Optimal Fiscal Policy in a Monetary Union*

Jordi Galí †   Tommaso Monacelli ‡

December 2004

Abstract

We lay out an optimizing multicountry framework suitable for fiscal policy analysis in a monetary union. We show that, for any given member country, the relinquishment of monetary policy independence, coupled with nominal price rigidity, generates a motive for fiscal stabilization beyond the optimal provision of public goods. This incentive depends on the degree of asymmetry between that country's and the union average natural rate of interest. Interestingly, this is shown to be the case despite fiscal policy being set, in each individual country, in order to maximize welfare at the level of the union as a whole.

Keywords: monetary union, optimal monetary and fiscal policy, sticky prices, fiscal gap.

JEL Classification Number: E52, F41, *E62.*

*Galí thanks the Fundacion Ramon Areces and the European Commission (MAPMU RTN). For comments, we thank participants to seminars at GIIS Geneve, University of Lausanne, and to the EUI conference on "Open Macro Models and Policy in the Development of the European Economy", and to the UPF conference on "Designing a Macroeconomic Policy Framework for Europe".

†CREI and Universitat Pompeu Fabra. Correspondence: CREI, Ramon Trias Fargas, 25-27 08005-Barcelona. URL: http://www.econ.upf.edu/crei/people/gali.

‡IGIER, Università Bocconi and CEPR. Correspondence to: IGIER, Via Salasco 5, 20136, Milan, Italy. URL: http://www.igier.uni-bocconi.it/monacelli.

1 Introduction

The creation of the European Monetary Union (EMU) has led to an array of new challenges for policymakers. Those challenges have been reflected most visibly in the controversies surrounding the implementation (and violation) of the Stability and Growth Pact (SGP), as well as in the frequent criticisms regarding the suitability of the interest rate decisions (or lack thereof) made by the European Central Bank (ECB) since its inception. From the perspective of macroeconomic theory, the issues raised by EMU have created an urgent need for an analytical framework that would allow us to evaluate alternative monetary and fiscal policy arrangements for EMU and other monetary unions that may arise in the future.

In our opinion that analytical framework has to meet several desiderata. First, it has to incorporate some of the main features characterizing the optimizing models with nominal rigidities that have been developed and used for monetary policy analysis in recent years. Secondly, it should contain a fiscal policy sector, with a motive for public consumption, and a purposeful fiscal authority. Thirdly, the framework should incorporate many interlinked open economies.

It is worth noticing that while several examples of optimizing sticky price models of the world economy can be found in the literature, tractability often requires that they be restricted to two-country world economies.[1]While such a framework may be useful to discuss issues pertaining to the links between two large economies (say, the U.S. and the euro area), it can hardly be viewed as a realistic description of the incentives and constraints facing policymakers in a monetary union like EMU, currently made up of twelve countries (each with an independent fiscal authority), but expected to accommodate as many as thirteen additional members over the next few years. Clearly, and in contrast with models featuring two large economies, the majority of the countries in EMU will be small relative to the union as a whole. As a result, some of its policy decisions will have very little impact on other countries. While it should certainly be possible, as a matter of principle, to modify some of the existing two-country models to incorporate an arbitrarily large number of countries (i.e., an N-country model, for large N), it is clear that such undertaking would render the resulting model virtually intractable.

In the present paper we propose a tractable framework for policy analysis in

[1] Just to name a few within the New Open Economy Macroeconomics stream, Obstfeld and Rogoff (1995), Corsetti and Pesenti (2000), Benigno and Benigno (2003), Bacchetta and van Wincoop (2000), Devereux and Engle (2003), Pappa (2003), Kollmann (2001), Chari , Kehoe and McGrattan (2003). Only a subset of these contributions feature a role for a fiscal sector. For a recent analysis of monetary-fiscal policy interaction in a two-country setting and flexible exchange rates see Lombardo and Sutherland (2004). For a two-country analysis more specifically tailored on a monetary union, see Ferrero (2005).

a monetary union that meets the three desiderata listed above. From a modelling perspective, the framework draws significantly from our earlier work on optimal monetary policy in a small open economy (Galí and Monacelli (2004); GM-I, henceforth), though suitably modified here to address a very different set of issues. In particular, as in GM-I, we model the world (union) economy as a continuum of small open economies; as shown below, this approach allows to overcome the likely tractability problems associated with "large N," by making each economy of negligible size relative to rest of the union.

The main differences relative to GM-I are twofold. First, we incorporate a purposeful fiscal policy sector, by having public consumption yield utility to domestic households, with the marginal value attached to that consumption being allowed to vary (stochastically) over time. Secondly, we focus our analysis on the optimal fiscal and monetary policies from the viewpoint of the monetary union as a whole. The case of the small open economy with an independent monetary policy, which was the focus of much of the analysis in GM-I, is re-visited here largely because of its usefulness as a benchmark.

Needless to say, our framework is highly abstract at this point. First, and largely in order to meet our self-imposed tractability requirement, we restrict ourselves to less-than-general parametric specifications for utility and technology, and ignore capital accumulation. Our model also ignores many aspects that are likely to be relevant for the design of optimal policies. Missing elements include, among others, the presence of sticky wages (along with sticky prices), the need to rely on distortionary taxes, the effects of government debt policies, and the likely existence of non-fully Ricardian behavior on the part of households. We plan to incorporate some of those features in future work.

The paper is organized as follows. Section 2 develops the basic model. Section 3 characterizes the optimal monetary and fiscal policies from the perspective of a small open economy that has its own independent central bank. Section 4 solves for the globally optimal fiscal and monetary policy for a continuum of economies that share the same currency but are subject to idiosyncratic shocks to technology and households' preference for public goods. Section 5 presents some simulation results. Section 6 concludes.

2 A Tractable Optimizing Multicountry Model for Monetary and Fiscal Policy Analysis

We model the world economy as a *continuum of small open economies* represented by the unit interval. Since each economy is of measure zero, its domestic policy decisions do not have any impact on the rest of the world. While different economies are subject to imperfectly correlated shocks, we assume that they share identical preferences, technology, and market structure.

Next we describe in detail the problem facing households and firms located in one such economy. Before we do so, a brief remark on notation is in order. Since our focus is on the behavior of a single economy and its interaction with the world economy, and in order to lighten the notation, variables *without* an i-index will refer to the small open economy being modeled. Variables with an $i \in [0,1]$ subscript refer to economy i, one among the continuum of economies making up the world economy. Finally, variables with a *star superscript* correspond to the world economy as a whole.

2.1 Households

A typical small open economy is inhabited by a representative household who seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \; U(C_t, N_t, G_t; \chi_t) \tag{1}$$

where N_t, C_t, and G_t respectively denote hours of work, private consumption, and public consumption, whereas χ_t is a shock to the preference for public goods.

More precisely, C_t is a composite consumption index defined by

$$C_t \equiv \frac{{C_{H,t}}^{1-\alpha} \; {C_{F,t}}^{\alpha}}{(1-\alpha)^{(1-\alpha)} \alpha^{\alpha}} \tag{2}$$

where $C_{H,t}$ is an index of consumption of domestic goods given by the CES function

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \; dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $j \in [0,1]$ denotes the good variety.[2] $C_{F,t}$ is an index of imported goods given by

$$C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\eta-1}{\eta}} \; di \right)^{\frac{\eta}{\eta-1}}$$

[2] As discussed below, each country produces a continuum of differentiated goods, represented by the unit interval.

where $C_{i,t}$ is, in turn, an index of the quantity of goods imported from country i and consumed by domestic households. It is given by an analogous CES function:

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

In addition we define G_t as a government spending index given by

$$G_t \equiv \left(\int_0^1 G_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

Notice that parameter $\varepsilon > 1$ denotes the elasticity of substitution between varieties (produced within any given country). Parameter $\alpha \in [0,1]$ is (inversely) related to the degree of home bias in preferences, and is thus a natural index of openness.

