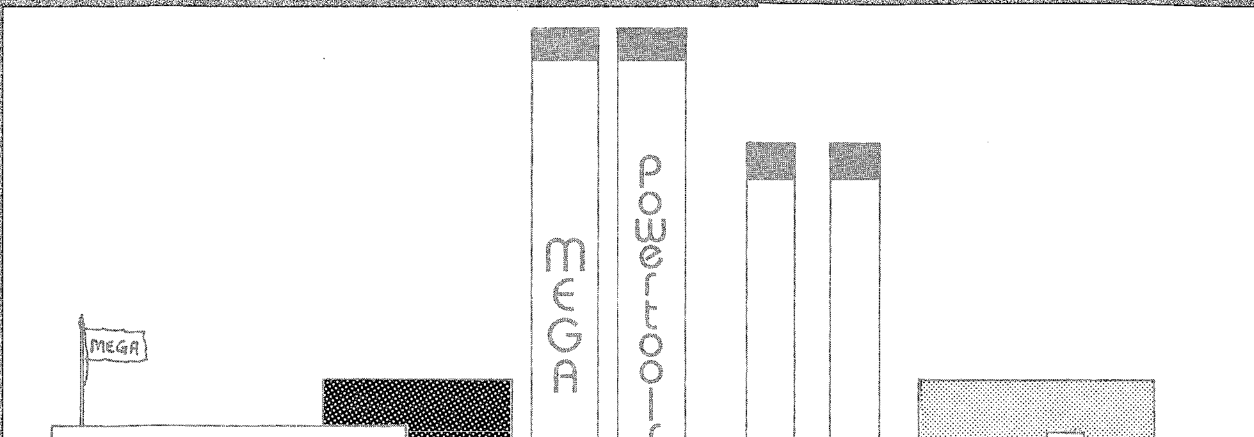
Federal Reserve Bank of San Francisco

Economic Review Number 1



Firm Size, Exchange Rates, and Interest Rates

Winter 1984

Indicators of Long-Term Real Interest Rates

Charles Pigott*

Longer term real interest rates cannot be measured directly, but their movements can be estimated from economic indicators they affect, particularly foreign exchange rates and nominal interest rates. An increase in U.S. nominal interest rates that is accompanied by a rising dollar indicates that U.S. longer term real interest rates probably also have risen. On this basis, the unprecedentedly high level of the dollar in recent years strongly suggests that U.S. long-term real interest rates remain very high by historical standards.

Since October 1979, when the Federal Reserve announced a major change in its operating procedures, interest rates here and abroad have fluctuated to a degree unprecedented in post-war experience. These fluctuations have generated great controversy, both about their origins and their consequences. Most perplexing of all have been the gyrations in longer-term interest rates, particularly their apparent tendency to vary with seemingly short-term disturbances in the markets.[1] This turmoil and confusion has come at a particularly unwelcome time, as financial innovation and deregulation sometimes have made it more difficult to predict the impact of the monetary aggregates targeted by the Federal Reserve, and hence increased the need for other indicators of the effect of policy on the economy.

These circumstances have underscored the need for measures of medium and long-term real interest rates and expected inflation. In theory, medium and long-term real interest rates are important determinants of investment and other real spending decisions. Knowledge of their level could be helpful in gauging the future course of economic activity, as well as the effect of current monetary and fiscal policies on the economy. Inflation anticipated over the next several years would provide an indication of public perceptions about the future course of monetary policy, and thus about the credibility of the authorities' public commitments to maintain price stability. Unfortunately, it is very difficult to measure longer-term real interest rates or expected inflation, mainly because inflation expected over the next several years need not depend in any predictable way on past trends.

The basic objective of this paper is to demonstrate a practical method for measuring medium and long-term real interest and expected inflation rates for the U.S. This method uses several economic variables affected by real interest rates and/or expected inflation as "indicators" of their movements. Included among these variables are the spot and forward exchange values of the dollar, which are shown to be closely related to long-term real interest rates and expected inflation. Estimates of longer-term real interest rates and expected inflation can then be calculated from weighted averages of the indicators. As explained in the next section, underlying this approach is the observation that real interest rates and expected inflation have very different impacts on certain other financial variables, such as exchange rates. Hence, the way in which these variables move when nominal interest rates vary provides a clue about the extent to which real interest rates and expected inflation have changed.

The next section describes the relations among

---

*Senior Economist. Federal Reserve Bank of San Francisco. Laura Leete and Mary-Ellen Burton-Christie provided valuable research assistance for this article.

real interest rates and expected inflation and the financial variables used as their indicators. An intuitive description of how the indicators can be used to measure real interest rates (and expected inflation as well) is also given. A more precise and technical description of the approach is given in the Appendix. Our estimates of the actual variations in monthly real interest rates over 1976—mid-1983 are given in Section II. One important finding is that the variability of long-term real interest rates has apparently increased dramatically since the change in Federal Reserve operating procedures in 1979. Another is that long-term real interest rates have remained relatively high in 1982 and 1983, despite a substantial fall in nominal interest rates.

I. Indicators of Real Interest Rates

Any interest rate can be conceptually divided into two parts: an *inflation premium* and a (before tax) *real interest rate*. The inflation premium is equal to the amount of inflation expected *over the life* of the investment and serves to compensate for the erosion of the purchasing power of the funds lent. For example, an individual who lends $100 for one year at a 10% rate is not really better off at the end of the year if inflation during the year is also 10%; then the $110 repaid at the end of the year buys the same amount of goods and services as the amount lent could have purchased a year earlier, and so no real return on the investment is gained.[2]

The real interest rate, which equals the nominal rate less the inflation premium, thus measures the amount of additional purchasing power an investment yields. So if the nominal interest rate were 12% while inflation was expected to be 10%, the real interest rate would be 2%. This relation can be written for reference as:

$$iu_t(n) = ru_t(n) + \Pi u_t(n) \qquad (1)$$

where iu(n) is the U.S. nominal interest rate on a n-year security, ru(n) is the corresponding U.S. real interest rate, and Πu(n) is the expected inflation rate over the next n years, that is, the inflation premium.

The real interest rate and the inflation premium are likely to have very different impacts on economic behavior. Economic theory implies that individuals' and businesses' real spending decisions are influenced by the real interest rate, but little, if at all, by the inflation premium. Inflation expectations, as reflected in the inflation premium, are likely to be important determinants of wages and prices set in contracts, besides serving as a gauge of the credibility of authorities' policies to ensure price stability. The effects of fluctuations in nominal interest rates on the economy thus will depend upon the extent to which they reflect changes in real rates or in expected inflation. For this reason, real interest rates and inflation premia are generally more useful to know than nominal rates alone.

Only nominal interest rates are actually quoted in financial markets, however. Their real and expected inflation components are not directly observable. Plainly, there is no way to determine from changes in nominal interest rates *alone* how much these components have varied.

Thus to measure real interest and expected inflation rates requires some information in addition to that provided by nominal interest rates. A common approach to this problem is to use an independent measure of expected inflation. For example, inflation over the near-term future is generally most closely related to that experienced in the recent past. The inflation premium on a short-term security can often be approximated by the current trend in actual inflation, providing a rough measure of the short-term real interest rate. On this basis it seems fairly clear that U.S. short-term real interest rates have fluctuated considerably over the last several years, much more so than during the 1970s.[3]

Unfortunately, this approach is not appropriate to the measurement of medium and longer-term real interest rates. Inflation expected over the coming *years* depends critically upon the macroeconomic policies authorities follow in the future. Public perceptions about these future policies—indeed any reasonable guess about them—need not be related in any obvious or dependable way to past trends, and therefore are apt to be extrememly difficult to gauge correctly. Who, after all, can pretend to know with any confidence what the stance of monetary and fiscal policies will be several years from now?

Indicators

An alternative approach is to look for economic variables whose movements provide clues about the variations in real interest rates and/or expected inflation, and hence serve as *indicators* of their values. In principle, any variable that is affected by real interest rates or expected inflation rates could serve as such an indicator. In this sense, nominal interest rates are indicators of their real and expected inflation components.

Particularly helpful as indicators are financial variables which react differently to real interest rates than to expected inflation. Suppose that a certain financial variable tended to increase when real interest rates rose, but generally was unaffected by fluctuations in expected inflation. Then a rise in nominal interst rates that was accompanied by an increase in this variable would suggest that real interest rates had increased. A variable that was affected by expected inflation but not by real interest rates could be used in an analogous fashion.

Admittedly, no particular indicator is likely to provide a completely accurate measure of either real interest rates or expected inflation. Still, it ought to be possible to use several such indicators to estimate, or approximate, these components of nominal interest rates. This approach, which attempts to 'read' the signals provided by financial markets, is the one taken here.

But What Indicators?

