# Do Analysts Trade off Bias and Uncertainty? Analyst Earnings Expectations at Different Forecast Horizons

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#### Abstract

Financial analysts' earnings forecasts are upwards biased with a bias that gets bigger, the longer the forecast horizon. One explanation of this bias is that it reflects asymmetric costs of positive and negative forecast errors: A positive bias may facilitate better access to companies' private information but also compromises the accuracy of analysts' forecasts. This paper proposes a simple theoretical model that relates the bias and accuracy of analysts' forecasts to the forecast horizon and studies its implications empirically.

<sup>\*</sup>The views, thoughts and opinions expressed in this paper are those of the authors in their individual capacity and should not in any way be attributed to Platinum Grove Asset Management LP or to Marco Aiolfi as a representative officer, or employee of Platinum Grove Asset Management LP.

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### 1 Introduction

The properties of forecasts generated under asymmetric loss has generated considerable interest in economics and finance and include work by Granger (1969, 1999), Varian (1974), Zellner (1986), Ito (1990), West, Edison and Cho (1993), Weiss (1996), Christoffersen and Diebold (1997), Batchelor and Peel (1998), Granger and Pesaran (2000), Pesaran and Skouras (2002), Artis and Marcellino (2001), Elliott, Komunjer and Timmermann (2005, 2007), Patton and Timmermann (2007a,b) and Capistran and Timmermann (2008). In many situations it can be difficult, however, to identify the economic source of the asymmetry.

One set of forecasts where the asymmetry is much better understood is financial analysts' forecasts of earnings. It is well known that financial analysts issue earnings forecasts that are systematically upwards biased. This could be due to analysts' irrational behavior and inefficient use of information. An alternative explanation is that the bias reflects analysts' economic incentives which lead to asymmetric costs of over- and underpredictions of earnings. Analysts are rewarded in part on the basis of the precision of their forecasts which is an explicit factor in the Institutional Investor magazine's All-American ranking of analysts and also matters to investors who base their stock transactions on such forecasts. However, analysts may also—implicitly or explicitly—be rewarded based on how favorable firms perceive their forecasts to be. This matters to investment banking and trading relationships with corporate clients and could influence whether analysts gain access to firms' private information (Lim (2001)). Both factors are likely to play a role in analysts' career prospects: Hong and Kubik (2003) find that analysts with more precise and more upward-biased earnings estimates stand a higher chance of experiencing favorable job separations.

Because biases affect forecast precision adversely, such economic incentives create a tradeoff that must be carefully balanced by the analysts. The nature of this trade-off and how it evolves across short-, medium and long forecast horizons is poorly understood, however. Our paper therefore studies the term structure of earnings expectations which reflects how the trade-off in analysts' incentives depends on the forecast horizon. We propose a simple theoretical framework that links asymmetries in analysts' costs of over- and underpredicting earnings to how the bias and precision of their forecasts depend on the forecast horizon.

<sup>&</sup>lt;sup>1</sup>See, e.g., Fried and Gilvoy (1982), O'Brien (1988), Butler and Lang (1991), Brous (1992), Kang, O'Brien and Sivaramakrishnan (1994), Easterwood and Nutt (1999), Lim (2001) and Hong and Kubik (2003).

<sup>&</sup>lt;sup>2</sup>Loh and Mian (2005) find that analysts with more accurate earnings forecasts also issue more profitable stock recommendations. However, Clement and Tse (2003) find that investors do not exploit analysts' forecasts efficiently since stock market returns correlate more strongly with the size of the analysts' brokerage house than with their past forecasting ability.

If financial analysts use information efficiently and their incentives for over- and underpredicting earnings are evenly balanced, then future revisions to their forecasts should not
be predictable by means of information known to the analysts at the time of the forecast.

Asymmetric costs of over- and underpredictions of earnings could, however, generate systematic biases in the forecasts that are consistent with optimizing behavior, see Basu and
Markov (2004). The mere presence of a bias is therefore not well suited to establish whether
analysts use information efficiently or whether the costs of overpredictions and underpredictions are simply not the same. Fortunately, this does not mean that we cannot test whether
analysts use information efficiently. Under asymmetric loss, analysts bias their forecasts so
as to minimize the probability of large, costly errors. The greater the uncertainty surrounding future earnings, the more cautious analysts become and hence the larger the optimal
bias. As the forecast horizon shrinks, uncertainty about the earnings figure is reduced so we
should expect the bias to diminish systematically.

By exploiting the unique 'fixed event' structure of analysts' earnings forecasts to study how the bias evolves as a function of the forecast horizon, we show that it is possible to test if analysts use information efficiently but have asymmetric loss as opposed to whether they use information inefficiently. Under the 'asymmetric loss' hypothesis, the bias as well as the variance of the forecast error should decline monotonically as the forecast horizon is reduced. If, however, analysts are simply acting irrationally, there is no reason to expect a systematic relationship between the forecast horizon and the bias. Studying data on analysts' forecasts at several horizons therefore allows us to construct a more robust test of forecast efficiency.

Our empirical results suggest that accuracy in analysts' earnings forecasts becomes increasingly important—and biases thus increasingly costly—as the end of the fiscal year approaches and uncertainty about earnings gets resolved. Using data on the firms included in the Dow Jones index, the standard deviation of the forecast error goes from 12 cents per share at the 12-month horizon to less than three cents per share at the 1-month horizon. Moreover, we find that analysts' forecasts change from being upward biased at long forecast horizons (e.g. 12 months) to being unbiased or slightly downward biased at short horizons. As pointed out by Hong and Kubik (2003), financial markets care about whether firms meet their earnings forecasts and so the bias needs to be reduced—or even reversed—as the time to the actual earnings announcement date draws closer. This finding is also consistent with the view that, late in the reporting period, analyst objectives are to issue forecasts that are easy to beat.

Our theoretical analysis further suggests that revisions to analysts' earnings forecasts

should be persistent (serially correlated) and predictable. Persistence in earnings revisions is a result of the presence of an upwards bias in analysts' earnings forecasts which gets reduced as the forecast horizon shrinks. We show that this tends to make negative revisions greater in magnitude, more likely and also more persistent than positive revisions. Predictability arises because of the link between the optimal bias in analysts' forecasts and the degree of uncertainty surrounding the earnings figure. This link means that predictability in earnings revisions can be generated by mean reverting volatility in the earnings process, a finding confirmed to hold empirically in the analysis by Aiolfi et al (2008).

