Comments on Eric Swanson's Paper, "On Signal Extraction and Non-Certainty Equivalence in Optimal Monetary Policy Rules"

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Eric Swanson's paper (2000) is a skillful and nicely presented study that clarifies several points regarding mismeasurement and signal extraction in the context of optimal monetary policy design. The basic analytical points have evidently been made before, but it is quite useful to have them emphasized and laid out in some detail, as Swanson has done. This detail is fully justified, in my opinion, by the importance of the practical topic that provides the paper's featured application, namely, the difficulty that monetary policymakers have in measuring their economy's potential output levels. That difficulty has led me, in work uninformed by optimal control analysis, to argue that it is probably undesirable for monetary policy to respond strongly to measured levels of the output gap (i.e., the relative excess of actual over potential output.)[1] It is from that perspective that I have approached the paper.

To begin with, I have two main comments. The first is that while the basic analytic results are undeniably interesting and elegant, they seem to be, to a considerable extent, inapplicable to actual practice. That is, actual central banks do not develop their measures of potential output within the context of a full optimal control exercise. Instead, they develop these measures separately and then use them in policy rule calculations that may or may not be of the optimizing type. For the latter type even to be possible, of course, the central bank must have a well-articulated and quantified model and objective function.

My second comment is that, with respect to mismeasurement of potential output, the actual problem is in large part <u>conceptual</u>—that is, it stems from the existence of various different concepts of the reference value that Swanson refers to as "potential output." That there are several distinct concepts is implicit in the terms used by different researchers. Some of these involve the words "trend," "capacity," "natural rate," "NAIRU," "market-clearing," and "flexible-price" output, rather than potential output. There are fewer distinct concepts than terms, probably, but there seem to be at least three fundamentally different ones in use: trend, NAIRU, and flexible-price concepts. And of course there are <u>many</u> ways of measuring trend output that are quite different in their effects. Furthermore, differences among these concepts are unlikely to possess the orthogonality properties of pure "noise."

These comments lead naturally into the question, <u>which</u> of the concepts is most appropriate theoretically? There is, I believe, a rather clear-cut answer. From the perspective of dynamic, optimizing analysis,[2] the appropriate concept is the third of those listed: the flexible-price concept—i.e., the output level that would prevail in the absence of nominal price stickiness. There have been very few attempts to implement this concept empirically, but there is one in McCallum and Nelson (1999a). Consequently, the next few paragraphs will present an improved and extended implementation. Then I will examine the quantitative consequences of use by the central bank of a simple <u>trend</u> concept if in fact the flexible-price concept is appropriate.

In our paper, Nelson and I assumed that output is produced according to a Cobb-Douglas production function, which in log terms could be written as

(1) $y_t = \alpha_0 + \alpha_1 t + \alpha_2 n_t + (1-\alpha_2)\, k_t + a_t,$

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[1] See, e.g., McCallum (1999a, pp. 1503-4). For some interesting historical evidence, see Orphanides (1999).
[2] See, for example, Clarida, Gali, Gertler (1999), Gali and Gertler (1999), Goodfriend and King (1997), Woodford (1999), and McCallum and Nelson (1999a).

where $0<\alpha_2<1$. Here $\alpha_1 t$ reflects a possible deterministic trend component of technological change, with the stochastic component being a_t. The variables y_t, n_t, and k_t are logs of output, labor input, and capital in per-person terms. Then the value of y_t that would prevail (given k_t) if there were full price flexibility would be

(2) $\quad \bar{y}_t = \alpha_0 + \alpha_1 t + \alpha_2 \bar{n}_t + (1-\alpha_2) k_t + a_t,$

where $\bar{n}_t$ is the market-clearing value of n_t. In our paper we used the simplifying assumption that $\bar{n}_t$ is a constant, but this is not necessary with this approach. Thus as a measure of the output gap we have

(3) $\quad y_t - \bar{y}_t = \alpha_2(n_t - \bar{n})$

and in the paper we measured n_t for the United States, 1955.1 - 1996.4, as total man-hours employed in non-agricultural private industry divided by the civilian labor force. We scaled the measure so that the average value of n_t equalled $\bar{n}$, a step whose necessity is undesirable, but there was no need to remove any deterministic trend from our $n_t - \bar{n}$ series. Then using $\alpha_2 = 0.7$, we constructed a series for $y_t - \bar{y}_t \equiv \tilde{y}_t$, which is shown in Figure 1. Its movements are there contrasted with one specific trend measure. The latter is a simple linear trend based on 1955-1996 data, but that particular example is shown principally for illustrative purposes.

