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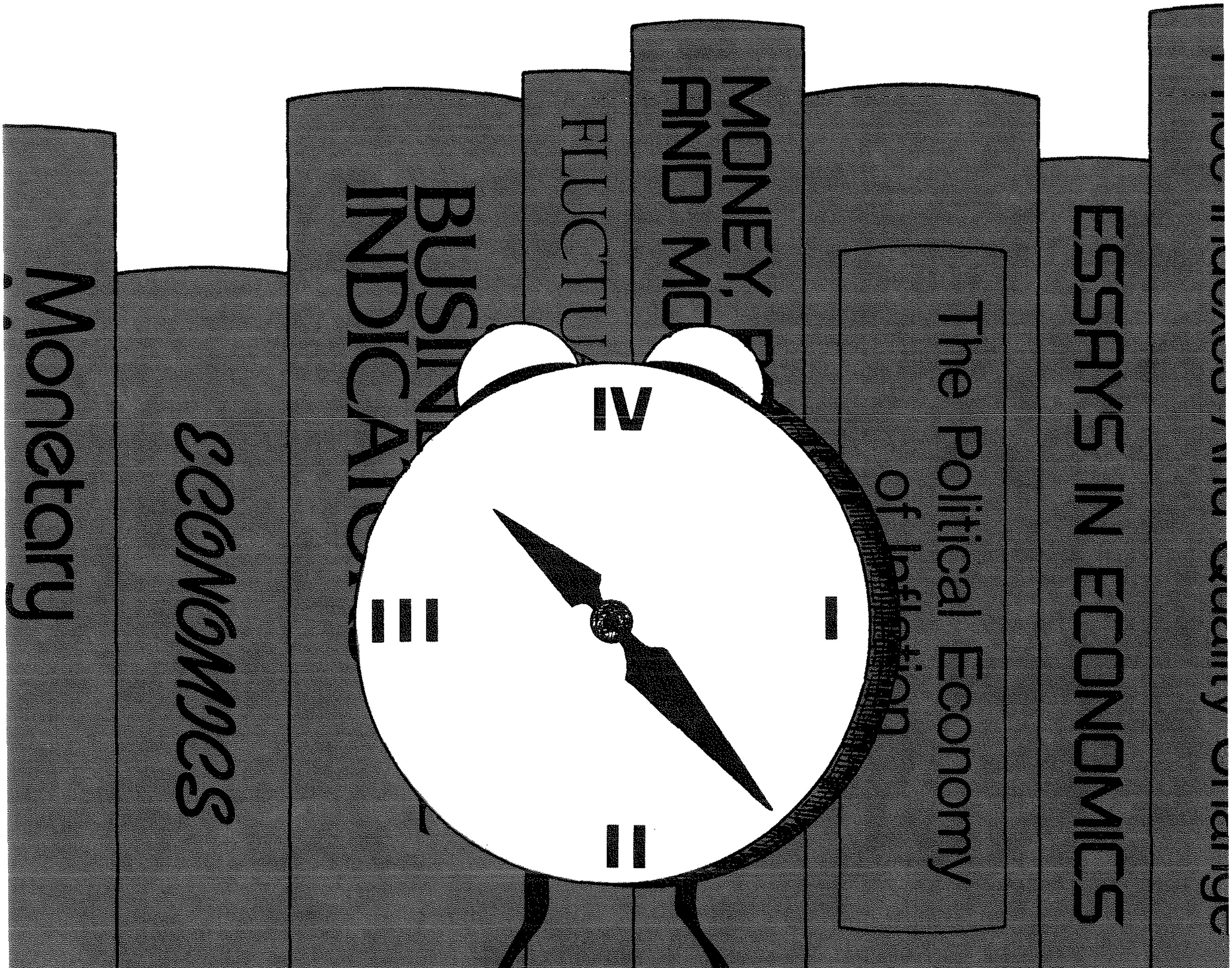

TIME AND ECONOMICS

Consumption, the Term Structure of Interest Rates and Inflation: An International Comparison

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This paper seeks to provide an answer to the question of how interest rates affect the level of real consumption. The framework used to solve this problem is one in which consumers make consumption plans using information about the entire term structure of interest rates. In theory, real consumption decisions are determined by relative prices and real interest rates, the latter defined as the nominal rate less the anticipated rate of inflation. Both determinants will be tested in the following analysis, the former, by testing whether the *level* of prices influences consumption, and the latter, by attempting to separate the effects on consumption of the term structure of nominal interest rates from the effects on consumption of the term structure of inflation. The same empirical analysis is undertaken for four countries: United States, United Kingdom, Canada and West German Federal Republic.

The plan of this paper is, first, to discuss the life cycle-permanent income model of consumption and the role of interest rates, in particular the spectrum of interest rates over all maturities, in influencing real consumption behavior. Second, several related theoretical issues are discussed, such as the concept of "money illusion" and the alternative ways a change in interest rates may influence consumption. Third, the form of the consumption equation to be empirically estimated is considered, followed by an analysis of related data measurement problems. Fourth, we discuss the estimated consumption equations for the U.S., U.K., Canada and Germany and, fifth, we conclude with the implications of the empirical findings, given the recent behavior of interest rates and inflation in the U.S. (Chart 1).

I. The Life Cycle-Permanent Income Model

The life cycle-permanent income model of consumption characterizes the consumption behavior of individuals as maximizing the utility or satisfaction derived from consumption over their entire lifetimes. The "constraint" on this implicit maximization we all perform in planning current consumption is not simply current income but current and all expected future income. So-called "life-cycle" models emphasize that aggregate wealth, defined as current wealth plus discounted expected income from labor and non-labor sources, is the variable which determines the "scale" of current consumption. Since an estimate of aggregate wealth involves an estimate of *discounted* future income, the models implicitly require some consideration of current and future interest rates. Thus, the term structure of interest rates is important for its influence on both the determination of the consumption allocation decision *and* the calculation of the wealth/income constraint.

As total wealth may be considered the "stock" constraint on the total lifetime consumption decision, the "flow constraint," that is, period-by-period, may be considered to be "permanent income." The concept of permanent income rests on the argument that consumers "smooth" the estimates of their income and base their consumption decisions on the non-transitory components of income. The basic idea to keep in mind, nonetheless, is the stock/flow distinction between wealth and permanent income. Both are constructed concepts to determine the lifetime and period-by-period constraints on the consumption decision.

