Endogenous Dollarization, Expectations, and Equilibrium Monetary Policy*

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1 Introduction

Emerging market countries have trouble floating, and many that claim to float do not deliver on such promises. That is a main conclusion of much recent empirical work, starting with the papers of Calvo and Reinhart (2002) and Stein et al (1999). The reason for this would seem to be dollarization of liabilities and balance sheet effects. Calvo (1999 and 2000), Krugman (1999 and 2000), Stein, Hausmann, Gavin, and Pagés-Serra (1999), and Aghion, Bachetta and Banerjee (1999), among others, make that case. If debts are denominated in dollars while firms depend on local currency revenues (or, more precisely, revenues increase with the relative price of goods produced at home), sharp and unexpected changes in relative prices matter for financial stability. The policy conclusion that emerges from this line of work is that flexible exchange rates can be destabilizing, and therefore emerging market nations would be well advised to design alternative arrangements, including currency boards and dollarization.

Such a view has become extremely influential. But even its most ardent advocates understand that it is only half the story. The claim is that floating is infeasible for a *given* dollarized debt portfolio. But of course portfolio choices – for instance, what shares of debt to hold in peso and dollar-denominated bonds– depend on the risk-return characteristics of these securities, which in turn depend on the structure of shocks and expected monetary and exchange rate policies. Following standard asset pricing and portfolio choice models, variances and covariances (especially with consumption) should matter. Several authors –Ize and Levy-Yeyati (2003), Ize and Parrado (2003) and Morón and Castro (2003) among others– have recently used this approach, in partial equilibrium, to model endogenous dollarization in emerging markets.

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So the message is that exchange rate policies depend on portfolio choices, and portfolio choices depend on anticipated exchange rate policies. Once one phrases the issue this way, a the inevitable next question is: what are the outcomes of this mutual interaction? In particular, how are policy regimes determined under this mutual causation? Is there a single outcome, or several ones? These are the issues that this paper focuses on.

We build an extremely simple model of a small open economy with an incomplete menu of assets: domestic residents can only borrow internationally using bonds denominated in domestic currency or foreign currency.[1] There are sticky wages, so that nominal exchange rate policies matter. One policy trade-off is the textbook one: in the presence of real external shocks, flexible rates stabilize labor supply and output at the expense of making the real exchange rate more volatile. The other trade-off is the one emphasized by the recent literature: unexpected changes in the real exchange rate affect wealth, and can exacerbate the volatility of domestic consumption.

In such a model, consider what happens if agents first write their contracts (involving wages and portfolio choices) and then the policymaker chooses the exchange rate regime (whether to fix the nominal exchange rate or to fix the domestic price level and let the exchange rate float). We show that floating is generally an equilibrium policy regime: if agents expect the central bank will float, they will arrange their wage and debt contracts accordingly, and given these the authorities will indeed find it optimal to float. But for certain parameters fixing is also an equilibrium: if agents expect fixing, the wages and portfolios they choose can make it optimal for the central bank to fix *ex post*. That is, we can have multiple equilibria in policy regimes.

The story in this paper is related to that in Chamon and Hausmann (2002). They consider a model with costly bankruptcy of domestic firms, which can be caused by large unexpected changes in the real exchange rate if firms have large dollar liabilities. The Central Bank can react to shocks by allowing the interest rate or the exchange rate to move. If domestic firms expect a policy of stable exchange rates they will borrow in dollars, which *ex post* may cause the monetary authority to validate such expectations for fear of bankrupting the firms. Hence one can have multiple equilibria in monetary policies.

A paper with self-validating policy regimes is Corsetti and Pesenti (2002). They study price-setting by firms and the choice of monetary policy by the government. There can be two equilibria. In one, firms preset prices in domestic currency only, and foreign-currency prices are determined by the law of one price. Floating exchange rates are then the optimal policy regime. In the second equilibrium firms preset prices in local currency, and a monetary union is the optimal policy choice.

The policy message here is similar to that in Caballero and Krishnamurty (2004). They consider in which financial market imperfections lead agents to under-provide insurance against liquidity shocks. In that model floating the exchange rate is powerless to ameliorate shocks once the quantity of insurance has

[1] Later we also consider an indexed bond.

been chosen, but can help *ex ante* to induce agents to take greater precautions against shocks. Hence Caballero and Krishnamurty argue for precommitting to a float, though for reasons very different from ours.

The next section outlines the basic model, and section 3 presents the basic results. Section 4 extends the analysis to the case of bonds indexed to the price of domestic output, while section 5 concludes.

2 The model

We consider a single-period, small open economy populated by two kinds of private agents, households and firms, and a central bank. The representative household owns the typical firm and receives its profits.

There are two goods, one produced at home and one produced abroad. The two goods are imperfect substitutes. Both are tradable. For simplicity, we assume that domestic households consume only the foreign good.