Changes in interest rates expected to prevail in the distant future ("long-run") are apt to be especially good indicators of *expected long-run* inflation. Consider the nominal (say one-year) interest rate now (at t) expected to prevail 'many,' or N, years in the future. This will be referred to as the *forward* interest rate and denoted fiu_t (it being understood that it is the forward rate corresponding to many years in the future).

$$fiu_t = E_t iu_{t+N}$$

($E_t iu_{t+N}$ stands for the "expected value" of iu_{t+N} based on information available at t). This can be *approximated* from the term structure of nominal interest rates, since long-term rates are approximately averages of expected future shorter-term rates.[4] As with any nominal interest rate, the forward rate is composed of the real interest and the inflation rates expected to prevail in the "long-run," that is, N years from now.

In the long-run though, the real interest rate is mainly determined by the productivity of capital, which in turn reflects the savings decisions of households, businesses, and government, the growth of the labor force, and the rate of progress of technology. Generally these conditions change slowly, so that the expected *long-run* real interest rate (as reflected in the forward interest rate) can be regarded as essentially constant when considering a period of several years. Variations in current real interest rates can be viewed as resulting from *non-persistent* imbalances in supply and demand in money and credit markets. For example, economic theory suggests that because of the lag between money and inflation, increases in money growth temporarily lower real interest rates by raising real balances; real interest rates return to their original values once inflation "catches up," however.[5]

This reasoning implies that changes in the long-term forward interest rate mainly reflect shifts in long-run expected inflation. So a rise in the forward rate corresponding to ten years in the future would measure the change in inflation expected to prevail ten years from now, or

$$\Delta fiu_t = \Delta E_t \Pi u_{t+N} \qquad (2)$$

where $E_t \Pi u_{t+N}$ is inflation expected to prevail *beginning* N years in the future. The forward rate also provides a more indirect indication of inflation expected to prevail over the *next* ten years, that is of the inflation premium in medium and long-term nominal interest rates. The reason is that an increase in inflation expected ten years from now suggests that inflation over the next several years has also increased. Thus it is more likely that an increase in, say, the 5 year nominal interest rate reflects an increase in expected inflation over the next five years if the forward interest rate has also increased than if it has not.

Conversely, a rise in the nominal n-year interest rate relative to the forward interest rate is more likely to signal an increase in the real interest rate than would an increase in the nominal interest rate taken by itself. Hence, the change in the *difference* between the rate on a n-period asset and the forward rate,

$I1: \Delta diu_t(n) = \Delta iu_t(n) - \Delta fiu_t$

is apt to be a better indicator of variations in the n-year real interest rate than changes in the nominal interest rate itself. This indicator is composed of changes in the real interest rate plus changes in the difference between inflation anticipated over the next n years and that expected in the long run.

$$\Delta diu_t(n) = \Delta ru_t(n) + \Delta d\Pi u_t(n), \quad (3)$$
$$\text{where } d\Pi u_t(n) = \Pi u_t(n) - E_t\Pi u_{t+N}.$$

Thus this new indicator effectively removes from nominal interest variations that portion of shifts in expected n-year inflation that simply reflect movements in anticipated long-run inflation. Since inflation expected over the next several years is apt to be closely related to long-run inflation expectations, this implies that real interest rates are likely to account for a larger proportion of the variations in the indicator I1 than those of the nominal interest rate. This suggests that I1 will be the better indicator of real interest rate movements (although by no means an exact one).[6]

Exchange Rate Indicators

The foreign exchange value of the dollar is closely related to U.S.—and foreign—interest rates simply because in comparing the yields on investments in different currencies an individual must take account of the expected change in the exchange rate between them. For example, if the interest rate on a one-year German-mark denominated-security were 5 percent while the mark were expected to appreciate by 3 percent over the year vis a vis the dollar, its expected yield *in dollars* would be 8 percent.

Because of the risk that exchange rates will not change by exactly the amount originally anticipated, the expected dollar yields on securities identical in all respects except the currencies they are denominated in may differ.[7] However available evidence suggests that in the absence of capital controls, such currency 'risk' premia are not very large, at least among the U.S. and other major industrial nations.[8] Thus the difference between U.S. and foreign interest rates for a given maturity can be viewed as a reasonable approximation of the expected change in foreign currency value of the dollar, expressed at an annual rate:

$$iu_t(n) - if_t(n) \cong (1/n)[e_t - E_te_{t+n}] \quad (4)$$

where e_t is the current foreign currency price of the dollar (expressed in logarithms) and E_te_{t+n} is its expected value n years from now.

This relation between nominal interest rates and the nominal exchange rate is easily converted to one between real interest rates and the real exchange rate. The *real* exchange rate, x_t, is simply the nominal exchange rate 'deflated' by the ratio of the foreign to the U.S. price level:

$$x_t = e_t + (pf_t - pu_t)$$

where pf and pu are the logarithms of the foreign and U.S. price levels.

The real exchange rate measures the value of foreign goods and services in terms of our own, or the rate at which U.S. and foreign products can be exchanged for one another. Suppose a 'basket' of U.S. goods sells for one dollar (our price level is one) while a 'basket' of German goods sells for one mark. Then if the nominal exchange rate is 2 marks/dollar, 2 baskets of German goods are needed to obtain one basket of U.S. products. Hence, the real exchange rate for the dollar is two.

As this suggests, the real exchange rate is a reflection of the relative value of U.S. versus foreign products. Ultimately, this rate will be determined by supply and demand conditions in product and factor markets. Furthermore, in the long-run, the level of the real exchange rate should be largely unaffected by inflation (since inflation's effect on *relative* prices is neutral, at least approximately) or by real interest variations (since these result from temporary disturbances in financial markets).

Subtracting the U.S. minus the foreign expected inflation rate from (2) and rearranging gives,

$$ru_t(n) - rf_t(n) = (1/n)[x_t - E_tx_{t+n}]$$

or

$$x_t = n[ru_t(n) - rf_t(n)] + E_tx_{t+n} \quad (5)$$

where E_tx_{t+n} is the *future* real exchange rate expected to prevail after n years. The relation shows that the n-year real interest differential effectively measures the divergence between the current real exchange rate and that expected to prevail *at maturity*.

This relation also implies that increases in the long-term U.S.–foreign real interest differential raise the *current* real exchange rate, x_t. For example, an increase of one percentage point in the (annualized) 5 year U.S. real interest relative to abroad, all other factors the same (that is, no change in the expected 'long-run' real exchange rate) will raise the real value of the dollar by five percent. In this sense, variations in long-term real interest rates can have very substantial impacts on actual real exchange rates.[9]. It follows that variations in the current real exchange rate are an indicator of the U.S. (and foreign) long-term real interest rates; a rise in x_t suggests that our real interest rate may have gone up.[10]

I2: $\Delta x_t = n[\Delta ru_t(n) - \Delta rf_t(n)] + \Delta E_t x_{t+n}$

Finally, the two indicators defined above can be combined with the foreign interest rate to yield an indicator of the change in our expected inflation—again expressed *relative* to that anticipated for the distant future. Define

$y_t = x_t + n[dif_t(n) - diu_t(n)]$

where the foreign interest rate indicator, $dif_t(n)$, is defined analogously to that for the U.S. Now using the expressions for the real exchange and interest rate indicators (see I1 and I2) gives:

I3: $\Delta y_t = n[\Delta d\Pi f_t(n) - \Delta d\Pi u_t(n)] + \Delta E_t x_{t+n}$

where again $d\Pi f_t(n)$ is analogous to the corresponding U.S. variable. The variable defined in I3 can be regarded as a third indicator of the U.S. long-term real interest rate. The reason is that its variations provide information about the expected inflation component of the U.S. nominal interest rate indicator, I1, and hence *indirectly* about its real interest component. In particular, a rise in this indicator suggests a *fall* in U.S. expected inflation, and therefore an increase in the U.S. real interest rate for any given value of the nominal interest rate indicator. This third indicator will be referred to as the deflated 'forward exchange rate,' since it is effectively the n-year forward exchange value of the dollar (the currently quoted value of the dollar for delivery n years from now) deflated by the current U.S.-foreign price level ratio, and expressed relative to the U.S.-foreign forward interest differential.[11]

How Do We Use Them?

The analysis has identified three potential indicators of the real and expected inflation portions of the long-term nominal interest rate, namely changes in: the n-year nominal interest rate relative to the forward interest rate; the current real exchange rate; and the (n-year) deflated forward exchange rate. The relations between these indicators and real interest rates are summarized in Table 1. The likely increase in the real interest rate accompanying a given rise in the nominal interest rate (iu) is greater:

(i) the larger the accompanying *rise* in the interest rate indicator I1;
(ii) the larger the accompanying *increase* in the real exchange value of the dollar, I2;
(iii) the smaller the *decline* in the forward exchange rate indicator, I3 (since a decline in y suggests a rise in expected U.S. inflation).

These observations suggest that movements in real interest rates and expected inflation can be estimated from variations in the indicators. An obvious course is to use weighted averages of the indicators as these estimates, say:

$$\Delta ru_t(n) = W1\Delta diu_t(n) + W2\Delta x_t + W3\Delta y_t. \quad (6)$$

Ideally the weights used should reflect the *average* degree to which the real interest rate changes with the indicators. For example, W1 should reflect the average change in the real interest rate corresponding to a given change in the interest rate indicator, all other indicators being constant.

