# Thoughts on Event Forecasting: Idiosyncratic and Systemic Aspects

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# A Difficult Problem (So Set Your Expectations Low)

Consider a currency collapse:

- Comparatively easy to identify overvalued exchange rates
- Notoriously difficult to predict whether/when a crash will occur

# Consideration: The Forecast Object

Time series

Cross section

*Event*

– Event timing

– Event outcome or magnitude

e.g., default, loss given default

# Financial and Economic Events

- Financial:
  - Corporate bond default
  - Sovereign bond default
  - Margin call
  - $\alpha$% VaR breach
  - Crisis: Stock market, currency, banking, current account, ...
  - Circuit breaker tripped
  - Yields hit zero lower bound
- Economic:
  - Two firms merge
  - Recession begins or ends
  - A firm fails to meet analysts' earnings expectations
  - Country X leaves EMU
  - EMU collapses
  - Europe collapses and resumes feudalism
- Broader: Marketing, insurance, politics, ...

# Consideration: The Forecast Statement

Point

Interval

Density

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Probability

– Event forecasts are probability forecasts

– Can be interpreted as a *point* forecast $p_t$ of a 0-1 indicator $I_t$

– Can be interpreted as a complete *density* forecast

$\Longrightarrow$ Cross-fertilization possibilities with density forecasting (construction, evaluation, combining)

# Consideration: The Loss function

Prob. forecast evaluation from a *point* forecasting perspective:

$$QPS = \frac{1}{T}\sum_{t=1}^{T}(p_t - I_t)^2$$

– Most relevant loss function? False alarms vs. missed calls.

Prob. forecast evaluation from a *density* forecasting perspective:

$$\{p_t\}_{t=1}^{T} = \{p_t^*\}_{t=1}^{T} \implies PIT \sim iidU(0,1)$$

– Subsumes measures of global calibration, local calibration, resolution, sharpness, etc.?

# Evaluation of Rare-Event Forecasts is Challenging

Events come at different frequencies:

- High: Transactions in liquid financial assets, ...
- Less high: Annual (seasonal) peaks in retail sales
- Medium: Business cycle expansions and recessions

- *Low: Government bond defaults, binding zero-lower-bound constraints, systemic financial collapses, depressions, apocalypse*

It's hard to assess the conditional calibration of rare event forecasts, precisely because of their rarity!

– Theory, Bayesian priors, introspection

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# Consideration: The Information Set

Covariates hold special appeal for event forecasting:

– Beyond "lagged dependent variables"

– Useful "leading indicators"?
(e.g., A leverage regime switch
may presage a credit market regime switch)

# A Model for Dynamic Event Forecasting

$$p(y_t|\pi_t) = \pi_t^{y_t}(1-\pi_t)^{1-y_t}$$
$$P(y|\pi) = \prod_{t=1}^{T} p(y_t|\pi_t)$$

$$\theta_t = log\left(\frac{\pi_t}{1-\pi_t}\right)$$
$$P(y_t|\theta_t) = exp[y_t\theta_t - log(1+exp(\theta_t))]$$
$$P(y|\theta) = \prod_{t=1}^{T} p(y_t|\theta_t)$$

$$\theta_t = \mu + x_t'\beta + \varepsilon_t$$
$$(1-L)^d\Phi(L)\varepsilon_t = \eta_t$$
$$\eta_t \sim iid(0, \sigma_\eta^2)$$

# Multivariate

Everything so far has been univariate

But we really want multivariate:

– Idiosyncratic aspects

– Common aspects

– Connectedness and systemic behavior

# Financial and Economic Connectedness

- Market Risk, Portfolio Concentration Risk
  (return connectedness)
- Credit Risk
  (default connectedness)
- Counterparty Risk, Gridlock Risk
  (bilateral and multilateral contractual connectedness)
- Systemic Risk
  (system-wide connectedness)
- Business Cycle Risk
  (local or global real output connectedness)