The paper is structured as follows. Section 2 presents a simple theoretical model to understand the behavior of analysts' earnings forecasts under asymmetric loss. Section 3 provides details of the data set and reports empirical results. Section 4 concludes.

## 2 Analyst Forecasts under Asymmetric Loss

This section explores the implications of asymmetries in analysts' objectives for their earnings forecasts. We show that asymmetries lead to a set of surprising predictions for analysts' behavior. Later sections of the paper carry out tests to see if these implications hold empirically.

There are strong economic reasons to believe that analysts' cost of overpredicting and underpredicting earnings are not the same. Brokerages employing sell side analysts may have investment banking relationships with firms whose earnings are being predicted, thus giving rise to an over-optimism bias in order to promote those firms' shares. Furthermore, analysts are likely to gain easier access to top executives if they present their firms' earnings prospects in a favorable light (Lim (2001)). There are limits to such biases, however, as forecast accuracy has also been found to affect analysts' career prospects (Hong and Kubik (2003)).

We will study how analysts' earnings forecasts evolve as a function of the forecast horizon, h. Let T be the earnings announcement date, so an h-period forecast is computed at time T-h. As time progresses from T-h to T-h+1, the forecast horizon shrinks from h to h-1 periods. Let  $f_{T,T-h}$  be the analyst's forecast of the actual earnings number,  $A_T$ , computed on the basis of information at time T-h, i.e. h periods prior to the earnings announcement date, T. The associated forecast error at the h-period horizon is then given by  $e_{T,T-h} = A_T - f_{T,T-h}$ .

We shall assume that analysts' objectives can be represented through a loss function,

L(e), that depends on the forecast error, e, and increases as the size of the forecast error (|e|) gets larger. The simplest way to account for asymmetries in analysts' objectives is to weight positive and negative forecast errors of equal size differently.<sup>3</sup> To this end, consider the linear-exponential (linex) loss function proposed by Varian (1974):

$$L(e) = \frac{1}{\psi} \left[ \exp(\psi e) - \psi e - 1 \right] \qquad (\psi \neq 0). \tag{1}$$

This loss function is convenient to work with but our results will not otherwise be dependent on this specific functional form. Asymmetric costs of over- and underpredictions are captured as follows. When  $\psi > 0$ , large positive forecast errors (underpredictions) are penalized more heavily than negative forecast errors of the same magnitude. The reverse holds when  $\psi < 0$ . As  $\psi \to 0$ , the standard symmetric, squared loss function,  $L(e) = e^2$  is obtained as a limiting case.

Suppose that the actual earnings figure,  $A_T$ , given analysts' information at time T - h,  $\Omega_{T-h}$ , is normally distributed  $N(\mu_{T,T-h}, \sigma_{T,T-h}^2)$ . Then the optimal forecast is given by (Zellner (1986), Christoffersen and Diebold (1997))

$$f_{T,T-h}^* = \mu_{T,T-h} + \frac{\psi \sigma_{T,T-h}^2}{2}.$$
 (2)

Note that as  $\psi \to 0$ , so the asymmetry in the loss function is reduced, the optimal forecast converges to the conditional expectation,  $\mu_{T,T-h}$ , and thus the optimal forecast is unbiased. When  $\psi > 0$ , so underpredictions are costlier than overpredictions, the optimal forecast is biased upwards, resulting in a negative mean forecast error:

$$bias_{T,T-h} = E[A_T | \Omega_{T-h}] - f_{T,T-h}^* = \mu_{T,T-h} - f_{T,T-h}^* = \frac{-\psi \sigma_{T,T-h}^2}{2}.$$
 (3)

The bias is larger in magnitude the greater is  $\psi$ , i.e. the greater the relative cost of underpredictions, and the greater the uncertainty surrounding the earnings figure. This is intuitive since the forecaster tries to avoid costly mistakes by moving the forecast error distribution further away from areas where mistakes are most costly.

Figure 1 shows the optimal bias when  $\psi = 1$  so the forecaster prefers to overpredict earnings in order to avoid costly positive forecast errors. Under low volatility ( $\sigma = 0.5$ ), the

<sup>&</sup>lt;sup>3</sup>Rodriguez (2006) develops a wage contract between the forecast provider and the forecast user in which the forecaster is assumed to be penalized asymmetrically for forecast errors of different signs and shows how this compensation scheme gives rise to an asymmetric loss function for the financial analyst. Granger and Machina (2006) demonstrate how loss functions can be derived from decision theory.

optimal bias is situated at -1/8. Doubling the uncertainty surrounding the earnings figure to  $\sigma = 1$ , the bias rises in magnitude (i.e. gets more strongly negative) and quadruples its value to -1/2.

The finding that the optimal bias increases as a function of the degree of uncertainty surrounding earnings is intuitively appealing. A large bias relative to the uncertainty would make it easy for forecast users to detect biases in analysts' earnings forecasts and could question their credibility. Our model has the desirable property that in the limit when corporate earnings uncertainty vanishes, no bias remains.

Another consequence of (3) is that the magnitude of the bias should increase as a function of the forecast horizon. Analysts' information improves as time progresses and the forecast horizon, h, shrinks. It follows from the convexity of the loss function (1) that, on average,  $\sigma_{T,T-h_S}^2 < \sigma_{T,T-h_L}^2$ , where  $h_S < h_L$  represent short and long forecast horizons, respectively. This result holds quite generally, independently of the specific process followed by actual earnings. Nevertheless, it is useful to study an explicit example. Suppose that actual earnings for fiscal year T,  $A_T$ , follow a random walk:<sup>4</sup>

$$A_T = A_{T-1} + \varepsilon_T, \quad \varepsilon_T \sim (0, \sigma_{\varepsilon}^2).$$
 (4)

Iterating h periods backwards on (4), we have

$$A_T = A_{T-h} + \varepsilon_{T-h+1} + \dots + \varepsilon_T$$

where future earnings shocks  $\varepsilon_{T-h+1}, ..., \varepsilon_T$  are unpredictable given the analyst's information at time T-h, whereas  $A_{T-h}$  is known given this information. Again this is easily modified and merely serves as a simplifying assumption.<sup>5</sup> For this example,  $\sigma_{e_{T,T-h}}^2 = h\sigma_{\varepsilon}^2$  and so the optimal bias (3) under the random walk for earnings (4) becomes

$$bias_{T,T-h}^* = \frac{-h\psi\sigma_{\varepsilon}^2}{2}.$$
 (5)

We summarize our discussion as follows.

