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# Modeling the Time-Series Behavior of the Aggregate Wage Rate


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*This paper looks at the time-series behavior of the real wage relative to that of productivity. Given an exogenous, nonstationary process for productivity, we use a simple model of dynamic labor demand to show that the real wage and the marginal product of labor will be cointegrated if the representative firm chooses the profit-maximizing level of employment. Data for the postwar period satisfy this condition. On the basis of this result we estimate a vector error correction model containing prices, wages, and productivity and examine the dynamic relationships among these variables. This specification provides a natural setting for looking at a number of issues of interest, including the role of the unemployment rate in the wage rate equation, issues of wage-price causality, and the effect of exogenous wage rate changes on productivity.*

This paper studies the behavior of wages relative to prices and productivity in a framework that places relatively few restrictions on the interactions among these variables. The use of the level of productivity as an anchor for the level of the real wage is a significant element of our analysis; it allows us to examine the behavior of wages without resorting to the common practice of arbitrarily detrending this variable and focusing on the residual. We use this specification to examine a number of key issues in the literature, including such questions as the relationship between wages and prices, the relationship between the unemployment rate and prices, and the way in which productivity shocks affect the wage rate.

We begin with a condition that can be found in any simple model of competitive firm behavior—specifically, the firm sets the real wage equal to the marginal product of labor. In a time-series context this suggests that real wages and the marginal product of labor should move together over time. Starting with the assumption that the exogenous productivity process is nonstationary, we use a simple model of dynamic labor demand to show that the real wage and the marginal product of labor will be cointegrated. Empirical tests over the post-war period reveal that the data are consistent with this condition.

We go on to show that there exists a single cointegrating relationship among the nominal wage, the price level, and labor productivity. Cointegration allows us to cast the relationship among these variables as a vector error correction model (VECM) and provides a natural framework for looking at a number of hypotheses about the wage rate. One issue has to do with the role of the unemployment rate in wage equations. We show that the role of the unemployment rate in such equations is sensitive both to the dynamic specification (in particular to whether the error correction term is included) and to the inclusion of a contemporaneous measure of productivity.

Another issue has to do with the causal relationship between wages and prices. We find that prices Granger cause wages but that wages do not Granger cause prices. This is evidence against models that specify prices as a markup on wages. There does exist a non-negligible contemporaneous correlation among the innovations to these variables, and wage innovations could have an impact on prices if firms were to complete their adjustment to a wage shock within the quarter in which the shock occurred.

Complete adjustment at such a rapid rate would appear to be at odds, however, with the large body of work on the sluggish behavior of prices.

The remainder of the paper is organized as follows. Section I focuses on the long-run relationship between wages and productivity. We begin by laying out a simple model that allows us to derive testable restrictions upon the evolution of the real wage and labor productivity. Turning to the data, we first establish the univariate properties of the series and then examine the joint behavior of the real wage and productivity, with a view to determining whether the representative firm can be said to be on its long-run demand curve. Section II presents the estimated VECM; three sub-sections use this model to analyze the issues raised above as well as to examine the dynamic interrelationships among wages, prices, and productivity more generally. Section III concludes.

## I. The Relationship between Wages and Productivity

### *A Simple Model*

In this section we use a simple model to determine the kinds of restrictions that can be placed upon the behavior of the wage rate. We begin by assuming that the labor market is perfectly competitive. The model we employ is a version of the kind of dynamic labor demand model found, for instance, in Sargent (1978) or Nickell (1986).

The representative firm produces a single output (whose price is taken as a numeraire) and maximizes the objective function given by

$$V = E_t \sum_{i=0}^{\infty} \beta^i \nu_{t+i},$$

where $E_t$ denotes the date $t$ expectation and $\beta$ is the discount factor for the firm. The one period profit function $\nu_t$ is

$$(1) \qquad \nu_t = g(L_t, a_t) - w_t L_t - h(\Delta L_t).$$

$g(\bullet)$ denotes the production function in which labor ($L$) is the only input. We assume that $\partial g / \partial L_t \equiv g_L > 0$ and $g_{LL} < 0$. Labor is augmented by the technology shock term $a_t$, which can be thought of as measuring increases in knowledge that make the same unit of labor more productive over time.[1] We assume that $a_t$ is nonstationary with positive drift, an assumption that is consistent with the empirical results below; $w_t$ denotes the real wage rate defined as $W_t/P_t$, where $W_t$ is the nominal wage and $P_t$ is the price of the firm's output; $\Delta$ denotes the first difference, so that the function $h(\bullet)$ measures the cost of changing employment, which is borne by the firm. We assume the function $h(\bullet)$ to be such that there are symmetrical costs to both hiring and firing.

The competitive solution can most easily be characterized as a situation in which the firm is on its dynamic labor demand curve $L^d$, which is implicitly defined by condition (2). The firm chooses $L$ to maximize $V$, taking stochastic processes $\{a_t, w_t\}$ as given, so that

$$(2) \qquad w_c \doteq g_L - [h_L(\Delta L_t) - \beta h_L(E_t \Delta L_{t+1})],$$

where the subscript $c$ denotes the real wage rate (in terms of the product price) determined in a competitive labor market, $h_L = \partial h / \partial L$, and the term in parentheses denotes the adjustment costs of changing the labor input in period $t$ as well as in $t+1$.

For our purposes, it suffices to note that, in equilibrium, (2) has to hold in each period. Equation (2) simply states that over time the marginal product of labor and the wage rate will move together, apart from deviations caused by the costs of changing employment. The nature of the adjustment costs will determine the relative behavior of the wage rate and productivity over time. By the symmetry assumption, both an increase and decrease in employment incur labor adjustment costs, which in turn depend only on the net amount of change ($\Delta L_t$). Thus, the term in parentheses in (2) above is likely to be small and temporary for both constant or trending levels of employment, because it is the difference between the marginal cost of adjustment in two adjacent periods.

Despite the apparent intuitive appeal of condition (2), there are alternative theories of the wage-employment determination mechanism that cannot be characterized in this way. The efficient wage bargaining theory that focuses on unionized labor markets is a case in point. Both the static version of the efficient wage bargaining model (e.g., Abowd 1987, MaCurdy and Pencavel 1986), and its dynamic extension (Espinosa and Rhee 1989), emphasize and focus on the strategic nature of the interaction between firms and workers in determining wages and employment. In these models, factors such as the relative strength of unions and firms and the union's preferences over wages versus employment are crucial determinants of the actual bargaining equilibrium outcome.