There is a domestic currency we call *peso* and a foreign currency we call *dollar*. Foreign goods have a constant price of one in terms of dollars, so we speak indistinctly of dollars and foreign goods.

To finance operations, at the beginning of the period the typical firm must borrow from the world market, here represented by a continuum of risk-neutral lenders. The key assumption in this section is that the firm can borrow or lend in pesos or dollars. Therefore, the firm's optimal borrowing policy determines the degree of "dollarization" in the economy, and will be influenced by the firm's expectations about equilibrium prices and the exchange rate. The latter are determined by the monetary policy chosen by the central bank, which in turn takes into account the degree of dollarization.

2.1 Firms

The representative firm has access to the technology

$$Y = AK^{\alpha}L^{1-\alpha} \tag{1}$$

where the capital stock is of fixed size K and labor is to be chosen. Households are heterogeneous in the labor services they provide, and the input L is an aggregate of the services of the different households in the economy:

$$L = \left[\int_0^1 L_i^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where we have indexed workers by i in the unit interval, L_i denotes the services purchased from household i, and $\theta > 1$ is the elasticity of demand for household i's services.

Let W_i denote the wage charged by worker i and W denote the aggregate wage, that is, the minimum cost of a unit of the L aggregate, expressed in terms of pesos. Cost minimization yields the demand for household i's labor:

$$L_i = \left(\frac{W_i}{W}\right)^{-\theta} L \tag{3}$$

The firm has no capital to start with, so it must finance capital purchases by borrowing abroad. To do this, at the beginning of the period, the firm sells bonds denominated either in pesos (B) or dollars (B^*). A peso (resp. dollar) bond is a promise to a peso (resp. dollar) at the end of the period. We assume that the world interest rate in dollars is zero, so a dollar bond must sell for one dollar. Letting Q denote the price in dollars of a peso bond, it follows that

$$QB + B^* = K, \tag{4}$$

End of period firm profits, in dollars, are denoted by Π and given by

$$S\Pi = PY - WL - SB^* - B, \tag{5}$$

where S is the exchange rate (in pesos per dollar) and P the peso price of home output. As usual, we assume that L is chosen at the end of the period, after uncertainty has been revealed.

Since the firm is owned by the representative household, it is natural to assume its objective function be $E\{u'(C)\Pi\}$, where $u'(C)$ is the marginal utility of the household's consumption, to be derived below, and $E\{.\}$ denotes the expectation at the beginning of the period. Hence the firm chooses B, B^*, and a contingent plan for L so maximize $E\{u'(C)\Pi\}$ subject to 3, 4, and 5. The solution is

$$E\left\{u'(C)\left(Q - \frac{1}{S}\right)\right\} = 0, \tag{6}$$

and

$$WL = (1-\alpha)PY, \tag{7}$$

which are standard.

2.2 Households

As mentioned before, households provide differentiated labor services, so each household enjoys some monopoly power in the labor market. We assume that, at the beginning of the period, each household sets a wage in pesos, and commits to satisfy demand forthcoming at that wage at the end of the period. The household consumes the dollar value of its labor income plus firm profits.

Formally, household i chooses its wage, W_i, to maximize

$$E\left\{u(C_i) - \left(\frac{\theta-1}{\theta}\right)v(L_i)\right\} \tag{8}$$

subject to

$$SC_i = S\Pi + W_i L_i \tag{9}$$

and to the labor demand function 3. The functions u and v satisfy usual assumptions. The optimal wage solves

$$WE\left\{\frac{u'(C)L}{S}\right\}=E\left\{Lv'(L)\right\}, \tag{10}$$

where we have imposed symmetry and eliminated i subscripts.

2.3 Foreign lenders

Foreign lenders are risk neutral and only care about foreign goods. Hence they will buy peso bonds if and only if their expected return, in dollars, equals the dollar world return of zero. Therefore,

$$Q=E\left\{\frac{1}{S}\right\} \tag{11}$$

2.4 Monetary and exchange rate policy

We consider exchange rate policies given by the rule

$$P^{\varepsilon}S^{1-\varepsilon}=z,\ 0\leq\varepsilon\leq 1. \tag{12}$$

where ε can be either 0 or 1, and z is a constant. Fixing occurs when $\varepsilon=0$ and therefore $S=z$; flexible rates plus domestic price-targeting occurs when $\varepsilon=1$ and therefore $P=z$. (In what follows we shall examine only those two polar alternatives, but adopting a generic formulation in which ε can take any value in the unit interval saves us from having to write certain expressions more than once.) Until further notice assume $z=1$.

2.5 Market clearing

Since local residents do not consume home goods, the demand for home output comes from foreigners. We assume that the value of the foreign demand for home produce is exogenous and given by a random variable X. This is the only source of uncertainty in the model.