**Proposition 1** Suppose that analysts' cost of underpredicting earnings exceeds their cost of overpredicting them according to the loss function (1) with  $\psi > 0$ . Then

<sup>&</sup>lt;sup>4</sup>Since we focus on forecasts of annual earnings we ignore seasonal components, but the model can easily be extended to account for additional components in earnings.

<sup>&</sup>lt;sup>5</sup>To the extent that  $\varepsilon_T$  is partially predictable and financial analysts receive a signal that has a correlation  $\rho$  with  $\varepsilon$ , it is possible to reduce the variance of the forecast error from  $\sigma_{\varepsilon}^2$  to  $\sigma_{\varepsilon}^2(1-\rho^2)$ .

- 1. The optimal forecast is upward biased and the magnitude of the bias is greater, the longer the forecast horizon and the greater the uncertainty surrounding future earnings.
- 2. The precision of the forecast (as measured by the inverse of the standard deviation of the forecast error) improves systematically as the forecast horizon is reduced;
- 3. Moreover, if earnings follow a random walk process (4), then the optimal bias will be proportional to the forecast horizon.

Proposition 1 makes it clear that the appropriate way to test whether analysts efficiently incorporate new information into their earnings forecasts is not through a test for unbiasedness in their forecasts. There are good reasons to expect analysts' forecasts to be biased even when they utilize information efficiently. Biases may simply reflect analysts' asymmetric costs of over- and underpredicting earnings.

This does *not* mean that we cannot test whether analysts use information efficiently. In fact, Proposition 1 suggests two separate tests. One is to inspect whether the standard deviation of the forecast error gets reduced as the forecast horizon (i.e. the time to the earnings announcement) shrinks. Moreover, the bias is a function of the uncertainty surrounding the earnings figure and so the magnitude of the bias should decline as the forecast horizon shrinks.

#### 2.1 Forecast Revisions

A further implication of asymmetric loss is that revisions to analysts' earnings forecasts, denoted  $\Delta f_{T,h} = f_{T,T-h+1} - f_{T,T-h}$ , may be serially correlated and (more generally) predictable over time. To see this in the context of the random walk model (4) note from (2) that the revision to analysts' earnings forecast between periods T - h and T - h + 1 is given by

$$\Delta f_{T,h} = \varepsilon_{T-h+1} - 0.5\psi(\sigma_{T,T-h}^2 - \sigma_{T,T-h+1}^2),\tag{6}$$

where  $\sigma_{T,T-h}^2$  and  $\sigma_{T,T-h+1}^2$  are the conditional volatility of earnings computed at time T-h and time T-h+1, respectively. When analysts' loss is symmetric,  $\psi \to 0$  and  $\Delta f_{T,h} \to \varepsilon_{T-h+1}$ , so revisions will be serially uncorrelated provided that analysts use their information efficiently. Conversely, if the uncertainty surrounding future earnings, as measured by  $(\sigma_{T,T-h}^2 - \sigma_{T,T-h+1}^2)$ , changes over time in a way that is itself predictable and  $\psi \neq 0$ ,

this will induce predictability in earnings revisions. Such predictability could occur when the volatility of earnings news clusters in time and is consistent with a large literature on volatility clustering in many financial and macroeconomic variables.

To illustrate this point, suppose that the volatility of the earnings process is driven by a state variable,  $S_t$ , which can take two values,  $s_t = 1$  or  $s_t = 2$ , corresponding to low volatility  $(\sigma_1)$  and high volatility  $(\sigma_2)$  states. Letting  $\sigma_{s_T}$  be the conditional volatility of  $\varepsilon$  in state  $s_T$ , the random walk process (4) is thus modified to

$$A_T = A_{T-1} + \sigma_{s_T} v_T, \quad v_T \sim IID(0, 1).$$
 (7)

To describe how the states evolve, let  $p_{11} = pr(s_t = 1|s_{t-1} = 1)$  and  $p_{22} = pr(s_t = 2|s_{t-1} = 2)$  be the probabilities of remaining in the low and high volatility states. Values of  $p_{11}$  and  $p_{22}$  greater than one-half are consistent with mean-reverting volatility.

Under these assumptions the conditional variance of future increments to earnings given information at time T - h becomes (assuming that h > 2)

$$\sigma_{T,T-h}^2 = E_{T-h}[(A_T - A_{T-h})^2] = E_{T-h}[\varepsilon_{T-h+1}^2 + \dots + \varepsilon_{T-1}^2 + \varepsilon_T^2]$$

$$= \frac{1}{2 - p_{11} - p_{22}} \sum_{m=1}^h \pi'_{T-h} \begin{bmatrix} 1 - p_{22} + \lambda_2^m (1 - p_{11}) & 1 - p_{22} - \lambda_2^m (1 - p_{22}) \\ 1 - p_{11} - \lambda_2^m (1 - p_{11}) & 1 - p_{11} + \lambda_2^m (1 - p_{22}) \end{bmatrix} \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \end{pmatrix},$$

where  $\lambda_2 = -1 + p_{11} + p_{22}$  is the eigenvalue of the transition probability matrix, P, that differs from unity and  $\pi_{T-h}$  is the  $2 \times 1$  vector of state probabilities at time T - h. Considering the same expression at time T - h + 1 and taking expectations given information at time T - h, we get an expression for the expected change in the earnings uncertainty between periods T - h and T - h + 1:

$$E_{T-h}[\sigma_{T,T-h}^2 - \sigma_{T,T-h+1}^2] = \frac{\pi'_{T-h}}{2 - p_{11} - p_{22}} \begin{bmatrix} 1 - p_{22} + \lambda_2(1 - p_{11}) & 1 - p_{22} - \lambda_2(1 - p_{22}) \\ 1 - p_{11} - \lambda_2(1 - p_{11}) & 1 - p_{11} + \lambda_2(1 - p_{22}) \end{bmatrix} \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \end{pmatrix}.$$