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In short, the consumer is pictured as maximizing his utility of consumption over his lifetime. The constraint on this exercise is his total wealth. Permanent income is then some fraction of total wealth which the consumer uses as the constraint on period-by-period consumption. Empirically one can use, with equal justification, either total wealth or permanent income in the consumption equation. For practical reasons one often finds in estimated consumption equations some estimate of permanent income, for example, some weighted average of recent income levels, rather than wealth because of the difficulty in estimating the latter with readily available data. In theory, at least, either permanent income or wealth can be used in consumption functions since they are part and parcel of the same theoretical paradigm.

II. A Simple Model of Consumer Choice

Individuals make consumption decisions about what individual goods to purchase in a temporal framework, now and in the future. Consumption decisions at any one point in time are determined by the prevailing relative prices. The intertemporal decision to consumer a good today or at some time in the future must necessarily involve interest rates, as well as the current and expected future prices of goods. The reason is simple enough: interest rates are the unit of exchange, the "prices," determining the rate at which a dollar of consumption today can be transformed into a dollar of consumption in some future period.

One problem of formulating and empirically testing a model of consumption decisions over time is that there is no unique interest rate upon which consumption decisions depend. Rather, there is a spectrum of interest rates depending on the maturities of financial assets.[1] (We abstract here from default risk and its effect on interest rates and assume default-free securities.) The interest rate for each maturity corresponds to the price at which today's consumption can be "traded" for consumption at some specific date in the future. Decisions to consume today or to postpone consumption thus depend on the entire structure of interest rates over the horizon of potential future consumption.

To achieve an optimal allocation of consumption expenditures in each time period, a consumer must consider relative prices by equating the desired rate of substitution between consumption today and consumption in each future period to the ratio of the

Chart 1

Three-Month Treasury Bill Rate and Inflation

*Consumer inflation is measured by the twelve-month percent change in the implicit price deflator for personal consumption expenditures.

discounted prices for each period. Consider the simplest two-period case. Here the consumer achieves an optimal allocation of his expenditures between the two periods by allocating his consumption according to the rule:

$$MRS_{1,2} = \frac{P_1}{P_2/(1+{}_1r_2)} = \frac{1+{}_1r_2}{1+{}_1\pi_2} = 1+{}_1Z_2 \qquad (1)$$

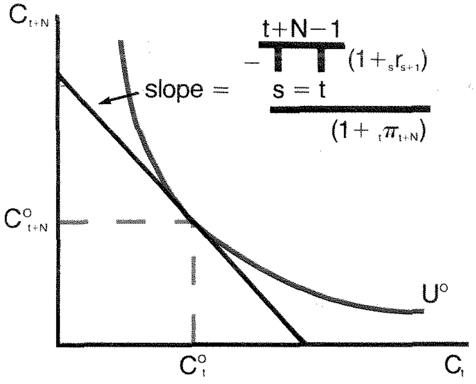
where $MRS_{1,2}$ is the marginal rate of substitution between consumption in period 1 and consumption in period 2; P_i the price level in period i, ${}_1r_2$ the short-term nominal interest rate between periods 1 and 2, and ${}_1\pi_2$ the rate of change in prices between periods 1 and 2. The marginal rate of substitution, $MRS_{1,2}$, is the ratio of the incremental satisfaction derived from consuming an additional unit of consumption in period 2 to that derived from consuming the good in period 1.

Equation (1) reveals that the consumption allocation decision between periods 1 and 2 depends on the "real interest rate," ${}_1Z_2$. Hence both nominal interest rates and the rate of inflation must be considered in the consumption decision. As equation (1) reveals, 1 plus the nominal interest rate divided by 1 plus the rate of change in prices is defined as 1 plus the real interest rate.

Equation (1) may be described graphically, as seen in Chart 2. Alternative combinations of consumption between any two periods leading to the same level of consumer utility describe the "indifference curve," U^O, along which the consumer is equally satisfied. Where optimal consumption takes place along this curve is determined by relative prices. In the simple two-period case this is given by equation (1). The marginal rate of substitution is the slope of the indifference curve at a given point. When the rate of consumption substitution is made equal to the ratio of discounted prices the consumer achieves the highest level of satisfaction for a given level of expenditure. In the case where the decision is between consumption today, period 1, and consumption in some future period beyond period 2, say, period k, the relevant relative price is given by the product of one-period interest rates, as seen in the expression given in Chart 2. This describes the role of the entire sequence of future interest rates in determining today's consumption decision vis-a-vis consumption in all future periods.

Chart 2

Representation of Consumer Equilibrium



Since consumption decisions involve more than the choice of consumption today versus consumption in the immediately adjacent period, equation (1) must be generalized to display the decision rule which permits an optimal allocation of consumption between the current period (say, period t) and any and all future periods over the consumers' consumption horizon. This equilibrium relationship can be expressed as simply

$$MRS_{t,t+N} = \frac{P_t}{P_{t+N}/[(1+{}_tr_{t+1})\,(1+{}_{t+1}r_{t+2})\cdots(1+{}_{t+N+1}r_{t+N})]}$$

$$= \frac{[(1+{}_tr_{t+1})\,(1+{}_{t+1}r_{t+2})\cdots(1+{}_{t+N-1}r_{t+N})]}{(1+{}_t\pi_N)}$$

$$\equiv (1+{}_tZ_{t+N}) \qquad (2)$$

Equation (2) states that the optimal allocation of consumption between period t and period t+N is given by the ratio of the discounted prices between the two period, where, for example, ${}_{t+1}r_{t+2}$ is simply the one-period interest rate between period t+1 and t+2. Rewritten we see this to be equal to the "real interest rate" prevailing over the period t to t+N.