There is a critical implication of this view that is relevant to our exercise. Typically, the solution set of wage-employment pairs for either the static or the dynamic efficient bargaining problem does not include points that lie on the representative firm's demand-for-labor curve. Thus, (2)

1. An alternative interpretation of this variable is that it represents the productive contribution of capital stock in the economy.

does not apply to this situation.[2] The fact that unions have played a significant role in the U.S. over our sample period provides a priori grounds for considering the efficient bargaining model seriously. In other words, there appears to be sufficient reason to believe that the behavioral prediction of (2) might not hold for our sample period.

### *A First Look at the Data*

While our discussion above has been in terms of the marginal product of labor, we have data only on average product. How closely does the latter approximate the former? The answer depends upon the underlying production function. If the production function is Cobb-Douglas, for instance, the log of the marginal product is just a constant plus the log of the average product. More generally, the results below will go through if there is a linear relationship between the log of the average and marginal products of labor. We can relate the two measures by introducing the elasticity of output with respect to labor, which is defined as the ratio between the marginal product and the average product. Thus, the log of the marginal product of labor equals the log of the average product plus the log of the elasticity, which is a constant for a wide range of production function specifications (e.g., linear and CES).

The measure of average product we use is the output per hour of all persons in the business sector, which is compiled by the Department of Labor. The log of this series is denoted by LYHR. For wages we use compensation per hour in the business sector, and denote the log of this variable by LNWAG. This measure seems most relevant to our purposes, since it includes total payments made by firms to all workers. Our focus upon labor productivity as an anchor for the real wage implies that we need a measure of the product wage, that is, a real wage measured in units of the firm's output. Consequently, we use the implicit price deflator for business sector output (whose log we denote by LDEF) to deflate the nominal wage. We use LRWAG to denote the log of the real wage. All series are available on the Citibase data tape.

The first order of business is to establish some facts about the long-run behavior of the individual series. Accordingly, Table 1 presents tests of the unit root hypothesis for each of these series against the alternative that it can be described as stationary around a linear trend over the 1948.Q2–1990.Q3 sample period. The results for the Dickey-Fuller test reveal that in no case are we able to reject the unit root hypothesis at even the 10 percent significance level.[3] Table 1 also contains results from Phillips' (1987) test. This test allows the error term to follow a more general process than the Dickey-Fuller test. It turns out that the second test leads to the same results as the first.

While we have allowed for a linear trend in these tests, visual inspection of the data suggests that the trend growth

TABLE 1

UNIT ROOT TESTS AGAINST ALTERNATIVE OF NO TREND BREAK

| Variable | Dickey–Fuller Test | Phillips |
|---|---|---|
| A. LOG LEVELS | | |
| LYHR | −1.74 | −1.19 |
| LNWAG | −1.27 | −1.84 |
| LDEF | −1.43 | −1.88 |
| LRWAG | −1.12 | −1.07 |
| B. DIFFERENCES | | |
| $\Delta$LYHR | −4.94** | −12.76** |
| $\Delta$LNWAG | −3.19* | −11.56** |
| $\Delta$LDEF | −3.26* | −9.33** |
| $\Delta$LRWAG | −4.43** | −12.74** |

NOTES:

Regressions in panel A contain a constant and a time trend. For both test statistics reported here, the 10 percent significance level is −3.15. Regressions in panel B contain a constant only; * denotes significance at 5 percent; ** denotes significance at 1 percent.

Dickey-Fuller test equations contain four lags of first difference of the dependent variable.

To compute the Phillips test statistics, we use Schwert's (1987) $l_{12}$ formula, which implies the use of thirteen autocovariances.

2. Among models of unionized labor markets, the monopoly union model provides an exception. According to the model, the monopoly union unilaterally chooses the wage rate, leaving the employment decision entirely up to the firm; the firm, in turn, takes the wage rate as given and determines employment using its demand-for-labor curve. However, contrary to this model, in practice bargaining usually takes place over both wages as well as employment. Further, Espinosa and Rhee (1989) show that the outcome described by the monopoly union model is unlikely to occur in a general dynamic bargaining game setup. There is a set of Pareto superior solutions that dominate such an outcome in a repeated game context.

3. In the case of the Dickey-Fuller test, the test statistic is calculated as the ratio of the coefficient of the lagged level to its standard error (from a regression where the first difference is regressed on a constant, a time trend, lagged first differences, and a lagged level); critical values are available in Fuller (1976).

rate of LYHR and LRWAG may have changed over the sample. Indeed, the productivity slowdown over this period has been widely noted. Consequently, we test whether the unit root specification (with no change in drift) can be rejected against an alternative that allows for a single change in a deterministic trend. This specification has been suggested as a reasonable alternative (in a somewhat different context) by both Rappoport and Reichlin (1989) and Perron (1989). Here we implement a procedure suggested by Christiano (1988). Specifically, we employ his "min-$t_\alpha$" procedure.[4]

The results from this procedure are shown in Table 2. For LYHR, for example, the procedure finds the most likely break date to be 1964.Q1, where the computed $t$ statistic has a value of $-3.45$. However, this value is considerably smaller than the expected value obtained under the null, and the computed $t$ statistic has a marginal significance level of .79. Similar results are obtained for the other variables. Thus, in no case are we even close to rejecting the unit root null against the alternative of a break in a deterministic trend.

TABLE 2

Unit Root Tests against Alternatives that Allow for Break in Trend

| Variables | Most Likely Break Date | Min-$t_\alpha$ | Significance Level/Expected Value |
|---|---|---|---|
| LYHR | 1964.Q1 | −3.45 | .79 / −4.02 |
| LNWAG | 1957.Q2 | −2.01 | 1.0 / −4.00 |
| LDEF | 1959.Q4 | −2.81 | 1.0 / −4.02 |
| LRWAG | 1969.Q2 | −3.45 | .80 / −4.00 |

Notes:

All regressions allow both the constant and time trend to change over the sample.