Table 1
The Indicators and Their Relations

Indicators

I1: $\Delta diu_t(n)$— — change in the n-year U.S. nominal interest rate (expressed in logarithms) less the (log of) the forward interest rate, fiu_t

I2: Δx_t— — change in the logarithm of the U.S. real exchange rate, calculated as the spot foreign currency/$ rate times the ratio of U.S. to foreign price level (using consumer prices)

I3: Δy_t— — change in the deflated forward exchange value of the dollar (relative to the U.S.-foreign forward interest rate differential), again in logarithms. $y_t = x_t + n\,(dif_t(n) - diu_t(n))$

Relations

(i) $\Delta diu_t(n) = \Delta ru_t(n) + \Delta d\Pi u_t(n)$
(ii) $\Delta x_t = n\,[\Delta ru_t(n) - \Delta rf_t(n)] + \Delta E_t x_{t+n}$
(iii) $\Delta y_t = n[\Delta d\Pi f_t(n) - \Delta d\Pi u_t(n)] + \Delta E_t x_{t+n}$

Of course such estimates of changes in the U.S. real interest rate cannot be expected to be exact, mainly because, as can be seen from Table 1, the indicators are affected by other variables as well. For example, the real exchange rate indicator is affected by the foreign real interest rate and the expected future real exchange rate, as well as the U.S. real interest rate. In fact, there are five "underlying" variables making up the set of indicators (the U.S. and foreign real interest rates, the U.S. and foreign expected inflation rates, and the expected future real exchange rate) none of which are directly observable. Given that there are only three indicators, none of these underlying variables can be determined exactly.

If direct observations of real interest rates were available, the weights, W, could be estimated simply by performing a *regression* of the form in (6), that is of changes in the real interest rate on the indicators. The problem then is, how can this regression be performed *without* any direct measurements of the dependent variable, namely the changes in the U.S. long-term real interest rate?

As explained in more detail below (and more completely in the Appendix), this regression, that is the estimation of the weights, can actually be carried out indirectly given certain additional assumptions discussed below. In effect, the weights can be inferred from clues provided by the relations among the indicators. Recall, for example, that an increase in the U.S. nominal interest rate indicator due to a rise in the real interest component, will, all other factors held constant, be associated with an increase in the real exchange rate indicator. This suggests that the greater the extent to which U.S. nominal interest rate and exchange rate indicators actually tended to move together, the greater the weight, W2, is apt to be.

Obtaining the Weights

To see more precisely what is involved in estimating these weights, let I_t stand for the vector of the indicators at 't.' (The following discusion is a bit technical; readers interested mainly in the results can skip to Section II: Empirical Results.)

$$I_t = [\Delta diu_t(n), \Delta x_t, \Delta y_t]$$

Then the weights given in (5) are defined by:

$$W = Cov[\Delta dru_t(n), I_t]Var(I_t)^{-1}, \qquad (7)$$

where W=(W1,W2,W3), Cov( ) stands for the covariance of changes in the U.S. real interest rate with the three indicators, and Var( ) is the variance matrix of those indicators.

The weights, W, are those that would be estimated if the regression (6) could actually be performed directly. Estimates of the change in the real interest rate using these weights are 'optimal' in the sense that they minimize the average (squared) divergence between the estimated and actual values of $\Delta ru_t(n)$ (in comparison with any other weighted average of the indicators).[12]

With no direct observations of the real interest rate, the technical problem becomes that of estimating the covariance of real interest rate changes with the indicators. (Clearly, the variance matrix of the indicators can be estimated directly). However it can be seen from Table I that these are determined by the relations—that is the variances and covariances—among the underlying variables that make up the indicators, the U.S. and foreign real interest and expected inflation changes, and changes in the expected future real exchange rate. For example, the covariance of the U.S. real interest rate change with the nominal interest rate indicator is determined by the variance of fluctuations in the real interest rate *and* its covariance with changes in U.S. expected inflation.

As suggested earlier, the relations among the indicators provide the primary source of information about the relations among the variables underlying them. A simplified example illustrates this. Suppose that changes in the U.S. real interest rate were independent of (uncorrelated with) the other underlying variables. Then the observed covariance between the interest rate and real exchange rate indicators is:

$$Cov[\Delta diu_t(n), \Delta x_t] = nVar[\Delta ru_t(n)]$$

In short, *under these assumptions*, the variance of the real interest rate, and hence its covariance with the nominal interest rate indicator, could be calculated from the observed relation between the U.S. nominal interest rate and the real exchange rate.

Proceeding in this way, it would appear possible to estimate the relations among all the underlying variables (their variances and covariances) from the relations among the indicators. This in turn would define the relation of real interest rate changes to the indicators, allowing the weights, W, to be estimated. This is the sense in which the approach taken here amounts to an 'indirect' regression.

The complication is that it cannot plausibly be assumed that real interest rates are independent of all other underlying variables. More generally, the relations among these variables could be fairly complex; for example, there might be complex interactions between U.S. and foreign real interest rates and expected inflation, and, if so, these would affect the way in which the relations between the interest rate and exchange rate indicators are interpreted. Once these possibilities are allowed for, the information provided by the relations among the indicators is no longer sufficient to determine those among the variables underlying them. The reason is that there are five underlying variables, and hence more relations among them than for the three indicators. Thus while the relations among the indicators will continue to be the primary basis for the estimation of weights, some additional assumptions, suggested by economic theory or other data, must be made.[13]

Assumptions

One assumption is suggested by the earlier discussion, where it was argued that the real exchange rate is unaffected in the *long-run* by inflation or real interest fluctuations. This implies (*assuming* that n-years is sufficient for this long-run condition to hold):[14]

(A1) Changes in the expected future (n-years from now) real exchange rate, $\Delta E_t x_{t+n}$, are uncorrelated with changes in the n-year U.S. and foreign real interest and expected inflation components.

It is also reasonable (and necessary) to restrict the cross-country relations among real interest rates and expected inflation by assuming that foreign expected inflation changes have no direct impact on U.S. real interest rates, and similarly for U.S. expected inflation and the foreign real interest rate. This can be stated as:

(A2) Foreign expected inflation affects the U.S. real interest rate only to the extent to which it affects U.S. expected inflation. Similarly, U.S. expected inflation affects the foreign real interest rate only via its impact on foreign expected inflation.[15]

Finally, the estimation also requires some assumption, that is, prior estimates, concerning the *average* response of U.S. and foreign real interest rates to their respective changes in expected inflation. These responses are measured by the 'coefficients' bu and bf defined as:

bu = average change in $ru_t(n)$ given a one percent change in $d\Pi u_t(n)$

bf = average change in $rf_t(n)$ given a one percent change in $d\Pi f_t(n)$

Similarly, some prior assumption must also be made about the average response of variations in foreign real interest rates to changes in the U.S. real interest rate, measured by the coefficient g defined as:

g = average change in $rf_t(n)$ given a one percent change in $ru_t(n)$.

Given these assumptions, the relations (covariances) among the five underlying variables can be expressed in terms of (see Appendix): their (5) variances; and the relation (covariance) between U.S. and foreign expected inflation. These parameters can then be calculated from the six independent variables provided by the covariance matrix of the indicators—once, that is, the values of bu, bf and g are specified. The way in which these coefficients are estimated is described briefly in the next section and in more detail in the Appendix.

II. Empirical Results

The analysis of the previous section will now be applied to estimate actual changes in U.S. real interest rates and expected inflation for the period 1976–mid-1983. These estimates will be based on the five year U.S. and *German* government bond rates (n=5), which are taken to be the 'long-term' nominal interest rates. The forward interest rates correspond to 7 years in the future for the U.S., and five years for Germany, while the exchange rate indicators are based on the foreign exchange value of the dollar vis a vis the German mark.[16] Separate estimates are calculated for the sub-periods before and after June 1979. The reason is that the variability of nominal interest rates changed dramatically in 1979 (especially after the change in Federal Reserve operating procedures in October of that year), as did their relation to exchange rates. This suggests that the behavior of real interest rates, and their relation to the indicators, also changed, and that the appropriate weights prior to mid-1979 are not the same as those applying after that date.[17]

A First Look

Its useful to begin by examining relations among the indicators to see what they tentatively suggest about the extent of fluctuations in real interest rates and expected inflation. For this purpose, Table 2 lists measures of the actual extent to which a given indicator tended to vary with a given change in each of the others (these are based on the variances of the indicators and correlations among them).

Several tentative conclusions are suggested by the figures in the table. First, the very weak relation between changes in the U.S. interest rate indicator and the real exchange rate for the earlier period suggests that the real interest rate's variability was low in comparison with that of the nominal interest rate itself. The analysis of the last section implies that a one-percent increase in the 5-year real interest rate will, all other factors held constant, raise the real exchange rate by 5 percentage points. Thus if real interest rates were the main source of changes in the U.S. nominal interest rate indicator, that indicator could be expected to be associated with more than proportionate changes in the real exchange rate in the same direction.