Equation (2) provides a link between optimal consumption allocation and the term structure of interest rates. Consumers do not know with certainty what short-term interest rate will prevail at some future time. However, because individuals arbitrage between yields available on short-term and long-term financial assets, there must necessarily be an equilibrium relationship between currently known long-term yields and current and expected short-term interest rates. If there are no market impediments to traders of financial assets of different maturities, and investors have similar risk preferences, the long-term interest rate will be approximately equal to the geometric average of the yield on current and expected future one-period securities, given by equation (3).

$$(1 + {}_t r_N) = [(1 + {}_t\bar{r}_{t+1})(1_{t+1}r_{t+2}) \cdots (1 + {}_{t+k-1}r_{t+N})]^{1/N} \quad (3)$$

A bar over a variable in equation (3) denotes a currently known value. Equation (3) is often referred to as the Hicksian "expectational model" of the term structure of interest rates because it defines an equilibium relationship between a known long-term interest rate and current and *expected* future short-term (one-period) interest rates. The r's in equation (3) without bars over them are implied "forward" (or expected) one-period interest rates. Equation (3) can be used to obtain any forward one-period rate by simply taking the ratio on known adjacent long-term bonds, as follows:

$$1 + {}_{t+j}r_{t+j+1} = \frac{(1 + {}_t\bar{r}_{t+j+1})^{j+1}}{(1 + {}_t\bar{r}_{t+j})^{j}} \quad (4)$$

Equations (2), (3), and (4) show the natural interrelationship between intertemporal consumption theory and term structure theory. A change in any long-term rate, or any expected future short-term rate, with given current and expected future prices, will change the right-hand side of equation (2). Hence the left-hand side must change, implying the consumer must marginally adjust his consumption between current consumption and consumption in the future in order to maintain an optimal allocation.

It has long been recognized that in theory, intertemporal consumption-saving decisions involve the entire maturity structure of interest rates. Empirically, however, investigators have, for the most part, swept the issue under the rug, often opting for the use of a particular interest rate for a given maturity, or attempting to model interest rate expectations by some ad hoc weighting of selected interest rates. These are but two of the reasons that the empirical literature on interest rate effects on consumption finds little agreement on the direction of the effect, let alone any agreement on the magnitude of such effects.

As seen in equation (2) above, real (price-deflated) consumption decisions are dependent on *real* interest rate. Thus, in principle, it is the term structure of real, and not nominal, interest rates whose influence on consumption we wish to determine. To uncover such a structure, however, presents several theoretical and empirical difficulties, since it requires the approximation of not only an anticipated rate of inflation over a given time period, but the approximation of the term structure of inflation for all maturities on the available financial assets. In principle, such an undertaking is required if we are to identify the effect on real consumption of changes in the term structure of real interest rates. As will be seen shortly, the procedure we chose will allow us to measure indirectly the effect of changes in the term structure of real interest rates on real (price deflated) consumption.

III. Other Theoretical Issues Regarding Consumption

It has been traditional in empirical studies to assume that the demand for real consumption is unaffected by the *level* of all prices. This proposition, called the "absence of money illusion," is held to apply in both the short-run and the long-run. In theory, this proposition is quite logical: a scaling of all nominal quantities, prices, income and wealth, should leave real demands unchanged. In recent years, however, changes in price levels have been argued to have short-run impacts on perceptions of changes in relative prices, and thereby to give rise to a real demand or supply response. Little empirical work has been attempted to isolate these so-called money illusion effects on consumption.[2] Our empirical estimation, however, will attempt to capture both the effects of the term structure of interest rates and the effects of the price level, or money illusion, on aggregate real consumption.

Above, we noted that inflation played a role in the consumption decision by determining the real interest rate. (See equation 2.) But aside from this role, is there any independent role inflation can play in determining real consumption? Such a role is possible in that, because they lack information on all prices, consumers may interpret a rise in a particular good's price with a change in its relative price. This confusion may lead to systematic changes in their consumption patterns. Thus, inflation has recently been given an additional separate role in consumption studies, that of confusing consumers' perceptions of relative prices.

Inflation can also have an independent role in consumption via three other avenues: real taxes, wealth and uncertainty. First, income taxes are not indexed in many countries, so inflation often results in a rise in real tax rates. This rise reduces the real returns on financial assets and may possibly induce increased expenditures on goods where the tax is either absent or sheltered, such as real estate. The deductibility of interest payments from taxable income is hence a greater haven from taxes when the real tax rate is raised by inflation. Second, while inflation reduces the real value of financial assets, it also decreases the real value of financial liabilities. It is thus possible for changes in consumption to result from changes in the real *net* wealth position of households. Such effects would not likely be captured with available income or wealth data series. Third, uncertain knowledge of future inflation makes future real income uncertain, and, it has been argued, reduces current consumption in some cases. That is, with an equal probability that future real income will be greater or less by the same amount, risk-averse individuals will choose to reduce current consumption.[3]

Is it possible, a priori, to establish what the sign of the effect of a rise in real interest rates will be on consumption? The answer in general is no.[4] The reason is well known and common to all empirical demand equations. A rise in an interest rate has two opposing effects on consumption: a positive income effect and a negative substitution effect. With a higher interest rate, individuals can increase current consumption without decreasing future consumption because a higher interest rate will generate greater future cash flow; this is the positive income effect. In contrast, a rise in the interest rate also induces a negative substitution effect on current consumption. The higher interest rate means that "forward prices," or, equivalently, discounted expected future prices, have fallen, creating the incentive to postpone current consumption in favor of purchasing goods in the future at lower effective prices. It is an empirical matter whether this latter negative substitution effect will dominate the positive income effect.

The ambiguity of the effect of interest rates on consumption is readily apparent in the empirical literature on consumption as well. To mention two recent studies, Michael Boskin and Warren Weber have estimated aggregate consumption functions with interest rates as explanatory variables and have obtained opposite results, Boskin finding that a rise in interest rates decreases consumption, and Weber finding that consumption is increased with a rise in interest rates.[5]

Weber's results are interesting because they hint at the importance of future interest rates. Weber finds that a rise in the weighted average of current and past nominal interest rates increases consumer expenditures on nondurables and services. He argues that the increase in the weighted average of current and past nominal interest rates suggests that nominal interest rates will be *lower* in the future. Hence the *expected* decline in interest rates would tend to increase the present value of future income. This rise in the present value of future income, Weber argues, causes consumers to increase current consumption. Weber's study suggests that understanding how changes in current interest rates may affect current consumption requires that one posit a relationship between *currently observed* interest rates and *expected future* interest rates. Such a relationship can resemble the expectational theory of the term structure, as was seen above. We now turn to the specification of our estimable consumption function.