Calculation of the Most Likely Break Date excludes three years of data at either end of the sample. The dates reported are those that lead to the smallest $t$ statistic on the lagged level of the dependent variable.

## *Are Firms on Their Demand Curves?*

The results in Tables 1 and 2 imply that the individual series contain unit roots. What can we say about the joint behavior of these series? Equation (2) implies that $g_L$ (labor's marginal product) and $w_c$ (the real wage in a competitive labor market) will be cointegrated. This result is intuitive; it says that if the firm stays on its demand curve, the marginal product of labor and the wage rate should move together over time, apart from temporary deviations caused by adjustment costs.

It should be noted that temporary deviations between labor productivity and the wage rate can occur for a number of reasons besides the costs of adjusting employment. For example, Bils (1990) shows that in sectors with long-term contracts, real wages increase significantly—relative to wages elsewhere in the economy—in the first year of the contract, but then decline over the life of the contract. Our empirical approximation of the marginal product of labor by the average product is another reason for not expecting an exact relationship between LYHR and LRWAG, even if the firm is on its demand-for-labor curve in the long run.

Turning to the data, a regression of the real wage on labor productivity over the 1947.Q1–1990.Q3 period leads to

$$\text{LRWAG}_t = -0.01 + 0.995\ \text{LYHR}_t.$$

The Augmented Dickey-Fuller (ADF) test (see Engle and Granger 1987) leads to a test statistic of $-5.02$, which compares to a 1 percent significance level of $-3.73$.[5] Use of Johansen's maximum likelihood procedure leads to the same result. Under the null hypothesis that there is no cointegrating vector, the computed value of the trace test is

4. The intuition behind the procedure is as follows. In attempting to determine whether a break has occurred, the date of any potential break is usually determined after looking at the same data. However, using a (formal or informal) search procedure to determine the break date implies that the distribution of the resulting test statistic will no longer be the same as it would be if the break date had been determined independently of the data at hand. Christiano suggests a number of alternative techniques to choose the most likely date for a break in the trend and then constructs empirical distributions which take this "pre-testing" into account.

The null hypothesis is that the process in question contains a unit root, while the alternative is

$$\Delta y_t = a_0 + a_1 D_\tau + b_0 {*}\text{trnd} + b_1 D_\tau {*}\text{trnd} + \alpha y_{t-1} + d(L)\Delta y_{t-1} + v_t$$

where
$$D_\tau = 0, \quad 0 < t < \tau$$
$$= 1, \quad \tau \leq t \leq T,$$

and trnd denotes a linear time trend. Thus, this specification allows for both a jump in the level of the variable and a change in the slope at date $\tau$. The value of $\tau$ is then allowed to vary over the entire sample (that is, we allow each date in the sample to be the break date), and we compute the value of $t_\alpha$ at each date. We then define the date at which $t_\alpha$ attains its minimum value as the most likely break date. An empirical distribution for this statistic is obtained by using the bootstrap to construct 1000 new series.

5. The critical values are from Engle and Yoo (1988). The equation used to estimate the test statistic contains one lag of the dependent variable. The lag length was arrived at by starting with six lags and eliminating

39.2, which is significant at the 1 percent level. The statistic for the maximal eigenvalue test (with a computed value of 31.0) also is significant at 1 percent.[6] (See Johansen and Juselius 1990 for a discussion of the tests and tabulated critical values.)

These results reveal that labor productivity and the observed real wage have shared the same stochastic trend component during our sample period and consequently are consistent with models that imply that the representative firm is on its demand-for-labor curve.

The test for cointegration between real wages and productivity is actually a test for cointegration among nominal wages, prices, and productivity, where the coefficient on (the log of) the price level is set equal but opposite in sign to the coefficient on wages. An alternative way to proceed is to estimate the cointegrating vector among these three variables without imposing any restrictions. Removing this restriction introduces prices explicitly into the model and allows us to look at a larger system in the analysis that follows. One advantage of looking at this system is that we can now allow for the possibility of exogenous shocks to the price level. For example, we can now allow a union's actions to affect LNWAG, while LDEF is influenced by the actions of the monetary authority or by OPEC. More generally, the point is that we can now allow for a greater number of disturbances to the model.

Repeating the cointegration test above leads to

$$\text{LNWAG}_t = -4.56 + 1.02\,\text{LDEF}_t + 0.97\,\text{LYHR}_t.$$

We obtain a test statistic of $-5.02$, which is again significant at 1 percent.[7] Increasing the number of variables in the model raises the possibility of more than one cointegrating relationship among these variables. We use Johansen's maximum likelihood approach to determine the number of cointegrating vectors. Under the null hypothesis that no cointegrating relationship exists among these variables, the value of Johansen's trace test is 48.5, which is significant at 1 percent.[8] By contrast, we cannot reject the null hypothesis that there is at most one cointegrating vector at the 10 percent level. (The computed value of the trace-test statistic is 15.3.) The maximal eigenvalue test yields the same results. These results indicate that there are two distinct stochastic trends driving these three variables, and that there is one cointegrating vector. With the variables ordered as $\text{LNWAG}_t$, $\text{LDEF}_t$, and $\text{LYHR}_t$, the estimated cointegrating vector is $(1\ -.995\ -.951)$.

Recent research has shown that the estimates obtained from the Johansen procedure are superior to those obtained from the OLS regression; consequently, this is the cointegrating vector that we will employ below.[9]

## II. THE ESTIMATED VECM AND ITS APPLICATIONS

The finding that the real wage rate and productivity are cointegrated implies that we can specify the model as a vector error correction model (VECM).[10] The equations of the estimated VECM are shown in Table 3. Notice that the error correction term ($EC_{t-1}$) enters significantly in the LNWAG equation and has a negative sign. This implies, for example, that the nominal wage falls whenever it gets too high relative to the price level and the productivity of labor. By contrast, $EC_{t-1}$ does not enter significantly into either the LDEF or the LYHR equation; it actually has the wrong sign in the LYHR equation. This implies that, in general, it is LNWAG that adjusts to correct the "error" among these variables. We will return to this issue below. Note also that the adjusted $R^2$ of the ΔLYHR equation is close to zero, implying that the model does not do a very good job of explaining changes in productivity.