In fact, during the earlier period, a one percentage point rise in the nominal U.S. interest rate indicator was, on average, associated with only a 0.9 percent increase in the real exchange rate. This suggests that changes in expected inflation, rather than in real interest rates, were the main sources of variations in U.S. nominal interest rates during this period—a conclusion supported by a number of previous studies of short-term interest rates.[18] Similar reasoning suggests that the variability of real interest rates rose substantially from the first to the second period: on average the real exchange rate increased by about 3 percent for each 1 percentage point rise in the U.S. interest rate indicator after mid-1979.

Second, the data suggest that the variability of (changes in) expected inflation may also have risen substantially from the first to the second period. This is suggested by the fact that the variability of the forward exchange rate indicator (which helps measure foreign relative to U.S. expected inflation) rose dramatically. (In addition, the U.S. forward interest rate variability also increased sharply after mid-1979).

Third, the data also suggest that there may be considerable variability in the expected future real

Table 2
Relations Among The Indicators[1]

	Standard Deviation (Basis Points)	Average response to 1 percentage point change in:[2]		
		$\Delta diu_t(5)$	Δx_t	Δy_t
I. First Period (1976.01–1979.06)				
a. $\Delta diu_t(5)$	17.3	—	.01	−.07
b. Δx_t	177.9	.92	—	.42
c. Δy_t	306.3	−2.50	1.24	—
Memo: $\Delta dif_t(5)$	37.3	−.37	.06	.10
II. Second Period (1979.07–1982.12)				
a. $\Delta diu_t(5)$	40.3	—	.05	−.05
b. Δx_t	300.7	3.06	—	.65
c. Δy_t	342.9	−2.48	.03	.04
Memo: $\Delta dif_t(5)$	26.0	.11	.03	.04

[1] For variable definitions, see Table I.

[2] This is the coefficient in a *bivariate* regression of the column variable on the row variable; for example, the response of $\Delta diu_t(5)$ to Δx_t is: $Cov(\Delta diu_t(5), \Delta x_t) / Var(\Delta x_t) = .01$ for the first period.

exchange rate, $E_t x_{t+5}$. The analysis in the last section (see Table 1) showed that movements in the actual real exchange rate reflect changes in the U.S.-foreign real interest differential, or shifts in the expected future real exchange rate, or both. The actual real exchange rate indicator was in fact highly variable in both periods. Yet, as argued above, the Table 2 figures do not point to much variability of the U.S. real interest rate, or indeed (similar reasoning would show) to much variability in the foreign real interest rate, over the first period. This suggests that much of the variability of the actual real exchange rate was due to changes in its expected future value. The same conclusion is suggested by the fact that the actual real exchange rate and deflated forward exchange rate indicators are very positively correlated. The expected future real exchange rate is the factor common to variations in these two indicators, and so if fluctuations in $E_t x_{t+5}$ were substantial, the real and deflated forward exchange rates could be expected to move closely together—as in fact they did.[19]

Variability of Real Interest Rate Changes

Table III lists estimates of the variability of the U.S. five year real interest and expected inflation changes obtained using the procedures outlined in the previous section. (The 'memo' lines in the Table are intended to provide an indication of how the estimates are affected by alternative choices of the prior-estimated parameters, bu, bf, and g.)

For the first period, the estimates are based on measures of the average response of U.S. and foreign real interest rates to their respective expected inflation rates estimated from observed *short-term* interest rates and expected inflation (see Appendix for details). This amounts to assuming that the average response of longer-term real interest rates to expected inflation (that is, bu and bf as defined earlier) is essentially the same as that for short-term rates and is plainly only an approximation.[20] It was also assumed for the period prior to mid-1979 that U.S. real interest rates had no *direct* impact on foreign real interest rates, which is consistent with previous studies suggesting that authorities abroad did *not* systematically vary their domestic interest rates in response to variations in U.S. (real) interest rates.[21]

The second period results are based on the assumption that the variability of changes in the expected future real exchange rate is the same as that estimated for the first period. (This leads to estimates that seem more plausible than those based on bu and bf estimates from short-term interest rates). This amounts in effect to assuming that all of the increased variability in the actual real exchange rate from the first to the second period is due to

Table 3
Estimates of the Variability of Real Interest Rates and Expected Inflation

	bu	bf	g^1	$\Delta ru(5)$	$\Delta d\Pi u(5)$	$\Delta \Pi u(5)$	$\Delta rf(5)$	$\Delta d\Pi f(5)$	$\Delta \Pi f(5)$	$\Delta E_t x_{t+5}$
				— Standard Deviation (Basis Points) —						
I. 1976.01–1979.06										
Estimates:	0	−.33	0	10.0	14.1	23.6	22.3	51.1	26.0	128.1
Memo: estimates with alternative bu, bf, g^3	−.30	−.33	−.10	13.4	16.8	17.2	22.9	50.5	25.3	113.1
II. 1979.07–1982.12										
Estimates:	−.63	−.63	.20	47.0	47.4	95.4	41.1	62.2	99.0	123.0
Memo: estimates with alternative bu, bf, g^3	−.44	−.20	.20	39.6	20.8	76.6	10.5	30.7	65.4	253.3

Notes:

1 'g' is the average change in the foreign real interest rate for a given change in the U.S. real interest rate;
$g = Cov[\Delta ru_t(5), \Delta rf_t(5)]/Var[\Delta ru_t(5)]$

2 See Appendix for details on how the estimated variance of the 5-year expected inflation change, $\Delta\Pi u_t(5)$ and $\Delta\Pi f_t(5)$, is obtained.

3 The 'memo' estimate for bu for the first period is taken from Mishkin's (1981) estimates; the 'memo' bu and bf for the second period are taken from Appendix Table A1, using short-term nominal interest rates and inflation.

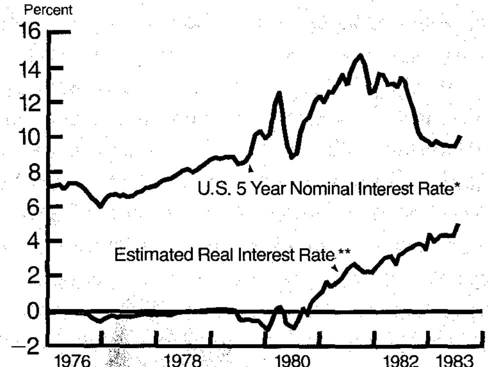
increased variations in real *interest* rates. In addition, the average response of foreign real interest rates to those in the U.S. is estimated from short-term interest rates for this period. The reason is that there is at least casual evidence to suggest that foreign authorities may at times have 'reacted' to interest rate changes in the U.S. after 1979.[22]

The results in Table 3 have three major implications. First, as suggested earlier, the variability of the U.S. real interest rate in the first period was relatively low compared to that of expected inflation. Apparently, movements in expected inflation dominated fluctuations in nominal long-term interest rates, as previous studies have suggested is the case for short-term rates. This conclusion seems to be reasonably robust, in the sense that it remains even if U.S. real interest rates and expected inflation are assumed to be substantially negatively correlated.

Second, the variability of U.S. real interest rates rose dramatically after the Federal Reserve stopped 'smoothing' nominal interest rates in 1979. This conclusion too is very robust, since it holds even if the prior-estimated parameters (bu, bf, g) are assumed the same as for the first period. More surprising, perhaps, is that the variability of U.S. expected inflation has also increased and apparently continues to be greater than that of the real interest rate.

Finally, the results imply that variations in real interest rates have *not* accounted for *all* the variations in the 'long-run' real exchange rate. For the first period, variations in the long-run real exchange rate accounted for about half of the variations in the current real exchange rate. This result is of interest, since it suggests that *purchasing power parity*, that is U.S. versus foreign inflation, is not the *sole* determinant of nominal exchange rates in the long-run, as is often asserted to be the case.[23]

Chart 1
Nominal and Estimated Real Interest Rates



* U.S. 5-year government bond rate.
** Real interest rate measured as the cumulative change since December 1975.