This expression shows that  $(\sigma_{T,T-h}^2 - \sigma_{T,T-h+1}^2)$ —and hence from (6) the revision in analysts' earnings forecasts—is predictable since this difference depends on whether the current volatility is high, low or normal as reflected in the initial state probabilities,  $\pi_{T-h}$ . Suppose that  $\sigma_2^2 > \sigma_1^2$  so uncertainty about earnings is greatest in the second state. Using an example with  $p_{11} = p_{22} = 0.8$ ,  $\sigma_2 = 2\sigma_1$ , h = 10 and  $\psi = 1$ , Figure 2 plots the expected forecast revision. First note that  $E_{T-h}[\sigma_{T,T-h}^2 - \sigma_{T,T-h+1}^2] > 0$ . This is a consequence of the fact that  $\sigma_{T,T-h}^2$  tracks the uncertainty about earnings at the h-period horizon, while  $\sigma_{T,T-h+1}^2$  tracks

the uncertainty at the shorter h-1-period horizon. As shown in (6), this is consistent with the average forecast revision being negative. As the probability of starting from the low volatility state (state 1) rises from zero to one, the change in the forecast revision predicted to occur between periods T-h and T-h+1 declines in magnitude (i.e., becomes less negative).

The pattern displayed in Figure 2 is a feature that holds not only for this particular example but is valid quite generally for earnings process with mean-reverting volatility as assumed by most models of time-varying volatility (Patton and Timmermann (2007b)). Such mean reversion implies that if the current (conditional) volatility is low, it can be predicted to rise next period. Conversely, if current volatility is unusually high, it can be predicted to decline. In both cases the change in the volatility is partially predictable. It follows from (6) that such predictability translates into predictability and serial correlation in earnings revisions.

Our simple model has the additional implication that the proportion of negative revisions should be greater than the proportion of positive revisions. To see this, note that under the simple random walk model with constant variance (4), forecast revisions are normally distributed with a negative mean of  $-0.5\psi\sigma_{\varepsilon}^2$ , so the probability of a negative revision is  $N(0.5\psi\sigma_{\varepsilon}) > 1/2$  when  $\psi > 0$ . Moreover, because the distribution of forecast errors is centered on a negative value with symmetrically distributed, mean-zero shocks, the magnitude of negative revisions—as measured by their average value—should be greater than that of positive revisions.

When the volatility of the earnings process is persistent but mean-reverting, it follows that the probability of negative revisions should also be persistent: Negative revisions are more likely to follow from previous negative revisions than from positive revisions. This happens because the bias increases in proportion with the variance of the earnings process. When the conditional variance is high, as a precaution analysts increase their upward bias (so as to lower the probability of a costly underprediction), which in turn increases the probability of observing a negative forecast error (an overprediction).

A similar effect also happens when volatility is unusually low. This makes the optimal bias small and increases the probability of observing positive revisions to analysts' forecasts. Assuming that periods with low earnings volatility are persistent, by an analogous argument, the probability of a positive earnings revision will be higher if the previous revision was positive. Furthermore, because the distribution of earnings revisions is centered on a negative value, negative earnings revisions should be more persistent than positive revisions.

We summarize this discussion in the following proposition:

**Proposition 2** Suppose that analysts' cost of underpredicting earnings exceeds their cost of overpredicting them according to the loss function (1) with  $\psi > 0$ . Then

- 1. The average revision to analysts' earnings forecasts is negative;
- 2. Negative revisions occur more frequently, are greater in magnitude and are more persistent than positive revisions.
  - Moreover, if the volatility of the earnings process is persistent and mean reverting, then
- 3. Revisions to analysts' earnings forecasts may be serially correlated and predictable.

The final property is a direct consequence of our finding that revisions to analysts' earnings forecasts should reflect changes to volatility forecasts. If volatility is mean-reverting, any patterns in the volatility should carry over to the optimal bias and hence give rise to predictability in the mean revisions.<sup>6</sup>

We next turn to some data to see if these predictions from our model can be validated empirically.

# 3 Empirical Results

While the relation between earnings revisions and stock price movements has been studied extensively, there has been little analysis of how the consensus earnings estimate evolves as a function of the forecast horizon. Our paper addresses thos issues by studying the properties of monthly revisions in analysts' earnings estimates for the 30 firms included in the Dow Jones index over a 20-year period.

#### 3.1 Data

Our data source on earnings forecasts is the summary tapes of the Institutional Brokers' Estimate System (I/B/E/S) and spans the period from January 1986 to December 2004, a total of 228 months. Using such a long time series gives us the ability to better document systematic patterns in revisions to analysts' earnings expectations. We are interested in the behavior of earnings revisions at the individual firm level as well as in the aggregate. To

<sup>&</sup>lt;sup>6</sup>We note that this implication of our theory is consistent with empirical findings of predictability in analysts' revisions reported by Kang, O'Brien and Sivaramakrishnan (1994) and Lys and Sohn (1990).

ensure sufficient analyst coverage throughout the long sample period studied here and keep the analysis manageable, we focus on the firms included in the Dow Jones 30 Index, all of which have excellent analyst coverage. Since individual analysts usually do not provide a complete series of forecast updates, we follow Klein (1990), Chaney, Hogan and Jeter (1999), Easterwood and Nutt (1999) and others in modeling the consensus forecast (i.e., the cross-sectional average) in order to get a long contiguous time series. The consensus forecast is generally viewed as highly influential and is the most widely accepted single measure of earnings expectations (see, e.g., Brown et al (1985)). While individual analysts' forecasts may differ from the consensus expectation, the latter still explains a large fraction of the time-series variation in individual analysts' views.<sup>7</sup>

Following Aiolfi et al (2008), time-series of monthly revisions to analysts' earnings estimates are constructed as follows. Every third Thursday of the month (t)—the so-called statistical date—I/B/E/S lists all analysts' earnings estimates entered since the third Thursday of the previous month (t-1). I/B/E/S then computes summary statistics (such as the consensus mean) over this set of individual analysts' estimates. We denote the consensus estimate of earnings for the current fiscal year recorded during month t for firm j by  $f_t^{j,8}$  At the end of the fiscal year, which we denote by T, firm j's actual earnings per share figure,  $A_T^j$ , is announced. Our analysis focuses on analysts' forecasts of earnings for the current fiscal year. When  $t+1 \leq T$ , the earnings revision for fiscal year T,  $\Delta f_{t+1}^j$ , is based on the difference between the earnings estimates on the statistical dates t and t+1. During months where the fiscal year changes we base the revision of the earnings forecast on a comparison of the previous month's forecast of earnings for fiscal year T+1 with the current month's forecast of this value. This allows us to create a contiguous time-series of monthly revisions to analysts' earnings forecasts.

