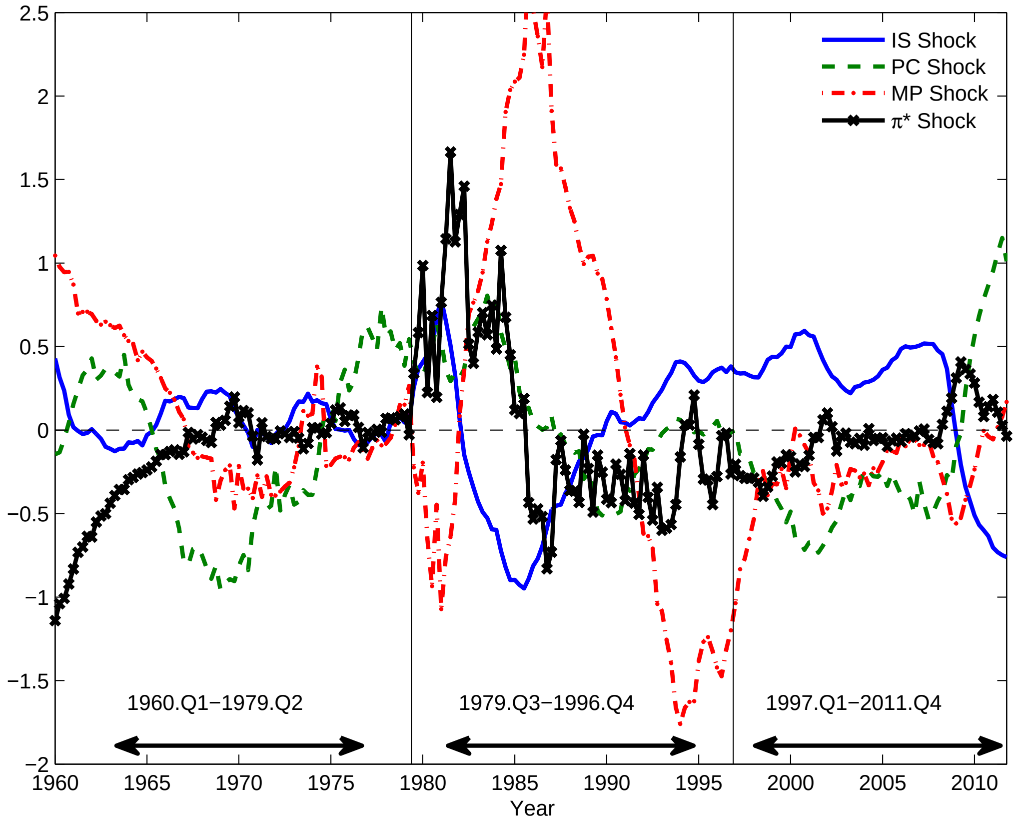
We minimize the objection function with respect to $\bar{\sigma}^{IS,2}$, $\bar{\sigma}^{PC,2}$, $\bar{\sigma}^{MP,2}$, and $\bar{\sigma}^{*,2}$ while holding constant all time-invariant parameters at the values shown in Table 5 in the main paper. The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments for that sub-period. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.

Figure A.2: (Panel C) Minimizing with Respect to Shock Volatilities: 1997.Q1-2011.Q4