The main point to be made here that non-zero realizations of a_t affect $\bar{y}_t$ one-for-one. By contrast, many detrending procedures, of the types used extensively by academics and to some extent by central banks, remove a_t almost entirely from each period's measure of $\bar{y}_t$.[3] The same is true, furthermore, for many NAIRU-based

[3] That is the case for linear or piecewise linear or quadratic or Hodrick-Prescott detrending of y_t, for example. Using Δy_t for $\Delta \bar{y}_t$ is also inappropriate unless n_t is a random walk.

procedures.[4] So the question at hand is whether this conceptual discrepancy is of quantitative importance—whether use of a mistaken concept will induce a large extent of suboptimality into policy rules that rely upon measures of the output gap.

There are two paragraphs in McCallum and Nelson (1999a) that briefly mention relevant results for two different models, in one of which the effects are not sizable. In our next paper we improved other aspects of our model's specification and since then I have altered the model still more. So let me quickly report some new and more extensive results.

The policy rule to be considered is

(4) $R_t = (1-\mu_3)[E_{t-1}\Delta p_{t+1} + \mu_1(E_{t-1}\Delta p_t - \pi^*) + \mu_2 \tilde{y}_t] + \mu_3 R_{t-1} + e_t,$

i.e., a Taylor-type rule with interest smoothing added, as in Clarida, Gali, and Gertler (1998). For μ_3, I use 0.8, as estimated by Clarida, Gali, and Gertler (1999), McCallum and Nelson (1999b), and many others. For μ_1, I will use 0.5 or 1.0 and then vary μ_2 over a wide range of values. The model used includes an optimizing or "expectational" IS function, with habit formation as in Fuhrer (1998).[5] The main change relative to the closed-economy model in McCallum and Nelson (1999b) is that in place of the P-bar price adjustment formulation, I have here used the second specification from McCallum (1999b), which is similar to Fuhrer and Moore (1995):

(5) $\Delta p_t = 0.5[E_t\Delta p_{t+1} + 0.5\Delta p_{t-1}] + \alpha \tilde{y}_t + u_t.$[6]

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[4] This would be true for any NAIRU-based procedure that used a constant NAIRU. It is also true for the procedure in Braun (1987), which is (I believe) basically the procedure used by the Fed for several years.

[5] The parameter, that equals the intertemporal elasticity of substitution in consumption when habit formation is absent, is raised from 0.2 to 0.4, and the habit parameter h is kept at 0.8.

[6] Here u_t is white noise. This is calibrated as in McCallum (1999b), with $\alpha = 0.0032$ and $\sigma_u = .002$.

Together, equations (4), (5), and the IS relation generate values for Δp_t, $\tilde{y}_t$, and R_t given an exogenous evolution of $\bar{y}_t$.[7] The stochastic process governing that evolution is of course crucial. But given our estimates of $\tilde{y}_t$ as explained above, $\bar{y}_t$ is obtained as $y_t - \tilde{y}_t$. The resulting U.S. time series data for 1955-1996 is well modeled as

(6) $\bar{y}_t - \bar{y}_{t-1} = .0073 + \varepsilon_t$,

where ε_t is white noise with $\sigma_\varepsilon = 0.007$.[8]

To examine the effects of one type of conceptual error in measuring $\bar{y}_t$, let us take the case in which the central bank uses instead of (2) the measure

(7) $\bar{y}_t^{TR} = \alpha_0 + \alpha_1 t + \alpha_2 k_t^{TR}$

where k_t^{TR} is a trend value for k_t. I will pretend that the central bank has accurate knowledge of α_1 and k_t^{TR}, so they can be left out of the analysis. Also, I will conduct simulations ignoring constants which implies that the central bank also knows α_0 and α_2 n. Thus the conceptual error, as implemented, is that the central bank neglects the influence of a_t on $\bar{y}_t$.

Results are reported in Table 1. The left hand panels assume that the output gap is measured correctly by the central bank; the right hand panels assume that it incorrectly uses the trend measure (7). On the left we see that as stronger feedback is applied to the gap—i.e., as μ_2 is increased—the variability of $\tilde{y}_t$ falls. The variability of inflation rises but not sharply; the sum of the two standard deviations ($\sigma_{\tilde{y}} + \sigma_{\Delta p}$) is approximately invariant to μ_2. In the right hand panels, by contrast, the variability of y_t is driven down

[7] Of course, actual $\bar{y}_t$ values are endogenous because k_t is variable. But here, as in much of the literature, I

as μ_2 is increased, but for the true gap $\tilde{y}_t$ the effect is small whereas the increase in inflation variability is very large. Instead of staying almost constant, the sum of the two standard deviations increases enormously as strong feedback response is applied to the incorrectly measured gap.

Clearly, there are many more comparisons of this type that one could make. The one just presented seems to me to be of special interest, but that is a matter of judgment. For this case, in any event, the results show this potential-output concept error to be of major importance quantitatively.