Following studies such as Klein (1990) and Lys and Sohn (1990), we scale the earnings revision by a firm's initial stock price,  $P_t^j$ , measured at the close on day t (the statistical date) and obtained from the CRSP daily files. The revision to the consensus estimate of firm j's earnings figure between months t and t+1 is thus computed as:

$$\Delta f_{t+1}^{j} = 100 \times \left( \frac{f_{t+1}^{j} - f_{t}^{j}}{P_{t}^{j}} \right). \tag{8}$$

<sup>&</sup>lt;sup>7</sup>This is in part due to herding among analysts, see, e.g., Hong, Kubik and Solomon (2000). Markov and Yan (2006) document asymmetries in individual analysts' loss and find that analysts working with the same employer tend to have similar asymmetries.

<sup>&</sup>lt;sup>8</sup>Without risk of confusion we have omitted the horizon subscript, T-h, since it is not needed here.

Hence we define the revision to the consensus earnings estimate as the change in the forecast of earnings per share from t to t+1,  $f_{t+1}^j - f_t^j$ , divided by the initial stock price per share,  $P_t^j$ , and multiplied by 100. Revision numbers can therefore be interpreted as a percentage of the stock price.

I/B/E/S reports the consensus estimate of earnings per share rounded to the nearest cent. For many of the months included in our sample, forecast revisions are zero since the arrival of new information between two neighboring months is insufficient to lead to an earnings revision in excess of one cent. While the earnings revision is unlikely to be exactly equal to zero, we follow common practice and record this as a zero observation. Such observations could be discarded, but doing so may lead to important biases. A 'no change' forecast may in fact contain valuable information about future revisions, particularly if periods with small revisions tend to be persistent (which we shall see is indeed the case).

Close to 50% of the monthly revisions to the consensus earnings forecasts are smaller than one cent per share and hence get recorded as zero. This grand average conceals substantial variations across firms, however. The proportion of 'no change' revisions exceeds 80% for firms such as General Electric and Pfizer, while this proportion falls below 20% for Alcoa, JP Morgan & Chase, General Motors and Exxon Mobil. This high proportion of zeros for the median forecast revision is in line with earlier results by Kang, O'Brien and Sivaramakrishnan (1994) who find that 19-35% of their forecast revisions are zero. Similarly, for their sample of annual earnings forecasts, Brown, Foster and Noreen (1985) find that the zeros in monthly forecast revisions range up to 80-90%.

### 3.2 Bias, Uncertainty and Forecast Horizon

To explore the implications of Proposition 1 on how the bias and precision of analysts' forecasts evolve as a function of the forecast horizon, we next study the relationship between the forecast horizon and properties of the forecast error,  $e_{T,T-h}$ . For each of the 30 Dow Jones firms Table 1 shows the mean forecast error, defined as the consensus estimate of earnings for the fiscal year minus the actual earnings figure,  $\bar{e}_{T,T-h} = \bar{A}_T^j - \bar{f}_{T,T-h}^j$ , as a function of the number of months remaining before the announcement of the actual earnings figure (h = 12, ..., 2, 1). For the vast majority of firms the forecast error starts out being negative at the 12-month horizon (representing overpredictions) but rises systematically and is close to zero—corresponding to largely unbiased forecasts—at the 1-month horizon. For most firms, analysts therefore tend to systematically overpredict earnings with a bias that is

greater the longer the forecast horizon.<sup>9</sup> Moreover, this bias is systematically reduced as the end of the current fiscal year draws closer. Few firms display the reverse pattern of initial underpredictions of earnings followed by upwards revisions to the earnings estimates.

These patterns in analyst biases are consistent with proposition 1. As the time to the earnings date draws closer and uncertainty gets reduced, the bias shrinks. Moreover, the bias has practically vanished at the shortest horizon. Indeed, for two-thirds of the firms the over-prediction bias observed at long horizons is reversed into a slight underprediction bias at the shortest horizon. This is consistent with an incentive for firms to meet earnings expectations. Another explanation of this finding is that, as the earnings announcement date approaches and the quality of analysts' information improves, any remaining biases in analyst forecasts become more obvious and hence more costly.

Next consider how the precision of analyst forecasts changes as the time to the end of the fiscal year draws closer. To this end Figure 3 plots the standard deviation of the forecast error, calculated as an average across firms with weights that are inversely proportional to the standard deviation of the forecast errors for the individual firms so that firms with lower standard deviations get greater weights. At the 12-month horizon the standard deviation is 12 cents per share. This gets steadily reduced as the fiscal year unfolds and the forecast horizon shrinks. The average standard deviation is nine, seven, four and three cents per share at the nine, six, three and one month horizons, respectively. Hence, the precision of analysts' earnings estimates clearly improves systematically as the fiscal year progresses, although it never tapers off completely.

Figure 4 presents standard deviation plots for a selection of individual firms. Patterns in the individual firms' plots are a bit noisier than the cross-sectional average since they are not smoothed out by taking cross-sectional averages. Even so, there is a clear decline in the standard deviation of individual firms' forecast errors as the horizon shrinks. Again these findings are consistent with Part 2 of Proposition 1 suggesting that the patterns observed in analysts' forecast errors reflect their economic incentives rather than inefficient use of information.

### 4 Conclusion

Key to understanding the evolution in consensus forecasts of corporate earnings is how analysts incorporate new information into their forecasts as the time to the earnings an-

<sup>&</sup>lt;sup>9</sup>This finding is consistent with studies such as Abarbanell (1991), Jain (1992), Kang, O'Brien and Sivaramakrishnan (1994) and Lim (2001).

nouncement date draws closer. As part of their revisions analysts must balance the need for forecast accuracy versus the benefits from issuing biased forecasts (Hong and Kubik (2003)). We presented a simple theoretical model for understanding how this trade-off evolves as a function of the forecast horizon and found that the implications of this theory—that the magnitude of the bias shrinks while the forecast accuracy increases as the forecast horizon is reduced—could be confirmed empirically. Forecast accuracy appears to become more important as the earnings announcement date is approached, more information is available and any remaining biases become both more easily detectable and (as a result) more costly.