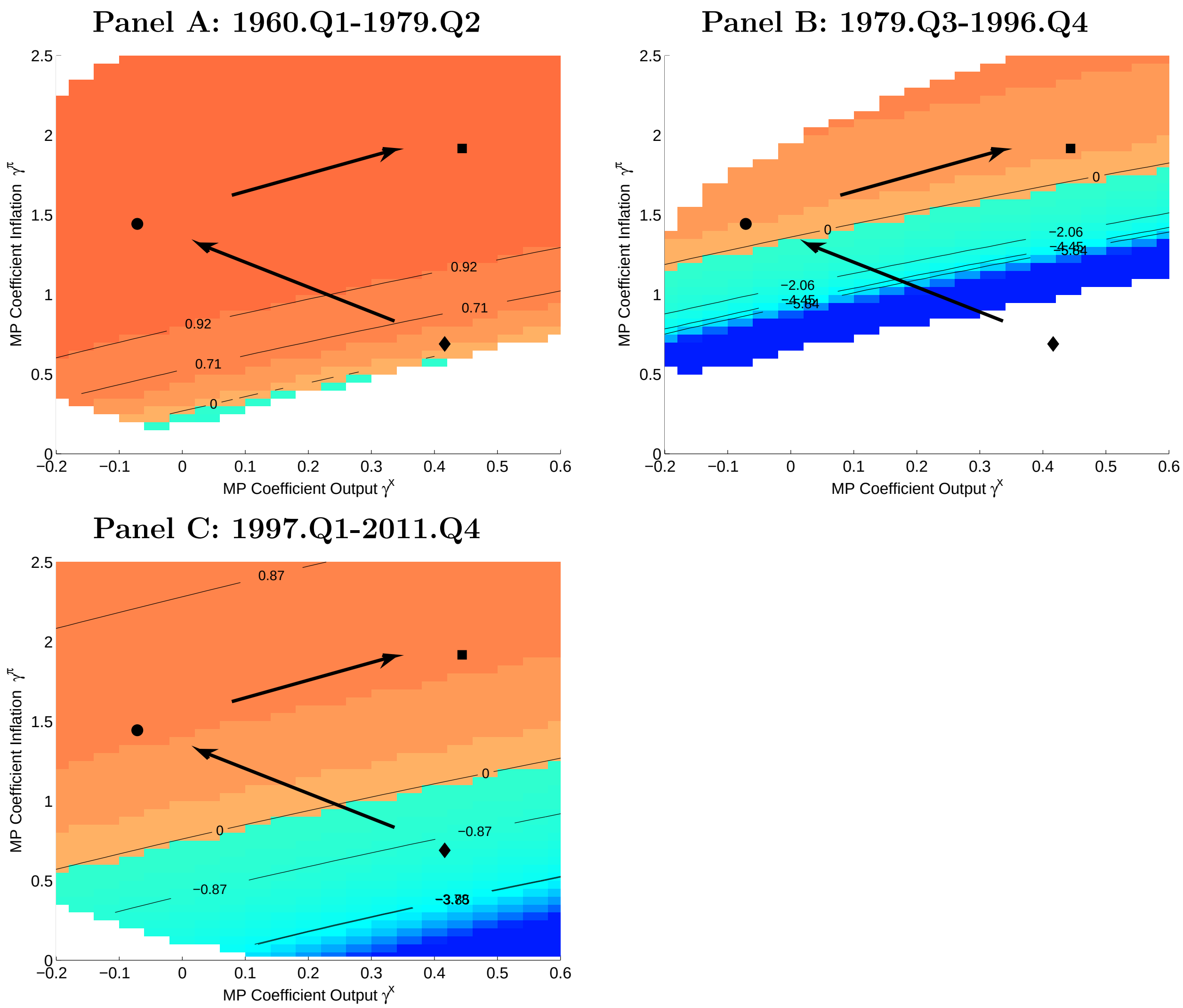
We minimize the objection function with respect to $\bar{\sigma}^{IS,3}$, $\bar{\sigma}^{PC,3}$, $\bar{\sigma}^{MP,3}$, and $\bar{\sigma}^{*,3}$ while holding constant all time-invariant parameters at the values shown in Table 5 in the main paper.The minimizing parameter values are indicated by circles. The objective function is the sum of squared differences between model and empirical moments for that sub-period. The considered moments are the slope coefficients of a VAR(1) in the log output gap, log inflation, log Fed Funds, and five year nominal log bond yield, the standard deviations of the VAR(1) residuals in annualized percent, equity return volatility and bond return volatility in annualized percent and the nominal bond beta. The equity and bond volatilities are scaled by 0.1 and the nominal bond beta is scaled by a factor of 10 to ensure that moments have roughly equal magnitudes.