The maximization of (1) is subject to a sequence of budget constraints of the form:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) \, dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) \, dj \, di \; + E_t\{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t - T_t \tag{4}$$

for $t = 0, 1, 2, ...,$ where $P_{i,t}(j)$ is the price of variety j imported from country i (expressed in domestic currency, i.e., the currency of the importing country whose economy is being modelled), D_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t (and which includes shares in firms), W_t is the nominal wage, T_t denotes lump-sum taxes and $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic household. All the previous variables are expressed in units of domestic currency.

We assume that households have access to a complete set of contingent claims, traded internationally, and that they take the level of government purchases as given.

The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \qquad ; \qquad C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\varepsilon} C_{i,t} \tag{5}$$

for all $i, j \in [0,1]$, where $P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}$ is the *domestic* price index (i.e., an index of prices of domestically produced goods) and $P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}$ is a price index for goods imported from country i (expressed in domestic currency), for all $i \in [0,1]$. It follows from (5) that $\int_0^1 P_{H,t}(j) \; C_{H,t}(j) \; dj = P_{H,t} \; C_{H,t}$ and $\int_0^1 P_{i,t}(j) \; C_{i,t}(j) \; dj = P_{i,t} \; C_{i,t}$.

Furthermore, the optimal allocation of expenditures on imported goods by country of origin implies:

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}} \right)^{-\eta} \; C_{F,t} \tag{6}$$

for all $i \in [0,1]$, and where $P_{F,t} \equiv \left(\int_0^1 P_{i,t}{}^{1-\eta} \, di \right)^{\frac{1}{1-\eta}}$ is the price index for *imported* consumption goods, also expressed in domestic currency. Notice that (6) implies that we can write total expenditures on imported goods as $\int_0^1 P_{i,t} \; C_{i,t} \; di = P_{F,t} \; C_{F,t}$

Finally, the optimal allocation of expenditures between domestic and imported goods is given by:

$$P_{H,t}C_{H,t} = (1-\alpha) \; P_t C_t \qquad ; \qquad P_{F,t}C_{F,t} = \alpha \; P_t C_t \tag{7}$$

where $P_t = P_{H,t}{}^{1-\alpha} P_{F,t}{}^{\alpha}$ is the consumer price index (CPI). Notice that parameter α corresponds to the share of domestic consumption allocated to imported goods. It is also in this sense that α represents a natural index of openness.

Accordingly, total consumption expenditures by domestic households are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t C_t$. Thus, the period budget constraint can be rewritten as:

$$P_t \; C_t + E_t\{Q_{t,t+1} \; D_{t+1}\} \leq D_t + W_t \; N_t + T_t \tag{8}$$

In what follows we specialize our specification of preferences in two ways. First, we let period utility take the form

$$U(C_t, N_t, G_t; \chi_t) \equiv \log C_t + \chi_t \; \log G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{9}$$

which is consistent with balanced growth. Secondly, we restrict our analysis to the case of a unit elasticity of substitution between goods produced in different foreign countries, i.e., $\eta = 1$. In that case we have

$$p_{F,t} = \int_0^1 p_{i,t} \; di$$

where $p_{F,t} \equiv \log P_{F,t}$ and $p_{i,t} \equiv \log P_{i,t}$.

Then we can rewrite the remaining optimality conditions for the household's problem as follows:

$$C_t \; N_t^{\varphi} = \frac{W_t}{P_t} \tag{10}$$

which is a standard intratemporal optimality condition, and

$$\beta \; \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1} \tag{11}$$

Taking conditional expectations on both sides of (11) and rearranging terms we obtain a conventional stochastic Euler equation:

$$\beta R_t \; E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{12}$$

where $R_t = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross return on a riskless one-period discount bond paying off one unit of domestic currency in $t+1$ (with $E_t\{Q_{t,t+1}\}$ being its price).

For future reference it is useful to note that (10) and (12) can be respectively written in log-linearized form as:

$$w_t - p_t = c_t + \varphi \; n_t$$

$$c_t = E_t\{c_{t+1}\} - (r_t - E_t\{\pi_{t+1}\} - \rho) \tag{13}$$

where lower case letters denote the logs of the respective variables, $\rho \equiv \beta^{-1} - 1$ is the time discount rate, and $\pi_t \equiv p_t - p_{t-1}$ is CPI inflation (with $p_t \equiv \log P_t$).

2.1.1 Some Definitions and Identities

Before proceeding with our analysis of the equilibrium we introduce several assumptions and definitions, and derive a number of identities that are extensively used below.

We start by defining the *bilateral terms of trade* between the domestic economy and country i as $\mathcal{S}_{i,t} = \frac{P_{i,t}}{P_{H,t}}$, i.e. the price of country i's goods in terms of home goods. The *effective terms of trade* are thus given by

$$\begin{aligned} \mathcal{S}_t &\equiv \frac{P_{F,t}}{P_{H,t}} \\ &= \exp \int_0^1 (p_{i,t} - p_{H,t}) \; di \\ &= \exp \int_0^1 s_{i,t} \; di \end{aligned}$$

where $s_{i,t} \equiv \log \mathcal{S}_{i,t}$. Equivalently, we have $s_t = \int_0^1 s_{i,t} \; di$, where $s_t \equiv \log \mathcal{S}_t$

Notice also that the CPI and the domestic price levels are related according to:

$$P_t = P_{H,t} \; \mathcal{S}_t^{\alpha}$$

which can be written in logs as

$$p_t = p_{H,t} + \alpha \; s_t \tag{14}$$

where $s_t \equiv p_{F,t} - p_{H,t}$ denotes the (log) *effective terms of trade*, i.e., the price of foreign goods in terms of home goods.

It follows that *domestic inflation* – defined as the rate of change in the index of domestic goods prices, i.e., $\pi_{H,t} \equiv p_{H,t+1} - p_{H,t}$ – and *CPI-inflation* are linked according to:

$$\pi_t = \pi_{H,t} + \alpha \, \Delta s_t \tag{15}$$

which makes the gap between our two measures of inflation proportional to the percent change in the terms of trade, with the coefficient of proportionality given by the index of openness α.

We assume that the *law of one price* holds for individual goods at all times (both for import and export prices), implying that $P_{i,t}(j) = \mathcal{E}_{i,t} \; P^i_{i,t}(j)$ for all $i, j \in [0,1]$, where $\mathcal{E}_{i,t}$ is the bilateral nominal exchange rate (the price of country i's currency in terms of the domestic currency), and $P^i_{i,t}(j)$ is the price of country i's good j expressed in the producer's (i.e., country i's) currency. Plugging the previous assumption into the definition of $P_{i,t}$ one obtains $P_{i,t} = \mathcal{E}_{i,t} \; P^i_{i,t}$, where $P^i_{i,t} \equiv \left(\int_0^1 P^i_{i,t}(j)^{1-\varepsilon} \; dj \right)^{\frac{1}{1-\varepsilon}}$. In turn, by substituting into the definition of $P_{F,t}$ we obtain:

$$\begin{aligned} p_{F,t} &= \int_0^1 (e_{i,t} + p^i_{i,t}) \; di \\ &= e_t + p^*_t \end{aligned}$$

where $e_t \equiv \int_0^1 e_{i,t} \; di$ is the (log) *nominal effective exchange rate*, $p^i_{i,t} \equiv \log P^i_{i,t}$ is the (log) domestic price index for country i (expressed in terms of its currency), and $p^*_t \equiv \int_0^1 p^i_{i,t} \; di$ is the (log) *world price index*. Notice also that for the world as a whole there is no distinction between CPI and domestic price level, nor for their corresponding inflation rates.[3]

Combining the previous result with the definition of the terms of trade we obtain the following expression:

$$s_t = e_t + p^*_t - p_{H,t} \tag{16}$$

Next, we derive a relationship between the terms of trade and the real exchange rate. First, we define the *bilateral real exchange rate* with country i as $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P^i_t}{P_t}$, i.e., the ratio of the two countries CPIs, both expressed in domestic currency. Let $q_t \equiv \int_0^1 q_{i,t} \; di$ define the (log) *effective real exchange rate*, where $q_{i,t} \equiv \log \mathcal{Q}_{i,t}$. It