IV. Consumption Function Specification

The arguments above have concentrated on why the term structure of real interest rates should influence consumption decisions. This section will briefly outline the functional specification of the consumption equation.

The basic life-cycle consumption argument is that consumption depends on total wealth. This relationship may be specified as simply:

$$c_t = k\,W_t \tag{5}$$

where c is real consumption, W real wealth, and k a parameter. Equation (5) represents current consumption as dependent on the stock of current real wealth. Assume that the consumer's planning horizon is infinite so that his stock of real wealth generates a flow of income each period proportional to his stock of wealth. This permanent income flow from wealth can be written as

$$y^P_t = iW_t \tag{6}$$

where y^P is permanent income and i a real interest rate. Solving for W in (6) and substituting into (5) yields

$$c_t = k \frac{y^P_t}{i} \equiv K(i)\ y^P_t \tag{7}$$

The proportionality factor K is now written as a function of the real interest rate. If we define the real interest rate i as the nominal interest rate, r, less the rate of inflation, and argue, as above, that real consumption depends on the term structure of real interest rates, the function K( ) would have to include more than one real interest rate. In addition, if individuals' real consumption decisions are altered by movements in the price level *and* the rate of inflation, these variables should also be incorporated in the K function. These considerations lead to the general specification of our real consumption equation.

$$C_t = K(r_{1t}, r_{2t}, \cdots, r_{mt}, \pi_{1t}, \pi_{2t}, \cdots, \pi_{mt}, P_{1t}, \cdots, P_{it})\ y^P_t \tag{8}$$

Equation (8) captures the effects of the term structure of real interest rates but includes both nominal interest rates and inflation variables. The first subscript on the interest rate and inflation variables represents the maturity of the financial assets. Thus, r_{mt} is the nominal interest rate at time t on an asset with m years to maturity and π_{mt} the *observed* inflation rate over the past m years. Equation (8) also includes the effects of changes in the price levels on disaggregate price indices. The reason for the inclusion of the disaggregate price indices is primarily statistical. Statistically, we can capture the presence or absence of money illusion with one aggregate price level variable only if all subaggregate indices move proportionately. Since this requirement is unlikely to be satisfied by the observed components of the aggregate index, we disaggregate the price level variable.

Recall that by definition the nominal interest rate, r, includes both a real interest rate and an anticipated inflation component. Equation (8) includes as independent variables both the level of nominal interest rates and the rate of inflation. This implies that in a regression equation the effect of the real interest rate on consumption is captured by the coefficient on the *nominal* interest rate while the independent effect of inflation on real consumption is captured by the *sum* of the coefficients on the nominal rate and the inflation variables. Since the nominal interest rate includes an anticipated inflation component its effect must be added to the separately measured observed inflation variable.

Three measurement problems remain before subjecting equation (8) to statistical estimation. The first is to devise some approximation for the time series data on permanent income. The other two measurement problems are related. Statistically, it is difficult to obtain reliable estimates of the effects of interest rates and inflation on consumption if more than a couple of these variables, for different maturities, are included in the equation. Hence, we devised proxy variables for the term structure of nominal interest rates and inflation.

V. Measurement Problems

In theory, what we wish to explain is consumption, not expenditures on consumer goods. The latter would include expenditures on durable goods, only a part of which can economically be considered current consumption. We therefore define consumption as expenditure on consumer nondurable goods and services plus a portion (the current service flow or depreciation) of the stock of consumer durables. This definition of current consumption was implemented where data provided a means of approximating the service flow of consumer durable goods, namely, in the U.S. and the U.K. (see Appendix). For Canada, our consumption variable is simply expenditure on consumer nondurable goods and services. In the case of Germany, disaggregated consumption data were not available and hence what we will seek to explain will be total consumer expenditures.

As described above, the hypothesis that consumption is dependent on permanent, and not currently observed income, derives basically from the argument that consumption is, in principle, dependent on wealth, where wealth is defined as discounted future income from both labor and non-labor sources. Wealth is thus a stock and permanent income, the flow; permanent income being some proportion of total wealth. Real consumption is thus dependent on the flow of current permanent income.[6]

Permanent income is assumed to adjust by some amount each period in proportion to the flow of current income. Hence, the estimate of permanent income is updated by the flow of observed income. To estimate permanent income according to this assumption, we use the following well-known approximation.[7]

$$y_t^P = \lambda y_t + (1-\lambda)(1+\delta)y_{t-i}^P \qquad (9)$$

In equation (9), y represents observed real per capita personal disposable income, y^P permanent income, δ the estimated growth rate (trend) of real per capita income and λ the permanent income adjustment coefficient, revealing what proportion of current observed personal disposable income is used to update the estimate of permanent income. The notion behind equation (9) is that future income is uncertain and variable, so the consumer obtains a "smoothed" estimate of his permanent income by updating it by some portion of recently observed income.

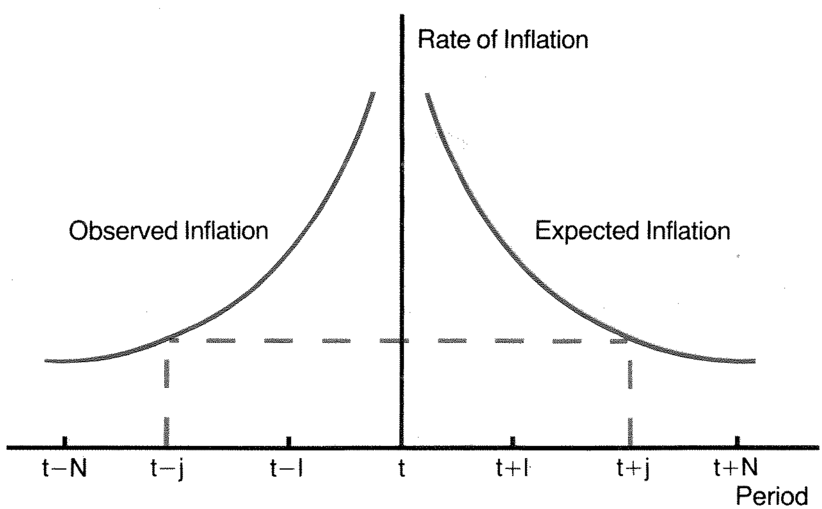
Equation (9) is nonlinear in the adjustment parameter λ. Hence alternative consumption functions are estimated for different measures of permanent income, iterating on values of λ between 0 and 1. That value of λ is selected with minimizes the standard error of the estimated consumption equation. δ is obtained by first regressing the log of income on a quadratic in time.

