Conditional probabilities for Euro area sovereign default risk

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Disclaimer: Not necessarily the views of ECB or ESCB.

Contributions

We propose a **novel modeling framework** to infer **conditional** and **joint probabilities** for sovereign default risk from observed CDS.

Novel framework? Based on a *dynamic GH skewed–t* multivariate density/copula with time-varying volatility and correlations.

Multivariate model is sufficiently flexible to be **calibrated daily** to credit market expectations. Not an "official opinion".

Analysis is based on **Euro area** CDS data from 2008M1 to 2011M6.
Event study: SMP/EFSF announcement & initial impact on risk.

Literature

1. **Sovereign credit risk:** e.g. Pan and Singleton (2008), Longstaff, Pan, Pedersen, and Singleton (2011), Ang and Longstaff (2011).
2. **Contagion**, see e.g. Forbes and Rigobon (2002), Caporin, Pelizzon, Ravazzolo, Rigobon (2012).
3. **Observation-driven time-varying parameter models**, see Creal, Koopman, and Lucas (2011, 2012), Zhang, Creal, Koopman, Lucas (2011), Creal, Schwaab, Koopman, Lucas (2011), Harvey (2012).
4. **Non-Gaussian dependence/copula/credit modeling**, see e.g. Demarta and McNeil (2005), Patton and Oh (2011).

Empirical questions

(**Q1**) Financial stability information: Based on credit market expectations, what is ...

Pr(two or more credit events in Euro area)?
Pr(i|j)-Pr(i), for any i,j?
Spillovers, e.g. Pr(PT|GR) - Pr(PT|not GR)?
Corr_t(i,j) at time t?

(**Q2**) Model risk: For answering (a), how important are parametric assumptions? *Normal* vs *Student-t* vs *GH skewed-t*.

(**Q3**) Event study: did the May 09, 2010 Euro area rescue package change risk dependence? How?

The GHST copula framework

Sovereign defaults iff benefits (v_{it}) exceed a cost (c_{it}), where

$$v_{it} = (\varsigma_t - \mu_\varsigma)\tilde{L}_{it}\gamma + \sqrt{\varsigma_t}\tilde{L}_{it}\epsilon_t, \quad i = 1, \ldots, n,$$

$\epsilon_t \sim$N$(0, I_n)$ is a vector of risk factors,
$\tilde{L}_{it}$ contains risk factor loadings,
$\gamma \in \mathbb{R}^n$ determines skewness,
$\varsigma_t \sim IG$ is an additional scalar risk factor for, say, *interconnectedness*.

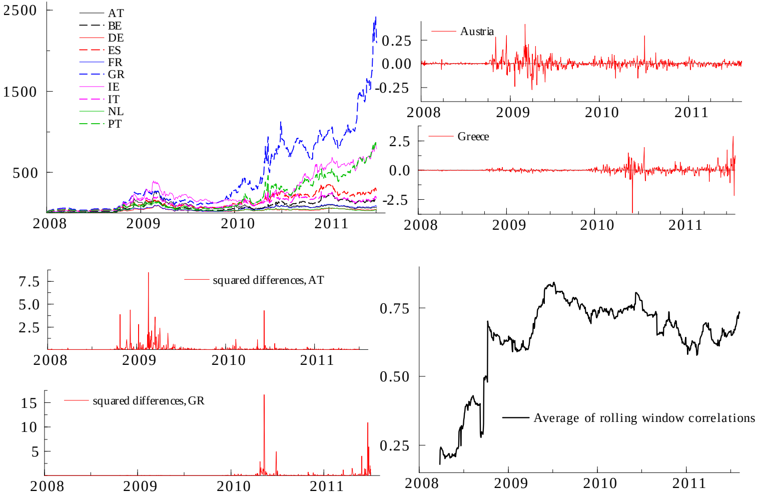
A default occurs with probability p_{it}, where

$$p_{it} = \Pr[v_{it} > c_{it}] = 1 - F_i(c_{it}) \Leftrightarrow c_{it} = F_i^{-1}(1 - p_{it}),$$

where F_i is the CDF of v_{it}.