[3]Notice that, while the law of one price holds at the level of each traded variety, bilateral PPP does not hold due to the presence of home bias in consumption.

follows that

$$\begin{aligned} q_t &= \int_0^1 (e_{i,t} + p_t^i - p_t) \; di \\ &= e_t + p_t^* - p_t \\ &= s_t + p_{H,t} - p_t \\ &= (1-\alpha) \; s_t \end{aligned}$$

2.1.2 International Risk Sharing

Under the assumption of complete securities markets, a first order condition analogous to (11) must also hold for the representative household in any other country, say country i:

$$\beta \left(\frac{C_t^i}{C_{t+1}^i} \right) \left(\frac{P_t^i}{P_{t+1}^i} \right) \left(\frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i} \right) = Q_{t,t+1} \tag{17}$$

Combining (11) and (17), together with the real exchange rate definition it follows that:

$$C_t = \vartheta_i \; C_t^i \; \mathcal{Q}_{i,t} \tag{18}$$

for all $i \in [0,1]$ and all t, and where ϑ_i is a constant which will generally depend on initial conditions regarding relative net asset positions. Henceforth, and without loss of generality, we assume symmetric initial conditions (i.e., zero net foreign asset holdings and an ex-ante identical environment), in which case we have $\vartheta_i = \vartheta = 1$ for all i.[4]

Taking logs on both sides of (18) and integrating over i we obtain

$$\begin{aligned} c_t &= c_t^* + q_t \\ &= c_t^* + (1-\alpha) \; s_t \end{aligned} \tag{19}$$

where $c_t^* \equiv \int_0^1 c_t^i \; di$ is our world consumption index. As usual, the assumption of complete markets at the international level leads to a simple relationship linking domestic consumption with world consumption and the terms of trade.

2.2 Allocation of Government Purchases

One of the central objectives of the present paper is to analyze the optimal determination of the level of government spending under alternative monetary policy and

[4]One can easily show that in the symmetric perfect foresight steady state we also have that $C = C^i = C^*$ and $\mathcal{Q}_i = \mathcal{S}_i = 1$ (i.e., purchasing power parity holds), for all i. See GM-I for details.

exchange rate regimes. Nevertheless, in all the cases considered below, it is assumed that whatever the aggregate level of government spending G_t, the quantities of each type of good purchased by the government are chosen in order to minimize total cost $\int_0^1 P_{H,t}(j)G_t(j)\ dj$. This yields the following set of government demand schedules:

$$G_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} G_t$$

Notice that government demand is completely biased towards domestically-produced goods.[5] An equivalent way to think of G_t is in terms of a bundle of differentiated goods assembled in a final good via the production function (3) and entering the utility function as an externality.[6] In addition, and in order to focus our attention on the determination of aggregate government spending, we assume that the latter is financed by means of lump sum taxes.

2.3 Firms

2.3.1 Technology

A typical firm in the home economy produces a differentiated good with a linear technology represented by the production function

$$Y_t(j) = A_t\ N_t(j) \tag{20}$$

where $a_t \equiv \log A_t$ follows the AR(1) process $a_t = \rho_a\ a_{t-1} + \varepsilon_t^a$, and $j \in [0,1]$ is a firm-specific index. Hence, the real marginal cost (expressed in terms of domestic prices) will be common across domestic firms and given by

$$mc_t = -\nu + w_t - p_{H,t} - a_t$$

where $\nu \equiv -\log(1-\tau)$, with τ being an employment subsidy whose role is discussed later in more detail.

Let $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}}\ dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$ represent an index for aggregate domestic output, analogous to the one introduced for consumption. It is useful, for future reference, to derive an approximate aggregate production function relating the previous index to aggregate employment. Hence, notice that

$$N_t \equiv \int_0^1 N_t(j)\ dj = \frac{Y_t\ Z_t}{A_t} \tag{21}$$

[5]For OECD countries, there is large evidence of home bias in government procurement. See for instance Trionfetti (2000) and Brulhart and Trionfetti (2004).

[6]For instance, one may think of the government buying intermediate goods, such as tanks and military gears, and assembling them in a final public good called "defense", which yields a utility flow.

where $Z_t \equiv \int_0^1 \frac{Y_t(j)}{Y_t}\, dj$. In the Appendix we show that equilibrium variations in $z_t \equiv \log Z_t$ around the perfect foresight steady state are of second order. Thus, and up to a first order approximation, we have an aggregate relationship

$$y_t = a_t + n_t \tag{22}$$

2.3.2 Price setting

We assume that firms set prices in a staggered fashion, as in Calvo (1983). Hence, a measure $1 - \theta$ of (randomly selected) firms sets new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. As shown in Galí and Monacelli (2004), the optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the (log-linear) rule:

$$\overline{p}_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t+k} + p_{H,t}\} \tag{23}$$

where $\overline{p}_{H,t}$ denotes the (log) of newly set domestic prices, and $\mu \equiv \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$, which corresponds to the log of the (gross) markup in the steady state (or, equivalently, the optimal markup in a flexible price economy).

Hence, we see that the pricing decision in our model (as in its closed economy counterpart) is a forward-looking one. The reason is simple and well understood by now: firms that are adjusting prices in any given period recognize that the price they set will remain effective for a (random) number of periods. As a result they set the price as a markup over a weighted average of expected future marginal costs, instead of looking at current marginal cost only. Notice that in the flexible price limit (i.e., as $\theta \to 0$), we recover the familiar markup rule $\overline{p}_{H,t} = \mu + mc_t + p_{H,t}$.

3 Equilibrium

3.1 Aggregate Demand and Output Determination

3.1.1 Consumption, Output and Government Spending in the Small Open Economy

The clearing of goods markets market requires

$$
\begin{aligned}
Y_t(j) &= C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)\ di + G_t(j) \\
&= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_t C_t}{P_{H,t}}\right) + \alpha \int_0^1 \left(\frac{\mathcal{E}_{i,t} P^i_t C^i_t}{P_{H,t}}\right)\ di + G_t\right]
\end{aligned} \tag{24}
$$

for all $j \in [0,1]$ and all t, where $C^i_{H,t}(j)$ denotes country i's demand for good j produced in our small open economy. Notice that the second equality has made use of (7) together with our assumption of symmetric preferences across countries, which implies $C^i_{H,t}(j) = \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_t}\right)^{-1} C^i_t$.

Plugging (24) into the definition of aggregate domestic output $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}}\ dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$ we obtain:

$$
\begin{aligned}
Y_t &= (1-\alpha)\left(\frac{P_t C_t}{P_{H,t}}\right) + \alpha \int_0^1 \left(\frac{\mathcal{E}_{i,t} P^i_t C^i_t}{P_{H,t}}\right)\ di + G_t \\
&= \mathcal{S}_t^{\ \alpha} \left[(1-\alpha)\ C_t + \alpha \int_0^1 \mathcal{Q}_{i,t}\ C^i_t\ di\right] + G_t \\
&= C_t\ \mathcal{S}_t^{\ \alpha} + G_t
\end{aligned} \tag{25}
$$

where the last equality follows from (18).

Before we proceed with our analysis we find it convenient to introduce the following variables. First, we let $\gamma_t \equiv \frac{G_t}{Y_t}$ denote the share of government spending in output. Much of the analysis below makes use of a monotonic transformation of γ_t given by

$$
g_t \equiv -\log(1-\gamma_t)
$$

The latter transformation allows us to derive a log-linear version of the market clearing condition (25)

$$
y_t = c_t + g_t + \alpha\ s_t \tag{26}
$$

Notice that a condition analogous to the one above will hold for all countries. Thus, for a *generic country* i, it can be written as $y^i_t = c^i_t + g^i_t + \alpha\ s^i_t$. By aggregating over all countries we can derive a world market clearing condition as follows

$$
\begin{aligned}
y^*_t &\equiv \int_0^1 y^i_t\ di \\
&= \int_0^1 (c^i_t + g^i_t + \alpha s^i_t)\ di \\
&= \int_0^1 (c^i_t + g^i_t)\ di \equiv c^*_t + g^*_t
\end{aligned} \tag{27}
$$

where y_t^*, c_t^*, and g_t^* are indexes for output, consumption, and government spending at the world level, and where the third equality follows from the fact that $\int_0^1 s_t^i \, di = \int_0^1 (e_t^i + p_t^* - p_{i,t}) \, di \equiv 0$.