Figure A.3: Time Series of Model Shocks



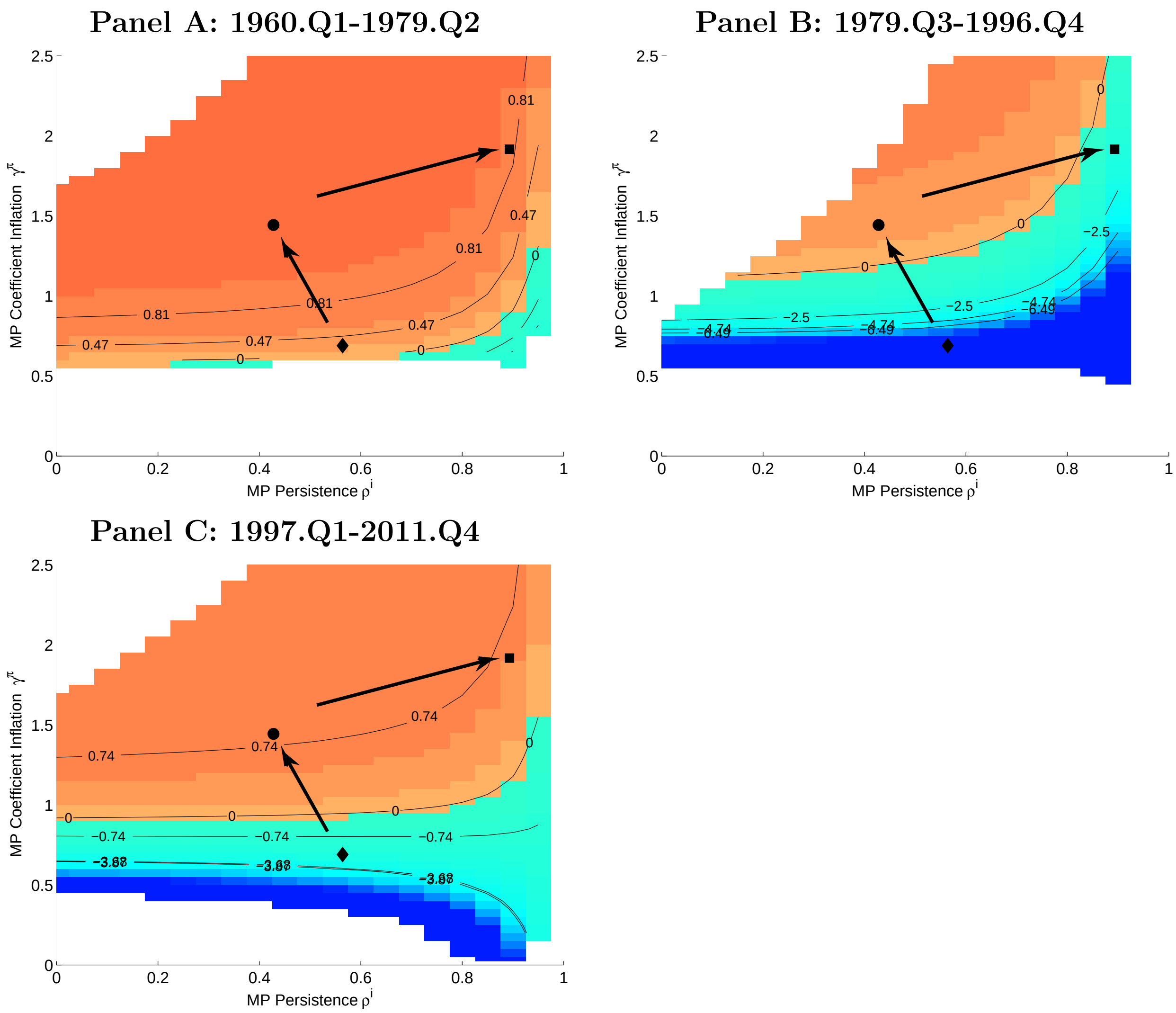
This figure plots the time series of smoothed IS, PC, MP and inflation target (π^*) shocks. IS shocks are in natural percent units, while PC, MP and inflation target shocks are in annualized percent units. The shocks are smoothed with a trailing exponentially-weighted moving average. The decay parameter equals 0.08 per quarter corresponding to a half life of 24 quarters.

