EXTREME EVENTS AND THE FED

by

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Extreme Value Theory

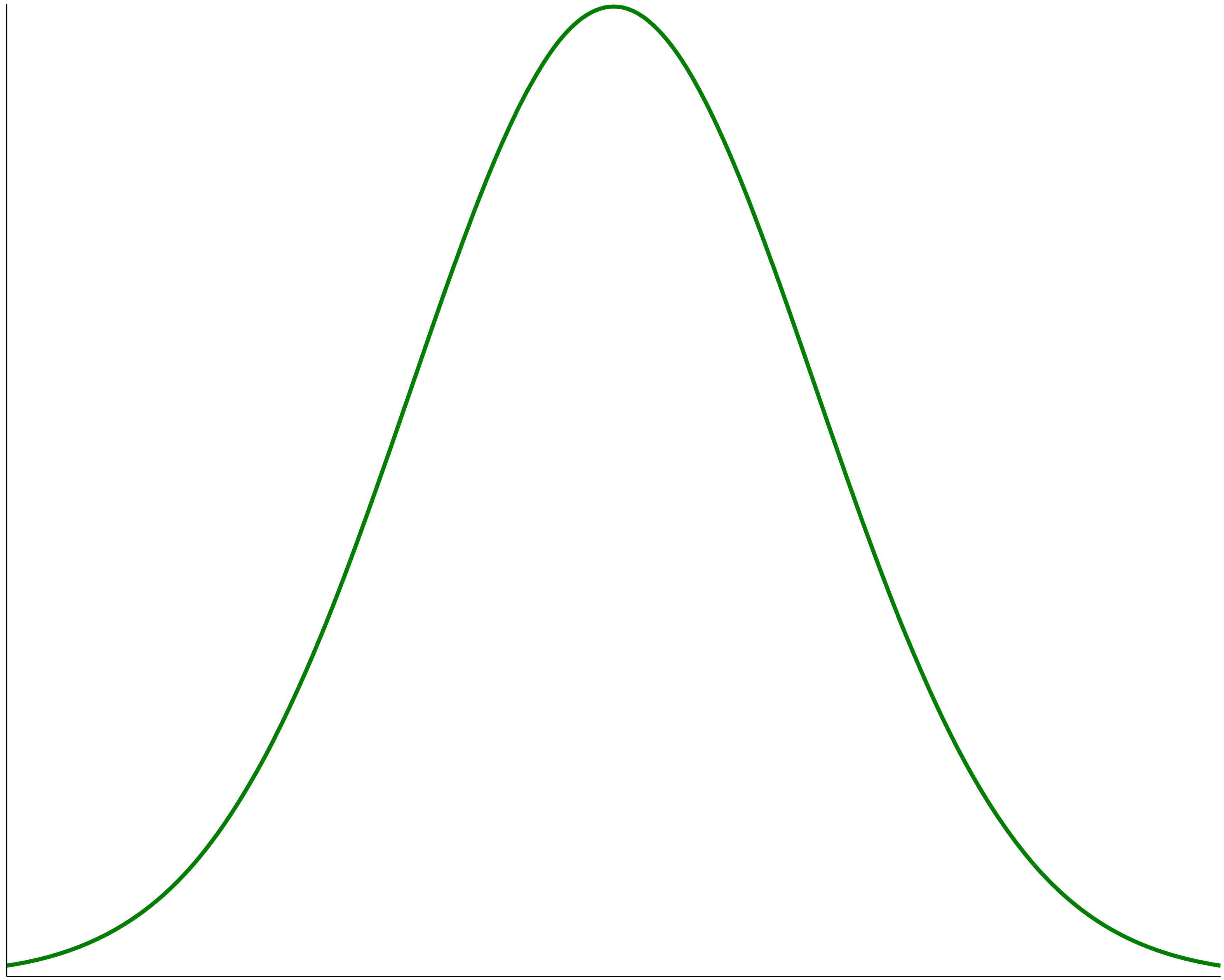
Branch of statistics concerned with extreme deviations from the median of probability distributions

Widely used in engineering, where designers seek to protect structures against infrequent, but potentially damaging, events

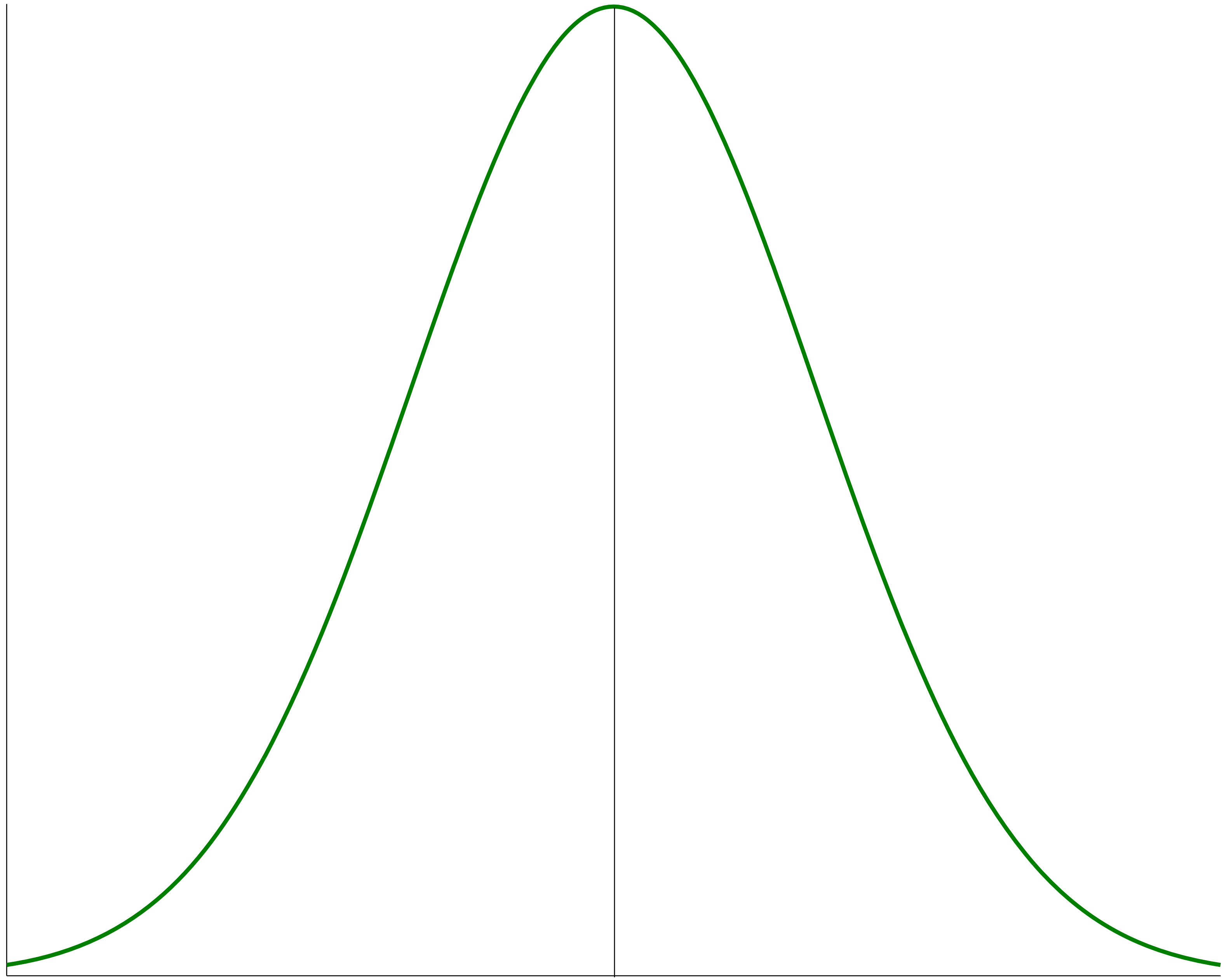
Economies are also subject to extreme shocks (e.g., oil shocks in the 1970s or the financial shocks in 2008)

It is important to design monetary policy with the possibility of extreme events in mind