# Factor Structure, Single Factor, Fully Orthogonal

$$r_{it} = \lambda_i f_t + \varepsilon_{it}$$

$$\implies \sigma_{it}^2 = \lambda_i^2 \sigma_f^2 + \gamma_i^2$$

$$i = 1, ..., N$$

Fraction of $i$'s variance coming from others:

$$\frac{\lambda_i^2 \sigma_f^2}{\lambda_i^2 \sigma_{ft}^2 + \gamma_i^2} = \frac{1}{1 + \frac{\gamma_i^2}{\lambda_i^2 \sigma_f^2}}$$

So take:

$$C_i = \frac{\lambda_i^2 \sigma_f^2}{\gamma_i^2}$$

Obtain system total by adding over $i$:

$$C = \sigma_f^2 \sum_{i=1}^{N} \left( \frac{\lambda_i}{\gamma_i} \right)^2$$

# Now Move to Factor Structure for Event Indicator

Linear probability model analog:

$$I(r_{it}) = \lambda_i I(f_t) + \varepsilon_{it}$$

$$\sigma_{it}^2 = \lambda_i^2 \sigma_{I(f)}^2 + \gamma_i^2$$

$$i = 1, ..., N$$

– Connectedness measures remain intact

Logit/probit analog:

$$I(r_{it}) = squash(\lambda_i I(f_t) + \varepsilon_{it})$$

– What happens to connectedness measures?

# What We Really Want

– General framework not necessarily assuming factor structure

– Based on conditional as opposed to unconditional variation

– General connectedness and systemic risk measures

– Links to stress testing?

# Two Natural Questions

A natural modeling question:
*What fraction of the H-step-ahead prediction-error variance of variable $i$ is due to shocks in variable $j, \forall i, j$?*

Variance decomposition: $d_{ij}^{H}, \forall i, j$

A natural financial/economic connectedness question:
*What fraction of the H-step-ahead prediction-error variance of variable $i$ is due to shocks in variable $j, \forall j \neq i$?*

**Non-own** elements of the variance decomposition: $d_{ij}^{H}, \forall j \neq i$

# Variance Decompositions and the Connectedness Table

$N$-Variable Connectedness Table

| | $x_1$ | $x_2$ | ... | $x_N$ | From Others to $i$ |
|---|---|---|---|---|---|
| $x_1$ | $d^H_{11}$ | $d^H_{12}$ | $\cdots$ | $d^H_{1N}$ | $\Sigma^N_{j=1} d^H_{1j}, j \neq 1$ |
| $x_2$ | $d^H_{21}$ | $d^H_{22}$ | $\cdots$ | $d^H_{2N}$ | $\Sigma^N_{j=1} d^H_{2j}, j \neq 2$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ |
| $x_N$ | $d^H_{N1}$ | $d^H_{N2}$ | $\cdots$ | $d^H_{NN}$ | $\Sigma^N_{j=1} d^H_{Nj}, j \neq N$ |
| To Others From $j$ | $\Sigma^N_{i=1} d^H_{i1}$ $i \neq 1$ | $\Sigma^N_{i=1} d^H_{i2}$ $i \neq 2$ | $\cdots$ | $\Sigma^N_{i=1} d^H_{iN}$ $i \neq N$ | $\Sigma^N_{i,j=1} d^H_{ij}$ $i \neq j$ |

Upper-left block is variance decomposition matrix, $D$

Connectedness involves the **non-diagonal** elements of $D$

# Connectedness Measures

- Pairwise Directional: $C^H_{i \leftarrow j} = d^H_{ij}$ ("$i$'s imports from $j$")
- Net: $C^H_{ij} = C^H_{j \leftarrow i} - C^H_{i \leftarrow j}$ ("$ij$ bilateral trade balance")

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- Total Directional:
  - From others to $i$: $C^H_{i \leftarrow \bullet} = \sum_{\substack{j=1 \\ j \neq i}}^{N} d^H_{ij}$ ("$i$'s total imports")
  - To others from $j$: $C^H_{\bullet \leftarrow j} = \sum_{\substack{i=1 \\ i \neq j}}^{N} d^H_{ij}$ ("$j$'s total exports")
- Net: $C^H_{i} = C^H_{\bullet \leftarrow i} - C^H_{i \leftarrow \bullet}$ ("$i$'s multilateral trade balance")