Estimates of Real Interest Rates

It is now straightforward to estimate the actual variations in U.S. long-term real interest rate. The weights on the indicators corresponding to the estimates in Table 3 are given in Table 4. In some cases, these weights are more easily interpreted by rewriting the estimating relations in terms of the U.S. interest rate, the real exchange rate, and the *foreign* interest rate, as is done in the last three columns of Table 4 (see the relations in Table 1). Note that in this rewritten form the coefficients of the U.S. nominal interest rate indicator are all positive, as are those on the real exchange rate.[24]

Chart 1 plots the estimates of the cumulative change in the U.S. real interest rate for January 1976 through July 1983 obtained from these weights

Table 4
Relations For Estimating Real Interest Variations

	Coefficient of $\Delta \hat{ru}_t$ on:				Implied Coefficients of:[2]		
	Δdiu_t	Δx_t	Δy_t	'R^2'[1]	Δdiu_t	Δx_t	Δdif_t
Period I (1976.01–1979.06)	.335	.012	.001	.39	.330	.013	.005
Period II (1979.07–1983.07)	−.583	.275	−.164	.60	.237	.111	−.816

Notes:

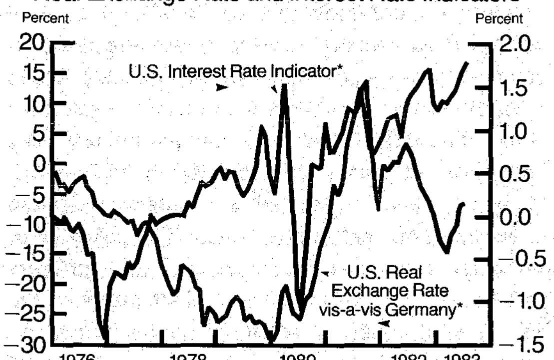
[1] *Estimate* of the fraction of variance of changes in the real interest rate accounted for by the indicators: $V(\Delta\hat{ru}_t)/V(\Delta ru_t)$ where $V(\Delta\hat{ru}_t)$ is the variance of the *estimates* of Δru_t calculated from the above relations.

[2] Obtained using the expression for Δy_t in Table 1.

and the indicators.[25] As expected, the estimates imply that real interest rates fluctuated little before 1979, but considerably more after then. Apparently, real interest rates rose from mid-1979 through April 1980, fell back through the following June, and then generally rose over the next several years. A particularly interesting implication of these results is that U.S. long-term real interest rates remained quite high over August 1981 through December 1982 even as our nominal interest rates declined sharply. The nominal 5-year interest rate fell by nearly 5 percentage points over this period, yet the estimates suggest that the real interest rate actually increased, by nearly one percent. This suggests that the decline in U.S. nominal interest rates during this period reflected a very sharp drop in expected inflation, rather than any substantial decline in real interest rates. Note also that the estimates of the long-term real interest rate generally

Chart 2A

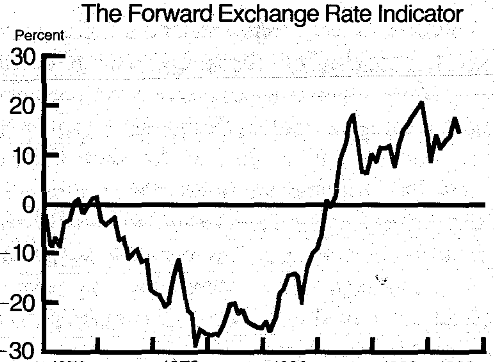
Real Exchange Rate and Interest Rate Indicators



* Both variables measured as the cumulative percentage change since December 1975.

Chart 2B

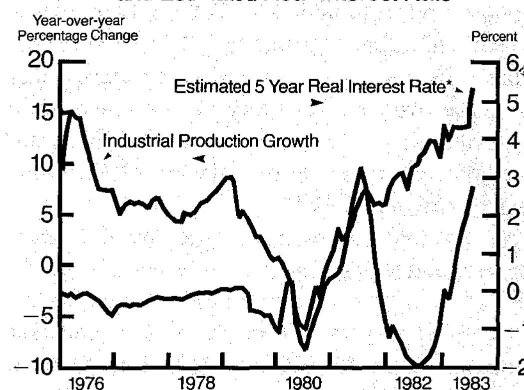
The Forward Exchange Rate Indicator



* Cumulative percentage change in the indicator since December 1975.

Chart 3

Industrial Production Growth and Estimated Real Interest Rate



* Real interest rate measured as the cumulative change (percentage points) since December 1975.

moved with short-term real interest rates until mid-1982, when the latter fell sharply.

That the U.S. real interest rate remained very high during 1982 is strongly suggested by the fact that both the real exchange rate and forward exchange rate indicators rose substantially during the period (the U.S. *forward* interest rate also fell by nearly as much as the 5 year nominal interest rate—see Chart 2.) Recall that increases in the real exchange rate suggest a rise in our real interest rate, while increases in the forward exchange rate signal a drop in our expected inflation. It is also interesting to note that the rise in the U.S. real interest rate from August 1981 through mid-1982—despite a nearly 200 basis point fall in the nominal rate—nearly coincided with a sharp drop in U.S. growth (Chart 3).

Less plausible, perhaps, is the results' implication that the long-term U.S. real interest rate increased by nearly two percentage points from mid-1982 to mid-1983. This result is a reflection of the sharp increase in the real value of the dollar during this period, as the other two indicators were essentially unchanged. Some increase in the U.S. real interest rate during 1983 is not implausible as nominal interest rates (and proxies for short-term real interest rates) did rise. However in view of the robust real growth during this period, it seems less reasonable to suppose that the real interest rate increased as much as the results here imply.[26]

Consider now the implications of these estimates for the behavior of expected inflation over the last several years. The estimates suggest that inflation anticipated for the next five years actually increased during 1980 and 1981, even though actual inflation began to decline in mid-1979. Is this pattern plausible? While inflation began to fall in 1980, actual inflation over 1980-1981 was actually higher than during the two previous years. Hence, the drop in actual inflation beginning in mid-1980 may not have affected longer-term inflation expectations much by the end of 1981.

Furthermore, actual and prospective U.S. government budget deficits rose substantially during 1980-1981, as the Administration's "supply-side" fiscal package was put in place. Many market commentators (although certainly not all) have argued that these developments substantially increased the risk that the Federal Reserve would have to raise money growth to accommodate huge deficits, and as a result raise inflation in the future. If so, inflation expected several years in the future could have been rising even as actual inflation was coming down. That expected future inflation did rise substantially over this period is also suggested by the fact that the forward U.S. interest rate increased by nearly 3 percentage points during this period.

However these results do conflict with survey evidence gathered by Richard Hoey that suggests a fall in the public's expected 5 year future inflation rate of about 1.5 percentage points during 1980-1981.[27] If the Hoey data is correct, the results here underestimate both the fall in expected inflation *and* the rise in real interest rates over this period.

The results also suggest a very dramatic decline in the expected inflation rate over the last eighteen months, indeed by nearly as much as the fall in the nominal U.S. interest rate. According to the estimates, expected inflation in mid-1983 was about 2 percentage points below its level in mid-1979. The downward trend in expected inflation (although not, perhaps, the implied magnitude of the decline) is very plausible in view of the dramatic drop in actual inflation during 1982. In addition, the substantial slowing of money growth from mid-1981 to mid-1982 may well have raised the credibility of the Federal Reserve's anti-inflation resolve, and so contributed to a further easing of market expectations of inflation. (Again, however, the Hoey survey suggests a milder—although still substantial—fall in expected inflation since the end of 1981, and a substantial fall in real interest rates as well).

Assessment

Overall, the results point to two conclusions about the 'indicator' approach to measuring real long-term interest rates taken here. First, the general pattern traced by the estimates for U.S. real interest rates and expected inflation seems generally plausible. The results suggest a substantial decline in expected five-year inflation over the last several years, as seems reasonable in view of the sharp drop in actual inflation and the course of Federal Reserve policy. The estimates also suggest that our longer-term real interest rates have remained very high over the last eighteen months in comparison to their level prior to the initiation of the Fed's anti-inflation drive in 1979. This too is very plausible since nominal longer-term interest rates are now much higher than in 1979, while, again, expected inflation almost certainly has fallen greatly.

Second, it is evident that the use of exchange rate indicators can greatly alter the impression of movements in longer-term real interest rates that would be conveyed by variations in nominal interest rates alone. The fall in long-term nominal interest rates here between August 1981 and March 1982 would, of itself, have suggested a substantial decline in longer-term real interest rates. The behavior of exchange rates, though, suggests a very different pattern, one which seems more plausible given the behavior of other economic variables. Thus exchange rate indicators do appear to provide useful information about long-term real interest variations in addition to that conveyed by nominal interest rates.

Needless to say, these results are highly experimental, and subject to substantial error in measurement. More accurate estimates may well be obtainable by using several exchange rates (rather than one), and by adding proxies for short-term real interest rates (or other variables related to long-term real interest rates) or expected inflation to the set of indicators. Nonetheless, the results do suggest that an indicator approach to measuring long-term real interest rate movements is practical and of potential use for policy-guidance.

III. Conclusion

The last several years have provided ample reminders that there are many factors that critically affect economic behavior that cannot be measured or observed directly. Economists and business analysts have long known that 'business confidence' is an important influence on investment, but they still have not found a way to measure this confidence with any precision. More recently, we have come to appreciate the impact of real interest rates and expected inflation on our economy, and thus to regret even more our inability to observe them.

In measuring longer-term real interest rates and expected inflation, this paper has attempted to apply systematically an approach long used implicitly by economists and others. That is, movements in variables that cannot be observed directly—in this case real interest rates and expected inflation—have been inferred from variations in other variables to which they are related, and which are directly measurable. The main basis for this analysis is economic theory, which specifies the relations that are likely to hold between the unobservable variables of interest and the indicators which are used to measure them. This process amounts to a bit of economic 'detective work,' with the observable indicators providing the 'clues' and economic analysis providing the rules by which they are used. The resulting estimates of real interest rates and expected inflation are, in effect the *most likely explanations* for the observed movements in the indicators, *given* the assumptions supplied by economic theory.