Economic theory provides little guidance as to which interest rates are appropriate to include in consumer demand equations. Intertemporal consumer choice implies that the entire term structure of interest rates is relevant, yet, since many interest rates, particularly those along the short-end of the maturity structure, move together, using more than a couple of interest rates in demand equations often leads to statistical problems of multicollinearity. Multicollinearity, in turn, results in the inability to distinguish statistically among the effects of different interest rates on aggregate demand.

Instead of including several interest rates in an aggregate demand equation, we adopt a parameterization suggested by Heller and Khan and used with some success in their study of money demand.[8] The procedure, for each time period, is to regress the vector of short-to-long-term interest rates, the term structure, against a quadratic in the maturities of the

Chart 3

Representation of the Term Structure of Observed and Expected Inflation



corresponding financial assets. The estimated coefficients are then saved and used as explanatory variables in the estimation of the consumption function. Formally, assuming that there are financial assets of N different maturities and T observations on the market yields, we estimated the following regression for each sample period:

$$\ell\eta\ r_{mt} = A_{Ot} + A_{1t}m + A_{2t}m^2 + v_t \qquad (10)$$
$$m = 1, \cdots, N; t = 1, \cdots, T$$

where r_{mt} is the market interest rate at time t on the asset with a maturity of m years. We thus approximate the maturity structure of rates with a quadratic in the maturities (v_t is the stochastic error for the term structure vector, $(r_1,\ldots,r_N)_t$, for which coefficients are to be estimated). The estimation of (10) will yield three term structure coefficients for each time period, $t=1,\ldots,T$. These coefficients are saved and used as explanatory variables in our consumption function.[9]

Since real interest rates determine real consumption decisions, we require either an estimate of the term structure of real rates or a direct estimate of the term structure of inflation. Given the effects of the latter on consumption, the former can be derived. We, therefore, follow a procedure similar to the above for estimating inflation. In this case, we estimate coefficients of the term structure of inflation over the immediate past and assume that this is the best directly available estimate of the future term structure of inflation. This assumption is represented graphically in Chart 3. The following equation is estimated for each sample period.

$$\ell\eta\pi_{mt} = B_{Ot} + B_{1t}m + B_{2t}m^2 + u_t \qquad (11)$$
$$m = 1, \cdots, N; t = 1, \cdots, T$$

where π_{mt} is the annual rate of inflation which has occurred over the immedediate past m years, and where the maturities of the measured past inflation rates correspond to the maturities of the financial assets. The coefficients B_O, B_1 and B_2 thus capture the prevailing (past) term structure of inflation.

Equations (10) and (11) may be viewed as an exponential approximation to the term structure of interest rates and inflation respectively. The coefficient A_O is the shift or intercept parameter for the interest rate term structure, A_1+2A_2m the slope of the term structure at maturity m and $2A_2$ its curvature. The intercept term does not by itself determine the average level of interest rates because a change in the other two terms can change the average level even with the intercept term held constant. (The same applies to the B coefficients for the term structure of inflation.) Note that the slope of the term structure is a function of its maturity. The signs of A_1 and A_2 will determine whether the term structure is positively or negatively sloped.

The estimated coefficients A_O, A_1, A_2 and B_O, B_1, B_2 will all be entered as explanatory variables to explain the behavior of real consumption. If real interest rates were important in determining consumption, the A coefficients would have to be of opposite signs from the B coefficients. The reason is as follows: Assume that a rise in real interest rates decreases current consumption. This implies that with inflation held constant (the B coefficients), a rise in the *nominal* structure of interest rates (the A coefficients) would cause real consumption to fall. Thus the estimated consumption function coefficients on the A variables should be negative. That is, a rise in the nominal term structure of interest rates, holding the term structure of inflation constant, increases the term structure of real interest rates, reducing consumption.

Alternatively, assume that the term structure of nominal interest rates remains fixed and the term structure of inflation rises. Real rates thus fall and, by assumption, should increase real consumption. This implies that the consumption function coefficients on the B variables should be positive. Moreover, the effect of the inflation rate on real consumption is given by the *sum* of the respective coefficients on the A_i and B_i variables. Since nominal interest rates include the anticipated rate of inflation, this effect, captured in the coefficient on the A variables, must be added to the impact of inflation captured by the coefficients of the B variables.

Because the forward-looking term structure of inflation is being proxied in the estimation of equation (11) by the observed past term structure of inflation, the assumption is that a reasonable estimate of future inflation, for all maturities, is the observed inflation rate. The term structure of past inflation is thus assumed to be symmetrical to the term structure of future inflation expectations. (See Chart 3.) Given this empirical assumption, the estimation of equation (11) required to obtain the three B coefficients, for each period over which the consumption function is to be estimated, limits the sample period for consumption function estimation. The reason is obvious enough: to estimate a consumption function which uses, for example, a ten-year interest rate term structure, requires that the term structure of inflation estimation use ten years of past inflation observations. Thus the availability of price data constrains the period for the estimation of the consumption function.