In the rest of this section we use this model to study several issues regarding the behavior of wages as well as the interaction of wages, prices, and productivity. The first extension is to introduce the unemployment rate into the model and to examine the relationship between unemployment and wages. This allows us to look at the "Phillips curve" in a framework that does not impose a potentially artificial separation between the short and long run on the data. Note that the inclusion of the unemployment rate in the equation for wages also can be motivated by appealing to efficiency wage theories of wage determination.

Next, we look at the relationship between wages and prices. Recent papers by Gordon (1988) and Mehra (1991) have looked at the causal relationships between these variables. Our model offers an alternative way of examining these issues. Finally, we will use the model to examine the dynamic relationships among wages, prices, and productivity.

the insignificant terms. (In no case do we fail to reject the null of no cointegration.)

6. The estimated model contains six lags of the first-differenced variables; the error terms from these equations are well-behaved.

7. The residual equation contains one lag.

8. The model included six lags as before. However, the residuals indicate that the normality assumption is violated because of too much kurtosis. Including up to four more lags does not solve this problem. Gonzalo (1989) points out that the Johansen procedure is robust to violations of the normality assumption.

9. See Gonzalo (1989), for example.

10. See Granger's Representation Theorem (Engle and Granger 1987).

## TABLE 3

### THE ESTIMATED VECM

| EXPLANATORY VARIABLES | $\Delta LNWAG_t$ | $\Delta LDEF_t$ | $\Delta LYHR_t$ |
|---|---|---|---|
| Constant | −0.54 | 0.19 | −0.22 |
| | (−3.0) | (1.1) | (1.0) |
| $\Delta LNWAG_{t-1}$ | 0.13 | −0.11 | 0.15 |
| | (1.3) | (−1.3) | (1.3) |
| $\Delta LNWAG_{t-2}$ | 0.11 | 0.21 | 0.04 |
| | (1.2) | (2.5) | (0.3) |
| $\Delta LNWAG_{t-3}$ | −0.12 | 0.08 | −0.12 |
| | (−1.2) | (0.9) | (−1.0) |
| $\Delta LNWAG_{t-4}$ | 0.04 | 0.01 | 0.10 |
| | (0.4) | (0.1) | (0.9) |
| $\Delta LNWAG_{t-5}$ | 0.14 | 0.02 | 0.01 |
| | (1.5) | (0.2) | (0.1) |
| $\Delta LNWAG_{t-6}$ | 0.07 | −0.02 | 0.08 |
| | (0.8) | (−0.2) | (0.8) |
| $\Delta LDEF_{t-1}$ | 0.37 | 0.46 | −0.24 |
| | (3.4) | (4.5) | (−1.8) |
| $\Delta LDEF_{t-2}$ | 0.22 | 0.19 | 0.05 |
| | (2.0) | (1.8) | (0.4) |
| $\Delta LDEF_{t-3}$ | −0.05 | −0.02 | −0.04 |
| | (−0.5) | (−0.2) | (−0.3) |
| $\Delta LDEF_{t-4}$ | −0.31 | −0.06 | −0.22 |
| | (−2.8) | (−0.6) | (−1.6) |
| $\Delta LDEF_{t-5}$ | 0.05 | −0.08 | −0.07 |
| | (0.4) | (−0.8) | (−5.0) |
| $\Delta LDEF_{t-6}$ | 0.06 | 0.10 | −0.06 |
| | (0.6) | (1.0) | (−0.5) |
| $\Delta LYHR_{t-1}$ | −0.14 | 0.15 | −0.10 |
| | (−1.7) | (1.9) | (−1.0) |
| $\Delta LYHR_{t-2}$ | 0.03 | 0.01 | 0.05 |
| | (0.3) | (0.1) | (0.5) |
| $\Delta LYHR_{t-3}$ | −0.11 | −0.06 | 0.02 |
| | (−1.3) | (−0.8) | (0.2) |
| $\Delta LYHR_{t-4}$ | −0.19 | 0.03 | −0.31 |
| | (−2.4) | (0.5) | (−3.1) |
| $\Delta LYHR_{t-5}$ | −0.13 | −0.17 | 0.02 |
| | (−1.7) | (−2.2) | (0.2) |
| $\Delta LYHR_{t-6}$ | 0.09 | 0.01 | −0.06 |
| | (1.1) | (0.1) | (−0.6) |
| $EC_t - 1$ | −0.12 | 0.04 | −0.05 |
| | (−3.0) | (1.1) | (−1.0) |
| $R^2$/adj. $R^2$ | .41/.33 | .49/.42 | .16/.06 |
| S.E.E. (x$10^2$) | 0.70 | 0.65 | 0.87 |
| Q(36)/SIG. LEVEL | 32.7/.63 | 9.1/.79 | 6.8/.87 |

NOTES:
$t$ statistics are in parentheses.
The error correction term is $EC_t$ = $LNWAG_t - .995\ LDEF_t - .951\ LYHR_t$.

### *Introducing the Unemployment Rate*

Perhaps the most straightforward way of introducing the unemployment rate in our wage equation is by appealing to Phillips (1958), who modeled the change in wages as a function of the unemployment rate. A recent example of an analysis carried out along these lines is Vroman and Abowd (1988), who regress the growth in hourly earnings on alternative measures of the civilian unemployment rate, the lagged consumer price index, and some other variables. The literature on efficiency wages provides another motivation. In Shapiro and Stiglitz (1984), for example, firms are unable to monitor workers' efforts fully. To offset this and provide the right incentive to workers, a firm has to pay a wage rate that meets a "no shirking constraint." The prevailing unemployment rate is a key determinant of such a wage rate and is inversely related to it, because the prevailing unemployment rate affects the probability of a worker finding a new job if he is found shirking and fired.[11] It is not difficult to motivate the inclusion of the unemployment rate in the price equation, either. For example, specifying prices as a cyclically sensitive markup on wages would imply a relationship between prices and the unemployment rate as well.

Our strategy is simply to include the unemployment rate into the VECM presented earlier.[12] While the resulting specification will not include many of the wrinkles of recent Phillips curve analyses, it improves upon conventional specifications in two ways. First, it allows changes in productivity to affect wages and prices directly. Second, it allows us to examine the cyclical relationship between wages and the unemployment rate in a framework that also models the long-run behavior of these variables, rather than assuming it away either by linear detrending or by first differencing the data.

