Vintage Capital as an Origin of Inequalities*

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Abstract

Does capital-embodied technological change play an important role in shaping labor market inequalities? This paper addresses the question in a model with vintage capital and search/matching frictions where costly capital investment leads to large heterogeneity in productivity among vacancies in equilibrium. The paper first demonstrates analytically how both technology growth and institutional variables affect equilibrium wage inequality, income shares and unemployment. Next, it applies the model to a quantitative evaluation of capital as an origin of wage inequality: at the current rate of embodied productivity growth a 10-year vintage differential in capital translates into a 6% wage gap. The model also allows a U.S.—continental Europe comparison: an embodied technological acceleration interacted with different labor market institutions can explain a significant part of the differential rise in unemployment and capital share and some of the differential dynamics in wage inequality.

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1 Introduction

Recently, we have witnessed striking changes in the technology used in the workplace. The "new economy" can arguably be characterized as follows: (i) technological improvements seem to be intimately connected with the introduction of new capital goods and (ii) these improvements proceed at a faster rate than before. Over the past two decades the productivity gains associated with new investment have represented the major source of growth in U.S. output per capita (Jorgenson 2001).

The fact that new technologies are "embodied" in capital opens the possibility of large discrepancies in worker productivity, both within and between firms. In this paper we argue that unless labor markets are perfectly competitive—so that identical workers are paid the same wage, independently of the capital they work with—rapid, capital-embodied technological change can be an important determinant of wage inequality. In addition, to the extent that there are frictions in matching capital to labor and that there are rents to be divided between them, the rate of unemployment and the income shares—other important aspects of inequality—can also be affected by technological progress.

Our analysis rests on a general-equilibrium framework with three building blocks: vintage capital, a frictional labor market, and wage bargaining. To model capital-embodied technological change we use a vintage capital framework where machines/jobs are costly-to-create units of capital of different ages, corresponding to technologies with different productivity levels. To model employment inequalities, we operate in the tradition of Diamond/Mortensen and Pissarides-style models, where an aggregate matching function determines the meeting rate between unemployed workers and vacant jobs. To model wage inequality and the division of income between labor and capital, we follow the standard approach in this literature whereby wages and profits are endogenously determined through Nash bargaining within the worker-firm pair. We show the existence and uniqueness of an equilibrium in our model and analyze how inequalities in the labor market are determined as a function of the economy's primitives: technology, frictions, and institutions. In particular, we study the role of the rate of technological change itself and its interaction with economic policy in the form of government intervention in the labor market.

We use our theory of labor market inequality implied by capital-embodied technological change to provide quantitative answers to two substantive questions. First, we use a calibrated version of the model to account for the contribution of vintage capital to observed wage inequality: how much of “residual” wage inequality, that is, inequality that cannot be attributed to observable characteristics of workers, might be due to differences in capital? Quantitative theory is useful here because we believe that it is very difficult to identify relative machine quality in the data; data on the age of capital is difficult to link to wage inequality, and firm or plant age are very crude and indirect measures; more disaggregated data is simply not available.[1]

The other question we take on is perhaps more ambitious. Over the past thirty years labor market outcomes in the United States and continental European countries have changed substantially and in very different ways. In the United States wage inequality jumped to the highest levels in the postwar period, the labor share of income declined slightly, and the unemployment rate remained remarkably stable. In sharp contrast, in most of the large continental European economies, the wage structure did not change much at all, while the labor share fell substantially and unemployment increased steadily. Over the very same period, impressive technological improvements embodied in new vintages of capital (especially in information and communication equipment and software) induced the adoption of new production technologies across virtually every developed economy. Can these facts be accounted for with our theory of capital-embodied technological change and labor market frictions? We study, in particular, whether the interaction between this growth channel and certain labor market institutions, whose strength differs between US and Europe, can explain quantitatively the different evolution of the various dimensions of labor market inequalities.

THE FACTS:

In Table 1 we report some key numbers on unemployment rate, wage inequality, and labor shares for several OECD countries at five-year intervals from 1965 to 1995. We are particularly interested in the comparison between United States and continental European countries (averaged in the row labelled Europe Average).[2]

In 1965 the unemployment rate in virtually every European country was lower than in the United States. Thirty years later, the opposite was true: the U.S. unemployment rate

[1] Any systematic relation between wages and capital quality in the data would also be hard to interpret, since workers' unobservable characteristics are likely to be correlated with the capital they are matched with (e.g., one might expect some degree of positive sorting).

[2] For completeness, we include data in Table 1 for the UK and Canada, whose behavior falls somewhere between that of the United States and Europe.

rose by 1.7% from 1965-1995, whereas the average rise for European countries is 8.4%. The labor share of aggregate income has declined only marginally in the United States, by 1.5% from 1965-1995, while on average it fell by almost 6 points in Europe. Wage inequality, measured by the percentage differential between the ninth and the first earnings deciles for male workers, rose only slightly in Europe by 4% in the past 15 years, and it even declined in some countries (Belgium, Germany, and Norway). The sharp surge of earnings inequality in the United States is well documented, see Katz and Autor 1999, and the OECD data confirm a rise of almost 30% since 1980. Interestingly, the European averages hide much less cross-country variation than one would expect given the raw nature of the comparison. For example, in 11 out of the 14 continental European countries, the increase in unemployment rate has been larger than 6%, and in 9 out of 14 countries the decline in the labor share has been greater than 5%.

THE QUALITATIVE ANALYSIS:

Aghion and Howitt (1994) and Mortensen and Pissarides (1998) pioneered the research on the relation between embodied productivity growth and unemployment in a frictional labor market.[3] In their standard models new capital is always costless to buy and, as a result, vacancies all consist of the newest capital. In contrast, the key new feature of our model is the existence of vacancy heterogeneity, i.e., vacancies differ with respect to the quality of the equipment on the job. This new feature is important for two reasons.

First, in the standard model the vintage structure is purely a frictional phenomenon: when the capital is matched with a worker, it ages until a break-up results from the capital becoming too obsolete relative to the worker's outside option. As matching becomes more and more instantaneous—as the friction is made weaker—separation occurs earlier and earlier; in the limit, with no matching friction, all capital is new, so vintage effects are absent.[4] Although our analysis has several features in common with these studies, we model capital

[3] Jovanovic (1998) investigates analytically the relation between embodied productivity growth and wage inequality in a competitive assignment model with a continuum of vintages of capital and of types of workers. Our introduction of frictions in the labor market allows a study of unemployment and induces a different wage determination mechanism with specific implications for wage inequality. Interestingly, some key mechanisms of the frictionless economy carry over to the frictional model, as will become clear below.

[4] Mortensen and Pissarides (1998) present also a model where firms can upgrade their capital without necessarily inducing the destruction of the match. Because upgrading the existing machine is costly, while destroying the job and opening a vacancy with the new capital entails only the search costs, it remains true that as the frictions disappear, so does the vintage structure. We return on the upgrading issue later in the paper.

differently. We view capital as costly to buy, and once capital has been purchased, it is natural to use it until it is so obsolete that the workers are more efficiently used elsewhere—since they can alternatively work with newer capital. Thus, a unit of capital has a natural life-cycle. Labor market frictions will make the life of capital longer because it is not costless for a worker to find new capital to work with—she may have to go through an unproductive period of unemployment. In contrast to the existing literature, in the frictionless version of our model, capital is used for a strictly positive time period before being scrapped. In this sense, our model is the most natural extension of the standard competitive vintage capital growth model (Solow 1960) to an economy with labor market frictions.

Second, the presence of a nontrivial distribution of vacancies introduces new economic forces in the standard model of equilibrium unemployment. First, the existence of a nonzero outside option for the firm reduces the match surplus proportionally to the firm's meeting rate. Thus, changes in the embodied productivity growth rate, which have an impact on the equilibrium meeting rates, will affect the surplus through this new channel. In addition, changes in the rate of technical progress will affect the equilibrium age distribution of vacancies and, through this channel, the worker's outside option of searching.

The main result of our qualitative analysis is that, notwithstanding the increased complexity that this heterogeneity introduces, we show that it is possible to maintain analytical tractability in characterizing the chief features of an equilibrium. In particular, we can represent the equilibrium of the economy with two curves (job creation curve and job destruction curve) in the two-dimensional space defined by the age of capital at destruction and the labor market tightness. The shifts of the two curves following a permanent rise in the rate of embodied productivity are unambiguous, which allows us to describe qualitatively the response of unemployment, inequality, and income shares. We show in particular that an economy with generous unemployment benefits is more likely to respond to such a faster productivity growth rate with a rise in unemployment duration, while a laissez-faire type economy is more prone to respond through a reduction in the life-length of capital and more job separations.

The intuition for this result is intimately related to the new features of our model: when capital is costly, there exists a minimum life-length of the job required to fully recover the set-up cost even in the absence of frictions. A U.S.-type economy with a minimal welfare state has low labor costs and, hence, "bad" jobs with very old capital are still profitable, so that the optimal scrapping age of capital is relatively high and far away from the technological minimum. In contrast, in a European-type economy with munificent welfare payments, firms are forced to scrap old capital earlier. An increase in the productivity of capital is in essence an "obsolescence" shock to which firms would like to respond by shortening the life of capital and adopting the new vintages more quickly. However, while this is possible in a U.S.-type economy, such margin of adjustment is not fully available to European-type economies, whose life of capital is already very close to the technological minimum. Since the scrapping age cannot decline enough, firms need to be compensated through a different margin—a higher meeting probability—which translates into longer unemployment durations for workers. This mechanism improves the bargaining power of firms and allows them to push workers closer to their outside option (which is constant across workers). The consequence is a larger fall in the labor share of output and a smaller rise in wage inequality in European-type economies. This qualitative analysis is one of the keys to deciphering the results of the quantitative exercise.

THE QUANTITATIVE EXERCISES:

The quantitative importance of capital-embodied technological change for residual wage inequality and unemployment has, as far as we know, not been studied before. The difference between the labor market experiences of the United States and continental Europe, however, has been the object of a quantitative analysis in a number of papers.[5] The divergent behavior of the two economies is explained in these papers through the interaction between different labor market institutions across regions and a common structural shock to the economic environment. In our view, the existing literature does not offer a satisfactory way to link the fundamental driving force behind the changes in the labor market to independent observable data. As a consequence, any calibration attempt matches one of the crucial elements of interest, such as the rise in inequality or the changes in income shares, by construction. We take the view that unemployment, inequality, and changes in the labor income share are of great importance and have to be explained jointly: they are dimensions along which the model should be evaluated rather than calibrated. An important advantage of our model is that the unique source of the shock is capital-embodied productivity, and the parameter regulating the speed of capital-embodied technological change can be measured through independent data—the change in the quality-adjusted relative price of equipment—as is done

[5]We summarize this literature in section 5.4.

in a number of previous papers that have applied this information to growth accounting and analyses of the labor market.[6]

The model suggests that vintage capital has a significant impact on wage inequality, although the implied level of wage inequality is small compared to the data in Table 1. This result is not surprising, as the only source of wage differentials in our economy is vintage capital within an ex ante equal set of workers. Because of the lack of detailed employer-employee matched data where one could sharply distinguish the role of workers' individual characteristics from the role of firms' characteristics in wage determination, we are not aware of any direct empirical estimate of the effect of differences in the vintage of capital on wage differentials. We can, however, use our calibrated model to give an answer to this question: we find that in a U.S.-type economy a difference of ten years in the vintage of capital used by the firm generates wage differentials around 6%. We argue that this represents about one fourth of residual wage inequality for ex-ante equal workers in the United States.

The quantitative U.S.–Europe exercise consists of an acceleration in the rate of embodied productivity growth in economies that differ according to the generosity of their welfare benefits and the strictness of employment protection legislation. The main result of our quantitative exercise is that the model is successful in generating the observed differential rise in unemployment and in the capital share between the United States and Europe. A permanent rise in the rate of capital-embodied productivity growth of 2 percentage points increases unemployment rate by less than 1 point in the U.S.-type economy and by over 8 points in the European-type economy, with all the increase taking place along the unemployment duration margin, as in the data. The labor share falls by over 6 points in both economies, but once we introduce a firing tax to capture variations in the degree of employment protection, the model generates a stronger fall in the labor share (by circa 3 points) in European-type economies with stricter firing restrictions.

Finally, the numerical simulations show that our model with vacancy heterogeneity displays a quantitative amount of technology-policy complementarity much larger than that of the standard Aghion-Howitt/Mortensen-Pissarides framework. We believe this complementarity helps in explaining the data.

The remainder of the paper is organized as follows. In Section 2, we start our analysis

[6] See Gordon (1990), Hornstein and Krusell (1996), Greenwood, Hercowitz, and Krusell (1997), Greenwood and Yorukoglu (1997), Krusell, Ohanian, Ríos-Rull, and Violante (2000), and Cummins and Violante (2002), among others.

with the frictionless environment, where workers are all paid the same wage and all are employed. In Section 3 we move to the frictional environment with heterogeneous vacancies, solve the model, and prove the existence and uniqueness of equilibrium. In Section 4 we characterize how equilibrium inequalities in employment, wages, and income shares respond qualitatively to a change in the speed of embodied technology, and we also study the role of different labor market institutions in an attempt to explain the distinct labor market performances of the United States and Europe. Section 5 presents the calibration of the model and the results of our quantitative exercises and discusses the related literature in detail. Section 5.5 compares our model with the standard matching model. Finally, Section 6 concludes the paper.

2 The frictionless economy

Time is continuous. The economy is populated by a stationary measure 1 of workers who are all alike, live forever, are risk-neutral, and discount the future at rate r. Technological progress is embodied in capital, and the productive capacity of new vintage machines grows at the rate $\gamma > 0$. A firm (or job, or production unit) can be created through an initial investment expenditure $I(t)$, and the cost of new vintage machines also grows at the rate γ. Firms can freely enter the market upon payment of the initial installation cost. At time t, firms can choose whether to purchase the newest vintage machine or a machine of any older existing vintage: newer vintages are relatively more expensive to set up, but they are also relatively more productive.

A firm is productive only when paired with a worker. There is no physical depreciation of machines and production of a firm remains constant through its lifetime. There is, however, economic depreciation. Older firms produce relatively less than newer firms because of embodied technological change, and firms with old enough capital will voluntarily exit the market.

In order to make the model stationary, we normalize all variables and define output relative to the newest production unit. The normalized cost of a new production unit is then constant at I, and the normalized output of a production unit of age a which is paired with a worker is $e^{-\gamma a}$. We will focus on the steady state of the normalized economy, which corresponds to a balanced growth path of the actual economy. Finally, we will assume that $r > \gamma$ to guarantee the boundedness of infinite sums.

We start by describing the competitive equilibrium for the frictionless economy. In the steady state the wage rate also grows at the rate γ and the normalized wage $w \leq 1$, now measured relative to the output of the newest vintage, is constant. Consider a price-taker firm that plans to set up a new vintage machine. The firm optimally chooses the exit age $\bar{a}$ that maximizes the present value of machine lifetime profits

$$\max_{\bar{a}} \int_0^{\bar{a}} e^{-ra}(1 - we^{\gamma a})da \equiv \Pi(w),$$

where Π is the profit function. Since flow profits are monotonically declining and eventually become negative, there is a unique exit age for new vintages. Profit maximization leads to the condition

$$w = e^{-\gamma \bar{a}}, \tag{1}$$

stating that the price of labor has to equal the productivity of the oldest machine, which is also the marginal productivity of labor. The higher the wage, the shorter the life-length of capital since (normalized) profits per period fall and thus reach zero sooner.

We next argue that profit-maximizing firms always choose the newest capital vintage. Suppose the labor required to operate new vintage machines was also increasing in the quality of machines at rate γ over time. Then firms would be indifferent between the newest and any older technology: an older vintage would simply scale down costs—both for the machine and wage expenses—and revenues by the same amount, leaving profits unchanged, and the time in operation would remain at the same level as that for new firms. The labor requirement, however, is not increasing over time, which is why new technologies are better; in fact, technological change is labor-augmenting here in the sense that it allows one worker to work with more and more efficiency units of capital over time by using newer and newer equipment. Thus, a firm choosing to invest in old capital would, once in operation, generate lower profits per period, and it would operate for a shorter period of time (since the time at which the wage equals the total product is reached sooner) than if it chose the newest capital. The lower cost of the old machine would compensate these losses only partially.[7]

Free entry of firms requires that in equilibrium $I = \Pi$. This is the key condition that determines exit age $\bar{a}$, and hence wages. Using the profit-maximization condition (1), the free entry condition can be written as

$$I = \int_0^{\bar{a}} e^{-ra}\left[1 - e^{-\gamma(\bar{a}-a)}\right] da. \tag{2}$$

[7] This argument is easy to verify mathematically, so we omit its proof in the text.

Equation (2) allows us to discuss existence and uniqueness of the equilibrium as well as comparative statics. It is straightforward to solve for efficient allocations and show that a stationary solution to the planner's problem reproduces the competitive allocations (see Appendix A.1).

The right-hand side of the equilibrium condition (2) is strictly increasing in the exit age $\bar{a}$ for two reasons. First, in an equilibrium with older firms, the relative productivity of the marginal operating firm is lower and therefore wages have to be lower and profits higher. Second, a longer machine life increases the duration for which profits are accumulated. The right-hand side of (2) increases from 0 to $1/r$ as $\bar{a}$ goes from 0 to infinity. Taken together, these facts mean that there exists a unique steady state exit age $\bar{a}^{CE}$ whenever $I < 1/r$. This condition is natural: unless you can recover the initial capital investment at zero wages using an infinite lifetime ($\int_0^\infty e^{-ra} da = 1/r$ being the net profit from such an operation), it is not profitable to start *any* firm. With a unit mass of workers, all employed, the firm distribution is uniform with density $1/\bar{a}^{CE}$, which is also the measure of entrant firms e_f.

Turning to comparative statics, we note that a larger interest rate r decreases present-value profits, thus lowering entry and increasing the life span of the machine. Conversely, an increase in the cost of a new machine I will raise the life span: fewer machines enter and they stay in operation longer to recover the fixed cost. An increased growth rate of capital-embodied technological change γ must decrease the life span of machines and increase the number of firms that enter at each point in time. Formally, the right-hand side of equation (2) is increasing in the growth rate γ: the higher the growth rate, the lower the relative productivity of the least productive firm, and therefore the lower the cost of hiring labor must be. Faster growth therefore means higher profits, implying an increase in entry at the expense of older machines that are forced to exit earlier. Thus, in the competitive economy when technological change accelerates, the rate of job turnover in the economy rises and, as a consequence of the decline in the wage rate, the labor share of aggregate income $\omega^{CE} = \gamma / \left(e^{\gamma \bar{a}^{CE}} - 1 \right)$ falls.

Although the prediction on the income shares qualitatively matches the facts of Table 1, it is worth remarking that the environment without frictions displays neither wage nor employment inequality, so it cannot serve as a tool to analyze the facts we described. For this reason, we now turn our attention to an environment with matching frictions.

3 The economy with matching frictions

In this section, we consider a slightly different economy. The demographics and the technological side of the model are unchanged, but the structure of the labor market is new. The labor market is no longer perfectly competitive: it is frictional. The matching process between workers and production units is random and takes place in one pool comprising all workers and all vacant firms; vacant firms are distinguished by the age of their capital. Throughout, and for tractability, we will focus on steady-state analysis; thus, the notation presumes no time-dependence. In particular, all distributions are stationary over time.