The above considerations lead us to the specifica-

Table 1
Estimated Real Consumption Equations*

Explanatory Variables	U.S.	Canada	U.K.	Germany
Constant	2.473 (4.38)	2.927 (3.61)	4.747 (4.08)	2.279 (.80)
Permanent Income	.621 (7.98)	.579 (5.59)	.406 (2.98)	.754 (2.30)
Term Structure of Interest Rate Variables				
A_0	−.028 (−3.46)	−.024 (−1.36)	−.080 (−1.63)	.001 (.03)
A_1	−.833 (−3.44)	−.421 (−1.76)	−3.136 (−2.25)	.111 (.45)
A_2	−16.694 (−2.94)	−5.785 (−2.17)	−77.540 (−2.15)	.942 (.53)
Term Structure of Inflation Variables				
B_0	.031 (1.77)	.058 (2.72)	−.295 (−.89)	−.049 (−3.13)
B_1	1.844 (4.31)	.815 (3.68)	−4.524 (−.63)	−.249 (−4.90)
B_2	42.159 (4.70)	8.726 (3.84)	−76.897 (−.54)	−1.167 (−5.09)
Price Level Variables				
$\ell\eta$ P-durables	.047 (5.37)	−.240 (−1.94)	−.069 (−.70)	−.446 (c) (−2.15)
$\ell\eta$ P-nondurables	.009 (.19)	.086 (.97)	.404 (1.94)	−.465 (b) (−5.14)
$\ell\eta$ P-services	.062 (1.73)	.332 (2.39)	−.019 (.15)	−.031 (−.15)
$\ell\eta$ P-semidurables	—	−.223 (−1.85)	—	.919 (a) (4.99)
Autoregressive Error Structure				
e (t-1)	.89 (7.08)	.89 (18.70)	—	−.78 (−4.47)
e (t-2)	−.67 (−5.55)	—	—	−.77 (−4.42)
Sample Period	1970.III-81.II	1959.IV-81.II	1973.IV-81.II	1974.II-80.IV
$\bar{R}^2$	.99925	.99906	.8557	.99616
Durbin-Watson	2.13	2.21	1.96	2.02
Std. Error of Regression	.002201	.00679	.01609	.00415

*Dependent and independent variables are in logs.
t-statistics below coefficients. The λ adjustment coefficients used in the construction of the permanent income series for the U.S., Canada, U.K., and Germany were, respectively, .26, .01, .84, and .0075.

(a) The price index variable here is for rent.
(b) The price index variable here is for food.
(c) The price index variable is defined for "industrial products."

tion of our consumption function with term structure effects, specified in logarithmic form:

$$\ell\eta\, c_t = \phi + \lambda\ell\eta y_t^P + \eta_1\ell\eta P_{1t} + \cdots + \eta_N\ell\eta P_{N_t} + \alpha_o A_{Ot} + \alpha_1 A_{1t} + \alpha_2 A_{2t} + \beta_o B_{Ot} + \beta_1 B_{1t} + \beta_2 B_{2t} + e_t \quad (12)$$

Recall in equation (12) that A_{Ot}, A_{1t}, and A_{2t} are the parameters derived from the regression of the log of the vector of interest rates on a quadratic in the maturity structure. Similarly, B_{Ot}, B_{1t} and B_{2t} are the parameters derived from the regression of the log of inflation on a quadratic in the maturity of inflation. The A's and B's in equation (12) are treated as explanatory variables representing the term structure of nominal interest rates and inflation, respectively.

To summarize, we argue that if real interest rates, more specifically the term structure of real interest rates, affect real consumption, then the coefficients on the interest rate and inflation proxy variables in (12) ought to be of opposite sign. If a rise in the term structure of *nominal* interest rates, represented by the A_t's (representing a rise in real interest rates with inflation held constant) decreases current consumption, a rise in the term structure of inflation, which implies a decline in real interest rates, should increase current consumption. We now turn to estimation of equation (12) for the U.S., Canada, the U.K. and Germany.

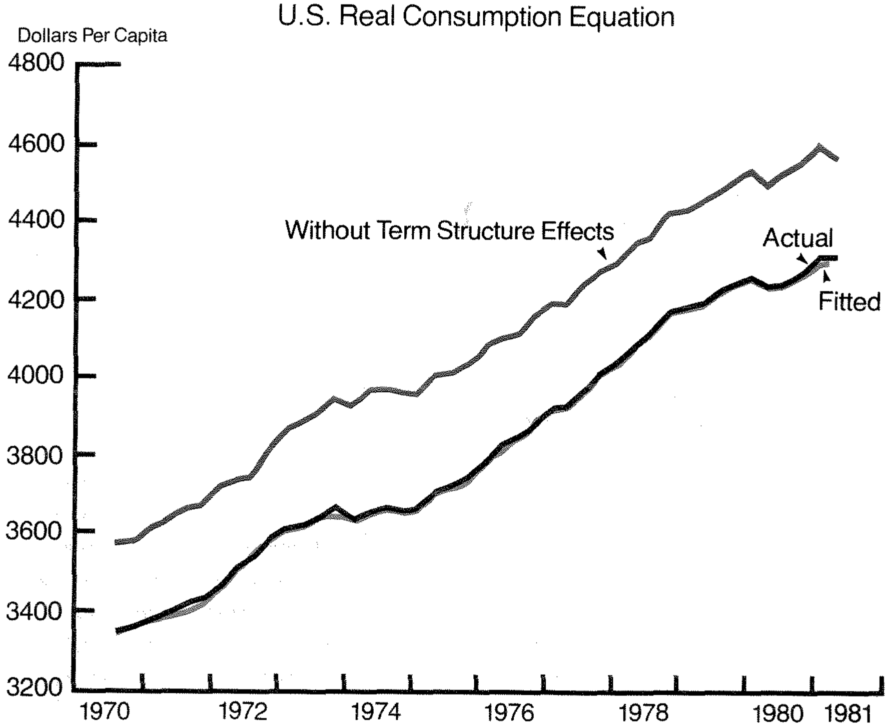
VI. The Empirical Consumption Function

Empirical estimates of equation (12) appear in Table 1. Note that the price level explanatory variables are the disaggregate components of consumer price indices. For the U.S. and the U.K., these components are the price indices for durables, nondurables and services; for Canada, these three plus a series for semi-durables; and for W. Germany, the addition of rent, food, consumer industrial products and services price components.

The next thing to note in the empirical estimates is that the coefficients of the term structure of nominal interest rates and of inflation are of opposite sign, as suggested above, for three of the four countries considered, the exception being the U.K. A

Chart 4

U.S. Real Consumption Equation



rise in the term structure of nominal interest rates, holding the term structure of inflation constant, is seen to *decrease* real consumption for the U.S., Canada and the U.K. The t-statistics reported below the coefficients in Table 1 suggest that the shape of the term structure, and not simply the general level of interest rates, helps determine current real consumption.