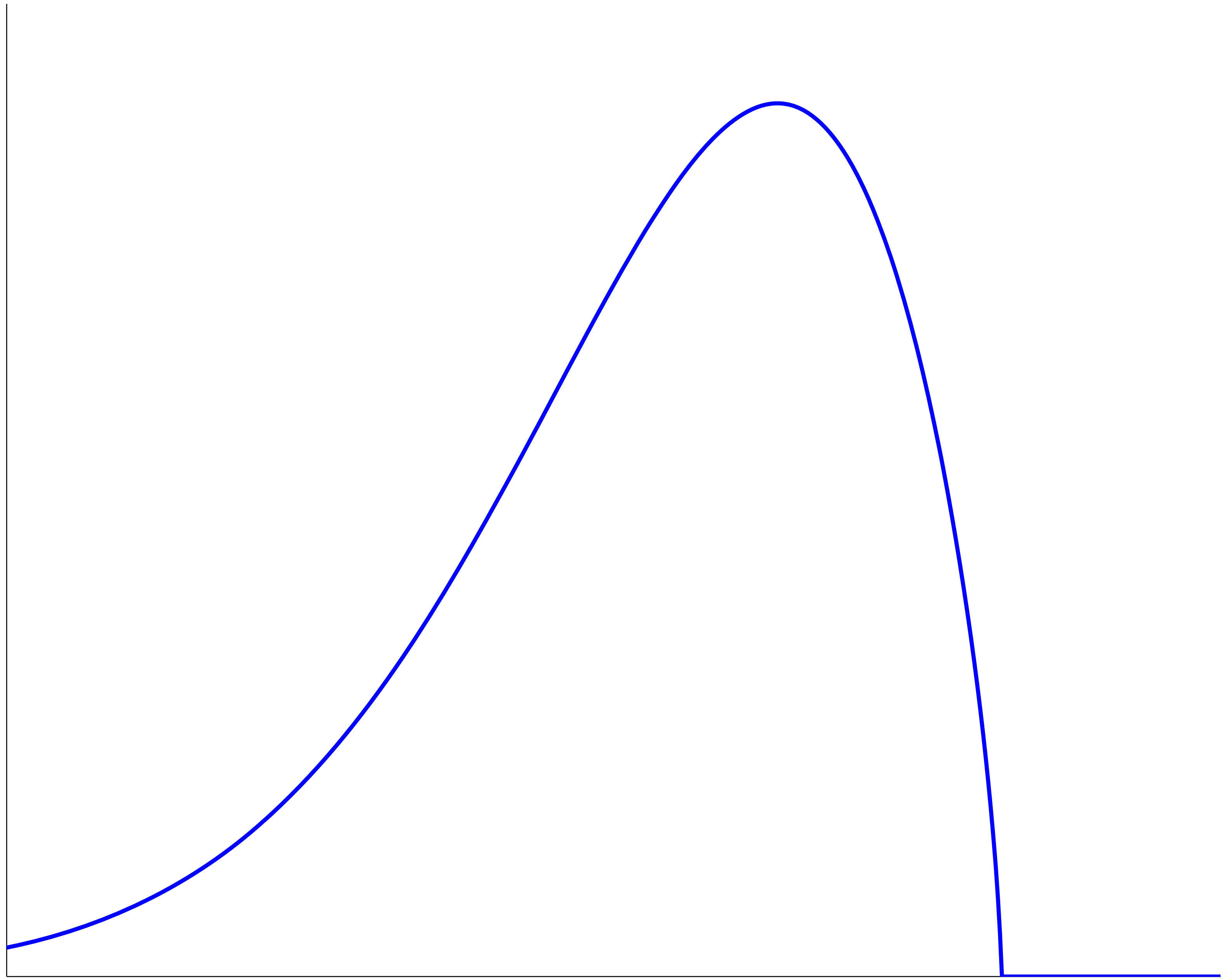
Normal Distribution



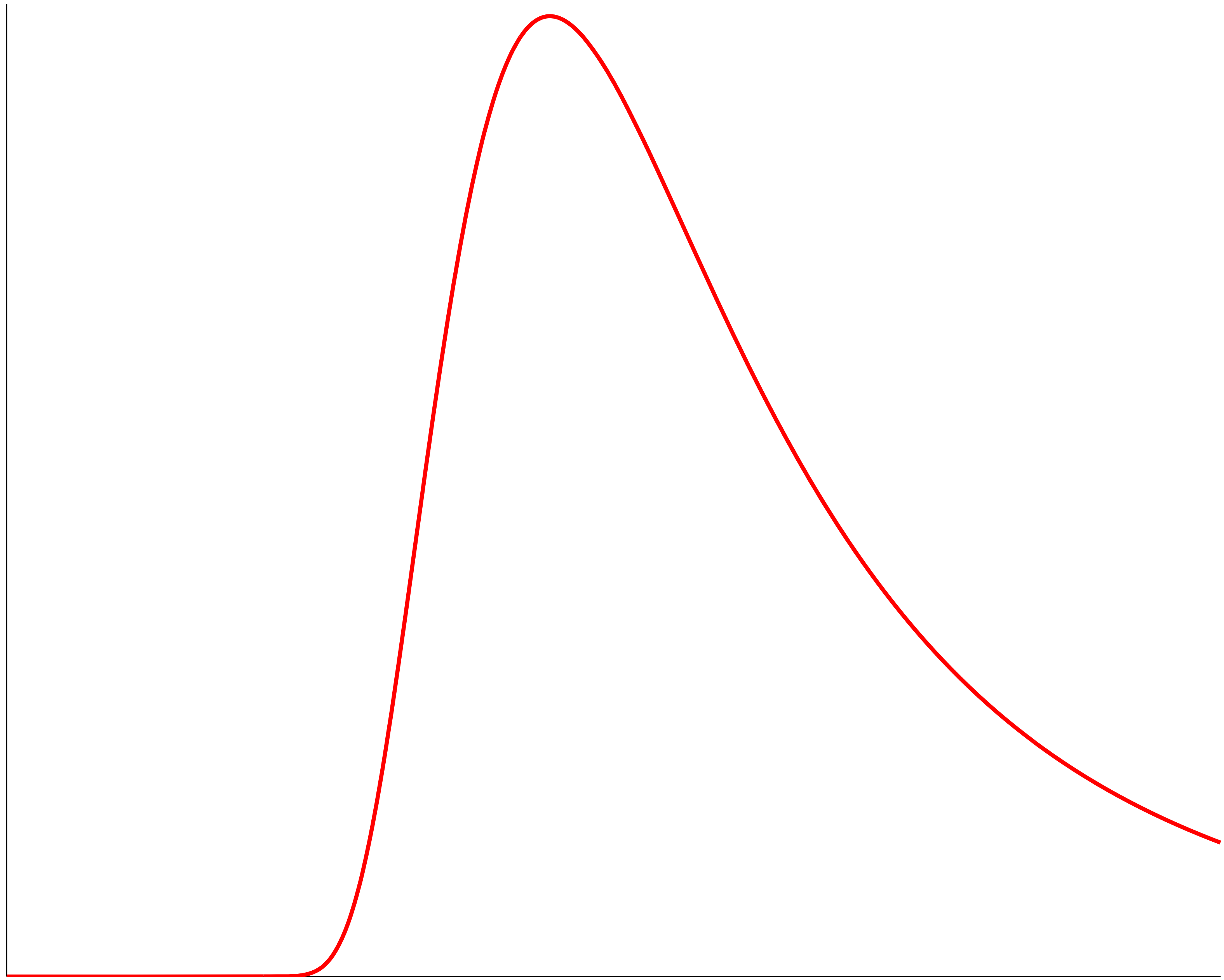
Normal Distribution



Weibull Distribution



Frechete Distribution



This Paper

We study the positive and normative implications of extreme events for monetary policy

We construct and estimate a non-linear dynamic model with rigid prices and wages

Derive implications under three policies:

Taylor

Ramsey

Strict inflation targeting

Evaluate the relative contribution of model nonlinearity and shock asymmetry

One Key Issue (Svensson, 2003)

Act prudently and systematically incorporate the possibility of extreme shocks into policy (e.g., by adjusting the inflation target)

or

Follow a wait-and-see approach

Preview of the Results

Structural estimates support the view that shock innovations are drawn from asymmetric distributions

Due to risk, there is (or there should be) a prudence motive in monetary policy making

However, optimal (net) inflation is close to zero because inflation costs paid every period override the precautionary benefits of having a non-zero inflation target (see Coibion *et al.*, 2012)

Under both the Taylor and Ramsey policies, the central bank responds non-linearly and asymmetrically to shocks

Sketch of the Model

Households

Monopolistic competitive power over their labor supply

Face convex cost to adjust nominal wages

Firms

Produce differentiated goods using labor only

Monopolistic competitive power

Face convex costs to adjust nominal prices

Monetary Authority (**the Fed**)

Selects monetary policy following a Taylor-type rule

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Ramsey Planner

Selects monetary policy to maximize households' welfare

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Strict Inflation Targeter

Selects monetary policy to achieve an inflation target

Households

Household $n \in [0,1]$ maximizes

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left(\frac{(c_t^h)^{1-\chi}}{1-\chi} - \frac{(n_t^h)^{1+\psi}}{z_t(1+\psi)} \right)$$

where

$$c_t^h = \left(\int_0^1 (c_{j,t}^h)^{1/\mu} dj \right)^{\mu}$$

Households have monopolistic power over their labor supply and, thus, their nominal wage is a choice variable

Labor market frictions induce a cost in the adjustment of nominal wages (Φ_t^n)

Budget Constraint

Two types of financial assets: one-period nominal bonds and a complete set of Arrow-Debreu securities

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Two types of financial assets: one-period nominal bonds and a complete set of Arrow-Debreu securities

The budget constraint is

$$c_t^h + \frac{Q_t A_t^h - A_{t-1}^h}{P_t} + \frac{B_t^h - i_{t-1} B_{t-1}^h}{P_t} = (1 - \Phi_t^h)\left(\frac{W_t^h n_t^h}{P_t}\right) + \frac{D_t^h}{P_t},$$

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where

$$\Phi_t^h = \left(\frac{\phi}{2}\right)\left(\frac{W_t^h}{W_{t-1}^h} - 1\right)^2$$

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where

$$\Phi_t^h = \left(\frac{\phi}{2}\right)\left(\frac{W_t^h}{W_{t-1}^h} - 1\right)^2$$

and

$$P_t = \left(\int_0^1 (P_{i,t})^{1/(1-\mu)} di\right)^{1/(1-\mu)}$$

is the price index

Firms

Firm $j \in [0,1]$ produces a differentiated good using the technology

$$y_{j,t} = x_t n_{j,t}^{1-\alpha}$$

where

$$n_{j,t} = \left(\int_0^1 (n_{j,t}^h)^{1/\varsigma} dh \right)^{\varsigma}$$

Firms have monopolistic power and, thus, their nominal price is a choice variable

Good market frictions induce a cost in the adjustment of nominal prices:

$$\Gamma_t^j = \left(\frac{\gamma}{2}\right)\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2,$$

Equilibrium

Symmetric equilibrium: all households and firms are identical *ex-post*

Arrow-Debreu securities and bonds are not held

Economy-wide resource constraint

$$c_t = y_t - (y_t \Gamma_t + w_t n_t \Phi_t)$$

The Fed

Sets the interest rate following the Taylor-type rule

$$\ln(i_t/i) = \eta_1 \ln(i_{t-1}/i) + \eta_2 \ln(\Pi_t/\Pi) + \eta_3 \ln(n_t/n) + e_t,$$

where $\eta_1 \in (-1,1)$, η_2 and η_3 are parameters

Shocks

Define

$$\xi_t = [\ln(z_t)\,\ln(x_t)\,\ln(e_t)]'$$

Then

$$\xi_t = \rho\xi_{t-1} + \varepsilon_t$$

where

$$\rho = \begin{bmatrix} \rho_z & 0 & 0 \\ 0 & \rho_x & 0 \\ 0 & 0 & \rho_e \end{bmatrix}$$

and $\varepsilon_t = [\varepsilon_{z,t}\ \varepsilon_{x,t}\ \varepsilon_{e,t}]'$ is a vector of i.i.d. innovations

Innovations are a generalized extreme value (GEV) distribution

GEV Distribution

According to the Fisher-Tippett (1928) theorem, the maximum of an i.i.d. series converges in distribution to either the Gumbel, Fréchet or Weibull distributions

Jenkinson (1955) shows that these three distributions can be represented in a unified way using the GEV distribution