The nature of the firm's decision process—buy a piece of capital, then match with a worker, and finally exit when the capital is so old that it no longer generates positive profit flows—remains the same as in the frictionless economy. In particular, firms in this economy will also choose to buy the newest form of capital when entering. Due to the matching frictions, some firms will also become idle, but idle firms have no option but to wait until they meet a worker.[8]

The rate at which a worker meets a firm with capital of age a is $\lambda_w(a)$ and the rate at which she meets any firm is $\lambda_w \equiv \int_0^{\bar{a}} \lambda_w(a)da$, where $\bar{a}$ is the job-destruction age. A firm meets a worker at the rate λ_f. Let $\nu(a)$ denote the measure of vacant firms of age a. We assume that the number of matches in any moment is determined by a constant returns to scale matching function $m(v,u)$, where $v \equiv \int_0^{\bar{a}} \nu(a)da$ is the total number of vacancies and u is the total number of unemployed workers. We also assume that $m(v,u)$ is strictly increasing in both arguments and satisfies some standard regularity conditions.[9] Using the notation $\theta = v/u$ to denote labor market tightness, we then have that

$$\lambda_w(a) = \frac{\nu(a)}{v} m(\theta, 1), \tag{3}$$

$$\lambda_f = \frac{m(\theta,1)}{\theta}. \tag{4}$$

[8] One can easily allow for an "upgrading" decision: in any given period, with some probability the firm has an opportunity to upgrade the machine to the newest capital and keep the worker at some cost. In the interest of keeping the model tractable for our qualitative analysis we abstract from this feature here.

[9] In particular,

$$\begin{aligned} m(0,u) &= m(v,0) = 0, \\ \lim_{u\to\infty} m_u(v,u) &= \lim_{v\to\infty} m_v(v,u) = 0, \\ \lim_{u\to 0} m_u(v,u) &= \lim_{v\to 0} m_v(v,u) = +\infty. \end{aligned}$$

The expression for the meeting probability in (4) provides a one-to-one (strictly decreasing) mapping between λ_f and θ. Thereafter, when we discuss changes in λ_f, we imagine changes in θ.

We assume that matches dissolve exogenously at the rate δ: upon dissolution, the worker and the firm are thrown into the pool of searchers.[10] Searching is costless: it only takes time. When unemployed, the worker receives a welfare payment b. The measure of matches with an a firm and a worker is denoted $\mu(a)$ and total employment μ.

Values for the market participants are $J(a)$ and $W(a)$ for matched firms and workers, respectively, $V(a)$ for vacant firms, and U for unemployed workers. Let $w(a)$ denote the wage paid to a worker from an a firm. The values solve the following differential equation system, which summarizes the flow payoffs of workers and firms:

$$(r-\gamma)V(a) = \max\{\lambda_f\left[J(a)-V(a)\right]+V'(a),0\} \tag{5}$$

$$(r-\gamma)J(a) = \max\{e^{-\gamma a}-w(a)-\delta\left[J(a)-V(a)\right]+J'(a),(r-\gamma)V(a)\} \tag{6}$$

$$(r-\gamma)U = b+\int_0^{\bar{a}}\lambda_w(a)\left[W(a)-U\right]da \tag{7}$$

$$(r-\gamma)W(a) = \max\{w(a)-\delta\left[W(a)-U\right]+W'(a),(r-\gamma)U\}. \tag{8}$$

The derivatives of the value functions with respect to a will be negative and are flow losses due to the aging of capital.[11]

In the presence of frictions, a bilateral monopoly problem between the firm and the worker arises, and thus wages are not competitive. As is standard in the literature, we choose a Nash bargaining solution for wages. With outside options as in the above equations, the wage is such that at every instant a fraction β of the total surplus $S(a)$ of a type a match goes to the worker and a fraction $1-\beta$ goes to the firm:

$$S(a) \equiv J(a)+W(a)-V(a)-U \tag{9}$$

$$W(a) = U+\beta S(a) \text{ and } J(a)=V(a)+(1-\beta)S(a). \tag{10}$$

The explicit solution for the wage is discussed in Section 4.2. Finally, we require that $V(0)=I$ so that there is no profitable entry by firms with new capital in equilibrium.

[10] We omitted this event from the description of the competitive equilibrium because, without frictions, it is immaterial to the firm whether the match dissolves exogenously or not as the worker can be replaced instantaneously at no cost.

[11] In Appendix A.2 we describe a typical derivation of the differential equations above.

3.1 Solving the matching model

We characterize the equilibrium of the matching model in terms of two variables: the rate at which vacant firms meet workers and the exit age: $(\bar{a}, \lambda_f)$. The two variables are jointly determined by two key conditions. The first condition, labelled the job destruction condition, expresses the indifference between carrying on and separating for a match with capital of age $\bar{a}$. The second condition, labelled the job creation condition, expresses the indifference for outside firms between creating a vacancy with the newest vintage and not entering. In the next main section, Section 3.2, we then demonstrate that a solution to these two equations exists and is unique.

In Section 3.1.1 we first derive closed-form solutions of the system of equations (5)–(10) that define the value/surplus functions. The specific solution for the surplus function depends on the pair $(\bar{a}, \lambda_f)$ and the unemployment value U. In Section 3.1.2 we apply the results of Section 3.1.1 to the optimal separation decision and derive the job destruction condition. The optimal separation decision does depend on the pair $(\bar{a}, \lambda_f)$ and the rates $\lambda_w(a)$ at which unemployed workers are matched with firms. In Section 3.1.3 we apply the results of Section 3.1.1 to the free entry requirement and derive the job creation condition which depends only on the pair $(\bar{a}, \lambda_f)$. In Section 3.1.4 we derive the rates $\lambda_w(a)$ at which unemployed workers are matched with firms in terms of the pair $(\bar{a}, \lambda_f)$.

3.1.1 The surplus function

In this class of models all decisions are surplus-maximizing. Thus, it is useful to start by stating the (flow version of the) surplus equation. Using (9) this equation can be described by

$$(r-\gamma)S(a) = \max\{e^{-\gamma a} - \delta S(a) - \lambda_f(1-\beta)S(a) - (r-\gamma)U + S'(a), 0\}. \qquad (11)$$

This asset-pricing-like equation is obtained by combining equations (5)-(10): the return on surplus on the left-hand side equals the flow gain on the right-hand side, where the flow gain is the maximum of zero and the flow difference between total inside minus total outside values. The inside value flows include (i) a production flow $e^{-\gamma a}$, (ii) a flow loss due to the probability of a separation of the match $\delta S(a)$, and (iii) changes in the value for the matched parties, $J'(a) + W'(a)$. The outside option flows are (i) the flow gain from the chance that a vacant firm matches $\lambda_f(1-\beta)S(a)$, (ii) the change in the value for the vacant firm $V'(a)$, and (iii) the flow value of unemployment $(r-\gamma)U$.

The solution of the first-order linear differential equation (11) is the function

$$S(a) = \int_a^{\overline{a}} e^{-(r+\delta+(1-\beta)\lambda_f)(\tilde{a}-a)} \left[e^{-\gamma a} - e^{\gamma(\tilde{a}-a)}(r-\gamma)U\right] d\tilde{a}, \tag{12}$$

where we have used the boundary condition associated with the fact that the surplus-maximizing decision is to keep the match alive until an age $\bar{a}$ such that $S(\bar{a}) = 0$. For lower a's the match will have strictly positive surplus, and for values of a above $\bar{a}$ the surplus will be equal to zero. Straightforward integration of the right-hand side in (12) and further differentiation shows that, over the range $[0, \bar{a})$, the function $S(a)$ is strictly decreasing and convex; moreover, $S(a)$ will approach 0 in such a manner that $S'(\bar{a})$ is defined and equals zero. Intuitively, the surplus is decreasing in age a for two reasons: first, the time-horizon over which the flow surplus accrues to the pair shortens with a; second, the outside option of the worker rises over time at rate γ – the pace of productivity growth of the new vacant jobs – while output is fixed.

Equation (12) contains a non-standard term due to the vacancy heterogeneity: the nonzero firm's outside option of remaining vacant with its machine reduces the surplus by increasing the "effective" discount factor through the term $(1-\beta)\,\lambda_f$. Everything else being equal, the quasi-rents in the match are decreasing as the bargaining power of the firm or its meeting rate is increasing.

3.1.2 The separation decision

The optimal separation rule $S(\bar{a}) = 0$ together with equation (12) implies that the exit age $\bar{a}$ satisfies

$$e^{-\gamma\bar{a}} = (r-\gamma)U, \tag{13}$$

for a given value of unemployment U. The idea is simple: firms with old enough capital shut down because workers are too expensive, since the average productivity of vacancies and, therefore, the workers' outside option of searching, is growing at the rate of the leading edge technology. Note that this equation resembles the profit-maximization condition in the frictionless economy, with the worker's flow outside option, $(r-\gamma)U$, playing the role of the competitive wage rate.[12]

[12]In fact, later we show that the lowest wage paid in the economy (on machines of age $\overline{a}$) exactly equals the flow value of unemployment.

We can now rewrite the surplus function (12) in terms of the two endogenous variables $(\bar{a}, \lambda_f)$ only, by substituting for $(r-\gamma)U$ from (13):

$$S(a;\bar{a},\lambda_f) = \int_a^{\bar{a}} e^{-(r+\delta+(1-\beta)\lambda_f)(\tilde{a}-a)} \left[e^{-\gamma a} - e^{\gamma(\tilde{a}-a-\bar{a})}\right] d\tilde{a}. \tag{14}$$

In this equation, and occasionally below, we use a notation of values (the surplus in this case) that shows an explicit dependence of $\bar{a}$ and λ_f. From (14) it is immediately clear that $S(a;\bar{a},\lambda_f)$ is strictly increasing in $\bar{a}$ and decreasing in λ_f. A longer life-span of capital $\bar{a}$ increases the surplus at each age for two reasons. First, it increases the surplus flow because it lowers the flow value of the worker's outside option, $(r-\gamma)\,U = e^{-\gamma\bar{a}}$. Second, it increases the duration for which a match receives a positive surplus flow. A higher rate at which firms meet workers, λ_f, reduces the surplus because it increases the outside option for a firm: a vacant firm meets workers at a higher rate.

The optimal separation (or *job destruction*) condition (13) requires that the lowest output in operation be equal to the flow value of unemployment. Using (7) and (10) we obtain

$$e^{-\gamma\bar{a}} = b + \beta \int_0^{\bar{a}} \lambda_w(a;\bar{a},\lambda_f) S(a;\bar{a},\lambda_f) da, \tag{JD}$$

which is an equation in the two unknowns $(\bar{a}, \lambda_f)$ and the rates $\lambda_w(a)$ at which unemployed workers are matched with firms. In Section (3.1.4) below, we explain how the two endogenous variables determine the workers' meeting rates.

3.1.3 The free-entry condition

We define the value of a vacancy of age a using the new expression (14) for the surplus of a match $S(a;\bar{a},\lambda_f)$ together with (10). The differential equation for a vacant firm (5) then implies that the net-present-value of a vacant firm equals

$$\lambda_f(1-\beta) \int_a^{\hat{a}} e^{-(r-\gamma)(\tilde{a}-a)} S\left(\tilde{a};\bar{a},\lambda_f\right) d\tilde{a}, \tag{15}$$

where $\hat{a}$ equals the age at which the vacant firm exits. Since vacant firms do not incur in any direct search cost, they will exit the market at an age such that this expression equals 0, from which it follows immediately that $\hat{a} = \bar{a}$. Since in equilibrium there are no profits from entry, we must have that $V(0;\bar{a},\lambda_f) = I$, and we thus have the free-entry (or *job creation*) condition, which becomes

$$I = \lambda_f(1-\beta) \int_0^{\bar{a}} e^{-(r-\gamma)a} S(a;\bar{a},\lambda_f) da. \tag{JC}$$

This condition requires that the cost of creating a new job I equals the value of a vacant firm at age zero, which is the expected present value of the profits it will generate —a share $(1-\beta)$ of the discounted future surpluses produced by a match occurring at the instantaneous rate λ_f. The job creation condition is the second equation in the two unknowns $(\bar{a}, \lambda_f)$.

3.1.4 The stationary distributions and measures

We now complete the characterization of the equilibrium and derive explicit expressions for the matching probabilities in terms of the endogenous variables $(\bar{a}, \lambda_f)$. The probabilities $\lambda_w(a)$ depend on the steady-state distributions of vacant firms. The inflow of new firms is $\nu(0)$: new firms acquire the new capital and proceed to the vacancy pool. Thereafter, these firms transit stochastically back and forth between vacancy and match, and they exit at $a = \bar{a}$, whether vacant or matched (after matched, a firm can always become vacant at age $a < \bar{a}$ at rate δ). This means that $\nu(a) + \mu(a) = \nu(0)$ for all $a \in [0, \bar{a})$. The functions $\nu(a)$ and $\mu(a)$ jump down to 0 discontinuously at $\bar{a}$. For $a \in [0, \bar{a})$, the evolution of $\mu(a)$ therefore follows

$$\dot{\mu}(a) = -\delta\mu(a) + \lambda_f \nu(a) = \lambda_f \nu(0) - (\delta + \lambda_f)\mu(a). \tag{16}$$

Exogenous separations $\delta\mu(a)$ reduce employment, and vacancies being filled $\lambda_f \nu(a)$ increases employment.[13] It is easy to demonstrate that

$$\frac{\mu(a)}{\mu} = \frac{1 - e^{-(\delta+\lambda_f)a}}{\bar{a} + \frac{1}{\delta+\lambda_f}(1 - e^{-(\delta+\lambda_f)\bar{a}})}, \text{ and} \tag{17}$$

$$\frac{\nu(a)}{v} = \frac{\delta + \lambda_f e^{-(\delta+\lambda_f)a}}{\bar{a}\delta + \frac{\lambda_f}{\delta+\lambda_f}(1 - e^{-(\delta+\lambda_f)\bar{a}})}, \tag{18}$$

where μ is the total mass of employed workers. The employment (vacancy) density is therefore increasing and concave (decreasing and convex) in age a. The reason for this is that for every age $a \in [0, \bar{a})$ there is a constant number of machines, and older machines have a larger cumulative probability of having been matched in the past. This feature distinguishes our model from standard-search vintage models where the distribution of vacant jobs is degenerate at zero and the employment density is decreasing in age a at a rate equal to the exogenous destruction rate δ.

With the vacancy distribution in hand, we now have the explicit expression for the value

[13] In Appendix A.2 we describe in detail how to derive (16), (17), and (18).

of $\lambda_w(a)$,

$$\lambda_w(a;\bar{a},\lambda_f) = m(\theta,1)\frac{\delta+\lambda_f e^{-(\delta+\lambda_f)a}}{\bar{a}\delta+\frac{\lambda_f}{\delta+\lambda_f}(1-e^{-(\delta+\lambda_f)\bar{a}})}, \tag{19}$$

which depends only on the pair of endogenous variables $(\bar{a},\lambda_f)$, given the relation between θ and λ_f.

3.2 Analysis of the equilibrium

We now proceed to show that there exists a unique steady state for the economy with frictions. We characterize the equilibrium in terms of the rate at which firms find workers, λ_f, and the exit age, $\bar{a}$. These two variables are jointly determined by the job creation condition (JC) and the job destruction condition (JD). We begin by studying each of the two steady-state equations in turn. Next, we turn to the comparative statics of changes in the unemployment benefits b, the growth rate γ, the interest rate r, and the efficiency of the matching process (a parameter of the matching function). The formal proofs of our arguments are contained in the Appendix.

3.2.1 The job creation condition (JC)

The job creation condition states that a potential entrant makes zero profits from setting up a new machine. We have

Lemma 1. *The job creation condition (JC) describes a curve that is negatively sloped in $(\bar{a},\lambda_f)$ space.*

Lemma 1 follows from the fact that the vacancy value of new firms is increasing in the exit age $\bar{a}$ and in the rate at which firms find workers λ_f. Keeping λ_f constant, a longer life-span of capital $\bar{a}$ increases the vacancy value of a new machine for two reasons: first, it raises the surplus in every match as explained above, and second, it prolongs the period over which the new firm can recoup the initial investment. Keeping $\bar{a}$ constant, a higher rate at which firms find workers λ_f also increases the vacancy value of a new machine. The reason is simply that, almost by definition, a match becomes more likely with a higher λ_f. Even though the surplus of a match declines in λ_f as discussed above, it is straightforward to prove that this indirect effect is always dominated by the direct effect. The job creation condition thus defines a curve in $(\bar{a},\lambda_f)$ space that has a *negative* slope: if the life-length of a machine goes up, the probability of finding a worker has to go down so that the value of entry remains at I. This condition is plotted in Figure 1.

Lemma 2. *As $\lambda_f \to \infty$, the (JC) curve asymptotes to the exit age of the frictionless economy, $\bar{a}^{CE}$.*

Suppose firms live for a very short period: $\bar{a}$ is very close to zero. Even if vacant firms meet workers for sure (with an arbitrarily high rate λ_f), the life-length of capital is too short for the initial investment I to pay off. That is, a minimum life-length is necessary to ensure that the free-entry condition can be satisfied with equality. The asymptote can be worked out to lie exactly at the destruction age for the competitive solution $\bar{a}^{CE}$. Intuitively, as $\lambda_f \to \infty$, the matching frictions disappear for vacancies and the firms' entry problem becomes the competitive problem (2) with solution $\bar{a}^{CE}$.

Lemma 3. *As $\bar{a} \to \infty$, the (JC) curve asymptotes to a strictly positive value*

$$\lambda_f^{\min} \equiv \frac{(r+\delta)\, rI}{(1-\beta)\,(1-rI)}. \tag{20}$$

Suppose that λ_f is very close to zero. Even if the life-length of capital is infinite, vacant firms meet workers with a probability that is too low for the initial investment to pay off in expected terms. The asymptote value $\lambda_f^{\min}$ is increasing in I and in the effective discount rate $r+\delta$, as they both make it more difficult to recover the initial investment, and it is decreasing in $1-\beta$, the surplus share accruing to the firm. Notice that if $rI > 1$ (recall that the condition for existence of the frictionless equilibrium is $rI < 1$), this asymptote would be negative.

3.2.2 The job destruction condition (JD)

The job destruction condition states that the productivity of the marginal match at the cutoff age $\bar{a}$ equals the flow value of the outside option for the worker.

Lemma 4. *If the matching function is Cobb-Douglas, $m(v,u) \equiv Av^{\alpha}u^{1-\alpha}$, with $\alpha > 1/2$, then the job destruction condition (JD) describes a curve that is positively sloped in $(\bar{a}, \lambda_f)$ space.*

The characterization of the job destruction condition (JD) turns out to be a bit more involved. After we multiply the (JD) equation with $e^{\gamma\bar{a}}$, we can show that the right-hand side of the equation is increasing in $\bar{a}$ and decreasing in λ_f. Note first that the capital value

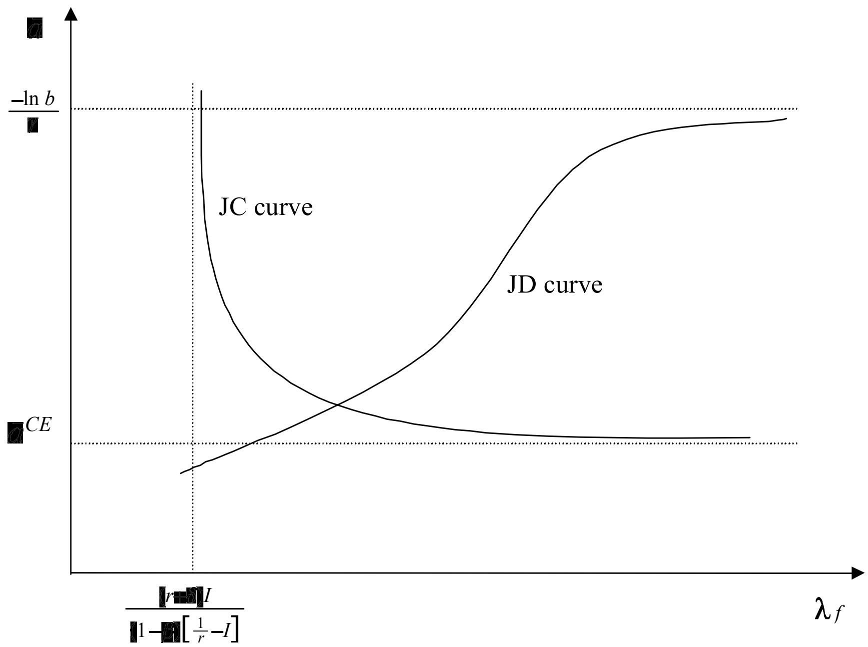

Figure 1: Job Creation and Job Destruction conditions, plotted in the $(\lambda_f, \bar{a})$-space.

of being unemployed depends on the expected surplus from a match, and we know that the surplus function decreases in λ_f, as explained earlier. Also, a higher λ_f decreases the unconditional meeting probability for workers λ_w by definition. But there is also a counter-acting effect that is unique to our model with a vacancy distribution: a faster meeting rate for vacant firms shifts the vacancy density towards younger vintages with larger potential surplus. We show that given the assumed Cobb-Douglas matching technology, the value of search is decreasing in λ_f because the decline of the unconditional probability becomes steep enough to overcome the counteracting shift in the vacancy distribution. Intuitively, one can write $\lambda_w \simeq (1/\lambda_f)^{\frac{\alpha}{1-\alpha}}$ so the larger is α, the steeper the decline in λ_w for a given rise in λ_f. For the cut-off age $\bar{a}$, a similar argument applies. First, the surplus is increasing with $\bar{a}$. However, the probability of meeting any given vintage—which the surplus function is weighted by—decreases as $\bar{a}$ goes up; in particular, it becomes relatively more probable to meet older vintages, and older vintages have lower surplus than younger ones. The latter effect is unambiguously dominated by the former effect with the assumed aggregate matching function. We conclude that the (JD) curve has a positive slope in $(\bar{a}, \lambda_f)$ space (see Figure 1).