Our theoretical analysis of analysts' optimal forecasts under asymmetric loss established a link between the optimal bias and the uncertainty about the underlying earnings process. Under asymmetric costs of over- and underpredicting earnings, loss averse agents respond to increased uncertainty by biasing their forecasts further away from the direction where errors are most costly. The greater the uncertainty, the higher the bias. This bias-precision relationship and its link to the analysts' forecast horizon has to our knowledge not previously been explored.

# References

- [1] Abarbanell, J.S., 1991, Do Analysts' Earnings Forecasts Incorporate Information in Prior Stock Price Changes? Journal of Accounting and Economics 14, 147-165.
- [2] Aiolfi, M., M. Rodriguez and A. Timmermann, 2008, Persistence, Asymmetry and Predictability of Analysts' Earnings Forecasts. Unpublished manuscript.
- [3] Artis, M. and M. Marcellino, 2001, Fiscal Forecasting: The Track Record of the IMF, OECD and EC. Econometrics Journal 4, S20-S36.
- [4] Basu, S. and S. Markov, 2004, Loss Function Assumptions in Rational Expectations Tests on Financial Analysts. Journal of Accounting and Economics 38, 171-203.
- [5] Batchelor, R. and D.A. Peel, 1998, Rationality Testing under Asymmetric Loss. Economics Letters 61, 49-54.
- [6] Brous, P., 1992, Common Stock Offering and Earnings Expectations: A Test of the Release of Unfavorable Information. Journal of Finance 4, 1517-1536.
- [7] Brown, P., G. Foster and E. Noreen, 1985, Security Analyst Multi-year Earnings Forecasts and the Capital Market. American Accounting Association, Sarasota, Florida.

- [8] Butler, K. and L. Lang, 1991, The Forecast Accuracy of Individual Analysts: Evidence of Systematic Optimism and Pessimism. Journal of Accounting Research 29, 150-156.
- [9] Capistran, C. and A. Timmermann, 2008, Disagreement and Biases in Inflation Expectations. Forthcoming in Journal of Money, Credit and Banking.
- [10] Chaney, P., C. Hogan and D. Jeter, 1999, The Effects of Reporting Restructuring Charges on Analysts' Forecast Revisions and Errors. Journal of Accounting and Economics 27, 261-284.
- [11] Christoffersen, P.F. and F.X. Diebold, 1997, Optimal Prediction under Asymmetric Loss. Econometric Theory 13, 808-817.
- [12] Clement, M. and S. Tse, 2003, Do Investors Respond to Analysts' Forecast Revisions as If Forecast Accuracy Is All That Matters? The Accounting Review, 78, 227-249.
- [13] Easterwood, J. C. and S. R. Nutt, 1999, Inefficiency in Analysts' Earnings Forecasts: Systematic Misreaction or Systematic Optimism? Journal of Finance 54-5, 1777-1797.
- [14] Elliott, G., I. Komunjer and A. Timmermann, 2005, Estimating Loss Function Parameters. Review of Economic Studies 72, 1107-1125.
- [15] Elliott, G., I. Komunjer and A. Timmermann, 2006, Biases In Macroeconomic Forecasts: Irrationality or Asymmetric Loss? Forthcoming in Journal of European Economic Association.
- [16] Fried, D. and D. Gilvoy, 1982, Financial Analysts' Forecasts of Earnings: A Better Surrogate for Market Expectations. Journal of Accounting and Economics 4, 85-107.
- [17] Granger, C.W.J., 1969, Prediction with a generalized cost function, OR, 20, 199-207.
- [18] Granger, C.W.J., 1999, Outline of Forecast Theory Using Generalized Cost Functions. Spanish Economic Review 1, 161-173.
- [19] Granger, C.W.J. and M. Machina, 2006, Forecasting and Decision Theory, In: Elliott, G, Granger, C.W.J., and A. Timmermann (Eds.), Handbook of Economic Forecasting, New York: Elsevier.
- [20] Granger, C.W.J. and M. H. Pesaran, 2000, A Decision Theoretic Approach to Forecast Evaluation, In: Chan, W. S., Li, W. K., and H. Tong (Eds.), Statistics and Finance: An Interface, London: Imperial College Press.

- [21] Hong, H. and J. D. Kubik, 2003, Analyzing the Analysts: Career Concerns and Biased Earnings Forecasts. Journal of Finance 58,1, 313-351.
- [22] Hong, H., J. D. Kubik, and A. Solomon, 2000, Security Analysts' Career Concerns and Herding of Earnings Forecasts. Rand Journal of Economics 31, 121-144.
- [23] Ito, T., 1990, Foreign Exchange Rate Expectations: Micro Survey Data. American Economic Review 80, 434-449.
- [24] Jain, P.C., 1992, Equity Issues and Changes in Expectations of Earnings by Financial Analysts. Review of Financial Studies 5, 669-683.
- [25] Kang, S., O'Brien, J. and K. Sivaramakrishnan, 1994, Analysts' Interim Earnings Forecasts: Evidence on the Forecasting Process. Journal of Accounting Research 32, 103-112.
- [26] Klein, A., 1990, A Direct Test of the Cognitive Bias Theory of Share Price Reversals. Journal of Accounting and Economics 13, 155-166.
- [27] Lim, T., 2001, Rationality and Analysts' Forecast Bias. Journal of Finance 56-1, 369-385.
- [28] Loh, R. K., and G. M. Mian, 2005, Do Accurate Earnings Forecasts Facilitate Superior Investment Recommendations? Forthcoming in Journal of Financial Economics.
- [29] Lys, T. and S. Sohn, 1990, The Association Between Revisions of Financial Analysts' Earnings Forecasts and Security Price Changes. Journal of Accounting and Economics 13, 341-363.
- [30] Markov, S. and M-Y. Tan, 2006, Loss function asymmetry and forecast optimality: Evidence from individual analysts' forecasts. Mimeo, Emory University.
- [31] O'Brien, P., 1988, Analysts' Forecasts as Earnings Expectation. Journal of Accounting and Economics 10, 53-83.
- [32] Patton, A. and A. Timmermann, 2007a, Testing Forecast Optimality Under Unknown Loss. Journal of American Statistical Association 102, 1172-1184.
- [33] Patton, A. and A. Timmermann, 2007b, Properties of Optimal Forecasts under Asymmetric Loss and Nonlinearity. Journal of Econometrics 140, 884-918.