Combining (26) with (19) and (27), we obtain:

$$y_t = c_t^* + g_t + s_t \tag{28}$$

Finally, combining (26) with Euler equation (13), we get:

$$\begin{aligned} y_t &= E_t\{y_{t+1}\} - (r_t - E_t\{\pi_{t+1}\} - \rho) - E_t\{\Delta g_{t+1}\} - \alpha \, E_t\{\Delta s_{t+1}\} \\ &= E_t\{y_{t+1}\} - (r_t - E_t\{\pi_{H,t+1}\} - \rho) - E_t\{\Delta g_{t+1}\} \end{aligned} \tag{29}$$

3.2 The Supply Side: Marginal Cost and Inflation Dynamics

3.2.1 Marginal Cost and Inflation Dynamics in the Small Open Economy

In the small open economy, the dynamics of *domestic* inflation in terms of real marginal cost are described by an equation analogous to the that associated with a closed economy. Hence,

$$\pi_{H,t} = \beta \, E_t\{\pi_{H,t+1}\} + \lambda \, \widehat{mc}_t \tag{30}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$.[7]

The determination of the real marginal cost as a function of domestic output in the small open economy differs somewhat from that in the closed economy, due to the existence of a wedge between output and consumption, and between domestic and consumer prices. We now have

$$\begin{aligned} mc_t &= -\nu + (w_t - p_{H,t}) - a_t \\ &= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -\nu + c_t + \varphi \, n_t + \alpha \, s_t - a_t \\ &= -\nu + c_t^* + \varphi \, y_t + s_t - (1+\varphi) \, a_t \end{aligned} \tag{31}$$

where the last equality makes use of (22) and (19). Thus, we see that marginal cost is increasing in the terms of trade and world output. Both variables end up influencing the real wage, through the wealth effect on labor supply resulting from their impact on domestic consumption. In addition, changes in the terms of trade have a direct effect on the product wage, for any given real wage. The influence of technology

[7]Details of the derivation can be found in GM-I.

(through its direct effect on labor productivity) and of domestic output (through its effect on employment and, hence, the real wage–for given output) is analogous to that observed in the closed economy.

Finally, using (28) to substitute for s_t, we can rewrite the previous expression for the real marginal cost in terms of domestic output and productivity, as well as world output:

$$mc_t = -\nu + (1+\varphi)\ y_t - g_t - (1+\varphi)\ a_t \tag{32}$$

Notice that, for a given level of output, an increase in g_t crowds out domestic consumption and generates a real appreciation, which in turn reduces the real marginal cost.

3.3 Equilibrium Dynamics: A Canonical Representation

In this section we show that the log-linear equilibrium dynamics for the small open economy have a representation in terms of output gap and domestic inflation analogous to that of its closed economy counterpart. That result, first shown in Galí and Monacelli (2004), is shown here to carry over to the case of a small open economy with government spending.

Let us define the domestic output gap $\widetilde{y}_t$ as the deviation of (log) domestic output y_t, from its natural level $\overline{y}_t$, where the latter is in turn defined as the equilibrium level of output in the absence of nominal rigidities, and conditional on the optimal choice of fiscal variable $\overline{g}_t$ (to be determined later), while taking as given world output y_t^*. Formally,

$$\widetilde{y}_t \equiv y_t - \overline{y}_t$$

The domestic natural level of output can be found after imposing $mc_t = -\mu$ for all t and solving for domestic output in equation (32):

$$\overline{y}_t = \Omega + a_t + \frac{1}{1+\varphi}\ \overline{g}_t \tag{33}$$

Under the assumption that $\overline{g}_t$ is a function of exogenous variables only (as shown below), it then follows from (32) that the domestic real marginal cost and the output gap will be related according to:

$$\widehat{mc}_t = (1+\varphi)\ \widetilde{y}_t - \widetilde{g}_t$$

where $\widetilde{g}_t \equiv g_t - \overline{g}_t$ denotes the gap between the fiscal variable g_t and its optimal value in the absence of nominal rigidities. For convenience we refer to $\widetilde{g}_t$ as the *fiscal gap.*

Combining the previous expression with (30) one can derive a so called New Keynesian Phillips Curve for the small open economy in terms of the output and fiscal gaps:

$$\pi_{H,t} = \beta \; E_t\{\pi_{H,t+1}\} + \kappa \; \widetilde{y}_t - \lambda \; \widetilde{g}_t \tag{34}$$

where $\kappa \equiv \lambda \, (1+\varphi)$.

Using (29) it is now straightforward to derive a version of the dynamic IS equation for the small open economy in terms of the output gap:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - (r_t - E_t\{\pi_{H,t+1}\} - \overline{rr}_t) - E_t\{\Delta \widetilde{g}_{t+1}\} \tag{35}$$

where

$$\begin{aligned} \overline{rr}_t &\equiv \rho + E_t\{\Delta \overline{y}_{t+1}\} - E_t\{\Delta \overline{g}_{t+1}\} \\ &= \rho + E_t\{\Delta a_{t+1}\} - \frac{\varphi}{1+\varphi} \; E_t\{\Delta \overline{g}_{t+1}\} \end{aligned}$$

is the small open economy's natural rate of interest (conditional on optimal provision of public consumption goods). Notice that under our assumptions $\{\overline{rr}_t\}$ is independent of the monetary or exchange rate regime.

Integrating forward one can express the current output gap as a function of the current fiscal gap and (as usual) the present discounted value of the deviations of the real interest rate from its natural level:

$$\widetilde{y}_t = \widetilde{g}_t - \sum_{k=0}^{\infty} E_t\{r_{t+k} - \pi_{H,t+k+1} - \overline{rr}_{t+k}\} \tag{36}$$

4 Optimal Fiscal and Monetary Policies for the Small Open Economy

In this section we derive and characterize the optimal fiscal and monetary policy regime for the small open economy, under the assumption of an independent, optimizing central bank. We want to think of that environment as a benchmark for our later analysis of the implications of the adherence of the same economy to a monetary union.

4.1 The Social Planner's Problem

Let us start by characterizing the optimal allocation from the viewpoint of a social planner *facing the resource constraints to which the small open economy is subject in equilibrium* (vis a vis the rest of the world), given our assumptions. In that case, the optimal allocation maximizes the utility of the domestic household subject to: (a) the set of technological constraints (20) and (b) the consumption/output possibilities set implied by the international risk sharing condition (18).

First, it is clear from (21) that the social planner will choose to equate the quantities produced of the different domestic goods, in which case the aggregate relationship $Y_t = A_t N_t$ will hold.

