The Macroeconomic Effects of a Carbon Tax to Meet the U.S. Paris Agreement Target: The Role of Firm Creation and Technology Adoption*

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Abstract

We analyze the labor market and aggregate effects of a carbon tax in a framework with pollution externalities and equilibrium unemployment. Our model incorporates labor force participation and two margins of adjustment influenced by carbon taxes: (1) firm creation and (2) green production-technology adoption. A carbon-tax policy that reduces carbon emissions by 35 percent—broadly consistent with Biden Administration's new Paris Agreement commitment—can generate mild positive long-run effects on consumption and output, an expansion in the number and fraction of firms that use green technologies, and greater labor force participation, with marginal changes in the unemployment rate. In the short term, the adjustment to a higher carbon tax need not be accompanied by losses in output and consumption or a substantial increase in unemployment. Abstracting from green technology adoption implies that the same policy has substantial adverse short- *and* long-term effects on labor income, consumption, and output. Our findings highlight the importance of considering endogenous technology adoption in assessments of the labor market and aggregate effects of a carbon tax.

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1 Introduction

The potential adverse effects of taxing carbon emissions on firms, job creation and employment, and aggregate economic activity are a central theme in current discussions of environmental policy (OECD, 2017; Hafstead and Williams III, 2019; Metcalf and Stock, 2020a,b). This topic has taken on greater importance with the Biden Administration's April 2021 Paris Agreement commitment to reducing greenhouse gas pollution by roughly 50 percent from 2005 levels by 2030 as part of the international climate negotiations—an ambitious target.[1] The growing interest in introducing a nationwide carbon tax in the U.S. raises three important questions: What are the quantitative effects on labor market and macroeconomic outcomes of using a carbon tax to meet the Administration's target? What role, if any, do carbon tax-induced changes in market structure (via firm entry and exit) and technology choices by firms play in shaping these outcomes? Finally, do the short-term effects of a carbon tax differ from the long-term effects?

We address these three questions in a general equilibrium framework with labor search frictions, an endogenous production structure, and pollution externalities. The labor market and endogenous production structure is based on the framework in Finkelstein Shapiro and Mandelman (2021) (henceforth FSM), who adapt the well-known Ghironi and Melitz (2005) trade framework to model technology adoption and analyze its impact on labor market outcomes (including labor force participation) amid endogenous firm entry. We extend FSM by introducing pollution externalities and focus on how firms' decisions over entry and technology adoption are influenced by the carbon tax. Our model therefore incorporates two margins of adjustment that have been jointly absent in existing quantitative analyses of carbon taxes: (1) firm entry and (2) firms' *choices* over (polluting vs. green) production technologies.[2]

[1] White House Fact Sheet on Greenhouse Gas Pollution Reduction Target, April 22, 2021, available at https://www.whitehouse.gov/briefing-room/statements-releases/2021/04/22/fact-sheet-president-biden-sets-2030-greenhouse-gas-pollution-reduction-target-aimed-at-creating-good-paying-union-jobs-and-securing-u-s-leadership-on-clean-energy-technologies/.

[2] One simple example of technology adoption is Amazon's move to adopt 100,000 electric delivery vehicles by 2030. Shifting to electric van delivery will reduce emissions but requires planning and significant start-up costs to make the transition. A *Motortrend* article highlights some of the issues involved in shifting from gas or diesel to electric vans for the company (see https://www.motortrend.com/news/2022-rivian-prime-delivery-van-first-look-review/).

These margins are important for a comprehensive assessment of the aggregate impact of carbon taxation for at least two reasons. First, the regulatory costs associated with environmental policy not only affect the labor and capital decisions of existing firms as well as their decisions over emissions abatement—an *intensive* margin of adjustment to a carbon tax—but also the incentive of potential firms to enter the market in the first place. In turn, firm entry and exit has direct implications for job creation and aggregate economic activity. Second, a carbon tax shapes firms' relative costs of production and, in doing so, influences the relative merits of adopting green technologies (which are not subject to carbon taxation)—an *extensive* margin of adjustment to this tax.

Based on modeling in the U.S. Energy Information Administration's (EIA) 2021 Annual Energy Outlook, emissions in 2030 in EIA's reference (no new policy) case will have fallen by nearly one-quarter from 2005 levels (4.583 billion metric tons relative to 2005 emissions of 6 billion metric tons). Thus, emissions will need to fall an additional 35 percent between now and 2030 to achieve the Biden Administration's goal.

Using our model under a carbon-tax scheme designed to reduce long-run emissions by 35 percent with carbon-tax revenue rebated lump-sum to households, we find that this policy can generate positive, though quantitatively limited, long-run effects on consumption, output, employment, and labor force participation; negligible long-run adverse effects on unemployment; and a long-run increase in both the number and the share of firms that adopt green technologies. In our simulations, the 35 percent reduction is achieved in five years making the 2030 target feasible. Moreover, the transition path to an economy with lower emissions need not entail short-term reductions in consumption, output, or labor force participation, even if the carbon tax generates a net reduction in the total number of firms in the economy. Since some of the output increase is used for fixed costs of adopting green technologies, increases in consumption or output does not necessarily imply welfare increases. To check that, we report measures of the welfare change following the imposition of the tax and, in general, find very modest welfare declines. The absence of significant adverse aggregate effects from a carbon tax are at odds with those documented in existing quantitative studies on the macroeconomic effects of carbon taxes in the macro literature. Indeed, these studies, which abstract from firms' ability to adopt different technologies in response to policy, find that for similar carbon tax-induced reductions in emissions, a carbon tax has non-trivial negative effects on labor, labor income, consumption, and output.

Our main findings may seem surprising given the distortionary nature of carbon taxation, but our results offer a way to reconcile recent empirical evidence that fails to find adverse effects on employment and output from carbon taxes (Metcalf and Stock, 2020a,b). In our model with endogenous technology adoption decisions along with firm entry and exit, a carbon tax triggers *endogenous* changes in both the market structure *and* in the economy's technological composition of production—that is, the *endogenous* prevalence of polluting versus green production technologies in the aggregate production process. These policy-induced endogenous changes improve the economy's average firm productivity and cost profile and, in doing so, lead to improved labor market and macroeconomic outcomes despite a carbon tax raising the cost of using technologies that generate emissions. A key finding of our analysis is that endogenous changes in the economy's dirty-clearn technological composition of production that arise as an indirect result of taxing emissions—changes that are at the core of policy discussions regarding the transition to a low carbon, greener economy but are nonetheless absent in existing quantitative studies—can play a decisive role in shaping labor-market and macroeconomic outcomes in response to a carbon tax, and can potentially generate positive (albeit small) macroeconomic and welfare effects from the policy.[3]

Our work contributes to a small but growing set of studies on the macroeconomic effects of carbon taxes and environmental policy, most of which are rooted in one-sector frameworks (see Fischer and Springborn, 2011, Heutel, 2012, Annicchiarico and Di Dio 2015, Annicchiarico, Correani, and Di Dio, 2017, and Annicchiarico and Diluiso, 2019, among others). A common finding across these studies, which generally abstract from considering labor market outcomes, is that carbon taxes have adverse effects on aggregate economic activity. Only recently has this literature started to explore the relationship between environmental policy, macroeconomic outcomes, and labor markets, with Aubert and Chiroleu-Assouline (2019), Gibson and Heutel (2020), and Castellanos and Heutel (2021), being the most prominent

[3]The market equilibrium in models with endogenous product variety and increasing returns to scale is not a social optimum in the absence of lump-sum transfers to firms such that firms price at marginal cost and earn non-negative firm profits (see Dixit and Stiglitz, 1977). As a result, a carbon tax can potentially raise welfare given this second-best economic environment.

ones using one-sector models, and Hafstead and Williams III (2018) and Férnandez Intriago (2020) being among the select few using two-sector (polluting and green) frameworks.[4] A parallel line of theoretical work has analyzed the link between firm entry and environmental policy (Annicchiarico, Correani, and Di Dio, 2018), how environmental policy shapes the firm size distribution and innovation in green technologies (Coria and Kyriakopoulou, 2018, and Fried, 2018, respectively), the role of endogenous green-product creation (Jondeau et al., 2022), and the interplay between research and development (R&D) across sectors, firm entry, and the adoption of green technologies (Acemoglu et al., 2016), all within a context of frictionless labor markets.[5] Our focus on green technology adoption is similar in nature to Acemoglu et al. (2016), though we abstract from modeling R&D and instead focus on unemployment and labor force participation. More broadly, these studies also find that carbon taxes generally have adverse effects on labor income and macroeconomic outcomes.

Our paper is also related to double-dividend and tax interaction literature. That literature, starting with Bovenberg and de Mooij (1994) and Parry (1995) note that the standard Pigouvian prescription to set a tax on pollution equal to the social marginal damages of pollution fails in a world with pre-existing tax distortions. In a world with pre-existing distortions, an environmental tax creates distortions on input uses that are of first-order importance. Thus the optimal tax will take account of the benefits of reducing pollution, the potential to use the environmental tax revenue to lower other distortionary taxes, and the efficiency costs of the environmental tax itself. In general, the optimal tax on pollution lies below the social marginal damages due to the first-order efficiency costs. The tax interaction literature went on to consider how the use of environmental tax revenue could affect welfare leading to a number of results such as a "weak double-dividend" where the welfare cost of an environmental tax is small when the revenue is used to lower pre-existing

[4]In recent work, Kanzig (2022) documents the short-run effects of a carbon pricing policy on emissions and aggregate economic activity in the European Union, finding that the policy temporarily reduces industrial production and GDP and increases unemployment. In addition, using U.K. data, he shows that the policy has asymmetric effects across households, with lower-income households facing a greater burden from the policy (reflected in lower household consumption) relative to higher-income households. He also provides evidence that the policy bolsters innovation in green technologies.

[5]For recent work on pollution, environmental regulation, firm entry, and trade, see Shapiro and Walker (2018) and Egger, Kreickemeier, and Richter (2021). There is an earlier literature on induced technological innovation (Romer, 1990, Goulder and Mathai, 2000, and Popp, 2002 among others). For a recent summary of the literature on innovation and climate policy, see Popp (2010).

distortionary taxes (on capital and labor, for example) than when the revenue is returned lump-sum. The literature finds no support for a strong double-dividend where welfare rises with the introduction of a carbon tax (when ignoring the environmental benefits of reduced pollution). See Goulder, 1995, for a fuller discussion of this topic. That a carbon tax has real costs, in the sense of the modest welfare reductions that we find in our quantitative analysis, is consistent with the rejection of the strong double dividend.[6]

We contribute to the literature on the labor market and macroeconomic consequences of carbon taxation in two ways. First and foremost, our work shows that the interaction of endogenous technology adoption by firms and firm entry—a mechanism that introduces the possibility of *endogenous* changes in the economy's underlying technological composition (polluting vs. green) of production, where these changes shape the economy's average firm productivity profile—play a decisive role in shaping the positive labor market and aggregate effects of the carbon tax in our analysis. This last finding and mechanism is, to the best of our knowledge, new and can explain why related studies that, by abstracting from the indirect effects that carbon taxes may have in shaping firms' choices over technologies and therefore the economy's endogenous production structure, generally document adverse labor and macroeconomic effects from carbon taxes. Moreover, our model-based results provide a plausible rationale behind recent empirical evidence on the absence of adverse employment and macroeconomic effects of carbon taxes (Metcalf and Stock, 2020a,b).

Second, the majority of existing models with frictional labor markets and pollution externalities focus exclusively on unemployment and, if they feature two sectors (as in Hafstead and Williams III, 2018), on the sectoral reallocation of workers, but they generally abstract from labor force participation. Moreover, these same models abstract from considering the role of firms' technology adoption decisions, which are at the heart of our model and main findings. Our framework expands on existing search models with pollution externalities by incorporating the search and labor force participation behavior of individuals, which plays an important role in shaping the composition of total unemployment. This composition is rele-

[6]While our model does include a damage function, a more realistic assessment of damages and the environmental benefits of reducing greenhouse gas emissions would take into account both irreversible threshold effects (tipping points) as well as heterogeneous damages suffered by different cohorts across time and space that are masked by focusing on an average damage measure.

vant in contexts where policymakers may consider the use of labor market policies to ease the transition of workers adversely affected by carbon taxes into new jobs.[7] More broadly, our work can be seen as bridging the gap between existing models of carbon taxation with equilibrium unemployment that abstract from endogenous changes in the economy's production structure, and models that focus on how carbon taxes can shape the production structure of the economy via technology adoption, but abstract from considering labor market outcomes (see, for example, Acemoglu et al., 2016).

The rest of the paper is structured as follows. Section 2 describes the model. Section 3 outlines our calibration strategy and presents the main results from our quantitative analysis. Section 4 concludes.

2 The Model

The economy is comprised of firms, a government, a population of unit mass, and a representative household with a measure one of household members that owns all firms. Search frictions in the labor market give rise to equilibrium unemployment. Households consume, make labor force participation decisions, and invest resources in firm creation so that firm entry is endogenous. As described below, once firms enter the market, they can choose the technological composition of their production process based on the availability of two production technologies. The first technology is "regular" and generates harmful carbon dioxide emissions as a by-product of production. Its adoption is costless but its usage entails the payment of a tax on emissions—a carbon tax—whose revenue is transferred lump-sum to households.[8] The second technology is "green" and does not generate emissions (and is therefore not subject to the carbon tax), but its adoption is subject to fixed costs.[9]

[7] Castellanos and Heutel (2021) considers sectoral labor force participation in a multi-sector computable general equilibrium model and highlights the role of mobility frictions in shaping unemployment outcomes, but abstracts from both firm entry and technology adoption, where the latter lie at the heart of our analysis and findings. Our framework allows for differential search costs across firm categories, which can represent barriers to mobility across sectors in a reduced-form way.

[8] While our model employs a carbon tax, our results are unchanged if we assume other forms of carbon pricing such as a cap and trade system. In the absence of uncertainty, a cap and trade system with auctioned allowances is equivalent to a carbon tax.

[9] This two-sector or two-firm-category structure is similar in nature to Acemoglu et al. (2016), Hafstead and Williams III (2018), and others who consider a production structure with polluting and non-polluting

The presence of firm creation and technology adoption decisions has two relevant implications for a comprehensive analysis of carbon taxation. First, as noted in Bilbiie, Ghironi, and Melitz (2012), firms can be interpreted as a form of capital, and therefore the costly creation of firms can be interpreted as a form of investment. Thus, the costs associated with firm entry and green technology adoption upon entry can be broadly interpreted as embodying, in a reduced-form way, the resource costs associated with investments in cleaner technologies (and their development, reflected partially in the costs of firm creation) that facilitate the transition to a low-carbon, greener economy-wide production structure.[10] Second, by allowing firms to choose their production technology in a context where firm entry is endogenous, the number of firms using each technology—and therefore the economy's underlying production structure—changes in response to policy and modifies, endogenously, the underlying carbon intensity of the economy. This last point differentiates our framework from existing models with polluting and non-polluting sectors that abstract from firm entry and green technology adoption, where resources can be reallocated across sectors but the economy's underlying production structure—and therefore the underlying carbon intensity of the economy—is ultimately exogenous.

Our model is an adaptation of the framework in Finkelstein Shapiro and Mandelman (2021) to an environment with pollution externalities and carbon taxation. The description below follows closely the general setup in that paper.

firms to account for the fact that only a subset of firms or sectors in the economy are responsible for the bulk of emissions.

[10] Per the International Energy Agency's May 2021 flagship report (IEA, 2021), the technologies to achieve the targeted emissions reductions by 2030 are already available (though new technologies will be needed to achieve further reductions by 2050). As such, assuming that green technology adoption is subject to fixed costs without explicitly modeling green-technology research and development (R&D) is a reasonable baseline assumption in the context of our analysis. As part of our robustness analysis, we experiment with alternative reduced-form specifications for the costs of entry and green-technology adoption (modeling the microfoundations of green-technology development and the barriers and frictions associated with their adoption, while relevant for understanding how new technologies needed to achieve emissions reductions beyond 2030 will be deployed and adopted, is outside the scope of our work). For a framework with frictionless labor markets and polluting and non-polluting technologies where R&D in these technologies is explicitly modeled, see Acemoglu et al. (2016). For macro models that microfound R&D investment and the lags and frictions associated with new technology adoption in a context with endogenous product creation, see Comin and Gertler (2006).

2.1 Firm and Production Structure

There is an unbounded number of monopolistically-competitive firm entrants whose entry is subject to a sunk entry resource cost φ_e. Once firms enter, they draw their idiosyncratic productivity a from a common distribution $G(a)$ with support $[a_{min}, \infty)$,where the resulting level of a remains unchanged until the firm exits with exogenous probability $0 < \delta < 1$. Each firm produces a single output variety ω based on a, where $y_t(\omega)$ denotes the output of a given firm producing variety ω. Thus, in the rest of the model description, we refer to a firm producing variety ω with productivity level a simply as firm a.

When a household decides to create a new firm, all it knows is the productivity distribution, $G(a)$, but not the firm's realized productivity. Upon entering and incurring the sunk entry cost, the new firm's productivity level is realized, allowing it to choose one of two technologies.

A regular (r) technology is available that generates carbon dioxide (harmful) emissions—emissions for short. These emissions are subject to a carbon tax but can be partly mitigated via expenditures on emissions abatement. A green (g) technology is also available that does not generate emissions. Using the green technology, however, entails incurring a fixed resource cost φ_g associated with the adoption of the technology.[11] Appendix A.1 formally shows that there is an endogenous threshold level of productivity such that firms with realized productivity below this threshold choose the r technology and firms with realized productivity above this threshold choose the g technology. Households will choose to create a new firm based on the new firm's expected future profits, which will depend on the distribution of the productivity parameter a and the resulting technology that is optimal for the firm to choose. The choice of technology makes the measure of firms in each category endogenous.

Both production technologies rely on labor, which is subject to search and matching frictions, and physical capital as inputs. Emissions from using the r technology add to the economy's stock of carbon dioxide pollution that, in turn, has negative externalities on

[11] For example, the fixed costs of adoption are well illustrated by the logistical and technology planning undertaken by Amazon in rolling out electric delivery vans (a technology that is already in existence). The presence of fixed costs of technology adoption is also reminiscent of Bustos (2011), who focuses on technology adoption among exporters in a trade setting.

production for all firms in the economy, as we detail below.

2.1.1 Total Output

Total output is given by $Y_t = \left(\int_{\omega\in\Omega} y_t(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega\right)^{\frac{\varepsilon}{\varepsilon-1}}$, where Ω is the potential measure of firms in the economy and $\varepsilon > 1$ is the elasticity of substitution across individual output varieties. In turn, the aggregate price index is $P_t = \left(\int_{\omega\in\Omega} p_t(\omega)^{1-\varepsilon} d\omega\right)^{\frac{1}{1-\varepsilon}}$. As in Ghironi and Melitz (2005), only a subset of firms $\Omega_t \subset \Omega$ are ultimately active in any given period.

2.1.2 Firm Structure

In what follows, we separate the production process from technology-adoption and pricing decisions by introducing intermediate goods producers and firms that use these intermediate goods. This facilitates the comparison of our framework to related models that abstract from firm entry and technology-adoption margins without affecting the general economic environment.[12]

Firm Profits and Threshold Productivity Level As noted earlier, we can think of a firm a as having access to two possible production lines that differ in their technology. Individual profits from producing with the r technology, $\pi^y_{r,t}(a)$, are given by

$$\pi^y_{r,t}(a) = \left[\rho_{r,t}(a) - \frac{mc_{r,t}}{a}\right] y_{r,t}(a),$$

while profits from producing with the g technology, $\pi^y_{g,t}(a)$, are given by

$$\pi^y_{g,t}(a) = \left[\rho_{g,t}(a) - \frac{mc_{g,t}}{a}\right] y_{g,t}(a) - \varphi_g,$$

[12] This separation of production from technology adoption is common in the macroeconomics literature. We can equivalently characterize the production process as one where firms use factors of production to produce a final good. Firms enter, learn their productivity level, and choose a production technology (regular or green). Having learned their productivity level, they match the technology choice to their productivity level appropriately (as discussed below). Firms use capital and labor to produce output with a constant-returns-to-scale production function. Marginal revenue implied for intermediate goods producers in Section 2.1.3 below simply becomes marginal cost for final goods producers just below.

where $\rho_{j,t}(a) \equiv p_{j,t}(a)/P_t$, $mc_{j,t}/a$, and $y_{j,t}(a)$ denote, respectively, the real output price, the real effective marginal cost, and the firm output associated with using technology $j \in \{g, r\}$, and φ_g is the fixed cost of g-technology adoption. Firm a is indifferent between production technologies when

$$\pi^y_{g,t}(a_{g,t}) = \pi^y_{r,t}(a_{g,t}), \tag{1}$$

where $a_{g,t}$ is the threshold idiosyncratic productivity level above which firms adopt the g technology.[13]

Optimal Pricing Given the aggregation of total firm output in Section 2.1.1, the demand function for firm a's output is given by $y_{j,t}(a) = (\rho_{j,t}(a))^{-\varepsilon} Y_t$ for $j \in \{g, r\}$. Then, firm a chooses $\rho_{j,t}(a)$ to maximize $\pi^y_{j,t}(a)$ subject to the demand function for $y_{j,t}(a)$. The resulting optimal real price for firm a is given by the standard markup condition under monopolistic competition: $\rho_{j,t}(a) = \frac{\varepsilon}{\varepsilon-1}\frac{mc_{j,t}}{a}$.

Evolution of Firms Denote by N_t the measure of total active firms and by $N_{e,t}$ the measure of new entrants. Then, the evolution of the total number of firms in the economy is

$$N_t = (1-\delta)\left[N_{t-1} + N_{e,t-1}\right]. \tag{2}$$

Recalling that firms draw their idiosyncratic productivity from a distribution $G(a)$ and that $a_{g,t}$ is the threshold level of productivity above which firms use the g technology, the number of r firms $N_{r,t}$ is given by $N_{r,t} = G(a_{g,t})N_t$ and the number of g firms $N_{g,t}$ is given by

[13] See FSM for an analogous indifference condition in the context of firms' decisions to adopt digital technologies, and Zlate (2016) in the context of firms' decisions to offshore production. It can be shown that if $a_{g,t}$ is not at the extreme ends of the support of the distribution, then the slope of $\pi^y_{g,t}(a_{g,t}) > \pi^y_{r,t}(a_{g,t})$, for g and r firms only intersect once (at $a_{g,t}$). See Appendix A.1 for a proof. In the model, given positive fixed costs of technology adoption, the g technology is associated with an endogenously higher average level of idiosyncratic productivity among firms using this technology. One way in which this outcome can be rationalized empirically is by considering the productivity-enhancing effects of research and development (R&D) in green technologies. Of course, polluting technologies can also become more productive with R&D, and R&D may be initially focused on these technologies given their larger relative base and adoption (see, for example, Acemoglu et al., 2016). However, amid environmental regulations (including carbon taxes) that affect production using polluting technologies, R&D becomes biased towards green technologies, which can enhance their productivity vis-à-vis polluting technologies.

$N_{g,t} = [1 - G(a_{g,t})]\, N_t$, where $G'(a_{g,t}) > 0$.

Firm Averages Denote by $\widetilde{a}_{r,t}$ the average idiosyncratic productivity level of r firms and by $\widetilde{a}_{g,t}$ the average idiosyncratic productivity level of g firms. Formally, these averages are given by $\widetilde{a}_{r,t} = \left[\frac{1}{G(a_{g,t})}\int_{a_{min}}^{a_{g,t}} a^{\varepsilon-1} dG(a)\right]^{\frac{1}{\varepsilon-1}}$ and $\widetilde{a}_{g,t} = \left[\left(\frac{1}{1-G(a_{g,t})}\right)\int_{a_{g,t}}^{\infty} a^{\varepsilon-1} dG(a)\right]^{\frac{1}{\varepsilon-1}}$. Then, we can define average individual-firm profits as $\widetilde{\pi}_t^y = \frac{N_{r,t}}{N_t}\widetilde{\pi}_{r,t}^y + \frac{N_{g,t}}{N_t}\widetilde{\pi}_{g,t}^y$ where $\widetilde{\pi}_{r,t}^y \equiv \pi_{r,t}^y(\widetilde{a}_{r,t})$ and $\widetilde{\pi}_{g,t}^y \equiv \pi_{g,t}^y(\widetilde{a}_{g,t})$ are average individual-firm profits from producing with the r and g technologies, respectively. Analogously, average real prices and average individual-firm output are given by $\widetilde{\rho}_{r,t} \equiv \rho_{r,t}(\widetilde{a}_{r,t})$ and $\widetilde{\rho}_{g,t} \equiv \rho_{g,t}(\widetilde{a}_{g,t})$ and by $\widetilde{y}_{r,t} \equiv y_{r,t}(\widetilde{a}_{r,t})$ and $\widetilde{y}_{g,t} \equiv y_{g,t}(\widetilde{a}_{g,t})$, respectively. As we show in Section 2.2, given that firms' idiosyncratic productivity is revealed only after incurring a sunk cost and entering the market, firm creation decisions are influenced by, among other factors, the expected value of $\widetilde{\pi}_t^y$.

2.1.3 Intermediate Goods Producers

There is a measure 1 of perfectly-competitive producers of intermediate goods for r and g firms. These producers use category-specific labor, which is subject to search and matching frictions, and capital. The production of intermediate goods for r firms generates pollution emissions e_t that add to the economy's stock of pollution x_t (where this stock is taken as given by firms). We follow the literature and assume that the stock of pollution evolves as $x_t = \rho_x x_{t-1} + e_t + e_t^{row}$, $0 < \rho_x < 1$, where e_t^{row} denotes exogenous emissions from the rest of the world.[14] Emissions e_t are subject to a carbon tax τ_t, but r firms can mitigate these emissions via abatement expenditures. In contrast, the production of intermediate goods for g firms does not generate pollution emissions and is not subject to the carbon tax. As in Nordhaus (2008) and others, the pollution stock leads to a loss of output for a given amount of capital and labor through a damages function $D(x_t)$ lying between 0 and 1, where $D(0) = 1$ and $D'(x_t) < 0$. The damages function, which is taken as given by producers,

[14] Similar to existing analyses of carbon taxes using closed-economy macroeconomic models, this assumption implies that our baseline framework abstracts from carbon leakage. Under plausible parameterizations for the degree of carbon leakage, assuming that a fraction of the tax-induced reduction in domestic emissions is partially offset by an increase in emissions from the rest of the world as a result of carbon leakage makes the positive effects of the carbon tax on output and consumption marginally smaller but does not change our main conclusions.

affects the production of intermediate goods for both r and g firms.

Formally, intermediate goods producers choose the number of vacancies $v_{g,t}$ and $v_{r,t}$ which are needed to hire workers to produce each category of intermediate goods; the total amount of capital k_{t+1}; the measures of g and r workers $n_{g,t}$ and $n_{r,t}$ producers would like to have; and the fraction of emissions abatement μ_t to maximize $\mathbb{E}_0 \sum_{t=0}^{\infty} \Xi_{t|0} \pi_t^i$ subject to[15]

$$\begin{aligned}\pi_t^i = & \left[D(x_t) mc_{r,t} H(n_{r,t}, k_{r,t}) - w_{r,t} n_{r,t} - \psi_r v_{r,t} - \tau_t e_t - \Gamma_t\right] \\ & + \left[D(x_t) mc_{g,t} F(n_{g,t}, k_{g,t}) - w_{g,t} n_{g,t} - \psi_g v_{g,t}\right] \\ & - \left[k_{t+1} - (1-\delta) k_t\right],\end{aligned}$$

the perceived evolution of each category of employment

$$n_{r,t} = (1-\varrho) n_{r,t-1} + v_{r,t} q(\theta_{r,t}), \tag{3}$$

and

$$n_{g,t} = (1-\varrho) n_{g,t-1} + v_{g,t} q(\theta_{g,t}), \tag{4}$$

and total physical capital

$$k_t = k_{g,t} + k_{r,t}, \tag{5}$$

where $\Xi_{t|0}$ is the household's stochastic discount factor (defined further below), the term

$$\Gamma_t = \gamma \mu_t^{\eta} D(x_t) H(n_{r,t}, k_{r,t}), \tag{6}$$

is the total cost of abating emissions from the production of intermediate goods for r firms, and

$$e_t = (1-\mu_t) \zeta \left[D(x_t) H(n_{r,t}, k_{r,t})\right]^{1-\nu}, \tag{7}$$

is the total amount of emissions generated by such production net of abatement, where $\gamma, \zeta > 0$, $\eta \geq 1$, and $0 < \nu \leq 1$.[16] Note that both the cost of abating emissions, Γ_t,

[15] Recall that households own all firms (and take their profits as given). Hence the joint profit maximization assumption below.