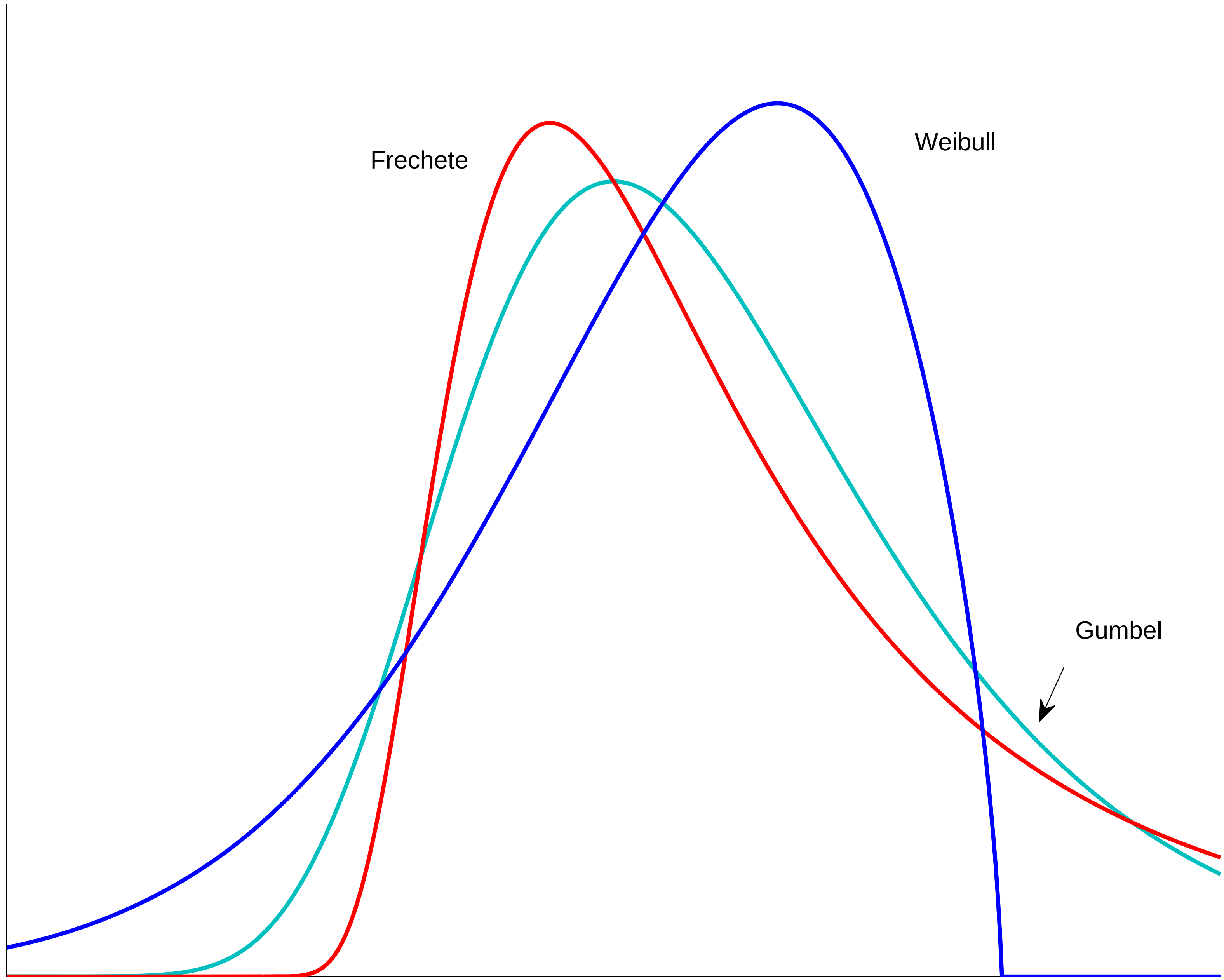
Three parameters: location, scale, and shape

The shape parameter controls the thickness of the tail of the distribution

Positive or negative skewness

Mean (variance) is not defined when shape parameter is larger than 1 (0.5)

GEV Distribution



Solution Method

Third-order approximation to policy functions (Jin and Judd, 2002)

In tensor notation

$$
\begin{aligned}
[f(x_t,\sigma)]^j = {} & [f(x,0)]^j + [f_x(x,0)]^j_a[(x_t - x)]^a \\
& + (1/2)[f_{xx}(x,0)]^j_{ab}[(x_t - x)]^a[(x_t - x)]^b \\
& + (1/6)[f_{xxx}(x,0)]^j_{abc}[(x_t - x)]^a[(x_t - x)]^b[(x_t - x)]^c \\
& + (1/2)[f_{\sigma\sigma}(x,0)]^j[\sigma][\sigma] \\
& + (1/2)[f_{x\sigma\sigma}(x,0)]^j_a[(x_t - x)]^a[\sigma][\sigma] \\
& + (1/6)[f_{\sigma\sigma\sigma}(x,0)]^j[\sigma][\sigma][\sigma],
\end{aligned}
$$

where x_t is a vector with the state variables

If innovation distributions are symmetric, $(1/6)[f_{\sigma\sigma\sigma}(x,0)]^j[\sigma][\sigma][\sigma] = 0$

Estimation

Simulated Method of Moments (SMM)

$$\widehat{\theta} = \underset{\{\theta\}}{\operatorname{argmin}} \, \mathbf{M}(\theta)' \mathbf{W} \mathbf{M}(\theta)$$

where

$$\mathbf{M}(\theta) = (1/T) \sum_{t=1}^{T} \mathbf{m}_t - (1/\lambda T) \sum_{\iota=1}^{\lambda T} \mathbf{m}_\iota(\theta)$$

T is the sample size, λ is a positive constant and $\mathbf{W}$ is a weighting matrix

Asymptotic Distribution

Under the regularity conditions in Duffie and Singleton (1993)

$$\sqrt{T}\,(\hat{\theta} - \theta) \to N(\mathbf{0}, (1 + 1/\lambda)(\mathbf{J}'\mathbf{W}^{-1}\mathbf{J})^{-1}\mathbf{J}'\mathbf{W}^{-1}\mathbf{S}\mathbf{W}^{-1}\mathbf{J}(\mathbf{J}'\mathbf{W}^{-1}\mathbf{J})^{-1})$$

where

$$\mathbf{S} = \lim_{T\to\infty} Var\left(\left(\frac{1}{\sqrt{T}}\right)\sum_{t=1}^{T}\mathbf{m}_t\right)$$

and

$$\mathbf{J} = E\left(\frac{\partial \mathbf{m}_t(\theta)}{\partial \theta}\right)$$

is a finite Jacobian matrix of full column rank

Data

Sample Period and Frequency

Quarterly from 1964Q2 to 2012Q4

Data

Sample Period and Frequency

Quarterly from 1964Q2 to 2012Q4

Series

Real per-capita consumption

Hours worked

Price inflation rate

Wage inflation rate

Nominal interest rate

Data

Sample Period and Frequency

Quarterly from 1964Q2 to 2012Q4

Series

Real per-capita consumption

Hours worked

Price inflation rate

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Nominal interest rate

Moments

Variances, covariances, autocovariances and skewness of all data series

SMM Estimates: Nominal Rigidity

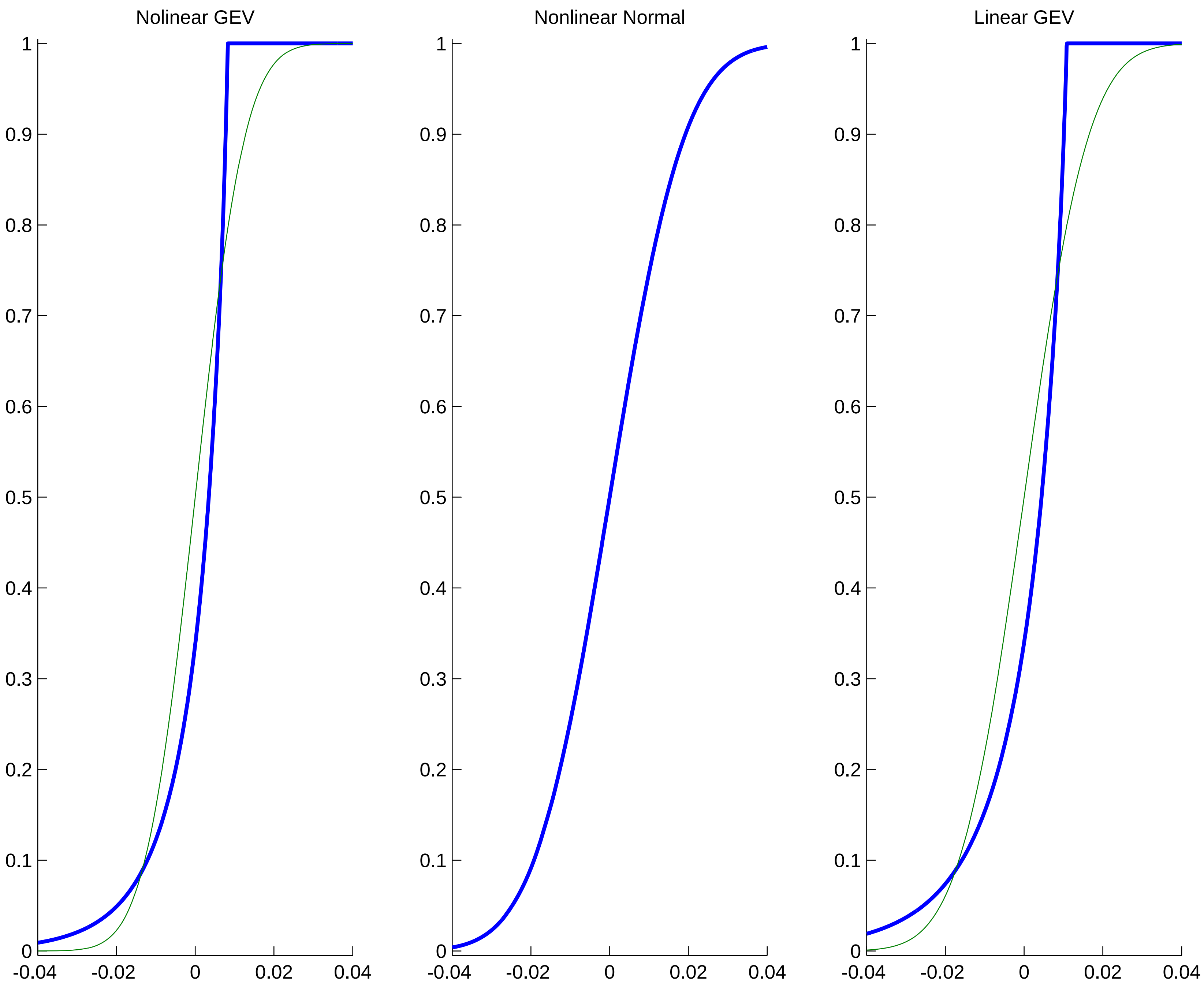
	Model					
	Nonlinear				Linear	
	GEV		Normal		GEV	
Parameter	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
Wages	230.7	0.001	282.3	0.002	9932.8	0.001
Prices	14.12	0.021	45.64	0.072	31.30	0.043

Note: s.e. are standard errors computed using a *k*-step block bootstrap with 5 steps and 19 replications. During the estimation $\beta = 0.995$, $\alpha = 1/3$, $\Pi = 1$, $\mu = 1.1$ and $\varsigma = 1.4$.