Consider first what happens when six lags of the level of the unemployment rate are included in our model. The null that the (lagged) unemployment rate does not belong in the price equation can be rejected only at a marginal significance level of 60 percent, while the null that it does not

11. Also see Blanchard and Fisher (1989) pp. 455–463 for a more general discussion of efficiency wage theories.

12. We began by considering the time-series properties of the unemployment rate. We carried out the two tests in Table 1 in order to test for stationarity. While it is possible to reject the null that the unemployment rate contains a unit root (at the 6 percent level of significance) when the augmented Dickey-Fuller test is used, we fail to reject when the Phillips test is used. Given this conflict, we chose to go with our prior, which is that the unemployment rate is stationary. Accordingly, we decided to include the level, and not the first difference, of the unemployment rate in the VECM. Another alternative would be to model the unemployment rate as being stationary around a shifted mean. See Evans (1989).

belong in the wage equation can be rejected only at a marginal significance level of 79 percent. The error correction term remains significant in the wage equation (with its estimated coefficient getting noticeably larger in absolute terms) and insignificant in the price equation even after the unemployment rate is introduced. As an alternative, we tried including the log levels of the unemployment rate in the VECM. Once again, the null that the unemployment rate does not belong in the equation cannot be rejected in either case at the 50 percent level of significance. Thus, we find no evidence to suggest that the lagged unemployment rate should be included in either of these equations.

The experiments described so far do not match up precisely with the Phillips curve literature, since we have omitted the contemporaneous unemployment rate. Note that introducing the contemporaneous unemployment rate into this system is not innocuous, since it begs the question of whether the unemployment rate is exogenous. However, we decided to include the contemporaneous term in order to allow a direct comparison with the Phillips curve literature. An $F$ test on the contemporaneous and six lagged values of the unemployment rate fails to reject the null that these terms are zero at the 35 percent level of significance. However, the contemporaneous unemployment rate term has a $t$ statistic of $-2.1$ in the wage equation (and $-1.8$ in the price equation).

In addition to the problem of exogeneity discussed above, another problem in trying to gauge the significance of this result is that the unemployment rate is the only contemporaneous variable included in the wage equation. Thus, its importance may result from the fact that it is the only way that contemporaneous developments are allowed to affect wages. There is an easy way around this problem in our model: Specifically, we introduce the contemporaneous change in productivity into the wage equation and see what effect this has on the significance of the unemployment rate.[13] It turns out that doing so reduces the $t$ statistic on the contemporaneous unemployment rate to $-1.5$, and we cannot reject the null that both contemporaneous and lagged unemployment terms are zero at the 70 percent level of significance. By contrast, the contemporaneous productivity term has a $t$ statistic that is close to 3. It also is worth mentioning that the error correction term remains significant in the wage rate equation through all the exercises described above. Finally, if we drop the error correction term from the specification just described, the $t$ statistic on the contemporaneous unemployment rate goes to $-2$ while the null that the current and lagged unemployment rate terms are zero can be rejected at 11 percent.

Our results demonstrate that inferences regarding the inclusion of the unemployment rate in an equation for wages are sensitive to how the dynamics of the wage rate are specified, as well as to whether the contemporaneous effects of changes in productivity are taken into account. While our search has not been exhaustive, we have shown that the unemployment rate is not very important in explaining the wage rate in a framework where the long-run behavior of wages is modeled explicitly. However, we do not wish to claim that unemployment can never matter for wages within our framework. Instead, we prefer to think of this exercise as an illustration of the usefulness of studying a "cyclical" relationship (between wages and the unemployment rate in this case) in the context of a model that ties down long-run behavior (here, of the wage rate).

### *Wage-Price Causality*

Our model also provides a straightforward way to study another set of issues, namely, the relationship between wages and prices. Causal relationships between these variables can be motivated in a number of ways. For example, Keynesian models commonly specify prices as a markup over wages. In these models a permanent change in the level of wages will have a permanent effect on the level of prices. Similarly, it is not hard to find models of the real wage rate in which nominal wages react to price innovations.

Two papers that recently looked at the empirical relationship between wages and prices are Gordon (1988) and Mehra (1991). Gordon looks at the relationship between prices and unit labor costs, with the latter variable defined as the difference between nominal wages and an exogenous (piecewise linear) trend in productivity. He concludes that wages and prices are determined independently of each other, though the evidence that wages do not have much effect on prices is stronger than the other way around. More recently, Mehra (1991) has carried out a similar analysis. In his model, prices are specified as markups over productivity-adjusted labor costs and are subject to various shocks. Wages are specified as a function of cyclical demand and expected prices. He then goes on to discuss how such equations imply that wages and prices must be related in the long run. Mehra carefully analyzes the time-series behavior of individual series and finds that the two are integrated of order 2, and that it is the first difference of wages that is cointegrated with the first difference of prices. He finds that the rate of inflation is Granger causally prior to the rate of change of wages, not vice versa.

The VECM specification we employ here allows us to look at the long-run relationship between wages and prices as well. In addition, our specification allows a potential

13. We consider the issue of whether LYHR is predetermined with respect to LNWAG below.

role for feedback from wage or price shocks to productivity (an issue we will return to below). Before going further, it is worth pointing out that the error correction term we employ has an interesting antecedent in a term used in Gordon (1988). Specifically, Gordon includes the difference between the lagged level of trend unit labor costs and the price level in equations for both the rate of change of prices and of trend unit labor costs, and he interprets this term as labor's income share. It turns out that this term does not enter significantly into either the inflation or unit labor cost equations.