[16] We follow Heutel (2012), Annicchiarico and di Dio (2015), among others, in modeling emissions and abatement cost as functions of net output (net of environmental damages). Nordhaus (2008), in contrast, models emissions and abatement cost as functions of gross output. We show in Table A3 of Appendix A.7

and the emissions themselves, e_t, are a function of the production of intermediate goods for r firms. $H(n_{r,t}, k_{r,t})$ and $F(n_{g,t}, k_{g,t})$ are constant-returns-to-scale and increasing and concave functions in each argument (we assume that aggregate productivity is constant and normalized to 1). $0 < \delta < 1$ is the capital depreciation rate and ψ_j and $w_{j,t}$ are, respectively, the flow cost of posting vacancies and the real wage of workers in category $j \in \{g, r\}$.[17] Recall that τ_t is the tax on emissions and $D(x_t)$ is the pollution-damages function whose properties were described above.

Turning to the evolution of each category of employment, $0 < \varrho < 1$ is the exogenous probability of job separation and $q(\theta_{j,t})$ is the endogenous job-filling probability in category j, which is a function of market tightness $\theta_{j,t}$. Finally, we follow the labor-market timing convention in Arseneau and Chugh (2012) whereby filled vacancies in period t become productive in the same period.

The first-order conditions yield an optimal emissions abatement rate μ_t

$$\tau_t \zeta \left(D(x_t) H(n_{r,t}, k_{r,t})\right)^{-\nu} = \gamma \eta \mu_t^{\eta-1}, \tag{8}$$

capital Euler equations

$$1 = \mathbb{E}_t \Xi_{t+1|t} \left[D(x_{t+1}) mc_{r,t+1} H_{k_r,t+1} - \tau_{t+1} e_{k_r,t+1} - \Gamma_{k_r,t+1} + (1-\delta)\right], \tag{9}$$

and

$$1 = \mathbb{E}_t \Xi_{t+1|t} \left[D(x_{t+1}) mc_{g,t+1} F_{k_g,t+1} + (1-\delta)\right], \tag{10}$$

as well as standard job creation conditions for employment in each category

$$\frac{\psi_r}{q(\theta_{r,t})} = \left[\begin{array}{c} D(x_t) mc_{r,t} H_{n_r,t} - \tau_t e_{n_r,t} \\ -\Gamma_{n_r,t} - w_{r,t} + (1-\varrho) \mathbb{E}_t \Xi_{t+1|t} \frac{\psi_r}{q(\theta_{r,t+1})} \end{array}\right], \tag{11}$$

that our results are not appreciably changed if we follow Nordhaus's approach.

[17] Following the macro literature on endogenous firm entry, we assume that the capital depreciation rate and the firm exit rate are the same. Introducing differences in firm exit and capital depreciation rates does not change our main conclusions.

and

$$\frac{\psi_g}{q(\theta_{g,t})} = \left[D(x_t)mc_{g,t}F_{n_g,t} - w_{g,t} + (1-\varrho)\mathbb{E}_t\Xi_{t+1|t}\frac{\psi_g}{q(\theta_{g,t+1})}\right], \tag{12}$$

where $e_{n_r,t}$ and $e_{k_r,t}$, denote the marginal increase in emissions from one more worker and one more unit of capital in the production of intermediate goods for r firms, respectively, and $\Gamma_{n_r,t}$ and $\Gamma_{k_r,t}$ denote the marginal increase in the resource cost of emissions abatement associated with having one more worker and one more unit of capital in the production of intermediate goods for r firms, respectively.[18]

Intermediate goods producers equate the marginal cost of emissions abatement—given by the resource cost incurred as a result of the marginal increase in emissions abatement—to the marginal benefit of emissions abatement—given by the marginal output gain (net of pollution damages) from not having to pay the carbon tax. The capital Euler equations are standard. Finally, the job creation conditions equate the marginal cost and the expected marginal benefit of posting a vacancy for each category of employment. In the case of posting a vacancy to hire workers who produce intermediate goods for r firms, producers take into account the regulation cost associated with emissions generation and the marginal resource cost of emissions abatement associated with having one more r worker. Note that the damages from pollution affect the expected marginal benefit of hiring workers across categories.

2.2 Households and Firm Creation

There is a representative household with a measure one of household members who can be employed, unemployed and searching for employment, or outside of the labor force. Households own all firms and spend resources to create firms. In addition, all proceeds from taxing emissions from the production of intermediate goods for r firms are transferred lump-sum to households.

Formally, households choose consumption c_t, the measures of searchers in each employment category $s_{g,t}$ and $s_{r,t}$, the measures of workers in each category $n_{g,t}$ and $n_{r,t}$ the household would like to have, the number of new firms $N_{e,t}$ and the desired total number of firms

[18] That is, $e_{n_r,t} = (1-\nu)\zeta(1-\mu_t)\left(D(x_t)H(n_{r,t},k_{r,t})\right)^{-\nu} D(x_t)H_{n_r,t}$ and $\Gamma_{n_r,t} = \gamma\mu_t^{\eta}D(x_t)H_{n_r,t}$. Analogous expressions hold for $k_{r,t}$.

N_{t+1} to maximize $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\mathbf{u}(c_t) - \mathbf{h}(lfp_{g,t}, lfp_{r,t})\right]$ subject to the budget constraint

$$c_t + \varphi_e N_{e,t} + T_t = w_{g,t} n_{g,t} + w_{r,t} n_{r,t} + \chi\left[(1 - f(\theta_{g,t}))s_{g,t} + (1 - f(\theta_{r,t}))s_{r,t}\right] + \widetilde{\pi}_t^y N_t + \pi_t^i + \tau_t e_t,$$

the perceived evolution of employment in each category $j \in \{g, r\}$

$$n_{j,t} = (1 - \varrho) n_{j,t-1} + s_{j,t} f(\theta_{j,t}), \tag{13}$$

and the evolution of final-goods firms

$$N_{t+1} = (1 - \delta)\left[N_t + N_{e,t}\right], \tag{14}$$

where labor force participation in each category is given by $lfp_{g,t} = n_{g,t} + (1 - f(\theta_{g,t}))s_{g,t}$ and $lfp_{r,t} = n_{r,t} + (1 - f(\theta_{r,t}))s_{r,t}$. The utility from consumption and disutility from labor force participation have standard properties, with $\mathbf{u}(c_t)$ being increasing and concave, and $\mathbf{h}(lfp_{g,t}, lfp_{r,t})$ being increasing and convex in $lfp_{j,t}$ for $j \in \{g, r\}$.[19] Total labor income is given by $w_{g,t} n_{g,t} + w_{r,t} n_{r,t}$. In turn, $\widetilde{\pi}_t^y N_t$ and π_t^i denote total average profits from firms and intermediate-goods producers, respectively. $\tau_t e_t$ are lump-sum transfers from taxing emissions, χ denote unemployment benefits, and $f(\theta_{j,t})$ is the job-finding probability in employment category $j \in \{g, r\}$ (defined in Section 2.3 below). In turn, $\varphi_e N_{e,t}$ represents the total resource cost from creating new firms. Finally, T_t are lump-sum taxes that finance unemployment benefits.

The first-order conditions yield labor force participation conditions for each employment category $j \in \{g, r\}$

$$\left(\frac{\mathbf{h}_{lfp_{j,t}} - \mathbf{u}'(c_t)\chi}{f(\theta_{j,t})\mathbf{u}'(c_t)}\right) = w_{j,t} - \chi + (1-\varrho)\mathbb{E}_t \Xi_{t+1|t}\left(1 - f(\theta_{j,t+1})\right)\left(\frac{\mathbf{h}_{lfp_{j,t+1}} - \mathbf{u}'(c_{t+1})\chi}{f(\theta_{j,t+1})\mathbf{u}'(c_{t+1})}\right), \tag{15}$$

[19] Note that our functional-form choice for $\mathbf{h}(lfp_{g,t}, lfp_{r,t})$ allows for potentially different weights in the (utility) costs of participating—that is, searching or working—in the r sector relative to the g sector. Since the only way for individuals to transition between sectors in the model is via search unemployment, a weight differential for the utility costs of participating can embody, in a reduced-form way, the relative costs of transitioning from one category of jobs to the other (this could be due to, for example, differences in the skills needed in each job category, which may make it more costly to search and work in one category relative to the other).

and a final-goods firm creation condition

$$\varphi_e = (1-\delta)\mathbb{E}_t \Xi_{t+1|t}\left[\widetilde{\pi}^y_{t+1} + \varphi_e\right], \tag{16}$$

where $\Xi_{t+1|t} \equiv \beta \mathbf{u}'(c_{t+1})/\mathbf{u}'(c_t)$. Intuitively, for each employment category, households equate the expected marginal cost of searching for a job, which is given by the marginal disutility from participating in the labor market net of unemployment benefits and adjusted by the probability of finding a job, to the expected marginal benefit, given by the wage net of unemployment benefits and the continuation value from staying employed in the future. In turn, households equate the marginal cost of creating a firm, given by the sunk entry resource cost, to the expected marginal benefit, which is given by expected average individual-firm profits and the continuation value if the firm survives into the next period with probability $(1-\delta)$.

2.3 Matching Processes and Wage Determination

Let $m(s_{g,t}, v_{g,t})$ and $m(s_{r,t}, v_{r,t})$ be standard constant-returns-to-scale matching functions for g and r employment that take vacancies and searchers in their respective categories as arguments. Then, the job-filling and job-finding probabilities for category $j \in \{g, r\}$ are given by $q(\theta_{j,t}) = m(s_{j,t}, v_{j,t})/v_{j,t}$ and $f(\theta_{j,t}) = m(s_{j,t}, v_{j,t})/s_{j,t}$, respectively, where market tightness is $\theta_{j,t} = v_{j,t}/s_{j,t}$. Following the search and matching literature, as a baseline, we assume that wages are determined via bilateral Nash bargaining between firms and workers. Using the value functions in Appendix A.2, we can show that the Nash real wages for each category are given by

$$w_{r,t} = \nu_n\left[D(x_t)mc_{r,t}H_{n_{r,t}} - \Gamma_{n_r,t} - \tau_t e_{n_r,t} + (1-\varrho)\mathbb{E}_t\Xi_{t+1|t}\psi_r\theta_{r,t+1}\right] + (1-\nu_n)\chi, \tag{17}$$

and

$$w_{g,t} = \nu_n\left[D(x_t)mc_{g,t}F_{n_{g,t}} + (1-\varrho)\mathbb{E}_t\Xi_{t+1|t}\psi_g\theta_{g,t+1}\right] + (1-\nu_n)\chi, \tag{18}$$

where $0 < \nu_n < 1$ is the bargaining power of workers.

2.4 Symmetric Equilibrium and Market Clearing

The price of aggregate output is given by the following expression: $1 = N_{r,t}\left(\widetilde{\rho}_{r,t}\right)^{1-\varepsilon} + N_{g,t}\left(\widetilde{\rho}_{g,t}\right)^{1-\varepsilon}$. Imposing symmetric equilibrium, market clearing in each output category implies that[20]

$$D(x_t)H(n_{r,t}, k_{r,t}) = N_{r,t}\left(\frac{\widetilde{y}_{r,t}}{\widetilde{a}_{r,t}}\right), \tag{19}$$

and

$$D(x_t)F(n_{g,t}, k_{g,t}) = N_{g,t}\left(\frac{\widetilde{y}_{g,t}}{\widetilde{a}_{g,t}}\right). \tag{20}$$

Turning to the government budget constraint, households pay lump-sum taxes to finance unemployment benefits and revenue from the carbon tax is transferred lump-sum to households.

Finally, the economy's resource constraint is given by

$$Y_t = c_t + \psi_r v_{r,t} + \psi_g v_{g,t} + \varphi_e N_{e,t} + \varphi_g N_{g,t} + k_{t+1} - (1-\delta)k_t + \Gamma_t, \tag{21}$$

where the costs of firm creation and green technology adoption are resource costs akin to the standard resource costs associated with physical capital investment. Appendix A.4.1 presents the full list of equilibrium conditions. Per the market clearing conditions for the production of intermediate goods, pollution damages are embedded in Y_t so that the resource constraint is inclusive of these damages. Using expressions (19) and (20) alongside the job creation, capital accumulation, and abatement decisions of intermediate goods producers imply that, as part of our model analysis in Section 3, we can refer to the decisions of producers of intermediate goods for j firms and the decisions of j firms interchangeably.

2.5 Key Model Mechanisms in a Simplified Model

Before turning to the quantitative analysis, we present some analytic results from a simplified version of our model that abstracts from labor search frictions, physical capital, and endogenous firm entry so that the total measure of firms is constant. Moreover, we fix $D(x)$

[20] The two market-clearing conditions below follow from equating the revenue $D(x_t)mc_{r,t}H(n_{r,t}, k_{r,t})$ from producing r intermediate goods with the average costs $\left(mc_{r,t}/\widetilde{a}_{r,t}\right)\widetilde{y}_{r,t}$ for the final goods firms using the r technology, and similarly for the g firms.

at a constant value to emphasize that any positive output effects from a carbon tax are not simply due to reduced damages from emissions. This simpler model allows us to focus on the green-technology adoption margin and highlight in a transparent way the key driving forces behind our main findings. In particular, we focus on the fundamental forces that determine the equilibrium impact of the carbon tax on total output. The full analytic model and the derivations that allow us to get to the results presented below are presented in Appendix A.5.

First, Section A.5.3 of Appendix A.5 shows that total output in the simplified model is given by

$$Y_t = D(x)\left(\nu_p N\right)^{\frac{1}{\varepsilon-1}}\left[\left(\alpha_{y,t}\right)^{\frac{1}{\varepsilon}}\left(H(n_{r,t})\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(1-\alpha_{y,t}\right)^{\frac{1}{\varepsilon}}\left(F(n_{g,t})\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{22}$$

where $D(x)$ and ν_p are positive constants, N is the total measure of firms (also a constant), $0 < \alpha_{y,t} < 1$ is endogenous and increasing in the the threshold productivity level above which firms decide to incur the fixed cost and adopt the g technology, $a_{g,t}$, and $H(n_{r,t})$ and $F(n_{g,t})$ denote the total output from each firm category (both linear in their respective labor).

Second, Section A.5.6 of Appendix A.5 derives the following result regarding the equilibrium impact of the carbon tax on total output:

$$\frac{dY}{d\tau} = \underbrace{\overbrace{\overbrace{(R_r - \epsilon_{Y,r})}^{<0}\overbrace{\frac{Y}{\Theta_{g,r}}\frac{d\Theta_{g,r}}{d\tau}}^{>0}}^{<0}}_{Input\ Reallocation\ Effect} + \underbrace{\overbrace{\overbrace{\frac{\partial Y}{\partial a_g}}^{<0}\overbrace{\frac{da_g}{d\tau}}^{<0}}^{>0}}_{Tech.\ Composition\ Effect}, \tag{23}$$

where $\Theta_{g,r}$ denotes the ratio of g labor to r labor, n_g/n_r, R_r denotes the share of r labor in total labor, and $\epsilon_{Y,r}$ is the elasticity of total output with respect to total r-firm output. This equation identifies two central and potentially opposing forces that shape the equilibrium impact of the carbon tax on total output: an *input reallocation effect* and a *technological composition effect.*

Input Reallocation Effect The input reallocation effect captures the adverse effect of the tax on total output via changes in the reallocation of inputs—in the simplified model, the reallocation of labor—across firm categories, and the resulting effects on firms' marginal productivities. This effect is determined broadly by three main elements: (1) the impact of the carbon tax on the relative share of g labor $\Theta_{g,r}$, $d\Theta_{g,r}/d\tau$; (2) the relative allocation of labor itself, $\Theta_{g,r}$; and most importantly (3) the differential between the share of r labor in total labor and the elasticity of total output with respect to r output, $(R_r - \epsilon_{Y,r})$. As we show in the Appendix, $\epsilon_{Y,r}$ embodies the contribution of r-firm output to total output. *This feature has a critical implication for the impact of carbon taxes on total output: as long as the contribution of* r*-firm output to total output is greater than the contribution of* r *labor to total labor,* $(R_r - \epsilon_{Y,r}) < 0$ *and the carbon tax-induced reallocation of labor towards* g *firms has adverse effects on total output.* We note that $(R_r - \epsilon_{Y,r}) < 0$ is a condition that holds under calibrations of the model that are consistent with U.S. data, even if we consider a more expansive mapping of the types of industries that are directly responsible for emissions.[21]

Technological Composition Effect The technological composition effect captures the positive effect of the tax on output via the endogenous shift in the economy's technological composition of production towards the technology used by g firms. This effect is determined by two elements: (1) the equilibrium impact of the carbon tax on the endogenous threshold productivity level a_g, $da_g/d\tau$; and (2) the impact of changes in a_g on total output, $\partial Y/\partial a_g$. Element (1) captures the fact that, when the carbon tax increases, the marginal cost of r firms increases, and more firms are willing to incur the fixed cost of adopting the g technology, resulting in an equilibrium reduction in a_g. Hence $da_g/d\tau < 0$. Element (2) captures how the carbon tax-induced reduction in a_g generates an endogenous shift in the technological composition of production towards the g technology.

Critically, while we can show that $\partial Y/\partial a_g < 0$, in the background of the changes in the technological composition of production towards the g technology are two interrelated forces that affect total output in opposing ways. The first force is reflected in an endogenous reduction in firms' average idiosyncratic productivity levels, $\widetilde{a}_r$ and $\widetilde{a}_g$, that stems from the fall in the threshold productivity level a_g (recall that both $\widetilde{a}_r$ and $\widetilde{a}_g$ are increasing in a_g)—a firm-productivity effect. All else equal, a reduction in firms' average productivities exerts

[21]In the U.S., the share of value added in GDP of industries that are commonly considered to contribute to emissions is always greater than the share of employment in those same industries.

downward pressure on these firms' output and, in turn, on total output. This force can be understood as working through the intensive margin since it affects the productivity of the average firm within each category for a given share of firms in each category.

The second force is reflected in the endogenous increase in the share of firms that adopt the g technology amid a fall in the threshold productivity level a_g—a production-composition effect. Formally, recall that in the context of our model, $N_r = G(a_g)N$ and $N_g = [1 - G(a_g)]\, N$, where $G'(a_g) > 0$. Thus, for a given measure of firms in the economy N, when a_g falls, the share N_g/N increases. This force—which is at the core of the change in the technological composition of production—can therefore be understood as working through the extensive margin. Moreover, this second force dominates the intensive-margin force described in the preceding paragraph, ultimately exerting upward pressure on total output and leading to $\partial Y/\partial a_g < 0$. To understand why, note that in the presence of technology adoption costs, g firms are endogenously more productive than r firms (i.e., $\widetilde{a}_g > \widetilde{a}_r$). Even though a reduction in a_g lowers both $\widetilde{a}_r$ and $\widetilde{a}_g$, a differential in average idiosyncratic productivity between categories remains. Then, when the reduction in a_g also increases the measure of firms that adopt the g technology—that is, the production-composition effect is operational—the average idiosyncratic productivity profile of the economy—$(N_r/N)\, \widetilde{a}_r + (N_g/N)\, \widetilde{a}_g$—improves, thereby bolstering total output. Under plausible parameterizations, the production-composition effect dominates, implying that the technological composition effect has a net positive effect on total output. We illustrate the two forces that shape the technological composition effect quantitatively within the context of the full model in Section 3.3.2.

Taken together, the fact that both $da_g/d\tau < 0$ and $\partial Y/\partial a_g < 0$ explains the positive equilibrium effect on total output that stems from the carbon-tax-induced reduction in a_g shown in expression (23). More broadly, expression (23) shows that as long as the *technological composition effect* is greater than the *input reallocation effect*, a carbon tax will have a net positive impact on total output.[22] Critically, the *technological composition effect* is

[22]Note that as shown in Table 2, in the benchmark model, a carbon tax leads to lower firm creation and therefore to a lower number of firms in the economy. By reducing the production capacity of the economy, a lower number of firms exerts downward pressure on total output. If we were to allow for endogenous firm creation in the simplified model, the technological composition effect would have to more than offset the adverse input reallocation effect as well as the adverse effect that the carbon tax has on total output via lower overall firm creation. At the same time, were we to allow for the damages function to be active, lower emissions would lower damages and bolster total output.

absent in a more standard environment where firms are unable to switch production technologies in response to the carbon tax. That is, absent endogenous technology adoption, $\partial Y/\partial a_g = da_g/d\tau = 0$, and a carbon tax has an unambiguously adverse impact on total output, as is the case in studies using models that abstract from this margin.

We next return to our full model to quantitatively determine the labor market and aggregate consequences of a carbon tax that reduces emissions by 35 percent from the baseline.

3 Quantitative Analysis

3.1 Calibration of Baseline Economy

3.1.1 Functional Forms

Household Utility, Production, and Matching Contemporaneous household utility in period t is $[\mathbf{u}(c_t) - \mathbf{h}(lfp_{g,t}, lfp_{r,t})] = \frac{c_t^{1-\sigma_c}}{1-\sigma_c} - \frac{[\kappa_g(lfp_{g,t})+\kappa_r(lfp_{r,t})]^{1+1/\phi_n}}{1+1/\phi_n}$, where σ_c,$\kappa_g, \kappa_r > 0$ and $\phi_n > 0$ dictates the elasticity of participation (see Arseneau and Chugh, 2012, for a similar functional form in a model with a single employment category). The production functions for intermediate goods are both Cobb-Douglas: $H(n_{r,t}, k_{r,t}) = (n_{r,t})^{1-\alpha_r}(k_{r,t})^{\alpha_r}$ and $F(n_{g,t}, k_{g,t}) = (n_{g,t})^{1-\alpha_g}(k_{g,t})^{\alpha_g}$, where $0 < \alpha_g, \alpha_r < 1$. Following the macro literature on endogenous firm entry, we use a Pareto distribution for $G(a) = \left[1 - (a_{min}/a)^{k_p}\right]$ with shape parameter $k_p > \varepsilon - 1$.[23] This implies that the number of r and g firms are $N_{r,t} = \left[1 - (a_{min}/a_{g,t})^{k_p}\right] N_t$ and $N_{g,t} = (a_{min}/a_{g,t})^{k_p} N_t$, and that the average idiosyncratic productivities can be written as $\widetilde{a}_{r,t} = \widetilde{a}_{g,t}\left(\frac{a_{g,t}^{k_p-(\varepsilon-1)} - a_{min}^{k_p-(\varepsilon-1)}}{a_{g,t}^{k_p} - a_{min}^{k_p}}\right)^{\frac{1}{\varepsilon-1}} a_{min}$ and $\widetilde{a}_{g,t} = \left(\frac{k_p}{k_p-(\varepsilon-1)}\right)^{\frac{1}{\varepsilon-1}} a_{g,t}$. Thus, $\widetilde{a}_{r,t}$ and $\widetilde{a}_{g,t}$ are both increasing in the endogenous threshold productivity level $a_{g,t}$. Finally, the matching functions for each category are given by $m(s_{j,t}, v_{j,t}) = s_{j,t}v_{j,t}/\left[s_{j,t}^{\xi} + v_{j,t}^{\xi}\right]^{1/\xi}$ where $\xi > 0$ for $j \in \{g, r\}$ (see den Haan, Ramey, and Watson, 2000).

[23] In addition to being highly tractable, as noted in Redding (2011), the Pareto distribution approximates the firm-size distribution reasonably well.

Pollution Damages and Abatement Costs The carbon dioxide pollution damages function is $D(x_t) = exp\left[-D_0(x_t - \bar{x})\right]$ where $D_0 > 0$ dictates the strength of the pollution externality and parameter $\bar{x}$ denotes pre-industrial atmospheric carbon dioxide concentration (see Annicchiarico, Correani, and Di Dio, 2018, Annicchiarico and Diluiso, 2019, for a similar functional form; alternative functional forms used in the literature deliver identical results). Total abatement costs Γ_t are proportional to the output of firms that are responsible for generating emissions, so that $\Gamma_t = \gamma\mu_t^{\eta}D(x_t)H(n_{r,t}, k_{r,t})$, where $\gamma > 0$ and $\eta > 1$ (see Heutel, 2012).

3.1.2 Parameters from Existing Literature

Production, Preferences, and Labor Market A period is a quarter. We set the capital shares $\alpha_g = \alpha_r = 0.32$, the household's subjective discount factor $\beta = 0.985$, the capital depreciation rate $\delta = 0.025$, and the relative risk aversion parameter $\sigma_c = 2$, which are commonly-adopted values in the macro literature. Based on micro estimates for the extensive-margin elasticity of participation from Chetty et al. (2011, 2013), we set $\phi_n = 0.26$ as a baseline and experiment with alternative values for robustness. Following Ghironi and Melitz (2005) and others, we normalize $a_{min} = 1$ and choose $\varepsilon = 3.8$ and $k_p = 4.2$ as baselines. These values deliver empirically-consistent markups and are commonly-adopted values in the literature. In turn, we set the separation rate $\varrho = 0.10$ and the worker bargaining power $\nu_n = 0.5$, both of which are consistent with standard values in the search and matching literature.

Parameters of Pollution Damages and Abatement Cost Functions Following Heutel (2012), we set the persistence of the pollution stock $\rho_x = 0.9979$, the parameter that dictates the elasticity of emissions with respect to r output $\nu = 0.304$, and the elasticity of abatement costs with respect to the abatement rate $\eta = 2.8$, which are consistent with estimates from Nordhaus (2008).[24] Following Hafstead and Williams III (2018) and others, we set

[24] The elasticity of abatement we adopt is based on Nordhaus's DICE-2013R model. In his most recent model (DICE-2016R2), the elasticity used is 2.6. Lowering the elasticity to 2.6 does not affect our main results or conclusions. See Column (6) of Table A2 in Appendix A.7, which compares our benchmark findings to results when we adopt an even lower elasticity of 2.2.

$\gamma = 1$. Finally, we set the carbon tax $\tau = 0$ as a baseline, reflecting the current absence of a nationwide carbon tax in the U.S.

3.1.3 Mapping of Production Categories to Data and Calibration Targets

Mapping with the Data Agriculture, construction, mining, utilities, transportation, and durable-goods manufacturing are commonly considered to be the main generators of carbon emissions. As such, these are the industries that are directly impacted by a carbon tax, but also the industries that would directly benefit from green technology adoption. We therefore assume that, in our model, these industries are represented by r firms, and choose targets for the employment and output shares of r firms in the model that are broadly consistent with the corresponding combined shares of employment and output in these industries in U.S. data (for a similar mapping, see Hafstead and Williams III, 2018).

Targets and Calibrated Parameters Absent evidence on differential vacancy-posting costs between the two employment categories, we assume that $\psi_r = \psi_g = \psi$ as a baseline (differences in these costs do not change our main conclusions). To obtain the value for $\bar{x}$ in the damages function $D(x_t) = exp\left[-D_0(x_t - \bar{x})\right]$, we assume that $\bar{x} = D_1 x$ where $0 < D_1 < 1$ and x is steady-state pollution. Given the growth in the atmospheric stock of greenhouse gases from roughly 280 parts per million at the beginning of the Industrial era to 401 parts per million in 2015, we set $D_1 = 280/401 = 0.6983$ (Annicchiarico, Correani, and Di Dio, 2018; Annicchiarico and Diluiso, 2019; Metcalf, 2019). Note that since x is endogenous, the value for $\bar{x}$ is obtained when we solve for the model's steady state.