Lemma 5. *As $\lambda_f \to \infty$, the (JD) curve asymptotes to* $\bar{a}^{\max} = -\ln(b)/\gamma > 0$.

This result tells us that when the meeting frictions disappear, the surplus goes to zero and output on the marginal job equals the wage, which, in turn, would equal the marginal value of leisure, given by the welfare benefit b. For the labor market to be viable, we need to impose the restriction $b < 1$, where "1" represents the normalized output on the best firm; otherwise no worker would accept any job.

3.2.3 Existence and uniqueness

Based on our characterization of the (JC) and (JD) curves we can now state a set of conditions that imply the existence and uniqueness of the steady-state equilibrium.

Proposition 1 *An equilibrium with finite values of the pair* $(\bar{a}, \lambda_f)$ *exists if and only if* $rI < 1$ *and* $\bar{a}^{\max} > \bar{a}^{CE}$. *If the matching function is Cobb-Douglas with* $\alpha > 1/2$*, then the equilibrium is unique.*

Proof. We first prove the necessity of each condition. If $rI > 1$, then no job is created and the job creation condition is not well defined. As $rI \to 1$, $\lambda_f^{\min} \to \infty$ and the (JC) and (JD) curves do not intersect for a finite value of λ_f. If $\bar{a}^{\max} \leq \bar{a}^{CE}$, then the (JC) curve lies strictly above the (JD) curve, and there is no intersection. Hence, if any of the two conditions of the Lemma is violated, no equilibrium will exist. To prove sufficiency, it is enough to consider that if $rI < 1$, then $\lambda_f^{\min}$ and $\bar{a}^{CE}$ are positive and finite, and if $\bar{a}^{\max} > \bar{a}^{CE}$, then the two curves intersect at least once in the positive orthant and an equilibrium $\left(\bar{a}^*, \lambda_f^*\right)$ exists. Furthermore, if the matching function is Cobb-Douglas with $\alpha > 1/2$, then the (JD) curve is monotonically increasing, and since the (JC) curve is monotonically decreasing, the intersection of the two curves (and the equilibrium) is unique. ■

4 Comparative statics: qualitative results

We now study how technological change and labor market institutions interact in the determination of the equilibrium income distribution and unemployment. In particular, we are interested in the role of the rate of embodied technological change γ, and the payments to workers when unemployed b. The parameter b represents the generosity of the welfare system and simultaneously captures the degree of downward wage rigidity, given the fact that wages in Nash bargaining have the workers' outside option as a lower bound. We also study the effects of changes in the interest rate r and in the efficiency of matching A. First, we analyze the effect of changes in the above mentioned parameters on the equilibrium pair $(\bar{a}^*, \lambda_f^*)$, using the job creation and the job destruction curves. We then study the implied changes for unemployment, wage inequality, and the labor share.

4.1 Comparative statics in $(\bar{a}, \lambda_f)$ space

Lemma 6. *A rise in b does not shift the (JC) curve but shifts the (JD) curve downward, inducing a fall in $\bar{a}^*$ and a rise in λ_f^*.*

The comparative statics of a rise in b are simple: the (JC) curve is unaffected by the worker's payoff determinants, and therefore by the unemployment benefit. A higher benefit will increase workers' outside options, so in order to restore the (JD) condition, output on the marginal job has to increase. Hence, for a given value of λ_f, the exit age $\bar{a}$ must fall, which induces a downward shift of the job-destruction curve. Workers become more expensive for firms without becoming more productive, and therefore machines are scrapped earlier. The upper panel of Figure 2 shows that the equilibrium moves along the (JC) curve and that both the life length of firms and labor market tightness thus fall unambiguously. In particular, the general equilibrium feedback weakens the fall in the life-length of capital, but transfers part of the impact of the shock on a reduction in firms' entry.[14]

Lemma 7. *A rise in γ shifts the (JC) curve and the (JD) curve downward, inducing a fall in $\bar{a}^*$. The change in λ_f^* is ambiguous.*

The comparative statics for γ are somewhat more complicated because an increase in γ has two counteracting effects on the surplus function (14). First, a higher γ means that a vintage's output relative to the frontier falls at a faster rate with age. This obsolescence effect decreases the surplus of a match. On the other hand, a higher γ reduces the relative output of the marginal technology of age $\bar{a}$ and thereby shrinks the outside option value of a worker. This worker's outside option effect increases the surplus of a match. The older a match is the stronger will be the obsolescence effect and the shorter the time period for which

[14]Upon impact, the higher b leads to a higher wage, lower profits, and shorter job duration; the reduction in firms' profits, in turn, decreases their incentive to enter the labor market with new machines (λ_f increases). The implied fall in the meeting rate for workers tends to reduce their outside option and hence their hiring costs and increase profits, therefore making a smaller fall in $\bar{a}$ necessary for the adjustment to the new equilibrium.

it will benefit from the worker's outside option effect. We show that there is a critical age such that for vintages younger (older) than this critical age the surplus rises (falls) with γ. In particular, at age zero there is no obsolescence effect, so the surplus of a new machine grows unambiguously with γ; this last remark will be important later, once we compare our model with the standard Aghion-Howitt/Mortensen-Pissarides framework where all vacancies are of age zero.

Notwithstanding this non-monotonicity of the surplus function, we can prove that the shifts of the (JC) and (JD) curves are unambiguous. A rise in γ increases the value of new vacancies and thus shifts the (JC) curve downward: for a given scrapping age, a lower meeting rate for vacant firms is necessary to bring the value of vacancies back in line with the constant set-up cost I. If we turn to the (JD) curve, a rise in γ reduces output on the marginal job, but also increases the value of search for an unemployed worker, as waiting is compensated by the expectation of being matched to a more productive firm. Both effects lead to the conclusion that for the (JD) condition to hold for a given market tightness, the marginal machine has to be scrapped earlier so the curve will shift downward. Taking these two shifts together, we see that the life length of firms declines unambiguously with a higher rate of technological change. Whether labor market tightness goes up or down depends on the relative slopes of the two curves and the initial position of the curves, that is, on the "starting values" for the parameters, including the initial growth rate (see the lower and central panels of Figure 2). This is an important factor for understanding the complementarity of growth and institutions, as explained below.

Lemma 8. *A rise in r shifts the (JC) and (JD) curves upward, inducing an increase in $\bar{a}^*$. The effect on λ_f^* is ambiguous.*

An increase in the interest rate lowers the weight on future profits and thus lowers surplus. This surplus reduction makes the job destruction curve shift up: for a given λ_f, the worker who is indifferent between staying on the job and leaving now needs a longer life on the job to counteract the fall in the surplus of the job. Similarly, the job creation curve shifts up: a lower surplus must be counteracted by a longer life in order for the firm to remain indifferent between entering and not entering. As a result, higher interest rates induce a rise in $\bar{a}$, whereas the impact on λ_f cannot be signed.

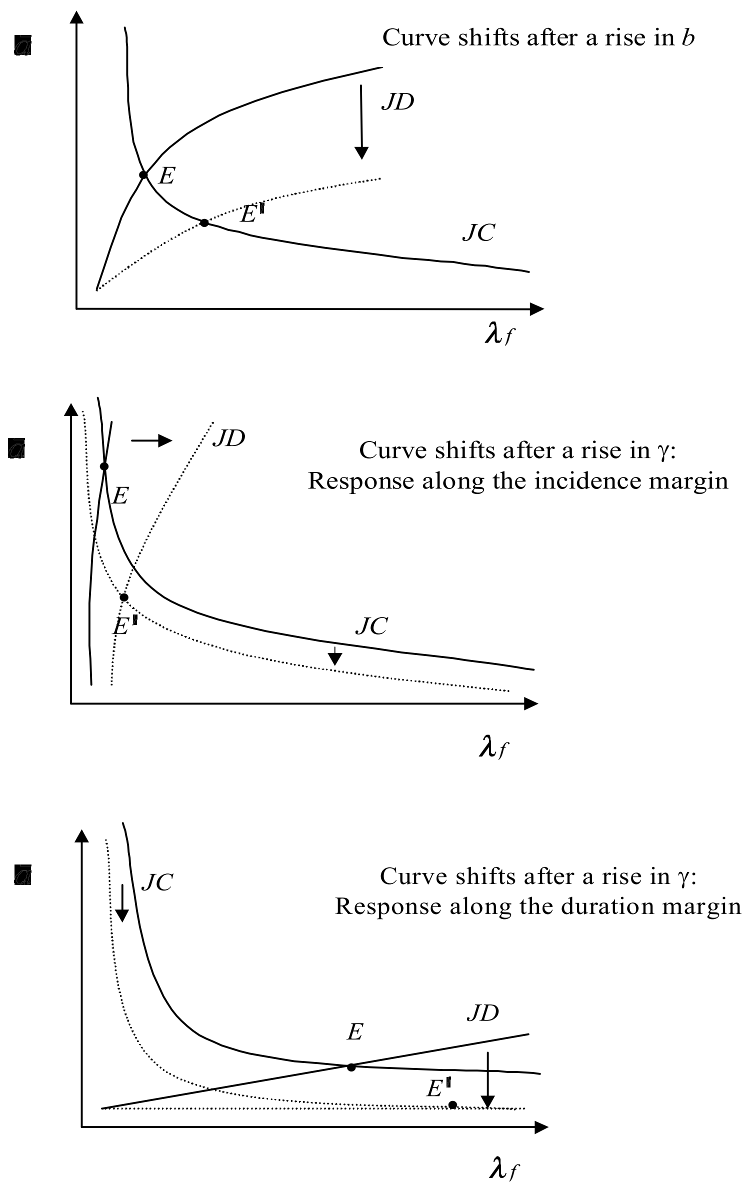

Figure 2: Qualitative comparative statics with respect to b (top graph) and γ (middle and bottom graph). The label E refers to the initial (pre-shock) equilibrium and the label E' to the final equilibrium. The dotted lines represent the JC and the JD curves after changes in b and γ.

The severity of the matching friction can be regulated with the level of the shift parameter of the matching function (A in the Cobb-Douglas formulation).

Lemma 9. *In the limit, as $A \to \infty$, the frictions disappear, and the equilibrium with frictions converges to the competitive equilibrium.*

As $A \to \infty$, the matching friction vanishes and the equilibrium of the economy entails $\overline{a} \to \overline{a}^{CE}$ and $\lambda_f \to \infty$. Recall that, when $\lambda_f \to \infty$ in the standard matching model without vacancy heterogeneity, $\overline{a} \to 0$: the vintage structure vanishes without frictions. In our model, the vintage capital structure survives in limiting frictionless equilibrium. Frictions extend the life of capital, but are not necessary for old machines to be operated by workers in equilibrium.

4.2 Unemployment, inequality, and labor share as functions of $(\bar{a}, \lambda_f)$

Having characterized the changes in the equilibrium pair $\left(\bar{a}^*, \lambda_f^*\right)$ for a given parameter change, we can study how the change in $\left(\bar{a}^*, \lambda_f^*\right)$ together with the underlying parameter change determines the labor market outcomes in which we are interested, namely unemployment, wage inequality, and the wage-income share.

In steady state, the flow into unemployment equals the flow out of unemployment. That is,

$$\delta\mu + \mu(\bar{a}) = \lambda_w u = m(\theta, 1)u. \tag{21}$$

To understand how unemployment responds to changes in the pair $(\bar{a}, \lambda_f)$, it is convenient to restate (21) as

$$\frac{u}{1-u} = \frac{\delta + \mu(\bar{a})/\mu}{m(\theta, 1)}, \tag{22}$$

which is simply the product of unemployment incidence and duration. The degree of endogenous job destruction $\mu(\overline{a})/\mu$, that is, the fraction of matched jobs destroyed at $\bar{a}$, can be read in (17). A rise in $\bar{a}$ reduces the unemployment rate, since endogenous job destruction is reduced. A rise in λ_f has two counteracting effects. First, a higher λ_f reduces the meeting probability for unemployed workers, which in turn increases unemployment duration. Second, a higher λ_f reduces endogenous job destruction, which in turn reduces unemployment incidence. As λ_f increases, vacant firms meet workers at a faster rate, so the employment distribution shifts towards younger machines, and there are relatively fewer machines at the exit age. We can show that for the Cobb-Douglas matching function with $\alpha > 1/2$, the first effect dominates and unemployment increases with λ_f.[15]

Wage payments support the surplus-sharing allocation in the economy with frictions. Embodied technical change therefore generates wage inequality since it implies productivity differences across vintages. Using the surplus-based definition (10) of the value of an employed worker $W(a)$ in equation (8) and rearranging terms, we obtain the wage rate as

$$w(a) = (r-\gamma)U + \beta\left[(r-\gamma+\delta)S(a) - S'(a)\right].$$

Using the differential equation for the surplus (11), we obtain the wage equation

$$w(a) = (r-\gamma)U + \beta\left[e^{-\gamma a} - (r-\gamma)U - \lambda_f(1-\beta)S(a)\right]. \tag{23}$$

The Nash wage rate exceeds the flow value of unemployment by a fraction β of the quasi-rents. This latter term is composed by the production flow $e^{-\gamma a}$, net of the worker's flow outside option $(r-\gamma)U$ and net of the firm's expected surplus share of being in an alternative match $\lambda_f(1-\beta)S(a)$. The last term is age-specific and it is intrinsically related to the value to older firms of becoming vacant. In standard models, this value is zero for every firm. The wage equation also confirms that at the separation age $\bar{a}$, the firm and the worker are indifferent between continuing the match and separating. Evaluating (23) at $\bar{a}$ together with the destruction condition shows that $w(\bar{a}) = e^{-\gamma\bar{a}}$, that is, the flow profits are zero. It also demonstrates that $w(\bar{a}) = (r-\gamma)U$, that is, the worker is indifferent between working and entering unemployment.

Wage inequality in the economy is determined by two factors: the maximum wage differential $w(0)/w(\bar{a})$ and the employment distribution. We can write the maximum wage differential as

$$\frac{w(0)}{w(\bar{a})} = (1-\beta) + \beta\frac{1-\lambda_f(1-\beta)S(0;\bar{a},\lambda_f)}{e^{-\gamma\bar{a}}}. \tag{24}$$

A longer life-span for the job increases the distance between the highest and lowest productivity in the economy, thus raising wage inequality: the *technological heterogeneity effect.* We explain in the proof of Lemma 9 that as the meeting rate for firms $\lambda_f \to \infty$, the flow outside option for the new firm grows towards the highest possible flow profit $1-e^{-\gamma\bar{a}}$, thus the term multiplied by β in (24) converges to one, implying (as in the competitive economy)

[15] To show this result, multiply both numerator and denominator of (22) by λ_f and differentiate $u/(1-u)$ with respect to λ_f.

perfect wage equality: the *firm's outside option effect*. The intuition is that as λ_f increases, firms meet at a faster rate and their bargaining position improves so much that gradually workers are squeezed against their outside option, which is constant, so wage inequality falls.

The changes in the employment distribution are also crucial for equilibrium inequality because, given that the wage rate (23) has a constant component and a component linked to the productivity of the vintage, younger vintages display larger inequality. As shown before, an increase in λ_f or a reduction in $\bar{a}$ shifts the employment distribution towards younger vintages, with more inequality: the *distributional effects*.

Finally, consider the labor share.[16] A shorter life-length of capital $\bar{a}$ unambiguously increases the wage share through a rise in the equilibrium outside option of the worker $e^{-\gamma\bar{a}}$. Instead, a larger firm's meeting rate λ_f improves the firm's threat point in the bargaining and reduces the wage share of output in each match. The distributional effects following an increase in λ_f or a reduction in $\bar{a}$ shift the employment distribution towards younger vintages, which have smaller labor share of output (recall that on matches of age $\bar{a}$ the labor share is one).

4.3 How inequalities are affected by growth and institutions

What are the qualitative effects on labor market variables of changes in the welfare system and the rate of technological change? For our model we find that a more generous welfare system increases unemployment, tends to reduce wage inequality, and is likely to have no impact on the labor share. We also find that a faster rate of technological change tends to increase unemployment and wage inequality and tends to reduce the labor share.

A more generous welfare system, higher b, decreases $\bar{a}^*$ and increases λ_f^*, which leads to a rise in the unemployment rate: both incidence and duration increase (see the upper panel of Figure 2). Economies with higher b should display less wage inequality. Because technological heterogeneity declines ($\bar{a}$ falls) and the firm's outside option increases (λ_f increases), the maximum wage differential across vintages falls, which tends to reduce inequality across vintages. There is a countervailing effect since the new equilibrium employment distribution gives more weight to younger vintages, which display more wage inequality. Barring strong changes in the employment distribution, however, inequality will fall with b. Welfare benefits

[16] A closed-form expression for the labor share in our model can be obtained, but it does not add much to the intuition we have built in the previous analysis on the wage rate and on inequality, so we omit it.

have conflicting effects on the labor share, as explained above, so we should not expect large differences across economies with different b.

A faster rate of embodied technological change, higher γ, lowers $\bar{a}^*$ and has an ambiguous effect on λ_f^*. The shorter lifetime of firms increases the unemployment rate through a higher unemployment incidence. The ambiguity with respect to the worker-finding rate can be understood by looking at the two extremes depicted in Figure 2. We could be in an economy that responds to the shock with a sharp fall in $\bar{a}$ but no significant change in λ_f, generating a higher unemployment incidence, but little change in unemployment duration (the central panel of Figure 2). Alternatively, we could be in an economy where job separation rates ($\bar{a}$) are barely affected and all the adjustment takes place through a lower entry rate of firms (λ_f), that is, unemployment duration rises with little impact on unemployment incidence (the lower panel of Figure 2).

A faster rate of embodied technological change directly increases wage inequality. This is counteracted by the implied decline of the exit age $\bar{a}$. Furthermore, if the firm's contact rate λ_f rises strongly, it will create an additional tendency towards lower inequality through the firm's outside option effect. Overall, we expect the direct effect to dominate, but wage inequality will tend to increase more in economies that respond through the unemployment incidence margin (λ_f) rather than the unemployment duration margin ($\bar{a}$).