Here it has been argued that exchange rates, spot and forward, are likely to be especially good sources of 'clues' about movements in longer-term real interest rates and expected inflation. The main reason is that real spot exchange rates are *directly* affected only by the real interest component of nominal interest rates, while (long-term) forward exchange rates are directly affected by expected inflation, but not by real interest rates. For this reason, movements in exchange rates provide information about how to separate changes in real interest and expected inflation rates that underlie observed movements in nominal interest rates. Similarly, the term structure of interest rates has been used to provide information about the source of changes in nominal interest rates, in the sense that when current nominal interest rates vary with long-term forward interest rates, the most likely cause is a change in expected inflation.

The analysis has also illustrated some of the practical difficulties of implementing what is, in theory, a fairly simple and straightforward idea. Of necessity, the estimates are based on certain assumptions which are not easily tested, and on parameters about which neither economic theory nor available evidence supply much definite information. This is one reason why the results must be regarded as provisional. Another is that more information, supplied by exchange rates vis a vis other countries, measures of short-term real interest rates, or other variables might well be added to the set of indicators to obtain more accurate and reliable estimates.

Nonetheless, the estimates are both plausible and surprising in ways that suggest that useful information can indeed be extracted from foreign exchange and other financial markets. Viewed *by itself*, the sharp fall in nominal interest rates over late 1981 and early 1982 would have suggested a significant drop in long-term real interest rates. Yet a very different impression, that real interest rates remained high and did not drop much, if at all, was suggested by the continued strength of the dollar in the foreign exchange markets, a conclusion supported by this paper's formal analysis based on both sets of indicators. And again, the actual behavior of the real sector of the U.S. economy during this period (although not that later in 1982 and during 1983) supports this latter impression more than that conveyed by nominal interest rates alone. This experience suggests that while using economic knowledge to 'read' the signals from financial markets is not an exact science, it is still of considerable potential use for policy guidance and worth further study.

Appendix

The following explains in more detail how the estimates of the real interest and expected inflation rates can be calculated from the variance-covariance relations among the indicators. In addition, Section C below explains how the estimates of bu, bf, and g are obtained from short-term interest rates and inflation.

A. As in the text define:

$$diu_t(n) = iu_t(n) - fiu_t = ru_t(n) + d\Pi u_t(n)$$

$$dif_t(n) = if_t(n) - fif_t = rf_t(n) + d\Pi f_t(n)$$

where dΠu and dΠf refer to the difference between the inflation premium on an n year asset and the forward interest rate. The results in the text are based on $n = 5$.

The basic assumptions discussed in Section III can be stated formally as:

(A1) Let $\Delta E_t x_{t+n}$ be denoted 's_t'. Then s_t, which refers to the change in the long-run expected real exchange rate ('long-run' being n years), is uncorrelated with changes in expected inflation or the real interest rates (including the zu and zf components of the latter).

(A2) $\Delta ru_t(n) = bu\Delta d\Pi u_t(n) + zu_t$;
$\Delta rf_t(n) = bf\Delta d\Pi f_t(n) + zf_t$
where zu_t and zf_t are both uncorrelated with $\Delta d\Pi u_t$ and $\Delta d\Pi f_t$.

The relation (A2) expresses the text assumption that any correlation between the U.S. real interest rate and foreign expected inflation be indirect, and similarly for the foreign real interest rate and U.S. expected inflation. In particular it implies:

$$Cov(\Delta ru_t(n), \Delta d\Pi f_t(n)) = buCov(\Delta d\Pi u_t(n), \Delta d\Pi f_t(n))$$

$$Cov(\Delta rf_t(n), \Delta d\Pi u_t(n)) = bfCov(\Delta d\Pi u_t(n), \Delta d\Pi f_t(n))$$

The definitions of bu and bf in the text also imply that:

$$Cov(\Delta ru_t(n), \Delta d\Pi u_t(n)) = buVar(\Delta d\Pi u_t(n));$$

$$Cov(\Delta rf_t(n), \Delta d\Pi f_t(n)) = bfVar(\Delta d\Pi f_t(n)).$$

B. The following relations are easily shown to hold among the variance-covariances of the three indicators, $\Delta diu_t(n)$, Δx_t, and $\Delta dif_t(n)$ (or, alternatively, Δyt, as defined in the text) and those of the underlying variables:

i) $Var(\Delta diu_t) = Var(\Delta ru_t) + (1+2bu)\, Var(\Delta d\Pi u_t)$

ii) $Var(\Delta dif_t) = Var(\Delta rf_t) + (1+2bf)\, Var(\Delta d\Pi f_t)$

iii) $Cov(\Delta diu_t, \Delta x_t)/n = Var(\Delta ru_t) - Cov(\Delta ru_t, \Delta rf_t) - bfCov(\Delta d\Pi u_t, \Delta d\Pi f_t) + buVar(\Delta d\Pi u_t)$

iv) $Cov(\Delta dif_t, \Delta x_t)/n = -Var(\Delta rf_t) + Cov(\Delta ru_t, \Delta rf_t) + buCov(\Delta d\Pi u_t, \Delta d\Pi f_t) - bfVar(\Delta d\Pi f_t)$

v) $Cov(\Delta diu_t, \Delta dif_t) = Cov(\Delta ru_t, \Delta rf_t) + (1 + bu + bf)\, Cov(\Delta d\Pi u_t, \Delta d\Pi f_t)$

vi) $Var(\Delta x_t)/n^2 = Var(\Delta ru_t) + Var(\Delta rf_t) - 2Cov(\Delta ru_t, \Delta rf_t) + Var(s_t)/n^2$

where the '(n)' have been dropped to simplify notation. These six relations—whose left hand sides are observable and come from the variance-covariance relations among the three indicators—are in terms of 7 variables, given estimates of bu and bf: the variances of the real interest and expected inflation rates (4); the variance of the change in the expected long-run real exchange rate (1); the covariance of U.S. and foreign real interest rates, and the covariance of U.S. and foreign expected inflation (2).

To close the model for their first period, it is assumed that zu and zf are uncorrelated, which implies:

vii) $Cov(\Delta ru_t, \Delta rf_t) = bubfCov(\Delta d\Pi u_t, \Delta d\Pi f_t)$

that is, the correlation between real interest rates entirely reflects the correlation between expected inflation rates across countries.

For the second period estimates, it is assumed that:

viii) $Cov(\Delta ru_t, \Delta rf_t) = gVar(\Delta d\Pi u_t)$

where g is independently estimated. To obtain the bu and bf estimates for the second period, let rd and dΠd refer to the U.S.-German real interest and expected inflation differentials, and did for the nominal interest differential. Then the relations (i)-(vi) can be combined to obtain:

ix) $Var(\Delta did_t) = Var(\Delta rd_t) + Var(\Delta d\Pi d_t) + 2Cov(\Delta rd_t, \Delta d\Pi d_t) = a1$

x) $Var(\Delta x_t)/n^2 - Var(s_t)/n^2 = Var(\Delta rd_t) = a2$

xi) $Cov(\Delta did_t, \Delta x_t)/5 = Var(\Delta rd_t) + Cov(\Delta rd_t, \Delta d\Pi d_t) = a3$

For the second period estimates, $Var(s_t)$ is taken equal to the estimated value for the first period. This allows a2 to be calculated, so that:

$$Var(\Delta rd_t) = a2; Cov(\Delta rd_t, \Delta d\Pi d_t) = a3 - a2;$$
$$Var(\Delta d\Pi d_t) = a1 + a2 - 2a3$$

The ratio of $Cov(\Delta rd_t, \Delta d\Pi d_t)/Var(\Delta d\Pi d_t)$ provides therefore an estimate of the 'average' value of bu and bf, and this average value is used for both countries for the estimates for the second period.

Finally, the variance of the actual expected five year inflation rate, $\Delta\Pi u_t(n)$ and $\Delta\Pi f_t(n)$, can be obtained as follows. Assume that changes in the forward interest rates (which measure changes in inflation anticipated many years from now, or changes in 'long-run' inflation) are related to changes in the n-year real interest rates only to the extent that they affect the expected inflation components, $\Delta d\Pi u_t(n)$ and $\Delta d\Pi f_t(n)$. This means, in effect, that changes in 'long-run' inflation are independent of the zu_t and zf_t components of the real interest rate defined earlier. This implies:

$$Cov[\Delta ru_t(n), \Delta fiu_t] = bu Cov(\Delta d\Pi u_t(n), \Delta fiu_t)$$

or,

$$Cov(\Delta d\Pi u_t(n), \Delta fiu_t) = (1+bu)^{-1} Cov(\Delta diu_t(n), \Delta fiu_t)$$

where the covariance on the right-hand-side of the latter expression is directly measurable. This allows the covariance of $\Delta d\Pi u_t(n)$ with changes in the forward interest rate, and hence its covariance with shifts in expected 'long-run' inflation, to be estimated. This, given that the observable variance of changes in fiu_t measures the variance of expected 'long-run' inflation, allows the variance of $\Delta\Pi u_t(n)$ given in Table 3 to be computed. The corresponding values for Germany are calculated similarly.