SMM Estimates: Productivity Shock

	Model					
	Nonlinear				Linear	
	GEV		Normal		GEV	
Parameter	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
Autoregresive coefficient	0.958	0.021	0.848	0.024	0.933	0.025
Scale ($\times 10^{-2}$)	0.899	0.227	–	–	1.174	0.226
Shape	–1.204	0.055	–	–	–1.189	0.212
Standard deviation ($\times 10^{-2}$)	0.999	0.230	1.502	0.143	1.292	0.223
Skewness	–2.655	0.182	0	–	–2.602	0.751

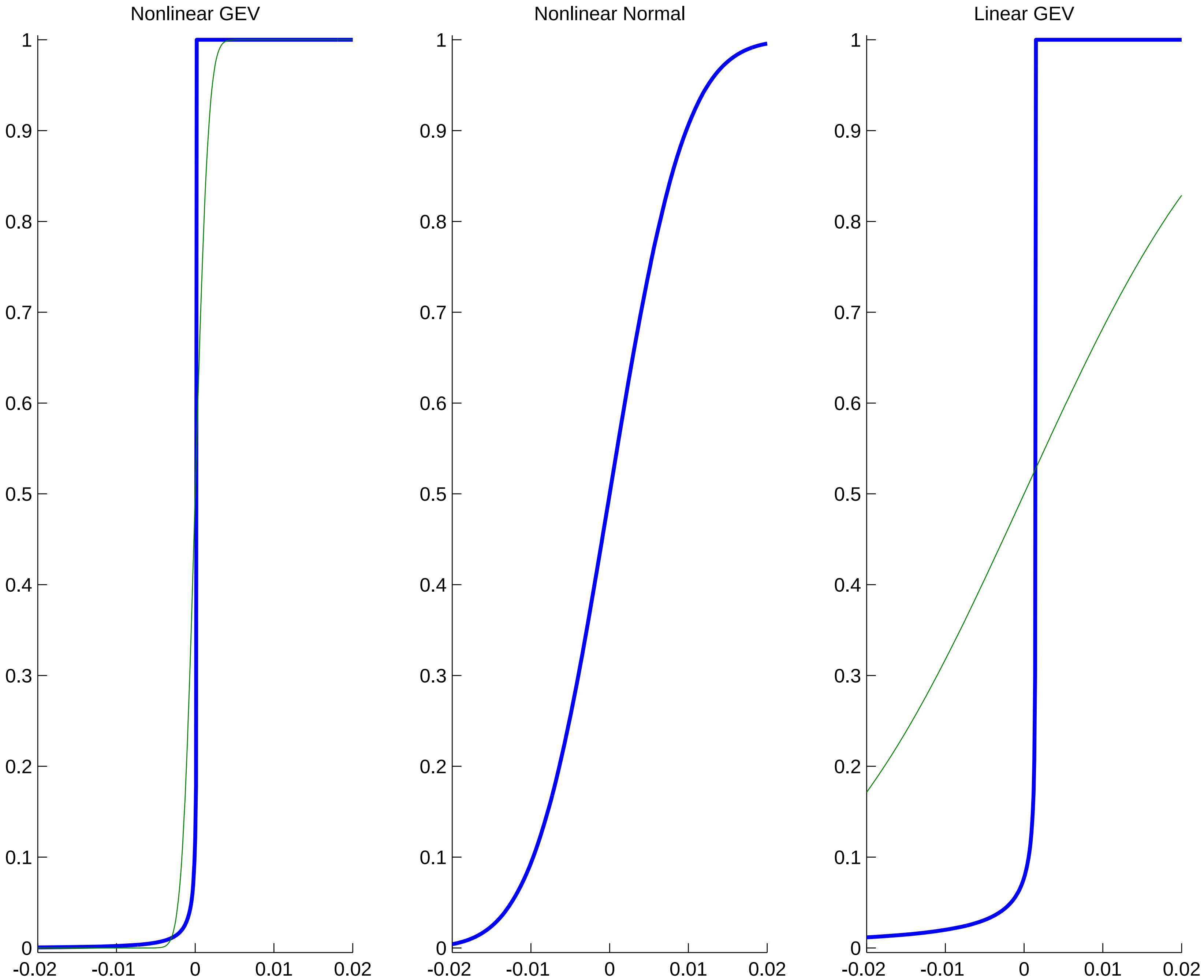
Figure 1: Estimated Cumulative Distribution Function of Productivity Shock



SMM Estimates: Labor Supply Shock

	Model					
	Nonlinear				Linear	
	GEV		Normal		GEV	
Parameter	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
Autoregresive coefficient	0.996	0.011	0.968	0.015	0.996	0.002
Scale ($\times 10^{-4}$)	0.418	0.615	–	–	0.715	0.644
Shape	–3.755	0.062	–	–	–4.881	0.182
Standard deviation ($\times 10^{-2}$)	0.132	0.106	0.758	0.401	2.107	1.243
Skewness	–45.50	3.252	0	–	–165.6	26.16

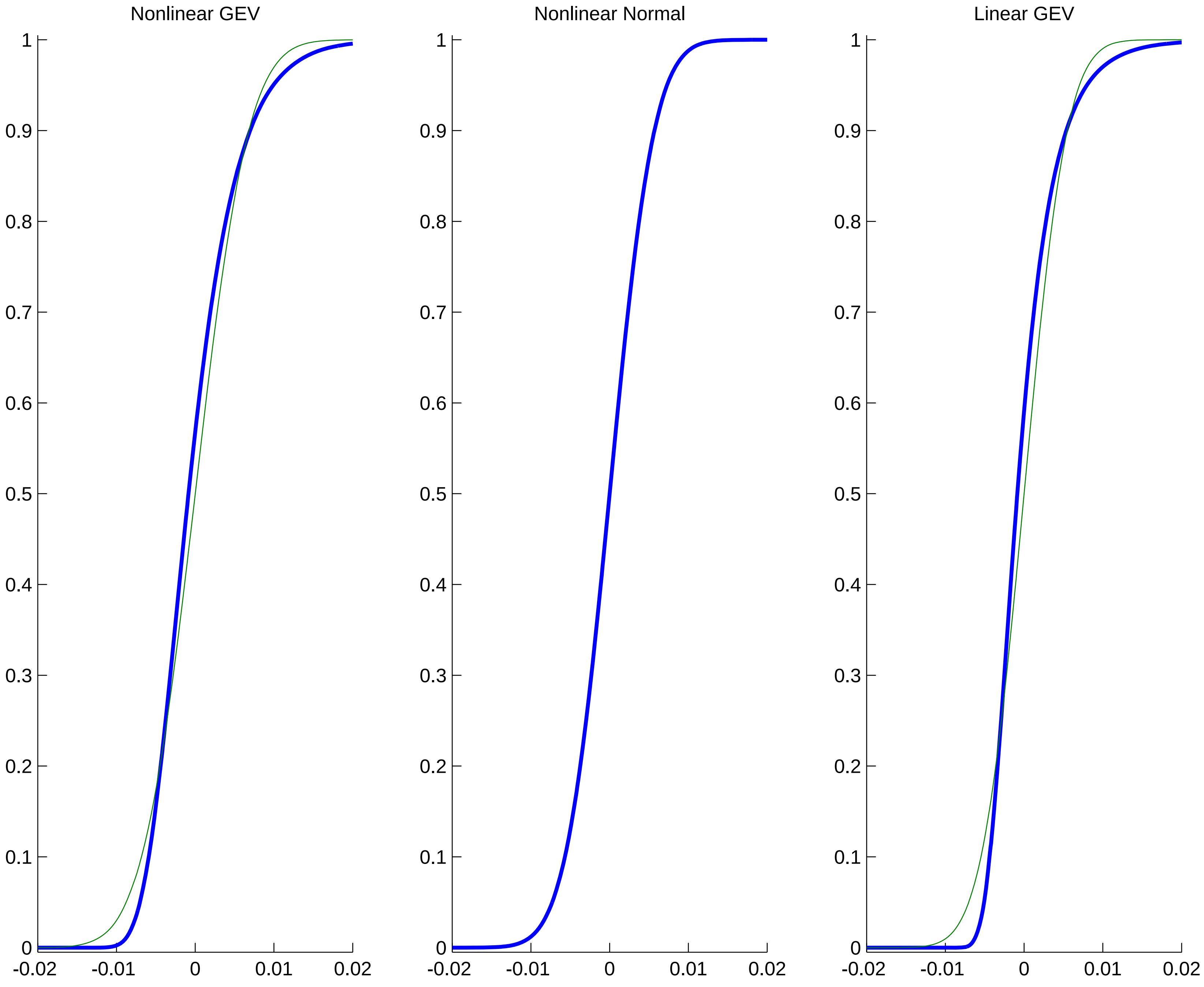
Figure 2: Estimated Cumulative Distribution Function of Labor Supply Shock