Table 1: Parameter Values, Description, and Sources or Targets in Benchmark Model

Parameters from Literature			
Parameter	Value	Description	Source
α_g, α_r	0.32	Capital share	Standard value in lit.
β	0.985	Discount factor	Standard value in lit.
σ_c	2	CRRA param.	Standard value in lit.
ϕ_n	0.26	Elast. of LFP	Chetty et al. (2011, 2013)
ε	3.8	Elast. substit. firm output	Ghironi and Melitz (2005)
k_p	4.2	Pareto shape param.	Ghironi and Melitz (2005)
a_{min}	1	Min. idiosyncratic prod.	Normalization
ϱ	0.05	Job separation probability	Search lit.
ν_n	0.5	Worker bargaining power	Search lit.
D_1	0.6983	Damages parameter	Annicchiarico, et al. (2018)
η	2.8	Elast. of abatement rate	Nordhaus (2008)
γ	1	Weight, abate. cost function	Hafstead and Williams III (2018)
ν	0.304	Elast. parameter, emissions	Heutel (2012)
ρ_x	0.9979	Persistence of pollution	Heutel (2012)

Calibrated Parameters			
Parameter	Value	Description	Target
D_0	0.0000095711	Damages parameter	Pollution damages/GDP = 0.0069
ψ_r, ψ_g	2.1063	Vacancy posting cost	Vacancy costs/GDP = 0.025
e^{row}	4	Emissions rest of world	$e^{row}/(e + e^{row}) = 0.80$
κ_r	0.7723	r LFP disutility param.	$lfp = 0.63$
κ_g	0.6618	g LFP disutility param.	$n_r/lfp = 0.165$
χ	26.5579	Unemployment benefits	$\chi = 0.50w$
φ_e	0.6821	Sunk entry cost	$\varphi_e/Y = 0.01$
φ_g	0.0051	Fixed cost tech. adoption	r-output share = 0.20
ξ	0.5098	Matching elasticity param.	Unempl. rate of 6 percent
ζ	1.4805	Weight of r output on em.	Normalization $e = 1$
$\bar{x}$	1662.51	Baseline pollution in $D(x)$	$\bar{x} = D_1 x$

All told, parameters $D_0, e^{row}, \psi, \kappa_r, \kappa_g, \xi, \chi, \varphi_e, \varphi_g$ and ζ are chosen to match the following targets based on U.S. data and related literature: a ratio of carbon dioxide pollution damages to GDP of 0.0069 (Heutel and Gibson, 2020); a cost of creating a firm of 1 percent of income per capita (consistent with data on the cost of creating a business in the U.S. per World Bank data); an average unemployment rate of 6 percent, an unemployment insurance (UI) replacement rate of 50 percent of average wages, and an average quarterly labor force participation (LFP) rate of 63 percent (all consistent with averages using quarterly U.S.

data from 1985 to 2019); a share of r employment in the labor force of 0.165; a ratio of the total cost of posting vacancies to GDP of roughly 2.5 percent; a share of total r-firm output in total output of 0.20 (similar to Hafstead and Williams III, 2018); steady-state business-as-usual (no policy) emissions normalized to 1; and a share of U.S. emissions in worldwide emissions of 0.20 (consistent with existing data on emissions for the U.S. and the rest of the world).[25] Table 1 summarizes all parameters, their values, and their sources or targets.

3.2 The Labor Market and Aggregate Effects of a Carbon Tax

Models with endogenous firm entry feature a love-of-variety component—stemming from having an endogenous measure of firms—that is not accounted for in empirical measures of the consumer price index (Ghironi and Melitz, 2005). This generates a discrepancy between the aggregate price index in the data and its counterpart in such models (the "welfare-based" aggregate price index). Therefore, if we want to compare real variables in the model to their counterparts in the data, real variables in the model, which are obtained by using the model-based aggregate price index, need to be adjusted so as to use the same aggregate price index that is used to deflate nominal variables in the data. We follow Ghironi and Melitz (2005) in making these adjustments. Specifically, denoting by λ_t^m a given real variable in the model that is obtained using the model-based aggregate price index, its data-consistent counterpart is given by $\lambda_t^d = \lambda_t^m (N_t)^{\frac{1}{1-\varepsilon}}$ (see Appendix A.3 for more details). Unless otherwise noted, all model-based real variables in our quantitative analysis below are expressed in data-consistent terms.

Recalling that our baseline calibration sets the tax on emissions τ to 0, we analyze an increase in τ such that emissions fall by 35 percent relative to their baseline level. This target for emission reductions is roughly consistent with the U.S. Paris Agreement commitment. Our analysis considers both the steady-state (or long) effects as well as the transition path to the new, lower-emissions steady state. In what follows, we first present our quantitative

[25] The target for the share of total r-firm output in total output corresponds to the average value added of agriculture, mining, utilities, transport, construction, chemicals, petroleum manufacturing, and durables manufacturing as a share of GDP based on annual data from 2005 to 2019 from the Bureau of Economic Analysis (BEA). The target for the share of r employment in the labor force is based on the average share of employment in these industries over the same time period (also from the BEA).

results, including those from simpler variants of our benchmark model, and then discuss the key economic mechanisms that are responsible for main findings.

3.2.1 Steady-State Effects

Benchmark Model Results Table 2 shows steady state values of select variables under the baseline (pre carbon tax) calibration described in Section 3.1, the values post carbon tax, and the resulting percent change (or, when appropriate, the percentage-point change) in these variables relative to their baseline-calibration values when we increase the carbon tax to reduce steady-state emissions by 35 percent.

In the steady state, total output and consumption increase by roughly 0.45 and 0.33 percent, respectively. Labor force participation increases by 0.34 percentage points (from a baseline rate of 63 percent to 63.34 percent), while the unemployment rate increases marginally by 0.034 percentage points (from a baseline rate of 6 percent to 6.034 percent). Employment in r firms falls by almost 19 percent, while employment in g firms increases by almost 5 percent. However, given the initial allocation of employment across categories, the measure of total employment increases by almost 0.50 percent. Real wages in both employment categories increase marginally. r firms' optimal abatement rate increases by 25 percentage points, while the equilibrium ratio of tax revenue to GDP is 0.18 percentage points.[26] Finally, the number of g firms increases by almost 15 percent while the total number of firms falls by roughly 1 percent, implying a reduction in net firm creation.

[26]The equilibrium abatement rate in the baseline calibration is effectively zero since r firms have no incentive to abate emissions if the carbon tax is zero.

Table 2: Steady State Changes in Response to Carbon Tax, Emissions Reduction of 35 Percent–Benchmark Model

Variable	Benchmark Model Values		**Percent Change** Rel.
	Before Tax (Baseline)	**After Tax**	to Baseline
Total Output	6.974	7.005	0.448
Consumption	4.517	4.533	0.334
Empl. r	0.104	0.085	-18.579
Empl. g	0.488	0.511	4.566
Total Empl.	0.592	0.595	0.503
Real Wage r	6.153	6.172	0.313
Real Wage g	5.277	5.293	0.313
Capital k_r	8.175	6.677	-18.325
Capital k_g	32.699	34.298	4.891
Firms (N)	592.991	587.183	-0.979
g Firms (N_g)	246.764	282.087	14.315
Ave. Idiosync. Prod. $\widetilde{a}_r$	1.099	1.084	-1.398
Ave. Idiosync. Prod. $\widetilde{a}_g$	1.824	1.763	-3.362
Overall Ave. Firm Prod.	1.401	1.410	0.657
			Percentage-Pt. Change Rel. to Baseline
Unempl. Rate	6.000%	6.034%	0.034
LFP Rate	63.000%	63.340%	0.340
Abate. Rate μ	0.000%	25.080%	25.080
Share of g-Firm Output	80.000%	83.539%	3.539
Share of g Firms	41.610%	48.041%	6.427
Tax Rev./Output	0.000%	0.180%	0.180

Notes: Overall Ave. Firm Prod. is defined as $(N_r/N)\,\widetilde{a}_r + (N_g/N)\,\widetilde{a}_g$. The first two columns show values rounded to two decimal places. All real variables are expressed in data-consistent terms.

All told, a carbon tax-induced reduction in emissions has positive effects on consumption and output, positive (though mild) effects on real wages across categories, and very limited adverse effects on unemployment given the magnitude of the reduction in emissions.[27] Using

[27]A possible concern is that the increase in output is driven by the policy-induced reduction in emissions and consequent reduction in damages to output. In the Appendix, we rerun the model fixing damages at the baseline level to control for this possible output effect and find that the change in damages is not driving the positive output result. See columns (4) and (6) in Table A4 of Appendix A.7.

2020 GDP as a baseline for comparison, a carbon tax revenue-GDP ratio equal to 0.18 percent translates into revenue of roughly \$42 billion annually, implying a tax of \$14 per metric ton of CO_2. This estimate is considerably lower than estimates from studies such as the U.S. Treasury study (Horowitz et al., 2017). As we discuss further below, the estimated carbon tax rate and revenue are both lower compared to an economic environment that is more consistent with existing macro models, which generally abstract from both firm creation and green technology adoption decisions.

The Effects of Technology Adoption and Firm Entry Anticipating our discussion of economic mechanisms in Section 3.3, Table 3 shows results from two model variants alongside those in the benchmark model. First, we consider a model version where we allow technology-adoption decisions but shut down the firm-entry margin (column (2) of Table 3).[28] Second, we consider a model version that abstracts from both firm entry and green technology adoption (column (3) of Table 3). This second variant maintains two firm categories—one that uses the regular, polluting technology and one that uses the green technology—and the ability to reallocate capital and labor to across firm categories in response to carbon taxation, but does not allow firms using the r technology to endogenously adopt the g technology. That is, there is no possibility for endogenous changes in the technological composition (polluting vs. green) of the economy's production structure, and therefore no possibility for endogenous changes in average idiosyncratic productivity levels across firm categories (recall that these changes stem from firms' optimal technology adoption decisions).[29] In this sense, the model in column (3) is closest to existing two-sector models with unemployment and pollution externalities that assume a fixed number of firms in each sector (the closest example is Hafstead and Williams III, 2018).

For comparability across models and when feasible given the adjustment margins in each

[28]Specifically, this variant of the benchmark model sets the sunk entry cost to 0 and normalizes the total number of firms to 1 so that there is a continuum of firms over the $[0, 1]$ interval. Within that $[0, 1]$ measure, an endogenous fraction decides to incur the fixed cost of green technology adoption and become g firms based on their idiosyncratic productivity. For completeness, Table A4 in Appendix A.7 discusses a related version of the benchmark model where we shut down the abatement margin while keeping both firm entry and technology adoption.

[29]This model variant can also be interpreted as a limiting case where firms' ability to adopt green technologies is severely restricted, implying a fixed set of firms in each technology category, but inputs can still be reallocated across categories.

model variant, we adopt the same baseline calibration targets as those used in the benchmark model. In the case of the model variant without firm entry and technology adoption, which by construction does not have idiosyncratic productivity, we do so by allowing for exogenous differences in average firm productivity between firm categories, which we determine using the benchmark model's calibration targets. This results in average—and in this case, exogenous—firm productivity differentials between the two firm categories that are identical to those in our benchmark model but, importantly, cannot endogenously change in response to the carbon tax given the absence of idiosyncratic productivity.

This means that any differences in outcomes between the benchmark model and this second variant are not due to baseline underlying firm-productivity differentials—both models exhibit the same baseline differentials—but rather due to endogenous adjustments in average firm idiosyncratic productivity in response to the carbon tax, where the endogenous changes in productivity stem from firms' optimal technology adoption decisions in the benchmark model (i.e., endogenous changes in the technological composition of the economy's production structure).

For each model, we show the percent change or, when appropriate, the percentage-point change, of select variables in response to the increase in carbon tax rates relative to the baseline calibration (i.e., when $\tau = 0$) of each respective model. This comparison across models illustrates the power that technology adoption has in offsetting the distortionary effects of the carbon tax by shifting the economy towards a lower-emissions (and endogenously more productive) production structure. The benchmark-model variant without endogenous firm entry (column (2)) of Table 3 delivers qualitative results similar to those of the benchmark model (column (1)), though the positive effects on real wages, the share of g firms, consumption, and total output are larger and the (quantitatively limited) increase in unemployment is smaller.

Table 3: Steady State Changes in Response to Carbon Tax–Benchmark Model vs. Simpler Model Variants

Variable	**Benchmark Model**	**No Firm Entry**	**No Firm Entry No Tech. Adopt.**
	(1)	(2)	(3)
	Percent Change Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline
Emissions e	-35	-35	-35
Total Output	0.448	1.061	-1.021
Consumption	0.334	0.395	-0.803
Empl. r	-18.579	-36.813	-24.038
Empl. g	4.566	9.108	5.611
Total Empl.	0.503	1.047	0.407
Real Wage r	0.313	0.740	-1.949
Real Wage g	0.313	0.743	-1.957
Capital k_r	-18.325	-36.326	-25.579
Capital k_g	4.891	9.960	3.438
Firms (N)	-0.979	–	–
g Firms (N_g)	14.315	–	–
Welfare Gain (% of Consumption)	**-0.018**	**0.401**	**-0.754**
	Percentage-Pt. Change Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline
Unempl. Rate	0.034	0.010	0.178
LFP Rate	0.340	0.667	0.376
Abate. Rate μ	25.080	10.782	21.020
Share of g-Firm Output	3.539	7.045	3.604
Share of g Firms	6.427	13.680	–
Tax Rev./Output	0.180	0.353	1.265

Notes: The welfare gain is computed in terms of consumption equivalent variation (CEV)—the percent increase in consumption that the household would need to receive in the baseline, no-policy scenario to be as well off as under the policy. A positive value denotes a welfare gain relative to the (within-model) no-policy baseline whereas a negative value denotes a welfare cost relative to the (within-model) no-policy baseline.

These larger positive quantitative changes are explained by the fact that in the benchmark model, the carbon tax depresses overall firm entry which, relative to an environment where the number of firms is fixed but firms can still make technology adoption choices, lowers input demand and ultimately output. Thus, accounting for firm creation is important to characterize the net quantitative impact of the carbon tax on output.

The most notable results in Table 3 pertain to the model that abstracts from both firm entry and technology adoption (column (3))—an environment that is closest to existing macro models in the literature. Indeed, for the same 35-percent reduction in emissions in the long run, the introduction of a carbon tax generates a *reduction* in real wages of roughly 2 percent, and reductions in consumption and total output of 0.80 and 1 percent, respectively. Moreover, the increase in the unemployment rate is more than five times greater than in the benchmark model—increasing by roughly one fifth of a percentage point. The clear adverse effects of carbon taxation on total output are not directly reflected in a large increase in unemployment. Instead, real wages take the brunt of the adjustment, which in turn leads to a reduction in labor income and ultimately in consumption.[30] The adverse macroeconomic effects from a carbon tax in the absence of endogenous technology adoption are consistent with and similar in magnitude to those in existing quantitative studies which, in contrast to our work, do not consider technology adoption decisions as a margin that firms can use to respond to policy (see, among others, Fischer and Springborn, 2011; Annicchiarico and Di Dio, 2015; and Annicchiarico, Correani, and Di Dio, 2018).

As noted earlier, in the benchmark model, the carbon tax collects revenue equal to 0.18 percent of GDP, implying a tax rate of $14 per metric ton of CO_2. In contrast, the 2016 U.S. Treasury study finds gross tax collections between 1 and 1.2 percent of GDP. Shutting off the firm entry channel in our model (column 2) increases the tax revenue to GDP ratio to 0.35 percent (implying a tax rate close to $30 per metric ton of CO_2) but it is still significantly below the Treasury estimate. When we also shut off the technology adoption channel (column 3), the revenue now jumps to 1.27 percent of GDP, implying a tax rate of $100 per metric ton of CO_2. The Treasury study estimates emission reductions on the

[30]Recalling that emissions are shaped by total r-firm output and the abatement rate μ, roughly 38 percent of the reduction in emissions in the benchmark model stems from the reduction in output from r firms. When we abstract from firm entry, the reduction in output from r firms accounts for almost 78 percent of the reduction in emissions. Finally, when we abstract from both firm entry and green technology adoption, the reduction in output from r firms accounts for 50 percent of the reduction in emissions.

order of 20 percent. When we rerun our model to achieve a 20 percent reduction instead of 35 percent, we find that the gross tax revenue equals 0.68 percent of GDP (implying a tax rate of \$54 per metric ton of CO_2) when we shut off the technology adoption and firm entry channels, close to the Treasury estimate. We conclude from this first that our model without firm entry and endogenous technology adoption tracks the U.S. Treasury model quite well; and second that modeling firm entry and technology adoption is crucial for identifying the carbon tax rate needed to achieve the Paris Agreement target (or any other emissions reduction target) as well as for estimating the revenue potential of carbon pricing. The result that the carbon tax needed to achieve a given reduction in emissions is lower once we allow for endogenous green technology adoption is consistent with the findings in Fried (2018), who uses a framework where innovation in polluting, green, and non-energy inputs takes place. An important difference between her framework and ours is that the production structure in our model allows for endogenous changes in the technological composition (polluting vs. green) of production in response to policy. As describe in detail in Section 3.3 below, these endogenous changes play a pivotal role in explaining the positive labor market and aggregate effects of carbon taxation.

Welfare Effects of Carbon Tax Following Fried (2018) and the macro literature on endogenous firm entry, we compute the long-term welfare cost of the policy in terms of consumption equivalent variation (CEV). More specifically, the welfare cost is implicitly given by

$$\left[\mathbf{u}\left(\left(1+\frac{\Delta}{100}\right)c^{base}\right)-\mathbf{h}\left(lfp_g^{base},lfp_r^{base}\right)\right]=\left[\mathbf{u}\left(c^{\tau}\right)-\mathbf{h}\left(lfp_{g,t}^{\tau},lfp_r^{\tau}\right)\right],$$

where (steady-state) variables with superscript *base* represent variables associated with the baseline, no-carbon-tax scenario and (steady-state) variables with superscript τ represent variables under the carbon tax. The welfare impact Δ represents the percent increase in steady-state consumption that the household would need to receive in the presence of a carbon tax in order to be as well off as in the baseline, no-carbon-tax scenario (thus, relative to the no-policy baseline, $\Delta > 0$ implies a welfare gain whereas $\Delta < 0$ implies a welfare loss).

We compute these welfare effects for the benchmark model and the two model variants.

Table 3 shows that for all intents and purposes, raising the carbon tax to reduce emissions entails no welfare costs in the benchmark model. Abstracting from firm entry while allowing firms to have a choice over green technology adoption delivers a small welfare gain, while abstracting from both firm entry and green technology adoption generates a more substantial welfare cost of roughly 0.75 percent of steady-state consumption. Again, the difference in welfare results between our benchmark model and a model that abstracts from green technology adoption and firm entry points to the importance of including these model features.

The Role of Endogenous Labor Force Participation Our framework differs from existing models that study the labor market and aggregate effects of environmental policy by introducing endogenous labor force participation (LFP) as a relevant labor market-based margin of adjustment to carbon taxes. As shown in Table 3, regardless of the model version, a carbon tax bolsters LFP. The increase in LFP traces back to the carbon tax-induced increase in the search for jobs in g firms, which more than offsets the drop in the search for r jobs. The greater search for g jobs ultimately leads to an expansion in the level of total employment in the economy, thereby contributing positively to total output.

To better understand the implications of this change in LFP, Table 4 presents results analogous to those in Table 3 for an experiment where we hold LFP constant at its baseline (pre carbon tax) level.[31] A comparison of Tables 3 and 4 shows that the increase in LFP as a result of the carbon tax plays an important role in limiting the adverse effects of the tax on total output and consumption. This result is intuitive: by increasing the relative cost of production of r firms and incentivizing job creation among g firms, the carbon tax not only reallocates labor away from r firms towards g firms but also attracts more searchers towards g firms, including individuals who were previously outside of the labor force and decide to enter the labor market. Given the quantitative response of job creation by g firms, the increase in participation is ultimately reflected in a higher employment level (see Table

[31] We maintain LFP at its baseline level by allowing parameter κ_i to adjust in response to the carbon tax so that LFP remains unchanged post tax. Given that κ_i is a parameter in the disutility of LFP, we do not present welfare results for this experiment.

3). With fixed participation, the pool from which g firms can hire workers is limited by the measure of those currently unemployed, thereby limiting the extent to which total output is bolstered by the expansion of employment in g firms.

Table 4: Steady State Changes in Response to Carbon Tax–Benchmark Model vs. Simpler Model Variants, Fixed Labor Force Participation at Baseline (Pre Carbon Tax) Level

Variable	**Benchmark Model**	**No Firm Entry**	**No Firm Entry No Tech. Adopt.**
	(1)	(2)	(3)
	Percent Change Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline
Emissions e	-35	-35	-35
Total Output	-0.239	-0.071	-1.578
Consumption	-0.303	-0.674	-1.358
Empl. r	-14.588	-34.696	-23.478
Empl. g	3.030	7.362	4.771
Total Empl.	-0.062	-0.021	-0.188
Real Wage r	-0.535	-0.670	-2.644
Real Wage g	0.157	0.656	-1.881
Firms (N)	-1.637	–	–
g Firms (N_g)	9.905	–	–
Welfare Gain (% of Consumption)	-0.602	-0.131	-1.000
	Percentage-Pt. Change Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline
Unempl. Rate	0.059	0.019	0.176
LFP Rate	–	–	–
Abate. Rate μ	27.351	12.519	21.293
Share of g-Firm Output	2.734	6.543	3.475
Share of g Firms	4.883	12.448	–
Tax Rev./Output	0.215	0.472	1.304

3.2.2 Transition Path to New Steady State

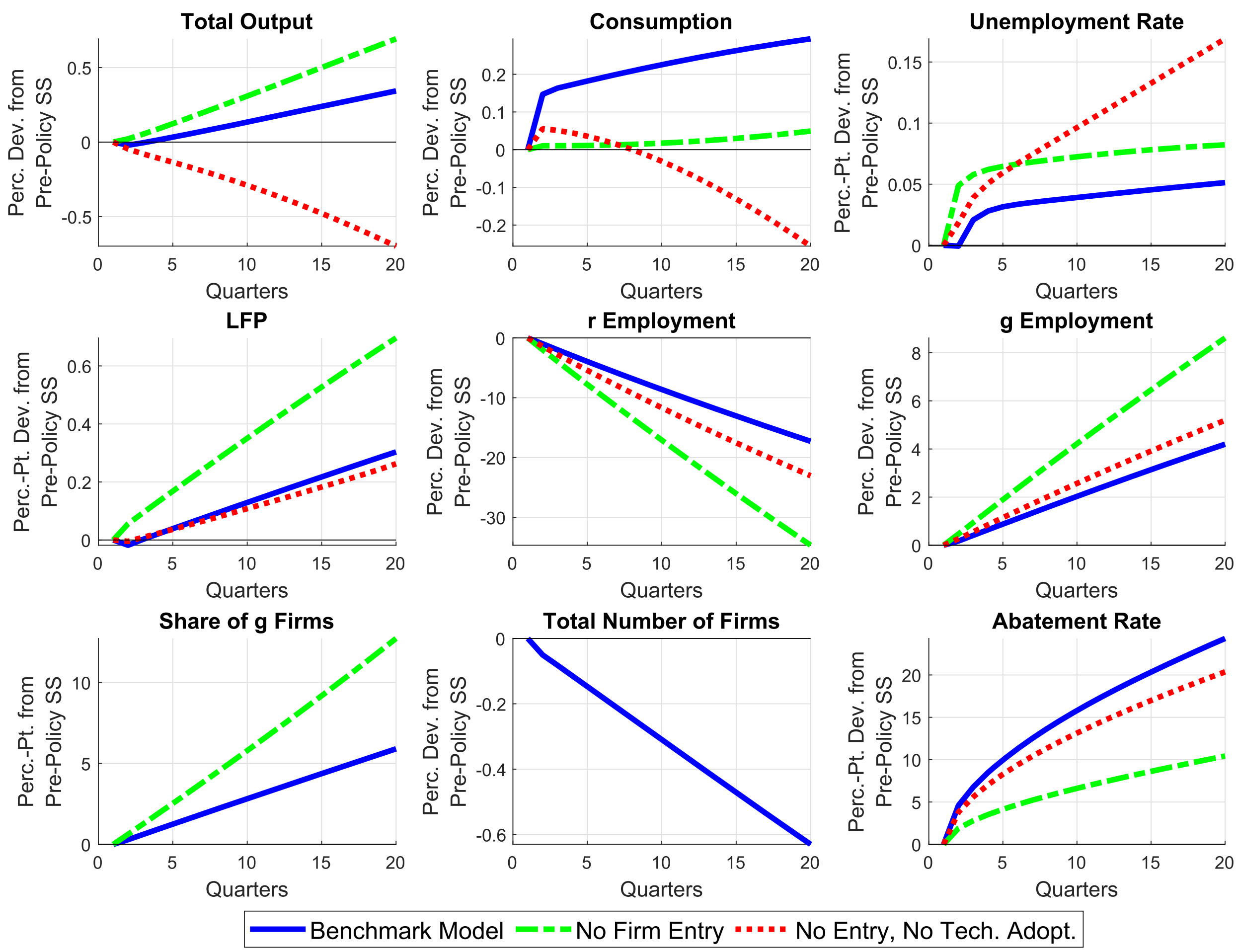
In the presence of search and matching frictions, costly firm creation, and costly technology adoption, the transition path to the new steady state may take time and could potentially entail short-term employment, consumption, and output costs. For illustrative purposes, we implement a gradual and uniform increase in the carbon tax that delivers the long-run decline in emissions of 35 percent relative to baseline as in Section 3.2.1. Specifically, we model the carbon tax to rise linearly from zero to its steady-state value at the end of 20 quarters (5 years) (Figure A6 in Appendix A.7 presents results under a more aggressive increase in the tax).

Figure 1 plots the transition path of select variables in response to the gradual increase in the carbon tax.[32] Given our steady-state results in Table 3 and for comparability, we show the transition path of: (1) the benchmark model (solid blue line), (2) the benchmark model variant without firm entry (dash-dotted green line), and (3) the benchmark model variant with neither firm entry nor technology adoption (dotted red line).

The discrepancy between the long-term changes in certain variables in Table 2 and the medium-term changes of the same variables in Figure 1 are due to the economy's underlying structure and frictions, which imply that the long-term (i.e. steady-state) effects of the carbon tax take time to fully materialize (Table A1 in Appendix A.6.1 compares the steady-state changes in response to the carbon tax in each model version to the policy-induced changes after 20 quarters). The key takeaway from Figure 1 is that a carbon tax need not have short- or medium-term adverse macroeconomic effects when firms are able to choose and adopt green technologies. This is the case even if the carbon tax reduces the number of firms in the economy (the upside of this reduction is that the resources that were previously used to cover the resource costs of firm creation are devoted to bolster consumption). Moreover, the adjustment of sectoral employment along the transition path is relatively smooth, with no meaningful overshooting in the unemployment rate taking place despite the presence of labor market frictions.

[32]Following the endogenous-firm-entry macro literature that analyzes transitional dynamics after a permanent change in policy, we solve the full non-linear version of the model under perfect foresight using the historical algorithm as described in Juillard (1996) (for an application of these methods to the analysis of labor-market and goods-market reforms, see Cacciatore and Fiori, 2016).

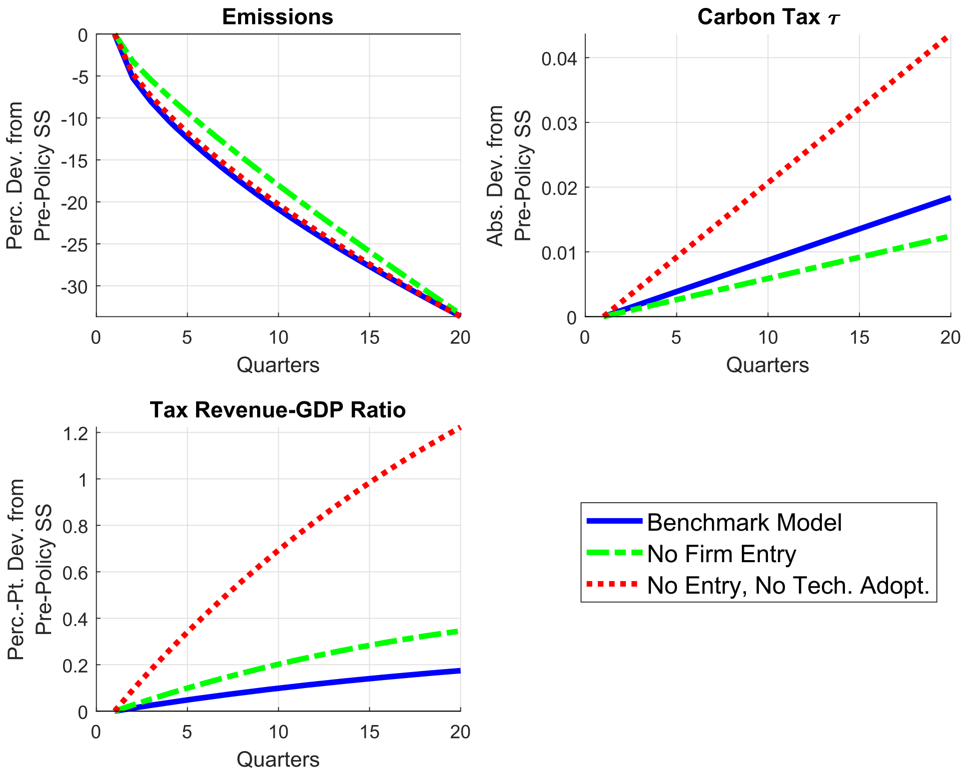
Figure 1: Transitional Dynamics in Benchmark Model and Model Variants (Gradual Reduction in Emissions via Carbon Tax)



Note: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations.