As a consequence, the demand function for domestic output is simply given by $PY=SX$ or

$$Y=\left(\frac{S}{P}\right)X, \tag{13}$$

Using 1, 7, and 12, the supply function of home output can be written as

$$Y=\beta\left(\frac{P}{S}\right)^{\eta}AK \tag{14}$$

where $\beta\equiv\left(\frac{(1-\alpha)A}{W}\right)^{\frac{1-\alpha}{\alpha}}$ and $\eta\equiv\left(\frac{1-\alpha}{\alpha}\right)(1-\varepsilon)$.

Combining demand and supply to eliminate Y we obtain a solution for the terms of trade $\left(\frac{S}{P}\right)$:

$$X = A\beta K \left(\frac{P}{S}\right)^{1+\eta} \tag{15}$$

For future use, 7 and 14 can also be combined to yield

$$L = \left(\frac{1-\alpha}{W}\right) SX, \tag{16}$$

which gives labor demand as a function of exports and prices.

Finally, equilibrium household consumption is obtained by combining 9, 4, 5, and 11

$$C = \left(\frac{P}{S}\right) Y - K + \left(E\left\{\frac{1}{S}\right\} - \frac{1}{S}\right) B \tag{17}$$

This says that consumption depends on the dollar value of output $(\left(\frac{P}{S}\right)Y)$, on the total dollar value of debt payments abroad (K) and on the capital gains or losses on peso debt associated with an exchange rate surprise (the last term on the RHS).

3 Equilibrium monetary policy

In describing the interaction between endogenous dollarization and monetary and exchange rate policy, assumptions about timing are crucial. Our assumption is that the central bank chooses the policy regime ($\varepsilon = 0$ or $\varepsilon = 1$) after private contracts have been written but before uncertainty is realized. We consider the equilibrium outcomes, in particular the degree of dollarization, under either flexible rates and fixed rates, and the conditions under which either policy is *credible* in the sense of Chari and Kehoe (1990) and Stokey (1991).

3.1 Flexible exchange rates

We use tildes to denote outcomes under flexible exchange rates. Here $\varepsilon = 1$ and $\tilde{P} = 1$, so 14 implies

$$\tilde{L} = \tilde{\beta}^{\frac{1}{1-\alpha}} K. \tag{18}$$

That is, labor supply is constant. Note $\tilde{\beta} \equiv \left(\frac{(1-\alpha)A}{\tilde{W}}\right)^{\frac{1-\alpha}{\alpha}}$ also gets a tilde, since it involves the nominal wage level $\tilde{W}$ set by households under the expectation of flexible exchange rates.

Since capital is also constant and there are no productivity shocks, it follows that output is constant too. Using 1 and 18 we have

$$\tilde{Y} = \tilde{\beta} A K. \tag{19}$$

In short, a policy of price-targeting *cum* flexible exchange rates manages to stabilize fully both the supply of labor and of domestic output.

The nominal exchange rate, however, is not constant. Using $\varepsilon = 1$ in 15 we have

$$\frac{1}{\tilde{S}} = \frac{X}{A\tilde{\beta}K}, \tag{20}$$

so the nominal exchange rate appreciates when exports are higher. Taking expectations of this expression we have

$$E\left\{\frac{1}{\tilde{S}}\right\} = \frac{E\{X\}}{A\tilde{\beta}K}, \tag{21}$$

so that, other things equal, the expected exchange rate depends both on the mean and the variance of exports, as well as on the contract wage.

What is the optimal portfolio allocation? The first-order condition 6, using 11, can be written as

$$E\left\{u'\left(\tilde{C}\right)\left[E\left\{\frac{1}{\tilde{S}}\right\} - \frac{1}{\tilde{S}}\right]\right\} = 0, \tag{22}$$

which implies

$$\text{Cov}\left\{\frac{1}{\tilde{S}}, u'(\tilde{C})\right\} = 0 \tag{23}$$

So equilibrium portfolios are such that the marginal utility of consumption is orthogonal to the terms of trade. The implication, as we show next, is that B and B^* are chosen so as to make consumption constant.

Consumption can be obtained by modifying 17 to incorporate the fact that under flexible exchange rates $\tilde{P} = 1$:

$$\tilde{C} = \left(\frac{1}{\tilde{S}}\right)\left(\tilde{Y} - \tilde{B}\right) - K + E\left\{\frac{1}{\tilde{S}}\right\}\tilde{B}, \tag{24}$$

So consumption is constant if

$$\tilde{B} = \tilde{Y} = \tilde{\beta}AK, \tag{25}$$

which is the optimal portfolio allocation. The intuition is straightforward. Given that labor effort is constant and so is the wage, the only risk the household faces is exchange rate risk, which can cause the price of domestic output in terms of consumption goods to fluctuate. The firm eliminates this risk on behalf of its owner, the household, by borrowing an amount in pesos equal to the (constant) value of output. This way the household is fully hedged against (real and nominal) exchange rate risk.