Notice that under our assumptions (19) and (25) imply the exact expression

$$C_t = [Y_t - G_t]^{1-\alpha} (C_t^*)^{\alpha} \tag{37}$$

Hence the optimal allocation from the viewpoint of the small open economy (i.e., taking world output as given) must maximize (9) subject to (37) and $Y_t = A_t N_t$. The optimality conditions are:

$$\frac{(1-\alpha)}{Y_t - G_t} - \frac{N_t^{1+\varphi}}{Y_t} = 0$$

$$-\frac{(1-\alpha)}{Y_t - G_t} + \frac{\chi_t}{G_t} = 0$$

which in turn imply

$$N_t = (1 - \alpha + \chi_t)^{\frac{1}{1+\varphi}}$$

$$\begin{aligned} G_t &= Y_t \, \frac{\chi_t}{1 - \alpha + \chi_t} \\ &\equiv Y_t \; \gamma(\chi_t) \end{aligned}$$

Using the transformation $g_t = -\log(1 - \frac{G_t}{Y_t})$, we can determine the form of the optimal fiscal rule

$$g(\chi_t) = \log\left(1 + \frac{\chi_t}{1-\alpha}\right)$$

4.2 The Flexible Price Equilibrium

Consider next the equilibrium with flexible prices. Letting variables with an upper bar denote the corresponding values under the flexible price equilibrium, we must have:

$$\begin{aligned} 1 - \frac{1}{\varepsilon} &= \overline{MC}_t \\ &= \frac{(1-\tau)}{A_t} \, \overline{C}_t \, \overline{N}_t^{\varphi} \, \overline{S}_t^{\alpha} \\ &= \frac{(1-\tau)}{A_t} \, \overline{C}_t \, \overline{N}_t^{\varphi} \, \frac{\overline{Y}_t - \overline{G}_t}{\overline{C}_t} \\ &= (1-\tau) \, (1 - \overline{\gamma}_t) \, \overline{N}_t^{1+\varphi} \end{aligned}$$

where $\overline{\gamma}_t = \frac{\overline{G}_t}{\overline{Y}_t}$

It is now easy to derive conditions that guarantee that the allocation under flexible prices will correspond to the optimal allocation derived above. First, the subsidy τ should satisfy

$$(1-\tau)(1-\alpha) = 1 - \frac{1}{\varepsilon}$$

Secondly, the share of government spending in output must satisfy the optimal rule, that is

$$\begin{aligned} \overline{\gamma}_t &= \frac{\chi_t}{1-\alpha+\chi_t} \\ &\equiv \gamma(\chi_t) \end{aligned} \tag{38}$$

If both conditions are satisfied, it is easy to check that the flexible price equilibrium will attain the optimal level of employment and output, while generating an efficient provision of public goods. The dependence of the steady state subsidy on $1 - \frac{1}{\varepsilon}$ (i.e., the reciprocal of the gross markup in the steady state) has to do with the desire to eliminate the distortion associated with the presence of market power, as in the closed economy setup of Rotemberg and Woodford (1999). Yet, as noted, among others, by Corsetti and Pesenti (2001), in an open economy there is an additional factor that distorts the incentives of the monetary authority (beyond the presence of market power): the possibility of influencing the terms of trade in a way beneficial to domestic consumers. This possibility is a consequence of the imperfect substitutability between domestic and foreign goods, combined with sticky prices (which render monetary policy non-neutral). That mechanism underlies the dependence of the required τ on the degree of openness α. As in Benigno and Benigno (2003) our subsidy exactly offsets the combined effects of market power and terms of trade distortions *in the steady state*. That assumption rules out the existence of an average inflation (or deflation) bias, and allows us to focus on policies consistent with zero average inflation, in a way analogous to the analysis for the closed economy found in the literature.[8]

4.3 Optimal Policy Implementation

From the analysis above it is clear that, in the absence of constraints on monetary policy, a small open economy policymaker seeking to maximize the utility of the domestic household (while taking as given external developments) will conduct fiscal and monetary policy according to the following principles:

[8]The fact that τ is decreasing in α suggests that the incentive to offset the market power distortion, on the one hand, and the terms of trade motive, on the other, tend to mutually offset.
.

(a) Fiscal policy must guarantee that the flexible price equilibrium allocation corresponds to the optimal one (from the viewpoint of the small open economy). That requires setting a constant subsidy/tax

$$\tau = 1 - \frac{1}{\mu(1-\alpha)}$$

and a value for the government spending index given by

$$\begin{aligned} g_t &= g(\chi_t) \\ &\simeq \overline{g} + \overline{g}_\chi \, \zeta_t \\ &\equiv \overline{g}_t \end{aligned}$$

where $\overline{g} \equiv g(\chi)$, $\overline{g}_\chi \equiv g'(\chi) = (1-\alpha+\chi)^{-1}$, and $\zeta_t \equiv \chi_t - \chi$.

(b) Monetary Policy must guarantee that the equilibrium allocation corresponds to the one associated with the flexible price equilibrium. As discussed in GM-I that objective can be achieved by following a rule of the form:

$$r_t = \overline{rr}_t + \phi_\pi \, \pi_{H,t}$$

where under our assumption of exogenous AR(1) processes for $\{a_t\}$ and $\{\zeta_t\}$ with corresponding AR coefficients ρ_a and ρ_ζ we have:

$$\overline{rr}_t = \rho - (1-\rho_a) \, a_t + \frac{\varphi \overline{g}_\chi}{1+\varphi} \, (1-\rho_\zeta) \, \zeta_t$$

In that case, and under the assumption that $\phi_\pi > 1$, the equilibrium will be locally unique and will correspond to the solution to the system of difference equations (34)-(35) given by $\widetilde{y}_t = \widetilde{g}_t = \pi_{H,t} = 0$ for all t.

5 Optimal Fiscal and Monetary Policy Design in a Monetary Union

So far we have analyzed the optimal setting of fiscal policy from the viewpoint of a small open economy endowed with an independent monetary authority. We now turn to the characterization of the optimal fiscal-monetary regime when each small open economy relinquishes its own monetary policy independence and, for reasons that we treat as exogenous, decides to join a monetary union. In this setting, while monetary policy is conducted in a centralized fashion, each member country is assumed to maintain the autonomy of its own fiscal policy. Importantly, we wish to analyze here the optimal joint policy regime that is able to maximize the welfare of the *currency union as a whole.*

5.1 The Social Planner's Problem

The union's optimal allocation problem can be described in terms of the following setup:

$$\max \int_0^1 U(C_t^i, N_t^i, G_t^i; \chi_t^i)\ di$$

subject to the technological and resource constraints

$$Y_t^i = A_t^i\ N_t^i$$

$$Y_t^i = C_{i,t}^i + \int_0^1 C_{i,t}^j\ dj + G_t^i$$

for all $i \in [0,1]$.

Under our specification of the utility function, the optimality conditions for the above problem are:

$$(N_t^i)^{\varphi} = A_t^i\ \frac{1-\alpha}{C_{i,t}^i} = A_t^i\ \int_0^1 \frac{\alpha}{C_{i,t}^j}\ dj = A_t^i\ \frac{\chi_t^i}{G_t^i}$$

for all $i \in [0,1]$.

Multiplying both sides by N_t^i we get

$$(N_t^i)^{1+\varphi} = Y_t^i\ \frac{1-\alpha}{C_{i,t}^i} = Y_t^i\ \int_0^1 \frac{\alpha}{C_{i,t}^j}\ dj = Y_t^i\ \frac{\chi_t^i}{G_t^i}$$

We guess and verify that for all $i \in [0,1]$ the solution is given by:

$$N_t^i = \left(1+\chi_t^i\right)^{\frac{1}{1+\varphi}} \tag{39}$$

$$C_{i,t}^i = \frac{1-\alpha}{1+\chi_t^i}\ Y_t^i = \frac{(1-\alpha)\ A_t^i}{(1+\chi_t^i)^{\frac{\varphi}{1+\varphi}}} \tag{40}$$

$$C_{i,t}^j = \frac{\alpha}{1+\chi_t^i}\ Y_t^i = \frac{\alpha\ A_t^i}{(1+\chi_t^i)^{\frac{\varphi}{1+\varphi}}} \qquad all\ j \neq i \tag{41}$$

$$G_t^i = \frac{\chi_t^i}{1+\chi_t^i}\ Y_t^i = \frac{\chi_t^i\ A_t^i}{(1+\chi_t^i)^{\frac{\varphi}{1+\varphi}}} \tag{42}$$

Notice that the latter condition implies that the optimal share of government spending in each member country from the view point of the union is now:

$$\gamma(\chi_t^i) = \frac{\chi_t^i}{1+\chi_t^i}$$

with the corresponding transformed fiscal stance indicator given by

$$g(\chi_t^i) = \log\left(1 + \chi_t^i\right)$$

5.2 The Flexible Price Equilibrium

Consider next the equilibrium with flexible prices. The analysis from the previous section implies

$$\begin{aligned} 1 - \frac{1}{\varepsilon} &= \overline{MC}_t^i \\ &= (1-\tau^i)\,(1-\overline{\gamma}_t^i)\,(\overline{N}_t^i)^{1+\varphi} \end{aligned}$$