Figure 2 plots the change in emissions, the carbon tax, and the carbon tax revenue-GDP ratio. Emissions fall by nearly 35 percent by the end of 20 quarters (five years) in all scenarios, assuring that the new U.S. Paris Agreement target is achievable by 2030. A comparison of the carbon tax in the benchmark model to the tax in the two model variants shows that a smaller absolute increase in the carbon tax is needed to achieve the same reduction in emissions when firms can use technology adoption as an adjustment margin to policy.

Figure 2: Transitional Dynamics in Benchmark Model and Model Variants (Gradual Reduction in Emissions via Carbon Tax, Continued)



Note: Perc. Dev. denotes percent deviations, Abs. Dev. denotes absolute deviations, and Perc.-Pt. Dev. denotes percentage-point deviations.

Figure 2 shows that the level of the tax needed to reduce emissions by 35 percent is reduced by roughly three-quarters relative to a model with no firm entry and technology adoption. Coupled with the policy-induced increase in output, the smaller increase in the carbon tax implies a smaller increase in the tax revenue-GDP ratio compared to an environment that abstracts from technology adoption.

For expositional brevity, we delegate a more extensive discussion of the transitional dynamics in Figures 1 and 2 to Appendix A.6.1 and simply summarize the main findings from our analysis. First, the positive long-run macroeconomic effects of carbon taxation in the benchmark model are not accompanied by short-term consumption or output costs along the transition path to the post carbon tax steady state. This occurs even as the carbon tax has a net adverse effect on the total number of firms. These findings are consistent with recent empirical evidence on the macroeconomic effects of these taxes, but stand in sharp contrast to existing quantitative studies in the literature that predict adverse effects on wages, aggregate consumption, and output. Second, firms' ability to adopt green technologies in response to the carbon tax plays a fundamental role in shaping the positive macroeconomic effects of this tax and limiting any adverse effects that policy may have on unemployment, and implies that a smaller increase in the carbon tax is needed to achieve a given emissions-reduction target.[33] An implication of this smaller carbon tax is that less revenue is raised compared to an environment that abstracts from technology adoption. Third, firm-entry decisions play an important role in generating positive short- and medium-term consumption effects in response to carbon taxation (even if this is accompanied by a net reduction in the number of firms in the economy), a result that is relevant for the short- and long-term welfare assessment of the tax.

3.3 Economic Mechanisms

3.3.1 Consumption, Output, and the Labor Market in the Benchmark Model

The positive macroeconomic and labor market effects of the carbon tax when firm creation and green technology adoption are both endogenous may appear surprising given the distortionary nature of the tax. Indeed, while beneficial from an environmental perspective, a carbon tax raises marginal costs and reduces productivity across firm categories. However, as we discussed in the Analytic Model section, the tax shifts production from r firms, which have (endogenously) lower productivity, to g firms, which have (endogenously) higher productivity, and leads to an endogenous restructuring of the aggregate production process. This restructuring ultimately results in an increase in overall (or economy-wide) average productivity and in a reduction in the overall marginal cost of production. This process is responsible for counteracting the sector-specific adverse macroeconomic impacts of the carbon tax and generating equilibrium increases in consumption and total output.[34]

[33] It is not straightforward to interpret the carbon tax rates in Figure 2. Emissions in the initial period have been normalized to 1 and the units of tax revenue are units of consumption. That is why we back out carbon tax rates from the tax revenue to GDP ratio in the text.

[34] Given the higher productivity of green technology, we conjecture that the unconstrained social optimum (using Dixit and Stiglitz's terminology) would contain more green firms. But given the fixed costs of adopting

Delving into the mechanisms of the benchmark model, note that r firms respond to the increase in the carbon tax by devoting more resources to costly abatement. Both the carbon tax and the resources devoted to abatement reduce r firms' marginal benefit to having a worker and accumulating capital. As a result, r firms use less capital, post fewer vacancies, and hire fewer workers, which leads to a reduction in r employment and r output. Both the increase in abatement and the reduction in r-firm output contribute to the reduction in emissions, but at the cost of lower r employment and r-firm output. However, by putting direct upward pressure on the marginal cost of using the r technology, the carbon tax also makes it relatively more attractive for firms to choose the g technology (which entails a fixed cost).

As shown in Table 2, g firms have a greater baseline (endogenous) average idiosyncratic productivity level compared to r firms. This positive average productivity differential between g and r firms, which is an outcome that stems partly from the presence of positive fixed costs associated with technology adoption, becomes narrower in response to the carbon tax. This takes place because, by making the r technology more expensive in relative terms, the carbon tax pushes firms to reduce the threshold productivity level above which they are willing to adopt the g technology. This results in both a greater number and a larger fraction of g firms as firms move away from the r technology, but also in a reduction in the levels of average idiosyncratic productivity in both firm categories as the policy changes the composition of firms in the economy. Despite this policy-induced reduction in r and g firms' average productivities, the increase in the number and share of g firms leads to greater labor demand by g firms (reflected in more g vacancy posting), to improved labor market conditions, and to greater g employment. Importantly, despite the reduction in average idiosyncratic productivity among g firms, economy-wide average idiosyncratic productivity—a weighted average of firms' average idiosyncratic productivity across firm categories, where the weights are the share of firms using each of the two technologies—increases. The increase in economy-wide productivity is mainly due to the fact that, as the share of g firms increases, the economy's endogenous production structure of the economy shifts towards firms that, on

the green technology, the market equilibrum discourages its adoption. The carbon tax can help move the market toward the unconstrained social optimum.

average, are endogenously more productive compared to r firms (even after the endogenous reduction in average idiosyncratic productivity among g firms due to the tax). At the same time, the reduction in searchers for jobs in r firms and improved labor market conditions for searchers for jobs in g firms contribute to the equilibrium rise in real wages across firm categories.

Given the presence of labor market frictions, the reallocation of employment away from r and into g firms is accompanied by a limited increase in the unemployment rate as well as an increase in labor force participation, where the latter is driven by both an increase in g employment and in the mass of g searchers. Note that the carbon tax induces a change in the composition of total unemployment, with unemployment among those searching for r jobs decreasing and unemployment among those searching for g jobs increasing. The relatively small increase in total unemployment is ultimately explained by the household's reallocation of searchers away from jobs in r firms and towards jobs in g firms.

3.3.2 Quantitative Strength of Technological Composition Effect

Section 3.3 used the simplified version of the model to show analytically how the positive effect of the carbon tax on output depends on the strength of the technological composition effect stemming from endogenous green-technology adoption. To unpack the forces that shape the technological composition effect in the full model, Appendix A.4.2 shows that we can write total output Y_t as

$$Y_t = D(x_t)\left(\nu_p N_t\right)^{\frac{1}{\varepsilon-1}}\left[\left(\alpha_{y,t}\right)^{\frac{1}{\varepsilon}}\left(H(n_{r,t},k_{r,t})\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(1-\alpha_{y,t}\right)^{\frac{1}{\varepsilon}}\left(F(n_{g,t},k_{g,t})\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \qquad (24)$$

where $\nu_p \equiv \left(\frac{k_p}{k_p-(\varepsilon-1)}\right)$ with $k_p > \varepsilon-1$, $0 < \alpha_{y,t} \equiv \left(1 - \frac{1}{a_{g,t}^{k_p-(\varepsilon-1)}}\right) < 1$, and $\alpha_{y,t}$ is increasing in $a_{g,t}$. Expression (24) is analogous to expression (22) of the simplified model presented in Section 3.3, the main differences being that now production uses both capital and labor and the total number of firms is endogenous. To see how the analytical results in Section 3.3 carry through to our benchmark model, note that the input reallocation effect is now embodied in the output of each firm category, $H(n_{r,t},k_{r,t})$ and $F(n_{g,t},k_{g,t})$. In turn, the technological composition effect is embodied in the term $\alpha_{y,t}$.

Recall that the carbon tax increases the marginal cost of r firms, thereby leading to reductions in r-firm capital and labor demand that translate into a reduction in r-firm production $H(n_{r,t}, k_{r,t})$. At the same time, the carbon tax triggers a reallocation of resources—both capital and labor—away from r firms and towards g firms that bolsters g-firm production $F(n_{g,t}, k_{g,t})$. For a given technological composition of total production $\alpha_{y,t}$, if the input decisions of r firms are sufficiently sensitive to the carbon tax, the adverse response of $H(n_{r,t}, k_{r,t})$ to the carbon tax can dominate the positive response of $F(n_{g,t}, k_{g,t})$, thereby exerting downward pressure on total output.

At the same time, the carbon tax-induced upward pressure on the marginal cost of r firms makes it more attractive for firms to incur the cost of adopting the g technology, leading to a reduction in the endogenous threshold $a_{g,t}$ and therefore to a reduction in $\alpha_{y,t}$ that shifts the composition of total output, and of firms, towards the g technology. Given the average productivity profile of r and g firms, this second mechanism exerts upward pressure on total output. That is, the two main effects described in the context of the simplified model in Section 2.5 are present in the benchmark model.

To illustrate these forces quantitatively, Table 5 shows the carbon tax-induced *percent changes* in the steady state of: the term $(H(n_r, k_r))^{\frac{\varepsilon-1}{\varepsilon}} + (F(n_g, k_g))^{\frac{\varepsilon-1}{\varepsilon}}$, whose elements shape total output in expression (24) and embody the input reallocation effect; the average idiosyncratic productivity of r and g firms ($\widetilde{a}_r$ and $\widetilde{a}_g$); the weight $(1 - \alpha_{y,t})$ on g-firm output in expression (24), which embodies the technological composition effect); the term $(\alpha_{y,t})^{\frac{1}{\varepsilon}} (H(n_r, k_r))^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \alpha_{y,t})^{\frac{1}{\varepsilon}} (F(n_g, k_g))^{\frac{\varepsilon-1}{\varepsilon}}$ in that same expression; and total output. For expositional simplicity, we abstract from showing the other elements that shape total output (the damages function and the total number of firms). We present results for the benchmark model (column (1) in Table 5) and the two variants we analyzed earlier (columns (2) and (3), respectively, of the same table).[35]

[35] Recall that by construction, the model variant that abstracts from firm entry and technology adoption does not feature endogenous idiosyncratic productivity. However, as noted in Section 3.2.1, in order to make this model variant comparable to the benchmark model at the pre carbon tax baseline, we introduce exogenous firm idiosyncratic productivity levels in the two categories that exactly match those that emerge, endogenously, in the benchmark model. As such, when abstracting from the technology-adoption margin, average firm idiosyncratic productivity remains fixed in response to carbon taxation, implying no changes in $\widetilde{a}_r$, $\widetilde{a}_g$, and $\hat{\alpha}_g$.

Table 5: The Importance of the Technological Composition Effect: Steady State Changes in Benchmark Model vs. Model Variants

	Benchmark	**No Firm Entry**	**No Firm Entry, No Tech. Adoption**	
			Baseline	**Exog. Changes in $\widetilde{a}_r$,$\widetilde{a}_g$ That Match (1)**
	(1)	(2)	(3)	(4)
	Percent Change Relative to Baseline	**Percent Change** Relative to Baseline	**Percent Change** Relative to Baseline	**Percent Change** Relative to Baseline
Emissions e	-35	-35	-35	-35
Input Reallocation Effect	-1.004	-1.998	-1.972	-2.584
Ave. Firm Prod.: r Category	-1.398	-2.797	–	-1.398*
Ave. Firm Prod.: g Category	-3.362	-6.543	–	-3.362*
Overall Ave. Firm Productivity	0.655	1.391	–	-2.462
Technological Composition Effect	4.904	9.937	–	–
Total Output	0.448	1.061	-1.021	-4.945

Notes: The Input Reallocation Effect is given by $(H(n_r, k_r))^{\frac{\varepsilon-1}{\varepsilon}} + (F(n_g, k_g))^{\frac{\varepsilon-1}{\varepsilon}}$. Ave. Firm Prod. (average firm productivity) in firm category $j \in \{g, r\}$ is given by $\widetilde{a}_j$. Overall Ave. Firm Productivity is given by $[(N_r/N)\,\widetilde{a}_r + (N_g/N)\,\widetilde{a}_g]$. The Technological Composition Effect is given by $(1-\alpha_y)$. Numbers in column (4) marked with a * are chosen to match the corresponding number in the respective row of column (1).

Column (1) of Table 5 confirms numerically that in the benchmark model, and consistent with the analytical results in the simplified model, the input reallocation effect stemming triggered by the carbon tax exerts downward pressure on total output. Similarly, the carbon tax-induced reduction in the threshold productivity level a_g lowers the average idiosyncratic productivity in both firm categories. These two adverse effects are counteracted by the endogenous shift in composition of total production towards the g technology, reflected in an increase in $(1-\alpha_y)$. Taken together, the technological composition effect dominates, leading to an equilibrium increase in total output.

Column (2) of Table 5 shows results for the model variant that abstracts from endogenous firm entry. Compared to the benchmark model, each of the forces described above remain qualitatively the same but are quantitatively stronger. These findings are expected given that, as shown in Section 3.2.1, the carbon tax reduces overall firm creation, which dilutes the quantitative strength of the technology adoption margin.

Column (3) of Table 5 shows results for the model variant that abstracts from both endogenous firm entry and technology adoption. In this scenario, firms' average idiosyncratic productivity is fixed and the economy cannot endogenous shift towards a production structure based on a more productive, and greener, technology in response to the carbon tax. Thus, the only force present is the input reallocation effect which, as discussed earlier, leads to an equilibrium reduction in total output.

Finally, Column (4) of Table 5 considers a version of the model variant without firm entry and technology adoption where the reduction in emissions induced by the carbon tax is accompanied by exogenous reductions in firms' average idiosyncratic productivity that match the reductions in the benchmark model. This experiment is meant to illustrate the fundamental role of endogenous changes in the technological composition of production in bolstering total output amid a carbon tax by effectively holding α_y fixed but letting the average productivity of firms adjust by the same magnitude as in the benchmark model. The results in column (4) show that without the most important element of the technological composition effect—the endogenous shift in the production structure towards the g technology—economy-wide average idiosyncratic productivity would fall, leading to an even larger contraction in total output (compare total output in columns (3) and (4)). More broadly, comparing the results in column (4) to those of the benchmark model in column (1) confirms that tax-induced endogenous changes in the technological composition of production are central to offsetting the adverse effects that the carbon tax would otherwise have on the labor market and economic activity, resulting in net positive aggregate effects from the tax.

3.4 Robustness Analysis: Baseline Parameters and Alternative Modeling Assumptions

For completeness, we revisit our quantitative findings under alternative baseline calibrations of the benchmark model and under an extensive set of alternative modeling assumptions. Results from these robustness checks are presented in Appendix A.7. On the baseline-calibration front, we revisit our policy experiments assuming: a higher value for parameter k_p (which reduces the dispersion in idiosyncratic productivity draws); lower and higher elasticities of labor force participation; a higher job separation rate; a lower elasticity in total emissions-abatement costs to changes in abatement rates; a lower bargaining power for workers; and a more aggressive implementation of the carbon tax that speeds up the transition to a lower-emissions steady state.

On the alternative-modeling-assumption front, we consider: a setting where emissions and abatement costs that are functions of gross versus net (of damages) output; a setting where the revenue from the carbon tax is used to subsidize the fixed cost of green technology adoption; versions of the benchmark model with real wage rigidities, capital adjustment costs, convex firm creation costs, and convex technology adoption costs; a version of the benchmark model with switching costs of searching for g jobs; and a richer version of our framework with two separate firm categories—one using a polluting production technology and the other using a non-polluting technology—each featuring endogenous firm entry, where only firms using the polluting technology can choose to adopt the green technology. The results from these experiments, which are summarized in Appendix A.7, confirm that the main conclusions from our benchmark model remain unchanged.

4 Conclusion

We explore the quantitative impact of a carbon tax that reduces emissions by 35 percent—a target consistent with the Biden Administration's new commitment under the Paris Agreement—on labor market and macroeconomic outcomes in a model with equilibrium unemployment and pollution externalities. In contrast to existing quantitative studies, our framework incorporates two key margins of adjustment to carbon taxation that are crucial for understanding the transition to a low-carbon, greener economy: firm entry and green technology adoption. Under a scheme where carbon-tax revenue is transferred lump-sum to households, we show that the tax bolsters labor income, consumption, output, and labor force participation, and has marginal adverse unemployment effects. In addition, the carbon tax does not entail short-term output or consumption costs as the economy adjusts to a higher carbon tax. Moreover, allowing for firm entry and green technology adoption reduces the tax rate needed to achieve the desired emissions reduction. As a corollary, modeling endogenous firm entry and technology adoption suggests the tax will collect less revenue.

Our analysis stresses the role of firm entry and green technology adoption decisions in shaping the net positive effects of a carbon tax on aggregate outcomes and the limited adverse effects on unemployment. Specifically, firms' ability to choose green production technologies leads to policy-induced *endogenous* changes in the economy's technological (regular vs. green) composition of aggregate production—an effect that is absent in models that abstract from green technology adoption. This technological composition effect is the central mechanism behind the positive effects of a carbon tax on consumption and output. Critically, abstracting from firm entry and technology adoption implies that a carbon tax has non-trivial adverse short- *and* long-term effects on labor income, consumption, and output, as well as comparatively larger adverse effects on unemployment. More broadly, our quantitative findings show that a carbon tax need not be accompanied by higher unemployment and lower consumption and output, a finding that reconciles recent cross-country evidence on the employment and macroeconomic effects of a carbon tax.

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A Appendix – Not For Publication

A.1 Technology Adoption Indifference Condition and Marginal Costs

Individual firm profits from producing with the r technology are

$$\pi^y_{r,t}(a) = \left[\rho_{r,t}(a) - \frac{mc_{r,t}}{a}\right] y_{r,t}(a),$$

while individual firm profits from producing with the g technology are given by

$$\pi^y_{g,t}(a) = \left[\rho_{g,t}(a) - \frac{mc_{g,t}}{a}\right] y_{g,t}(a) - \varphi_g.$$

Taking into account the demand curve each firm in category $j \in \{g, r\}$ faces, $y_{j,t}(a) = (\rho_{j,t}(a))^{-\varepsilon} Y_t$, a given firm sets its optimal price such that

$$\rho_{j,t}(a) = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{mc_{j,t}}{a}.$$

Then, using this last expression along the optimal demand function, individual firm profits for a given firm in category r can be written as

$$\begin{aligned}
\pi^y_{r,t}(a) &= \left[\rho_{r,t}(a) - \frac{mc_{r,t}}{a}\right] (\rho_{r,t}(a))^{-\varepsilon} Y_t \\
&= \left[\left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{mc_{r,t}}{a} - \frac{mc_{r,t}}{a}\right] \left(\left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{mc_{r,t}}{a}\right)^{-\varepsilon} Y_t \\
&= \left(\frac{1}{\varepsilon - 1}\right) \frac{mc_{r,t}}{a} \left(\left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{mc_{r,t}}{a}\right)^{-\varepsilon} Y_t \\
&= \left[\left(\frac{1}{\varepsilon - 1}\right) \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{-\varepsilon} Y_t\right] \left(\frac{mc_{r,t}}{a}\right)^{1-\varepsilon} \\
&= B_t \left(\frac{a}{mc_{r,t}}\right)^{\varepsilon - 1},
\end{aligned}$$

where $B_t > 0$. Following analogous steps, we have

$$\pi^y_{g,t}(a) = B_t \left(\frac{a}{mc_{g,t}} \right)^{\varepsilon - 1} - \varphi_g.$$

Now, denote by $a_{g,t}$ the threshold idiosyncratic productivity level such that a firm is indifferent between production technologies:

$$\pi^y_{r,t}(a_{g,t}) = \pi^y_{g,t}(a_{g,t}).$$

This can be rewritten as

$$B_t \left(\frac{a_{g,t}}{mc_{r,t}} \right)^{\varepsilon - 1} = B_t \left(\frac{a_{g,t}}{mc_{g,t}} \right)^{\varepsilon - 1} - \varphi_g,$$

Thus, if $\varphi_g > 0$, it must be that

$$\left(\frac{a_{g,t}}{mc_{g,t}} \right)^{\varepsilon - 1} > \left(\frac{a_{g,t}}{mc_{r,t}} \right)^{\varepsilon - 1},$$

or

$$mc_{r,t} > mc_{g,t}.$$

This makes intuitive sense since firms would never adopt the green technology and pay the fixed cost of doing so unless there is a benefit (here, in the form of lower marginal costs of production).

Now consider the slope of the individual-firm profit function in category $j \in \{g, r\}$ with respect to idiosyncratic productivity a:

$$\frac{\partial \pi^y_{j,t}}{\partial a} = B_t \left(\varepsilon - 1 \right) \left(\frac{1}{mc_{j,t}} \right)^{\varepsilon - 1} a^{\varepsilon - 2}.$$

Given the result about marginal cost above, at any given value of a, we have

$$\frac{\partial \pi^y_{g,t}}{\partial a} > \frac{\partial \pi^y_{r,t}}{\partial a}.$$

A.2 Value Functions: Labor Market

Denote by $\mathbf{W}_{j,t}$ the *net* value to the household of having a worker employed in the production of j intermediate goods for $j \in \{g, r\}$. It is easy to show that

$$\mathbf{W}_{j,t} = \frac{\mathbf{h}'(lfp_{j,t}) - \chi}{f(\theta_{j,t})\mathbf{u}'(c_t)}, \tag{25}$$

for $j \in \{g, r\}$ (see Arseneau and Chugh, 2012). Analogously, denote by $\mathbf{J}_{g,t}$ and $\mathbf{J}_{t,t}$ the net values to intermediate-goods firms of having workers employed in the production of g and r intermediate goods. These values are given by

$$\mathbf{J}_{g,t} = D(x_t)mc_{g,t}F_{n_{g,t}} - w_{g,t} + (1-\varrho)\mathbb{E}_t\Xi_{t+1|t}\mathbf{J}_{g,t+1}, \tag{26}$$

and

$$\mathbf{J}_{r,t} = D(x_t)mc_{r,t}H_{n_{r,t}} - \tau_t e_{n_{r,t}} - \Gamma_{n_{r,t}} - w_{r,t} + (1-\varrho)\mathbb{E}_t\Xi_{t+1|t}\mathbf{J}_{r,t+1}. \tag{27}$$

Real wages are determined via bilateral Nash bargaining between firms and workers. Formally, the real wage $w_{j,t}$ in employment category $j \in \{g, r\}$ is the solution of the following maximization problem:

$$\underset{w_{j,t}}{max} \left(\mathbf{W}_{j,t}\right)^{\nu_n} \left(\mathbf{J}_{j,t}\right)^{1-\nu_n},$$

where $0 < \nu_n < 1$ is the bargaining power of workers and we impose free vacancy entry. Two simple steps of algebra show that the real Nash wage w_j for employment category $j \in \{g, r\}$ is implicitly given by

$$\mathbf{W}_{j,t} = \left(\frac{\nu_n}{1-\nu_n}\right)\mathbf{J}_{j,t}. \tag{28}$$

A.3 Data-Consistent vs. Model-Consistent Variables

Recall that the aggregate price level is given by $P_t = \left(\int_{\omega\in\Omega} p_t(\omega)^{1-\varepsilon}d\omega\right)^{\frac{1}{1-\varepsilon}}$. In a symmetric equilibrium, the aggregate price can be written as

$$P_t = \left[G(a_{g,t})N_t\widetilde{p}_{r,t}^{1-\varepsilon} + (1-G(a_{g,t}))N_t\widetilde{p}_{g,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{29}$$

or

$$P_t = N_t^{\frac{1}{1-\varepsilon}}[G(a_{g,t})\widetilde{p}_{r,t}^{1-\varepsilon} + (1 - G(a_{g,t}))\widetilde{p}_{g,t}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \tag{30}$$

where the average nominal prices associated with g and r firms are $\widetilde{p}_{g,t} \equiv p_{g,t}(\widetilde{a}_{g,t})$ and $\widetilde{p}_{r,t} \equiv p_{r,t}(\widetilde{a}_{r,t})$. Then, we can write

$$P_t = N_t^{\frac{1}{1-\varepsilon}}\widetilde{P}_t, \tag{31}$$

where $\widetilde{P}_t \equiv [G(a_{g,t})\widetilde{p}_{r,t}^{1-\varepsilon} + (1 - G(a_{g,t}))\widetilde{p}_{g,t}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$. Finally, if λ_t^m denotes a real variable in the model, following Ghironi and Melitz (2005), the data-consistent version of this variable is given by $\lambda_t^d = \lambda_t^m \frac{P_t}{\widetilde{P}_t} = \lambda_t^m (N_t)^{\frac{1}{1-\varepsilon}}$.