Figure A.4: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and γ^{x} - Alternative Monetary Policy Rule



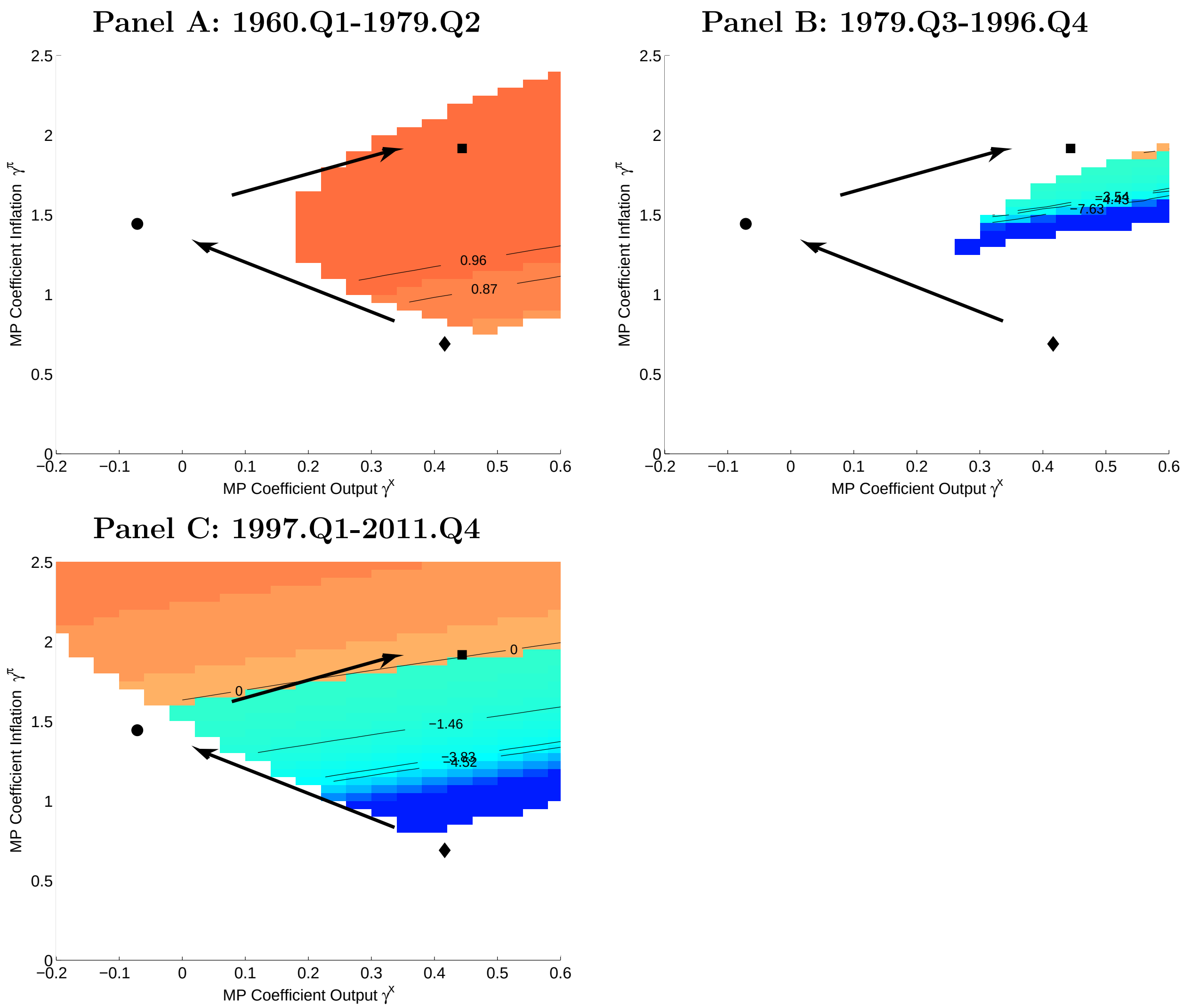
This figure re-creates Figure 3 in the main text for a model with the alternative monetary policy rule (135). All parameters are equal to the values in Table 5 and are not re-calibrated to fit moments. The curves in the figure are less smooth than those in Figure 3 in the main paper, because this figure was constructed with fewer pixels.