We can now easily derive the conditions that guarantee that the allocation under flexible prices will correspond to the union's optimal allocation. First, the subsidy τ^i must satisfy

$$\tau^i = \frac{1}{\varepsilon} \tag{43}$$

Secondly, the share of government spending in output must satisfy the optimal rule, that is

$$\begin{aligned} \overline{\gamma}_t^i &= \frac{\chi_t^i}{1+\chi_t^i} \\ &\equiv \gamma(\chi_t^i) \end{aligned} \tag{44}$$

If both conditions are satisfied for all $i \in [0,1]$, the flexible price equilibrium will yield the level of employment and output in each country that is *optimal from the union's perspective.*

Two aspects are worth emphasizing concerning conditions (43) and (44). First, in this case the size of the subsidy is not affected by any desire to influence the terms of trade in one's favor, for that goal cannot be attained by all countries simultaneously, a constraint that is now internalized. Second, by comparing (44) with (38), one notices that, in any given country, the share of government spending which is optimal from the individual country's perspective is *larger* than the one perceived to be optimal from the perspective of the union as a whole. This positive spending bias is the result of the lack of coordination among each member country's fiscal authority. In fact, when $\alpha > 0$, and taking other countries' fiscal authority behavior as given, each country has an incentive to use government spending to generate a welfare-improving appreciation of its own terms of trade. Yet in a Nash equilibrium where all the fiscal authorities behave in identical fashion, all bilateral terms of trade remain unchanged, thereby producing an inefficient level of government spending.

Notice also that optimal government spending from the individual country perspective coincides with the one that is optimal from the perspective of the union as a whole only in the case of $\alpha = 0$, namely when the absence of trade linkages among member countries precludes relative price adjustments. As a result, in our context, a social planner responsible for maximizing welfare at the level of the union wishes to implement fiscal policy in each individual country as if each same country corresponded to a closed economy.

5.3 Optimal Policy in the Monetary Union in the Presence of Nominal Rigidities

When all countries in our world economy adopt a single currency (i.e., when they form a monetary union) and relinquish an independent monetary policy it will generally be impossible to attain the optimal allocation when price rigidities are present. The reason is simple and well understood: the sluggish adjustment of prices, combined with the impossibility of any adjustment in the nominal exchange rate, implies that the changes in relative prices required to support the optimal allocation will not occur instantaneously. As a result, the union as a whole will experience some deviations from that optimal allocation which will generate some welfare losses.

In the Appendix we show that a second order approximation to the *sum* of those welfare losses, expressed as a fraction of steady state consumption can be written as follows:

$$\mathbb{W} \equiv -\frac{1}{2}(1+\chi)\sum_{t=0}^{\infty}\beta^t \int_0^1 \left[\frac{\varepsilon}{\lambda}\,(\pi_{i,t}^i)^2 + (1+\varphi)\,(\widetilde{y}_t^i)^2 + \left(\frac{1}{\chi}\right)\,(\widetilde{g}_t^i)^2\right]\,di + t.i.p. + o(\|a\|^3) \tag{45}$$

where $t.i.p$ denote terms that are independent of policy and $o(\|a\|^3)$ represents terms that are of order third or higher in the bound $\|a\|$ on the amplitude of the relevant shocks. In the notation above, $\pi_{i,t}^i, \widetilde{y}_t^i$ and $\widetilde{g}_t^i$ denote, respectively, domestic (producer) inflation, output gap and fiscal gap in a generic country i belonging to the union.

It follows that the optimal (monetary and fiscal) policy at the level of the union will consist in a set of processes $\left\{\pi_{i,t}^i, \widetilde{y}_t^i, \widetilde{g}_t^i\right\}$ maximizing (45) subject to the set of constraints:

$$\pi_{i,t}^i = \beta\ E_t\{\pi_{i,t+1}^i\} + \kappa\ \widetilde{y}_t^i - \lambda\ \widetilde{g}_t^i \tag{46}$$

$$\widetilde{y}_t^i = E_t\{\widetilde{y}_{t+1}^i\} - (r_t^* - E_t\{\pi_{i,t+1}^i\} - \overline{rr}_t^i) - E_t\{\Delta\widetilde{g}_{t+1}^i\} \tag{47}$$

for all $i \in [0,1]$

5.3.1 Discretion

In the absence of a commitment device, optimal policy requires minimizing the period loss function

$$\int_0^1 \left[\frac{\varepsilon}{\lambda} \, (\pi_{i,t}^i)^2 + (1+\varphi) \, (\widetilde{y}_t^i)^2 + \left(\frac{1}{\chi}\right) \, (\widetilde{g}_t^i)^2 \right] \, di$$

subject to the set of constraints

$$\pi_{i,t}^i = \lambda(1+\varphi) \, \widetilde{y}_t^i - \lambda \, \widetilde{g}_t^i + f_t^i \tag{48}$$

$$\widetilde{y}_t^i = \widetilde{g}_t^i + h_t^i \tag{49}$$

for all $i \in [0,1]$, where $f_t^i \equiv \beta \, E_t\{\pi_{i,t+1}^i\}$ and $h_t^i \equiv E_t\{\widetilde{y}_{t+1}^i\} - E_t\{\widetilde{g}_{t+1}^i\} + E_t\{\pi_{i,t+1}^i\} - \psi_t^i$ are taken as given by the policymaker in the discretion case, and where

$$\psi_t^i \equiv (r_t^* - \overline{rr}_t^i) \tag{50}$$

is the *interest rate gap*, a measure of the asymmetry between the nominal interest rate prevailing in the monetary union and the natural rate of interest in the representative domestic economy.

The first order conditions of this problem read as follows:

$$(1+\varphi) \, \widetilde{y}_t^i + \left(\frac{1}{\chi}\right) \, \widetilde{g}_t^i + \varphi\varepsilon \, \pi_{i,t}^i = 0 \tag{51}$$

and

$$r_t^* = \int_0^1 \overline{rr}_t^j \, dj \equiv \overline{rr}_t^* \tag{52}$$

Rearranging (51) one obtains a set of fiscal policy rules for all $i \in [0,1]$:

$$\widetilde{g}_t^i = -(1+\varphi)\chi \, \widetilde{y}_t^i - \varepsilon\varphi\chi \, \pi_{i,t}^i \tag{53}$$

Hence the set of conditions (52) and (53) characterize a jointly optimal monetary-fiscal regime for the union as a whole under a Markov-perfect solution. Equation (52) requires that the nominal interest rate at the level of the union be set in order to equate the *average* natural rate of the member countries. Equation (53) describes the optimal fiscal rule under discretion. That rule implies that public spending should deviate from its natural level (i.e., the level consistent with the optimal provision of public goods) whenever inflation or the output gap (or both) deviate from their optimal levels. Notice that in that case, optimal fiscal policy involves leaning against the wind, thus generating a "stabilizing" role, beyond that of an efficient provision of public goods.

It is worth noticing that public consumption yielding utility is a necessary condition for generating, in equilibrium, deviations from the optimal provision of public goods. In fact, from equation (53), one can see that in the case of $\chi = 0$ the optimal fiscal rule collapses to the special case $\widetilde{g}_t^i = 0$ for all t and i, exactly like in the small open economy (and independent monetary authority) case analyzed above.[9] On the other hand, the latter example suggests that public consumption yielding utility, although necessary, is not a sufficient condition for generating a role for fiscal stabilization as a part of an optimal policy. This requires the additional constraint of monetary policy independence being relinquished.