A.4 Equilibrium Conditions: Benchmark Model

A.4.1 Definition of Equilibrium

Taking the exogenous process e_t^{row} as given, the allocations and prices $\{\widetilde{\pi}_{r,t}^y, \widetilde{\pi}_{g,t}^y, \widetilde{\rho}_{r,t}, \widetilde{\rho}_{g,t}, mc_{r,t}\}$, $\{mc_{g,t}, \widetilde{a}_{r,t}, \widetilde{a}_{g,t}, a_{g,t}, \widetilde{\pi}_t^y, N_{g,t}, \Gamma_t, e_t, \mu_t, v_{r,t}, v_{g,t}, n_{r,t}, n_{g,t}, N_t, N_{e,t}, s_{r,t}, s_{g,t}, w_{r,t}, w_{g,t}, \widetilde{y}_{r,t}, \widetilde{y}_{g,t}\}$, and $\{x_t, c_t, Y_t, N_{r,t}, k_t, k_{g,t}, k_{r,t}\}$ satisfy:

$$\widetilde{\pi}_{r,t}^y = \left[\widetilde{\rho}_{r,t} - \frac{mc_{r,t}}{\widetilde{a}_{r,t}}\right]\widetilde{y}_{r,t}, \tag{32}$$

$$\widetilde{\pi}_{g,t}^y = \left[\widetilde{\rho}_{g,t} - \frac{mc_{g,t}}{\widetilde{a}_{g,t}}\right]\widetilde{y}_{g,t} - \varphi_g, \tag{33}$$

$$1 = N_{r,t}\left(\widetilde{\rho}_{r,t}\right)^{1-\varepsilon} + N_{g,t}\left(\widetilde{\rho}_{g,t}\right)^{1-\varepsilon}, \tag{34}$$

$$\widetilde{\rho}_{g,t} = \frac{\varepsilon}{\varepsilon - 1}\frac{mc_{g,t}}{\widetilde{a}_{g,t}}, \tag{35}$$

$$\widetilde{\rho}_{r,t} = \frac{\varepsilon}{\varepsilon - 1}\frac{mc_{r,t}}{\widetilde{a}_{r,t}}, \tag{36}$$

$$\widetilde{a}_{g,t} = \left(\frac{k_p}{k_p - (\varepsilon - 1)}\right)^{\frac{1}{\varepsilon-1}} a_{g,t}, \tag{37}$$

$$\widetilde{a}_{r,t} = \widetilde{a}_{g,t}\left(\frac{a_{g,t}^{k_p-(\varepsilon-1)} - a_{min}^{k_p-(\varepsilon-1)}}{a_{g,t}^{k_p} - a_{min}^{k_p}}\right)^{\frac{1}{\varepsilon-1}} a_{min}, \tag{38}$$

$$N_{g,t} = \left(\frac{a_{min}}{a_{g,t}}\right)^{k_p} N_t, \tag{39}$$

$$N_{r,t} = N_t - N_{g,t}, \tag{40}$$

$$\widetilde{\pi}_t^y = \frac{N_{r,t}}{N_t}\widetilde{\pi}_{r,t}^y + \frac{N_{g,t}}{N_t}\widetilde{\pi}_{g,t}^y, \tag{41}$$

$$\pi_{g,t}^y(a_{g,t}) = \pi_{r,t}^y(a_{g,t}), \tag{42}$$

$$\Gamma_t = \gamma \mu_t^\eta D(x_t) H(n_{r,t}, k_{r,t}), \tag{43}$$

$$e_t = (1-\mu_t)\zeta \left[D(x_t)H(n_{r,t}, k_{r,t})\right]^{1-\nu}, \tag{44}$$

$$\tau_t \zeta \left[D(x_t)H(n_{r,t}, k_{r,t})\right]^{-\nu} = \gamma \eta \mu_t^{\eta-1}, \tag{45}$$

$$\frac{\psi_r}{q(\theta_{r,t})} = \left[\begin{array}{c} D(x_t) mc_{r,t} H_{n_r,t} - \tau_t e_{n_r,t} \\ -\Gamma_{n_r,t} - w_{r,t} + (1-\varrho)\mathbb{E}_t \Xi_{t+1|t} \frac{\psi_r}{q(\theta_{r,t+1})} \end{array}\right], \tag{46}$$

$$\frac{\psi_g}{q(\theta_{g,t})} = \left[D(x_t) mc_{g,t} F_{n_g,t} - w_{g,t} + (1-\varrho)\mathbb{E}_t \Xi_{t+1|t} \frac{\psi_g}{q(\theta_{g,t+1})}\right], \tag{47}$$

$$n_{r,t} = (1-\varrho) n_{r,t-1} + s_{r,t} f(\theta_{r,t}), \tag{48}$$

$$n_{g,t} = (1-\varrho) n_{g,t-1} + s_{g,t} f(\theta_{g,t}), \tag{49}$$

$$N_{t+1} = (1-\delta)\left[N_t + N_{e,t}\right], \tag{50}$$

$$\varphi_e = (1-\delta)\mathbb{E}_t \Xi_{t+1|t}\left[\widetilde{\pi}_{t+1}^y + \varphi_e\right], \tag{51}$$

$$\left(\frac{\mathbf{h}_{lfp_{r,t}} - \chi \mathbf{u}'(c_t)}{f(\theta_{r,t})\mathbf{u}'(c_t)}\right) = w_{r,t} - \chi + (1-\varrho)\mathbb{E}_t \Xi_{t+1|t}\left(1 - f(\theta_{r,t+1})\right)\left(\frac{\mathbf{h}_{lfp_{r,t+1}} - \chi \mathbf{u}'(c_{t+1})}{f(\theta_{r,t+1})\mathbf{u}'(c_{t+1})}\right), \tag{52}$$

$$\left(\frac{\mathbf{h}_{lfp_{g,t}} - \chi \mathbf{u}'(c_t)}{f(\theta_{g,t})\mathbf{u}'(c_t)}\right) = w_{g,t} - \chi + (1-\varrho)\mathbb{E}_t \Xi_{t+1|t}\left(1 - f(\theta_{g,t+1})\right)\left(\frac{\mathbf{h}_{lfp_{g,t+1}} - \chi \mathbf{u}'(c_{t+1})}{f(\theta_{g,t+1})\mathbf{u}'(c_{t+1})}\right), \tag{53}$$

$$w_{r,t} = \nu_n \left[D(x_t) mc_{r,t} H_{n_{r,t}} - \Gamma_{n_r,t} - \tau_t e_{n_r,t} + (1-\varrho)\mathbb{E}_t \Xi_{t+1|t} \psi_r \theta_{r,t+1}\right] + (1-\nu_n)\chi, \tag{54}$$

$$w_{g,t} = \nu_n \left[D(x_t) mc_{g,t} F_{n_{g,t}} + (1-\varrho)\mathbb{E}_t \Xi_{t+1|t} \psi_g \theta_{g,t+1}\right] + (1-\nu_n)\chi, \tag{55}$$

$$D(x_t) H(n_{r,t}, k_{r,t}) = N_{r,t}\left(\frac{\widetilde{y}_{r,t}}{\widetilde{a}_{r,t}}\right), \tag{56}$$

$$D(x_t) F(n_{g,t}, k_{g,t}) = N_{g,t}\left(\frac{\widetilde{y}_{g,t}}{\widetilde{a}_{g,t}}\right), \tag{57}$$

$$\widetilde{y}_{r,t} = (\widetilde{\rho}_{r,t})^{-\varepsilon} Y_t, \tag{58}$$

$$\widetilde{y}_{g,t} = (\widetilde{\rho}_{g,t})^{-\varepsilon} Y_t, \tag{59}$$

$$x_t = \rho_x x_{t-1} + e_t + e_t^{row}, \tag{60}$$

$$k_t = k_{g,t} + k_{r,t}, \tag{61}$$

$$1 = \mathbb{E}_t \Xi_{t+1|t} \left[D(x_{t+1}) mc_{r,t+1} H_{k_r,t+1} - \tau_{t+1} e_{k_r,t+1} - \Gamma_{k_r,t+1} + (1-\delta) \right], \tag{62}$$

$$1 = \mathbb{E}_t \Xi_{t+1|t} \left[D(x_{t+1}) mc_{g,t+1} F_{k_g,t+1} + (1-\delta) \right], \tag{63}$$

$$Y_t = c_t + \psi_r v_{r,t} + \psi_g v_{g,t} + \varphi_e N_{e,t} + \varphi_g N_{g,t} + k_{t+1} - (1-\delta) k_t + \Gamma_t, \tag{64}$$

where all other relevant variables in these conditions are defined in the main text.

A.4.2 Total Output in Equilibrium

To obtain the expression for total output presented in Section A.4.1, start with the equilibrium condition

$$1 = N_{r,t} (\widetilde{\rho}_{r,t})^{1-\varepsilon} + N_{g,t} (\widetilde{\rho}_{g,t})^{1-\varepsilon} .$$

Using the demand functions for each firm category, we can rewrite the above condition as

$$Y_t = \left[N_{r,t} (\widetilde{y}_{r,t})^{\frac{\varepsilon-1}{\varepsilon}} + N_{g,t} (\widetilde{y}_{g,t})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} .$$

Then, recall that market clearing for each firm category is given by $\widetilde{a}_{r,t} D(x_t) H(n_{r,t}, k_{r,t}) = N_{r,t} \widetilde{y}_{r,t}$ and $\widetilde{a}_{g,t} D(x_t) F(n_{g,t}, k_{g,t}) = N_{g,t} \widetilde{y}_{g,t}$, implying that we can write total output as

$$Y_t = \left[N_{r,t} \left(\frac{\widetilde{a}_{r,t} D(x_t) H(n_{r,t}, k_{r,t})}{N_{r,t}} \right)^{\frac{\varepsilon-1}{\varepsilon}} + N_{g,t} \left(\frac{\widetilde{a}_{g,t} D(x_t) F(n_{g,t}, k_{g,t})}{N_{g,t}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} ,$$

which we can rewrite as

$$Y_t = D(x_t) \left[N_{r,t}^{\frac{1}{\varepsilon}} (\widetilde{a}_{r,t} H(n_{r,t}, k_{r,t}))^{\frac{\varepsilon-1}{\varepsilon}} + N_{g,t}^{\frac{1}{\varepsilon}} (\widetilde{a}_{g,t} F(n_{g,t}, k_{g,t}))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} .$$

Now, recall that $N_{r,t} = G(a_{g,t})N_t = \left[1 - \left(\frac{a_{min}}{a_{g,t}}\right)^{k_p}\right] N_t$ and $N_{g,t} = (1 - G(a_{g,t}))\, N_t = \left(\frac{a_{min}}{a_{g,t}}\right)^{k_p} N_t$. Letting $a_{min} = 1$ without loss of generality, we have $N_{r,t} = \left(\frac{a_{g,t}^{k_p}-1}{a_{g,t}^{k_p}}\right) N_t$ and $N_{g,t} = \left(\frac{1}{a_{g,t}^{k_p}}\right) N_t$. Using these expressions, we can write

$$Y_t = D(x_t)\left[\left(\underbrace{\left(\frac{a_{g,t}^{k_p}-1}{a_{g,t}^{k_p}}\right)N_t}_{N_{r,t}}\widetilde{a}_{r,t}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon}}(H(n_{r,t},k_{r,t}))^{\frac{\varepsilon-1}{\varepsilon}} + \left(\underbrace{\left(\frac{1}{a_{g,t}^{k_p}}\right)N_t}_{N_{g,t}}\widetilde{a}_{g,t}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon}}(F(n_{g,t},k_{g,t}))^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

or

$$Y_t = D(x_t)\,(N_t)^{\frac{1}{\varepsilon-1}}\left[\left(\left(\frac{a_{g,t}^{k_p}-1}{a_{g,t}^{k_p}}\right)\widetilde{a}_{r,t}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon}}(H(n_{r,t},k_{r,t}))^{\frac{\varepsilon-1}{\varepsilon}} + \left(\left(\frac{1}{a_{g,t}^{k_p}}\right)\widetilde{a}_{g,t}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon}}(F(n_{g,t},k_{g,t}))^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Now, using the average idiosyncratic productivity levels of each firm category

$$\widetilde{a}_{g,t} = \left(\frac{k_p}{k_p-(\varepsilon-1)}\right)^{\frac{1}{\varepsilon-1}} a_{g,t},$$

and

$$\begin{aligned}\widetilde{a}_{r,t} &= \underbrace{\left(\frac{k_p}{k_p-(\varepsilon-1)}\right)^{\frac{1}{\varepsilon-1}} a_{g,t}}_{\widetilde{a}_{g,t}}\left(\frac{a_{g,t}^{k_p-(\varepsilon-1)} - a_{min}^{k_p-(\varepsilon-1)}}{a_{g,t}^{k_p} - a_{min}^{k_p}}\right)^{\frac{1}{\varepsilon-1}} \\ &= \left(\frac{k_p}{k_p-(\varepsilon-1)}\right)^{\frac{1}{\varepsilon-1}} a_{g,t}\left(\frac{a_{g,t}^{k_p-(\varepsilon-1)}-1}{a_{g,t}^{k_p}-1}\right)^{\frac{1}{\varepsilon-1}}.\end{aligned}$$

we can show that the terms

$$\begin{aligned}
\left(\frac{a_{g,t}^{k_p}-1}{a_{g,t}^{k_p}}\right)\widetilde{a}_{r,t}^{\varepsilon-1} &= \left(\frac{a_{g,t}^{k_p}-1}{a_{g,t}^{k_p}}\right)\left(\frac{k_p}{k_p-(\varepsilon-1)}\right)a_{g,t}^{\varepsilon-1}\left(\frac{a_{g,t}^{k_p-(\varepsilon-1)}-1}{a_{g,t}^{k_p}-1}\right)\\
&= \left(\frac{k_p}{k_p-(\varepsilon-1)}\right)\left(\frac{a_{g,t}^{k_p}-a_{g,t}^{\varepsilon-1}}{a_{g,t}^{k_p}}\right)\\
&= \left(\frac{k_p}{k_p-(\varepsilon-1)}\right)\left(1-\frac{1}{a_{g,t}^{k_p-(\varepsilon-1)}}\right),
\end{aligned}$$

and

$$\left(\frac{1}{a_{g,t}^{k_p}}\right)\widetilde{a}_{g,t}^{\varepsilon-1} = \left(\frac{1}{a_{g,t}^{k_p}}\right)\left(\frac{k_p}{k_p-(\varepsilon-1)}\right)a_{g,t}^{\varepsilon-1} = \left(\frac{k_p}{k_p-(\varepsilon-1)}\right)\left(\frac{1}{a_{g,t}^{k_p-(\varepsilon-1)}}\right).$$

It follows that

$$Y_t = D(x_t)\left(\nu_p N_t\right)^{\frac{1}{\varepsilon-1}}\left[\left(1-\frac{1}{a_{g,t}^{k_p-(\varepsilon-1)}}\right)^{\frac{1}{\varepsilon}}\left(H(n_{r,t},k_{r,t})\right)^{\frac{\varepsilon-1}{\varepsilon}}+\left(\frac{1}{a_{g,t}^{k_p-(\varepsilon-1)}}\right)^{\frac{1}{\varepsilon}}\left(F(n_{g,t},k_{g,t})\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\nu_p \equiv \left(\frac{k_p}{k_p-(\varepsilon-1)}\right)$, $k_p > \varepsilon-1$, and $0 < \frac{1}{a_{g,t}^{k_p-(\varepsilon-1)}} < 1$. Finally, defining $\alpha_{y,t} \equiv \left(1-\frac{1}{a_{g,t}^{k_p-(\varepsilon-1)}}\right)$,which is increasing in $a_{g,t}$, we can write

$$Y_t = D(x_t)\left(\nu_p N_t\right)^{\frac{1}{\varepsilon-1}}\left[\left(\alpha_{y,t}\right)^{\frac{1}{\varepsilon}}\left(H(n_{r,t},k_{r,t})\right)^{\frac{\varepsilon-1}{\varepsilon}}+\left(1-\alpha_{y,t}\right)^{\frac{1}{\varepsilon}}\left(F(n_{g,t},k_{g,t})\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

A.5 Simplified Model: Analytical Results and Model Mechanisms

To highlight the fundamental mechanisms behind the positive impact of carbon taxes on total output in the benchmark model, we adopt several simplifying assumptions to obtain a set of analytical results. First, we abstract from physical capital and from labor search frictions. Second, given that technology adoption is a key factor behind our main findings, we simplify the firm creation side of the model and assume that new firm entrants produce without a lag, and all active firms exit after producing each period ($\delta = 1$). This assumption implies that the evolution of firms collapses to $N_t = (1-\delta)N_{t-1} + N_{e,t} = N_{e,t}$, i.e., all operating firms are new. We also assume zero persistence of the pollution stock ($\rho_x = 0$) and zero emissions from the rest of the world ($e_t^{row} = 0$), so that the pollution stock is only determined by emissions: $x_t = \rho_x x_{t-1} + e_t + e_t^{row} = e_t$. We also eliminate the damage function by setting $D(x) = 1$. Finally, normalizing the household's time endowment to 1, we assume that leisure is fixed and normalized to zero, so that the time constraint is given by $1 = n_g + n_r$. This allows us to focus on labor reallocation between firm categories.

Turning to the functional forms and parameterization of the model, risk aversion is not essential for the main mechanisms of the model. Therefore, we let $\mathbf{u}(c_t) = c_t$ and $\mathbf{h}(n_{j,t}) = n_{j,t}$ so that $\mathbf{u}'(c_t) = 1$ and $\mathbf{h}'(n_{j,t}) = 1$ for $j \in \{g, r\}$, and total utility each period is equal to $\mathbf{u}(c_t) - (\mathbf{h}(n_{g,t}) + \mathbf{h}(n_{r,t})) = c_t - (n_{g,t} + n_{r,t})$. These assumptions greatly simplify the optimal labor supply conditions: with a frictionless labor market and the functional forms and parameterization we adopt, the optimal labor supply conditions are simply given by $1 = w_{r,t}$ and $1 = w_{g,t}$. To simplify the model further, we assume that $k_p = \varepsilon = 3.8$, which continues to satisfy the condition $k_p > \varepsilon - 1$ and implies that $k_p - (\varepsilon - 1) = 1$ (this simplifies the expressions for average idiosyncratic productivity for each firm category). We also set $a_{min} = 1$, $\gamma = 1$, $\varphi_e = 1$, $\zeta = 1$, and $\nu = 0$. This implies that in the absence of physical capital, total abatement costs $\Gamma_t = \gamma\mu_t^\eta D(x_t) n_{r,t} = \mu_t^\eta D(x_t) n_{r,t}$ and emissions $e_t = (1-\mu_t)\zeta \left[D(x_t) n_{r,t}\right]^{1-\nu} = (1-\mu_t) D(x_t) n_{r,t}$ are both linear in labor used by r firms. It then follows that $\Gamma_{n_r,t} = \mu_t^\eta D(x_t)$ and $e_{n_r,t} = (1-\mu_t) D(x_t)$.

Making use of the two simplified optimal labor supply conditions, the optimal labor demand conditions can be written as $mc_{r,t} = \frac{\left(\tau_t(1-\mu_t)+\mu_t^\eta\right)D(x_t)+w_{r,t}}{D(x_t)} = \frac{\left(\tau_t(1-\mu_t)+\mu_t^\eta\right)D(x_t)+1}{D(x_t)}$ and

$mc_{g,t} = \frac{w_{g,t}}{D(x_t)} = \frac{1}{D(x_t)}$. Under the above simplifying assumptions, the equilibrium conditions of the modified benchmark model are given by:

$$\widetilde{\rho}_{r,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{mc_{r,t}}{\widetilde{a}_{r,t}}, \tag{65}$$

$$\widetilde{\rho}_{g,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{mc_{g,t}}{\widetilde{a}_{g,t}}, \tag{66}$$

$$\widetilde{\pi}^y_{r,t} = \frac{1}{\varepsilon} \left(\widetilde{\rho}_{r,t}\right)^{1-\varepsilon} Y_t, \tag{67}$$

$$\widetilde{\pi}^y_{g,t} = \frac{1}{\varepsilon} \left(\widetilde{\rho}_{g,t}\right)^{1-\varepsilon} Y_t - \varphi_g, \tag{68}$$

$$\widetilde{a}_{g,t} = \left(k_p\right)^{\frac{1}{\varepsilon-1}} a_{g,t}, \tag{69}$$

$$\widetilde{a}_{r,t} = \widetilde{a}_{g,t} \left(\frac{a_{g,t} - 1}{a_{g,t}^{k_p} - 1} \right)^{\frac{1}{\varepsilon-1}}, \tag{70}$$

$$\widetilde{\pi}^y_{g,t} = k_p \left(\frac{a_{g,t}}{\widetilde{a}_{r,t}} \right)^{\varepsilon-1} \widetilde{\pi}^y_{r,t} + (\varepsilon - 1)\,\varphi_g, \tag{71}$$

$$\tau_t = \eta \mu_t^{\eta-1}, \tag{72}$$

$$\widetilde{y}_{r,t} = \left(\widetilde{\rho}_{r,t}\right)^{-\varepsilon} Y_t = \frac{\widetilde{a}_{r,t} D(x_t) n_{r,t}}{\left[1 - \left(\frac{1}{a_{g,t}}\right)^{k_p}\right] N_t}, \tag{73}$$

$$\widetilde{y}_{g,t} = \left(\widetilde{\rho}_{g,t}\right)^{-\varepsilon} Y_t = \frac{\widetilde{a}_{g,t} D(x_t) n_{g,t}}{\left(\frac{1}{a_{g,t}}\right)^{k_p} N_t}, \tag{74}$$

$$1 = \left[1 - \left(\frac{1}{a_{g,t}}\right)^{k_p}\right] N_t \left(\widetilde{\rho}_{r,t}\right)^{1-\varepsilon} + \left(\frac{1}{a_{g,t}}\right)^{k_p} N_t \left(\widetilde{\rho}_{g,t}\right)^{1-\varepsilon}, \tag{75}$$

$$x_t = e_t = (1 - \mu_t) D(x_t) n_{r,t}, \tag{76}$$

$$N_t = N_{e,t}, \tag{77}$$

$$Y_t = c_t + \left[1 + \varphi_g \left(\frac{1}{a_{g,t}}\right)^{k_p}\right] N_t + \mu_t^{\eta} D(x_t) n_{r,t}, \tag{78}$$

where, in writing these conditions, we use the expressions for $mc_{r,t}$ and $mc_{g,t}$, as well as the fact that $N_t = N_{e,t}$, $\varphi_e = 1$, $N_{g,t} = \left(\frac{1}{a_{g,t}}\right)^{k_p} N_t$ and $N_{r,t} = \left[1 - \left(\frac{1}{a_{g,t}}\right)^{k_p}\right] N_t$.

In what follows, we separately derive each of the key conditions that we use to analytically characterize the fundamental mechanisms that shape the equilibrium impact of carbon taxes on total output. Specifically, we make use of the conditions that (1) determine equilibrium labor and market clearing in each firm category and (2) the condition that pins down the threshold idiosyncratic productivity level above which firms decide to adopt the g technology. These are the main margins of adjustment to the carbon tax in the simplified version of the benchmark model.

A.5.1 Equilibrium Labor, Sectoral Market Clearing, and Carbon Taxes

Recall that in the model, pollution enters via a damages function $D(x_t)$ that reduces firms' total factor productivity, where firms take $D(x_t)$ as given. To focus on the role of input reallocation between firm categories and green technology adoption in the model, in what follows and without loss of generality, we turn off the damage function, setting $D(x) = 1$, We note, though, that our main conclusions remain unchanged if we explicitly take into account the link between the reduction in production by r firms, the reduction in emissions and pollution, and the resulting increase in firms' total factor productivity via lower damages when we analyze the impact of a carbon tax on labor, technology adoption, and output.

Recall that under our simplifying assumptions, the optimal abatement rate μ_t is pinned down by condition $\tau_t = \eta \mu_t^{\eta-1}$ where $\eta > 1$. The abatement rate can therefore be rewritten as $\mu_t = \left(\frac{\tau_t}{\eta}\right)^{\frac{1}{\eta-1}}$. Using this last expression, we can write the term $\left(\tau_t\left(1-\mu_t\right)+\mu_t^\eta\right) D(x_t)$ in the optimal labor demand for r labor as $\left(\tau_t + (1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x_t)$. Then, using the optimal pricing condition for each firm category, we can write the average relative prices $\widetilde{\rho}_{r,t}$ and $\widetilde{\rho}_{g,t}$ as follows:

$$\begin{aligned}
\widetilde{\rho}_{r,t} &= \frac{\varepsilon}{\varepsilon-1}\frac{mc_{r,t}}{\widetilde{a}_{r,t}} \\
&= \frac{\varepsilon}{\varepsilon-1}\frac{\left(\tau_t + (1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x_t) + 1}{D(x_t)\widetilde{a}_{r,t}},
\end{aligned}$$

and

$$
\begin{aligned}
\widetilde{\rho}_{g,t} &= \frac{\varepsilon}{\varepsilon-1}\frac{mc_{g,t}}{\widetilde{a}_{g,t}} \\
&= \frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x_t)\widetilde{a}_{g,t}}.
\end{aligned}
$$

Using the demand functions and market clearing conditions for the output of the two firm categories alongside the expressions for $\widetilde{\rho}_{r,t}$ and $\widetilde{\rho}_{g,t}$, we can then write

$$
\underbrace{\left[1-\left(\frac{1}{a_{g,t}}\right)^{k_p}\right]}_{N_{r,t}} N_t \underbrace{\left(\underbrace{\frac{\varepsilon}{\varepsilon-1}\frac{\left(\tau_t+(1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x_t)+1}{D(x_t)\widetilde{a}_{r,t}}}_{\widetilde{\rho}_{r,t}}\right)^{-\varepsilon}}_{\widetilde{y}_{r,t}} Y_t = \widetilde{a}_{r,t}D(x_t)n_{r,t},
$$

and

$$
\underbrace{\left(\frac{1}{a_{g,t}}\right)^{k_p}}_{N_{g,t}} N_t \underbrace{\left(\underbrace{\frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x_t)\widetilde{a}_{g,t}}}_{\widetilde{\rho}_{g,t}}\right)^{-\varepsilon}}_{\widetilde{y}_{g,t}} Y_t = \widetilde{a}_{g,t}D(x_t)n_{g,t}.
$$

The market clearing condition for total r-firm output can be rewritten as

$$
\frac{\left[1-\left(\frac{1}{a_{g,t}}\right)^{k_p}\right]N_t}{D(x_t)\widetilde{a}_{r,t}}\left(\frac{\varepsilon}{\varepsilon-1}\frac{\left(\tau_t+(1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x_t)+1}{D(x_t)\widetilde{a}_{r,t}}\right)^{-\varepsilon} Y_t = n_{r,t},
$$

or using the fact that the average idiosyncratic productivities given our simplifying assumptions are $\widetilde{a}_{r,t} = \widetilde{a}_{g,t}\left(\frac{a_{g,t}-1}{a_{g,t}^{k_p}-1}\right)^{\frac{1}{\varepsilon-1}}$ and $\widetilde{a}_{g,t} = (k_p)^{\frac{1}{\varepsilon-1}}a_{g,t}$,

$$
\frac{N_t}{D(x_t)}k_p\left(1-\frac{1}{a_{g,t}}\right)\left(\frac{\varepsilon}{\varepsilon-1}\frac{\left(\tau_t+(1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x_t)+1}{D(x_t)}\right)^{-\varepsilon} Y_t = n_{r,t}. \tag{79}
$$

Following similar steps, the market clearing condition for total g-firm output can be rewritten as

$$\frac{\left(\frac{1}{a_{g,t}}\right)^{k_p} N_t}{D(x_t)\widetilde{a}_{g,t}} \left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x_t)\widetilde{a}_{g,t}}\right)^{-\varepsilon} Y_t = n_{g,t},$$

or, using the expressions for $\widetilde{a}_{r,t}$ and $\widetilde{a}_{g,t}$, we have

$$\frac{N_t}{D(x_t)} k_p \left(\frac{1}{a_{g,t}}\right) \left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x_t)}\right)^{-\varepsilon} Y_t = n_{g,t}. \tag{80}$$

Note that by dividing equation (79) by equation (80) and rearranging terms, we obtain

$$\frac{n_{g,t}}{n_{r,t}} = \frac{\left(\left(\tau_t + (1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x_t) + 1\right)^{\varepsilon}}{(a_{g,t}-1)}, \tag{81}$$

where $\varepsilon > 1$. Given the normalization of the minimum level of idiosyncratic productivity, $a_{min} = 1$, it must be that $a_{g,t} > 1$. Condition (81) makes clear that all else equal, an increase in the carbon tax τ_t or a reduction in the endogenous threshold productivity level $a_{g,t}$ increase the ratio $\frac{n_{g,t}}{n_{r,t}}$. Note that in the absence of a green technology adoption margin, the threshold idiosyncratic productivity level $a_{g,t}$ becomes a constant.[36]

A.5.2 Endogenous Productivity, Technological Composition of Production, and Carbon Taxes

We now derive the condition that pins down the threshold idiosyncratic productivity level, $a_{g,t}$.

[36] This constant can be chosen so that the (exogenous) productivity differential between firm categories in the absence of technology adoption is the same as the baseline (pre carbon tax) productivity differential in the presence of technology adoption.