- [34] Pesaran, M.H. and S. Skouras, 2002, Decision-based Methods for Forecast Evaluation. In Clements, M.P. and D. F. Hendry (Eds.), A Companion to Economic Forecasting. Blackwell, Oxford.
- [35] Rodriguez, M., 2006, Financial Analysts' Incentives and Forecast Biases. Manuscript, UC San Diego.
- [36] Varian, H. R., 1974, A Bayesian Approach to Real Estate Assessment, in Studies in Bayesian Econometrics and Statistics in Honor of Leonard J. Savage, eds. S.E. Fienberg and A. Zellner, Amsterdam: North Holland, pp. 195-208.
- [37] Weiss, A. A., 1996, Estimating Time Series Models Using a Relevant Cost Function, Journal of Applied Econometrics 11, 539-560.
- [38] West, K. D., Edison, H. J., and D. Cho, 1993, A Utility Based Comparison of Some Models of Exchange Rate Volatility, Journal of International Economics, 35, 23-45.
- [39] Zellner, A., 1986, Bayesian Estimation and Prediction Using Asymmetric Loss Functions, Journal of the American Statistical Association, 81, 446-451.

Figure 1: Linex loss function and the optimal bias  $(\mu)$  under forecast error distributions with high and low volatility.

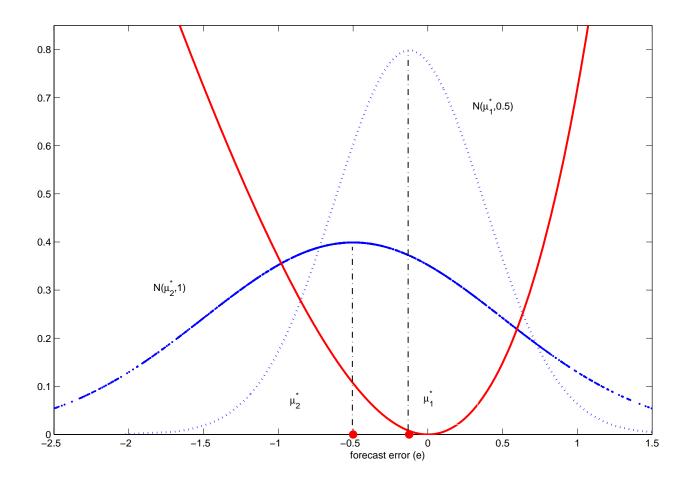


Figure 2: Magnitude of the expected revisions in analysts' earnings forecasts as a function of the state probability.

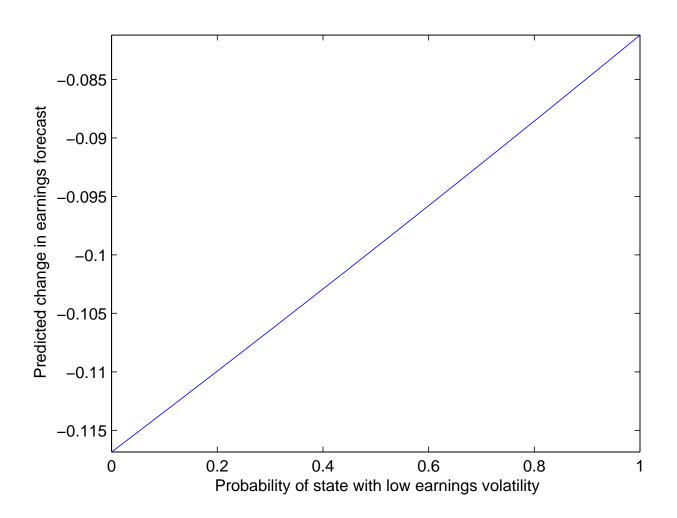


Figure 3: Average standard deviation of the consensus forecast errors as a function of forecast horizon (h). The weights used in computing the average are inversely proportional to standard deviation of the individual firms' consensus forecasts errors.

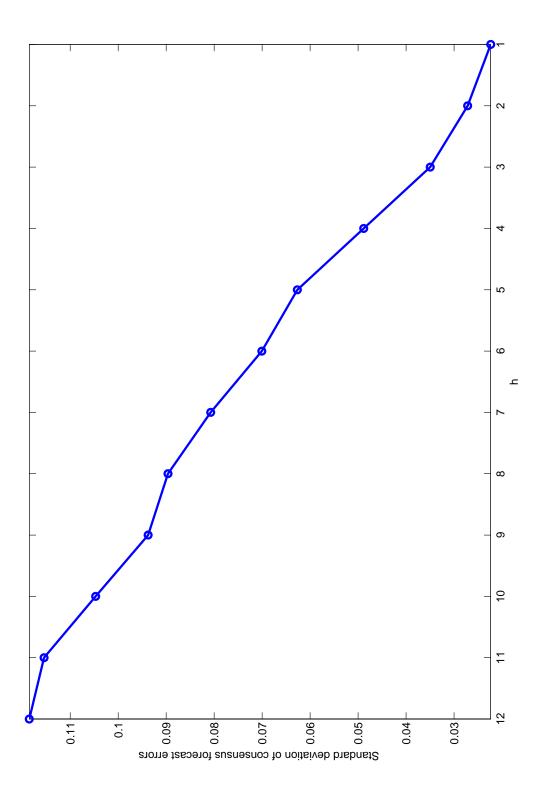


Figure 4: Standard deviation of the consensus forecast errors as a function of the forecast horizon (h) for the individual firms in the Dow Jones Index.