SMM Estimates: Taylor Rule

	Model					
	Nonlinear				Linear	
	GEV		Normal		GEV	
Parameter	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
Smoothing	0.844	0.060	0.862	0.063	0.693	0.043
Inflation	0.384	0.077	0.385	0.083	0.384	0.059
Output	0.143	0.037	0.137	0.048	0.063	0.050
Scale ($\times 10^{-2}$)	0.420	0.098	–	–	0.298	0.082
Shape ($\times 10^{-1}$)	–0.917	1.887	–	–	0.775	0.851
Standard deviation ($\times 10^{-2}$)	0.532	0.133	0.443	0.114	0.428	0.136
Skewness	1.086	1.881	0	–	1.698	1.194

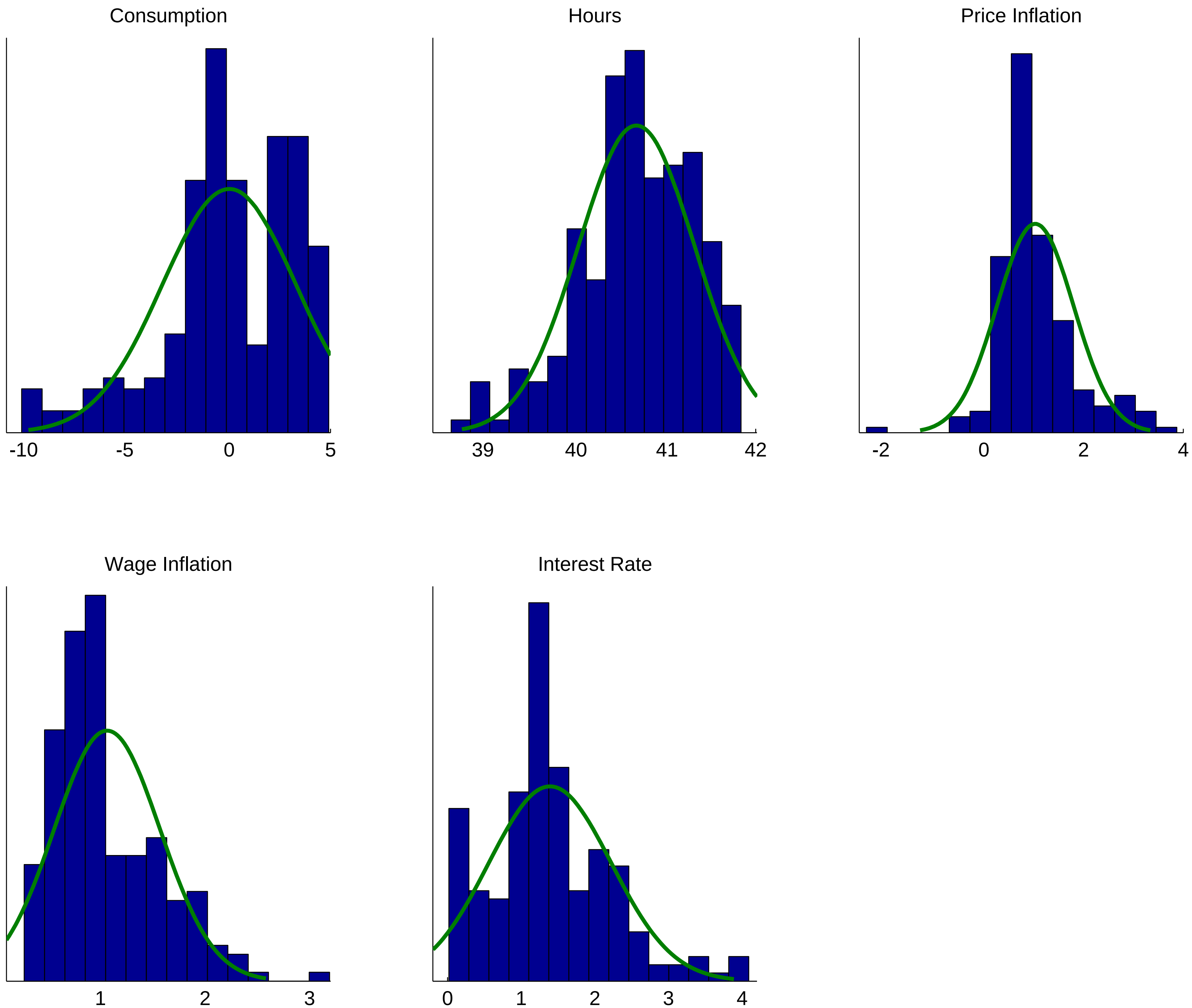
Figure 3: Estimated Cumulative Distribution Function of Monetary Policy Shock



Skewness

	U.S. Data	Model: Nonlinear GEV	Model: Nonlinear Normal	Model: Linear GEV
Consumption	–0.874			
Hours	–0.580			
Price inflation	0.656			
Wage inflation	1.023			
Nominal interest rate	0.641			

Figure 4: Asymmetry of U.S. Macroeconomic Data



Skewness

	U.S. Data	Model: Nonlinear GEV	Model: Nonlinear Normal	Model: Linear GEV
Consumption	–0.874	–0.566		
Hours	–0.580	–0.625		
Price inflation	0.656	1.150		
Wage inflation	1.023	0.899		
Nominal interest rate	0.641	0.703		

Skewness

	U.S. Data	Model		
		Nonlinear GEV	Nonlinear Normal	Linear GEV
Consumption	–0.874	–0.566	0.014	
Hours	–0.580	–0.625	0.078	
Price inflation	0.656	1.150	0.094	
Wage inflation	1.023	0.899	–0.066	
Nominal interest rate	0.641	0.703	0.044	

Skewness

		Model		
	U.S. Data	Nonlinear GEV	Nonlinear Normal	Linear GEV
Consumption	–0.874	–0.566	0.014	–0.725
Hours	–0.580	–0.625	0.078	–0.538
Price inflation	0.656	1.150	0.094	0.987
Wage inflation	1.023	0.899	–0.066	0.991
Nominal interest rate	0.641	0.703	0.044	0.659

Kurtosis

	U.S. Data	Model: Nonlinear GEV	Model: Nonlinear Normal	Model: Linear GEV
Consumption	3.581			
Hours	3.190			
Price inflation	6.051			
Wage inflation	3.996			
Nominal interest rate	3.767			

Kurtosis

		Model		
	U.S. Data	Nonlinear GEV	Nonlinear Normal	Linear GEV
Consumption	3.581	3.419		
Hours	3.190	3.574		
Price inflation	6.051	5.701		
Wage inflation	3.996	4.312		
Nominal interest rate	3.767	3.972		

Kurtosis

		Model		
	U.S. Data	Nonlinear GEV	Nonlinear Normal	Linear GEV
Consumption	3.581	3.419	2.720	
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Wage inflation	3.996	4.312	3.062	
Nominal interest rate	3.767	3.972	2.663	

Kurtosis

	U.S.	Model		
	Data	Nonlinear GEV	Nonlinear Normal	Linear GEV
Consumption	3.581	3.419	2.720	3.796
Hours	3.190	3.574	3.092	3.720
Price inflation	6.051	5.701	3.114	4.715
Wage inflation	3.996	4.312	3.062	4.227
Nominal interest rate	3.767	3.972	2.663	3.695

Jarque-Bera Test

		Model		
Series	U.S. Data	Nonlinear GEV	Nonlinear Normal	Linear GEV
Consumption	0.001			
Hours	0.011			
Price inflation	0.001			
Wage inflation	0.001			
Nominal interest rate	0.003			

Jarque-Bera Test

		Model		
Series	U.S. Data	Nonlinear GEV	Nonlinear Normal	Linear GEV
Consumption	0.001	0.001		
Hours	0.011	0.001		
Price inflation	0.001	0.001		
Wage inflation	0.001	0.001		
Nominal interest rate	0.003	0.001		

Jarque-Bera Test

		Model		
Series	U.S. Data	Nonlinear GEV	Nonlinear Normal	Linear GEV
Consumption	0.001	0.001	0.037	
Hours	0.011	0.001	0.249	
Price inflation	0.001	0.001	0.130	
Wage inflation	0.001	0.001	0.402	
Nominal interest rate	0.003	0.001	0.008	

Jarque-Bera Test

		Model		
	U.S.	Nonlinear	Nonlinear	Linear
Series	Data	GEV	Normal	GEV
Consumption	0.001	0.001	0.037	0.001
Hours	0.011	0.001	0.249	0.001
Price inflation	0.001	0.001	0.130	0.001
Wage inflation	0.001	0.001	0.402	0.001
Nominal interest rate	0.003	0.001	0.008	0.001

Dynamics

Since model is nonlinear, impulse responses depend on sign, size, and timing (see Gallant, Rossi and Tauchen, 1993, and Koop, Pesaran, and Potter, 1996)

