Sunspot Equilibrium

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Early History of Sunspots at Penn

- Dave Cass
- Karl Shell
- Costas Azariadis
- Roger Farmer
- Yves Balasko

Mea Culpa

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- Excess Volatility (Shiller)

$$\text{Gain} = \frac{\text{Volatility of Outcome}}{\text{Volatility of Fundamentals}} = \frac{+}{0} \text{ in SSE}$$

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- SSE combines ideas from micro, macro, and game theory

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- Not merely randomizations over certainty equilibria (more later)

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  - Choice between money taxation and commodity taxation

Money Taxation : Example of Source of SSE

- 1 commodity, $l = 1$, chocolates
- 3 guys, $h = 1, 2, 3$
- money taxes $\tau = (\tau_1, \tau_2, \tau_3)$, dollars
- $\tau_1 + \tau_2 + \tau_3 = 0$, dollars
- endowments $\omega = (\omega_1, \omega_2, \omega_3) > 0$, chocolates
- allocations $x = (x_1, x_2, x_3) > 0$, chocolates

Certainty Economy

- max $u_h(x_h)$
  s.t. $x_h = \omega_h - P^m \tau_h = \tilde{\omega}_h$

  where P^m is the chocolate price of money

- $x_1 + x_2 + x_3 = \omega_1 + \omega_2 + \omega_3$,
  or
- $x_1 + x_2 + x_3 = \tilde{\omega}_1 + \tilde{\omega}_2 + \tilde{\omega}_3$

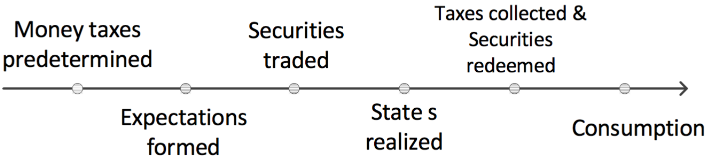
- $0 \leq P^m < \bar{P}^m$

Certainty Economy: Example

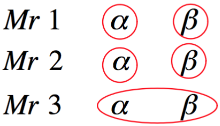
- $\omega = (20, 10, 5)$
- $\tau = (5, 0, -5)$
- $0 \leq P^m < 4 = \bar{P}^m$
- Equilibrium:

  $\{x = (x_1, x_2, x_3) \in \mathbb{R}^3_{++} | x_1 = 20 - 5P^m, x_2 = 10, x_3 = 5 + 5P^m, P^m \geq 0\}$

Sunspots Economy



- Extrinsic random variable: $s \in \{\alpha, \beta\}, \quad \pi(\alpha) + \pi(\beta) = 1$
- Information:



- $\tau_h(\alpha) = \tau_h(\beta) = \tau_h$, incomplete instruments
- $\omega_h(\alpha) = \omega_h(\beta) = \omega_h$, extrinsic uncertainty

Sunspots Economy: Example

- $\omega = (20, 10, 5)$
- $\tau = (5, 0, -5)$
- $u_h = \log$
- $\pi(\alpha) = 3/4, \pi(\beta) = 1/4$
- $P^m(\alpha) = 1, P^m(\beta) = 2$
- α is inflationary state, β is deflationary
- Mr 1 is taxed. He fears deflation. Mr 3 fears inflation. Mr 2 is a banker. He can only gain from volatility.

Sunspots Economy: Example

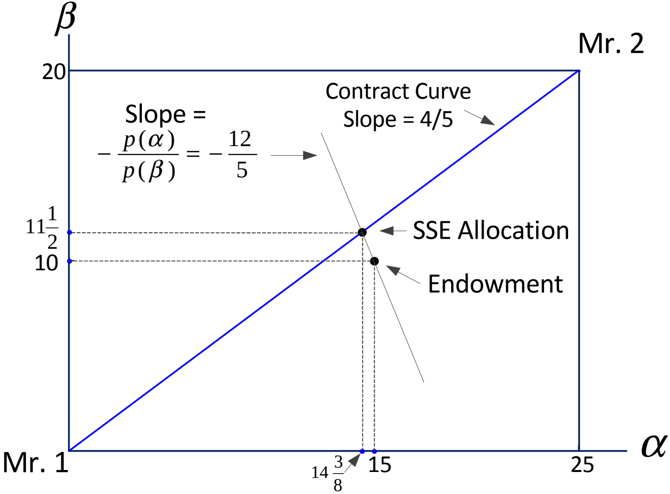
- $(\widetilde{\omega}_1(\alpha), \widetilde{\omega}_1(\beta)) = (15, 10)$
- $(\widetilde{\omega}_2(\alpha), \widetilde{\omega}_2(\beta)) = (10, 10)$
- $(\widetilde{\omega}_3(\alpha), \widetilde{\omega}_3(\beta)) = (10, 15)$

- $\widetilde{x}_3(\alpha) = \widetilde{\omega}_3(\alpha) = 10$
- $\widetilde{x}_3(\beta) = \widetilde{\omega}_3(\beta) = 15$

- Therefore,
$$\widetilde{x}_1(\alpha) + \widetilde{x}_2(\alpha) = 35 - 10 = 25$$
$$\widetilde{x}_1(\beta) + \widetilde{x}_2(\beta) = 35 - 15 = 20$$

- TA-EB is a proper rectangle, 25×20.

Tax-Adjusted Edgeworth Box



The 25 × 20 Tax-Adjusted Edgeworth Box

- Not mere randomization over CE

$$x_2(\alpha) = 10\frac{5}{8} > 10$$
$$x_2(\beta) = 20 - 11\frac{1}{2} = 8\frac{1}{2} < 10$$

- Mr 2 ("banker") gains from volatility
- Mr 3 ("passive") loses from volatility
- Mr 1 and Mr 2 in aggregate lose
- Hence Mr 1 is a loser