A faster rate of embodied technological change tends to reduce the labor income share. The direct effect of a rise in γ reduces the equilibrium value of unemployment, $e^{-\gamma\bar{a}}$, and the share of output going to labor in each job. The indirect effect through $\bar{a}$ and λ_f depends crucially on the margin of adjustment. In economies that adjust through the unemployment incidence margin, the substantial shortening of job durations tends to counteract the direct effect and increases the labor share. In economies that adjust through the unemployment duration margin, the substantial increase of the firm's contact rate improves the firm's outside option value and reduces the worker's share in production. Finally, as explained, changes in the employment distribution always reinforce the fall in the labor share. Thus, we should expect a more dramatic fall in the labor share in economies responding through the duration margin.

Finally, consider the interaction of labor market institutions and technological change. We argue that in our model labor market institutions can, at least qualitatively, account for a differential response in labor market variables to the same acceleration in embodied technological change. Consider first a low-benefits economy (the United States). An acceleration in the rate of productivity growth of new vintages represents an obsolescence shock that makes installed capital obsolete faster –labor costs grow at a swifter pace over the life of a job with fixed productivity. Firms respond to the obsolescence shock by adopting new technologies more rapidly, and in order to do that firms must shorten the optimal life-length of machines. The U.S. economy reduces the lifetime of machines and adjusts along the unemployment incidence margin discussed above (see the central panel in Figure 2). Now consider the response of a high-benefits economy (Europe). In our previous discussion we argued that higher benefits move the initial equilibrium down the job creation curve towards its flat region (see the bottom panel in Figure 2). With an initial position where the job creation curve is very flat, a rise in the growth rate γ will induce a much larger rise in λ_f. The logic for this result is straightforward. High benefits and high labor costs have already pushed the optimal life-length of capital very close to its technological minimum $\bar{a}^{CE}$, and the life-time of a machine cannot be reduced much further. Since operating firms cannot decrease $\bar{a}$ any more, they need to be compensated in a different way, i.e. through an increase in their contact rate when positions are vacant. The corresponding stronger decrease in the worker's meeting rates induces a larger rise in unemployment duration, a smaller increase in wage inequality, and a larger decline in the labor share of aggregate income. To conclude, the level of the policy determines the location of the pre-shock equilibrium and this, in turn, determines the nature of the adjustment.

5 The quantitative role of technology-policy complementarity

We now go beyond a purely qualitative analysis. We will organize the analysis around the United States–Europe comparison; a by-product of this calibration analysis will be the answer to our first quantitative question: what is the role of the mechanism we study for residual wage inequality and unemployment? The United States–Europe question is: can our simple model account quantitatively for the differential behavior of unemployment, wage inequality, and income shares in the United States and Europe over the past thirty years? In our experiment we calculate the steady-state responses of the model economies to the observed increase of the rate of embodied technological change γ. The model economies differ with respect to the policy measure b, which we interpret as a form of welfare benefit and/or downward wage rigidity. We find that the same increase of the rate of technological change implies a larger increase of the unemployment rate and a smaller increase of wage inequality in economies with high welfare benefits. Quantitatively, the differential unemployment response of high and low welfare payment economies calibrated to continental Europe and the United States is remarkably similar in magnitude to the actual differential response of these economies.

The benchmark model does not match the differential response of the labor income share: although it predicts that the labor income share will decline, the magnitude of the decline is the same for all economies. We consider the potential of another difference in labor market institutions—employment protection—to account for the differential response in the shares of labor income. Employment protection legislation tends to be much stricter in continental Europe than in the United States. In Section 5.3, we study the effect of a simple version of employment protection and we find that stricter employment protection in continental Europe can account for the relatively larger decline of the labor income share in Europe.

5.1 Calibration

The quantitative analysis requires calibration of the model economy. In the calibration, we choose to match U.S. averages for the pre-1970 period since the technological shock we model is likely to have hit the economy around the early-mid 1970s.[17] Moreover, initially we choose to represent Europe as an economy that differs from the United States only in terms of policy. This choice simplifies the interpretation of the results since the different outcomes are entirely attributable to different policies. Later, we relax this assumption. Finally, it is important to point out that in the experiment we treat the data for the late 1960s and mid-1990s as both representing steady states.

Given the choice of a Cobb-Douglas matching function with constant elasticity of matching with respect to unemployment equal to α and scale parameter A (that we normalize to 1), the model has seven parameters: $\{r, \delta, \alpha, \beta, I, b, \gamma_L\}$. We set r to match an annual interest rate of 7%.[18] We set δ in order to match an annual worker's separation rate from

[17]See Hornstein and Krusell (1998) and Acemoglu (2000) for discussions of the timing of the technological acceleration.

[18]This value of r is slightly larger than the one commonly used in the literature, but it is necessary to keep $r > \gamma$ in every experiment so that the infinite discounted sums are all well defined. In Section 5.4 we

employment to unemployment equal to 25%, as reported in CEPR (1995, page 10).[19] We choose α to match an average unemployment duration of approximately 8-9 weeks (as reported by Abrahams and Shimer [2001]), which together with the above separation rates gives us an unemployment rate of 4%, the U.S. value for the early 1970s (as reported in Table 1). The Nash bargaining parameter β is chosen to match a labor share of 0.69, and the cost of setting up a production unit I is chosen to reproduce an average age of capital of about 11.4 years, as reported by the Bureau of Economic Analysis (1994) for the late 1960s.

The speed of embodied technological change γ is matched to the inverse of the rate of decline of quality-adjusted relative price of equipment before and after the mid-1970s. This procedure implies a value of 3.5% per year for the first steady state with a low rate of embodied technical change (γ_L). As documented in Gordon's (1990) influential work on quality-adjusted prices for durable goods, and more recently by Cummins and Violante (2002), in the last two decades the speed of embodied technical change has increased substantially to reach 6.5% in the years 1995-2000.[20] How reasonable is it to assume that the shock is common between the U.S. and Europe? A recent OECD study (Colecchia and Schreyer 2001) measures the decline in relative price for several high-tech equipment items across various countries in Europe from 1980 to 2000. Table 2 shows that in the last decade large European countries experienced an acceleration quantitatively comparable to the United States. Since high-tech goods drove the technological acceleration in the aggregate price index, we can be confident that the aggregate indexes should display similar patterns.[21]

discuss the impact of changes in r.

[19]In the model, the separation rate, i.e., the unconditional probability that a worker separates from a job within the period, is defined as $[\delta + \mu(\bar{a})]/\mu$. Note that it would be incorrect to match this variable to job destruction rates (i.e., job flows rather than worker flows, as we do) since the event occurring at rate δ involves only a separation of workers and machines, but not the destruction of the job.

[20]Other authors, using measurement techniques different from quality-adjusted relative prices, arrived at very similar conclusions on the pace of embodied technical change in the postwar era (see for example Hobjin 2000) for the United States.

[21]Ideally, one would like to compare growth rates in the 1970s as well, but these are not available for European countries. Table 1 shows that in some continental European countries the measured acceleration is even larger than in the United States, but one should keep in mind that the high-tech goods' share of aggregate equipment in these same countries is likely to be smaller than in the United States.

Table 2: Acceleration of capital-embodied technical change

	U.S.	UK	France	Germany	Italy
Computers	7.6	7.3	7.4	4.8	8.7
Communications	4.4	5.1	5.5	2.6	7.4
Software	1.9	2.8	3.0	2.2	5.1

Note: Difference between the rate of decline of quality-adjusted relative price of equipment-capital type in the period 1990-2000 and the period 1980-1990.

Source: Table 4, Colecchia and Schreyer (OECD 2001)

In our experiment we gradually increase the annual growth rate γ from 3.5% to 6.5% and study the response of unemployment, wage inequality, and the labor share for economies with different values of the benefits parameter b. This simple parameter is supposed to summarize a wide variation of benefit policies with respect to unemployment duration, family situation (none of which we model), and country. The OECD Employment Outlook (1996) computes average replacement rates from unemployment benefits in OECD countries from 1961 to 1995 for two earnings levels, three family types, and three durations of unemployment. In the mid-1970s the OECD average replacement rates for the United States were 11%, whereas for many European countries the replacement rates were 40% or higher in the same time period (Chart 2.2, page 29).[22]

The OECD replacement rates for Europe understate the measure of benefits that we use in our model, for many European countries offer long-term social assistance schemes in addition to unemployment benefits. Our parameter should reflect these policies since most of them are not earnings-related and have indefinite duration. Hansen (1998) computes corrected replacement ratios to account for social assistance and finds much larger values for a set of European countries, all between 45% and 72% (Hansen 1998, Graph 3, page 29).[23] In the baseline economy, which we interpret as the U.S. economy before the technological acceleration, we set $b = 0.05$, which implies a ratio of welfare benefits to average wage of roughly 10%. To model European-type economies, we gradually increase b to 0.4, which implies a ratio of welfare benefits to average wage of roughly 70%. Finally, although in the data there is some time-series variation in these replacement rates, we model them as constant through time, that is, the only source of shock is the rate of productivity growth of capital. Bertola, Blau, and Kahn (2001) and Blanchard and Wolfers (2000) find that

[22] The same OECD data source documents that average replacement rates reached peaks of 50% in the Netherlands, 45% in Belgium, 37% in France, 35% in Spain, and 30% in Germany.

[23] These replacement rates are calculated for a 40-year old single male production worker.

time-variation in labor market institutions is small compared to cross-country differences and empirically accounts for a minor fraction of unemployment rate differentials.

The calibrated parameter values are summarized in the following table:

Table 3: Calibration of the Model Economy

Parameter	Value	Moment to match
r	0.017	interest rate
δ	0.0515	separation rate (CEPR 1995)
β	0.50	labor share (Cooley 1995)
α	0.55	unemployment duration (Abrahams and Shimer 2001)
I	14.5	average life of capital (BEA 2001)
b	0.05-0.4	welfare benefits (OECD 1996)
γ_L	0.009-0.016	relative price of equipment (Krusell et al. 2000)

Note: A unit time period represents one quarter.

Finally, we should stress that we have not used any of the parameters to try to match the initial level of wage inequality in the data. The reason is that wage inequality in this model is purely due to vintage capital effects and we are not aware of data counterparts measuring the extent of inequality that can be attributed to this source. One contribution of our work is that we can use the calibrated model as a measurement tool to find out how much wage inequality is generated by this mechanism in a United States-like economy. We return to this point in the next section.

5.2 Results

The main quantitative results of our experiment are reported in Figure 3, where a number of equilibrium outcomes of the model (unemployment rate, unemployment duration, separation rate, maximum age of capital, wage inequality, and labor share) are plotted for different rates of embodied technical change (from 3.5% to 6.5%) and for different levels of the policy variable of interest b (from 0.05 to 0.40).

The contribution of vintage capital and matching frictions to wage inequality.

Let us start with wage inequality, measured by the 90-10 log-wage differential. As explained, the model is designed to generate inequality among ex ante equal workers, which originates from a combination of labor market frictions and vintage capital differentials. Figure 3 shows that in the final steady state of the baseline economy the 90-10 log-wage differential is around 7%. An alternative way to measure inequality induced by this mechanism is to say that a vintage differential of 10 years in the capital used by firms translates into a wage gap of about 6% in our model economy. How reasonable is this number? We are not aware of any exact data counterpart, but Doms, Dunne, and Troske (1998) provide an interesting benchmark of comparison. They examine a sample of U.S. plants in 1988 for which one can observe both the degree of technological advancement of the plant (i.e., the technologies recently adopted) and the characteristics of the workers (like education) and conclude that the wage differential induced only by the technological gap between a plant in the top quartile and a plant in the bottom quartile of the technological scale is 8.4% for production workers and 12.7% for technical and non-production workers (Table III, page 267). These figures seem to suggest that our estimates are of the right size.

How much do vintage capital and matching frictions contribute to overall wage inequality among ex ante identical workers in the U.S. economy? According to Katz and Autor (1999) the 90-10 log-wage differential of "residual" wage inequality, that is inequality not related to observable characteristics, such as age and education, is about 90%. Gottschalk and Moffitt (1994) argue that the fraction of residual inequality accounted for by permanent unobservable characteristics of individuals, that is "innate ability", is roughly 2/3. Thus, the model tells us that in the United States labor market frictions, together with vintage capital, account for almost 25% of wage inequality among ex-ante equal workers. The remaining inequality can be the result of match quality and skill dynamics within and between jobs.

The impact of a faster rate of technological change on labor market inequalities.

The effect of a faster rate of technological change on wage inequality is ambiguous and depends on the magnitude of the policy variable b. Figure 3 shows that wage inequality in the U.S.-type economy rises by 1 point. As explained earlier, this increase in inequality is brought about essentially by an increase in technological heterogeneity through the interplay between a higher γ (a large productivity differential across successive vintages of machines) and a lower $\bar{a}$ (a lower age gap between the youngest and the oldest machine; see Figure 5). In the simulations, the first effect dominates, and the distributional effects are always fairly small. The firm's outside option effect restrains inequality from a sharp surge. In economies with a large b, the increase in λ_f is massive, so this latter effect is very strong and in some extreme cases (e.g., $b = 0.4$), can dominate the larger technological heterogeneity

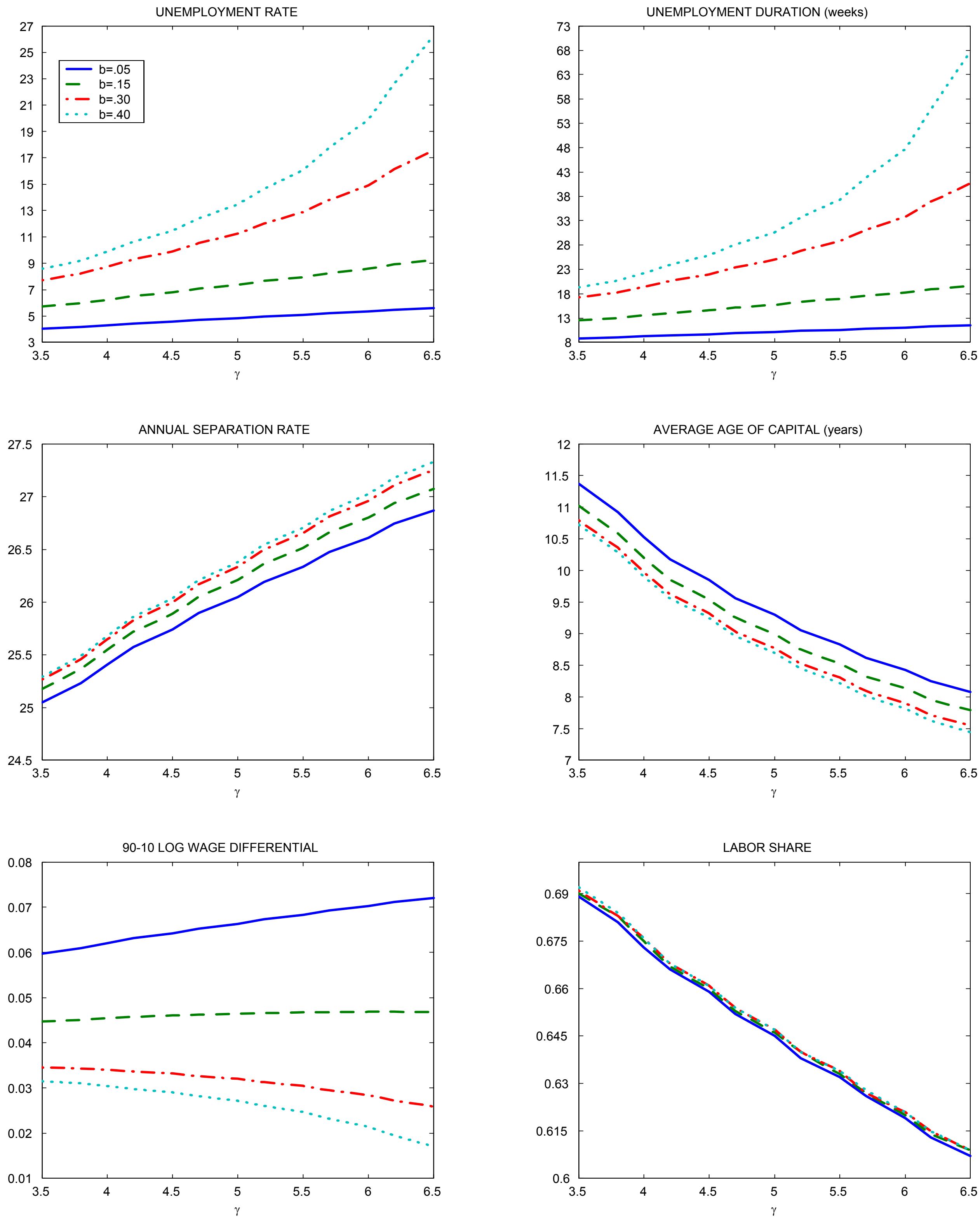

Figure 3: The effects of a rise in the rate of embodied technical change γ in the vintage-capital model when the economies differ only via the level of welfare benefits b.

and lead wage inequality to fall.[24] The model suggests that wage inequality did not increase in continental Europe as a response to the shock because the sharp increase in unemployment duration improved the bargaining position of the firms so as to squeeze workers against their outside option, which is invariant. It is worth remarking that this argument, based on the firm's outside option, is a unique feature of our model with vacancy heterogeneity.

A faster rate of technological change increases unemployment for all values of the welfare benefits, but the increase is much more pronounced for European-type economies with high b. If we take the cautious view that the new steady-state level of capital-embodied productivity growth is 5.5%, then the model predicts that in the baseline economy unemployment moves very little, by less than 1 point, whereas for $b = 0.40$ unemployment jumps by 6 percentage points. It is clear from Figure 3 that although both unemployment duration and the separation rate rise with γ, the bulk of the differential increase in unemployment is explained by the former: the separation rate changes only very marginally, by roughly 1% in all the economies considered, while the increase in unemployment duration is small in the U.S.-type economy (from 8.5 to 10.5 weeks) but is substantial in the economies with high b, from 19 to 38 weeks when $b = 0.4$. This result is consistent with the recent experience of European labor markets, where labor turnover did *not* increase significantly, and where most of the increase in the unemployment rate is associated to longer durations (Machin and Manning 1999). Overall, quantitatively, the differential rise in unemployment rate is of an order of magnitude comparable to the data in Table 1.

The labor income share (last panel of Figure 3) declines as γ increases, independently of the magnitude of the policy parameter b. Note also that the model generates labor shares of a similar size independently of the magnitude of the policy parameter b, whereas in Europe the labor share has been slightly above the US value until 1980. This independence reflects the offsetting effects of b on the labor share discussed in Section 4.3. As expected, the labor share falls with γ: a rise from 3.5% to 5.5% reduces the labor share in every economy by 6%. The magnitude of this decline of the labor share is in line with the data for continental European economies in Table 1 (6 points). On the other hand, the size of the US labor income share decline in the data is substantially smaller than is predicted by the model. In the next section, we argue that the inclusion of another important labor market policy,

[24]Note that this decline in wage inequality is not inconsistent with the European data. Table 1 shows that wage inequality fell on average in Europe from 1980 to 1990, and in some countries such as Germany and Norway kept falling until the mid 1990s.

employment protection legislation, helps the model match the differential decline in labor shares.

One limitation of our experiment is that the model economies initially (i.e., for a low value of γ) display a large unemployment differential since they only differ through the policy b. The data in Table 1 show that the unemployment rates in continental Europe and the United States were quite similar in the early 1970s. Machin and Manning (1999) report that although they had a similar unemployment rate in that period, unemployment duration was already much longer in continental Europe (Machin and Manning 1999, Table 4, page 3100). To account for this observation, we modify the experiment and change the separation rate δ with the policy parameter b to keep the initial unemployment rate constant at 4%. The results are in Figure 4: the rise in unemployment is still magnified by the policy b by an amount which is in line with the data, and the bulk of the rise is once again due to longer durations, with the separation rates changing very little. The changes in wage inequality and labor share remain of the same magnitude.