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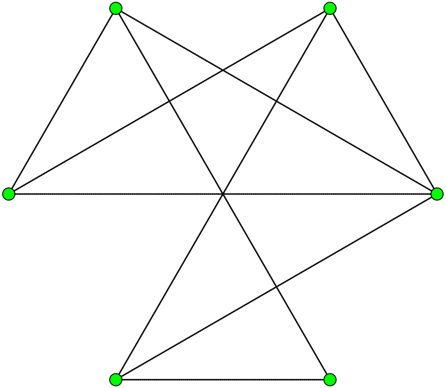
- Total: $C^H = \frac{1}{N} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} d^H_{ij}$ ("total world exports")

# Networks I: Representation



$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Adjacency Matrix (Symmetric)
$A_{ij} = 1$ if nodes $i$, $j$ linked
$A_{ij} = 0$ otherwise

# Networks I: Degree

*Degree of node i*, $d_i$:

$$d_i = \sum_{j=1}^{N} A_{ij}$$

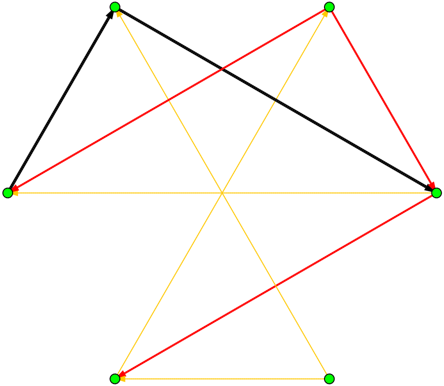
Discrete *degree distribution*, $P(d)$, on 0, ..., $N-1$

*Mean degree*, $E(d)$, is the key connectedness measure

Beautiful results (e.g., "small world") involve the mean degree:

$$\textit{diameter} \approx \frac{\ln N}{\ln E(d)}$$

# Networks II : Representation (Weighted, Directed)



$$A = \begin{pmatrix} 0 & .5 & .7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .3 & 0 \\ 0 & 0 & 0 & .7 & 0 & .3 \\ .3 & .5 & 0 & 0 & 0 & 0 \\ .5 & 0 & 0 & 0 & 0 & .3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

"to $i$, from $j$"

# Networks II: Degree (Weighted, Directed)

$A_{ij} \in [0, 1]$ depending on connection strength

Two degrees:

$$d_i^{from} = \sum_{j=1}^{N} A_{ij}$$

$$d_j^{to} = \sum_{i=1}^{N} A_{ij}$$

Continuous "from" and "to" degree distributions on $[0, N-1]$

Mean degrees $E(d)$ remain key for connectedness

# Central Observation: $D$ is a Weighted, Directed Network

Connectedness Table

| | $x_1$ | $x_2$ | ... | $x_N$ | From Others |
|---|---|---|---|---|---|
| $x_1$ | $d_{11}^H$ | $d_{12}^H$ | $\cdots$ | $d_{1N}^H$ | $\sum_{j\neq 1} d_{1j}^H$ |
| $x_2$ | $d_{21}^H$ | $d_{22}^H$ | $\cdots$ | $d_{2N}^H$ | $\sum_{j\neq 2} d_{2j}^H$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ |
| $x_N$ | $d_{N1}^H$ | $d_{N2}^H$ | $\cdots$ | $d_{NN}^H$ | $\sum_{j\neq N} d_{Nj}^H$ |
| To Others | $\sum_{i\neq 1} d_{i1}^H$ | $\sum_{i\neq 2} d_{i2}^H$ | $\cdots$ | $\sum_{i\neq N} d_{iN}^H$ | $\sum_{i\neq j} d_{ij}^H$ |

$$C_{i\leftarrow\bullet}^H = \sum_{\substack{j=1\\ j\neq i}}^N d_{ij}^H, \text{ are the “from degrees”}$$

$$C_{\bullet\leftarrow j}^H = \sum_{\substack{i=1\\ i\neq j}}^N d_{ij}^H, \text{ are the “to degrees”}$$

$$C^H = \frac{1}{N}\sum_{\substack{i,j=1\\ i\neq j}}^N d_{ij}^H, \text{ is the mean degree (to or from)}$$

# Relationships to Other Market-Based Measures I: Marginal Expected Shortfall (MES)

$$MES^{j|mkt}_{T+1|T} = E_T \left[ r_{j,T+1} | \mathbb{C} \left( r_{mkt,T+1} \right) \right]$$