Consider innovations in the 1st and 99th percentiles

Innovations take place when system is at the stochastic steady state

Figure 8: Interest Rate Policy Function

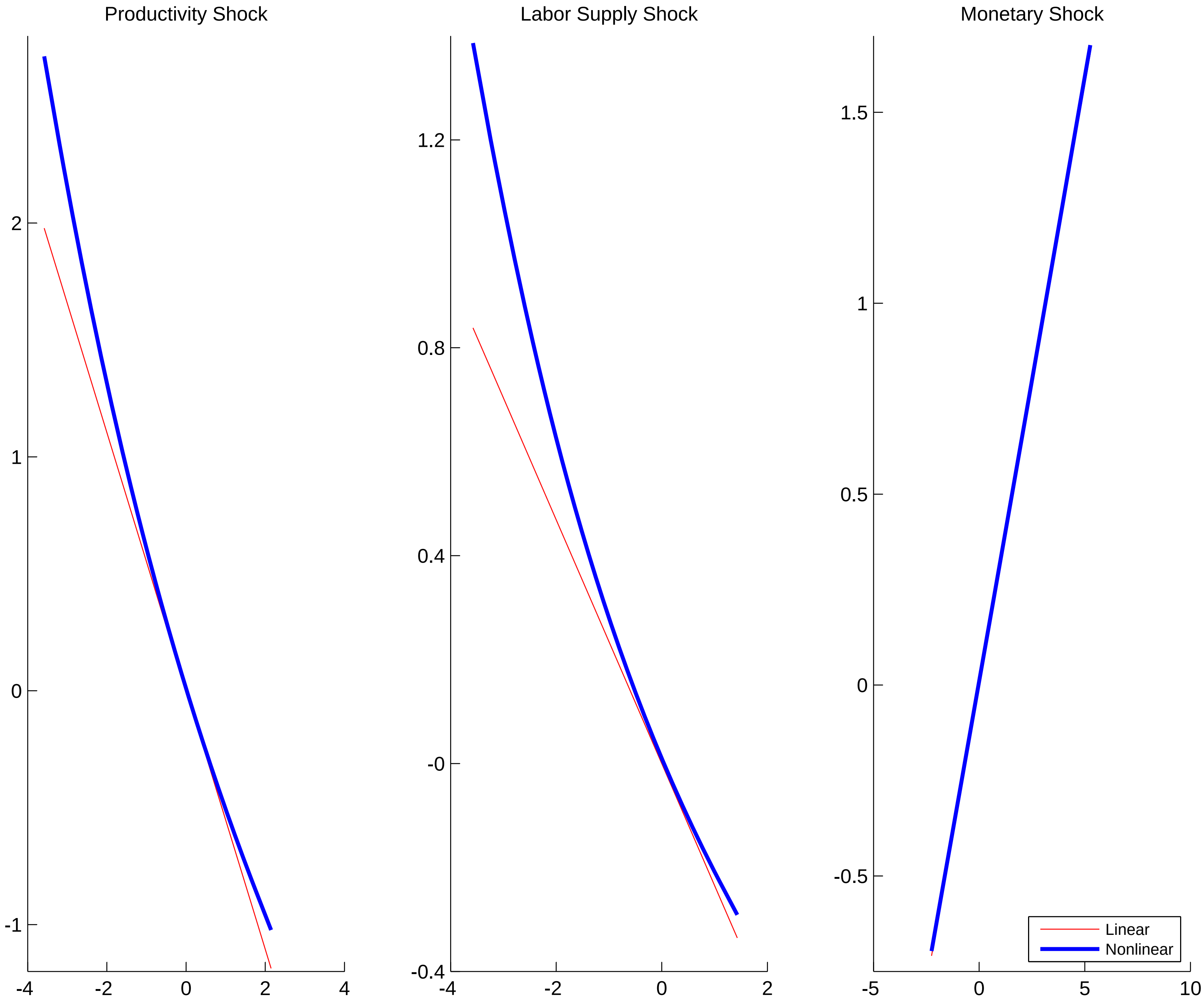


Figure 5: Responses to Extreme Productivity Shocks under Taylor Rule Policy

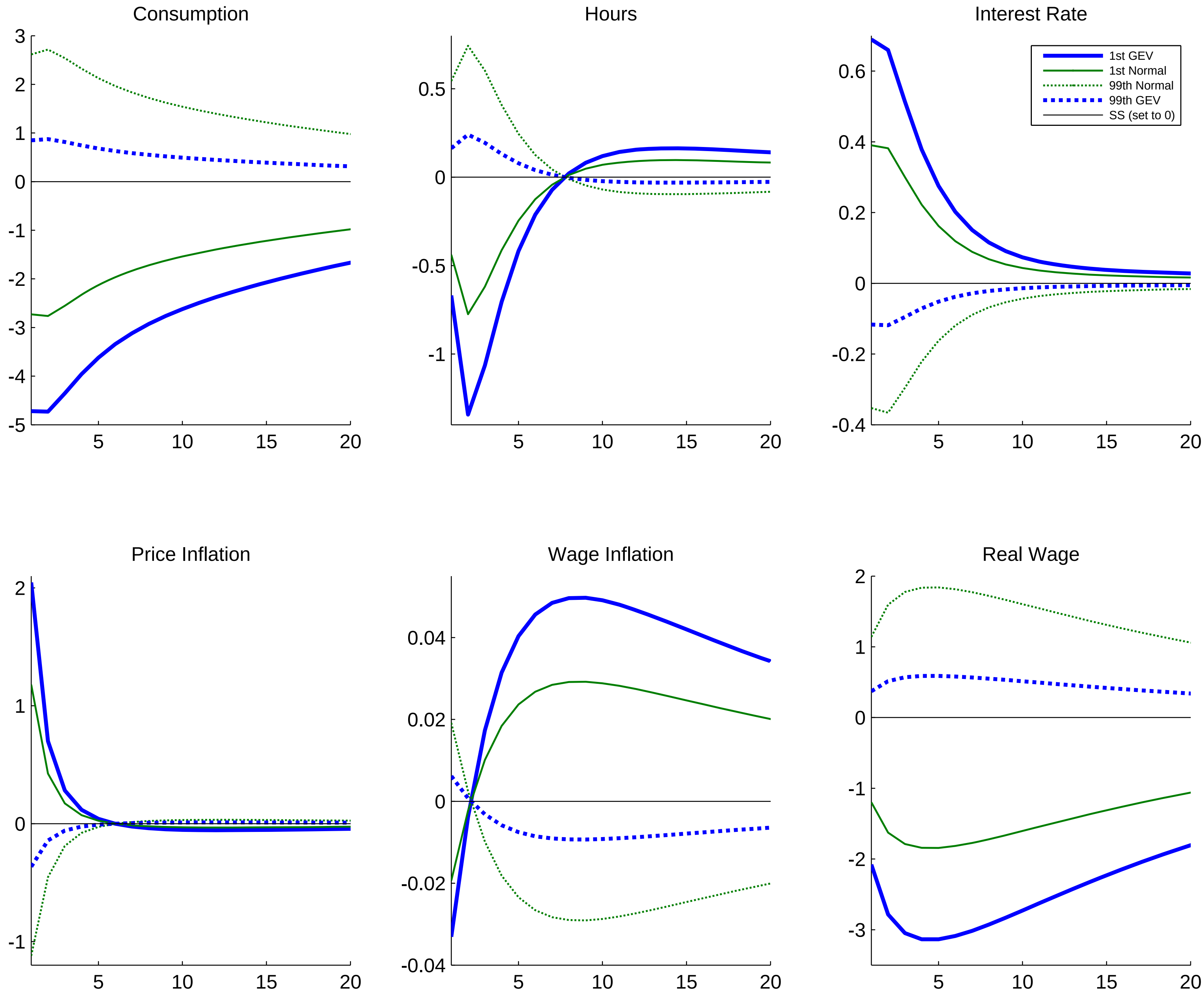


Figure 6: Responses to Extreme Labor Supply Shocks under Taylor Rule Policy

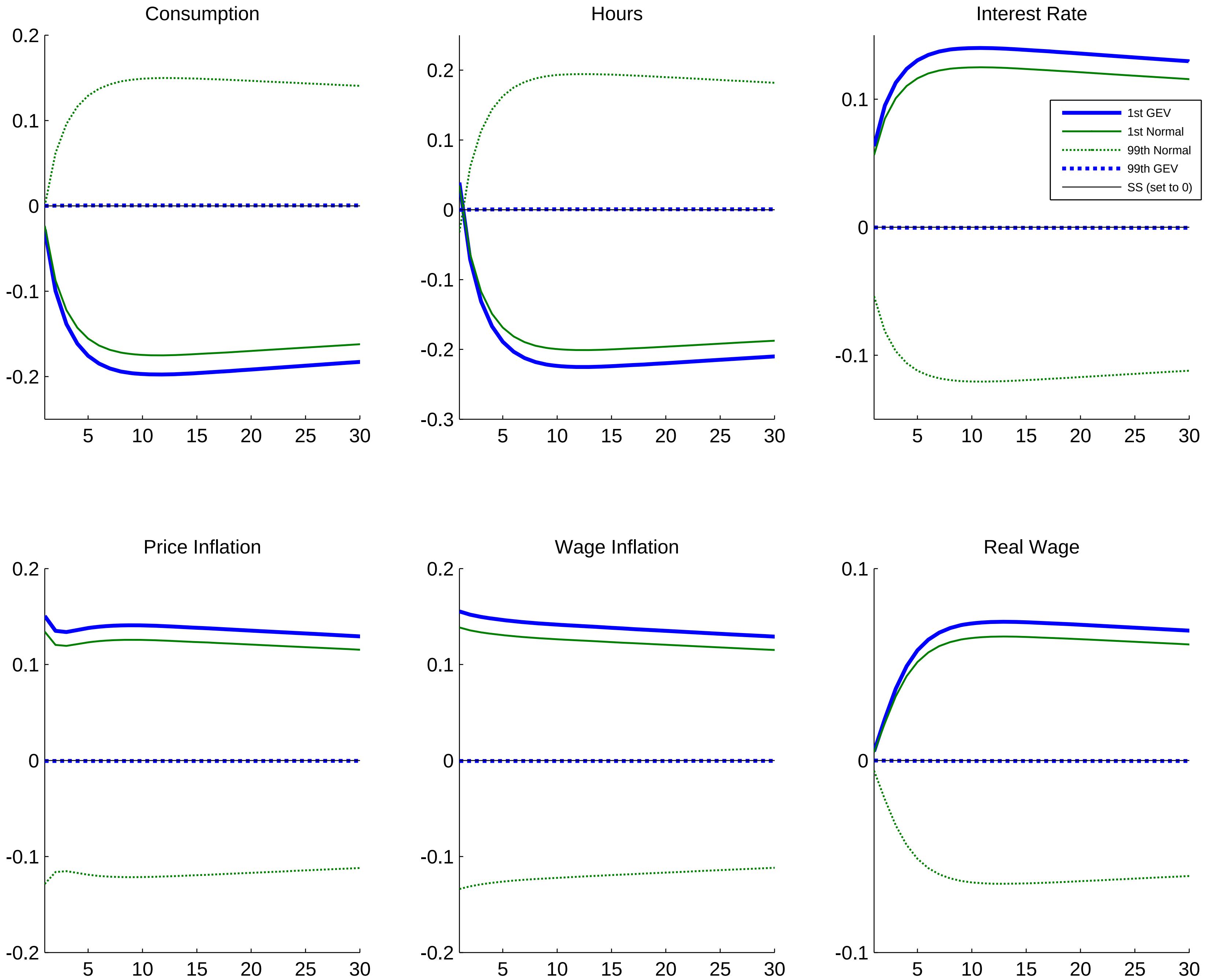
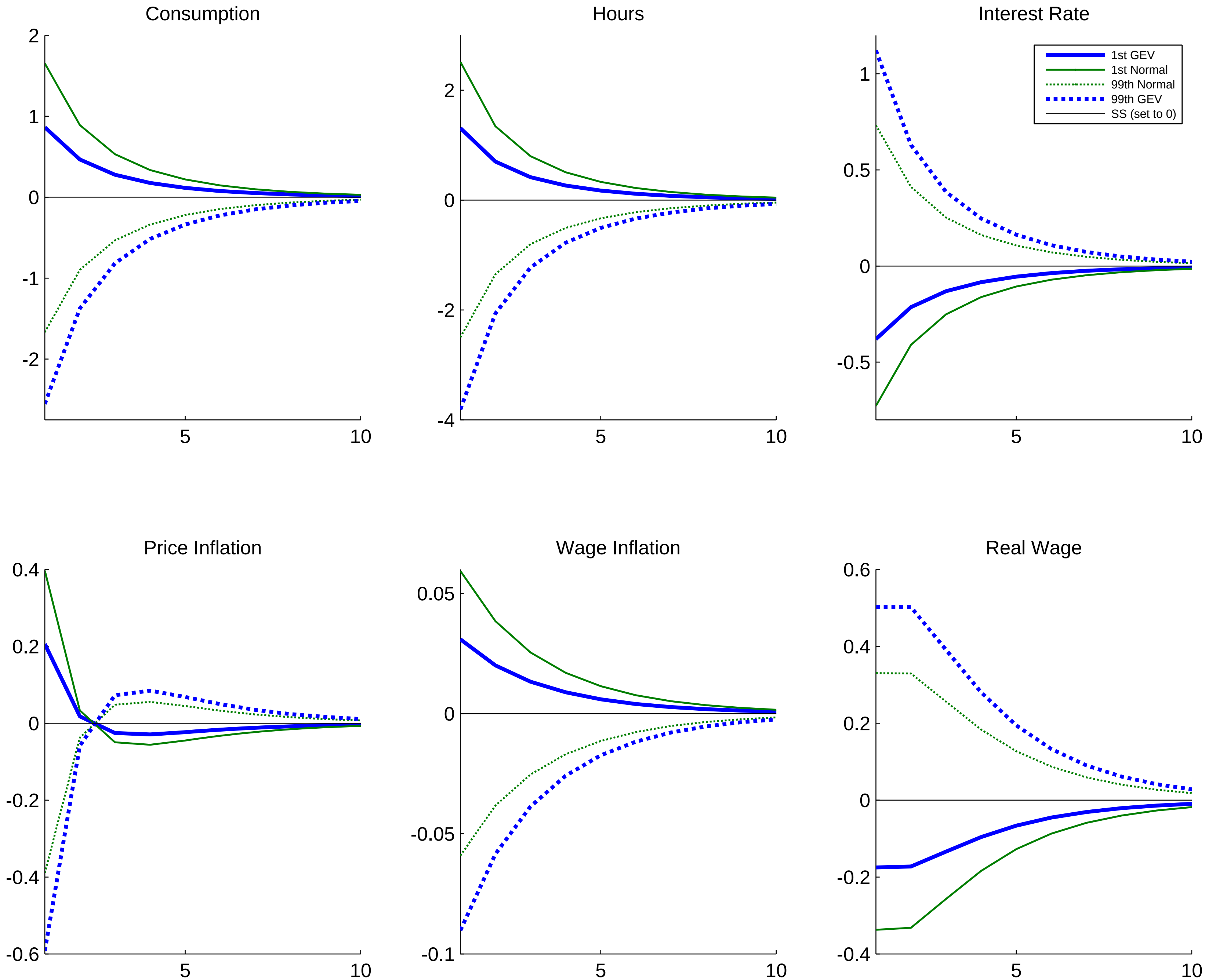