The corresponding constant consumption level is

$$\tilde{C} = E\left\{\frac{1}{\tilde{S}}\right\}\tilde{Y} - K = E\{X\} - K, \tag{26}$$

where we have used the expression above for 21. So, in equilibrium with flexible rates, the household consumes the expected dollar value of home output minus the cost of capital.

Note that under this allocation $\tilde{Q}\tilde{B} = E\left\{\frac{1}{S}\right\}\tilde{Y} = E\{X\} > K$ is necessary for consumption to be positive. This means that initially the firm must sell peso bonds with a higher value than its total foreign liability K, and devote some of the proceeds to buying dollar assets. In other words, the firm optimally chooses to be a net creditor in dollars and a net debtor in pesos.

The equilibrium wage can now be calculated from the optimal wage setting condition 10, which reduces to[2]

$$\tilde{W} = \frac{v'\left(\tilde{L}\right)}{u'\left(\tilde{C}\right)E\left\{\frac{1}{S}\right\}} = \frac{v'\left(\tilde{L}\right)\tilde{Y}}{u'\left(\tilde{C}\right)E\{X\}}. \tag{27}$$

Equilibrium labor supply is given by 16, and therefore expected labor supply is

$$\tilde{L} = \left(\frac{1-\alpha}{\tilde{W}}\right)\tilde{Y} = \left(\frac{1-\alpha}{\tilde{W}}\right)A\tilde{\beta}K. \tag{28}$$

3.2 Time consistency of flexible exchange rates

If the monetary authority announces that it will follow a price-targeting/flexible rates policy, and the private sector makes its nominal wage and portfolio choices accordingly, will the monetary authority deliver that policy ex-post? In other words, is floating a rational expectations equilibrium?

Suppose that the central bank has announced a policy of the form 12 with $\varepsilon = 1$ and $z = 1$; if the announcement is credible and the central bank sticks to it, the outcomes are as given in the last subsection. Now we want to consider a deviation to a fixed exchange rate policy ($\varepsilon = 0$). An immediate question is: what is the value of z –and hence of the exchange rate– under the deviation? In order to separate domestic policy concerns from time inconsistency issues associated with foreign debt, we assume that the central bank is constrained to deviations that leave the expected dollar value of pesos unchanged (at its pre-deviation level). This is the natural restriction, since then a deviation implies no expected expropriation of foreign lenders.

Denote outcomes under a deviation to fixed exchange rates by an overbar. Our assumption is that, if the central bank deviates, it sets $S = \bar{S}$ such that the (post-deviation) value $1/\bar{S}$ is given by 21. This requires

$$\frac{1}{\bar{S}} = \frac{E\{X\}}{A\tilde{\beta}K}. \tag{29}$$

It turns out that it is easy to characterize the effects of the deviation on labor supply and consumption, and hence on domestic welfare. After the switch $S = \bar{S}$, so equilibrium labor supply 16 reduces to

$$\bar{L} = \left(\frac{1-\alpha}{\tilde{W}}\right)\bar{S}X, \tag{30}$$

[2] Note that this is not a closed form solution but an implicit equation in $\tilde{W}$, since $\tilde{L}$ and $\tilde{Y}$ depend on $\tilde{W}$.

where, the nominal wage rate is that associated with flexible rates, since it was set before the deviation. Using the definition of $\bar{S}$ from 29 in 30 we obtain

$$\bar{L} = \left(\frac{1-\alpha}{\tilde{W}}\right)\left(A\tilde{\beta}K\right)\frac{X}{E\{X\}}. \tag{31}$$

Hence labor supply is a linear function of X and is random, but expected labor supply is

$$E\left\{\bar{L}\right\} = \left(\frac{1-\alpha}{\tilde{W}}\right)A\tilde{\beta}K = E\left\{\tilde{L}\right\}. \tag{32}$$

In other words, the deviation to fixed exchange rates keeps expected labor supply the same, but increases the variability of labor effort. The latter obtains because fixing the exchange rate means that P, and hence the real wage, must fluctuate in order to accommodate shocks to export demand.

Using 17 evaluated at $\tilde{B} = \tilde{\beta}AK$, consumption under a deviation to fixed rates is given by

$$\bar{C} = X - K + \left[E\left\{\frac{1}{\bar{S}}\right\} - \frac{1}{\bar{S}}\right]\tilde{\beta}AK. \tag{33}$$

Taking expectations of this expression we again have that

$$E\left\{\bar{C}\right\} = E\{X\} - K = \tilde{C} \tag{34}$$

In other words, the deviation also causes a mean-preserving spread in consumption.