Optimal Pricing, Marginal Costs, and Average Firm Profits by Firm Category Average individual-firm profits for each firm category can be written as

$$\begin{aligned}\widetilde{\pi}^{y}_{r,t} &= \frac{1}{\varepsilon}\left(\widetilde{\rho}_{r,t}\right)^{1-\varepsilon} Y_t \\ &= \frac{1}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\frac{\left(\tau_t + (1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x_t) + 1}{D(x_t)\widetilde{a}_{r,t}}\right)^{1-\varepsilon} Y_t, \end{aligned} \tag{82}$$

and

$$\begin{aligned}\widetilde{\pi}^{y}_{g,t} &= \frac{1}{\varepsilon}\left(\widetilde{\rho}_{g,t}\right)^{1-\varepsilon} Y_t - \varphi_g \\ &= \frac{1}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x_t)\widetilde{a}_{g,t}}\right)^{1-\varepsilon} Y_t - \varphi_g. \end{aligned} \tag{83}$$

Endogenous Productivity Threshold The condition that implicitly pins down the threshold productivity level $a_{g,t}$ is $\pi^{y}_{g,t}(a_{g,t}) = \pi^{y}_{r,t}(a_{g,t})$ (that is, a firm is indifferent between producing using the r or the g technology). We can show that this indifference condition can be equivalently expressed as a function of average individual-firm profits:

$$\widetilde{\pi}^{y}_{g,t} = (k_p)\left(\frac{a_{g,t}}{\widetilde{a}_{r,t}}\right)^{\varepsilon-1}\widetilde{\pi}^{y}_{r,t} + (\varepsilon-1)\,\varphi_g,$$

Plugging in the expressions for $\widetilde{\pi}^{y}_{g,t}$ and $\widetilde{\pi}^{y}_{r,t}$ derived above, we can rewrite this last expression as

$$\begin{aligned}\frac{1}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x_t)\widetilde{a}_{g,t}}\right)^{1-\varepsilon} Y_t - \varphi_g = (k_p)\left(\frac{a_{g,t}}{\widetilde{a}_{r,t}}\right)^{\varepsilon-1}\frac{1}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\frac{\left(\tau_t + (1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x_t) + 1}{D(x_t)\widetilde{a}_{r,t}}\right)^{1-\varepsilon} Y_t \\ + (\varepsilon-1)\,\varphi_g,\end{aligned}$$

or

$$\frac{1}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x_t)\widetilde{a}_{g,t}}\right)^{1-\varepsilon}Y_t=(k_p)\left(\frac{a_{g,t}}{\widetilde{a}_{r,t}}\right)^{\varepsilon-1}\frac{1}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\frac{\left(\tau_t+(1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x_t)+1}{D(x_t)\widetilde{a}_{r,t}}\right)^{1-\varepsilon}Y_t$$
$$+\varepsilon\varphi_g,$$

$$\left(\frac{1}{D(x_t)\widetilde{a}_{g,t}}\right)^{1-\varepsilon}Y_t=(k_p)\left(\frac{a_{g,t}}{\widetilde{a}_{r,t}}\right)^{\varepsilon-1}\left(\frac{\left(\tau_t+(1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x_t)+1}{D(x_t)\widetilde{a}_{r,t}}\right)^{1-\varepsilon}Y_t$$
$$+\left(\frac{\varepsilon-1}{\varepsilon}\right)^{1-\varepsilon}\varepsilon^2\varphi_g,$$

$$\left(\frac{1}{\widetilde{a}_{g,t}}\right)^{1-\varepsilon}Y_t=(k_p)\left(\frac{a_{g,t}}{\widetilde{a}_{r,t}}\right)^{\varepsilon-1}\left(\frac{\left(\tau_t+(1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x_t)+1}{\widetilde{a}_{r,t}}\right)^{1-\varepsilon}Y_t$$
$$+\left(D(x_t)\left(\frac{\varepsilon-1}{\varepsilon}\right)\right)^{1-\varepsilon}\varepsilon^2\varphi_g.$$

Using the expression for $\widetilde{a}_{g,t}=(k_p)^{\frac{1}{\varepsilon-1}}a_{g,t}$, we can write the above condition as

$$\left(\frac{1}{(k_p)^{\frac{1}{\varepsilon-1}}a_{g,t}}\right)^{1-\varepsilon}Y_t=(k_p)\left(\frac{a_{g,t}}{\widetilde{a}_{r,t}}\right)^{\varepsilon-1}\left(\frac{1}{\widetilde{a}_{r,t}}\right)^{1-\varepsilon}\left(\left(\tau_t+(1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x_t)+1\right)^{1-\varepsilon}Y_t$$
$$+\left(D(x_t)\left(\frac{\varepsilon-1}{\varepsilon}\right)\right)^{1-\varepsilon}\varepsilon^2\varphi_g,$$

$$Y_t=\left((k_p)^{\frac{1}{\varepsilon-1}}a_{g,t}\right)^{1-\varepsilon}(k_p)\left(a_{g,t}\right)^{\varepsilon-1}\left(\left(\tau_t+(1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x_t)+1\right)^{1-\varepsilon}Y_t$$
$$+\left((k_p)^{\frac{1}{\varepsilon-1}}a_{g,t}\right)^{1-\varepsilon}\left(D(x_t)\left(\frac{\varepsilon-1}{\varepsilon}\right)\right)^{1-\varepsilon}\varepsilon^2\varphi_g,$$

$$Y_t = \left(\left(\tau_t + (1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x_t) + 1\right)^{1-\varepsilon} Y_t$$
$$+ \frac{\left(a_{g,t} D(x_t)\left(\frac{\varepsilon-1}{\varepsilon}\right)\right)^{1-\varepsilon} \varepsilon^2 \varphi_g}{k_p},$$

or

$$\left(1 - \left(\left(\tau_t + (1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x_t) + 1\right)^{1-\varepsilon}\right) Y_t = \frac{\left(a_{g,t} D(x_t)\left(\varepsilon - 1\right)\right)^{1-\varepsilon} \left(\varepsilon\right)^{\varepsilon+1} \varphi_g}{k_p}.$$

Finally, recalling that $k_p = \varepsilon$, we have

$$\left(1 - \left(\left(\tau_t + (1-\eta)\left(\frac{\tau_t}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x_t) + 1\right)^{1-\varepsilon}\right) Y_t = \left(a_{g,t} D(x_t)\left(\frac{\varepsilon - 1}{\varepsilon}\right)\right)^{1-\varepsilon} \varepsilon \varphi_g. \quad (84)$$

This expression links the endogenous threshold productivity level $a_{g,t}$ to firms' marginal costs (which in the case of r firms, include the carbon tax τ_t and implicitly the abatement rate μ_t) and total output Y_t.[37]

A.5.3 Total Output

Having derived a condition that pins down the optimal amount of labor in each firm category and the threshold idiosyncratic productivity level $a_{g,t}$, we turn to total output which, as we show below, ultimately depends on sectoral labor and $a_{g,t}$.

Using the demand functions $\widetilde{y}_{r,t} = \left(\widetilde{\rho}_{r,t}\right)^{-\varepsilon} Y_t$ and $\widetilde{y}_{g,t} = \left(\widetilde{\rho}_{g,t}\right)^{-\varepsilon} Y_t$, we can write the equilibrium condition $1 = N_{r,t}\left(\widetilde{\rho}_{r,t}\right)^{1-\varepsilon} + N_{g,t}\left(\widetilde{\rho}_{g,t}\right)^{1-\varepsilon}$ as

$$Y_t = \left[N_{r,t}\left(\widetilde{y}_{r,t}\right)^{\frac{\varepsilon-1}{\varepsilon}} + N_{g,t}\left(\widetilde{y}_{g,t}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Then, noting that market clearing in the goods market for each firm category is given by

[37] Given that $\varepsilon > 1$ and $\varphi_g > 0$, $\tau_t \to 0$ implies that $a_{g,t}$ becomes larger. Of note, even for values of τ_t that approach 0, $a_{g,t}$ remains finite.

$\widetilde{y}_{r,t} = \frac{\widetilde{a}_{r,t} D(x_t) n_{r,t}}{N_{r,t}}$ and $\widetilde{y}_{g,t} = \frac{\widetilde{a}_{g,t} D(x_t) n_{g,t}}{N_{g,t}}$, we have

$$Y_t = D(x_t) \left[N_{r,t} \left(\frac{\widetilde{a}_{r,t} n_{r,t}}{N_{r,t}} \right)^{\frac{\varepsilon-1}{\varepsilon}} + N_{g,t} \left(\frac{\widetilde{a}_{g,t} n_{g,t}}{N_{g,t}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

or

$$Y_t = D(x_t) \left[N_{r,t}^{\frac{1}{\varepsilon}} \left(\widetilde{a}_{r,t} n_{r,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} + N_{g,t}^{\frac{1}{\varepsilon}} \left(\widetilde{a}_{g,t} n_{g,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Using the conditions $N_{g,t} = \left(\frac{1}{a_{g,t}} \right)^{k_p} N_t$ and $N_{r,t} = \left[1 - \left(\frac{1}{a_{g,t}} \right)^{k_p} \right] N_t$, we can write the above expression as

$$Y_t = D(x_t) \left[\left(\left(1 - \left(\frac{1}{a_{g,t}} \right)^{k_p} \right) N_t \right)^{\frac{1}{\varepsilon}} \left(\widetilde{a}_{r,t} n_{r,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\left(\frac{1}{a_{g,t}} \right)^{k_p} N_t \right)^{\frac{1}{\varepsilon}} \left(\widetilde{a}_{g,t} n_{g,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

or

$$Y_t = D(x_t) N_t^{\frac{1}{\varepsilon-1}} \left[\left(1 - \frac{1}{a_{g,t}^{k_p}} \right)^{\frac{1}{\varepsilon}} \left(\widetilde{a}_{r,t} n_{r,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{1}{a_{g,t}^{k_p}} \right)^{\frac{1}{\varepsilon}} \left(\widetilde{a}_{g,t} n_{g,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Now, using the fact that average idiosyncratic productivities for each firm category are given by $\widetilde{a}_{g,t} = (k_p)^{\frac{1}{\varepsilon-1}} a_{g,t}$ and $\widetilde{a}_{r,t} = (k_p)^{\frac{1}{\varepsilon-1}} a_{g,t} \left(\frac{a_{g,t} - 1}{a_{g,t}^{k_p} - 1} \right)^{\frac{1}{\varepsilon-1}}$, and after a few steps of algebra, we can write

$$Y_t = D(x_t) \left(k_p N_t \right)^{\frac{1}{\varepsilon-1}} \left[\left(\alpha_{y,t} \right)^{\frac{1}{\varepsilon}} \left(H(n_{r,t}) \right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(1 - \alpha_{y,t} \right)^{\frac{1}{\varepsilon}} \left(F(n_{g,t}) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{85}$$

where $\alpha_{y,t} \equiv (1 - 1/a_{g,t})$ in the absence of physical capital, $H(n_{r,t}) = n_{r,t}$ and $F(n_{g,t}) = n_{g,t}$. Expression (99) shows that total output depends on the damages function $D(x_t)$,the total number of firms N_t, the endogenous threshold productivity level $a_{g,t}$,and labor in each firm category, $n_{r,t}$ and $n_{g,t}$.

A.5.4 Equilibrium Effects of Carbon Taxes

Recall that we want to determine the circumstances under which the carbon tax τ_t increases total output, and what the role of technology adoption is in shaping the response of total output. To do so, consider the following equilibrium conditions of the simplified model in the steady state:

$$n_g = \frac{N}{D(x)} k_p \left(\frac{1}{a_g}\right) \left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x)}\right)^{-\varepsilon} Y,$$

$$n_r = \frac{N}{D(x)} k_p \left(1-\frac{1}{a_g}\right) \left(\frac{\varepsilon}{\varepsilon-1}\frac{\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1}{D(x)}\right)^{-\varepsilon} Y.$$

and

$$\left(1-\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)^{1-\varepsilon}\right)Y = \left(a_g D(x)\left(\frac{\varepsilon-1}{\varepsilon}\right)\right)^{1-\varepsilon}\varepsilon\varphi_g.$$

Thus, we can write the ratio of sectoral employment as

$$\Theta_{g,r} \equiv \frac{n_g}{n_r} = \frac{\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)^{\varepsilon}}{(a_g-1)}. \tag{86}$$

In turn, note that using $n_g = \frac{N_t}{D(x)} k_p \left(\frac{1}{a_g}\right)\left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x)}\right)^{-\varepsilon} Y$, the condition that pins down a_g can be expressed as

$$\left(1-\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)^{1-\varepsilon}\right) = \frac{(a_g)^{-\varepsilon}}{n_g}\frac{k_p N\left(D(x)\left(\frac{\varepsilon-1}{\varepsilon}\right)\right)^{1-\varepsilon}\varepsilon\varphi_g}{D(x)\left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x)}\right)^{\varepsilon}}. \tag{87}$$

In what follows, we continue to assume that $D(x)$ is fixed. Moreover, to clarify the role of technology adoption, we further assume that the total number of firms N is constant (i.e., firm entry is fixed). Allowing both $D(x)$ and N to adjust in response to changes in the carbon tax cloud make the analytical expressions we derive more complex without fundamentally altering the main mechanisms and findings that we describe when we assume that $D(x)$ and N as constants.

A.5.5 Equilibrium Effects of Carbon Tax on Sectoral Labor and Technology Adoption

First, focusing on the ratio $\Theta_{g,r} \equiv \frac{n_g}{n_r}$, take the total derivative of condition (86), which yields

$$
\begin{aligned}
d\Theta_{g,r} = & \frac{\varepsilon\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)^{\varepsilon}}{(a_g-1)\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)}\left(1-\left(\frac{\tau}{\eta}\right)^{\frac{1}{\eta-1}}\right)D(x)d\tau \\
& -\frac{\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)^{\varepsilon}}{(a_g-1)^2}da_g.
\end{aligned}
$$

Dividing both sides by $d\tau$, we have

$$
\begin{aligned}
\frac{d\Theta_{g,r}}{d\tau} = & \frac{\varepsilon\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)^{\varepsilon}}{(a_g-1)\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)}\left(1-\left(\frac{\tau}{\eta}\right)^{\frac{1}{\eta-1}}\right)D(x) \\
& -\frac{\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)^{\varepsilon}}{(a_g-1)^2}\frac{da_g}{d\tau},
\end{aligned}
$$

which we can write more succinctly as

$$
\frac{d\Theta_{g,r}}{d\tau} = \Psi_\tau + \Psi_{a_g}\frac{da_g}{d\tau}, \tag{88}
$$

where, making use of the fact that $\Theta_{g,r} = \frac{\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)^{\varepsilon}}{(a_g-1)}$,we have

$$
\Psi_\tau \equiv \frac{\varepsilon\Theta_{g,r}\left(1-\left(\frac{\tau}{\eta}\right)^{\frac{1}{\eta-1}}\right)D(x)}{\left(\left(\tau+(1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right)D(x)+1\right)}. \tag{89}
$$

Note that $\Psi_\tau > 0$ since the optimal abatement rate $0 \leq \mu < 1$ is given by $\mu = \left(\frac{\tau}{\eta}\right)^{\frac{1}{\eta-1}}$ with $\eta > 1$, implying that $0 \leq \left(1 - \left(\frac{\tau}{\eta}\right)^{\frac{1}{\eta-1}}\right) < 1$. In turn, we have

$$\Psi_{a_g} \equiv -\frac{\Theta_{g,r}}{(a_g - 1)} < 0, \tag{90}$$

since $a_g > 1$ given that $a_{min} = 1$ and $\varphi_g > 0$.

Now, take the total derivative of condition 87, which yields

$$\begin{aligned} -(1-\varepsilon)\left(\left(\tau + (1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x) + 1\right)^{-\varepsilon} n_g \left(1 - \left(\frac{\tau}{\eta}\right)^{\frac{1}{\eta-1}}\right) D(x) d\tau \\ + \left(1 - \left(\left(\tau + (1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x) + 1\right)^{1-\varepsilon}\right) \frac{\partial n_g}{\partial \Theta_{g,r}} d\Theta_{g,r} \\ = -\varepsilon \left(a_g\right)^{-\varepsilon-1} \frac{k_p N \left(D(x)\left(\frac{\varepsilon-1}{\varepsilon}\right)\right)^{1-\varepsilon} \varepsilon \varphi_g}{D(x)\left(\frac{\varepsilon}{\varepsilon-1}\frac{1}{D(x)}\right)^{\varepsilon}} da_g, \end{aligned}$$

where note that $\frac{\partial n_g}{\partial \Theta_{g,r}} = n_r$. We can rewrite this as

$$\frac{da_g}{d\tau} = \Omega_\tau + \Omega_{\Theta_{g,r}} \frac{d\Theta_{g,r}}{d\tau}, \tag{91}$$

where, using the condition that pins down a_g, we define

$$\Omega_\tau \equiv \frac{a_g \left(\frac{1-\varepsilon}{\varepsilon}\right) \left(\left(\tau + (1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x) + 1\right)^{-\varepsilon} \left(1 - \left(\frac{\tau}{\eta}\right)^{\frac{1}{\eta-1}}\right) D(x)}{\left(1 - \left(\left(\tau + (1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x) + 1\right)^{1-\varepsilon}\right)} < 0, \tag{92}$$

and

$$\Omega_{\Theta_{g,r}} \equiv -\frac{n_r}{n_g}\frac{a_g}{\varepsilon} < 0. \tag{93}$$

Then, going back to condition (88) and inserting condition (91), we have

$$\frac{d\Theta_{g,r}}{d\tau} = \Psi_\tau + \Psi_{a_g} \overbrace{\left(\Omega_\tau + \Omega_{\Theta_{g,r}} \frac{d\Theta_{g,r}}{d\tau}\right)}^{=\frac{da_g}{d\tau}},$$

or

$$\frac{d\Theta_{g,r}}{d\tau} = \frac{\Psi_\tau + \Psi_{a_g}\Omega_\tau}{\left(1 - \Psi_{a_g}\Omega_{\Theta_{g,r}}\right)}. \tag{94}$$

Note that in the absence of endogenous green technology adoption, $\Psi_{a_g} = 0$ and therefore $\frac{d\Theta_{g,r}}{d\tau} = \Psi_\tau > 0$. With endogenous technology adoption, we can show that

$$\left(1 - \Psi_{a_g}\Omega_{\Theta_{g,r}}\right) > 0,$$

as long as $a_g > \frac{\varepsilon}{(\varepsilon-1)}$, which holds for empirically plausible calibrations of the model, and[38]

$$\Psi_\tau + \Psi_{a_g}\Omega_\tau > 0,$$

so that

$$\frac{d\Theta_{g,r}}{d\tau} = \frac{\Psi_\tau + \Psi_{a_g}\Omega_\tau}{\left(1 - \Psi_{a_g}\Omega_{\Theta_{g,r}}\right)} > 0. \tag{95}$$

Finally, we can show that

$$\frac{da_g}{d\tau} = \Omega_\tau + \Omega_{\Theta_{g,r}} \left(\frac{\Psi_\tau + \Psi_{a_g}\Omega_\tau}{1 - \Psi_{a_g}\Omega_{\Theta_{g,r}}}\right) < 0, \tag{96}$$

as long as $a_g > \frac{\varepsilon}{(\varepsilon-1)}$ and $\left(\left(\tau + (1-\eta)\left(\frac{\tau}{\eta}\right)^{\frac{\eta}{\eta-1}}\right) D(x) + 1\right)^{1-\varepsilon} < \varepsilon$, where these conditions hold under empirically plausible calibrations of the model and for $\tau \geq 0$. Therefore, conditions (88) and (91) show that the carbon tax increases the ratio of g-category labor to r-category labor, $\Theta_{g,r}$, and lowers the threshold productivity level above which firms adopt the green technology, a_g.

[38] The condition $a_g > \frac{\varepsilon}{(\varepsilon-1)}$ emerges in an environment where, for simplicity only, we have assumed that $k_p = \varepsilon$ (recall that this assumption satisfies the standard assumption of $k_p > \varepsilon - 1$ in the literature). Letting $k_p \neq \varepsilon$ (while continuing to assume that $k_p > \varepsilon - 1$) implies that $\left(1 - \Psi_{a_g}\Omega_{\Theta_{g,r}}\right) > 0$ as long as $a_g > 1$, which is the case for any empirically plausible calibration of the model where, without loss of generality and following the literature, we assume that $a_{min} = 1$.

A.5.6 Equilibrium Impact of Carbon Tax on Total Output

We have just shown that $\frac{d\Theta_{g,r}}{d\tau} > 0$ and $\frac{da_g}{d\tau} < 0$. With these results in mind and recalling our simplifying assumptions that $D(x)$ and N are constants, consider the expression for steady-state total output, Y:

$$Y = D(x)\,(k_p N)^{\frac{1}{\varepsilon-1}} \left[\left(1-\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_r)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_g)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Take the total derivative of Y to obtain

$$\frac{dY}{d\tau} = \frac{\partial Y}{\partial n_g}\frac{dn_g}{d\Theta_{g,r}}\frac{d\Theta_{g,r}}{d\tau} + \frac{\partial Y}{\partial n_r}\frac{dn_r}{d\Theta_{g,r}}\frac{d\Theta_{g,r}}{d\tau} + \frac{\partial Y}{\partial a_g}\frac{da_g}{d\tau},$$

or, combining common terms,

$$\frac{dY}{d\tau} = \left(\frac{\partial Y}{\partial n_g}\frac{dn_g}{d\Theta_{g,r}} + \frac{\partial Y}{\partial n_r}\frac{dn_r}{d\Theta_{g,r}}\right)\frac{d\Theta_{g,r}}{d\tau} + \frac{\partial Y}{\partial a_g}\frac{da_g}{d\tau}.$$

First, note that

$$\begin{aligned}\frac{\partial Y}{\partial n_g} =& \frac{Y}{n_g}\frac{\left(\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_g)^{\frac{\varepsilon-1}{\varepsilon}}}{\left(1-\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_r)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_g)^{\frac{\varepsilon-1}{\varepsilon}}} \\ =& \epsilon_{Y,g}\frac{Y}{n_g} > 0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial Y}{\partial n_r} =& \frac{Y}{n_r}\frac{\left(1-\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_r)^{\frac{\varepsilon-1}{\varepsilon}}}{\left(1-\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_r)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_g)^{\frac{\varepsilon-1}{\varepsilon}}} \\ =& \epsilon_{Y,r}\frac{Y}{n_r} > 0,\end{aligned}$$

where $\epsilon_{Y,g} \equiv \frac{\left(\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_g)^{\frac{\varepsilon-1}{\varepsilon}}}{\left(1-\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_r)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_g)^{\frac{\varepsilon-1}{\varepsilon}}} \in (0,1)$ and $\epsilon_{Y,r} \equiv \frac{\left(1-\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_r)^{\frac{\varepsilon-1}{\varepsilon}}}{\left(1-\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_r)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_g)^{\frac{\varepsilon-1}{\varepsilon}}} \in (0,1)$ represent the elasticities of total output with respect to g-category and r-category output, respectively (recall that in the simplified version of our benchmark model, each output category is linear in its respective labor). We can also show that

$$\frac{\partial Y}{\partial a_g} = \frac{\left(\frac{1}{\varepsilon-1}\right)\frac{Y}{(a_g)^2}\left(\left(1-\frac{1}{a_g}\right)^{\frac{1-\varepsilon}{\varepsilon}}(n_r)^{\frac{\varepsilon-1}{\varepsilon}} - \left(\frac{1}{a_g}\right)^{\frac{1-\varepsilon}{\varepsilon}}(n_g)^{\frac{\varepsilon-1}{\varepsilon}}\right)}{\left(1-\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}}(n_r)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}}(n_g)^{\frac{\varepsilon-1}{\varepsilon}}} < 0,$$

as long as $\frac{1}{(a_g-1)} < \frac{n_g}{n_r}$, where $a_g > 1$. Given the mapping between the two firm categories in the model and the data, the condition $\frac{1}{(a_g-1)} < \frac{n_g}{n_r}$ always holds under ratios of g-category labor to r-category labor, $\frac{n_g}{n_r}$,that are empirically consistent. Finally, using the fact that $\Theta_{g,r} \equiv \frac{n_g}{n_r}$ and the labor-market constraint $1 = n_r + n_g$, we can show that $\frac{dn_g}{d\Theta_{g,r}} = \frac{n_r}{\left(1+\frac{n_g}{n_r}\right)} = \frac{(n_r)^2}{(n_r+n_g)} > 0$ and $\frac{dn_r}{d\Theta_{g,r}} = -\frac{(n_r)^2}{(n_r+n_g)} < 0$.

With the above results in mind and after a few steps of algebra, we can write

$$\frac{dY}{d\tau} = \left(\epsilon_{Y,g}\frac{n_r}{n_g} - \epsilon_{Y,r}\right)\left(\frac{n_r}{n_r+n_g}\right)Y\frac{d\Theta_{g,r}}{d\tau} + \frac{\partial Y}{\partial a_g}\frac{da_g}{d\tau}.$$

Recall that per our simplifying assumptions, leisure is fixed and normalized to zero and households have a time endowment of 1, all of which implies that $n_r + n_g = 1$. Therefore, the term $\left(\frac{n_r}{n_r+n_g}\right)$ simply becomes n_r. In addition, given the properties of aggregation of firm-category output into total output, $\epsilon_{Y,g} + \epsilon_{Y,r} = 1$. Then, it follows that we can write

$$\left(\epsilon_{Y,g}\frac{n_r}{n_g} - \epsilon_{Y,r}\right) = \left((1-\epsilon_{Y,r})\frac{n_r}{n_g} - \epsilon_{Y,r}\right) = \left(\frac{n_r}{n_g} - \left(\frac{n_g+n_r}{n_g}\right)\epsilon_{Y,r}\right) = \left(\frac{n_r-\epsilon_{Y,r}}{n_g}\right). \quad (97)$$

Finally, using the fact that $\Theta_{g,r} \equiv \frac{n_g}{n_r}$, we can rewrite the expression for $\frac{dY}{d\tau}$ as

$$\frac{dY}{d\tau} = (R_r - \epsilon_{Y,r})\frac{Y}{\Theta_{g,r}}\frac{d\Theta_{g,r}}{d\tau} + \frac{\partial Y}{\partial a_g}\frac{da_g}{d\tau}, \quad (98)$$

where $R_r \equiv \left(\frac{n_r}{n_r+n_g}\right) = n_r$ since $n_r + n_g = 1$. The term $(R_r - \epsilon_{Y,r})$ plays a key role in determining the net effect of the carbon tax on total output. In particular, we can show that $(R_r - \epsilon_{Y,r}) < 0$ holds if $\frac{1}{(a_g-1)} < \frac{n_g}{n_r}$ where $a_g > 1$, which is the same condition that allows us to determine that, in a calibration of the relative allocation of labor across firm categories that is consistent with the data, $\frac{\partial Y}{\partial a_g} < 0$. All told, we can show that

$$\frac{dY}{d\tau} = \underbrace{\overbrace{\overbrace{(R_r - \epsilon_{Y,r})}^{<0} \overbrace{\frac{Y}{\Theta_{g,r}} \frac{d\Theta_{g,r}}{d\tau}}^{>0}}^{<0}}_{Input\ Reallocation\ Effect} + \underbrace{\overbrace{\overbrace{\frac{\partial Y}{\partial a_g}}^{<0} \overbrace{\frac{da_g}{d\tau}}^{<0}}^{>0}}_{Tech.\ Composition\ Effect} \tag{99}$$

where recall that $\epsilon_{Y,r} = \frac{\left(1-\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_r)^{\frac{\varepsilon-1}{\varepsilon}}}{\left(1-\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_r)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{1}{a_g}\right)^{\frac{1}{\varepsilon}} (n_g)^{\frac{\varepsilon-1}{\varepsilon}}}$. Thus, $\epsilon_{Y,r}$ depends fundamentally on the contribution of r-firm output—which in this simplified model depends on r labor—to total output. Expression (99) explicitly identifies two central and potentially opposing forces that shape the equilibrium impact of the carbon tax on total output. The first force operates via a *input reallocation effect*: this force captures the adverse effect of the tax on total output via changes in the reallocation of labor across firm categories, and the resulting effects on firms' marginal productivities. The second force operates via a *technological composition effect*: this force captures the positive effect of the tax on output via the endogenous shift in the technological composition of total output towards the technology used by g firms. Critically, the *technological composition effect* is absent in a more standard environment where firms are unable to switch production technologies in response to the carbon tax. That is, absent endogenous technology adoption, $\frac{\partial Y}{\partial a_g} = \frac{da_g}{d\tau} = 0$, and therefore $\frac{dY}{d\tau} < 0$.

A.6 Transitional Dynamics and Model Mechanisms

A.6.1 Discussion of Transitional Dynamics

Recall that column (1) of Table 2 in the main text showed that a higher carbon tax generates higher steady-state consumption and output, and marginally higher unemployment. Intuitively, as the tax on emissions increases gradually, r firms respond by devoting more resources to abatement. Both the carbon tax and the additional resources devoted to abatement reduce r firms' marginal benefit to having a worker and accumulating capital. As a result, r firms use less capital, post fewer vacancies and hire fewer workers, which leads to a reduction in r employment and r output. All told, both the increase in abatement and the reduction in r-firm output contribute to the reduction in emissions.

At the same time, by increasing the relative cost of using the r technology, the increase in the carbon tax makes it relatively more attractive for r firms to incur the fixed cost of adopting the g technology. As a result, the number (and fraction) of g firms increases. This leads to greater labor demand by g firms and to greater g employment and real wages in equilibrium. Given the presence of labor market frictions, the reallocation of employment away from r and into g firms is accompanied by a gradual but limited increase in the unemployment rate as well as an increase in labor force participation, where the latter is driven by both an increase in g employment and in the mass of g searchers (note that the marginal increase in unemployment is explained by the reallocation of searchers away from jobs in r firms to jobs in g firms). The increase in total output takes time as resources are reallocated towards abatement, the creation of g firms, and capital accumulation for these firms. In contrast, the increase in consumption materializes earlier compared to output. There are two reasons underlying the transition path of consumption. First, carbon-tax revenue is transferred lump-sum to the household, which bolsters household consumption. Second, the incentive to create firms amid a carbon tax is lower, which frees up household resources for consumption that would otherwise be used to cover the resource costs of firm creation.

Similar mechanisms are at play when we abstract from endogenous firm entry, the only difference being the magnitude of the responses with respect to the benchmark model. In particular, the absence of a firm-creation margin implies that in the short- and medium term, more resources are devoted to cover the fixed costs of green technology adoption as opposed to consumption, leading to a larger increase in the share of g firms. Hence the smaller increase in consumption and the larger increase in output as the carbon tax steadily rises. At the same time, the larger expansion in the share of g firms increases the incentive for household members to search for employment in these firms, thereby leading to a larger short-run increase in g searchers and therefore in unemployment. Ultimately, though, recall that the long-term unemployment rate increases by less compared to the benchmark model due to the larger long-term expansion in the share of g firms.