Focus on *conditional* probability $\Pr[v_{it} > c_{it} | v_{jt} > c_{jt}]$, $i \neq j$.

Data: skewed, fat tailed, tv vol's and correlation



The GH skewed-t multivariate distribution

$$y_t = \mu + L_t e_t, \quad t = 1, \ldots, T, \; e_t \sim \text{GHST}, \; \mathrm{E}[e_t e_t'] = I_n,$$

$$p(y_t; \cdot) = \frac{\upsilon^{\frac{\upsilon}{2}} 2^{1-\frac{\upsilon+n}{2}}}{\Gamma\left(\frac{\upsilon}{2}\right) \pi^{\frac{n}{2}} \left|\tilde{\Sigma}_t\right|^{\frac{1}{2}}} \cdot \frac{K_{\frac{\upsilon+n}{2}}\left(\sqrt{d(y_t) \cdot (\gamma'\gamma)}\right) e^{\gamma' \tilde{L}_t^{-1}(y_t - \tilde{\mu}_t)}}{\left(d(y_t) \cdot (\gamma'\gamma)\right)^{-\frac{\upsilon+n}{4}} d(y_t)^{\frac{\upsilon+n}{2}}},$$

where

$$d(y_t) = \upsilon + (y_t - \tilde{\mu}_t)' \tilde{\Sigma}_t^{-1} (y_t - \tilde{\mu}_t),$$

$$\tilde{\mu}_t = -\upsilon/(\upsilon - 2)\; \tilde{L}_t \gamma,$$

$$\tilde{\Sigma}_t = \tilde{L}_t \tilde{L}_t' \quad \text{is scale matrix}$$

If $\gamma = 0$, then GH skewed-t simplifies to Student's t density.
If in addition $\upsilon^{-1} \to 0$, then multivariate Gaussian density.
$\tilde{\Sigma}_t(f_t) = \tilde{L}_t(f_t)\tilde{L}_t(f_t)'$ is driven by 1st and 2nd derivative of the pdf.

The model with time varying parameters

Assume that $\Sigma_t = D_t R_t D_t = L_t(f_t) L_t(f_t)'$ and that

$$f_{t+1} = \omega + \sum_{i=0}^{p-1} A_i s_{t-i} + \sum_{j=0}^{q-1} B_j f_{t-j},$$

$$\begin{aligned} \text{where} \quad s_t &= S_t \nabla_t \text{ is the scaled score} \\ \nabla_t &= \partial \ln p(y_t; \tilde{\Sigma}(f_t), \gamma, \upsilon) / \partial f_t \\ S_t &= E_{t-1}[\nabla_t \nabla_t' | y_{t-1}, y_{t-2}, \ldots]^{-1}, \end{aligned}$$

Scaling matrix S_t is inverse conditional Fisher information matrix.

Time varying parameters: score

Important: first two derivatives are available in closed form.

$$\begin{aligned}
\nabla_t &= \partial \ln p(y_t; \tilde{\Sigma}(f_t), \gamma, \upsilon) / \partial f_t \\
&= \frac{\partial \text{vech}(\Sigma_t)'}{\partial f_t} \frac{\partial \text{vech}(L_t)'}{\partial \text{vech}(\Sigma_t)} \frac{\partial \text{vec}(\tilde{L}_t)'}{\partial \text{vech}(L_t)} \frac{\partial \ln p_{GH}(y_t | f_t)}{\partial \text{vec}(\tilde{L}_t)} \\
&= \ldots \\
&= \Psi_t' H_t' \text{vec} \left\{ w_t y_t y_t' - \tilde{\Sigma}_t - \left(1 - \frac{\upsilon}{\upsilon - 2} w_t\right) \tilde{L}_t \gamma y_t' \right\}
\end{aligned}$$

$$\begin{aligned}
\text{where } \Psi_t &= \partial \text{vech}(\Sigma_t) / \partial f_t' \\
H_t &= \text{messy} \\
w_t &= \frac{\upsilon + n}{2 \cdot d(y_t)} - \frac{k'_{\frac{\upsilon+n}{2}}\left(\sqrt{d(y_t) \cdot (\gamma'\gamma)}\right)}{\sqrt{d(y_t)/\gamma'\gamma}}; \; k'_a(b) = \frac{\partial \ln K_a(b)}{\partial b}.
\end{aligned}$$

Extracting marginal pd's from CDS

We equate the premium and default leg of CDS given a default intensity.