The conclusion: by deviating from flexible rates to fixed rates when households had expected the former, the monetary authority induces volatility into labor supply and consumption without changing the expected value of either variable. Since volatility decreases expected utility, the policymaker can only decrease expected utility by switching. It follows that the policy of price-targeting/flexible rates is always a time consistent outcome. This is intuitive, since this policy stabilizes both consumption and labor supply.

3.3 Fixed Exchange Rates

Now consider a policy of fixing the exchange rate at $S = \bar{S} = 1$ (again using overbars to denote fixed rates). Then nominal demand is given by

$$\bar{P}\bar{Y} = X \tag{35}$$

and 16 reduces to

$$\bar{L} = \frac{(1-\alpha)X}{\bar{W}}, \tag{36}$$

Labor supply is hence a linear function of X.

Expression 17 for consumption becomes

$$\bar{C} = X - K. \tag{37}$$

Note that $E\left\{\bar{C}\right\} = E\{X\} - K$. The variance of consumption is equal to the variance of exports.

Note also that B and B^* are indeterminate, since bonds in pesos and dollars are now perfect substitutes. This means that portfolio composition may be pinned down by other things outside the model.

To complete the characterization of this allocation use the optimal wage-setting condition 10, which reduces to

$$\bar{W} = \frac{E\left\{\bar{L}v'(\bar{L})\right\}}{E\left\{u'(\bar{C})\bar{L}\right\}} = \frac{E\left\{Xv'(\bar{L})\right\}}{E\left\{u'(\bar{C})X\right\}}, \tag{38}$$

where the equilibrium levels of L and C can be obtained from 36 and 37 above.

3.4 Time consistency of fixed exchange rates

To check whether fixing is an equilibrium exchange rate policy, consider deviating to a fixed P policy such that $E\left\{\frac{1}{\tilde{S}}\right\} = 1$. Again, the justification for such a restriction is to ensure that the deviation imposes no expected expropriation on foreigners.

In the case of a switch to floating and $P = \tilde{P}$, expression 15 for the exchange rate can be written as

$$A\bar{\beta}K\tilde{P} = \tilde{S}X. \tag{39}$$

Taking expectations and imposing $E\left\{\frac{1}{\tilde{S}}\right\} = 1$, $\tilde{P}$ must be given by

$$\tilde{P} = \frac{E\{X\}}{A\bar{\beta}K} \tag{40}$$

Using this in 15 again yields the exchange rate after the deviation,

$$\tilde{S} = \frac{E\{X\}}{X}. \tag{41}$$

Applying this to 16 we have that labor supply is given by

$$\tilde{L} = E\left\{\bar{L}\right\} = \frac{(1-\alpha)E\{X\}}{\bar{W}}. \tag{42}$$

Comparing this last expression with 36 we see that the switch to floating has the effect of rendering L constant at its expected value under fixing, so labor supply is no longer variable. Hence there is a "temptation" to abandon fixed rates.

To find the effect on consumption of a switch to flexible rates, use 41 in 17 evaluated at $E\left\{\frac{1}{\tilde{S}}\right\} = 1$. This yields

$$\tilde{C} = X\left(1 - \frac{\bar{B}}{E\{X\}}\right) + \bar{B} - K. \tag{43}$$

Hence the effect of the deviation on consumption depends on the degree of dollarization, which is indeterminate under fixed rates. But notice that $E\left\{\tilde{C}\right\} = E\{X\} - K$, so the deviation keeps the expected value of consumption constant. Therefore, whether the deviation increases or reduces the expected utility of consumption depends on the implications for the variability of consumption.

From 43 we see that after a switch the variance of consumption is equal to the variance of exports times $\left(1 - \frac{B}{E\{X\}}\right)^2$. So the switch makes the variance of consumption fall if $0 < B < 2E\{X\}$, and it makes it rise otherwise. If the variance falls, the policymaker will unambiguously want to switch, since that would reduce the variance of both consumption and labor supply while preserving their expected values. Notice, for instance that by fluke agents could have adopted the portfolio that corresponds to the expectation of flexible rates: with $E\left\{\frac{1}{S}\right\} = 1$, $\tilde{B} = E\{X\}$. In that case the variance of consumption after the switch would be zero, and switching would be the optimal action for the policymaker.

It follows that a necessary condition for fixed rates to be an equilibrium is either $B < 0$ or $B > 2E\{X\}$. The first case is perhaps the more interesting one: the representative agent has gross assets in pesos and gross debts in dollars. In equilibrium, this currency mismatch makes no difference to him nor to lenders. But it deters the government from abandoning fixed exchange rates. Dollarization of liabilities gives rise to fear of floating.