Figure A.5: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and ρ^{i} - Alternative Monetary Policy Rule



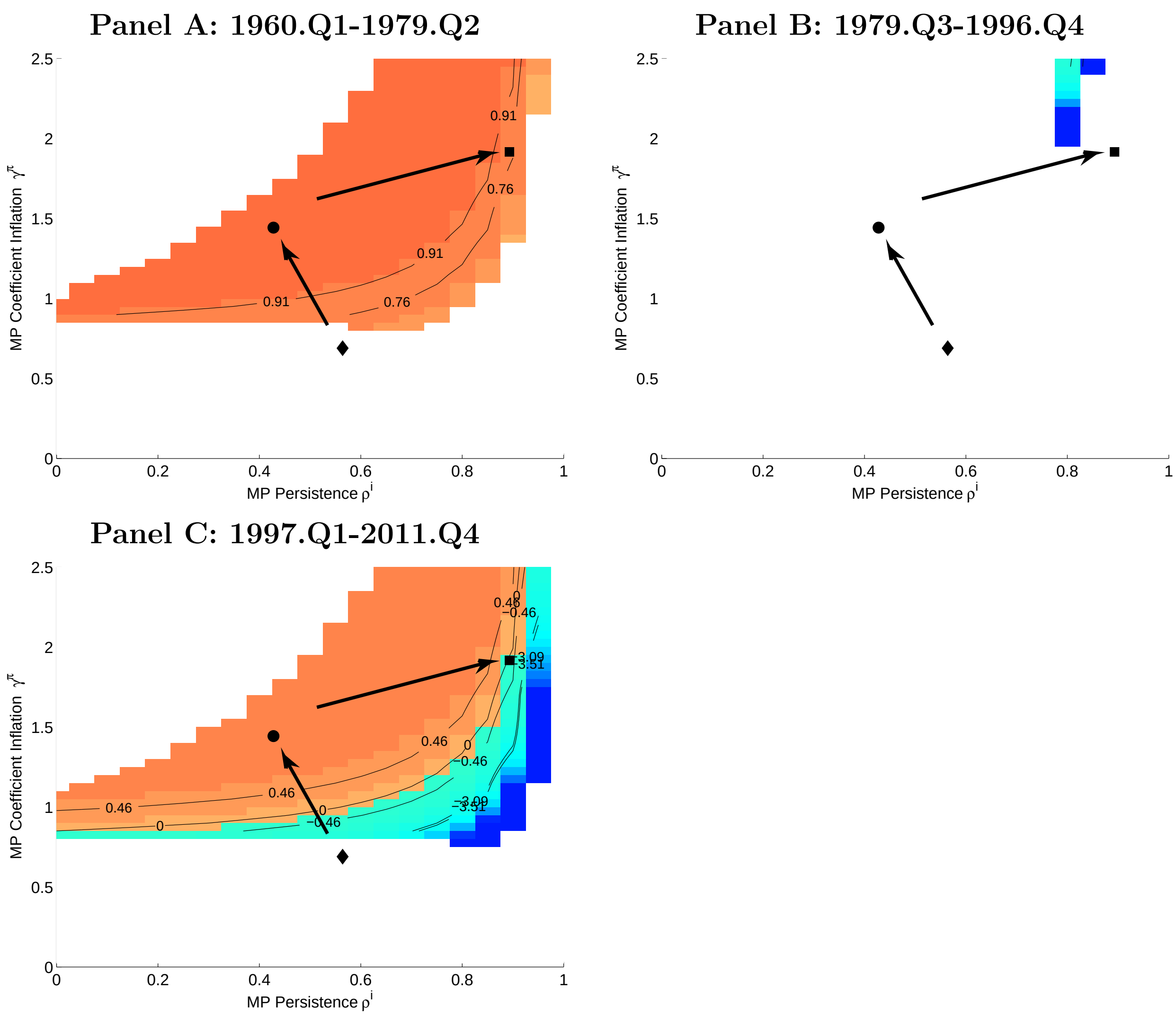
This figure re-creates Figure 4 in the main text for a model with the alternative monetary policy rule (135). All parameters are equal to the values in Table 5 and are not re-calibrated to fit moments. The curves in the figure are less smooth than those in Figure 4 in the main paper, because this figure was constructed with fewer pixels.