As suggested earlier, the coefficients on the A_0, A_1 and A_2 variables can be interpreted as representing the impact of the term structure of real interest rates on consumption. All three coefficients on these variables are negative for the consumption equations estimated for the U.S., Canada and the U.K. These results suggest that a rise in the term structure of real interest rates will *decrease* real consumption. The estimated coefficients are significant at the 5-percent significance level for a two-tail test except for the A_0 coefficients for Canada and the U.K. This last result suggests that it may be the shape of the term structure, as well as of the general level of interest rates, which is affecting real consumption. In the case of Germany the coefficients on the interest rate variables are all positive, but none are significant. We conclude from this that neither the level nor the shape of the term structure has any impact on real consumption in Germany.

The estimated equations capture expected inflation in two ways. First, the B_0, B_1 and B_2 variables act as proxies for inflation. Second, since the nominal interest rate is assumed to contain an expected inflation premium, the expected inflation rate is also captured in the coefficients of the A variables. Hence, summing the respective coefficients on the A and B variables yields the total impact of expected inflation on real consumption. Carrying out this exercise, we find that a rise in expected inflation increases real consumption in the U.S. and Canada and reduces real consumption in the U.K. and W. Germany. The coefficients on the directly estimated expected inflation proxy variables for the W. German equation are all negative and very significant at conventional significance levels.

The assumption that real consumption can be represented with a permanent income hypothesis is supported by the estimated equations. The permanent income variable coefficients are all significant. The elasticities of real consumption with respect to permanent income are very similar for Canada and the U.S.

Except for the German case, the appearance of widespread "money illusion," displayed by significant coefficients in the price *level* variables, is not confirmed by the equations for the U.S., Canada and the U.K. While one or more individual price level coefficients are found to be significant, we do not establish the degree of money illusion in consumption which other investigators have uncovered.[10] Only in the case of Germany is significant money illusion evident, and here it may be due to a data measurement problem, since the dependent variable includes expenditures on durable goods which cannot accurately be called consumption.

The general impression one obtains from viewing the results in Table 1 is that, within the context of a permanent income consumption model, the term structure of real interest rates plays a significant role. This result has not previously been noted in other empirical consumption functions studies but it is implicit in much of the theoretical literature on intertemporal consumption. Furthermore, in three of the four countries considered, a rise in the term structure of nominal interest rates, holding constant the term structure of inflation, was found to decrease real consumption. In the U.S. and Canada, inflation appears to increase real consumption, but it appears to decrease real consumption in the U.K. and W. Germany.

As we have stated above, it is not possible to determine a priori whether a rise in real interest rates will increase or decrease current consumption. The sign of the effect depends on whether the negative substitution effect is greater or less than the positive income effect. Our empirical results appear to indicate that for the U.S., Canada and the U.K. the substitution effect dominates the income effect—*Real interest rates have a negative impact on real consumption.*

The quantitative impact of the term structure of interest rates and inflation can be summarized for the U.S. by viewing Chart 4. Here, the estimated equation in Table 1 is fitted over the estimation period with and without the effects of the interest rate and inflation term structure variables, along with the actual values of real per capita consumption. As can readily be seen, real per capita consumption is considerably overstated when the term structure effects are dropped from the equation.[11]

VII. Conclusion

The aim of this paper was to determine whether real consumption was influenced by the term structure of interest rates in a model which captured the influence of permanent income and the impact of price level movements. The answer was found generally to be in the affirmative for the four countries considered. A rising term structure of real interest rates was found to decrease current consumption in the case of the U.S., U.K., and Canada, and to increase current consumption in the case of Germany. By holding constant the effect of inflation over varying maturities, it was found that the impact of the term structure comes from the impact of changes in real interest rates. Holding constant the term structure of real interest rates, a rise in inflation appears to increase real consumption in the U.S. and Canada and to reduce consumption in the U.K. and Germany. Although not reported here, it was also found that the coefficients on the term structure variables were often more robust than the coefficients on permanent income or price levels, that is, the term structure coefficients changed much less when alternative functional specifications or sample periods were chosen for estimation.

Consider now the policy significance of the empirical results. From June to August 1982, interest rates in the U.S. declined dramatically. The three-month Treasury bill rate fell from an average 12.47 percent in June to 7.92 percent in September. From the results of this analysis, this rapid decline in interest rates in the U.S. can be expected to contribute to growth in real consumption spending in the U.S. Since we found that inflation generally had a positive impact on real consumption in the U.S., we can expect the declining inflation rate to weaken real consumption. Quantitatively, however, the major decline in real interest rates, particularly at the short-end of the maturity spectrum, should provide a major boost to consumption and to the general economic recovery.

Data Appendix

This appendix briefly describes the source of the data and the construction of variables used in the test. The constructed variables include the real per capita permanent personal disposable income for the U.S., U.K., Canada and Germany, the A_0, A_1 and A_2 and B_0, B_1 and B_2 variables used to approximate the term structure of interest rates and inflation, respectively, for each country, the generation of the service flow from durable goods for the U.S., and the quarterly interpolation from annual data on capital consumption at current replacement cost for the U.K. personal sector. Population series for the U.K. and Germany are obtained by interpolating annual to quarterly observations.

I. Data Sources

(A) **Germany**

Disposable income, private consumption and the cost of living index for all households are taken from Statistical Supplement, Series 4, to the **Monthly Reports of the Deutsche Bundesbank**. Annual population series were obtained from the annual O.E.C.D. country reports for Germany. Disaggregate prices for Germany were obtained from the Data Resources, Inc., databank. Interest rates are Eurocurrency deposit rates for 1-, 3-, 6- and 12-month, and 2-, 3-, 4- and 5-year maturities, taken from the Data Resources, Inc., databank.

(B) **United Kingdom**

Capital consumption at current replacement cost for the personal sector is taken from the Central Statistical Office's **National Income and Expenditures**, 1981 Edition (the Blue Book), an annual series, and interpolated to quarterly figures. Series for personal disposable income, consumer expenditures and prices are taken from the Central Statistical Office's **Monthly Digest of Statistics**, various issues. An annual population series was obtained from the GSO's **National Income and Expenditure**, 1981 Edition, interpolated to get quarterly series. Interest rates used to construct the term structure variables included the three-month interbank rate, the gross redemption yield on local authority one-year bonds, and the gross redemption yield on 5-, 10-, and 20-year gilts. These interest rate data were kindly provided by Mr. Paul Temperton, Economics Division, Bank of England.