Summarizing: if is either $B < 0$ or $B > 2E\{X\}$, the switch to flexible rates induces a mean-preserving spread on consumption relative to fixed exchange rates. Since the deviation labor effort at its mean value under fixed rates, fixed exchange rates may or may not be an equilibrium. This depends on the parameters of the model and, in particular, of the utility cost associated with consumption fluctuations relative to labor effort fluctuations (determined by the shape and curvature of u and v). But also, and importantly for our purposes, the credibility of fixed exchange rates depends on the degree of dollarization, which is not determined by equilibrium considerations.

If fixed exchange rates are in fact an equilibrium, then there are multiple equilibria in policy regimes, since flexible exchange rates are always an equilibrium. In that case, animal spirits play a role: if agents expect fixed rates (and the currency composition of portfolios satisfies $B < 0$ or $B > 2E\{X\}$.) the government will indeed deliver fixed rates; if agents expect flexible rates, the government will choose flexible rates.

Example: Suppose

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma} \tag{44}$$

and

$$v(L) = \frac{\kappa}{2} L^2. \tag{45}$$

The equilibrium wage is then given by inserting 37 and 36 in 38:

$$\bar{W}^2 = \frac{\kappa(1-\alpha)E\left\{X^2\right\}}{E\left\{X\right\}(X-K)^{-\sigma}} \tag{46}$$

One can then calculate that

$$E\left\{v(\bar{L})\right\} - E\left\{v(\tilde{L})\right\} = \frac{(1-\alpha)}{2}\left[E\left\{X\right\}(X-K)^{-\sigma}\right]\frac{\text{Var}\,(X)}{E\left\{X^2\right\}} > 0 \tag{47}$$

which gives the temptation to abandon fixed rates.

One can also calculate

$$E\left\{u(\bar{C})\right\} - E\left\{u(\tilde{C})\right\} = \frac{1}{1-\sigma}\left[E\left\{(X-K)^{1-\sigma}\right\} - E\left\{X-K+B\left(1-\frac{X}{E\left\{X\right\}}\right)\right\}\right]^{1-\sigma} \tag{48}$$

The arguments above imply that $E\left\{u(\bar{C})\right\} - E\left\{u(\tilde{C})\right\} > 0$ if $B < 0$ or $B > 2E\left\{X\right\}$. Assume that is the case. Then, note that fixed exchange rates are an equilibrium if

$$E\left\{u(\bar{C})\right\} - E\left\{u(\tilde{C})\right\} - \left(\frac{\theta-1}{\theta}\right)\left[E\left\{v(\bar{L})\right\} - E\left\{v(\tilde{L})\right\}\right] > 0 \tag{49}$$

For any given B, this condition is satisfied if either θ or α are close enough to one.

4 Indexed bonds

In emerging markets bonds denominated in domestic currency are often indexed. To assess the consequences of having that kind of security available to domestic agents, next we modify the model by replacing peso bonds by bonds with payoffs indexed to the price of the domestic good. More precisely, here we assume that the representative firm sells dollar bonds and indexed bonds. An indexed bond is a promise to P pesos at the end of the period.

Rather than developing the model from scratch, we simply write down here the equilibrium conditions that are different from the earlier formulation. Foreign lenders again arbitrage the returns on both kinds of loans, so the initial price of an indexed bond, in dollars, must be $E\left\{Z\right\}$, where we have defined the relative price of home goods (the terms of trade) as

$$Z \equiv \frac{P}{S}. \tag{50}$$

The first order condition 10 for nominal wages remains the same, while first order condition 6 for optimal portfolio shares becomes

$$E\left\{u'(C)\left(E\left\{Z\right\} - Z\right)\right\} = 0, \tag{51}$$

Equilibrium conditions 13, 14, 15 and 16 are the same, while expression 17 for consumption becomes

$$C = ZY - K + B\left(E\left\{Z\right\} - Z\right) \tag{52}$$

where B now denotes the home output value of stock of indexed bonds.

4.1 Flexible exchange rates redux

Consider first the case of price targeting (with $\tilde{P} = 1$) plus flexible exchange rates. Assuming that this policy is credible and indeed carried out, indexed bonds become identical to peso bonds. Hence the equilibrium outcomes are just the same as flexible exchange rates in the model with peso bonds, and given by 3.1. Notice in particular that 25 still holds, so that the optimal portfolio involves

$$\tilde{B} = \tilde{Y}, \tag{53}$$

so that indexed bond-holdings are equal to the value of home output.

Indexed bonds do make a difference, however, in considering the implications of a deviation towards fixed exchange rates. Again to prevent expected expropriation of foreign lenders, we assume that such a deviation leaves the expected terms of trade, Z, unchanged. In other words, using overbars once more to denote the consequences of a deviation to fixed rates, we impose

$$E\left\{\bar{Z}\right\} = E\left\{\tilde{Z}\right\} = \frac{E\left\{X\right\}}{\tilde{Y}}, \tag{54}$$

where the last equality follows from 13.