Figure A.6: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and γ^{x} - Alternative Phillips Curve Parameterization



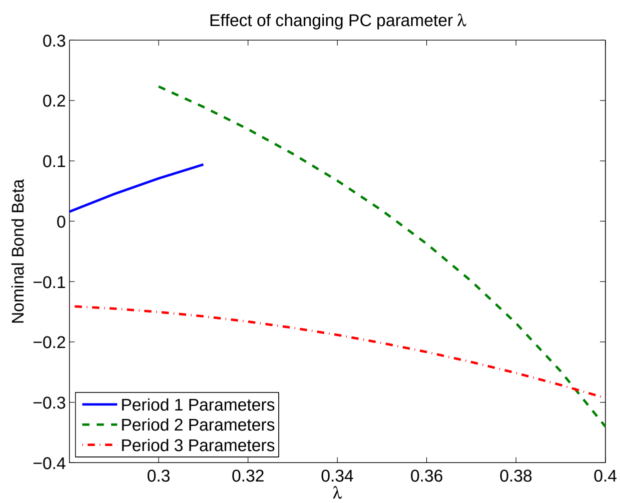
This figure re-creates Figure 3 in the main text for a model with the alternative Phillips curve parameter $\rho^{\pi} = 0.7$, which is smaller than the backward looking component in our main calibration. All remaining parameters are equal to the values in Table 5 and are not re-calibrated to fit moments. The curves in the figure are less smooth than those in Figure 3 in the main paper, because this figure was constructed with fewer pixels.

Figure A.7: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and ρ^{i} - Alternative Phillips Curve Parameterization