Finally, abstracting from both firm entry and technology adoption generates a short- and medium-term contraction in both consumption and output, as well as a more steady and sustained increase in unemployment (even if the increase in absolute terms remains limited given the sizable reduction in emissions). That is, absent these two margins of adjustment, increasing the carbon tax entails both short- and long-term consumption and output costs. This occurs because the only way for r firms to adjust to the carbon tax is via costly abatement since firms cannot substitute away from the technology that, as a result of the tax, has become more expensive to use. Both the tax and abatement increase these firms' marginal costs, leading to a reduction in output that cannot be offset by the increase in output by g firms. Hence the equilibrium reduction in total output.

Table A1: Transition and Steady State Changes in Response to Carbon Tax–Benchmark Model vs. Simpler Model Variants

Variable	**Benchmark Model**	**Benchmark Model**	**No Firm Entry**	**No Firm Entry**	**No Firm Entry, No Tech. Adopt.**	**No Firm Entry, No Tech. Adopt.**
	Steady State	Transition After 20 Quarters	Steady State	Transition After 20 Quarters	Steady State	Transition After 20 Quarters
	Percent Change Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline
Emissions e	-35	-33.631	-35	-33.375	-35	-33.670
Total Output	0.448	0.343	1.061	0.693	-1.021	-0.698
Consumption	0.334	0.294	0.395	0.049	-0.803	-0.256
Empl. r	-18.579	-17.289	-36.813	-34.674	-24.038	-23.050
Empl. g	4.566	4.199	9.108	8.616	5.611	5.194
Firms (N)	-0.979	-0.630	–	–	–	–
g Firms (N_g)	14.315	13.499	–	–	–	–
	Percentage-Pt. Change Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline
Unempl. Rate	0.034	0.051	0.010	0.082	0.178	0.169
LFP Rate	0.340	0.304	0.667	0.696	0.376	0.263
Abate. Rate μ	25.080	24.313	10.782	10.427	21.020	20.370
Tax Rev./Output	0.180	0.175	0.353	0.345	1.265	1.223

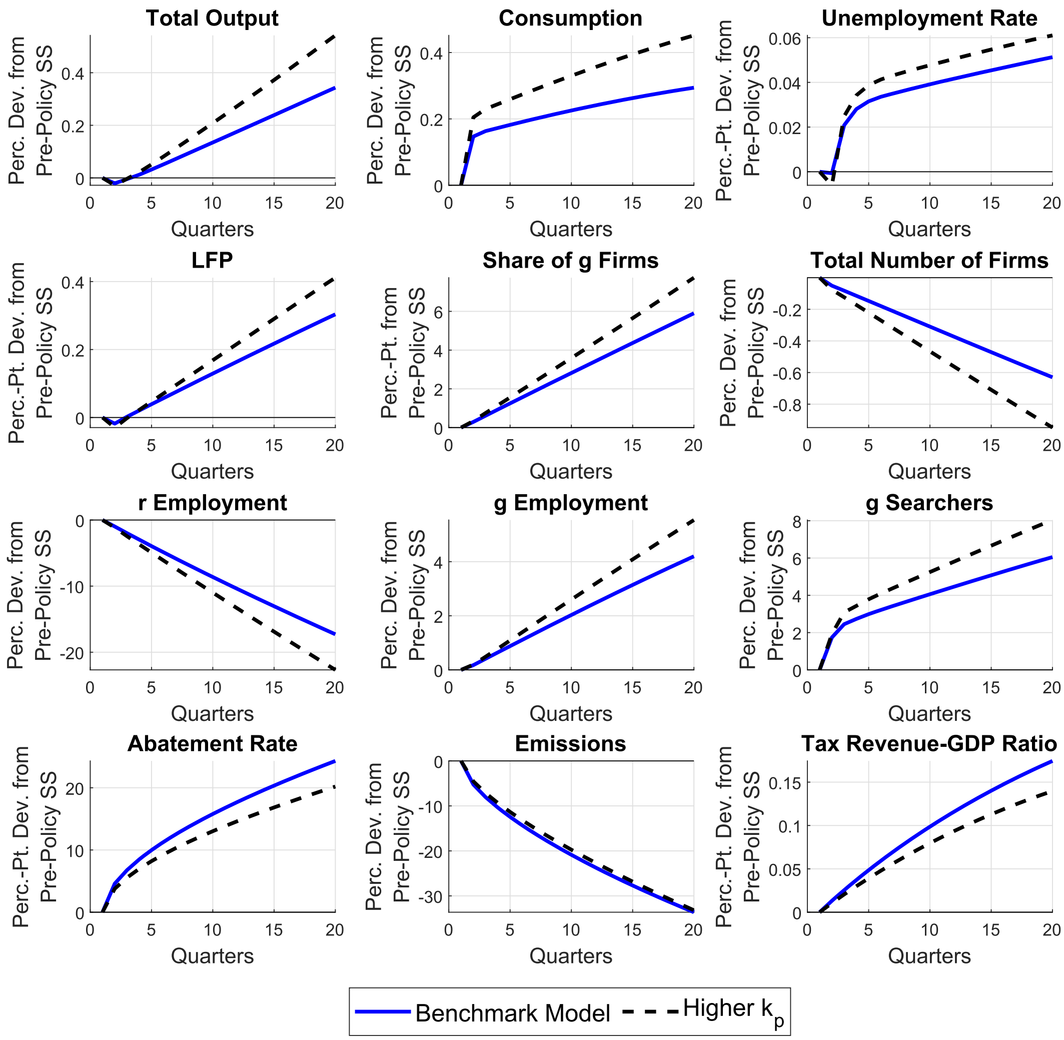
A.7 Robustness Analysis and Additional Results

Table A2 and Figures A1, A2, A3, A4, and A5 below confirm that our main findings in the benchmark model remain unchanged under alternative baseline calibrations. In particular, we consider calibrations where: (1) k_p is higher ($k_p = 5.2$ vs. $k_p = 4.2$ in the benchmark calibration; this reduces the dispersion in productivity draws); (2) lower and higher elasticities of labor force participation ($\phi_n = 0.17$ and $\phi_n = 0.50$ vs. $\phi_n = 0.26$ in the benchmark calibration per Chetty et al., 2013); (3), the job separation rate is lower ($\varrho = 0.05$); (4) the elasticity of emissions-abatement costs with respect to abatement rates is lower ($\eta = 2.2$ vs. $\eta = 2.8$ in the benchmark calibration); and (5) the elasticity of substitution between firm output within each category is higher ($\varepsilon = 4$ vs. $\varepsilon = 3.8$ in the benchmark calibration). Figure A6 presents results for the benchmark model under a more aggressive carbon tax implementation whereby the carbon tax reaches its steady state level in 4 quarters as opposed to 20 quarters. Figure A7 presents results for the benchmark model where the carbon tax revenue is used to subsidize the fixed cost of green technology adoption (column (3) of Table A4 presents steady state results from this alternative assumption regarding tax revenue usage). Finally, Table A3 shows results when we assume that emissions and abatement costs depend on gross rather than net (of damages) output.

Table A2: Steady State Changes in Response to Carbon Tax–Benchmark Model under Alternative Baseline Calibrations

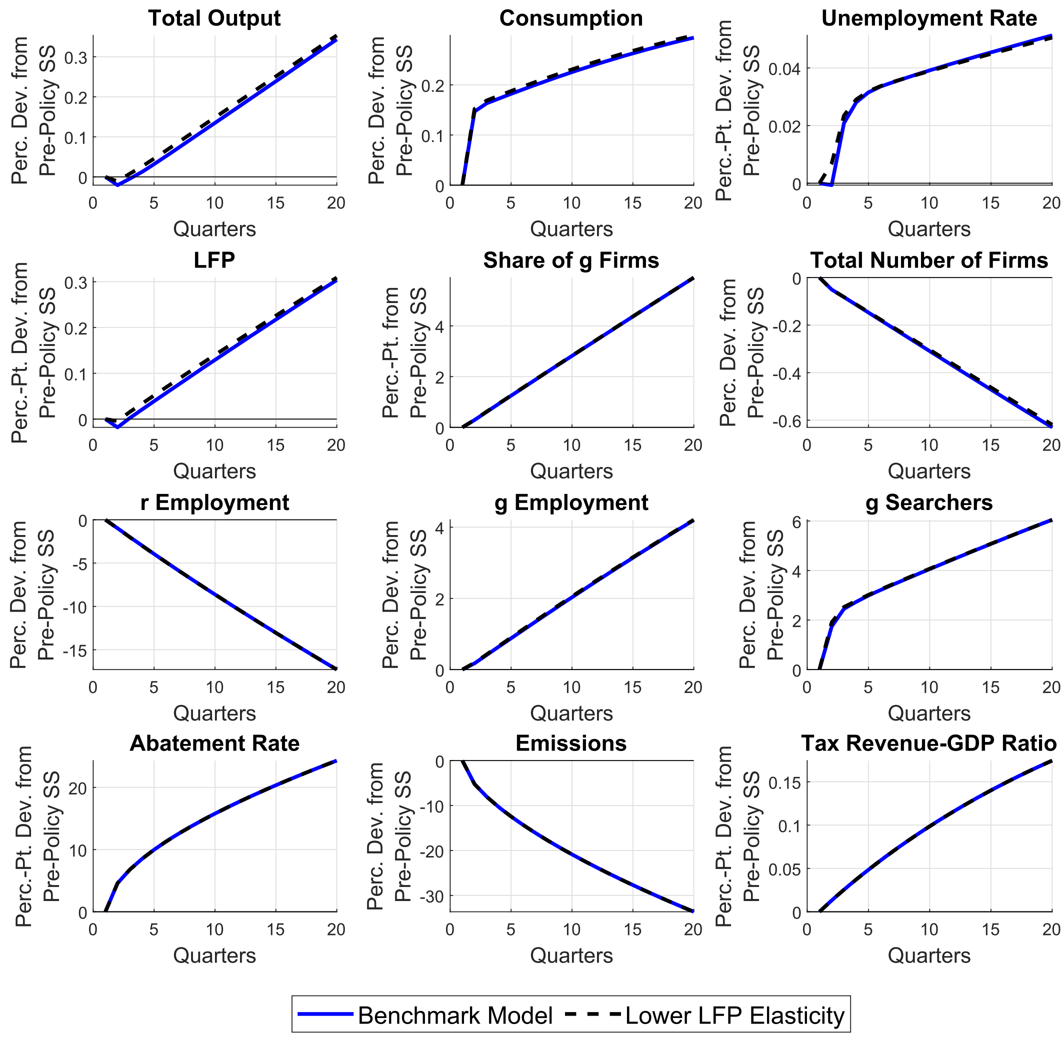
Variable	**Benchmark Model**	**Benchmark Higher** k_p	**Benchmark Lower LFP Elasticity** ϕ_n	**Benchmark Higher LFP Elasticity** ϕ_n	**Benchmark Job Sep. Rate** $\varrho = 0.05$	**Benchmark Lower** $\eta = 2.2$	**Benchmark Higher** ε
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Percent Change Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline
Emissions e	-35	-35	-35	-35	-35	-35	-35
Total Output	0.448	0.657	0.448	0.448	0.460	0.541	0.379
Consumption	0.334	0.483	0.334	0.334	0.337	0.380	0.282
Empl. r	-18.579	-24.747	-18.579	-18.579	-18.574	-26.023	-17.852
Empl. g	4.566	6.121	4.566	4.566	4.576	6.432	4.378
Total Empl.	0.506	0.703	0.503	0.503	0.512	0.735	0.476
Real Wage r	0.313	0.553	0.313	0.313	0.315	0.349	0.218
Real Wage g	0.313	0.553	0.313	0.313	0.415	0.348	0.218
Firms N	-0.979	-1.392	-0.979	-0.980	-0.961	-1.575	-1.012
g Firms N_g	14.315	14.500	14.315	14.314	14.338	20.450	15.455
Welfare Gain (of Consumption)	-0.018	-0.036	-0.018	-0.018	-0.006	-0.217	-0.056
	Percentage-Pt. Change Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline
Unempl. Rate	0.034	0.038	0.034	0.034	0.029	0.058	0.038
LFP Rate	0.340	0.468	0.340	0.339	0.342	0.502	0.326
Abate. Rate μ	25.080	20.874	25.080	25.080	25.085	19.877	25.528
Share of g Firms (N_g/N)	6.427	8.556	6.427	6.427	6.428	9.312	5.866
Tax Rev./Output	0.180	0.143	0.180	0.180	0.180	0.238	0.215

Figure A1: Transitional Dynamics in Response to Carbon Tax–Benchmark Model, Calibration with Higher k_p



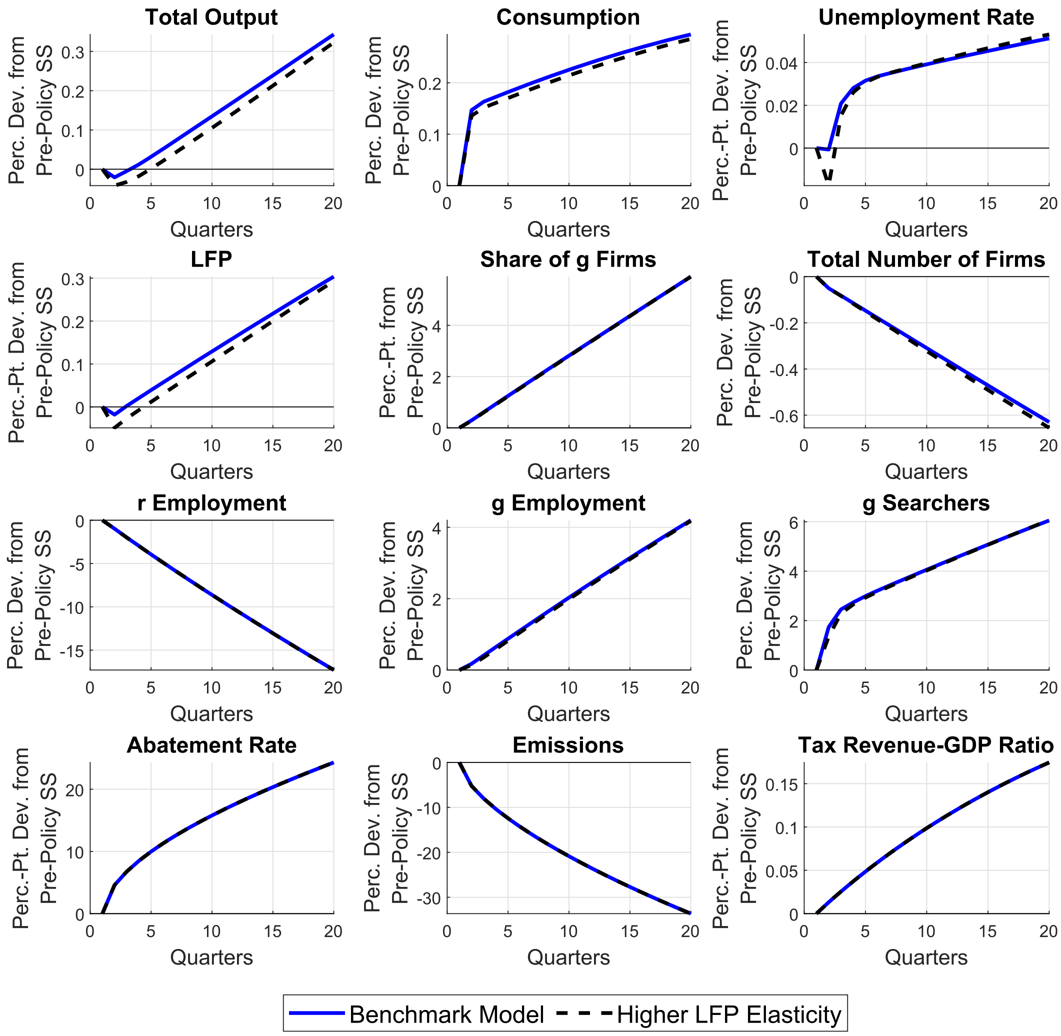
Note: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations.

Figure A2: Transitional Dynamics in Response to Carbon Tax–Benchmark Model, Calibration with Lower Elasticity of Labor Force Participation



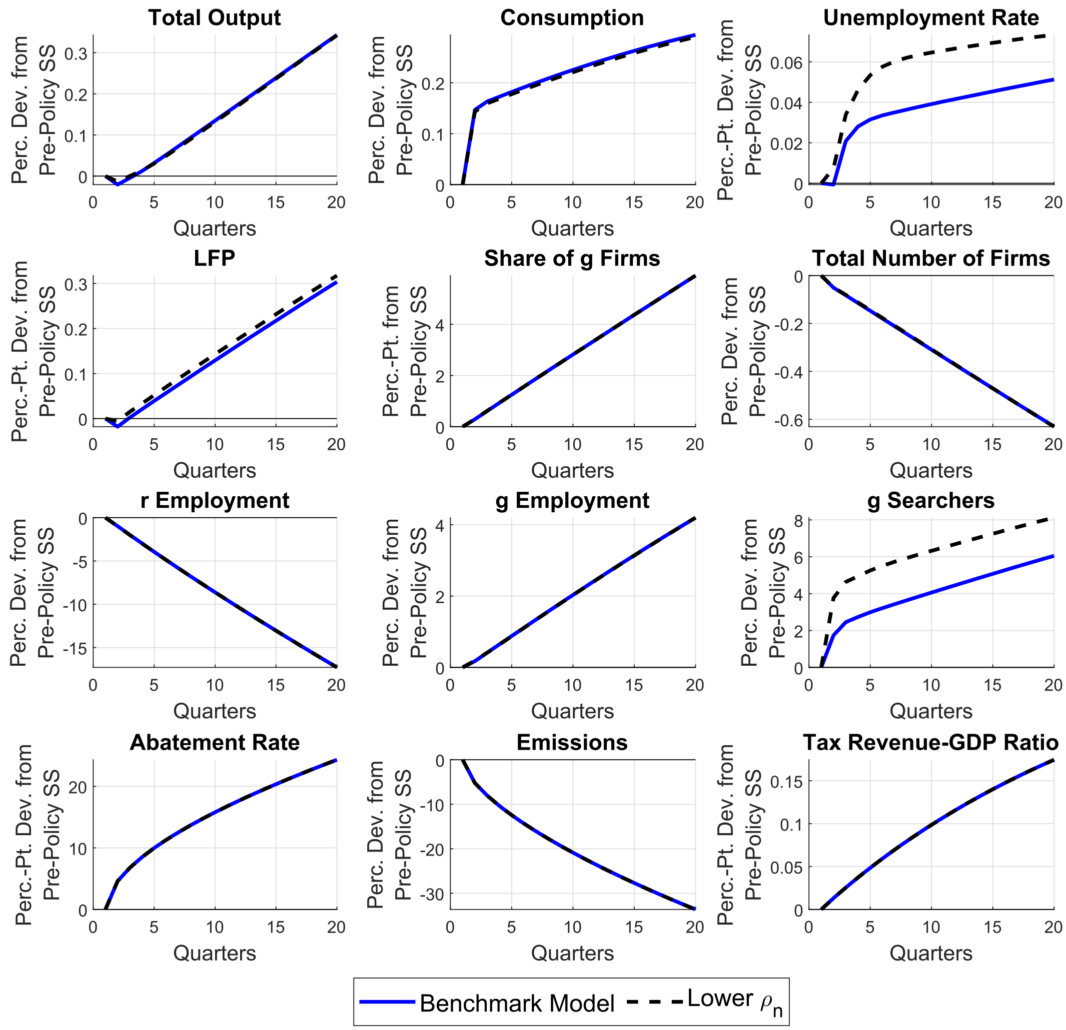
Note: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations.

Figure A3: Transitional Dynamics in Response to Carbon Tax–Benchmark Model, Calibration with Higher Elasticity of Labor Force Participation



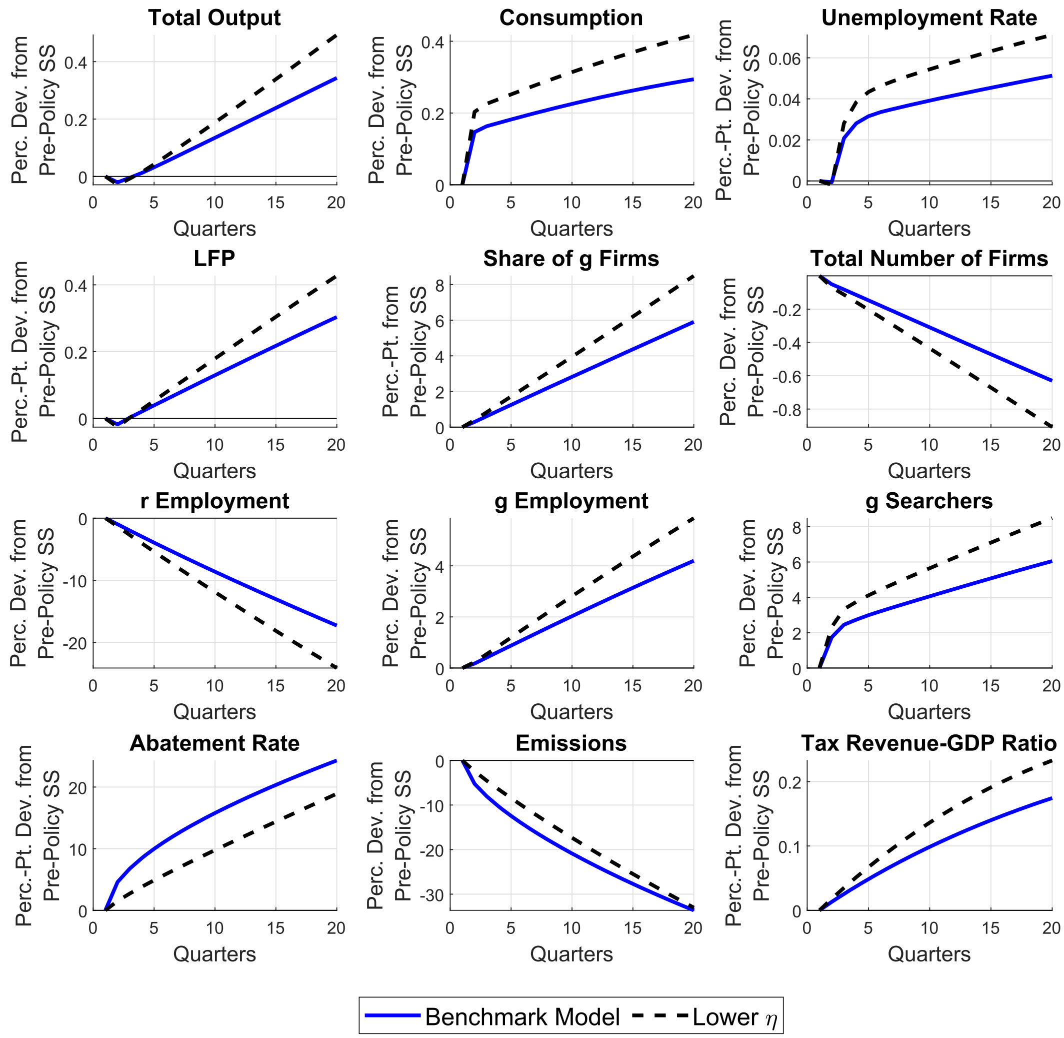
Note: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations.

Figure A4: Transitional Dynamics in Response to Carbon Tax–Benchmark Model, Calibration with Lower Job Separation Rates



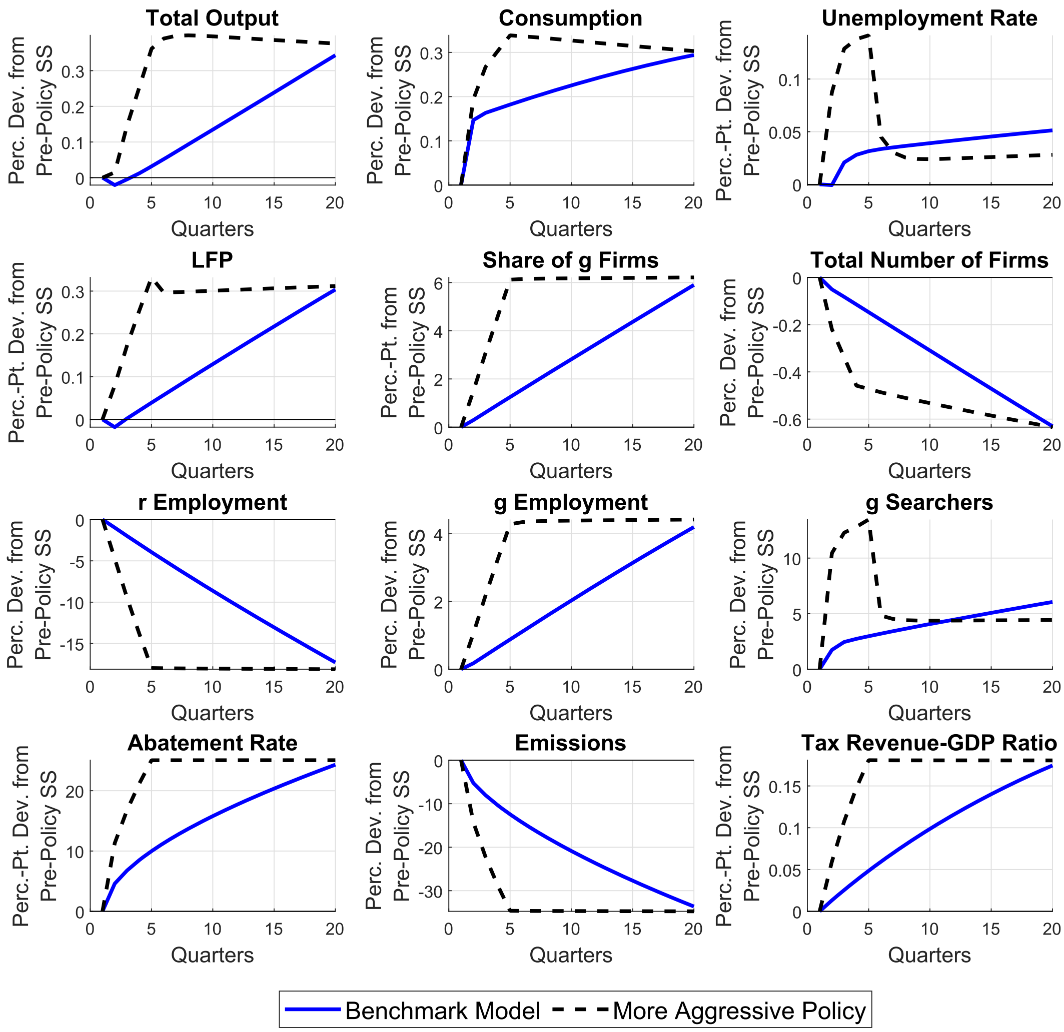
Note: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations.