Note that these latter estimates do *not* affect the estimates of the weights, and hence the estimates of actual real interest variations. The reason is that the weights depend only upon the estimated variances and covariances of the variables underlying the indicators, that is $\Delta d\Pi u_t(n)$ and $\Delta d\Pi f_t(n)$, as well as of the real interest and expected future real exchange rate changes.

C. As indicated in the text, one way to derive estimates of bu and bf, and g, is to examine the relations between short-term interest rates and proxies for short-term expected inflation—the latter being much easier to obtain than proxies for longer-term expected inflation.

Let iu_t stand for the one-month U.S. interest rate, Πu_t for the actual one-month U.S. inflation rate for the month ending at t (at an annual rate), and $E_t\Pi u_{t+1}$ for the inflation rate anticipated to prevail over the *next* month. Then bu could be estimated from the regression,

$$\hat{ru}_t = iu_t - \Pi u_{t+1} = C + bu[E_t\Pi u_{t+1} - fiu_t]$$

(recall that fiu_t equals the expected 'long-run' nominal interest rate, which consists of a *constant* 'long-run' real interest rate and the expected 'long-run' inflation rate). The $\hat{ru}_t$ is the ex-post, or realized, real interest rate and is an unbiased measure of the

Table A.1.
Estimates of Real Interest-Expected Inflation Relation

	Using as Expected Inflation Proxy: 1-Month Inflation	6-Month Average Inflation
Earlier Period:		
bu	.04	.32
bf	−.33	.33
g²	(.13)³	(.04)³
Later Period:		
bu	−.44	−.78
bf	−.20	−.80
g²	.17	.20

[1]bu (bf) estimated from regression:
$iu_t - \Pi u_{t+1} = C + bu\,[E_t\Pi u_{t+1} - fiu_t]$ (similarly for bf)
where iu_t is the one-month eurodollar deposit rate, Πu_{t+1} is the actual (consumer price) inflation rate from month t to t+1 (at an annual rate), and $E_t\Pi u_{t+1}$ is a proxy for the expected value of that inflation rate as of t. This expectation is approximated as either the one month actual inflation for the period ending in month t, or the average of the six months inflation for the period ending t. Thus, the dependent variable is the 'ex-post' real interest rate.

[2]g estimated from regression:
$if_t - E_t\Pi f_{t+1} = C + g[iu_t - E_t\Pi u_{t+1}]$
Variable definitions are as in note 1.

[3]Recall that this value is not used for the first period estimates in the text. Neither value is significantly different from zero, however.

actual real interest rate, assuming that market expectations are rational. This form is used because the bu in the text refers to the relation between the long-term real interest rate and the *difference* between the n-year expected inflation rate and that in the 'long-run'. Estimates of the value of bu, using one and three month past inflation rates (consumer price) as proxies for the next period's rate, are given in Table A.1.*

The fact that the German interest rate indicator (that is, $\Delta dif_t(n)$) and real exchange rate changes are positively correlated (see Table 2) strongly suggests that variations in real interest and expected inflation rates are *negatively* correlated for that country, that is that bf must be negative for both periods.** The estimates in Table A.1. also suggest a negative correlation between expected inflation and real interest rates for the U.S. for the *second* period. On this basis, the estimates in the first column, using last month's inflation rate as the proxy for that expected the next month, are most plausible, since these lead to negative estimates of bf, and since the second column estimates for the second period, while negative, give a *negative* estimate of the variance of the expected future real exchange rate change (this means that they are too large in absolute value). The actual (bu, bf) values used in the text estimates for period two lie between the first and second column estimates.

Finally, the value of 'g' for the second period is estimated from a similar regression of the form:

$$if_t - E_t\Pi f_{t+1} = C + g(iu_t - E_t\Pi u_{t+1})$$

where the dependent and independent variables are proxies for the foreign and U.S. real interest rates. As before, one and six months past inflation are used as proxies for that expected over the next month. Note again that the value of g estimated in this way is used in the text estimates for the second period only (where I have taken the column two value). However, the estimates for the first period are reasonably close to the zero value assumed in the text.

*Admittedly, the relation should, ideally, be estimated in first-differenced form to be stricly consistent with the text analysis. However, the estimates of bu and bf (and g) obtained with a first differenced form of the above yield implausible estimates for the variances of the underlying variables (that is, one or more are negative).

**Recall that, ceteris paribus, a rise in the German real interest rate *lowers* the real exchange rate indicator. The positive correlation between the German nominal interest rate and real exchange rate changes thus implies a negative correlation between German nominal and real interest rate variations. This is most likely to occur if German real interest and expected inflation rate changes are strongly negatively correlated.

FOOTNOTES

1. See, for example, Cornell (1982) and the article by Joseph Bisignano in the Fall 1983 issue of this Review.

2. As defined here, the real interest rate simply refers to the expected real return of the investment. It thus includes any allowance for risk—from inflation, future interest rate changes, or default—investors demand, since these help determine that real rate.

3. For example, the standard deviation of the one-month U.S. interest rate less the average of the past six months inflation increased more than three-fold from the period 1976-mid 1979 to the period mid-1979 through 1982.

4. That is, we can suppose that approximately,

$$iu_t(N+1) = (1/N+1)\sum_{j=0}^{N} E_t iu_{t+j}$$

so, again approximately,

$$E_t iu_{t+N} = (N+1)iu_t(N+1) - Niu_t(n).$$

This must be regarded as an approximation for two reasons. First, longer-term interest rates may well deviate from an average of expected future shorter-term rates by a "risk" premium that compensates for uncertainties about the future. There is much evidence for the existence of such a premium, although there remains considerable dispute as to its size. Second, even in the absence of this premium, the formula holds exactly only for pure discount (zero coupon) bonds and can be regarded as only an approximation—sometimes a rough one—when there are coupons.

5. For an illustration, and the effect on exchange rates, see Dornbusch (1976).

6. Frankel (1979a) used the difference between the short-term and long-term nominal interest rates as a proxy for the real interest rate. This is, however, equal to the difference

between the short and long-term real interest rates **plus** the short-term-long-term inflation **differential**, and is unlikely to be a very good measure of the long-term real interest rate.

It is very important to note that in no sense can changes in $diu_t(n)$ be regarded as **exact** measures of variations in the n-year real interest rate. This would only be the case if changes in expected inflation were the same for all horizons, that is if shifts in the term-structure of expected inflation were "flat." This will not always be the case, as numerous past instances of **temporary** increases in inflation due to supply-demand imbalances in primary commodity markets suggest. However, if inflation increases tend to be fairly persistent, much of the variance in the expected inflation component of the nominal interest, $iu_t(n)$, will be "removed" by the transformation to $diu_t(n)$. The expected inflation component of this latter indicator **will** generally be correlated with shifts in expected "long-run" inflation, however.

7. The circumstances under which such premia will exist (and what they depend on) are described in Frankel (1979b).

8. See, for example, Frankel (1982).

9. In effect, the expected real depreciation of the dollar, which is the percentage difference between its current level and that expected in the long-run, compensates for the **real** difference in the total interest earned on the U.S. versus a foreign asset **over the entire life of that investment**. For example, an increase relative to abroad in the U.S. real ten year rate of one percentage point **annualized** implies that the U.S. instrument now earns ten percent more in real terms that its foreign counterpart over its ten-year life; hence the real value of the dollar must rise above its value expected ten years from now by 10 percent. If in addition the real value of the dollar expected ten years from now is unaffected—which means at the least that real interest fluctuations are expected to have ceased by then—then the current real dollar itself rises by ten percent. Note however that the above (and text) relations between exchange rates and the maturity of an asset's yield are strictly true only for pure discount instruments; for coupon instruments, the 'scale' factor is proportional to the *duration*.

As this suggests, only changes in long-term real interest rates are likely to have unambiguous impacts on the current real exchange rate. For any given term, the corresponding real interest differential measures the deviation of the current real exchange rate from the value expected to prevail at maturity. However it is only for longer-term maturities, that is for periods far enough into the future that real interest fluctuations and other temporary influences on exchange rates have ceased, that the value expected at maturity can be expected to be unaffected.

The impact of shorter-term real interest fluctuations on the current real exchange rate thus depends upon how real interest rates expected in the future are affected. As an example, suppose that the current 1-month U.S. real interest rate increases by one percentage point (annualized) above its long-run level, but is expected to fall one percentage point below that level next month, and then return to the long-run level in all subsequent months. The current long-run real interest rate is thus unaffected, and hence the current real exchange rate will not change. However, it is easy to see that the real exchange rate expected to prevail a month from now must **fall**, and that the actual real exchange rate a month from now will fall from its current level.

10. Of course, increases in x_t may also reflect declines in foreign real interst rates, or an increase in the expected future real exchange rate.