We begin by asking about the nature of the long-run adjustments between these variables. First, does either of these variables adjust to maintain the long-run relationship estimated above? Table 3 shows that the estimated error correction term does not enter significantly into the price level equation. We obtain the same result when we use the test discussed in Johansen and Juselius (1990); the $\chi^2$ statistic calculated under the null that the error correction term does not belong in the price equation has a marginal significance level of .3. By contrast, restricting the error correction term to be zero in the wage equation leads to a $\chi^2(1)$ statistic of 19, which is significant at any reasonable level. Thus, it is the level of wages—and not the price level—that adjusts to maintain the cointegrating relationship in our model. This influence of prices on wages through the error correction term means that the common practice of estimating a single equation where the price level is regressed on the contemporaneous wage rate (and other variables) is inappropriate. (See Banerjee, et al., 1993 for a discussion of the issues involved.) The appropriate way to proceed in studying this issue would be to estimate this equation as part of a system that also includes an equation for the wage rate.

It is, of course, still possible that changes in the growth rate of wages temporarily affect the growth rate of prices. However, the data do not provide much support for this hypothesis either (the computed $F$ statistic has a marginal significance level of 18 percent). By contrast, we cannot reject the hypothesis that changes in inflation affect the growth rate of wages (we obtain an $F$ statistic with a marginal significance level of 1 percent). Thus, we find that prices Granger cause wages but that wages do not Granger cause prices. Our evidence against the wage markup model echoes the results of both Gordon and Mehra, although the results are not exactly the same.[14]

14. These differences probably reflect both differences in specification (including the precise variables used) as well as differences in modeling strategy. Of the latter, it is worth noting that our specification does not include dummies or other exogenous variables.

### *The Dynamics of the Wages-Productivity Relationship*

We now look at how wages, prices, and productivity respond to various disturbances to the system. Our VECM can be used to analyze a number of interesting issues, some of which are related to the issues raised above. For instance, we can use the model to estimate the responses to a permanent change in productivity. Do firms react to productivity shocks by raising the nominal wage, or is the resulting long-run increase in real wages achieved by falling prices? Similarly, do shocks to productivity have a significant effect on the real wage or are nominal wage innovations more important for the real wage?

To study these and related questions we use the unrestricted VECM presented in Table 3 above. As a robustness check, we also looked at two simplified versions of our model. First, we used a statistical criterion to select lag lengths; however, the resulting shorter lag lengths did not lead to noticeable changes in the dynamic responses obtained from the model. Second, we also estimated a model that imposed the long-run restrictions that we tested for above; specifically, the model excluded the error correction term from the price and productivity equations. We will point out any difference in the dynamics below.

There is still the matter of identification. While there are a number of alternative ways of imposing identifying restrictions on vector autoregressions, none is completely unproblematic; see Hansen and Sargent (1989) for a recent critique. Here, we present results using the earliest such method of identification, suggested by Sims (1980), with the hope that we can get at some of the issues we are interested in by using relatively simple restrictions.[15] Given our concern with productivity and the wage rate, we examined two orderings that alternatively place productivity and nominal wages at the top of the system. Our discussion below focuses on the case where productivity is placed first; however, we also discuss how the results differ in the case where wages are placed first.[16]

Figure 1 shows impulse responses from the system where productivity is placed first, prices are placed second, and wages are placed last. The top left panel shows the response of LRWAG and LYHR to productivity shocks,

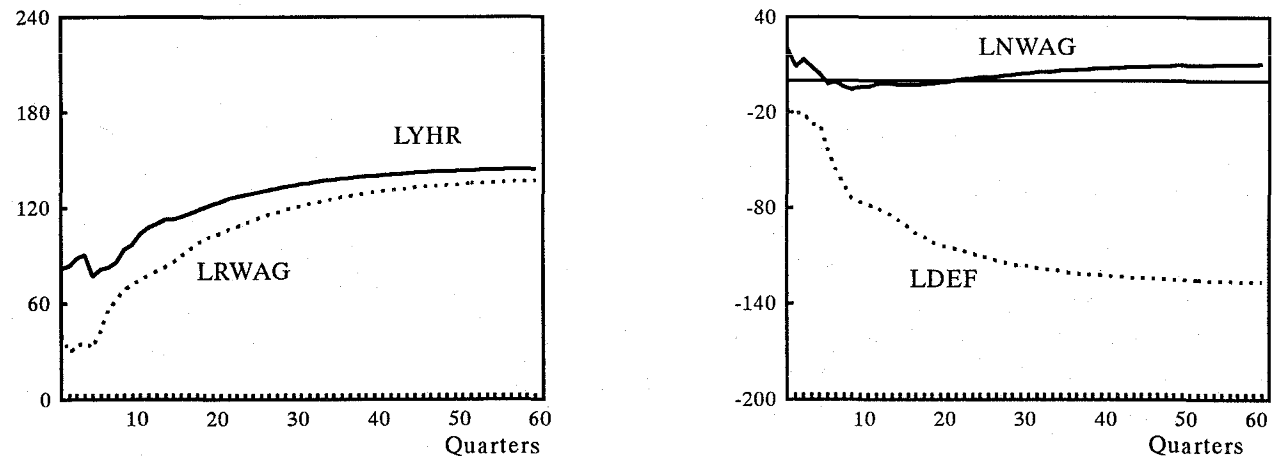
15. Given two variables $X$ and $Y$, for example, one could leave the residuals from the equation for $X$ unchanged and transform the residuals from the $Y$ equation so that they are orthoganal to those from the $X$ equation. Thus, the covariance between the estimated error terms is attributed to the innovation in $X$, and $X$ is said to be ordered first.

16. The correlation between LDEF and LYHR residuals is $-.33$, that between LDEF and LNWAG residuals is .43, and that between LYHR and LNWAG residuals is .30.