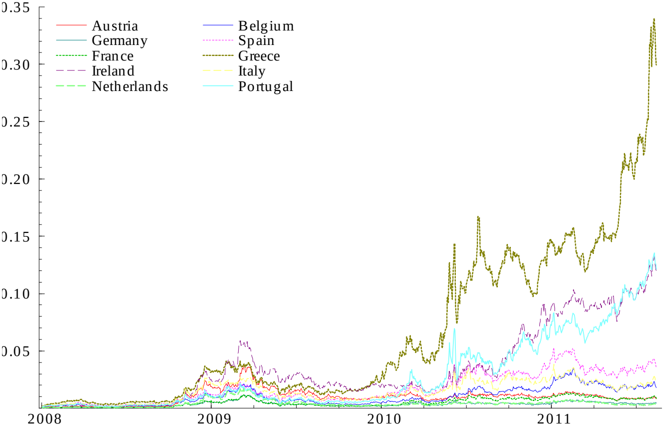
$$p_{it} \approx \frac{s_{it}(1 + r_t)}{1 - rec_i}, \qquad (*)$$

where

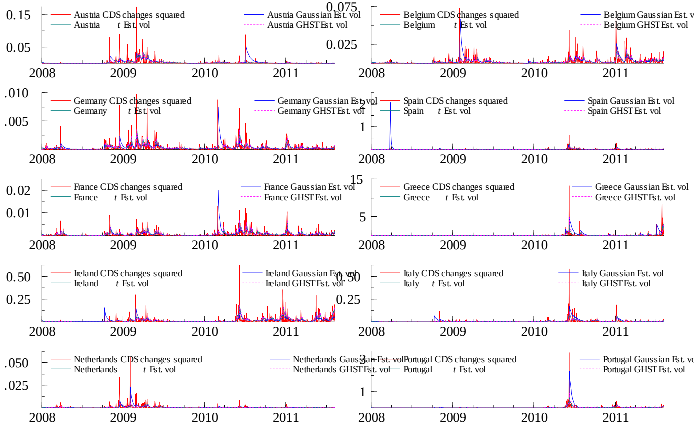
$$\begin{aligned} s_{it} &= \textit{CDS} \text{ annual fee, country } i\text{, time } t \\ r_t &= \textit{LIBOR} \text{ 1 year rate, flat} \\ rec_i &= 25\% \text{ expected recovery, stressed.} \end{aligned}$$

Eqn $(*)$ is exact if the term structures for pd's and interest rates are flat, s_{it} is paid to the seller continuously, and there is no counterparty credit risk, see Brigo and Mercurio (2007).