To solve for the consequences of a deviation to fixed rates, note that 16 implies

$$\bar{L} = \frac{(1-\alpha)\bar{S}X}{\tilde{W}} \tag{55}$$

Moreover, from 13, we have

$$\bar{P} = \frac{\bar{S}X}{\bar{Y}} = \frac{\bar{S}X}{AK^{\alpha}\bar{L}^{1-\alpha}} = \left(\frac{\bar{S}X}{\tilde{Y}}\right)^{\alpha}, \tag{56}$$

where the last equality follows from 55 and 25. It follows that

$$\bar{Z} = \frac{\bar{P}}{\bar{S}} = \left(\frac{X}{\tilde{Y}}\right)^{\alpha} \bar{S}^{\alpha-1} \tag{57}$$

Taking expectations and using 54 one obtains

$$\bar{S} = \left(\frac{E\left\{X^{\alpha}\right\}}{E\left\{X\right\}}\right)^{1-\alpha} \tilde{Y} \tag{58}$$

This is the value at which the exchange rate needs to be fixed if the central bank deviates from flexible rates. Inserting this value in 56 and simplifying one gets

$$\bar{P} = X^{\alpha} \left(\frac{E\{X^{\alpha}\}}{E\{X\}} \right)^{\frac{\alpha}{1-\alpha}}. \tag{59}$$

So, in particular,

$$E\{\bar{P}\} = \left(\frac{E\{X^{\alpha}\}}{[E\{X\}]^{\alpha}} \right)^{\frac{1}{1-\alpha}} < 1. \tag{60}$$

Before the switch the price level was 1. After the switch it is given by 60. Hence the expected price falls. It follows that the expected real wage rises, and expected labor effort falls. This last point can be seen in the following way. From 55, 58 and the definition of $\tilde{Y}$ one can derive

$$\bar{L} = \tilde{L} \frac{X}{E\{X\}} E\{\bar{P}\}. \tag{61}$$

Taking expectations we have

$$E\{\bar{L}\} = \tilde{L} E\{\bar{P}\} < \tilde{L}. \tag{62}$$

Hence, the deviation to fixed rates implies that labor effort becomes variable but, in contrast with the case of peso bonds, the mean value of labor effort falls. The reduction in mean labor effort is welfare-improving, making a switch towards fixed rates attractive.[3]

As in the case of peso bonds, the switch from flexible to fixed exchange rates causes a mean-preserving spread in consumption (the proof is similar to the one in the case of peso bonds and left to the interested reader.) Additional consumption variability makes expected welfare fall and reduces the desirability of the deviation, as in the case of peso bonds. However, with indexed bonds, mean labor effort falls. Flexible rates are an equilibrium if the utility benefit associated with the smaller labor effort is less than the cost associated with increased variability in both consumption and labor.

4.2 Fixed exchange rates redux

Consider next the policy of fixing the exchange rate at $\bar{S} = 1$. Condition 36 still gives labor effort, and from 35 and the production function we obtain the price of home output

$$\bar{P} = \frac{X}{\bar{Y}} = \frac{X^{\alpha}}{AK^{\alpha}} \left(\frac{\bar{W}}{1-\alpha} \right)^{1-\alpha}. \tag{63}$$

[3] The fact that an increase in labor supply is welfare-decreasing might seem surprising, since the model features imperfect competition in the labor market, which causes equilibrium labor supply to be too low relative to the planner's solution. But in this model the dollar value of domestic production is given by 13. Hence working more just causes the terms of trade to turn against the country, without any benefit for consumption.

Consumption in this case is

$$\bar{C} = X - K + \bar{B}\left(E(\bar{P}) - \bar{P}\right) \tag{64}$$

So, in general, consumption is variable and depends on $\bar{B}$, because of the presence of indexed bonds instead of peso bonds. And in contrast with the case of peso bonds, $\bar{B}$ is not indeterminate. Instead, it must be set to satisfy the condition 51, which here reduces to

$$\text{Cov}\left(u'(\bar{C}), \bar{P}\right) = 0 \tag{65}$$

This case turns out to be fairly complex, so for concreteness we assume from now on that $u(C)$ is quadratic (at least in the relevant range). Then u' is linear in C, and the previous expression reduces to

$$\text{Cov}\left(\bar{C}, \bar{P}\right) = 0 \tag{66}$$

In other words, equilibrium portfolios must be set so that consumption is orthogonal to the terms of trade. Using the previous expressions for $\bar{P}$ and $\bar{C}$ one readily finds that the stock of indexed bonds in the equilibrium portfolio is

$$\bar{B} = \frac{\text{Cov}(X, X^{\alpha})}{\text{Var}\,(X^{\alpha})} AK^{\alpha} \left(\frac{1-\alpha}{\bar{W}}\right)^{1-\alpha} \tag{67}$$

Using 15 evaluated at $\eta = \frac{1-\alpha}{\alpha}$ and the definition of β, 67 can also be written

$$\frac{\bar{B}}{E\left\{\bar{Y}\right\}} = \frac{\text{Cov}(X, X^{\alpha})}{\text{Var}\,(X^{\alpha})\, E\left\{X^{1-\alpha}\right\}} \tag{68}$$

Hence again the stock of bonds is proportional to expected output.