In terms of experimental design, it could be quite worthwhile to do away with the treatment of capital as fixed. This would make the variation of $\bar{y}_t$ endogenous, although still largely dependent upon technology shocks. Quite recently, Casares (2000) has developed a nice tractable way of endogenizing capital accumulation that matches the data reasonably well.

In this regard it should be noted that if $\bar{y}_t$ is endogenous, it becomes undesirable from a theoretical perspective to include the output gap as the aggregate demand variable in the IS function. If the latter is derived in an optimizing fashion, it is y_t not $\tilde{y}_t$ that appears. If the latter is used instead, that throws ($\bar{y}_t$ - $E_t\bar{y}_{t+1}$) into the composite disturbance term. This is probably harmless if $\bar{y}_t$ is exogenous—especially if it is close to a random walk, as Nelson and I have estimated—but is wrong in principle and could be harmful in some cases.

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keep k_t fixed. On this, see comments below.

[8] Here $\varepsilon_t = \Delta a_t$ if k_t is constant but not otherwise.

These comments have strayed from Swanson's useful paper, primarily because my only objection to his analysis concerns its applicability. It is agreeable, then to be able to conclude with agreement on two practical issues. Specifically, my approach agrees with Swanson's discussion on the following points:

(i) It seems likely that, because of imperfect measurement of $\bar{y}_t$, monetary policymakers should attenuate policy-rule coefficients on the measured output gap;

(ii) The basic problem does not stem primarily from the distinction between real-time and revised data per se. (Rather, it stems largely from uncertainty concerning reference-value concepts.)

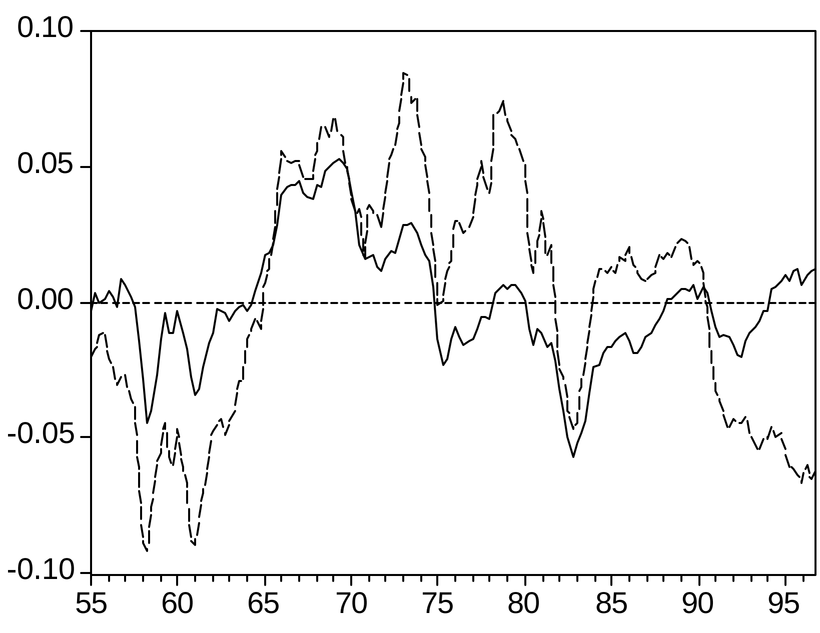
Table 1 - Simulation Results

Standard deviations of Δp_t, $\tilde{y}_t$, y_t, R_t respectively

	With $\tilde{y}_t$ in policy rule (4)						With y_t replacing $\tilde{y}_t$ in (4)				
	value of μ_2						value of μ_2				
μ_1, μ_3	0.0	0.5	1.0	2.0	5.0		0.0	0.5	1.0	2.0	5.0
0.5, 0.8	2.57	2.85	3.10	3.56	4.19		2.55	4.50	7.46	13.89	35.01
"	2.07	1.60	1.36	1.10	0.85		2.07	1.89	2.05	2.26	2.59
"	4.20	4.03	3.83	3.81	3.59		4.25	3.75	3.57	3.51	3.63
"	2.23	2.25	2.42	2.83	3.57		2.23	4.11	7.14	13.70	35.04
1.0, 0.8	2.31	2.61	2.77	3.15	3.74		2.37	3.25	4.48	7.37	17.31
"	2.35	1.98	1.67	1.36	0.97		2.46	2.06	1.99	1.99	2.12
"	4.32	3.89	3.95	3.85	3.77		4.28	4.23	3.77	3.54	3.51
"	2.48	2.47	2.49	2.66	3.22		2.58	3.11	4.25	7.12	17.20

Figure 1

Output Gap Measures



Solid line: Derived from labor input
Dashed line: 1955-1996 trend removed

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