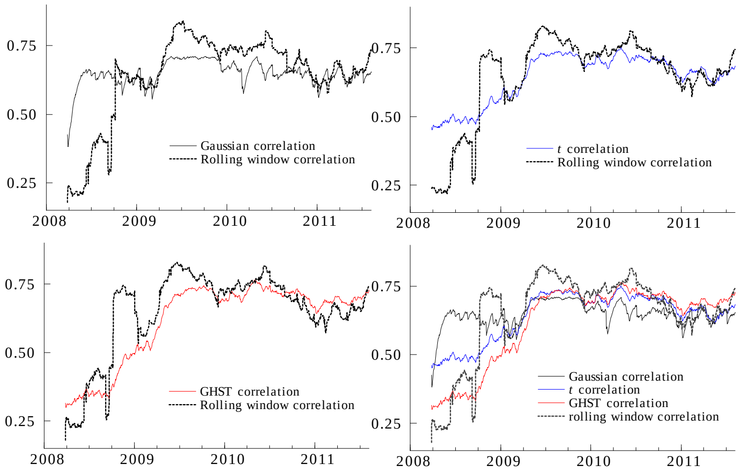
Marginal pd's from CDS



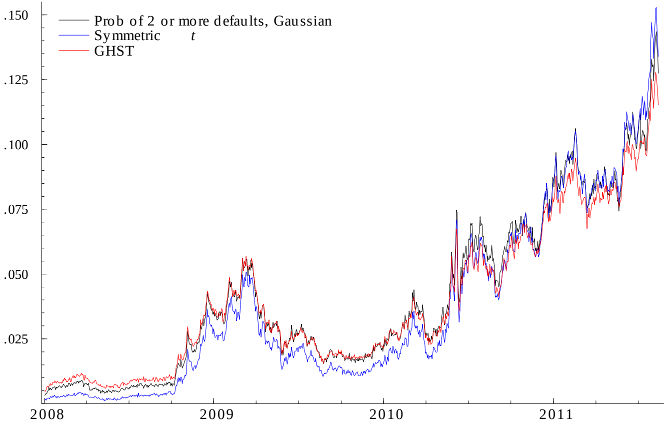
Volatility estimates



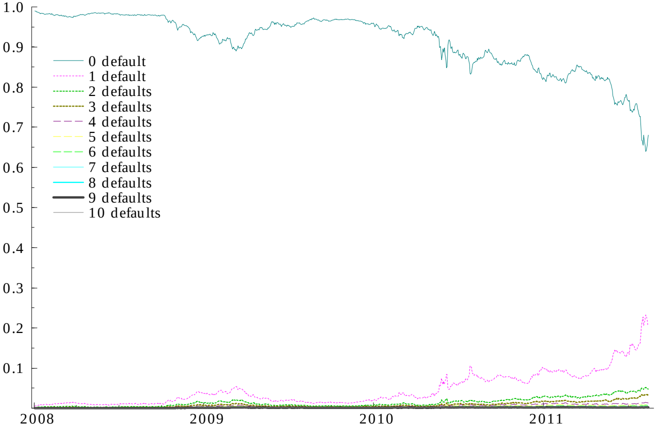
Dynamic correlations



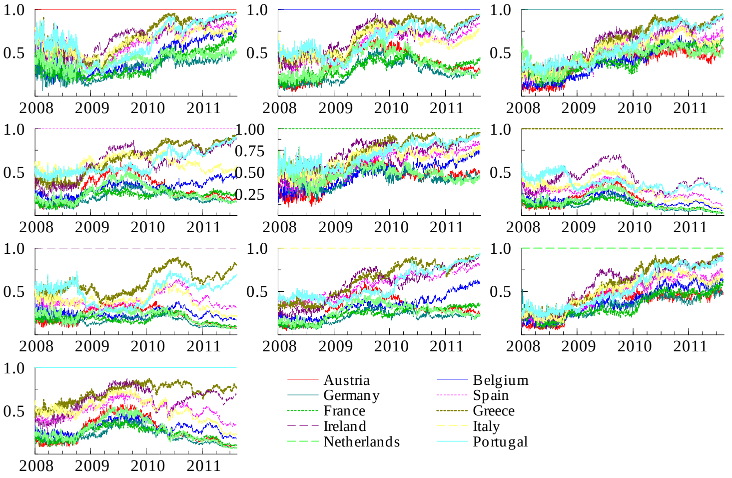
The probability of two or more failures



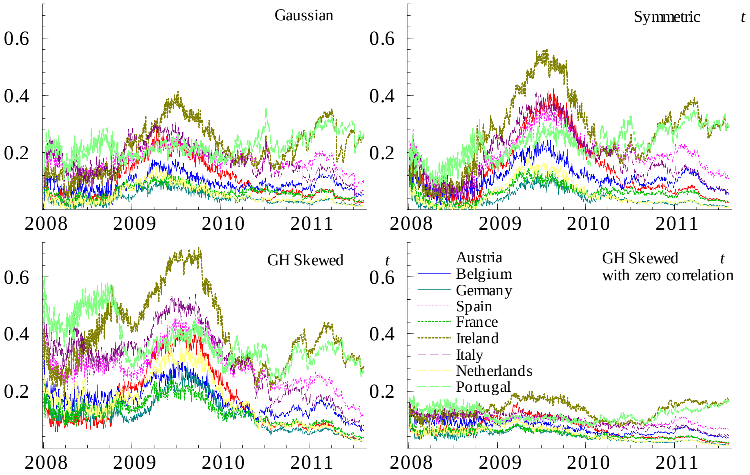
The probability of k=0,1,2,... failures



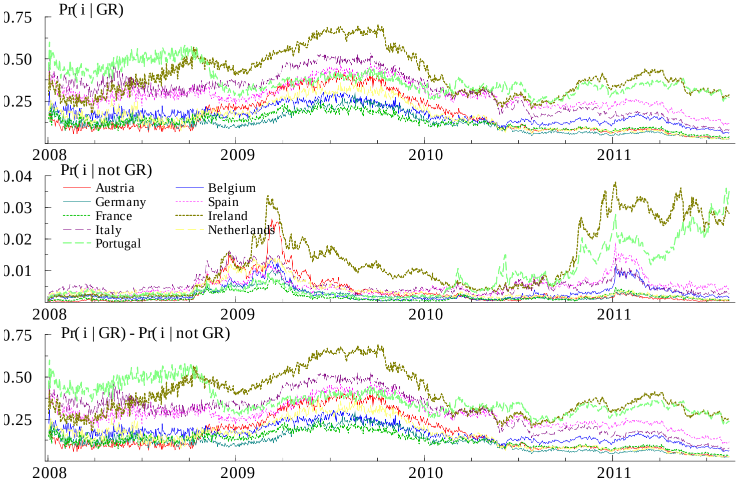
Conditional pds: Pr(all i| all j)



Conditional pds: Pr(i|GR)



GHST spillovers: Pr(i|GR) - Pr(i|not GR)



The May 09, 2010 package

Joint risk, $\Pr(i \bigcap j)$						
	Thu 06 May 2010			Tue 11 May 2010		
	PT	GR	DE	PT	GR	DE
AT	1.1%	1.1%	0.6%	0.6%	0.7%	0.4%
BE	1.2%	1.4%	0.7%	0.9%	1.0%	0.6%
DE	1.0%	1.1%		0.8%	0.8%	
ES	3.0%	3.3%	0.9%	1.5%	1.6%	0.6%
FR	1.0%	1.0%	0.6%	0.8%	0.9%	0.6%