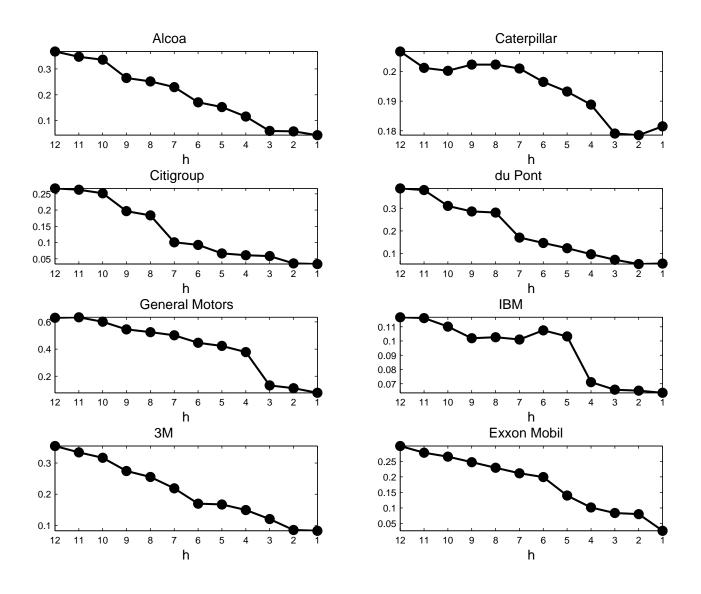


Table 1: Bias as a function of the forecast horizon

For each firm the table reports the bias in the forecast error, calculated as the actual minus the predicted value, as a function of the forecast horizon (h), measured in months. Predicted values are based on consensus forecasts reported by IBES.

	h = 12	h = 11	h = 10	h = 9	h = 8	h = 7	h = 6	h = 5	h = 4	h = 3	h=2	h = 1
AA	-0.189	-0.188	-0.174	-0.165	-0.152	-0.135	-0.117	-0.115	-0.088	-0.058	-0.050	-0.035
AIG	-0.045	-0.040	-0.042	-0.048	-0.048	-0.050	-0.053	-0.057	-0.042	-0.035	-0.037	-0.038
ALD	-0.127	-0.108	-0.100	-0.087	-0.083	-0.042	-0.041	-0.019	-0.013	-0.011	0.001	0.004
AXP	-0.169	-0.163	-0.132	-0.116	-0.106	-0.071	-0.075	-0.068	-0.050	-0.025	-0.012	-0.013
BA	-0.256	-0.201	-0.161	-0.153	-0.145	-0.120	-0.099	-0.104	-0.071	0.002	-0.004	0.013
$\operatorname{BEL}$	-0.063	-0.061	-0.057	-0.054	-0.054	-0.048	-0.027	-0.031	-0.006	0.003	0.004	0.006
CAT	-0.032	0.006	0.011	-0.022	-0.036	-0.021	-0.033	-0.033	-0.031	-0.015	0.000	0.006
CCC2	0.001	0.022	0.022	0.007	-0.010	-0.012	-0.026	-0.012	-0.002	-0.010	-0.013	0.006
$\operatorname{CHL}$	-1.183	-0.773	-0.771	-0.747	-0.686	-0.391	-0.331	-0.296	0.020	0.067	0.101	0.148
DD	-0.105	-0.082	-0.102	-0.130	-0.126	-0.104	-0.069	-0.044	-0.033	-0.013	-0.011	0.006
DIS	-0.017	-0.015	-0.019	-0.025	-0.015	-0.008	-0.006	-0.004	0.001	-0.001	0.005	0.009
GE	-0.014	-0.013	-0.012	-0.014	-0.013	-0.012	-0.012	-0.013	-0.010	-0.008	-0.000	-0.001
GM	-0.425	-0.397	-0.475	-0.617	-0.699	-0.555	-0.484	-0.390	-0.319	-0.143	-0.041	0.067
$_{ m HD}$	0.030	0.029	0.024	0.017	0.016	0.008	0.005	0.010	0.003	0.003	0.006	0.007
HWP	-0.083	-0.063	-0.070	-0.092	-0.092	-0.073	-0.057	-0.040	-0.036	-0.005	0.001	0.008
$_{\mathrm{IBM}}$	-0.405	-0.299	-0.248	-0.198	-0.196	-0.170	-0.156	-0.156	-0.123	-0.080	-0.033	-0.006
INTC	-0.008	-0.013	0.016	0.014	0.010	0.018	0.010	0.011	0.010	0.013	0.011	0.013
JNJ	0.010	0.012	0.010	0.008	0.007	0.005	0.005	0.005	0.000	-0.001	-0.001	-0.001
KO	-0.030	-0.022	-0.015	-0.015	-0.015	-0.013	-0.015	-0.005	0.003	0.003	0.006	0.003
MCD	-0.015	-0.013	-0.010	-0.010	-0.010	-0.006	-0.007	-0.002	-0.007	-0.007	-0.003	-0.003
MMM	-0.041	-0.020	-0.023	-0.026	-0.020	-0.015	-0.016	-0.014	-0.008	-0.008	-0.003	0.002
MO	-0.053	-0.056	-0.031	-0.031	-0.032	-0.032	-0.027	-0.027	-0.013	-0.005	-0.001	-0.004
MRK	-0.043	-0.043	-0.041	-0.043	-0.043	-0.039	-0.041	-0.034	-0.006	0.003	0.003	0.002
MSFT	0.036	0.042	0.042	0.035	0.029	0.029	0.023	0.016	0.015	0.008	0.005	0.005
PFE	-0.006	-0.001	-0.002	-0.002	0.001	0.004	0.004	0.003	0.004	0.004	0.002	0.003
PG	0.003	0.001	-0.002	-0.002	-0.005	-0.010	-0.011	-0.005	-0.006	-0.008	-0.006	-0.005
SBC	-0.141	-0.126	-0.119	-0.118	-0.118	-0.115	-0.115	-0.116	-0.112	-0.097	-0.096	-0.102
UTX	-0.056	-0.042	-0.038	-0.025	-0.016	-0.012	-0.008	-0.008	-0.003	-0.002	0.000	0.002
WMT	0.001	0.008	0.008	0.005	0.004	0.005	-0.001	0.001	0.003	0.003	0.004	0.005
XON	0.161	0.150	0.146	0.115	0.092	0.079	0.080	0.071	0.065	0.063	0.063	0.046
Average	-0.109	-0.082	-0.079	-0.085	-0.085	-0.064	-0.057	-0.049	-0.028	-0.012	-0.003	0.005