Figure 7: Responses to Extreme Monetary Shocks under Taylor Rule Policy



Ramsey Policy

A benevolent central bank chooses $\{c_t,\ h_t,\ w_t,\ i_t,\ \Omega_t,\ \Pi_t\}_{t=\tau}^{\infty}$ to maximize the households welfare subject to:

The social resource constraint

First-order conditions of firms and

First-order conditions of households

Dynamics

Consider innovations in the 1st and 99th percentiles

Innovations take place when system is at the stochastic steady state

Figure 11: Optimal Interest Rate Policy Function

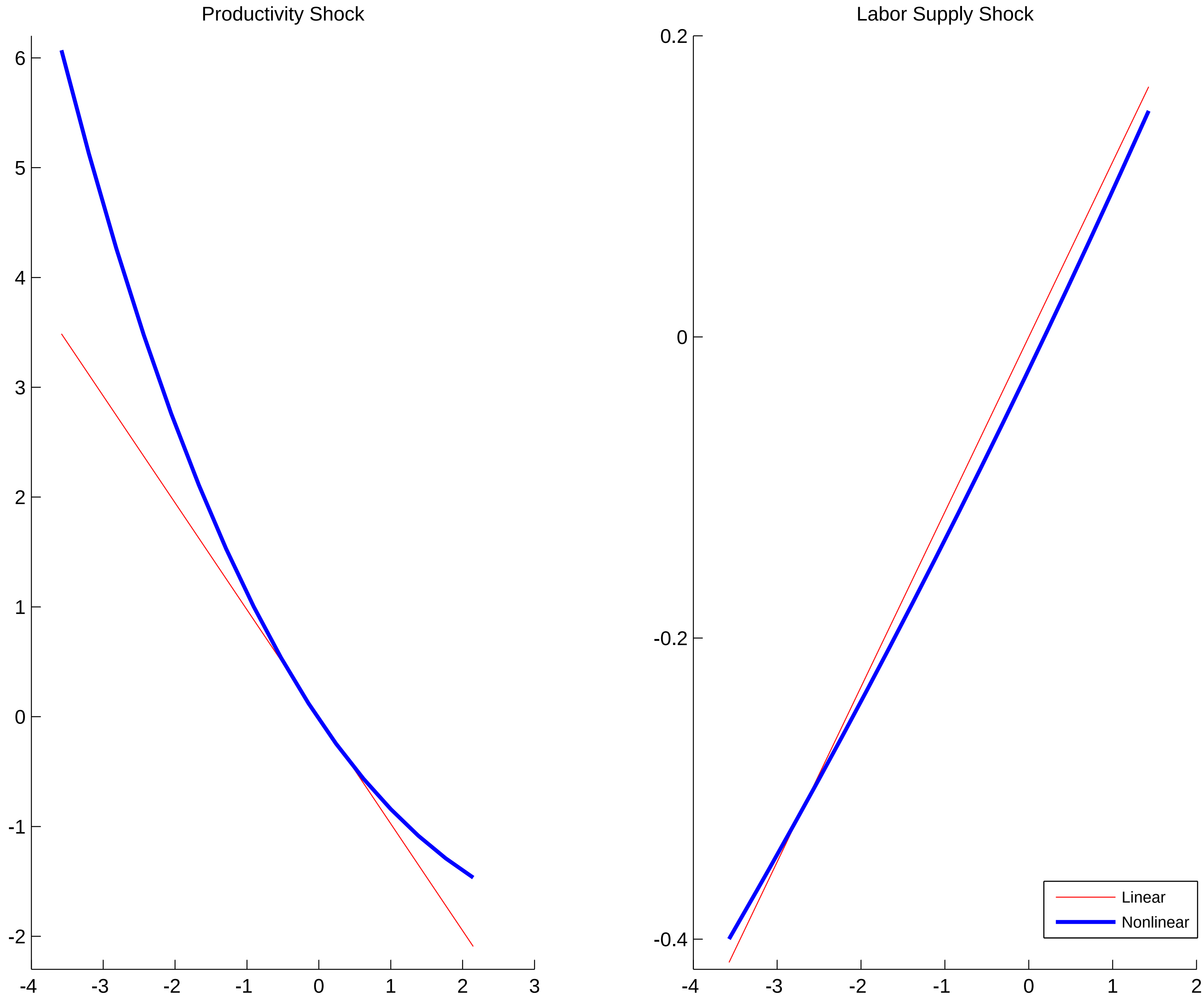


Figure 9: Optimal Responses to Extreme Productivity Shocks

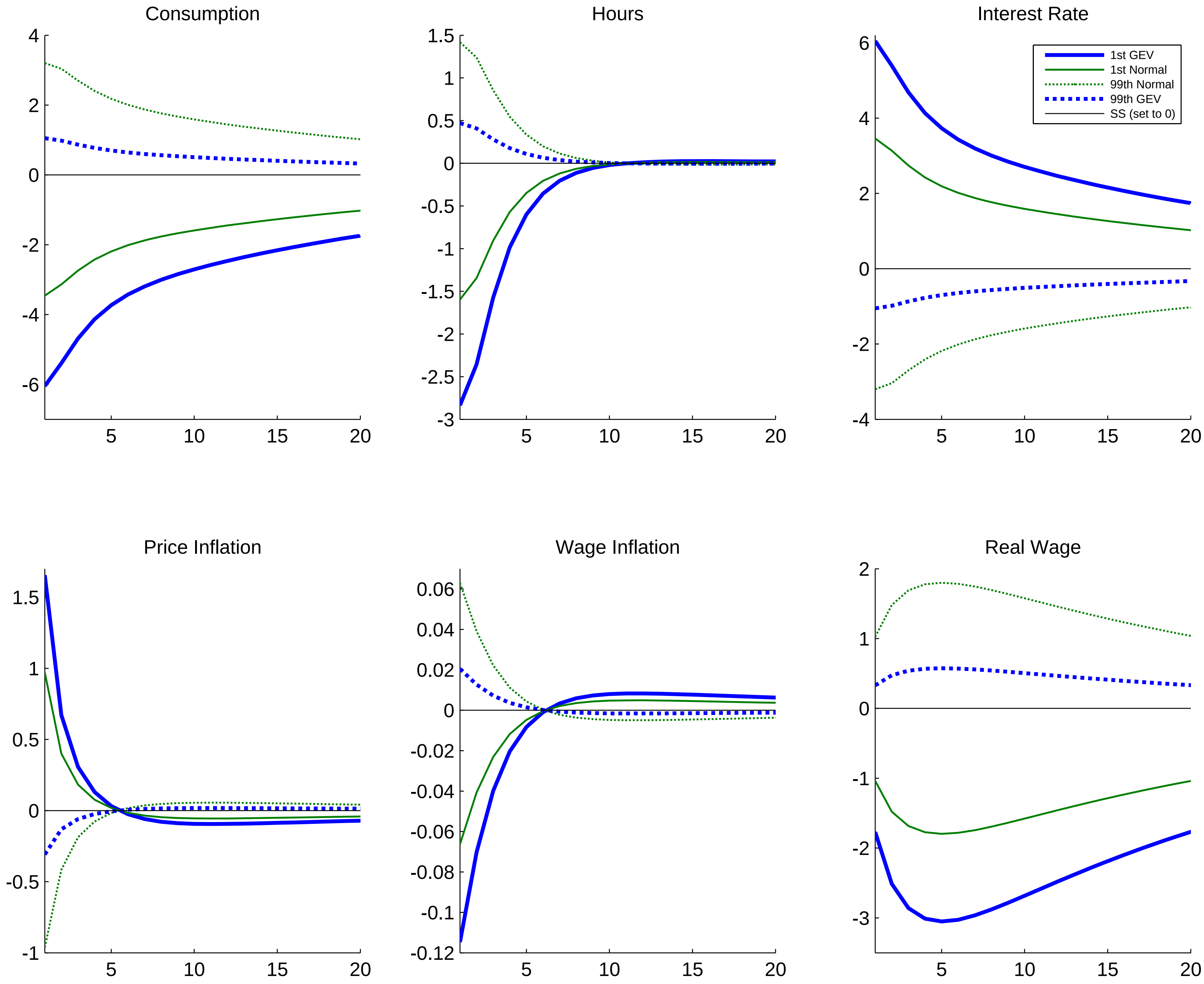
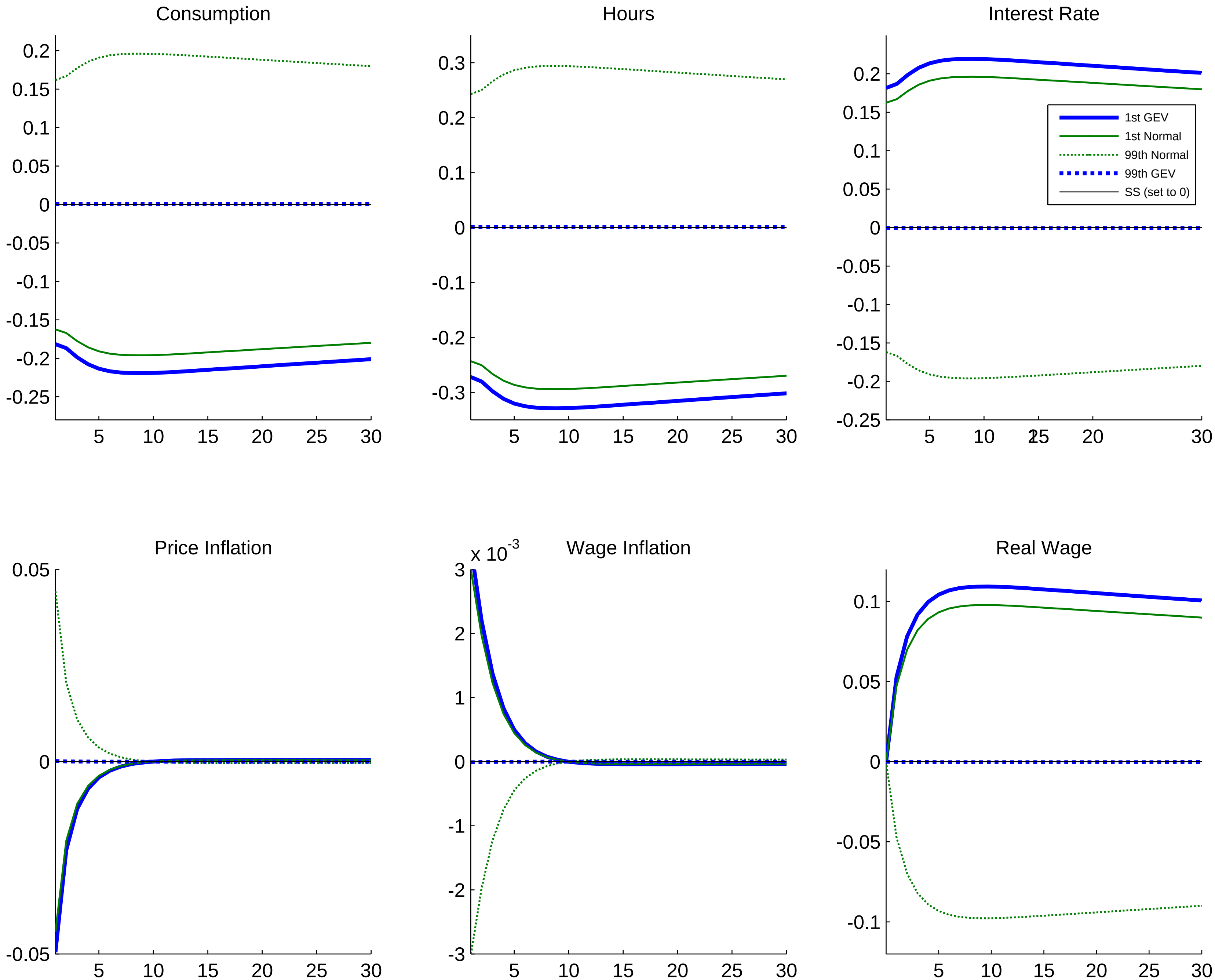


Figure 10: Optimal Responses to Extreme Labor Supply Shocks



Optimal Inflation

In the deterministic steady state, (gross) optimal inflation $= 1.0$

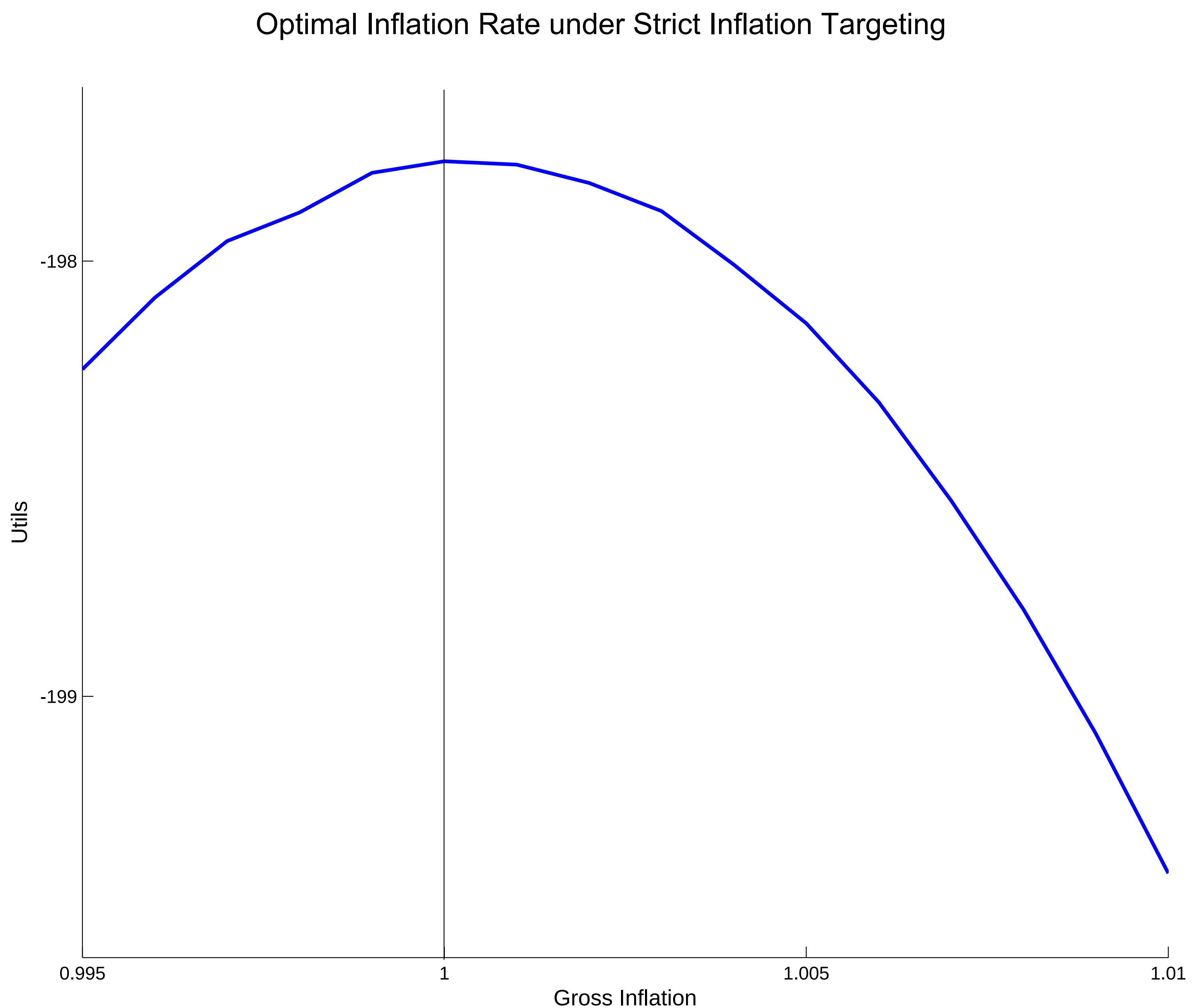
In the stochastic steady state, (gross) optimal inflation $= 1.001$

Comparison with Strict Inflation Targeting

Inflation targeter has less knowledge and flexibility than Ramsey

Optimal inflation may be different from that under Ramsey

In the stochastic steady state, (gross) optimal inflation ≈ 1.0



Optimal Inflation Rate under Strict Inflation Targeting
Utils
-198
-199
0.995
1
1.005
1.01
Gross Inflation

Summary

In an economy where extreme events can occasionally happen:

There is (or there should be) a prudence motive in monetary policy making

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In an economy where extreme events can occasionally happen:

There is (or there should be) a prudence motive in monetary policy making

However, optimal (net) inflation is close to zero because inflation costs paid every period override the precautionary benefits of having a non-zero inflation target

Summary

In an economy where extreme events can occasionally happen:

There is (or there should be) a prudence motive in monetary policy making

However, optimal (net) inflation is close to zero because inflation costs paid every period override the precautionary benefits of having a non-zero inflation target

Under both the Taylor and Ramsey policies, the central bank responds non-linearly and asymmetrically to shocks