GR	**4.8%**		1.1%	**2.3%**		0.8%
IR	**2.6%**	**3.1%**	0.8%	**1.4%**	**1.8%**	0.6%
IT	2.8%	2.9%	0.9%	1.4%	1.5%	0.6%
NL	0.9%	0.9%	0.5%	0.6%	0.7%	0.5%
PT		4.8%	1.0%		2.3%	0.8%
Avg	**2.0%**	**2.2%**	0.8%	**1.1%**	**1.2%**	0.6%

The May 09, 2010 package

Conditional risk, Pr(i | j)						
	Thu 06 May 2010			Tue 11 May 2010		
	PT	GR	DE	PT	GR	DE
AT	17%	8%	53%	22%	10%	46%
BE	20%	10%	60%	32%	15%	61%
DE	16%	8%		26%	12%	
ES	49%	25%	78%	50%	23%	63%
FR	16%	8%	58%	28%	12%	62%
GR	**78%**		99%	**80%**		86%
IR	**43%**	**23%**	75%	**49%**	**26%**	68%
IT	45%	22%	77%	49%	21%	64%
NL	14%	7%	49%	21%	10%	50%
PT		36%	91%		33%	81%
Avg	**33%**	**16%**	71%	**40%**	**18%**	64%

Bottom line: joint risks ↓↓, but dependence ↑. "Firewall"-analogy?

Conclusion

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Thank you