Notice that the implied inflation dynamics under the optimal discretionary policy take the form:

$$(1 - \lambda\varepsilon\varphi\chi)\ \pi_{i,t}^i = \beta\ E_t\{\pi_{i,t+1}^i\} + \kappa(1+\chi)\ \widetilde{y}_t^i \tag{54}$$

On the other hand we can combine the optimal discretionary rule with the IS equation to obtain:

$$\widetilde{y}_t^i = E_t\{\widetilde{y}_{t+1}^i\} - \left(\frac{\varepsilon\varphi\chi}{1+(1+\varphi)\chi}\right)\ \pi_{i,t}^i + \left(\frac{1+\varepsilon\varphi\chi}{1+(1+\varphi)\chi}\right)\ E_t\{\pi_{i,t+1}^i\} - \left(\frac{1}{1+(1+\varphi)\chi}\right)\ \Psi_t^i \tag{55}$$

where

$$\Psi_t^i \equiv \overline{rr}_t^* - \overline{rr}_t^i \tag{56}$$

is the *natural interest rate gap*, a measure of the deviations of the natural rate of interest in country i from its *average* counterpart in the union.

5.3.2 Fluctuations in the Natural Rate Gap

It is of particular interest, in our context, to analyze the optimal behavior of fiscal policy in response to fluctuations in the natural interest rate gap Ψ_t^i.[10] With that goal, and without loss of generality, we assume that Ψ_t^i follows a univariate autoregressive process

[9] Notice that in the independent small economy case it does not matter whether or not prices are sticky. In that case, in fact, the flexible price allocation is always feasible thanks to exchange rate flexibility (matters would of course be different if nominal wages, as well as prices, were sticky). Hence, in the monetary union case, movements in the fiscal gap act as a substitute of nominal exchange rate flexibility.

[10] Alternatively, one may think of variations in Ψ_t^i as originating from idiosyncratic shocks to productivity a_t^i and/or to the marginal utility of government consumption χ_t^i. These shocks would equivalently generate an asymmetry between the union-wide natural rate and the natural rate of interest of the representative member economy.

$$\Psi_t^i = \rho^{\psi} \; \Psi_t^i + \varepsilon_t^{\psi} \tag{57}$$

where ε_t^{ψ} is an iid shock. In our analysis, movements in the natural rate gap act as a proxy of asymmetric disturbances. The presence of idiosyncratic deviations of the natural rate in country i from the union's average is central in generating the need for relative price adjustments in equilibrium.

In our simulation analysis we employ the following parameterization. We assume $\varphi = 1$, which implies a unitary labor supply elasticity, and a value for the steady-state markup $\mu = 1.2$, which implies that ε, the elasticity of substitution between differentiated goods, is 6. Parameter θ is set equal to 0.75, a value consistent with an average period of one year between price adjustments. We assume $\beta = 0.99$, which implies a riskless annual return of about 4 percent in the steady state. We set a baseline value for α (or degree of openness) of 0.4. Finally we set $\gamma = 0.25$, which corresponds to the average value of the ratio of government consumption to GDP in European countries.

Figure 1 displays, for alternative values of the persistence parameter ρ_{ψ}, impulse responses of domestic inflation $\pi_{i,t}^i$, output gap $\widetilde{y}_t^i$ and fiscal gap $\widetilde{g}_t^i$ to a one percent positive innovation in the natural rate gap.

[Figure 1 about here]

Thus we can see that, as a result of the natural rate of interest falling below the union average, the output gap in the representative economy falls below its efficient value, thereby inducing a fall in inflation. For both output gap and inflation are now below their efficient values, the fiscal policy response in each country calls for an expansion in the fiscal gap, which gradually conveys output gap and inflation back to their steady state (efficient) values.

These dynamics are representative of one of our main results. Hence we see that an optimal fiscal policy requires, in the presence of idiosyncratic variations in the natural rate of interest, persistent deviations of government spending from its efficient value (consistent with flexible prices and optimal provision of public goods). Interestingly, these deviations inherit the degree of persistence in the natural rate gap Ψ_t^i.

6 Conclusions

We believe two are the main conclusions that can be drawn from the present analysis. First, the paper offers a methodological contribution towards the definition of a new paradigm suitable for the study of macroeconomic policy in a multicountry setting, and - in particular - in the presence of a currency area regime characterized by centralized monetary policy and decentralized fiscal authorities. Second, it shows that despite this joint monetary-fiscal regime being designed in order to maximize the welfare of the union as a whole, optimal fiscal policy in each member country calls, in equilibrium, for a macroeconomic stabilization role that goes beyond the one consistent with the optimal provision of public goods.

Our setting suggests a natural series of extensions to be addressed in future research. We foresee this as an important challenge for the definition of a new theoretical framework for policy analysis in a monetary union. Such extensions should allow for a role of sticky wages (in particular when addressing policy issues related to EMU), as well as for a richer characterization of the fiscal sector, featuring a role for distortionary taxes and/or government debt policy.

Appendix: Derivation of the Welfare Loss Function for the Union

In the present appendix we derive a second order approximation of the sum of the union's consumers utility about the optimal allocation for the union (which, as shown above, corresponds to the flexible price allocation with an appropriate employment subsidy in all countries). For notational convenience we ignore country superscripts when not needed.

First, notice that utility derived from consumption can be expressed as

$$\begin{aligned}
\log C_t^i &= \overline{c}_t^i + \widetilde{c}_t^i \\
&= \overline{c}_t^i + (1-\alpha)\,(\widetilde{y}_t^i - \widetilde{g}_t^i) + \alpha \int_0^1 \widetilde{c}_t^j dj
\end{aligned}$$

where $o(\|a\|^3)$ refers to terms of third or higher order. Notice that in deriving the second equality we have made use of the logarithmic version of (37), and the fact that y_t^* is taken as exogenous by the small economy's monetary authority.

Similarly, turning to the disutility of labor we have

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \frac{\overline{N}_t^{1+\varphi}}{1+\varphi} + \overline{N}_t^{1+\varphi} \left[\widetilde{n}_t + \frac{1}{2}(1+\varphi)\,\widetilde{n}_t^2\right] + o(\|a\|^3)$$

The next step consists in rewriting the previous expression in terms of the output gap. Using the fact that $N_t = \left(\frac{Y_t}{A_t}\right) \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di$, we have

$$\widetilde{n}_t = \widetilde{y}_t + z_t$$

where $z_t \equiv \log \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di$. The following lemma shows that z_t is proportional to the cross-sectional distribution of relative prices (and, hence, of second order).

Lemma 1: $z_t = \frac{\varepsilon}{2}\, var_i\{p_{H,t}(i)\} + o(\|a\|^3)$.

Proof: Let $\widehat{p}_{H,t}(i) \equiv p_{H,t}(i) - p_{H,t}$. Notice that,

$$\begin{aligned}
\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon} &= \exp\left[(1-\varepsilon)\,\widehat{p}_{H,t}(i)\right] \\
&= 1 + (1-\varepsilon)\,\widehat{p}_{H,t}(i) + \frac{(1-\varepsilon)^2}{2}\,\widehat{p}_{H,t}(i)^2 + o(\|a\|^3)
\end{aligned}$$

Furthermore, from the definition of $P_{H,t}$, we have $1 = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon} di$. Hence, it follows that

$$E_i\{\widehat{p}_{H,t}(i)\} = \frac{(\varepsilon - 1)}{2}\; E_i\{\widehat{p}_{H,t}(i)^2\}$$

In addition, a second order approximation to $\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}$, yields:

$$\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} = 1 - \varepsilon\; \widehat{p}_{H,t}(i) + \frac{\varepsilon^2}{2}\; \widehat{p}_{H,t}(i)^2 + o(\|a\|^3)$$

Combining the two previous results, it follows that

$$\begin{aligned}
\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di &= 1 + \frac{\varepsilon}{2}\; E_i\{\widehat{p}_{H,t}(i)^2\} \\
&= 1 + \frac{\varepsilon}{2}\; var_i\{p_{H,t}(i)\}
\end{aligned}$$

from which follows that $z_t = \frac{\varepsilon}{2}\; var_i\{p_{H,t}(i)\} + o(\|a\|^3)$.