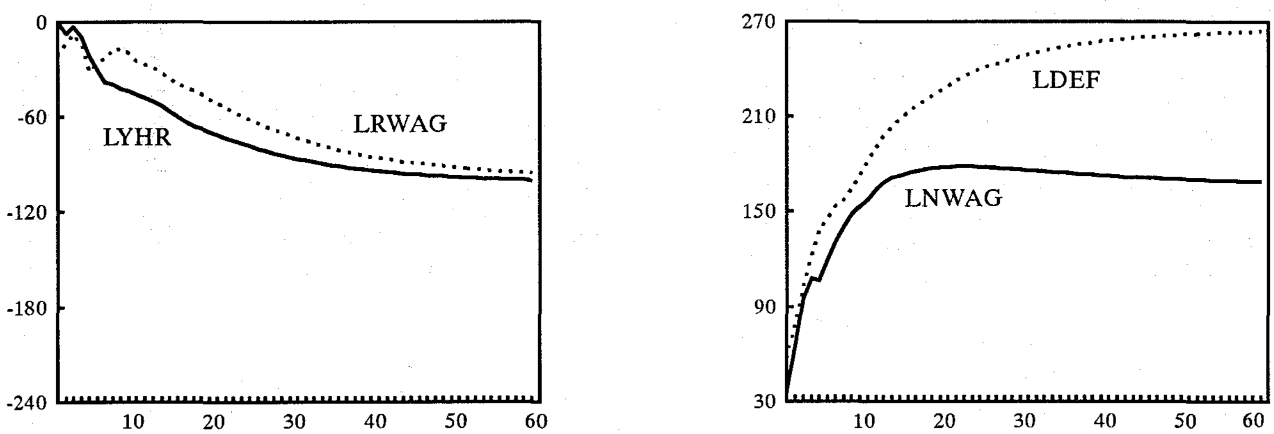
FIGURE 1

DYNAMIC RESPONSES
(ORDERING: LYHR, LDEF, LNWAG)

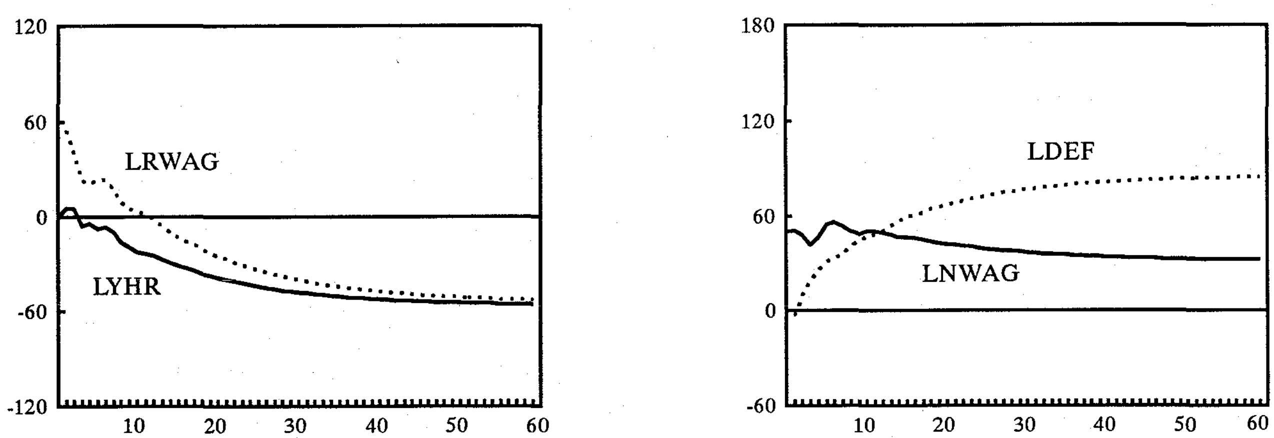
LYHR INNOVATIONS




LDEF INNOVATIONS




LNWAG INNOVATIONS



while the top right panel shows the individual responses of LNWAG and LDEF. (The LRWAG response is obtained as the difference between the LNWAG and the LDEF responses.) The left panel shows that the effects of productivity innovations grow over time; the real wage changes little over the first four to six quarters but does catch up with the change in productivity after a while. The right panel reveals that nominal wages do not go up very much following a positive shock to productivity; instead, the required increase in the real wage is achieved by a fall in the price level. Such a response might occur, for instance, if firms tend to introduce improved products or technology at the old prices. In the alternative ordering, where wages are placed first, productivity second, and prices third, positive productivity shocks also lead to permanently higher real wages because of a reduction in the price level; however, anomalously, the nominal wage falls somewhat.

The middle right panel shows that price level shocks are persistent as well, and that they tend to grow over time. It also shows that while the nominal wage does go up in response to the price shock, it never catches up. The outcome, shown in the middle left panel, is a permanently lower real wage (LRWAG). Thus, price level surprises are associated with lower productivity and real wages. A similar result is obtained with the alternative ordering. Such a response might result, for instance, if a negative supply shock manifests itself first as an increase in prices.

The panels at the bottom show the effects of a positive nominal wage shock. While the nominal wage is persistently higher as a result, the price level increases by more than the increase in wages, so that the ultimate outcome is a reduction in both the real wage and productivity. We obtain a similar result in the system with the alternative ordering, although the nominal wage shock has a larger effect on LNWAG and LDEF than in the first ordering.

As might be expected, the effects of the nominal wage shock are sensitive to whether the error correction term is included in the price and productivity equations, regardless of the ordering. If this term is not included, both the real and nominal wage return to zero over a six to seven year horizon following a nominal wage shock (in the LYHR, LDEF, LNWAG ordering). The initial response of the price level in that model also is much smaller than that shown in the bottom panels.

The variance decompositions associated with Figure 1 are shown in Table 4. LYHR appears to be largely exogenous. Note also that in the long run, LNWAG is driven largely by LDEF innovations. LDEF is driven largely by its own innovations, with LYHR innovations playing a small role. The bottom panel shows that while nominal wage innovations have a substantial impact on real wages in the short run, they become less important as the time horizon lengthens, while productivity innovations become more and more important.

TABLE 4

VARIANCE DECOMPOSITIONS
ORDERING: LYHR LDEF LNWAG

| | Quarters Ahead | LYHR | LDEF | LNWAG |
|---|---|---|---|---|
| LYHR | 1 | 100 | 0 | 0 |
| | 4 | 99 | 1 | 0 |
| | 8 | 92 | 7 | 1 |
| | 12 | 87 | 11 | 2 |
| | 60 | 66 | 26 | 8 |
| LDEF | 1 | 10 | 90 | 0 |
| | 4 | 5 | 94 | 1 |
| | 8 | 9 | 88 | 3 |
| | 12 | 12 | 84 | 4 |
| | 60 | 16 | 77 | 7 |
| LNWAG | 1 | 9 | 31 | 60 |
| | 4 | 2 | 72 | 23 |
| | 8 | 1 | 81 | 19 |
| | 12 | 0 | 86 | 14 |
| | 60 | 0 | 94 | 6 |
| LRWAG | 1 | 34 | 10 | 56 |
| | 4 | 36 | 7 | 57 |
| | 8 | 54 | 12 | 34 |
| | 12 | 71 | 10 | 19 |
| | 60 | 66 | 25 | 8 |

NOTE: This table reports the percentage of forecast error variance that is attributed to each of the three shocks.