The model's implications for labor market inequalities are closely related to its predictions for the economic lifespan of capital. In the wake of a technological acceleration the age of capital declines in the model: firms scrap their machines earlier in response to a faster obsolescence rate.The Bureau of Economic Analysis publishes data on the age of capital for the U.S. economy since 1925. In Figure 5 we plot the average age of private fixed assets (BEA 2002, Table 2.10) for our sample period 1965-1995. Average age in the United States falls from 11.4 years in 1965 to 8.6 in 1985 and then it rises again to 9.5 years in 1995. The model's age of capital (Figures 3 and 4) falls to 8.7 as γ approaches 5.5%. Overall, the size of the age decline in the model is quite similar to that in the United States, with the U.S. data showing a fall by 17% and the model by 25%.

5.3 Extension: employment protection legislation

Our analysis of differences in labor market institutions has focused so far on the role of welfare benefits/unemployment insurance. This analysis has successfully accounted for the differential response on unemployment and wage inequality in Europe and the United States to an increase of the rate of technological change. Differences in welfare benefits, however, cannot account for the differential response of the labor income share in these countries. We now assess whether differences in employment protection legislation, in particular a firing

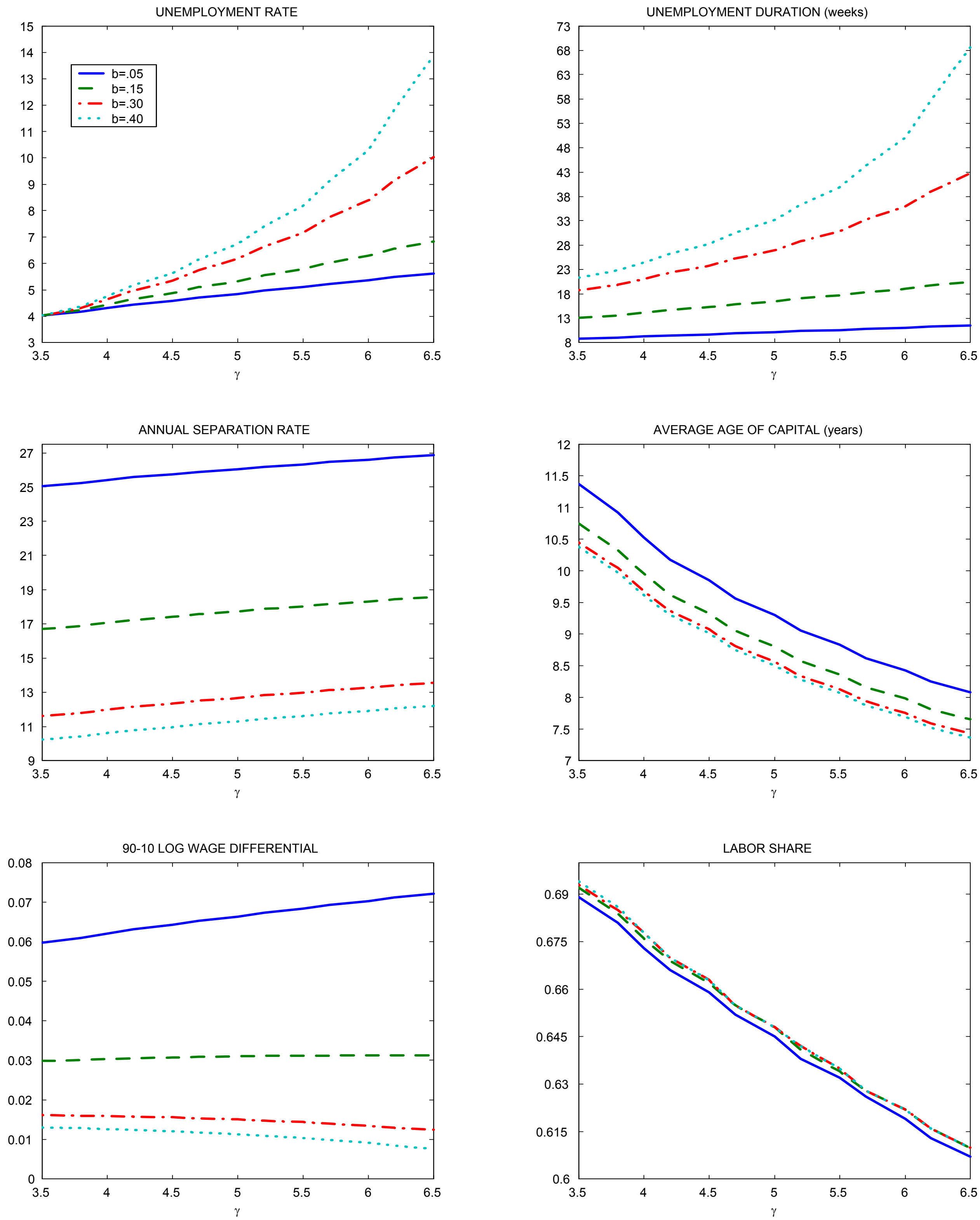

Figure 4: The effects of a rise in the rate of embodied technical change γ in the vintage-capital model when the economies differ both via the level of welfare benefits b and the exogenous separation rate δ.

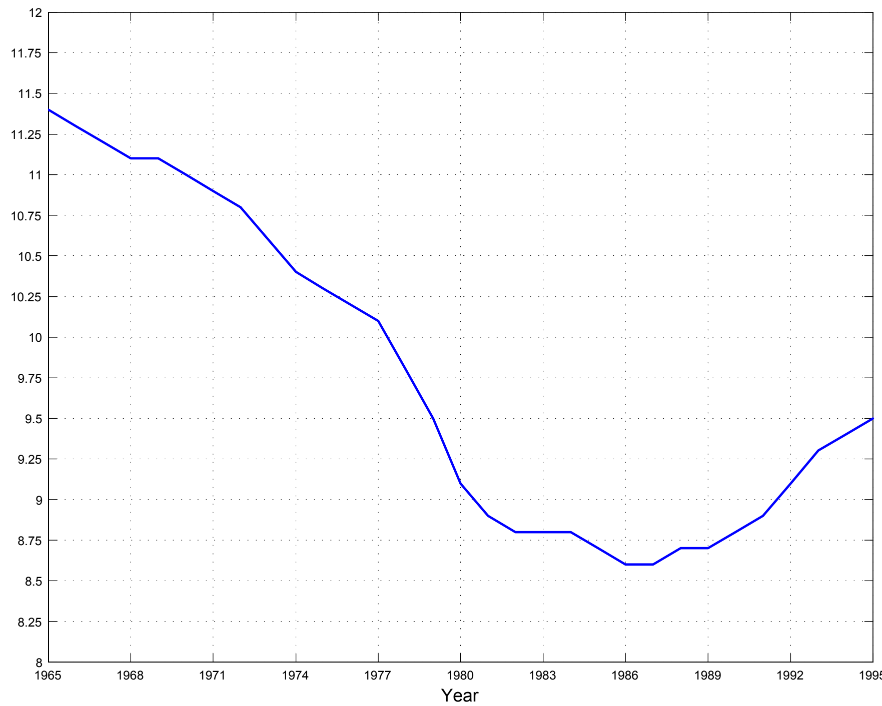

Figure 5: Average age of capital in the U.S. economy (in years), 1965-1995. Source: Bureau of Economic Analysis (2002), Table 2.10.

tax, can account for the differential response of the labor income share.

Introducing a pure firing tax complicates the analysis significantly. The destruction age for a vacancy, $\hat{a}$, would no longer be the same as the destruction age for an existing match, $\bar{a}$. This is because prior to matching, the cost of the dissolution of the match in the future—the firing tax—is not a liability, and it will not become one until the match is formed. An existing match treats this cost as sunk and the match dissolves at a capital age such that the total surplus of the match (which includes a liability T to the government) equals $-T$: at this point the marginal profit flow from production is equal to zero. A vacancy would not be posted at such an age because the total surplus is negative, so vacant capital withdraws from the market at an earlier age than matched capital: $\hat{a} < \bar{a}$. Formally, the presence of a firing tax leads to two different notions of surplus: surplus upon hiring, where the disagreement in the bargaining does not imply the payment of the tax, and surplus during the match, where it does. As a result, we have a two-tier labor market (with two wage functions) and two destruction thresholds. The structure of the model with a simple firing tax is therefore quite different from the one studied above.

Fortunately, there is a simple way to introduce an employment protection policy without affecting the structure of our equilibrium. Consider an employment protection policy that combines a firing tax with a hiring subsidy. In particular, assume that in an existing match the firm pays a firing tax T on separation, and a vacant firm that hires a worker receives a hiring subsidy T.[25] In Appendix A.6 we solve the model and show that the modified destruction rule for a match is

$$e^{-\gamma\bar{a}} + (r-\gamma)T = (r-\gamma)U. \tag{25}$$

The intuition behind this equation is easy to grasp once it is understood that the policy T has the form of a zero-coupon government bond from which the firm is entitled to receive the growth-adjusted return, $r-\gamma$, for the duration of the match. In other words, every period the firm's payoff from the match is augmented by the amount $(r-\gamma)\,T$, which will tend to extend the life-length of capital.

In the Appendix we show that the new wage function is

$$w(a) = (r-\gamma)\,U + \beta\left[e^{-\gamma a} + \lambda_f(1-\beta)S(a) - (r-\gamma)(U-T)\right]. \tag{26}$$

As before, we have $w(\bar{a}) = (r-\gamma)\,U$, and a separation will occur when the marginal operating profit of the firm $e^{-\gamma\bar{a}} + (r-\gamma)T$ is entirely paid into the wage bill. The job creation and job destruction equations become

$$e^{-\gamma\bar{a}} + (r-\gamma)\,T = b + \beta\int_0^{\bar{a}} \lambda_w(a;\bar{a},\lambda_f)S(a;\bar{a},\lambda_f)da, \tag{JD'}$$

$$I = \lambda_f(1-\beta)\int_0^{\bar{a}} e^{-(r-\gamma)a}S(a;\bar{a},\lambda_f)da. \tag{JC'}$$

One can see that the comparative statics of the job creation curve (JC') are unchanged. The comparative statics of the job destruction curve (JD') with respect to γ are also qualitatively unchanged. Now, however, a rise in γ reduces the LHS of (JD') by an amount that increases with the size of T, so the policy amplifies the downward shift of the (JD') curve. Here the policy-technology complementarity is very stark. Equation (26) makes clear that the larger the tax/subsidy T, the more the wage will fall as γ increases: through bargaining, the worker can appropriate a share β of this additional return to the match, and the fall in this quasi-rent due to an increase in the growth rate γ is proportional to T. This mechanism will tend to

[25] This tax/subsidy scheme implies that every period the policy satisfies a balanced budget constraint for the government because total separations equal newly created matches in steady-state.

reduce the labor share more severely in economies with more generous employment protection policies. Finally, as we should expect, a rise in the firing tax (for a given γ) shifts the (JD') curve upward, inducing longer job tenures and more firm entry, which unambiguously lowers unemployment.

We now move to the quantitative analysis. Based on the data on firing costs reported in the OECD Employment Outlook (1999), we choose a conservative range for the tax T running from zero to one year of salary.[26] The results are displayed in Figure 6. In economies with a high firing tax T, the labor share starts at a higher level and falls much faster as γ increases, following the same pattern as in the data: the firing tax can account for a 4.5 percentage point differential decline across economies, which is very close to the number implied by Table 1, 4.3%. Although the model economy is able to generate this differential fall, in absolute terms it overpredicts the decline in the labor share. For the United States, it predicts a fall of 6%, whereas the decline in the labor share in the U.S. data is only 1.5%. Interestingly, the firing tax is much less important than the welfare benefits in the determination of cross-country differences in the evolution of the unemployment rate: independently of the level of T, unemployment duration and separation rates change by very similar amounts in the model.

In conclusion, employment protection does not seem to be responsible for different patterns of unemployment between U.S. and Europe, but it might be important in understanding the different evolutions of the distribution of income between capital and labor.

5.4 Discussion of related results in the literature

Ljungqvist and Sargent (1998) are among the first to study quantitatively the channel of technology-policy complementarity to explain the relative labor market performances of the U.S. and Europe. They model the common shock as a rise in the degree of skill depreciation of unemployed workers and analyze the impact of this shock in a search model where workers receive unemployment benefits linked to their past earnings (and their past skills) and receive wage offers linked to their current skills. More rapid skill depreciation worsens the value of the average wage offer compared to the value of unemployment and increases unemployment duration. This model has a fixed wage distribution and fixed number of jobs; thus, the

[26] In virtually all OECD economies, firing costs are proportional to the wage at separation, so we model $T = (r - \gamma) U \cdot t$. We have verified that this choice has no impact on the results. The other parameters of the model are unchanged with respect to the benchmark calibration. In particular, we set $b = 0.05$.

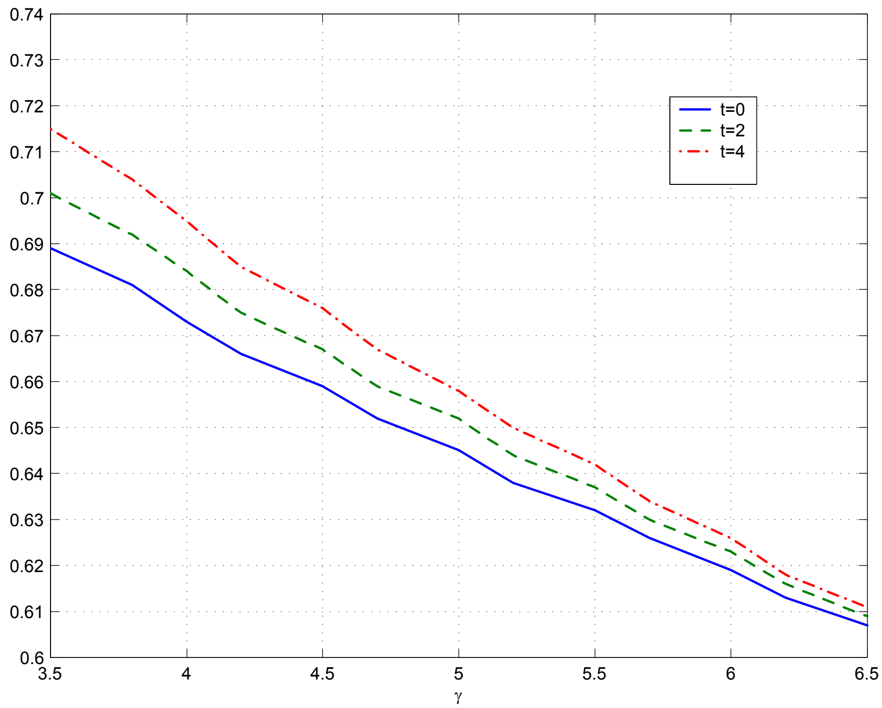

Figure 6: The response of the labor income share to a rise in the rate of embodied technical change γ in the vintage-capital model when economies differ with respect to the firing tax t.

mechanism operates entirely on the labor supply side. In our model, in contrast, workers accept every job offer, but both wages and labor demand (i.e., the number of jobs) are endogenous. In this sense, the two papers highlight the importance of the complementarity between technological shocks and welfare benefits along two parallel margins: labor supply and labor demand.

Blanchard and Wolfers (1999) attribute unemployment differentials across countries and over time in a panel of OECD countries to the interaction between shocks and labor market policies. They consider three types of shocks—a productivity growth slowdown, a rise in the interest rate, and technological change biased against labor—together with several types of policies, including unemployment insurance and employment protection legislation. Our embodied productivity acceleration can be interpreted as the source of technological change biased against labor, measured by Blanchard and Wolfers directly off the fall in the labor share. The authors find significant evidence of interactions between shocks and institutions: in line with our findings, they report that a shock that increases unemployment by 1% in the country with the lowest welfare benefits would have an impact 5 times larger in the country with the most generous welfare payments, whereas this "multiplier" effect for employment protection legislation is only 2.[27] Bentolila and Saint-Paul (1999) also study the evolution of the labor share across OECD countries since 1970. They find that in the presence of institutions that promote wage rigidity, shocks that reduce employment significantly also reduce the labor share of income. One can view our quantitative study as the "structural" counterpart of these empirical analyses.

In a recent paper, den Haan, Haefke, and Ramey (2001) study the quantitative implications of interest-rate and TFP shocks within a calibrated version of the traditional Mortensen and Pissarides (1994) framework. In this class of models, a rise in the real interest rate or a fall in TFP have identical effects: the equilibrium unemployment rate increases through a rise in the "effective discount factor," as demonstrated already in Pissarides (1990). Like us, den Haan et al. also ask if institutions can account for the differential response of labor markets to these shocks. They find that labor market institutions are important only if the United States and Europe differ substantially in their cross-sectional distributions of match-specific productivities, a dimension of the data that the authors do not attempt to calibrate. In our model economies, the effects of an interest rate shock are negligible. Following an interest rate hike, fewer jobs are created and unemployment duration rises, but at the same time the destruction age increases, which reduces the separation rate (see Lemma 8). We find that the net effect on unemployment is small and the policy-shock interactions are much less pronounced than in our experiments. For example, as the real interest rate grows from 1% per year to 6%, the differential rise in the unemployment rate between the economy with the highest benefits and the economy with the lowest benefits is only by 2%.[28]

Our paper relates to the argument advanced for example in Blanchard (1997) and Caballero and Hammour (1998) whereby European unemployment is largely due to expensive labor services and a fall in labor demand associated to firms' adoption of ever more labor-saving technologies. Our model does not allow for any substitutability between capital and labor at the level of the production unit, but some form of substitution takes place at the aggregate level: as capital becomes cheaper and cheaper in efficiency units, the European economy produces with more productive capital per worker and fewer workers.

[27] We reached this conclusion from their Table 1, where shocks are all bundled into a time effect. An ideal comparison with our model would be a measure of this institutional multiplier for their observable "labor demand" shock, but this number is not directly available in the paper.

[28] In this experiment we set $\gamma = 5\%$ per year, the average of the period considered. All the other parameters are unchanged.

Last but not least, our approach has the advantage over virtually all of the existing literature that formalizes the U.S.–Europe comparison of being able to measure the source of the shock independently, through capital-embodied productivity. Thus we do not calibrate the source of the shock to the labor market equilibrium outcomes of interest, such as wage inequality or labor share.[29] As a result, the focus of the analysis is limited to the different unemployment experience of United States and Europe. We maintain the view that unemployment, wage inequality and changes in the labor income share have been produced by the same fundamental shock and should be explained jointly: they are all dimensions along which the model needs to be evaluated rather than calibrated.

5.5 A comparison with the Aghion-Howitt/Mortensen-Pissarides setup

In contrast to other search models with vintage capital (in particular, see Aghion and Howitt 1994, and Mortensen and Pissarides 1999) the pool of vacant firms in our model economies is heterogenous. Vacancies are heterogeneous because of random matching and because the bulk of the cost associated with creating a job is related to the purchase of the capital needed in production. It is precisely this vacancy heterogeneity that contributes to the amplification of the effects of shocks in our economies.

Traditional search-matching frameworks (e.g., Mortensen and Pissarides 1999) assume that a new machine can be created at zero cost and that only posting a vacancy is costly. This assumption implies that the pool of vacancies consists of the newest machines only, and that only machines in existing matches age over time. In our setup we instead assume that once a machine has been acquired at a cost, recruiting costs are "small compared to

[29] In Ljungqvist and Sargent (1998) the skill depreciation shock is calibrated to the increase in U.S. earnings instability. In Mortensen and Pissarides (1999a), the shock is a mean-preserving spread in the skill distribution, calibrated to the increase in U.S. wage inequality. In Blanchard (1997), the main source of the shock is the degree of technological bias in favor of capital (against labor) and is identified through changes in the capital share of income. At the extreme of the spectrum the shock is completely unobservable. In Marimon and Zilibotti (1999) the shock is an increase in the degree of mismatch between workers and jobs, while Caballero and Hammour (1998) model an "appropriability" shock that changes the division of quasi-rents between capital and labor. Because of the intrinsic unobservability of the shock, no attempt is made to calibrate the shock in these two papers.