Figure A5: Transitional Dynamics in Response to Carbon Tax–Benchmark Model, Calibration with Lower Elasticity of Abatement Costs with Respect to Abatement Rate



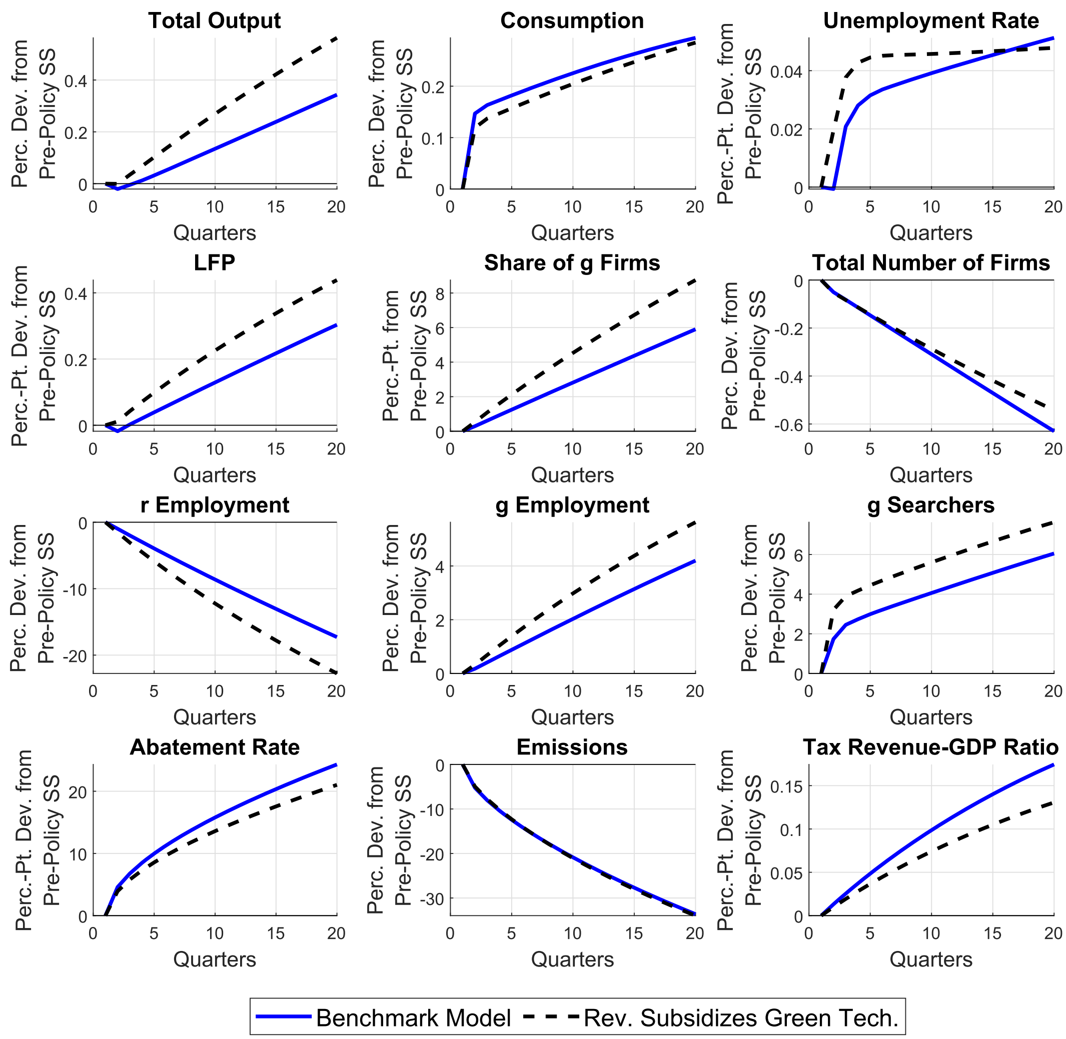
Note: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations.

Figure A6: Transitional Dynamics in Response to Carbon Tax–Benchmark Model, More Aggressive Carbon Tax (Carbon Tax Increase Over 4 Quarters) Policy



Note: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations.

Figure A7: Transitional Dynamics in Response to Carbon Tax–Benchmark Model, Tax Revenue Subsidizes Fixed Cost of Green Technology Adoption



Note: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations.

Net Output in Emissions and Abatement Costs Following Heutel (2012) and others in the literature, our baseline model assumes that emissions and abatement costs are inclusive of pollution damages $D(x_t)$ and given by $e_t = (1-\mu_t)\left[D(x_t)H(n_{r,t},k_{r,t})\right]^{1-\nu}$ and $\Gamma_t = \gamma\mu_t^{\eta}D(x_t)H(n_{r,t},k_{r,t})$, respectively. The DICE model (Nordhaus, 2008), in contrast, makes emissions and abatement functions of gross output. As shown in Table A3 below, assuming that emissions and abatement costs depend on gross r-firm output and are instead given by $e_t = (1-\mu_t)\left[H(n_{r,t},k_{r,t})\right]^{1-\nu}$ and $\Gamma_t = \gamma\mu_t^{\eta}H(n_{r,t},k_{r,t})$ does not change our main findings.

Table A3: Steady State Changes in Response to Carbon Tax–Benchmark Model vs. Model with Gross Output in Emissions and Abatement Costs

Variable	**Benchmark Model**	**Benchmark Model Gross Output in** e**,** Γ
	(1)	(2)
	Percent Change Rel. to Baseline	**Percent Change** Rel. to Baseline
Emissions e	-35	-35
Total Output	0.448	0.460
Consumption	0.334	0.337
Empl. r	-18.579	-18.572
Empl. g	4.566	4.575
Total Empl.	0.503	0.512
Real Wage r	0.313	0.315
Real Wage g	0.313	0.315
Firms (N)	-0.979	-0.961
g Firms (N_g)	14.315	14.336
Welfare Gain (% of Consumption)	-0.018	-0.006
	Percentage-Pt. Change Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline
Unempl. Rate	0.034	0.029
LFP Rate	0.340	0.342
Abate. Rate μ	25.080	25.003
Share of g Firms (N_g/N)	6.427	6.427
Tax Rev./Output	0.180	0.180

Alternative Values for Parameter k_p Recall that we assume that the idiosyncratic productivity of firms is drawn from a Pareto distribution $G(a) = \left[1 - (a_{\min}/a)^{k_p}\right]$ with shape parameter $k_p > \varepsilon - 1$. Following the macro literature on endogenous firm entry (Bilbiie, Ghironi, and Melitz, 2012), we choose $\varepsilon = 3.8$. In turn, as a baseline, we choose $k_p = 4.2$ (for a similar value in a context of production offshoring decisions, see Zlate, 2016).

The value of k_p has direct implications on the average firm productivity differential between g and r firms. In particular, values of k_p smaller than $\varepsilon = 3.8$ deliver implausibly large productivity differentials. For example, setting $k_p = 3.4$ implies that average g-firm productivity is more than 50 percent greater than average r-firm productivity. In contrast, the larger is k_p relative to ε, the smaller—and more plausible—is the average firm productivity differential.[39] Importantly, the larger is the value of k_p relative to the value of ε, the smaller is the average firm productivity differential between g and r firms, and the larger are the positive effects of a carbon tax on real wages and output. As such, our baseline results can be seen as a lower bound on the positive effects of a carbon tax on labor market and macroeconomic outcomes.

Abatement Decisions Amid Firm Creation and Green Technology Adoption Table A4 below compares the steady-state outcomes of the benchmark model (column (1) of the table) to those of the same model shutting down the abatement margin (column (2) of the table). Recall that emissions abatement can be interpreted as an *intensive* margin of adjustment to a carbon tax, whereas green technology adoption can be interpreted as an *extensive* margin of adjustment. While the qualitative impact of a carbon tax remains unchanged without the possibility to abate emissions, the quantitative effects are noticeable: absent abatement, r employment would drop by more than 45 percent, the total number of firms would fall by 3.5 percent, and the unemployment rate would increase by 0.13 percentage points. The intuition behind these results is simple: without the ability to abate emissions, higher carbon tax rates are needed to hit the emission reduction target. These higher rates increase the marginal cost of r firms, which puts additional downward pressure on firm profits, further reducing the incentive to create firms compared to an environment where r firms can abate emissions. Surprisingly, despite the larger reduction in the number of firms, the increase in both consumption and output is larger compared to the benchmark model. These two outcomes are solely due to the sharper reallocation of resources towards g firms in the absence of abatement by r firms (note the larger expansion in the number and share of g firms compared to the benchmark model). All told, being able to abate emissions

[39] For example, a value of $k_p = 6$ implies that average g-firm productivity is 25 percent greater than average r-firm productivity, and a value of $k_p = 9$ implies that average g-firm productivity is 15 percent greater than average r-firm productivity.

reduces the sensitivity of the economy to a carbon tax: it limits the consumption and output gains from resource reallocation but, importantly, also contributes to limiting the adverse effects of a carbon tax on unemployment.

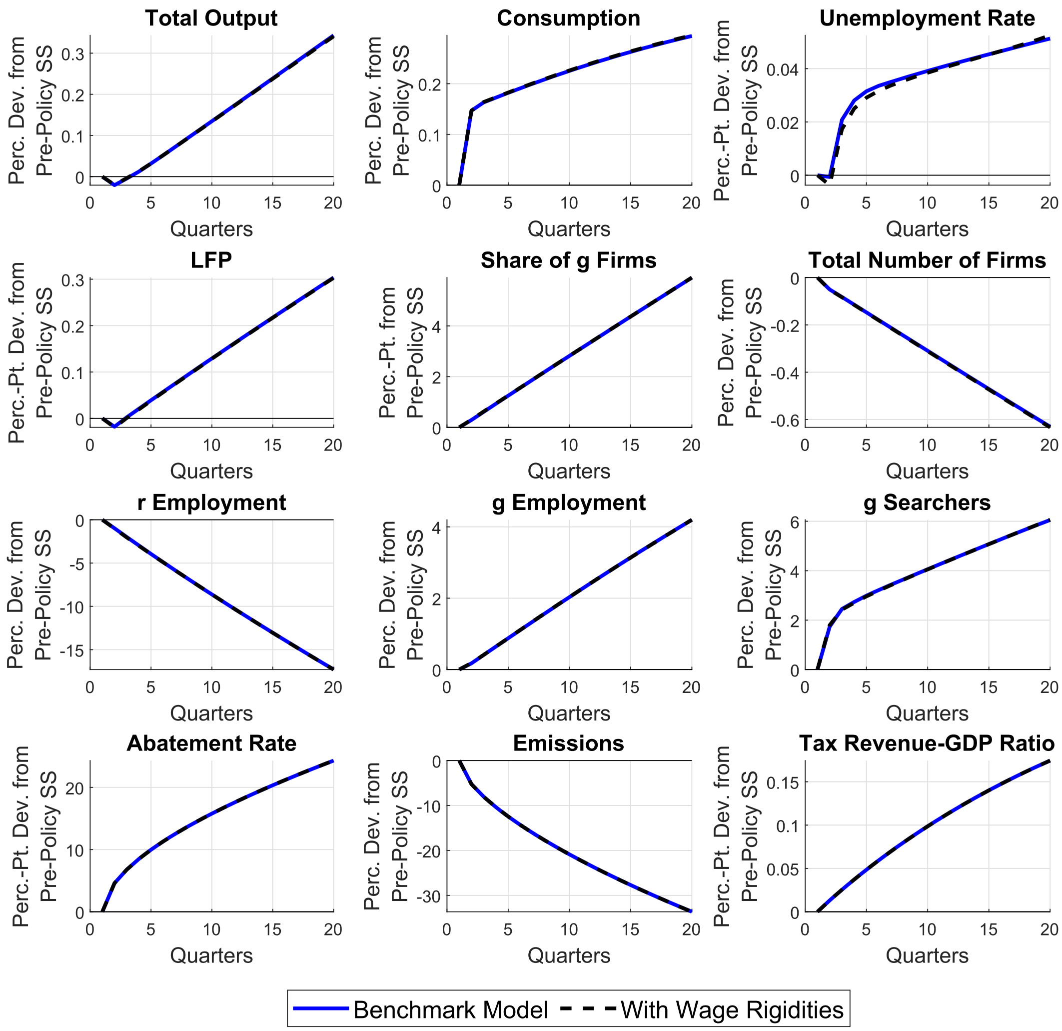
Carbon Tax Revenue Subsidizes Green Technology Adoption Cost Table A4 compares the steady-state outcomes of the benchmark model (column (1) of the table) to those of the same model where we assume that carbon tax revenue is used to subsidize the cost of green technology adoption (column (3) of the table). The positive outcomes in our benchmark model are enhanced as we accelerate the (development and) adoption of green technology. Not surprisingly, a lower carbon tax rate is needed to achieve our targeted emission reduction. These results complement those presented in Figure A7 above.

Productivity Penalty from Green Technology Adoption Table A4 compares the steady-state outcomes of the benchmark model (column (1) of the table) to those of the same model where we assume that, in addition to incurring a fixed cost, firms adopting the green technology also face a productivity penalty (column (5) of the table). Absent empirical evidence on productivity losses from technology adoption, for illustrative purposes, we consider a productivity penalty from green-technology adoption of 5 percent. Columns (4) and (6) provide a check for the concern that lowering emissions and the consequent reduced damages is driving the increase in output in the benchmark model and the No Firm Entry variant of Table 3. In both cases, output still rises even when we fix damages at the baseline level - effectively removing any environmental benefits from reducing emissions.

Table A4: Steady State Changes in Response to Carbon Tax–Benchmark Model vs. Model without Abatement Margin vs. Benchmark Model where Tax Revenue Subsidizes the Fixed Cost of Green Technology Adoption vs. Model without Policy-Induced Change in Damages Function vs. Model with Tech.-Adoption Productivity Penalty

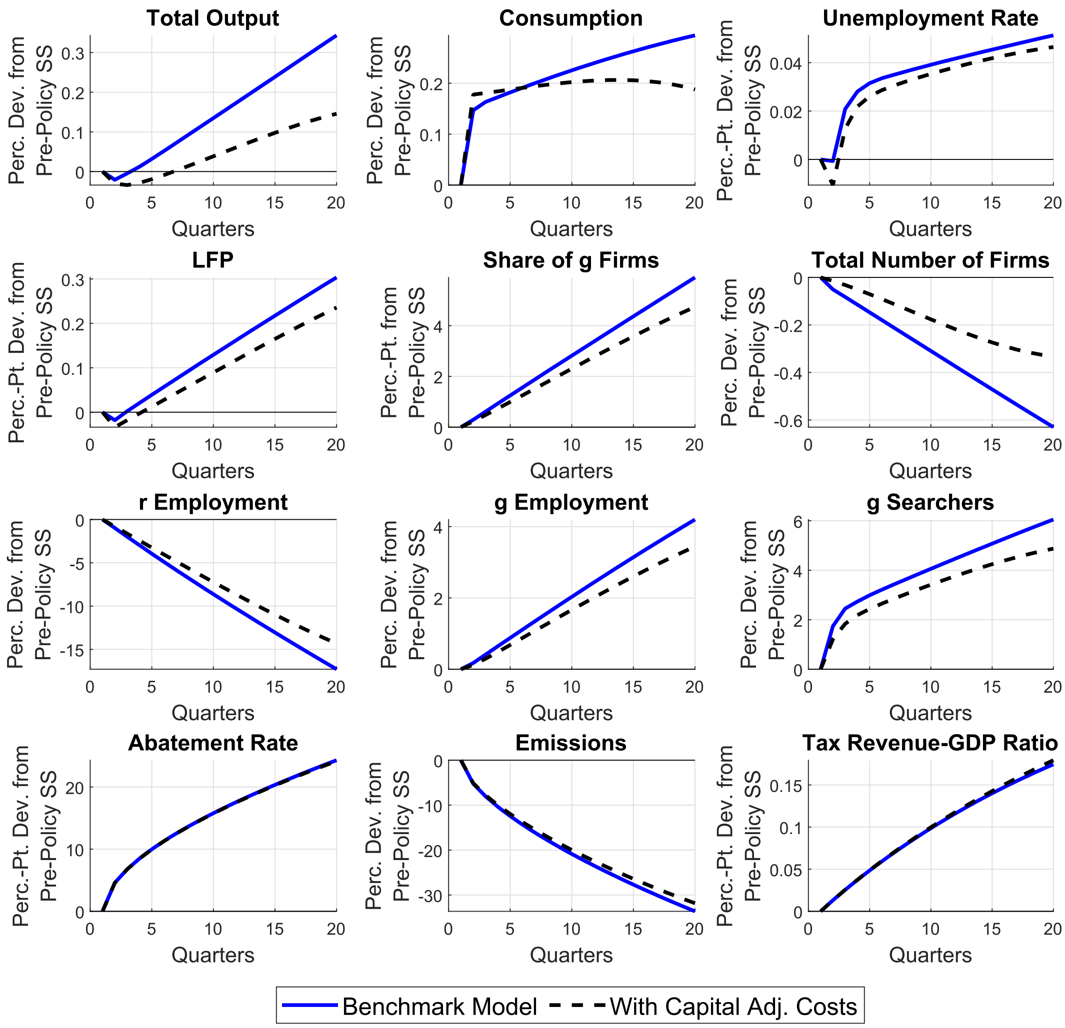
Variable	**Benchmark Model**	**Benchmark Model No Abatement**	**Benchmark Model, Tax Rev. Subsidizes Green Tech. Adopt.**	**Benchmark Model Damages $D(x)$ Held at Baseline**	**Productivity Penalty from Tech. Adoption**	**No Firm Entry $D(x)$ Held at Baseline**
	(1)	(2)	(3)	(4)	(5)	(6)
	Percent Change Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline
Emissions e	-35	-35	-35	-35	-35	-35
Total Output	0.448	0.818	0.795	0.202	0.394	0.847
Consumption	0.334	0.557	0.462	0.101	0.307	0.187
Empl. r	-18.579	-46.107	-23.764	-18.500	-25.825	-36.640
Empl. g	4.566	11.480	5.861	4.613	6.357	9.103
Total Empl.	0.503	1.372	0.661	0.556	0.708	1.073
Real Wage r	0.313	0.518	0.684	0.037	0.341	0.506
Real Wage g	0.313	0.516	0.686	0.035	0.341	0.508
Firms (N)	-0.979	-3.382	-0.363	-1.353	-0.730	–
g Firms (N_g)	14.315	38.874	21.601	13.851	20.854	–
Welfare Gain (% of Consumption)	-0.018	-0.789	0.353	-0.445	0.046	0.167
	Percentage-Pt. Change Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline
Unempl. Rate	0.034	0.126	0.002	0.062	0.037	0.026
LFP Rate	0.340	0.950	0.417	0.392	0.471	0.694
Abate. Rate μ	25.080	–	21.677	24.976	20.081	10.805
Share of g Firms (N_g/N)	6.427	18.200	9.173	6.413	10.034	13.583
Tax Rev./Output	0.180	0.395	0.135	0.179	0.120	0.355

Figure A8: Transitional Dynamics in Response to Carbon Tax–Benchmark Model with Real Wage Rigidities



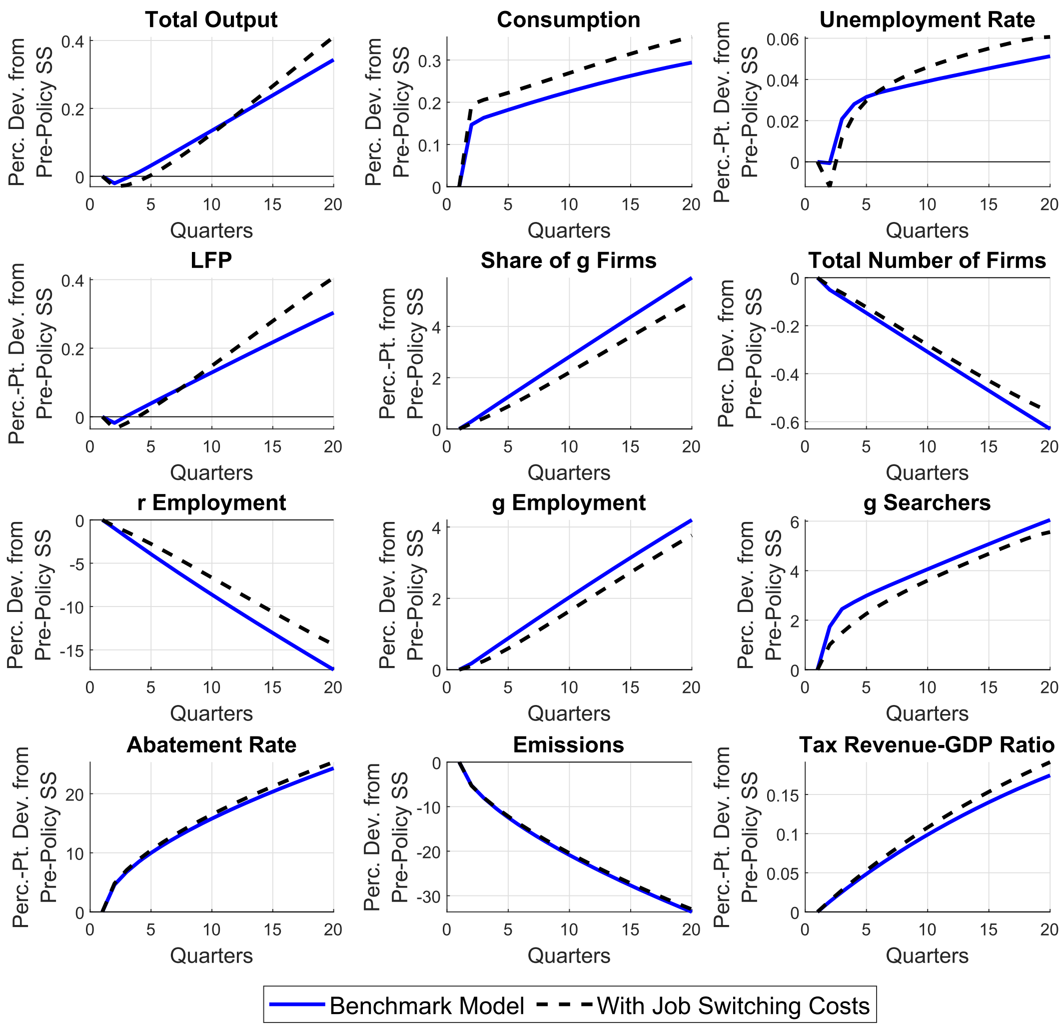
Notes: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations. We introduce wage rigidities by assuming that $w_{j,t} = \left(w_{j,t}^{nash}\right)^{1-\gamma_w} (w_{j,t-1})^{\gamma_w}$ for $j \in \{r, g\}$, and set $\gamma_w = 0.90$ following the literature.

Figure A9: Transitional Dynamics in Response to Carbon Tax–Benchmark Model with Capital Adjustment Costs in Firm-g Category



Notes: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations. The capital adjustment cost function is given by $\frac{\varphi_k}{2}\left(\frac{k_t}{k_{t-1}}-1\right)^2 k_t$. For illustrative purposes, we set $\varphi_k = 2$.

Figure A10: Transitional Dynamics in Response to Carbon Tax–Benchmark Model with Switching Costs for Searching for g Jobs



Notes: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations. For illustrative purposes, we assume that switching costs for g jobs represent 0.5 percent of GDP.

Table A5: Steady State Changes in Response to Carbon Tax–Benchmark Model Specification vs. Alternatives

Variable	**Benchmark Model**	**Convex Firm Creation and Tech. Adoption Costs**	**Lower Worker Bargaining Power** $\nu_n = 0.05$	**Switching Costs of Searching for** g **Jobs**	**Reallocation Costs of Capital for** g **Firms**	**Fixed and Variable Abatement Costs**
	(1)	(2)	(3)	(4)	(5)	(6)
	Percent Change Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline	**Percent Change** Rel. to Baseline
Emissions e	-35	-35	-35	-35	-35	-35
Total Output	0.448	0.384	0.306	0.752	0.341	0.448
Consumption	0.334	0.289	0.308	0.595	0.237	0.335
Empl. r	-18.579	-18.655	-18.678	-16.809	-21.253	-18.621
Empl. g	4.566	4.577	4.460	4.438	5.250	4.576
Total Empl.	0.506	0.499	0.399	0.709	0.598	0.504
Real Wage r	0.313	0.215	0.326	0.105	0.225	0.313
Real Wage g	0.313	0.215	0.325	0.379	0.225	0.313
Firms N	-0.979	-1.023	-1.197	-0.368	-0.981	-0.984
g Firms N_g	14.315	13.530	14.074	13.685	16.610	14.347
Welfare Gain (% of Consumption)	-0.018	-0.080	-0.140	0.628	-0.019	-0.018
	Percentage-Pt. Change Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline
Unempl. Rate	0.034	0.041	0.020	0.014	0.044	0.034
LFP Rate	0.340	0.342	0.265	0.455	0.406	0.340
Abate. Rate μ	25.080	25.012	25.000	26.196	23.302	25.053
Share of g Firms (N_g/N)	6.427	6.119	6.432	5.870	7.810	6.443
Tax Rev./Output	0.180	0.222	0.179	0.195	0.158	0.180

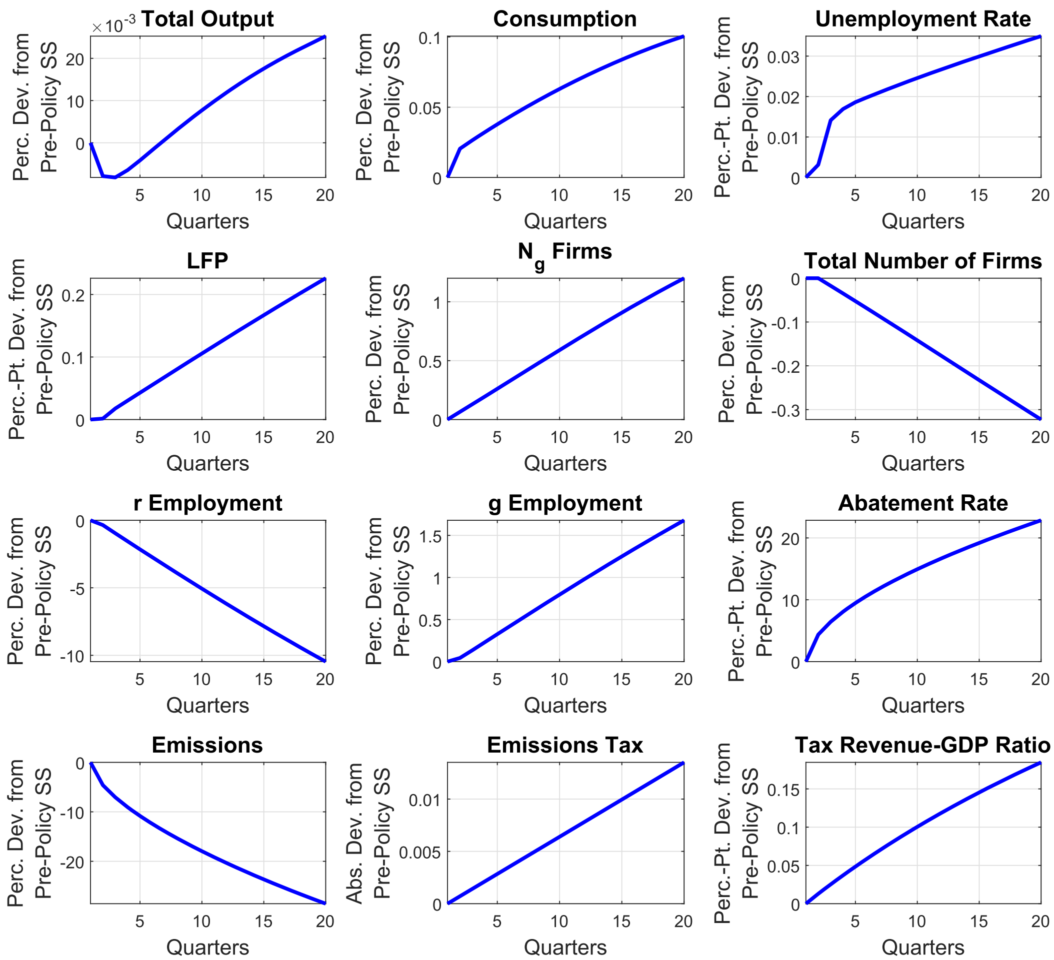
Notes: Results in column (3) are based on a worker bargaining power of $\nu_n = 0.05$ (vs. $\nu_n = 0.50$ in the benchmark calibration). Results in column (4) are based on the assumption that switching costs for g jobs represent 0.5 percent of GDP. Results in column (5) are based on the assumption that capital reallocation costs represent 1 percent of GDP.

Table A6: Steady State Changes in Response to Carbon Tax–Benchmark Model vs. Model with Separate Sectors (r and g), Endogenous Firm Entry in Each Sector, and Green Technology Adoption Margin in r Sector

Variable	**Benchmark Model***	**Two-Sector Model, Green Tech. Adopt. in r Sector**
	(1)	(2)
	Percent Change Rel. to Baseline	**Percent Change** Rel. to Baseline
Emissions e	-35	-35
Total Output	0.384	0.314
Consumption	0.289	0.396
Empl. r	-18.655	-17.093
Empl. g	4.577	4.241
Total Empl.	0.499	0.726
Real Wage r	0.215	0.254
Real Wage g	0.215	0.253
Firms (N)	-1.023	-1.050
Total g Firms	13.530	3.364
Welfare Gain (% of Consumption)	-0.018	0.010
	Percentage-Pt. Change Rel. to Baseline	**Percentage-Pt. Change** Rel. to Baseline
Unempl. Rate	0.041	0.029
LFP Rate	0.342	0.477
Abate. Rate μ	25.012	23.973
Share of Total g Firms	6.119	3.389
Tax Rev./Output	0.222	0.177

Notes: *For comparability, both models have convex costs of firm creation and technology adoption.

Figure A11: Transitional Dynamics in Response to Carbon Tax–Model with Separate Sectors (r and g), Endogenous Firm Entry in Each Sector, and Green Technology Adoption Margin in r Sector



Note: Perc. Dev. denotes percent deviations and Perc.-Pt. Dev. denotes percentage-point deviations.