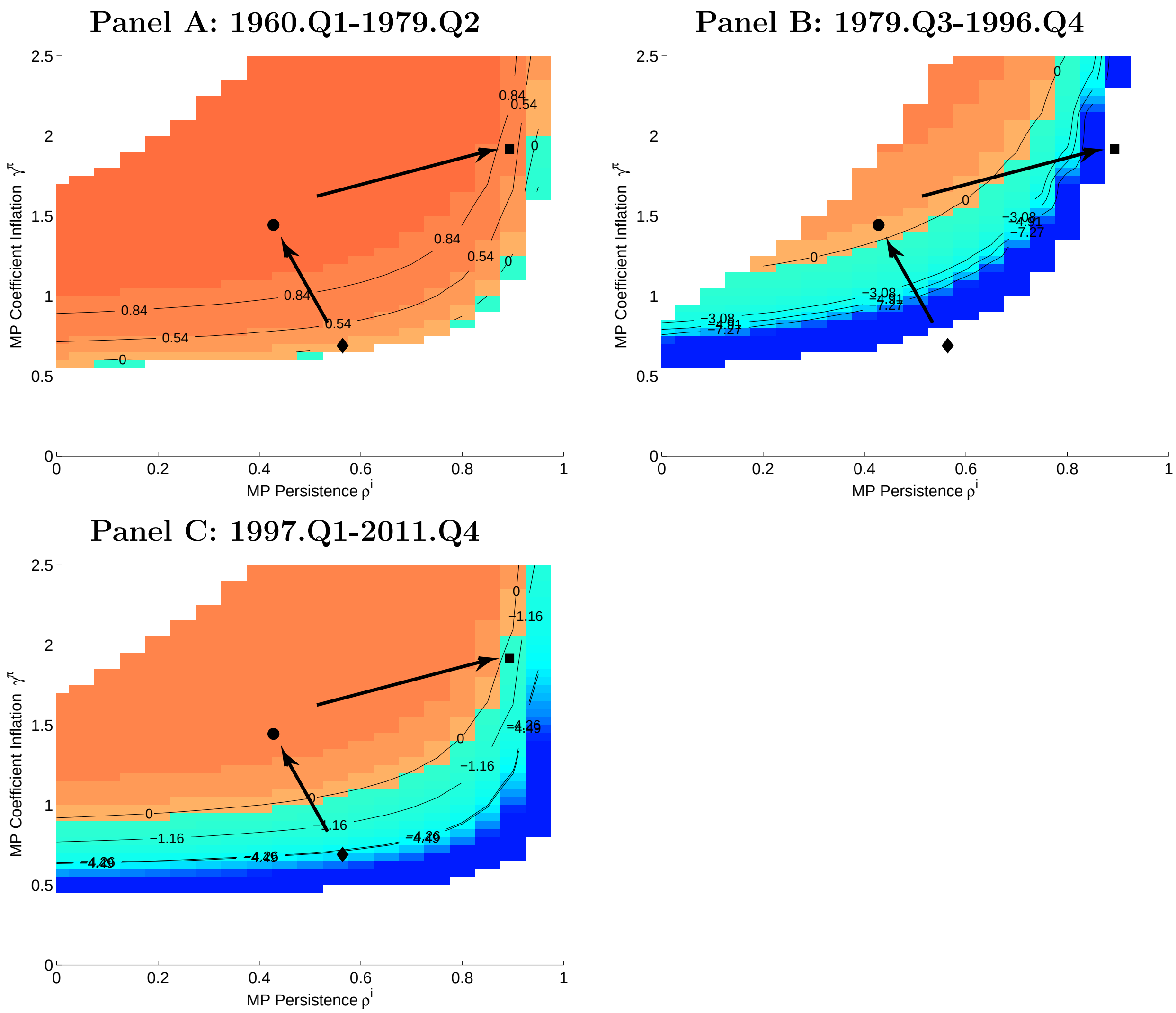
This figure re-creates Figure 4 in the main text for a model with the alternative Phillips curve parameter $\rho^{\pi} = 0.7$, which is smaller than the backward looking component in our main calibration. All remaining parameters are equal to the values in Table 5 and are not re-calibrated to fit moments. The curves in the figure are less smooth than those in Figure 4 in the main paper, because this figure was constructed with fewer pixels.