A recent exception is the paper by den Haan et al. (2001), where several candidates for the shock are considered (the decline in total factor productivity and the rise in the real interest rate), all measured from independent data. However, the authors focus on unemployment and do not examine the dynamics of wage inequality or income shares.

that cost" (zero in our model).[30] This explicit distinction between a "large" purchase/setup cost for the machine – which is sunk when the vacant firm start searching– and a "smaller" recruiting cost fits naturally with a vintage capital growth model, whose emphasis is on capital investment expenditures as a way of improving productivity. Aghion and Howitt (1994) also describe a vintage capital model with large setup costs for capital, but they assume that matching is "deterministic": at the time a new machine is set up, a worker queues up for the machine, and after a fixed amount of time the worker and firm start operations. Hence, in the matching process, all vacant firms are equal (although they do not embody the leading-edge technology).

The equilibrium of the Mortensen-Pissarides (MP) model is characterized by the following modified surplus, job creation, and job destruction equations:

$$\widetilde{S}(a;\bar{a}) = \int_a^{\bar{a}} e^{-(r+\delta)(\tilde{a}-a)}(e^{-\gamma a} - e^{\gamma(\tilde{a}-a-\bar{a})})d\tilde{a}, \tag{27}$$

$$e^{-\gamma\bar{a}} = b + m(\theta)\beta\widetilde{S}(0;\bar{a}), \tag{JC$''$}$$

$$I = \lambda_f\,(1-\beta)\,\widetilde{S}(0;\bar{a}). \tag{JD$''$}$$

The cost I now represents a flow search cost. Notice two differences between our setup and the MP model. First, in the MP model old machines from an existing match are discarded once the match dissolves. This means that there is no longer a non-zero outside option to the firm, and the term $(1-\beta)\,\lambda_f$ is eliminated from the effective discount factor in the surplus function (27). Second, in the MP model workers always face the newest vintage in the pool of vacancies. This means that in the job destruction condition (JD''), only the surplus of the most recent vintage is relevant and not a weighted average of the surpluses across vintages.

The qualitative comparative statics of the MP model with respect to b and γ are the same as in our model, but there are important quantitative differences. Compared to the MP model, the just mentioned differences between (JC, JD) and (JC'', JD'') dampen the positive effect of a rise in γ on the right-hand sides of JC'' and JD'' in our setup for two reasons. First, as discussed in Section 4.1, the equilibrium density of vacancies shifts towards older machines and for sufficiently old matches the surplus declines with the rate of embodied technological change. Second, from (14), it is clear that the larger discount factor weakens the effect of γ on the surplus. In conclusion, a given rise in γ increases the value

[30] At the risk of being redundant, let us restate that vacancy heterogeneity will survive the addition of a flow search cost c, as long as this cost is strictly less than the initial set-up cost I.

of vacant firms and the value of search in the standard model by a larger amount, which implies ampler downward shifts of the two curves and a larger reduction of $\bar{a}$. Thus, the Mortensen-Pissarides economy responds to shocks mainly through the incidence margin and not through the duration margin.

We now turn to wage inequality and the income shares. In the MP model the ratio of the highest to the lowest wage is $w(0)/w(\bar{a}) = (1-\beta) + \beta e^{\gamma \bar{a}}$; thus, changes to wage inequality induced by a rise in γ will only take place through the technological heterogeneity channel. In relation to the labor share, since the Mortensen-Pissarides economy responds to the shock mainly through the incidence margin, that is, through a large reduction in the destruction age $\bar{a}$ rather than a rise in unemployment duration, it will have a smaller change in the labor share, as explained in Section 4.3. Finally, note the absence of any effect due to the firms' outside option both for inequality and for the labor share, contrary to what we found in Section 4.2. These effects, which originate precisely from the vacancy heterogeneity, play an important role in the quantitative analysis above.

Figure 7 shows that the MP model displays a weak interaction between changes in the rate of embodied technological change γ and benefits, for empirically plausible values of b (until $b = 0.6$).[31] Unemployment duration barely responds to a change in the growth rate γ, whereas the life-length of capital is reduced substantially, and in fact the separation rate increases much more than in the baseline economy. The only case where an interaction is evident is for $b = 0.7$, the extreme parametrization. However, the significant rise in unemployment (4 points) is still well below its data counterpart. Moreover, as anticipated this rise takes place along the "wrong" margin: it is unemployment incidence that increases by 8%, while duration goes up by a small amount (only 5 weeks). Wage inequality increases by less than 0.5% in U.S.-type economies and is essentially constant in Europe-type economies. In particular, the absence of the firm's outside option effect prevents inequality from falling in high b economies. Finally, the reduction in the labor share is insignificant, in both low- and high-benefits economies.

[31] We have tried to calibrate the MP model to the same set of moments we selected for our economy in Section 5. In the process we have found that the MP model cannot match the average age of capital in the data. It can generate at most an average age of capital equal to 7.5 years conditionally on matching all the other moments. We let γ vary in the same range, while we change the values of b to generate similar benefits-wage ratio as in the baseline experiment. The value $b = 0.10$ implies a benefits-wage ratio of 15%, $b = 0.4$ of 50%, $b = 0.6$ of 80% and $b = 0.7$ of 90%. The case $b = 0.7$ is not empirically plausible, but it is useful to interpret the results. The other model parameters are set as follows: $r = 0.017, \delta = 0.061, \beta = 0.1, \alpha = 0.55, I = 2.7$, and A is normalized to 1.

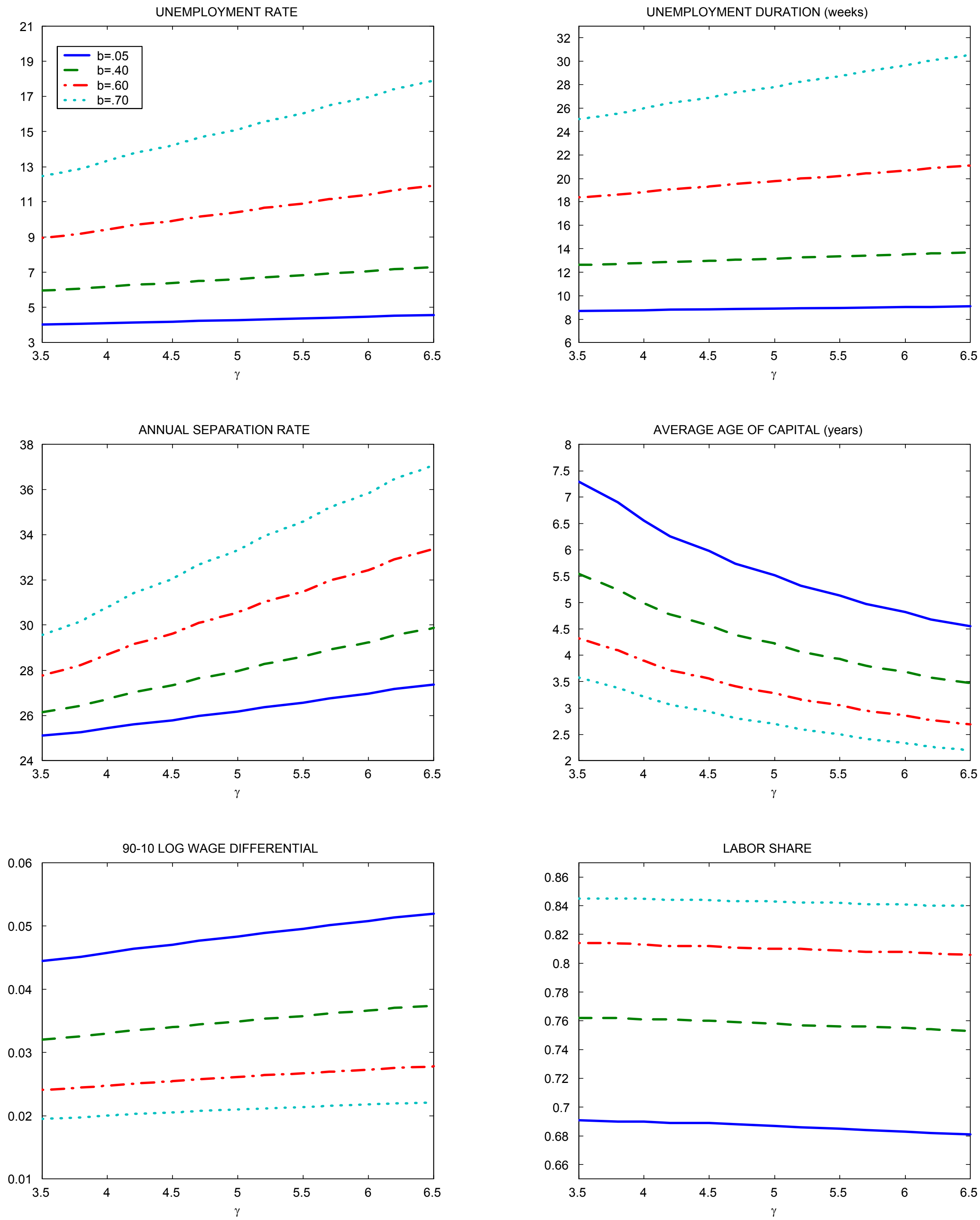

Figure 7: The effects of a rise in the rate of embodied technical change γ in the standard Mortensen-Pissarides model, when the economies differ only via the level of welfare benefits b.

6 Concluding remarks

The past twenty years have been marked by very rapid capital-embodied growth. In this paper, we have made an attempt to understand how the benefits of this type of technological change are shared among labor market players: who are the winners and the losers when the relative productivity of new capital increases? In a generalized version of the standard vintage capital model with labor market frictions, we showed qualitatively that the answer depends crucially on the institutions and policies of the economy considered. With European-style generous welfare benefits, faster technical change reduces the wage inequalities among employed workers. It magnifies the differences between employed and unemployed workers, as unemployment duration rises substantially with little change in job tenures, and it shrinks the share of income going to labor earners, more strongly so where employment protection legislation is particularly strict.

Quantitatively, the model suggests (i) that embodied technical change accounts for a small but significant part of wage inequality, and (ii) that an acceleration of embodied technical change together with different labor market policies can account for the differential labor market outcomes in continental Europe and the United States. Given the absence of studies on the impact of vintage effects on wage differentials, we have not calibrated the model along this dimension, but rather we have used it to obtain an estimate of this elasticity. The calibrated model suggests that embodied technical change accounts for one fourth of residual wage inequality in the United States. The calibrated model is also successful in generating the differential rise in unemployment between the United States and Europe following an acceleration of technological change. When firing costs are embedded, the model can also match the differential fall in the labor share, but it tends to overstate the fall in the U.S. labor share. An acceleration in the rate of embodied technical progress leads to a small increase in wage inequality in the U.S.-like economy and to a small fall in the Europe-like economy with generous welfare benefits.

We conjecture that the two main shortcomings of the quantitative analysis (too low a rise in inequality and too rapid a decline in the labor share in U.S.-type economies) result mainly from the simplifying assumption that workers are ex ante equal. In Hornstein, Krusell, and Violante (2002), we extend our environment to two skill levels. In particular, we investigate whether such a model can generate much larger increases in wage inequality and a stable labor share. Accounting for differences in ex ante labor quality might be important, because in the United States the fall in the wage bill for unskilled workers seems to have been accompanied by a rise in the demand for skilled workers and in their labor share.

Finally, we have compared our generalized version of the model to the standard version with a trivial equilibrium distribution of vacancies to show that vacancy heterogeneity is an important driving force behind the results. In the process, we realized that the standard model predicts no change in labor shares. Although this is a shortcoming of the standard model, recall that our model fails in the opposite direction since it generates too large a fall in the U.S. labor share. A useful lesson that can be learned from the comparison of the two models is that the larger is the initial set-up cost I, the steeper is the decline in the labor share as γ rises. One could perhaps argue that certain policy differences between the U.S. and Europe that affect I (e.g., red tape associated with starting new businesses) can also help explaining the differential evolution of the labor shares.

Appendix

A.1 In the frictionless economy the competitive equilibrium allocation and the Pareto-optimal allocations are the same.

The planner maximizes the discounted value of future consumption, i.e., output net of investment, subject to the constraint that the total number of machines in operation in each period cannot exceed the aggregate labor force:

$$\max_{\left\{e_f(t),\bar{a}(t)\right\}} \int_0^\infty e^{-rt}\left\{e^{\gamma t}\int_0^{\bar{a}(t)} e_f\left(t-a\right)e^{-\gamma a}da - e_f\left(t\right)Ie^{\gamma t}\right\}dt$$
$$\text{s.t.} \int_0^{\bar{a}(t)} e_f(t-a)da \leq 1, \text{ for all } t.$$

The planner chooses the measure of entrants $e_f(t)$ of vintage t and the maximal age $\bar{a}\left(t\right)$ of vintages which are operating at t. Since there are no frictions, the planner can use arbitrary firm measures of any vintage, and since more recent vintages are more productive, the planner uses all firms of the most recent vintages first. At time t the planner operates vintages in $\left[t-\bar{a}\left(t\right),t\right]$. We can write the Lagrangian for the constrained optimization problem in terms of the contributions of the different vintages as

$$\begin{aligned}\mathcal{L} &= \int_0^\infty e^{-(r-\gamma)t}e_f\left(t\right)\left\{\int_0^{\hat{a}(t)} e^{-ra}da - I\right\}dt \\ &\quad - \int_{-\bar{a}(0)}^\infty e_f\left(t\right)\left\{\int_t^{t+\hat{a}(t)} \varphi\left(\tau\right)e^{-(r-\gamma)\tau}d\tau\right\}dt + \int_0^\infty e^{-(r-\gamma)t}\varphi\left(t\right)dt\ ,\end{aligned}$$

where $\varphi\left(t\right)$ is the Lagrange multiplier on the labor endowment constraint and $\hat{a}\left(t\right) \equiv \bar{a}\left[t-\hat{a}\left(t\right)\right]$ denotes the age at which a vintage t machine is retired. The first order conditions with respect to $e_f(t)$ and $\hat{a}(t)$ read, respectively,

$$\begin{aligned}\int_0^{\hat{a}(t)} e^{-ra}da - I - \int_0^{\hat{a}(t)} e^{-(r-\gamma)a}\varphi(t+a)da &= 0, \\ e^{-r\hat{a}(t)}\left\{1-e^{\gamma\hat{a}(t)}\varphi\left[t+\hat{a}\left(t\right)\right]\right\} &= 0.\end{aligned}$$

The first condition states that the planner will add new firms until the benefits (present value of additional output) equal the direct installation costs and the indirect costs. This could follow from the fact that the creation of new firms requires the destruction of others, given the fixed amount of labor available. The second condition states that a marginal increase in the destruction age raises the expected output of existing firms but once again requires a reduction in the total number of operating firms.

In steady state, the time subscripts can be omitted and $\hat{a} = \bar{a}$. Moreover, we can impose the condition $e_f = 1/\bar{a}$ which guarantees that the distribution is stationary and all the labor is employed. From the second condition, we obtain that $\varphi = e^{-\gamma\bar{a}}$. This expression is easily interpretable: φ is the multiplier on the total labor force constraint, and $e^{-\gamma\bar{a}}$ is exactly the value of slackening this constraint, i.e., the marginal contribution of an extra unit of labor (recall that this is also the equilibrium wage rate). Using this result in the first condition, we arrive at

$$I = \int_0^{\bar{a}} e^{-ra}\left[1 - e^{-\gamma(\bar{a}-a)}\right] da$$

which is the key equilibrium condition (2) in the decentralized economy.

A.2 Derivations of value functions and employment distributions.

The value functions and distributions of our continuous-time model can be derived as limits of a discrete time formulation. A typical derivation of the differential equations for value functions (5)-(8) goes as follows. Consider the value of a vacant firm with capital of age a at time t, $\tilde{V}(t,a)$. For a Poisson matching process, the probability that the vacant firm meets a worker over a small finite time interval $[t, t+\Delta]$ is $\Delta\lambda_f$. We can define the vacancy value recursively as

$$\tilde{V}(t,a) = \Delta\lambda_f\left[\tilde{J}(t+\Delta, a+\Delta) - \tilde{V}(t+\Delta, a+\Delta)\right] + e^{-r\Delta}\tilde{V}(t+\Delta, a+\Delta),$$

where the first term is the expected capital gain from becoming a matched firm with value $\tilde{J}$ and the second term is the present value of remaining vacant at the end of the time interval. On a balanced growth path all value functions increase at the rate γ over time, i.e., $\tilde{V}(t,a) = e^{\gamma t}V(a)$ and $\tilde{J}(t,a) = e^{\gamma t}J(a)$. Subtracting $\tilde{V}(t+\Delta, a)$ from both sides, substituting the balanced growth path expressions for $\tilde{V}$ and $\tilde{J}$, and dividing by $\Delta e^{\gamma(t+\Delta)}$, we can rearrange the value equation into

$$\begin{aligned} -e^{-\gamma\Delta}V(a)\frac{e^{\gamma\Delta}-1}{\Delta} &= \lambda_f\left[J(a+\Delta) - V(a+\Delta)\right] + \frac{e^{-r\Delta}-1}{\Delta}V(a+\Delta) \\ &\quad + \frac{V(a+\Delta) - V(a)}{\Delta}. \end{aligned}$$

As we shorten the length of the time interval and take the limit for $\Delta \to 0$, we obtain the differential equation (5):

$$-\gamma V(a) = \lambda_f\left[J(a) - V(a)\right] - rV(a) + V'(a).$$

The equations describing employment dynamics are derived as follows. Consider the measure of matched vintage a firms at time t. Over a short time interval of length Δ, the approximate change in the measure is

$$\mu(t+\Delta, a) = \mu(t, a-\Delta)(1-\Delta\delta) + \Delta\lambda_f \nu(t, a-\Delta).$$

Subtracting $\mu(t,a)$ from both sides and dividing by Δ we obtain

$$\frac{\mu(t+\Delta, a) - \mu(t,a)}{\Delta} = -\frac{\mu(t,a) - \mu(t, a-\Delta)}{\Delta} - \delta\mu(t, a-\Delta) + \lambda_f \nu(t, a-\Delta).$$

Taking the limit for $\Delta \to 0$ one obtains

$$\mu_t(t,a) = -\mu_a(t,a) - \delta\mu(t,a) + \lambda_f \nu(t,a).$$

At steady state, these measures do not change with t, and we obtain the result stated in (16).

Similarly, the differential equation for unemployment can be derived as follows. Over a short time period of length Δ the change in unemployment is

$$u(t+\Delta) = u(t)\left[1 - \int_0^{\bar{a}} \Delta\lambda_w(a)da\right] + \Delta\delta\int_0^{\bar{a}} \mu(t,a)da + \int_0^{\Delta} \mu(t, \bar{a}-x)dx$$

The first two terms on the right-hand side are standard: they are flows assuming a Poisson process and these flows are approximately linear in the length of the interval, since the interval is small. The third term sums all those matches that will reach $\bar{a}$ by the end of the period and therefore separate. Subtracting $u(t)$ on both sides, dividing by Δ, taking limits as Δ approaches 0, and assuming steady state yields the result (21). To find $\lim_{\Delta\to 0}\left[\int_0^{\Delta} \mu(\bar{a}-x)dx\right]/\Delta$, use l'Hôpital's rule.