We can thus rewrite the second order approximation to the disutility of labor as:

$$\begin{aligned}
\frac{N_t^{1+\varphi}}{1+\varphi} &= \frac{\overline{N}_t^{1+\varphi}}{1+\varphi} + \overline{N}_t^{1+\varphi}\; \left[\widetilde{y}_t + z_t + \frac{1}{2}(1+\varphi)\; \widetilde{y}_t^2\right] + o(\|a\|^3) \\
&= \frac{\overline{N}_t^{1+\varphi}}{1+\varphi} + (1+\chi_t)\; \left[\widetilde{y}_t + z_t + \frac{1}{2}(1+\varphi)\; \widetilde{y}_t^2\right] + o(\|a\|^3)
\end{aligned}$$

where the second equality uses the fact that under the optimal subsidy scheme assumed, $\overline{N}_t^{1+\varphi} = 1 - \alpha + \chi_t$ holds for all t.

Similarly, we have

$$\begin{aligned}
\log G_t &= \log\left(\frac{G_t}{Y_t}\right) + \widetilde{y}_t + t.i.p \\
&= \log\left(1 - \exp\{-g_t\}\right) + \widetilde{y}_t + t.i.p \\
&= \frac{1-\overline{\gamma}_t}{\overline{\gamma}_t}\; \widetilde{g}_t - \frac{1}{2}\frac{1-\overline{\gamma}_t}{\overline{\gamma}_t^2}\; \widetilde{g}_t^2 + \widetilde{y}_t + t.i.p. + o(\|a\|^3)
\end{aligned}$$

Accordingly, the utility generated by public spending can be approximated by

$$\begin{aligned}
\chi_t \log G_t &= \frac{\overline{\gamma}_t}{1-\overline{\gamma}_t}\; \log G_t \\
&= \widetilde{g}_t - \frac{1}{2}\frac{1}{\overline{\gamma}}\; \widetilde{g}_t^2 + \chi_t\; \widetilde{y}_t + t.i.p. + o(\|a\|^3)
\end{aligned}$$

Collecting terms we obtain:

$$U(C_t^i, N_t^i, G_t^i; \chi_t^i) \ = \ -\alpha\ (\widetilde{y}_t^i - \widetilde{g}_t^i) + \alpha \int_0^1 \widetilde{c}_t^j\ dj - (1+\chi_t^i)\ \left[z_t + \frac{1}{2}(1+\varphi)\ \widetilde{y}_t^2\right] - \frac{1}{2}\frac{1}{\gamma}\ \widetilde{g}_t^2$$
$$+t.i.p. + \ o(\|a\|^3)$$

Furthermore we have:
Lemma 2: $\sum_{t=0}^{\infty} \beta^t\ var_i\{p_{H,t}(i)\} = \frac{1}{\lambda} \sum_{t=0}^{\infty} \beta^t\ \pi_{H,t}^2$, where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$
Proof: Woodford (2001, NBER WP8071), pp 22-23.

Collecting all the previous results, we can write the second order approximation to the utility of the representative consumer in economy i as follows:

$$\mathbb{W}^i \ \equiv \ -\alpha\ (\widetilde{y}_t^i - \widetilde{g}_t^i) + \alpha \int_0^1 \widetilde{c}_t^j\ dj - \frac{1}{2}(1+\chi) \sum_{t=0}^{\infty} \beta^t\ \left[\frac{\varepsilon}{\lambda}\ (\pi_{i,t}^i)^2 + (1+\varphi)\ (\widetilde{y}_t^i)^2 + \left(\frac{1}{\chi}\right)\ (\widetilde{g}_t^i)^2\right]$$
$$+t.i.p. + \ o(\|a\|^3)$$

Integrating over all economies and using the world resource constraint $\int_0^1 (\widetilde{y}_t^i - \widetilde{c}_t^i - \widetilde{g}_t^i)\ di = 0$ we obtain equation (45) in the text.

References

Bacchetta, Philippe, and Eric van Wincoop (2000): “Does Exchange Rate Stability Increase Trade and Welfare?”, *American Economic Review*, 90:5, 1093-1109.

Benigno, Gianluca and Pierpaolo Benigno (2003): “Price Stability in Open Economies”, *Review of Economic Studies*, December

Blanchard O. and C. Khan (1980): ”The Solution of Linear Difference Models with Rational Expectations”, *Econometrica* 48: 1305-1311.

Brulhart M. and F. Trionfetti (2004), ”Public Expenditure, International Specialisation and Agglomeration”, *European Economic Review* 48, 851-881.

Calvo, Guillermo, 1983, “Staggered Prices in a Utility Maximizing Framework,” *Journal of Monetary Economics*, 12, 383-398.

Chari, V.V., Patrick Kehoe, and Ellen McGrattan (2002): “Monetary Shocks and Real Exchange Rates in Sticky Price Models of International Business Cycles,” *Review of Economic Studies* 69, 533-563

Clarida, Richard, Jordi Galí, and Mark Gertler (2000): “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory,” *Quarterly Journal of Economics*, vol. 105, issue 1, 147-180.

Clarida, Richard, Jordi Galí, and Mark Gertler (2001): “Optimal Monetary Policy in Open vs. Closed Economies: An Integrated Approach,” *American Economic Review*, vol. 91, no. 2, 248-252.

Clarida, Richard, Jordi Galí, and Mark Gertler (2002): “A Simple Framework for International Monetary Policy Analysis,” *Journal of Monetary Economics*, vol. 49, no. 5, 879-904.

Corsetti, Giancarlo and Paolo Pesenti (2001): “Welfare and Macroeconomic Interdependence,” *Quarterly Journal of Economics* vol. CXVI,issue 2, 421-446.

Devereux, Michael B. and Charles Engel (2003): ”Monetary Policy in the Open Economy Revisited: Exchange Rate Flexibility and Price Setting Behavior”, *Review of Economic Studies,* December.

Ferrero, A. (2005) ”Fiscal and Monetary Rules for a Currency Union”, mimeo New York University.

Galí, Jordi (2003): “New Perspectives on Monetary Policy, Inflation, and the Business Cycle,” in *Advances in Economics and Econometrics*, volume III, edited by M. Dewatripont, L. Hansen, and S. Turnovsky, Cambridge University Press

Galí, Jordi and Mark Gertler (1999): “Inflation Dynamics: A Structural Econometric Analysis,” *Journal of Monetary Economics*, vol. 44, no. 2, 195-222.

Galí, Jordi, and Tommaso Monacelli (2004): “Monetary Policy and Exchange Rate Volatility in a Small Open Economy”, forthcoming *Review of Economic Studies.*

Kollmann, Robert (2001): “The Exchange Rate in a Dynamic Optimizing Current Account Model with Nominal Rigidities: A Quantitative Investigation,” *Journal of International Economics* vol.*55, 243-262.*

Lombardo G. and A. Sutherland (2004), ”Monetary and Fiscal Interactions in Open Economies”, *Journal of Macroeconomics*, 26, 319-348.

Obstfeld, Maurice and Kenneth Rogoff (1995): “Exchange Rate Dynamics Redux,” *Journal of Political Economy* 103, no. 3, 624-660.

Obstfeld, Maurice and Kenneth Rogoff (1999): “New Directions for Stochastic Open Economy Models,” *Journal of International Economics*, vol. 50, no. 1, 117-153.

Pappa, Evi (2003): “Should the Fed and the ECB Cooperate ? Optimal Monetary Policy in a Two-Country World,” *Journal of Monetary Economics* forthcoming.

Rotemberg, Julio and Michael Woodford (1999): “Interest Rate Rules in an Estimated Sticky Price Model,” in J.B. Taylor ed., *Monetary Policy Rules*, University of Chicago Press.

Trionfetti F. (2000), ”Discriminatory Public Procurement and International Trade”, Blackwell Publishers.

Vestin, D. (1999): “Price level Targeting versus Inflation Targeting in a Forward Looking Model,” unpublished manuscript, IIES, Stockholm University.

Woodford, Michael (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

Yun, Tack (1996), “Monetary Policy, Nominal Price Rigidity, and Business Cycles”, *Journal of Monetary Economics*, 37:345-70.

Figure 1. Impulse Responses to a Rise in the Natural Interest Rate Gap under Discretion