Figure A.8: Nominal Bond Betas and the Phillips Curve Slope λ



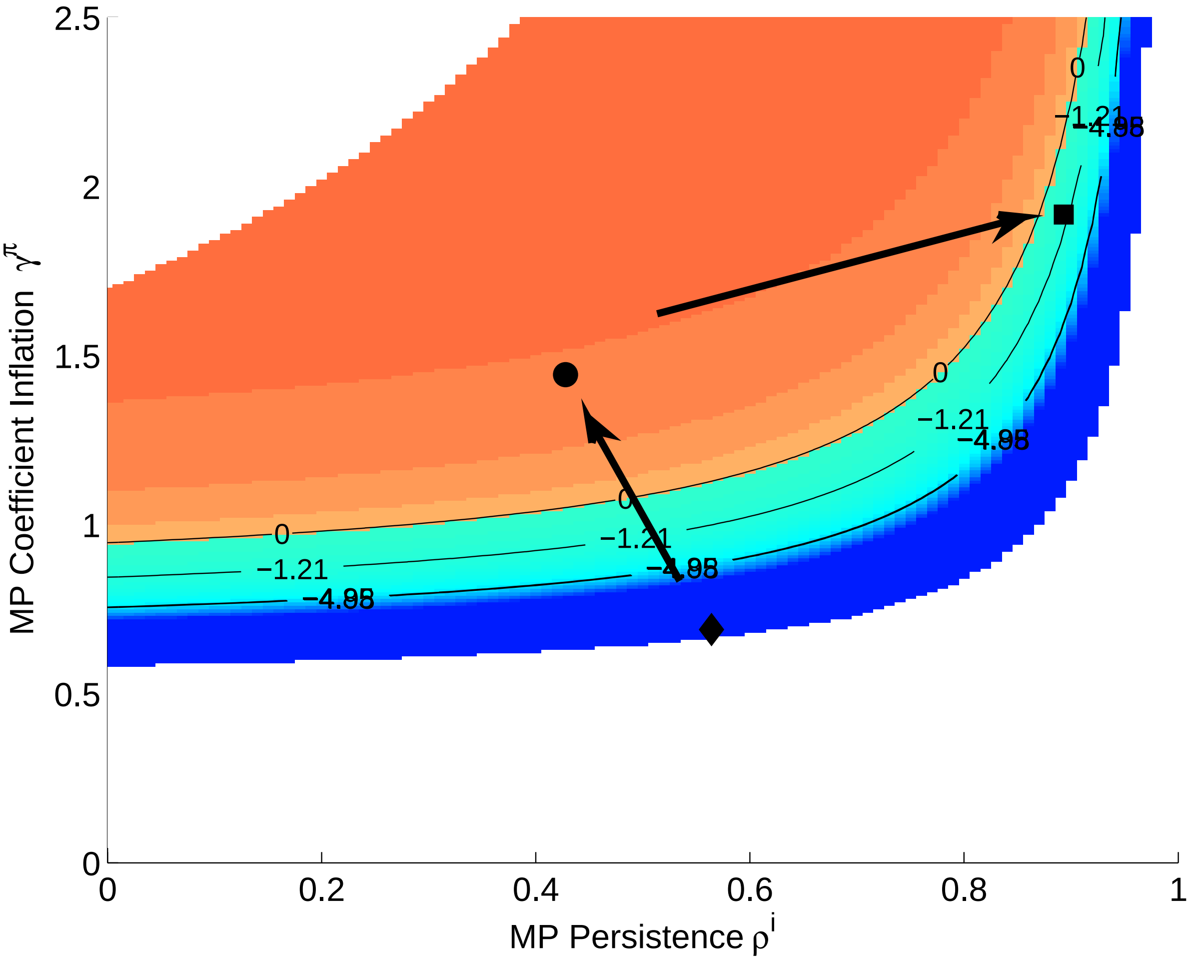
This figure shows how the nominal bond beta varies with the Phillips curve slope parameter λ while all other parameters are held constant at their period 1 values (blue solid), period 2 values (green dash), or period 3 values (red dash-dot).

Figure A.9: Nominal Bond Betas Against Monetary Policy Parameters γ^{π} and ρ^{i} - Alternative Leverage Parameter



This figure re-creates Figure 4 in the main text for a model with the leverage parameter $\delta = 1$, corresponding to a firm leverage ratio of 0%. The leverage parameter in our main calibration is $\delta = 2.43$ corresponding to a firm leverage ratio of 59%. All remaining parameters are equal to the values in Table 5 and are not re-calibrated to fit moments. The curves in the figure are less smooth than those in Figure 4 in the main paper, because this figure was constructed with fewer pixels.

Figure A.10: Real Bond Betas Against Monetary Policy Parameters γ^{π} and ρ^{i} - 1960.Q1-1979.Q2 Calibration



This figure is analogous to Figure 4, Panel A in the main paper, except that this figure plots the beta of real bonds instead of nominal bonds.