Given the differential equation for employment (16), we can easily determine that

$$\mu(a) = \frac{\lambda_f \nu(0)}{\delta+\lambda_f}\left[1 - e^{-(\delta+\lambda_f)a}\right] \text{ and that} \tag{28}$$

$$\nu(a) = \frac{\nu(0)}{\delta+\lambda_f}\left[\delta + \lambda_f e^{-(\delta+\lambda_f)a}\right]. \tag{29}$$

Thus, the total number of vacancies, v, satisfies

$$v = \int_0^{\bar{a}} \nu(a)da = \frac{\nu(0)}{\delta+\lambda_f}\left\{\bar{a}\delta + \frac{\lambda_f}{\delta+\lambda_f}\left[1 - e^{-(\delta+\lambda_f)\bar{a}}\right]\right\}. \tag{30}$$

Integrating both sides of the equation $\nu(a)+\mu(a)=\nu(0)$ over the support $[0,\overline{a})$, we conclude that the total number of matched pairs (employment), μ, satisfies $\mu=\nu(0)\bar{a}-v$, or

$$\mu=\frac{\nu(0)\lambda_f}{\delta+\lambda_f}\left\{\bar{a}-\frac{1}{\delta+\lambda_f}\left[1-e^{-(\delta+\lambda_f)\bar{a}}\right]\right\}. \tag{31}$$

Solving (21) for u, and substituting in for $\mu(\bar{a})/\mu$, we arrive at

$$u=\frac{1+\delta\left(\frac{\bar{a}}{1-e^{-(\delta+\lambda_f)\bar{a}}}-\frac{1}{\delta+\lambda_f}\right)}{1+\left[\delta+m(\theta,1)\right]\left(\frac{\bar{a}}{1-e^{-(\delta+\lambda_f)\bar{a}}}-\frac{1}{\delta+\lambda_f}\right)}. \tag{32}$$

Having found the unemployment rate u, the entry of firms $\nu(0)$ is simply found from equation (31), using the fact that $\mu=1-u$. Equations (17) and (18) in the main text can be derived simply using (28) together with (31) and (29) together with (30), respectively.

A.3 Proof of Lemmas 1,2,3 (the job creation curve).

Lemma 1 (the downward sloping (JC) curve): The (JC) curve is implicitly defined by the equation

$$I=(1-\beta)\lambda_f\int_0^{\bar{a}}e^{-(r-\gamma)a}S\left(a;\bar{a},\lambda_f,\gamma\right)da. \tag{JC}$$

We show that the RHS of this expression is increasing in $\bar{a}$ and λ_f, which implies that the (JC) curve is downward sloping in the $(\lambda_f,\bar{a})$ space.

Straightforward integration of the equation (14) defining the surplus equation yields

$$S\left(a;\bar{a},\lambda_f\right)=e^{-\gamma a}\left(1-e^{-\sigma_0(\bar{a}-a)}\right)/\sigma_0-e^{-\gamma\bar{a}}\left(1-e^{-\sigma_1(\bar{a}-a)}\right)/\sigma_1 \tag{33}$$

with $\sigma_0=r+\delta+(1-\beta)\lambda_f$ and $\sigma_1=\sigma_0-\gamma$. It is immediate that the surplus function is decreasing in a and increasing in $\bar{a}$. Since the surplus function is increasing in the exit age $\bar{a}$, it is immediate that the RHS of (JC) is increasing in the exit age $\bar{a}$.

To show that the RHS of (JC) is increasing in λ_f, rewrite the integral as

$$I=e^{-\gamma\bar{a}}\int_0^{\bar{a}}e^{-(r-\gamma)a}\left\{\int_0^{\bar{a}-a}\hat{\lambda}_f e^{-\left(\rho+\hat{\lambda}_f\right)\tilde{a}}\left[e^{\gamma(\bar{a}-a-\tilde{a})}-1\right]d\tilde{a}\right\}da \tag{34}$$

with $\rho=r-\gamma+\delta$ and $\hat{\lambda}_f=(1-\beta)\lambda_f$. We now show that the integral of the function $f(\tilde{a};\lambda_f)=\lambda_f e^{-\left(\rho+\lambda_f\right)\tilde{a}}$ with respect to the weighting function $g(\tilde{a})=e^{\gamma(\bar{a}-a-\tilde{a})}-1$ is increasing in λ_f. The function f is increasing (decreasing) with respect to λ_f for $\tilde{a}<(>)\,\hat{a}=1/\lambda_f$.

The integral of the function f, however, is increasing with λ_f, as

$$\begin{aligned} \int_0^{\bar{a}} f(\tilde{a};\lambda_f)\, d\tilde{a} &= \left[1 - e^{-\left(\rho+\lambda_f\right)\bar{a}}\right] \frac{\lambda_f}{\rho+\lambda_f} \\ \frac{\partial}{\partial\lambda_f}\int_0^{\bar{a}} f(\tilde{a};\lambda_f)\, d\tilde{a} &= \frac{\rho}{\rho+\lambda_f}\int_0^{\bar{a}} f(\tilde{a};\lambda_f)\, d\tilde{a} + \frac{\lambda_f}{\rho+\lambda_f} e^{-\left(\rho+\lambda_f\right)\bar{a}} > 0. \end{aligned}$$

The integral of f with respect to g is also increasing with λ_f, since the weighting function g is monotonically decreasing in $\tilde{a}$,

$$\begin{aligned} & \frac{\partial}{\partial\lambda_f}\int_0^{\bar{a}-a} f(\tilde{a};\lambda_f)\, g(\tilde{a})\, d\tilde{a} \\ =\ & \int_0^{a_+} f_{\lambda_f}(\tilde{a};\lambda_f)\, g(\tilde{a})\, d\tilde{a} + \int_{a_+}^{\bar{a}-a} f_{\lambda_f}(\tilde{a};\lambda_f)\, g(\tilde{a})\, d\tilde{a} \\ >\ & \int_0^{a_+} f_{\lambda_f}(\tilde{a};\lambda_f)\, g(a_+)\, d\tilde{a} + \int_{a_+}^{\bar{a}-a} f_{\lambda_f}(\tilde{a};\lambda_f)\, g(a_+)\, d\tilde{a} \\ =\ & g(a_+)\int_0^{\bar{a}-a} f_{\lambda_f}(\tilde{a};\lambda_f)\, d\tilde{a} > 0, \end{aligned}$$

with $a_+ = \min\{\hat{a}, \bar{a}-a\}$.

Lemmas 2 and 3 (the asymptotes of the (JC) curve): Integrating equation (JC) yields

$$\begin{aligned} I =\ & \frac{(1-\beta)\lambda_f}{r+\delta+(1-\beta)\lambda_f} \\ & \left\{\frac{1-e^{-r\bar{a}}}{r} - \frac{\sigma_0}{\sigma_1}e^{-\gamma\bar{a}}\frac{1-e^{-(r-\gamma)\bar{a}}}{r-\gamma} + e^{-r\bar{a}}\frac{1-e^{-\left(\delta+(1-\beta)\lambda_f\right)}}{\delta+(1-\beta)\lambda_f}\frac{\gamma}{\sigma_1}\right\}. \end{aligned} \tag{35}$$

Taking the limit of expression (35) as $\lambda_f \to \infty$, we get

$$\begin{aligned} I &= \frac{1-e^{-r\bar{a}^{\min}}}{r} - e^{-\gamma\bar{a}^{\min}}\frac{1-e^{-(r-\gamma)\bar{a}^{\min}}}{r-\gamma} \\ &= \int_0^{\bar{a}^{\min}} e^{-ra}\left[1-e^{-\gamma(\bar{a}^{\min}-a)}\right] \Rightarrow \bar{a}^{\min} = \bar{a}^{CE}, \end{aligned}$$

where $\bar{a}^{CE}$ is the age cut-off of the frictionless economy, implicitly defined by (2). Alternatively taking the limit of expression (35) as $\bar{a} \to \infty$, we get

$$I = \frac{(1-\beta)\lambda_f^{\min}}{r+\delta+(1-\beta)\lambda_f^{\min}}\frac{1}{r} \Rightarrow \lambda_f^{\min} = \frac{rI}{1-rI}\frac{r+\delta}{1-\beta}.$$

A.4 Proof of Lemmas 4 and 5 (the job destruction curve).

Lemma 4 (the upward-sloping (JD) curve): The (JD) curve is implicitly defined by the equation

$$1 = be^{\gamma\bar{a}} + \beta \int_0^{\bar{a}} \lambda_w(a; \bar{a}, \lambda_f) e^{\gamma\bar{a}} S(a; \bar{a}, \lambda_f, \gamma) da. \tag{JD}$$

We show that the RHS of this expression is increasing in $\bar{a}$ and decreasing in λ_f, which implies that the (JD) curve is upward-sloping in $(\lambda_f, \bar{a})$ space.

(4a) The RHS of (JD) is increasing in $\bar{a}$: The first term is clearly increasing in $\bar{a}$. Now take the derivative of the function to be integrated in the second term, and express it in terms of elasticities

$$\left\{ \frac{\partial \lambda_w}{\partial \bar{a}} \frac{\bar{a}}{\lambda_w} + \frac{\partial \tilde{S}}{\partial \bar{a}} \frac{\bar{a}}{\tilde{S}} \right\} \frac{\tilde{S} \lambda_w}{\bar{a}} \tag{36}$$

where $\tilde{S} = e^{\gamma\bar{a}} S$. The elasticity of the density λ_w is given by

$$\frac{\partial \lambda_w}{\partial \bar{a}} \frac{\bar{a}}{\lambda_w} = -\frac{\delta \bar{a} + \lambda_f \bar{a} e^{-\left(\delta + \lambda_f\right)\bar{a}}}{\delta \bar{a} + \lambda_f \int_0^{\bar{a}} e^{-\left(\delta + \lambda_f\right) a} da}.$$

Thus the first term in (36) is negative, but its absolute value is less than one since $e^{-\left(\delta+\lambda_f\right)a} > e^{-\left(\delta+\lambda_f\right)\bar{a}}$ for $a \leq \bar{a}$. We will now show that the elasticity of the modified surplus function $\tilde{S}$ with respect to $\bar{a}$ is positive and greater than or equal to one. It will therefore follow that the integral in (JD) is increasing in $\bar{a}$.

We proceed in three steps. First, we show that the elasticity of $\tilde{S}$ with respect to $\bar{a}$ is increasing in a for a given $\bar{a}$. That is, if the elasticity is greater than one at $a = 0$, then it is greater than one for all a. Second, we show that for small enough $\bar{a}$ the elasticity is greater or equal to zero at $a = 0$; in particular, we show that $\lim_{\bar{a} \to 0} \left(\partial \tilde{S} / \partial \bar{a} \right) \left(\bar{a} / \tilde{S} \right) \geq 1$. Third, we show at $a = 0$ the elasticity is increasing in $\bar{a}$. The three steps together imply that the elasticity is greater or equal to one for all $a \leq \bar{a}$.

The elasticity of $\tilde{S}$ with respect to $\bar{a}$ is given by

$$\frac{\partial \tilde{S}}{\partial \bar{a}} \frac{\bar{a}}{\tilde{S}} = \frac{\gamma \bar{a}}{1 - H\left(\bar{a} - a\right)} \text{ with } H\left(x\right) \equiv e^{-\gamma x} \frac{\left(1 - e^{-\sigma_1 x}\right) / \sigma_1}{\left(1 - e^{-\sigma_0 x}\right) / \sigma_0}.$$

The sign of the derivative of the function H is given by

$$sign\left(H'\right) = \sigma_0 \gamma e^{-(\sigma_0 + \gamma)} \left\{ \int_0^x e^{\gamma y} dy - \int_0^x e^{\sigma_0 y} dy \right\}.$$

Since $\sigma_0 = r + \delta + \left(1 - \beta\right) \lambda_f$ and by assumption $r > \gamma$, the function H is decreasing in x. Therefore the elasticity is increasing in a.

The limit of the elasticity at $a = 0$ as $\bar{a}$ converges to zero is greater or equal to one. To see this note that for $\bar{a} \to 0$, the numerator and denominator converge to zero, and by l'Hôpital's rule

$$\lim_{\bar{a}\to 0} \frac{\bar{a}\partial\tilde{S}/\partial\bar{a}}{\tilde{S}} = \lim_{\bar{a}\to 0} \frac{\bar{a}\left(\partial^2\tilde{S}/\partial\bar{a}^2\right) + \partial\tilde{S}/\partial\bar{a}}{\partial\tilde{S}/\partial\bar{a}} = 1 + \lim_{\bar{a}\to 0} \frac{\left(\partial^2\tilde{S}/\partial\bar{a}^2\right)\bar{a}}{\partial\tilde{S}/\partial\bar{a}} \geq 1$$

since $\partial\tilde{S}/\partial\bar{a}$, $\partial^2\tilde{S}/\partial\bar{a}^2 \geq 0$.

Finally we need to show that at $a = 0$, the elasticity is increasing in $\bar{a}$, that is $G(\bar{a}) = \bar{a}/\left[1 - H(\bar{a})\right]$ is increasing in $\bar{a}$. First, multiply numerator and denominator by $\sigma_1(1-e^{-\sigma_0\bar{a}})$. This delivers

$$G(\bar{a}) = \frac{\sigma_1\bar{a}(1-e^{-\sigma_0\bar{a}})}{\sigma_1(1-e^{-\sigma_0\bar{a}}) - \sigma_0(1-e^{-\sigma_1\bar{a}})e^{-\gamma\bar{a}}} = \bar{a}\frac{\sigma_1(1-e^{-\sigma_0\bar{a}})}{\sigma_1 - \sigma_0 e^{-\gamma\bar{a}} + \gamma e^{-\sigma_0\bar{a}}}.$$

Notice that the denominator of this expression is positive: at $\bar{a} = 0$, it equals 0, and its derivative equals $\gamma\sigma_0(e^{-\gamma\bar{a}} - e^{-\sigma_0\bar{a}})$, which is positive because of the assumption that $\gamma < r < \sigma_0$. For large $\bar{a}$, the expression is large: $\lim_{\bar{a}\to\infty} G(\bar{a}) = \lim_{\bar{a}\to\infty} \bar{a} = \infty$.

The derivative of G equals

$$\begin{aligned}
\frac{G'(\bar{a})}{\sigma_1} &= \frac{1-e^{-\sigma_0\bar{a}}}{\sigma_1 - \sigma_0 e^{-\gamma\bar{a}} + \gamma e^{-\sigma_0\bar{a}}} \\
&\quad -\bar{a}\frac{\sigma_0 e^{-\sigma_0\bar{a}}\left(\sigma_1 - \sigma_0 e^{-\gamma\bar{a}} + \gamma e^{-\sigma_0\bar{a}}\right) - \left(1-e^{-\sigma_0\bar{a}}\right)\gamma\sigma_0\left(e^{-\gamma\bar{a}} - e^{-\sigma_0\bar{a}}\right)}{\left(\sigma_1 - \sigma_0 e^{-\gamma\bar{a}} + \gamma e^{-\sigma_0\bar{a}}\right)^2} \\
&= \frac{1-e^{-\sigma_0\bar{a}}}{\sigma_1 - \sigma_0 e^{-\gamma\bar{a}} + \gamma e^{-\sigma_0\bar{a}}} \\
&\quad -\bar{a}\sigma_0\frac{\sigma_1 e^{-\sigma_0\bar{a}} - \sigma_0 e^{-(\gamma+\sigma_0)\bar{a}} + \gamma\left(e^{-2\sigma_0\bar{a}} - e^{-\gamma\bar{a}} + e^{-\sigma_0\bar{a}} + e^{-(\gamma+\sigma_0)\bar{a}} - e^{-2\sigma_0\bar{a}}\right)}{\left(\sigma_1 - \sigma_0 e^{-\gamma\bar{a}} + \gamma e^{-\sigma_0\bar{a}}\right)^2}.
\end{aligned}$$

Substituting for $\sigma_1 = \sigma_0 - \gamma$ and simplifying yields

$$\frac{G'(\bar{a})}{\sigma_1} = \frac{1-e^{-\sigma_0\bar{a}}}{\sigma_1 - \sigma_0 e^{-\gamma\bar{a}} + \gamma e^{-\sigma_0\bar{a}}} - \bar{a}\sigma_0\frac{\sigma_0\left(e^{-\sigma_0\bar{a}} - e^{-(\gamma+\sigma_0)\bar{a}}\right) + \gamma\left(e^{-(\gamma+\sigma_0)\bar{a}} - e^{-\gamma\bar{a}}\right)}{\left(\sigma_1 - \sigma_0 e^{-\gamma\bar{a}} + \gamma e^{-\sigma_0\bar{a}}\right)^2}.$$

Thus, it is sufficient to study

$$\left(1-e^{-\sigma_0\bar{a}}\right)\left(\sigma_1 - \sigma_0 e^{-\gamma\bar{a}} + \gamma e^{-\sigma_0\bar{a}}\right) - \bar{a}\sigma_0\left[\sigma_0\left(e^{-\sigma_0\bar{a}} - e^{-(\gamma+\sigma_0)\bar{a}}\right) + \gamma\left(e^{-(\gamma+\sigma_0)\bar{a}} - e^{-\gamma\bar{a}}\right)\right].$$

This expression equals

$$\left(1-e^{-\sigma_0\bar{a}}\right)\left(\sigma_1 - \sigma_0 e^{-\gamma\bar{a}} + \gamma e^{-\sigma_0\bar{a}}\right) - \bar{a}\sigma_0\left[\sigma_0 e^{-\sigma_0\bar{a}}\left(1-e^{-\gamma\bar{a}}\right) - \gamma e^{-\gamma\bar{a}}\left(1-e^{-\sigma_0\bar{a}}\right)\right].$$

Factorizing, we are left with

$$\left(1-e^{-\sigma_0\bar{a}}\right)\left(\sigma_1-\sigma_0 e^{-\gamma\bar{a}}+\gamma e^{-\sigma_0\bar{a}}-\bar{a}\sigma_0^2 e^{-\sigma_0\bar{a}}\frac{1-e^{-\gamma\bar{a}}}{1-e^{-\sigma_0\bar{a}}}+\bar{a}\gamma\sigma_0 e^{-\gamma\bar{a}}\right).$$

The left factor is larger than zero (it starts at zero and increases). After substituting for $\sigma_1=\sigma_0-\gamma$, the right factor can be rewritten as

$$\sigma_0\left(1-e^{-\gamma\bar{a}}\right)-\gamma\left(1-e^{-\sigma_0\bar{a}}\right)+\sigma_0\gamma\bar{a}\left(e^{-\gamma\bar{a}}-e^{-\sigma_0\bar{a}}\frac{1-e^{-\gamma\bar{a}}}{1-e^{-\sigma_0\bar{a}}}\frac{\sigma_0}{\gamma}\right).$$

Utilizing a simple integral formula, this becomes

$$\sigma_0\gamma\left(\int_0^{\bar{a}}e^{-\gamma x}dx-\int_0^{\bar{a}}e^{-\sigma_0 x}dx\right)+\sigma_0\gamma\bar{a}\left(e^{-\gamma\bar{a}}-e^{-\sigma_0\bar{a}}\frac{\int_0^{\bar{a}}e^{-\gamma x}dx}{\int_0^{\bar{a}}e^{-\sigma_0 x}dx}\right).$$

The first of these two terms is positive, since $\sigma_0>\gamma$ by assumption. If we can show that the second term is positive, we are done. That term has two sub-terms; we will prove that their ratio exceeds one. The ratio reads

$$\frac{e^{-\gamma\bar{a}}\int_0^{\bar{a}}e^{-\sigma_0 x}dx}{e^{-\sigma_0\bar{a}}\int_0^{\bar{a}}e^{-\gamma x}dx}=\frac{\int_0^{\bar{a}}e^{-(\gamma+\sigma_0)x}e^{-(\bar{a}-x)\gamma}dx}{\int_0^{\bar{a}}e^{-(\gamma+\sigma_0)x}e^{-(\bar{a}-x)\sigma_0}dx}.$$

But since the weighting function $e^{-(\bar{a}-x)\gamma}$ is everywhere above the weighting function $e^{-(\bar{a}-x)\sigma_0}$, again because $\sigma_0>\gamma$ (and $\bar{a}>x$), and the rest of the integrand is positive, the ratio indeed must exceed 1.