11. Alternatively, the third indicator can be thought of as the foreign interest rate indicator, $dif_t(n)$. Movements in foreign interest rates provide information about variations in our real interest rate, in part by helping to "interpret" changes in the actual real exchange rate: the U.S. real interest rate is more likely to have increased, given a rise in the real value of the dollar, if the foreign interest rate has remained unchanged than if it has fallen.

12. More precisely, relation (6) with the weights given in (7) gives the conditional expectation of changes in the real interest rate, given the observed changes in the indicators. The statement that the estimates are "optimal" is strictly true only when we have an exact, or correct, measure of $Cov(\Delta ru_t, I_t)$. In practice, we have to estimate it.

13. The procedure for estimating the variances and covariances of the indicators from those of the underlying variables is described in detail in the Appendix.

14. Note that this assumption is strictly valid only for n long enough for real interest rates to have returned to their (constant) long-run values. Real interest rates on shorter-term assets generally will be correlated with the real exchange rate expected at maturity. Moreover, **permanent** shifts in the real interest rate, caused by changes in capital productivity or other factors, generally will lead to changes in long-run relative commodity prices, and hence will be correlated with the long-run real exchange rate. Therefore, the assumption that real interest rate fluctuations are the result of temporary financial market disturbances is closely related to (A1).

15. This does **not necessarily** imply that, say, changes in U.S. expected inflation are uncorrelated with changes in foreign real interest rates. An "indirect" correlation could arise if U.S. and foreign changes in expected inflation were correlated, **and** changes in foreign expected inflation and real interest rates were also related.

16. All exchange rate and interest rate data refer to monthly **averages**. The forward interest rate for the U.S. is effectively the 3-year bond rate expected to prevail seven years from now, while for Germany it is the 2-year rate expected to prevail five years from now. For example, for the U.S.

$$fiu_t = 1/3[\text{Log}\,[(1+iu_t(10))^{10}] - \text{Log}\,[(1+iu_t(7))^7]]$$

where the iu_t are expressed in decimals.

Ideally, in calculating the 5-year forward exchange rate from the current exchange rate, interest rates on assets that are identical except for their currency of denomination should be used. Since the government bonds used here are not strictly identical in this sense (their tax treatment, for example, will differ), the forward exchange rate as calculated from the bond rates will differ from the 'true' value somewhat. The analysis in the text implicitly assumes that the difference is **constant** over time, so that it does not affect the calculated **changes** in the forward exchange rate.

17. The break is made in mid-1979, rather than in October of that year, since the Fed began slowing money growth somewhat before its official change in monetary targeting procedures in October 1979.

18. See, for example, Mishkin (1981), Fama and Gibbons (1982), and Cornell (1982).

19. This positive association **could** also reflect a strong **negative** relation between the U.S.-German real interest differentiatial and their expected inflation differential (see Table 1). This illustrates that a negative real interest/expected inflation relation has many of the same implications for the behavior of the indicators as does a high degree of variability in the expected future real exchange rate. This is the reason that the more negative the estimated relation of real interest rates and expected inflation, generally the **lower** the estimated variability of the expected future real exchange rate, and the higher the estimated variability of real interest rates.

20. See Appendix for more details on how bu and bf are estimated.

The effect of estimating bu and bf from shorter term interest rates can be seen from the "memo" item for the second period in Table 3. Since the alternative (bu, bf) are lower in absolute value, the estimates of the variation in U.S. and foreign real interest rates are also lower. The resulting estimate also implies a very large increase in the variance of the expected future real exchange rate over the first period. Indeed, the "memo" estimates imply that the variance of changes in $E_t x_{t+n}$ is about twice as great as that of the actual real exchange rate during the first period—which does not seem very plausible.

The "memo" for the first period uses a value for bu suggested by a study by Mishkin (1981) of U.S. shorter-term real interest rates during the 1960's and 1970's. This suggests that the short-term real rate increased by about 30 basis points for a 100 basis point increase in short-term inflation.

21. See Throop (1980) and Bisignano (1983). The Appendix again explains how the relation between U.S. and foreign real interest rates is estimated for the two periods.

22. Again, see Bisignano (1983).

23. Strictly speaking, of course, the results suggest only that the real exchange rate expected to prevail five years from **now** varies substantially; it does **not** rule out the possibility that purchasing power parity might hold over some longer period. However, examination of correlations among longer-term interest rates and exchange rates suggest that the basic conclusion would not be substantially altered by considering, say, 10 year interest rates. Meyer and Startz (1982) use an inference approach analogous to that here and find that most of the error in short-term predictions of the nominal exchange rate reflects error in predicting the **real** exchange rate. Besides being applied to short-term interest-exchange rate relations, their analysis is somewhat more restrictive than that here. In particular, it assumes that real interest and expected inflation variations are uncorrelated.

24. In interpreting the individual coefficients, it is very important to note that a change in one indicator will typically imply a change in some other variable related to the U.S. real interest rate. For example, a rise in the U.S. nominal interest rate with **no** change in the current real or forward exchange rate indicators—whose implied impact on the estimate of the U.S. real interest rate change is given by W1—can **only** occur if the foreign interest rate has also risen. Thus W1 can, in effect, be interpreted as the increase in the U.S. real interest rate given an equal change in the U.S. and foreign nominal interest rates. The U.S. real and foreign nominal interest rates apparently are **negatively** associated for the second period, which is why W1 appears to have the "wrong" sign. (This negative association reflects the apparent negative correlation between the foreign nominal and real interest rates, combined with the positive association of U.S. and foreign real interest rates). The fact that the coefficient of the foreign interest rate indicator is generally negative in the "rewritten" equation given in the right portion of the table has an analogous interpretation.

25. Data on the 5-year and 7-year German bond rates were not available after 1982. The change in the 5-year German bond rate was then taken to equal the change in the 5-year euro-DM deposit rate during 1983. To 'extrapolate' the 7-year German bond rate (to estimate fif_t), this rate was regressed on the German long-term government bond rate (which corresponds to an average of several long-term maturities) over the period 1979.07-1982. The relation was then used to estimate the seven year rate for 1983.

26. Interestingly, if the forward exchange rate indicator is dropped, the resulting estimates of the real interest rate behave very similarly to those shown in Chart 1 for 1982, but increase by considerably less during 1983 (about 60 basis points).

The "memo" estimates for the second period (using bu and bf estimated from short-term interest rates) imply a somewhat different pattern for the U.S. real interest rate during 1982 and 1983. These suggest that real interest rates fell nearly 1 percentage point during 1982, ending the year at about the level of mid-1979; the same estimates suggest that the real interest rate fell further during 1983. However the estimates also imply that expected future inflation in mid-1983 was actually several percentage points higher than it was in mid-1979, a period over which the actual inflation rate declined by nearly half. This is another reason why these estimates do not seem so plausible.

27. For a compilation of Hoey's estimates, see Peter Isard, "What's Wrong with Empirical Exchange Rate Models...," Discussion Paper #226 (August 1983) of The International Finance Division of The Board of Governors of the Federal Reserve.

REFERENCES

Bisignano, Joseph, "Monetary Policy Regimes and International Term Structures of Interest Rates," **Economic Review**, of the Federal Reserve Bank of San Francisco, Fall 1983, No. 4.

Cornell, Bradford, "Money Supply Announcements and Interest Rates: Another View," Working Paper, UCLA, 1982.

Dornbusch, R., "Expectations and Exchange Rate Dynamics," **Journal of Political Economy**, December 1976.

Engel, Charles and Jeffrey Frankel, "Why Money Announcements Move Interest Rates: An Answer from the Foreign Exchange Markets," **Conference Supplement: Sixth West Coast Academic/Federal Reserve Economic Research Seminar**, November 1982. (November 1983).

Fama, Eugene and Michael R. Gibbons, "Inflation, Real Returns, and Investment," **Journal of Monetary Economics**, Volume 9, No. 3 (May 1982).

Frankel, Jeffrey, "On the Mark: A Theory of Floating Exchange Rates Based on Real Interest Differentials," **American Economic Review**, September 1979a.

________________, "The Diversifiability of Exchange Risk," **Journal of International Economics**, September 1979b.

________________, "In Search of the Exchange Risk Premium: A Six-Currency Test Assuming Mean-Variance Optimization," **Journal of International Money and Finance**, Volume 1, No. 3, December 1982.

Judd, John and Rose McElhatten, "The Behavior of Money and the Economy in 1982-1983," FRBSF **Economic Review**, Summer 1983, No. 3.

Meyer, Stephen and Richard Startz, "Real Versus Nominal Forecast Errors in the Prediction of Foreign Exchange Rates," **Journal of International Money and Finance**, Volume 1, No. 2, August 1982.

Mishkin, Frederic, "The Real Interest Rate: An Empirical Investigation," **Carnegie-Rochester Conference on Public Policy**, 15 (1981).

Throop, Adrian, "Managed Floating and the Independence Interest Rates," **Economic Review** of the Federal Reserve Bank of San Francisco, Summer 1980.