Replacing 67 in 64 yields equilibrium consumption:

$$\bar{C} = X - K + \frac{\text{Cov}(X, X^{\alpha})}{\text{Var} X^{\alpha}} \left[E\left\{X^{\alpha}\right\} - X^{\alpha}\right] \tag{69}$$

Now consider the possibility of a deviation to fixed rates, imposing once more the restriction of no expropriation to foreign lenders, which requires that the post-deviation expected value $E\left\{\frac{\tilde{P}}{\tilde{S}}\right\}$ must equal $E\left\{\bar{P}\right\}$.

After the switch, 7 must hold, which together with the production function yields labor effort:

$$\tilde{L} = A\bar{\beta}K\left(\frac{(1-\alpha)}{\bar{W}}\right)\tilde{P}^{\frac{1}{\alpha}}, \tag{70}$$

where $\tilde{P}$ is the fixed value of P after the deviation, to be determined shortly.

Since 13 must hold, $\tilde{P}/\tilde{S} = X/\tilde{Y}$. Taking expectations on both sides and using the production function and 70 one obtains

$$E\left\{\frac{\tilde{P}}{\tilde{S}}\right\} = \frac{E\left\{X\right\}}{KA^{\frac{1}{\alpha}}}\left(\frac{(1-\alpha)\tilde{P}}{\bar{W}}\right)^{-\frac{1-\alpha}{\alpha}}. \tag{71}$$

But this has to be equal to $E\left\{\bar{P}\right\}$, where $\bar{P}$ is given by 63. So, taking expectations in 63, equating the result to the preceding equation and rearranging gives the required value of $\tilde{P}$:

$$\tilde{P} = \left[\frac{E\left\{X\right\}}{E\left\{X^{\alpha}\right\}}\right]^{\frac{\alpha}{1-\alpha}} \frac{1}{AK^{\alpha}} \left(\frac{\bar{W}}{1-\alpha}\right)^{1-\alpha} \tag{72}$$

Replacing in the equation for $\tilde{L}$ above we obtain

$$\tilde{L} = \left[\frac{\left(E\left\{X\right\}\right)^{\alpha}}{E\left\{X^{\alpha}\right\}}\right]^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\bar{W}}\right) E\left\{X\right\} > \left(\frac{1-\alpha}{\bar{W}}\right) E\left\{X\right\} = E\left\{\bar{L}\right\} \tag{73}$$

The inequality follows from Jensen's inequality. Switching to flexible rates stabilizes labor effort, but at a level that is higher than the mean value of L under fixed rates. The sum of these two effects on the representative household's welfare is ambiguous and depends on the parameters of the model.

But those are not the only effects of a possible switch. The effect of the deviation on consumption can be calculated from

$$\tilde{C} = X - K + \bar{B}\left[E\bar{P} - \frac{\tilde{P}}{\tilde{S}}\right]. \tag{74}$$

Recalling that $\frac{\tilde{P}}{\tilde{S}} = \frac{X}{Y}$ and after some tedious algebra one obtains

$$\tilde{C} = X - K + \frac{\mathrm{Cov}(X, X^{\alpha})}{\mathrm{Var}\left(X^{\alpha}\right)} \frac{E\left\{X^{\alpha}\right\}}{E\left\{X\right\}} \left[E\left\{X\right\} - X\right]. \tag{75}$$

Recalling 69, one readily notices that $E\bar{C} = E\tilde{C}$. Hence the deviation leaves the expected value of consumption unchanged. But the effect on consumption variance is unclear, although the expressions for $\bar{C}$ and $\tilde{C}$ reveal that it depends solely on α and the distribution of X.

(This part to be completed)

5 Final Remarks

One limitation of the analysis is that dollarization is endogenous, but only given the exogenous restrictions on the menu of assets. While we have allowed for an asset menu that included more than the usual non-contingent world currency bonds, it may be desirable and useful to derive market incompleteness from more fundamental assumptions on the environment. That remains a substantial task, however, and at this point we can only leave it for future research.

A second limitation, of course, is that we have imposed strong restrictions on the environment and policy options. These restrictions were justified on the basis of tractability and analytical convenience, but obviously they will have to be relaxed if the model is to be the basis for actual policy evaluation.

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