The alternative ordering does lead to a greater role for LNWAG innovations in both the LNWAG and LDEF forecasts. For instance, at a horizon of 60 quarters, wage innovations are somewhat more important than price innovations for predicting wages and are roughly as important for predicting prices. However, LNWAG innovations explain almost none of the forecast error variance of LYHR or LRWAG in the long run. And neither LDEF nor LNWAG innovations account for much of the variation in LYHR.

Overall, despite differences between the two orderings, they share a number of features. Thus, in both orderings productivity shocks affect the real wage rate through price level adjustments; price level innovations lower the real wage, and nominal wage shocks have little effect on the real wage in the long run.

Before concluding this section it also is worth reviewing some of the evidence presented here in light of the results discussed in prior sections. Our results indicate that shocks to the price level have a significant effect on wages. By contrast, the proportion of the price level forecast error attributable to the nominal wage shock is relatively small, even when the nominal wage is placed first.

There is, of course, significant contemporaneous correlation between the innovations to these two variables; if this correlation were assumed to be the result of shocks to the wage rate, then wage shocks could be said to have a non-negligible effect on prices.[17] However, this inference can be reconciled with the Granger causality tests presented above only if firms complete the required price adjustment (to wage shocks) within the quarter in which the wage shocks occur. Such rapid adjustment seems rather unlikely to us.

Finally, there is no evidence to suggest that shocks to the nominal wage rate have any permanent effect on either real wages or productivity.

## III. Summary and Conclusions

We have argued that the time-series behavior of aggregate wages should be studied in relation to the time-series behavior of productivity. If productivity is nonstationary, relatively tight restrictions on the joint behavior of these variables can be derived from models in which the representative firm is on its demand curve in the long run. We find that data for the postwar U.S. economy are consistent with the hypothesis that the representative firm is on its demand-for-labor curve in the long run.

This finding—which in terms of the empirics is that productivity, wages and prices are cointegrated—allows us to cast the data in the form of a vector error correction model. Using this specification we find that (Granger) causality runs from prices and productivity to wages but not the other way around. Further, our analysis reveals that the measured impact of cyclical variables, such as the unemployment rate, is sensitive both to how the long run is modeled and to the inclusion of a measure of productivity in the wage equation. Our analysis also reveals that nominal wage innovations have little, if any, influence on the long-run behavior of real wages (or, by implication, of productivity). Instead, the long-term behavior of the real wage rate is determined largely by innovations to productivity, and these innovations act almost entirely through changes in the price level.

17. The one exception to this statement is the case where we restrict the error correction term to be zero in the price and productivity equations. In this case, wage shocks have almost no long-run effect on prices.

## References


Abowd, J.M. 1987. "Collective Bargaining and the Division of the Value of the Enterprise." NBER Working Paper No. 2137.

Banerjee, Anindya, Juan Dolado, John W. Galbraith, and David F. Hendry. 1993. *Cointegration, Error-Correction, and the Econometric Analysis of Non-stationary Data.* New York: Oxford University Press.

Bils, Mark. 1990. "Wage and Employment Patterns in Long-Term Contracts when Labor Is Quasi-Fixed." *NBER Macroeconomics Annual*, pp. 187–226.

Blanchard, Olivier, and Stanley Fischer. 1989. *Lectures on Macroeconomics*. Cambridge: MIT Press.

Christiano, Lawrence J. 1988. "Searching for a Break in Real GNP." NBER Working Paper No. 2695.

Engle, Robert F., and C. W. J. Granger. 1987. "Cointegration and Error Correction: Representation, Estimation and Testing." *Econometrica* (March) pp. 251–276.

Engle, Robert F., and Byung Sam Yoo. 1988. "Forecasting and Testing in Co-integrated Systems." *Journal of Econometrics*, pp. 143–159.

Espinosa, Maria Paz, and Changyong Rhee. 1989. "Efficient Wage Bargaining as a Repeated Game." *Quarterly Journal of Economics* (August) pp. 565–588.

Evans, George W. 1989. "Output and Unemployment Dynamics in the United States:1950-1985." *Journal of Applied Econometrics* 3, pp. 213–237.

Fuller, Wayne A. 1976. *Introduction to Statistical Time Series*. New York: John Wiley & Sons.

Gonzalo, Jesus. 1989. "Comparison of Five Alternative Methods of Estimating Long-Run Equilibrium Relationships." Unpublished manuscript. University of California San Diego (November).

Gordon, Robert J. 1988. "The Role of Wages in the Inflation Process." *American Economic Review Papers and Proceedings*, pp. 276–283.

Hansen, Lars P., and Thomas J. Sargent. 1989. "Two Difficulties in Estimating Vector Autoregressions." In *Rational Expectations Econometrics*. Boulder: Westview Press.

Johansen, Søren, and Katarina Juselius. 1990. "Maximum Likelihood Estimation and Inference on Cointegration—with Applications to the Demand for Money." *Oxford Bulletin of Economics and Statistics*, pp. 169–210.

MaCurdy, Thomas E., and John H. Pencavel. 1986. "Testing between Competing Models of Wage and Employment Determination in Unionized Markets." *Journal of Political Economy*, pp. S3–S39.

Mehra, Yash. 1991. "Wage Growth and the Inflation Process." *American Economic Review*, pp. 931–937.

Nickell, Stephen. 1986. "Dynamic Models of Labor Demand." In *Handbook of Labor Economics*, eds. Orley Ashenfelter and Richard Layard. Amsterdam: Elsevier Science Publishers B.V.

Perron, Pierre. 1989. "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis." *Econometrica*, pp. 1361–1401.

Phillips, A.W. 1958. "The Relationship between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957." *Econometrica*, pp. 283–299.

Phillips, Peter C. B. 1987. "Time Series Regressions with a Unit Root." *Econometrica*, pp. 277-301.