- Sensitivity of firm *j*'s return to extreme market event $\mathbb{C}$
- Market-based "stress test" of firm *j*'s fragility
- Like "total directional connectedness *from*" (from degree)

# Relationships to Other Market-Based Measures II: CoVaR and ΔCoVaR

$$p = \Pr_T \left( r_{mkt,T+1} < -CoVaR^{mkt|i}_{T+1|T} \mid \mathbb{C}\left(r_{i,T+1}\right) \right)$$

- Measures tail-event linkages
- Leading choice of $\mathbb{C}\left(r_{i,T+1}\right)$ is that firm $i$ breaches its VaR
- Like "total directional connectedness *to*" (to degree)

$$\Delta CoVaR^{mkt|i}_{T+1|T} = CoVaR^{mkt|VaR(i)}_{T+1|T} - CoVaR^{mkt|Med(i)}_{T+1|T}$$

# Estimating Connectedness

Thus far we've worked under correct specification, in population:

$$C(x, H, B(L))$$

Now we want:

$$\widehat{C}\left(x, H, B(L), M(L; \hat{\theta})\right),$$

and similarly for other variants of connectedness

# Many Interesting Issues

- $x$ objects: Returns? **Return volatilities**? Real activities?
- $x$ universe: How many and which ones?
  ($\approx$ **15 major financial institutions**)
- $x$ frequency: **Daily?** Monthly? Quarterly?

- $H$: **Match VaR horizon**? Holding period?

- $M$: **VAR**? Structural?

- Identification of variance decompositions:
  Cholesky? **Generalized**? Structural?

- Estimation: **Classical**? Bayesian?

# Connectedness of Major U.S. Financial Institutions

$$\widehat{C}\left(x, H, B(L), M(L;\hat{\theta})\right)$$

- $x$: Thirteen daily realized stock return volatilities

  Commercial banks: JP Morgan Chase (JPM), Bank of America (BAC), CitiGroup (C), Wells Fargo (WFC), Bank of New York Mellon (BK), U.S. BankCorp (USB), PNC Bank (PNC)

  Investment Banks: Goldman Sachs (GS), Morgan Stanley (MS)

  GSEs: Fannie Mae (FNM), Freddie Mac (FRE)

  Insurance: AIG (AIG)

  Specialized: American Express (AXP)

- $H$: 12 days

- $M(L;\theta)$: logarithmic $VAR(3)$, generalized identification, 5/4/1999 - 4/30/2010