(4b) The RHS of (JD) is decreasing in λ_f for a Cobb-Douglas matching function with $\alpha>1/2$: We rewrite equation (JD) as

$$1=be^{\gamma\bar{a}}+\beta\left[m\left(\theta,1\right)\lambda_f\right]\int_0^{\bar{a}}\left[\frac{\lambda_w(a;\bar{a},\lambda_f)/\lambda_f}{m\left(\theta,1\right)}\right]\tilde{S}(a;\bar{a},\lambda_f,\gamma)da.$$

It is immediate that the two terms under the integral, the modified density λ_w and the modified surplus function $\tilde{S}$, are decreasing in λ_f. Assuming a Cobb-Douglas matching function and substituting for θ, the term pre-multiplying the integral becomes

$$A\lambda_f^{(1-2\alpha)/(1-\alpha)}$$

which is decreasing in λ_f for $\alpha>1/2$.

Lemma 5 (The asymptote of the (JD) curve): For λ_f large, the density λ_w converges to a uniform density on $[0,\bar{a}]$, $\lim_{\lambda_f\to\infty}\lambda_w\left(a\right)=\dfrac{1}{\bar{a}+1/\delta}$, and the surplus function converges to zero, $\lim_{\lambda_f\to\infty}S\left(a;\lambda_f,\gamma\right)=0$. Therefore equation (JD) converges to

$$1=be^{\gamma\bar{a}^{\max}}\Rightarrow\bar{a}^{\max}=-\ln\left(b\right)/\gamma.$$

A.5 Proof of Lemma 6,7,8, and 9 (comparative statics).

Lemma 6 (b): Obvious from inspection of (JC) and (JD).

Lemma 7 (γ): The RHS of (JC) is increasing in the rate of embodied technical change γ, because the function $f(\gamma) = e^{\gamma a} S(a; \bar{a}, \lambda_f, \gamma)$ is increasing in γ. The derivative of f with respect to γ is

$$\frac{\partial f}{\partial \gamma} = e^{-\gamma x} \left(\sigma_1 x + e^{-\sigma_1 x} - 1 \right) / \sigma_1^2$$

with $x = \bar{a} - a$. Notice that $\partial f / \partial \gamma = 0$ at $x = 0$, and that $\partial f / \partial \gamma$ is increasing in x

$$\frac{\partial^2 f}{\partial \gamma \partial x} = \sigma_1 \left(1 - e^{-\sigma_1 x} \right) \geq 0 \text{ for } x \geq 0.$$

Since the RHS of (JC) is increasing γ and $\bar{a}$, the (JC) curve shifts downward as γ increases.

The RHS of (JD) is increasing in the rate of embodied technical change γ because the function $\tilde{S} \equiv e^{\gamma \bar{a}} S$ is increasing in γ. The derivative of $\tilde{S}$ with respect to γ is

$$\frac{\partial \tilde{S}}{\partial \gamma} = x e^{\gamma x} \left(1 - e^{-\sigma_0 x} \right) / \sigma_0 + \left[e^{-\sigma_1 x} \left(1 + \sigma_1 x \right) - 1 \right] / \sigma_1^2$$

with $x = \bar{a} - a$. Notice that $\partial \tilde{S} / \partial \gamma = 0$ at $x = 0$ and that $\partial \tilde{S} / \partial \gamma$ is increasing in x, i.e.,

$$\frac{\partial^2 \tilde{S}}{\partial \gamma \partial x} = e^{\gamma x} \left(1 + \gamma x \right) \left(1 - e^{-\sigma_0 x} \right) / \sigma_0 \geq 0 \text{ for } x \geq 0.$$

Therefore $\partial \tilde{S} / \partial \gamma \geq 0$ for all $x \geq 0$. Since $\tilde{S}$ is increasing in γ for all $a \in [0, \bar{a}]$, the RHS of (JD) is increasing in γ. Since the RHS of (JD) is increasing in γ and $\bar{a}$, the (JD) curve shifts downward as γ increases.

Lemma 8 (r): Obvious from inspection of (JC) and (JD).

Lemma 9 (A): When the frictions disappear, both meeting probabilities tend to infinity. We proved above that as $\lambda_f \to \infty$, the job creation condition converges to the competitive condition (2). Consider now the wage equation (23). It is easy to compute that as $\lambda_f \to \infty$, the term $\lambda_f (1 - \beta) S(a)$ converges to $e^{-\gamma a} - e^{-\gamma \bar{a}}$, implying $w(a) = (1 - \beta)(r - \gamma) U + \beta e^{-\gamma \bar{a}} = e^{-\gamma \bar{a}}$, where the last equality follows from condition (13). Thus, the job destruction condition converges to the competitive one as well, which proves the Lemma.

A.6 Solution of the model with the firing tax.

The value equations in the model with the firing tax/hiring subsidy are:

$$
\begin{aligned}
(r-\gamma)V(a) &= \max\{\lambda_f\,[T+J(a)-V(a)]+V'(a),0\} \\
(r-\gamma)J(a) &= \max\{e^{-\gamma a}-w(a)-\delta\,[J(a)-V(a)+T]+J'(a),(r-\gamma)[V(a)-T]\} \\
(r-\gamma)U &= b+\int_0^{\bar{a}}\lambda_w(a)\,[W(a)-U]\,da \\
(r-\gamma)W(a) &= \max\{w(a)-\delta\,[W(a)-U]+W'(a),(r-\gamma)U\},
\end{aligned}
$$

where it is clear that the firm receives from the government the subsidy T upon hiring and pays a tax of the same amount T upon separation. The total surplus $S(a)$ of a type a match is now $S(a)\equiv J(a)+W(a)-V(a)-U+T$ both for a new meeting and for an ongoing relationship. The term T plays the role of the subsidy in the first case, and the role of the tax in the second case (with the subsidy being sunk): we can avoid the two-tier structure. The surplus is split according to the Nash rule $\beta[W(a)-U]=(1-\beta)[T+J(a)-V(a)]$ and using this rule with the definition of the surplus, we obtain a differential equation for the surplus with solution

$$
S(a;\bar{a},\lambda_f)=\int_a^{\bar{a}}e^{-[r-\gamma+\delta+\lambda_f(1-\beta)](\tilde{a}-a)}\left[e^{-\gamma a}-(r-\gamma)(U-T)\right]d\tilde{a}, \tag{37}
$$

where the associated destruction rule for a match is (25) in the main text. Using the value equation for the employed worker and the Nash rule, we arrive at the expression for the wage function (26) in the main text. Using the Nash splitting rule into the equation for the value of a vacancy and solving the associated differential equation yields exactly the same job creation condition as in the benchmark model with the implication that the destruction rule for a vacancy $V(a)=0$ implies $S(\hat{a};\bar{a},\lambda_f)=0$; thus, $\hat{a}=\bar{a}$. Using the value equation for the unemployed worker into the destruction condition (25) yields the equilibrium condition (JD') in the main text. Finally, it is easy to see that the expressions for all the distributions are unchanged.

References

[1] **Acemoglu, Daron** (2000); "Technical Change, Inequality and the Labor Market," NBER working paper 7800.

[2] **Abrahams, Katharine G. and Robert Shimer** (2001); "Changes in Unemployment Duration and Labor Force Attachment", NBER working paper 8513.

[3] **Aghion, Philippe and Peter Howitt** (1994), "Growth and Unemployment, *Review of Economic Studies*, 61, pages 477-494.

[4] **Bentolila, Samuel and Gilles Saint-Paul** (1999); "Explaining Movements in the Labor Share, mimeo, CEMFI.

[5] **Blanchard, Olivier** (1997); "The Medium Run", *Brookings Papers of Economic Activity*, 2, pages 89-158.

[6] **Blanchard, Olivier and Justin Wolfers** (2000); "The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence", mimeo, MIT.

[7] **Bertola, Giuseppe, Francine D. Blau, and Lawrence M. Kahn** (2001); "Comparative Analysis of Labor Market Outcomes: Lessons for the US from International Long-Run Evidence", NBER working paper 8526.

[8] **Breshnahan, Timothy F., Erik Brynjolfsson and Laurin M. Hitt** (2002); "Information Technology, Workplace Organization and the Demand for Skilled Labor: Firm-Level Evidence," *Quarterly Journal of Economics* vol. 117, pages 339-376.

[9] **Bureau of Economic Analysis** (2002), "Fixed Assets and Consumer Durable Goods for 1925-2000," U.S. Department of Commerce: Washington D.C., `www.bea.doc.gov`.

[10] **Caballero, Ricardo J. and Mohamad L. Hammour** (1998); "Jobless growth: appropriability, factor substitution, and unemployment," *Carnegie-Rochester Conference Series On Public Policy* (48)1, pages 51-94.

[11] **CEPR** (1995); "Unemployment: Choices for Europe", *Monitoring European Integration series*, vol.5. London: Centre for Economic Policy Research.

[12] **Colecchia Alessandra and Paul Schreyer** (2002); "ICT Investment and Economic Growthin the 1990s: Is the United States a Unique Case? A Comparative Studies of Nine OECD Countries", *Review of Economic Dynamics*, vol. 5(2) pages 408-442.

[13] **Cooley, Tom** (ed.), *Frontiers of Business Cycle Research*, Princeton University Press, 1995.

[14] **den Haan, Wouter, Christian Haefke and Garey Ramey** (2001); "Shocks and Institutions in a Job Matching Model", mimeo.

[15] **Cummins, Jason and Giovanni L. Violante** (2002); "Investment-Specific Technological Change in the US (1947-2000): Measurement and Macroeconomic Consequences", FRB working paper 2002-10.

[16] **Doms, Mark, Timothy Dunne and Kenneth R. Troske** (1997); "Workers, wages and technology," *The Quarterly Journal of Economics,* Vol. 112(1), pages 253 – 290.

[17] **Gordon, Robert. J.** (1990); *The Measurement of Durable Good Prices*, NBER Monograph Series, University of Chicago press.

[18] **Gordon, Robert J.** (1997); "Is There a Trade-off between Unemployment and Productivity Growth?" in Snower, Dennis J.;de la Dehesa, Guillermo, eds. *Unemployment policy: Government options for the labour market.* Cambridge; New York and Melbourne: Cambridge University Press, pages 433-63.

[19] **Gottschalk, Peter, and Robert Moffitt** (1994); "The Growth of Earnings Instability in the U.S. Labour Market," *Brookings Papers of Economic Activity*, 2, pages 217-272.

[20] **Greenwood, Jeremy and Memhet Yorukoglu** (1997); "1974", *Carnegie-Rochester Conference Series on Public Policy*, vol. 46, pages 49-95.

[21] **Greenwood, Jeremy, Zvi Hercowitz and Per Krusell** (1997);"Long-Run Implications of Investment-Specific Technological Change", *American Economic Review*, 87(3), pages 342-62.

[22] **Hansen, Hans** (1998); "Transition from unemployment benefits to social assistance in seven European OECD countries," *Empirical Economics*, vol. 23(1-2), pages 5-30.

[23] **Hobjin, Bart** (2000); "Identifying Sources of Growth", mimeo, Federal Reserve Bank of New York.

[24] **Hornstein, Andreas and Per Krusell**, (1996); "Can Technology Improvements Cause Productivity Slowdowns?," *NBER Macroeconomics Annual*, pages 209-259.

[25] **Hornstein, Andreas, Per Krusell, and Giovanni L. Violante** (2002), "Capital-Skill Complementarity as an Origin of Inequalities", mimeo.

[26] **Jorgenson, Dale W.** (2001); "Information Technology and the US Economy," *American Economic Review*, vol. 91(1), pages 1-32.

[27] **Jovanovic, Boyan** (1998); "Vintage Capital and Inequality", *Review of Economic Dynamics*, 1(2), pages 497-530.

[28] **Juhn, Chinhui, Kevin Murphy, and Robert Topel** (1991); "Why Has the Natural Rate of Unemployment Increased over Time?", *Brookings Papers on Economic Activity* (2), pages 75-126.

[29] **Katz, Lawrence and David Autor** (1999); "Changes in the Wage Structure and Earnings Inequality," chapter 26, Volume 3, in *Handbook of Labor Economics*, (edited by Orley Ashenfelter and David Card).

[30] **Krusell, Per, Lee Ohanian, José-Víctor Ríos-Rull, and Giovanni L. Violante** (2000); "Capital Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, 68, pages 1029-1054.

[31] **Ljunqvist, Lars and Thomas J. Sargent** (1998); "The European Unemployment Dilemma", *Journal of Political Economy*;106(3), pages 514-50.

[32] **Machin, Steve and Alan Manning** (1999); "The Causes and Consequences of Longterm Unemployment in Europe," chapter 47, Volume 3, in *Handbook of Labor Economics*, (edited by Orley Ashenfelter, and David Card).

[33] **Marimon, Ramon and Fabrizio Zilibotti** (1999); "Unemployment vs. Mismatch of Talents: Reconsidering Unemployment Benefits", *Economic Journal*,109(455), pages 266-91.

[34] **Mortensen, Dale and Christopher Pissarides** (1998); "Technological Progress, Job Creation, and Job Destruction", *Review of Economic Dynamics*;1(4), pages 733-53.

[35] **Mortensen, Dale and Christopher Pissarides** (1999a); "Unemployment Responses to 'Skill-Biased' Technology Shocks: The Role of Labour Market Policy", *Economic Journal*, 109(455), pages 242-65.

[36] **OECD** (1996), "OECD Employment Outlook", OECD, Paris.

[37] **OECD** (1999), "OECD Employment Outlook", OECD, Paris.

[38] **Pissarides, Christopher** (1990); *Equilibrium Unemployment Theory*, MIT Press: Cambridge, MA.

[39] **Solow, Robert** (1960); "Investment and technological progress," in *Mathematical Methods in the Social Sciences*, (edited by Kenneth Arrow, Samuel Karlin and Patrick Suppes), pages 89-104, Stanford University Press: Stanford, CA.

Table 1: Cross-country labor market data

		1965	1970	1975	1980	1985	1990	1995	Change
	Unemp. Rate	0.018	0.011	0.017	0.029	0.045	0.054	0.061	0.043
Austria	Labor share	0.698	0.679	0.717	0.694	0.665	0.646	0.645	-0.053
	Inequality				0.820	0.790	0.870	0.880	0.060
	Unemp. Rate	0.023	0.022	0.064	0.114	0.111	0.110	0.142	0.120
Belgium	Labor share		0.667	0.729	0.730	0.682	0.685	0.676	0.009
	Inequality					0.660	0.650	0.640	-0.020
	Unemp. Rate	0.014	0.016	0.061	0.093	0.085	0.112	0.103	0.089
Denmark	Labor share	0.736	0.723	0.732	0.706	0.677	0.635	0.605	-0.131
	Inequality				0.760	0.770	0.770		0.010
	Unemp. Rate	0.025	0.021	0.050	0.051	0.047	0.121	0.167	0.142
Finland	Labor share	0.738	0.711	0.762	0.730	0.723	0.733	0.680	-0.058
	Inequality				0.890	0.920	0.940	0.930	0.040
	Unemp. Rate	0.020	0.027	0.049	0.079	0.101	0.105	0.115	0.095
France	Labor share	0.688	0.674	0.707	0.710	0.645	0.618	0.603	-0.085
	Inequality				1.210	1.210	1.240	1.230	0.020
	Unemp. Rate	0.010	0.011	0.037	0.060	0.075	0.078	0.099	0.089
Germany	Labor share	0.685	0.703	0.703	0.704	0.667	0.658	0.637	-0.048
	Inequality				0.870	0.830	0.830	0.810	-0.060
	Unemp. Rate	0.019	0.025	0.044	0.089	0.091	0.086	0.079	0.060
Greece	Labor share	0.693	0.699	0.698	0.694	0.690	0.712	0.692	-0.002
	Inequality								
	Unemp. Rate	0.047	0.055	0.078	0.112	0.164	0.146	0.120	0.073
Ireland	Labor share	0.828	0.842	0.835	0.833	0.763	0.715	0.645	-0.183
	Inequality								
	Unemp. Rate	0.041	0.043	0.051	0.070	0.099	0.096	0.120	0.079
Italy	Labor share	0.669	0.687	0.711	0.690	0.656	0.653	0.606	-0.063
	Inequality				0.850	0.830	0.770	0.970	0.120
	Unemp. Rate	0.010	0.018	0.038	0.080	0.081	0.062	0.071	0.061
Netherlands	Labor share	0.656	0.687	0.705	0.661	0.623	0.619	0.624	-0.032
	Inequality					0.920	0.960	0.950	0.030
	Unemp. Rate	0.016	0.015	0.018	0.026	0.030	0.056	0.049	0.034
Norway	Labor share	0.750	0.771	0.782	0.757	0.739	0.713		-0.037
	Inequality				0.720	0.720	0.680		-0.040
	Unemp. Rate	0.040	0.024	0.065	0.079	0.070	0.051	0.073	0.033
Portugal	Labor share	0.562	0.615	0.873	0.751	0.673	0.679	0.680	0.118
	Inequality								
	Unemp. Rate	0.028	0.030	0.059	0.161	0.200	0.196	0.230	0.202
Spain	Labor share	0.763	0.780	0.788	0.756	0.679	0.669	0.616	-0.147
	Inequality								
	Unemp. Rate	0.018	0.022	0.019	0.028	0.021	0.052	0.079	0.061
Sweden	Labor share	0.724	0.716	0.745	0.711	0.691	0.693	0.630	-0.095
	Inequality				0.750	0.760	0.730	0.790	0.040
	Unemp. Rate	0.019	0.025	0.044	0.089	0.091	0.086	0.079	0.060
UK	Labor share	0.693	0.699	0.698	0.694	0.690	0.712	0.692	-0.002
	Inequality				0.920	1.050	1.150	1.200	0.280
	Unemp. Rate	0.040	0.058	0.076	0.099	0.089	0.103	0.096	0.056
Canada	Labor share	0.716	0.660	0.652	0.634	0.630	0.666	0.659	-0.057
	Inequality				1.240	1.390	1.380	1.330	0.090
	Unemp. Rate	0.038	0.054	0.070	0.083	0.062	0.066	0.055	0.017
USA	Labor share	0.685	0.695	0.675	0.678	0.665	0.666	0.670	-0.015
	Inequality				1.180	1.350	1.380	1.470	0.290
Europe	Unemp. Rate	0.023	0.024	0.046	0.077	0.087	0.095	0.108	0.084
Average	Labor share	0.707	0.711	0.749	0.723	0.684	0.673	0.641	-0.058
	Inequality				0.859	0.841	0.844	0.900	0.040

Note: Data on unemployment rates are from Blanchard and Wolfers (2000). Data on labor shares are from Blanchard and Wolfers (2000) except the 1995 entry for Austria, Denmark, Ireland and Portugal which was computed directly from OECD data. Inequality is measured as the 90-10 log-wage differential for male workers. The data are taken from the OECD Employment Outlook (1996, Table 3.1). Austria: the measure is the 80-10 differential and data in the 1985 column are for 1987. Belgium: the measure is the 80-10 differential and data in the 1995 column are for 1993. Denmark: 1985 and 1990 columns are for 1983 and 1991 respectively. Finland: data in the 1985 column are for 1986. Germany: data in the 1985 and 1995 columns are for 1983 and 1993 respectively. Italy: data in the 1985, 1990 and 1995 columns are for 1984, 1991 and 1993 respectively. Netherlands: the measure of inequality is for males and females. Norway: data in the 1985 and 1990 columns are for 1983 and 1991 respectively. Moreover, the measure of inequality is for males and females. Portugal: data in the 1990 and 1995 columns are for 1989 and 1993 respectively. Canada: data in the 1980 and 1985 columns are for 1981 and 1986 respectively. For all countries, except US and UK, data in the 1995 column are for 1994. Europe average: unweighted mean of European countries, except UK.