(C) **United States**

Personal disposable income, consumer expenditure series for durable and nondurable goods and services, the personal consumption expenditure price deflator, and its components, are taken from the **National Income and Product Accounts**. Interest rate data used to approximate the term structure included the 3- and 6- month Treasury bill yields and the yields on constant maturity 1-, 3-, 5-, 10- and 20-year Treasury bonds, available from the Federal Reserve **Bulletin**. The benchmark stock of consumer durables used to derive the service flow from durable goods held by the household sector was obtained from the **Flow of Funds Accounts**.

(D) **Canada**

Data for personal disposable income, personal expenditures on consumer and services goods (services and nondurables), and price indices for personal expenditures on consumer goods and services were obtained from the **National Income and Expenditure Accounts**, Volume 2 (the quarterly estimates 1947-1974), and quarterly issues of the same publication. The population series came from the **Canadian Statistical Review**, various issues. Canadian term structure of interest rate data on Canadian Government securities: 3- and 6-month Treasury bills, bonds of 1–3, 3–5, 5–10 and 10-years-and-over maturities were kindly provided by the Bank of Canada.

II. Construction of the Service Flow from Durable Goods for the U.S.

To accurately reflect total quarterly consumption of durable goods, it is necessary to separate the service flow derived from these goods in each period from total expenditures on durable goods in each period. To do so requires a benchmark estimate of the stock of durable goods held by households. This stock was taken as the end of the year 1953 value from the Flow of Funds Accounts of October 1981. For the period 1954.I to 1981.II we then solved the following two equations:

$$DD_t = .02761(ED_t) + .21784(SD_{t-1})$$
$$SD_t = .25(ED_t - DD_t) + SD_{t-1}$$

where

ED_t = expenditure on durable goods in period t

SD_t = the stock of durables in period t

DD_t = the service flow from durables in period t

This methodology is that used in the Board of Governors staff's quarterly econometric model. Personal consumption is then defined as expenditures on nondurables and services plus the derived DD variable.

III. Construction Quarterly Data from Annual Observations

Where interpolation is used to derive quarterly series from annual observations, the method of cubic splines interpolation was employed.

FOOTNOTES

1. There are several ways of formulating the simple intertemporal consumption problem. Two of the clearest presentations can be seen in J. M. Henderson and R. E. Quandt, **Microeconomic Theory**, 2nd edition, pp. 293–309, and E. F. Fama and M. H. Miller, **The Theory of Finance**, Holt, Rinehart and Winston. The role of discounted prices in intertemporal consumption analysis is emphasized in Henderson and Quandt and in A. Deaton and J. Muelbauer, **Economics and Consumer Behavior**, Cambridge University Press, (1980).

2. An early attempt to estimate money illusion effects on real consumption is W. H. Branson and A. K. Klevorick, "Money Illusion and the Aggregate Consumption Function," **American Economic Review**, (December 1969). See also A. Deaton, "Involuntary Saving Through Unanticipated Inflation," **American Economic Review**, (December 1977), and J. C. Townsend, "The Personal Saving Ratio," **Bank of England Quarterly Bulletin**, (March 1976). The role of the misperception of the price level on aggregate demand is considered in a theoretical model by R. J. Barro, "Rational Expectations and the Role of Monetary Policy," **Journal of Monetary Economics**, (1976).

3. See especially Deaton (1977), cited above, with respect to the role of inflation in causing confusion over true relative price changes. With respect to the effect of uncertain real income on consumption, see J. Dreze and F. Modigliani, "Consumption Decisions Under Uncertainty," **Journal of Economic Theory**, (December 1972).

4. An illustration of the theoretical ambiguity in determining alternative interest rate effects on consumption can be seen in J. I. Bernstein and D. Fisher, "Consumption, the Term Structure of Interest Rates and the Demand for Money," Working Paper #1976-8, Concordia University. See also Henderson and Quandt, cited above.

5. See M. J. Boskin, "Taxation, Saving and the Rate of Interest," **Journal of Political Economy**, (April 1978, Part 2), and W. E. Weber, "Interest Rates, Inflation and Consumer Expenditures," **American Economic Review**, (December 1975).

6. See Milton Friedman, **A Theory of the Consumption Function**, Princeton University Press, (1956). A recent test of the permanent income hypothesis which utilizes the concept of rational expectations is given in M. A. Flavin, "The Adjustment of Consumption to Changing Expectations About Future Income," **Journal of Political Economy**, (1981).

7. See M. R. Darby, "The Permanent Income Theory of Consumption—A Restatement," **Quarterly Journal of Economics**, (May 1974), and J. J. Seater, "On the Estimation of Permanent Income," **Journal of Money, Credit and Banking**, (February 1982).

8. H. R. Heller and M. S. Khan, "The Demand for Money and the Term Structure of Interest Rate," **Journal of Political Economy**, (February 1979).

9. Quarterly term structure of interest rate data were used to estimate equation (10) for each quarterly sample period over which the consumption function was to be estimated. Data for the U.S. included yields on the three-month and six-month Treasury bills and the constant maturity 1-, 3-, 5-, 10-, and 20-year government bonds. In the case of Germany the term structure data were yields on 1-, 3-, and 6-month and 1-, 2-, 3-, 4-, and 5-year Eurocurrency deposit rates from 1973.IV to 1981.III. Data for the U.K., supplied by the Bank of England, included the three-month interbank rate, the gross redemption yield on local authority one-year bonds, and the gross redemption yields on 5-, 10-, and 20-year gilts. While the U.K. data are not all yields on government securities, they were suggested to be representative market yields for the given maturities. For Canada yield data were for 3- and 6-month Treasury bills and government bond yields for 1–3-year, 3–5-year, 5–10-year and 10-year-and-over maturities, provided by the Bank of Canada.

10. The arguments for introducing the disaggregated components of the general index of consumer prices instead of the aggregate index were offered by A. Cukierman, "Money Illusion and the Aggregate Consumption Function: Comment," **American Economic Review**, (March 1972). Cukierman argues that the "plim" of the money illusion coefficient is dependent on the individual components of the aggregate price index. J. van Daal argues that the appearance of money illusion in macro consumption functions may likely be due to aggregation bias. See J. van Daal, "Money Illusion and Aggregation Bias," **De Economist**, (1980).

11. Both fitted series plotted in Chart 3 utilized the second order autocorrelated error structure estimated for the U.S. in Table I.