# Full-Sample Connectedness Table

| | AXP | BAC | BK | C | GS | JPM | MS | PNC | USB | WFC | AIG | FNM | FRE | **FROM** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AXP | 20.0 | 8.5 | 7.1 | 10.3 | 5.8 | 9.8 | 8.8 | 5.1 | 8.0 | 7.8 | 3.2 | 2.6 | 3.0 | 80.0 |
| BAC | 8.3 | 19.1 | 6.0 | 10.6 | 5.8 | 8.0 | 7.4 | 6.1 | 7.1 | 9.2 | 4.2 | 3.5 | 4.6 | 80.9 |
| BK | 8.4 | 8.3 | 18.8 | 8.4 | 6.2 | 9.3 | 8.5 | 5.7 | 8.4 | 8.3 | 4.2 | 2.4 | 3.0 | 81.2 |
| C | 9.5 | 9.6 | 5.4 | 20.4 | 4.9 | 8.7 | 7.8 | 5.2 | 7.0 | 8.0 | 5.4 | 3.5 | 4.7 | 79.6 |
| GS | 8.2 | 8.6 | 6.8 | 7.6 | 22.1 | 8.8 | 13.3 | 4.0 | 6.0 | 7.6 | 2.4 | 1.9 | 2.6 | 77.9 |
| JPM | 10.2 | 8.6 | 7.1 | 10.6 | 6.2 | 18.8 | 9.5 | 5.2 | 7.8 | 7.3 | 3.6 | 2.5 | 2.6 | 81.2 |
| MS | 9.2 | 8.3 | 7.1 | 8.9 | 9.8 | 9.7 | 20.5 | 4.2 | 5.5 | 7.1 | 3.4 | 2.8 | 3.6 | 79.5 |
| PNC | 7.7 | 8.8 | 7.4 | 8.5 | 4.6 | 7.6 | 6.6 | 18.1 | 7.6 | 8.8 | 5.2 | 4.2 | 4.9 | 81.9 |
| USB | 9.3 | 9.9 | 7.6 | 9.9 | 5.7 | 8.7 | 6.4 | 5.4 | 20.1 | 8.5 | 4.3 | 1.6 | 2.7 | 79.9 |
| WFC | 8.3 | 10.2 | 6.5 | 9.8 | 6.2 | 7.6 | 7.1 | 5.9 | 7.3 | 18.0 | 3.8 | 3.8 | 5.3 | 82.0 |
| AIG | 5.3 | 7.3 | 4.9 | 8.8 | 2.6 | 5.2 | 4.9 | 6.2 | 6.0 | 5.6 | 27.5 | 6.6 | 9.0 | 72.5 |
| FNM | 4.2 | 5.4 | 2.5 | 6.0 | 2.3 | 3.5 | 3.8 | 5.5 | 1.9 | 6.8 | 6.5 | 29.6 | 22.0 | 70.4 |
| FRE | 4.3 | 6.3 | 2.9 | 6.5 | 2.6 | 3.3 | 4.1 | 5.2 | 2.9 | 7.3 | 7.4 | 17.6 | 29.6 | 70.4 |
| **TO** | 92.9 | 99.7 | 71.3 | 106.1 | 62.7 | 90.2 | 88.2 | 63.7 | 75.5 | 92.2 | 53.8 | 53.1 | 68.1 | **78.3** |

# Estimating Time-Varying Connectedness

Before:

$$C\left(x, H, B(L), M(L;\theta)\right)$$
$$\widehat{C}\left(x, H, B(L), M(L;\hat{\theta})\right)$$

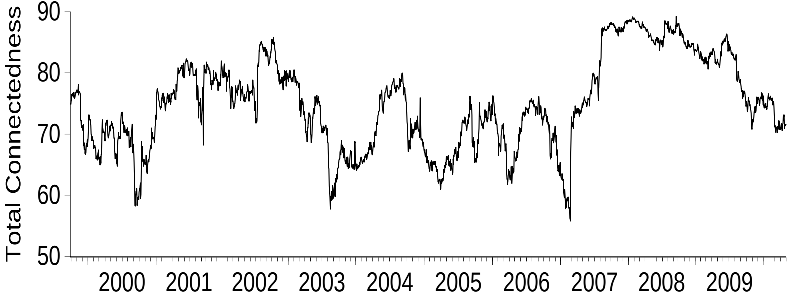
Now:

$$C_t\left(x, H, B_t(L), M(L;\theta_t)\right)$$
$$\widehat{C}_t\left(x, H, B_t(L), M(L;\hat{\theta}_t)\right)$$

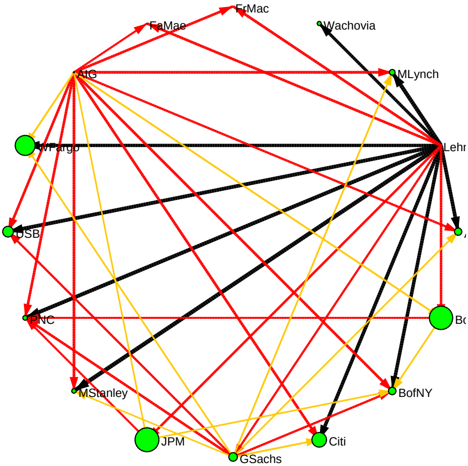
- Time-varying parameters: **Rolling estimation**? Smooth TVP model? Regime-switching?

  **(100-day estimation window)**

# Rolling Total Connectedness



# Net Pairwise Directional Connectedness: The Lehman Bankruptcy, September 17, 2008



# Rare Event Forecasting: Swallow Hard and March Onward (There's no Alternative)

– Probability forecasts and their evaluation

– Flexible multivariate modeling

– Systemic risk and network connectedness

[– Theory, Bayesian priors, model averaging]