Discussion of

Futures Prices as Risk-Adjusted Forecasts of Monetary Policy

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Summary of Paper

- Question: is it true that

$$\widetilde{F}_t^{(n)} = \mathbb{E}_t[\widetilde{R}_{t+n}]? \tag{1}$$

  Here: $\widetilde{R}_t$ is Fed Funds Rate, $\widetilde{F}_t^{(n)}$ is Fed Funds Future Rate.

- Answer: no! In order to obtain $\mathbb{E}_t[\widetilde{R}_{t+n}]$ futures rates have to be risk-adjusted.

- Findings:

  - Adjustments are substantial: $\widetilde{F}_t^{(4)} = \mathbb{E}_t[\widetilde{R}_{t+4}]+50$ basis points. (1 quarter horizon)

  - Adjustments increase in recessions (negatively correlated with employment).

  - Risk adjustments improve Fed Funds forecasts

- Very interesting empirical work!

Context (I)

- Reduced Form VAR:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + u_t, \quad u_t \sim (0, \Sigma_u) \tag{2}$$

- Structural VAR:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \Sigma_u^{chol} \Omega \epsilon_t, \quad \epsilon_t \sim (0, I) \tag{3}$$

  ϵ_t's are "structural", Ω is orthonormal and cannot be estimated.

- Alternative:

$$\text{Monetary Policy Shock } = \epsilon_{R,t} \equiv R_t - \mathbb{E}_{t-\delta}[R_t] \stackrel{?}{=} F_{t-\delta}^{\delta} \tag{4}$$

Context (II)

- Suppose monetary policy follows:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \left(\frac{X_t}{X_t^*}\right)^{\psi_2}\right]^{(1-\rho_R)} e^{\epsilon_{R,t}}. \quad (5)$$

- Then (take log deviations: $\widetilde{R}_t = \ln(R_t/R^*)$):

$$\widetilde{R}_t - I\!E_{t-1}[\widetilde{R}_t] = \epsilon_{R,t} + (1-\rho_R)\psi_1(\widetilde{\pi}_t - I\!E_{t-1}[\widetilde{\pi}_t]) + (1-\rho_R)\psi_2(\widetilde{X}_t - I\!E_{t-1}[\widetilde{X}_t]) \quad (6)$$

- Ideally we want

$$\widetilde{R}_t - I\!E[\widetilde{R}_t | \mathcal{F}_{t-1}, \pi_t, \widetilde{X}_t] \quad (7)$$

  to identify policy shock. Generates some delicate timing issues.

- E.g., Faust, Swanson, and Wright (JME, 2004). But Monika and Eric's paper does not focus on VAR identification...

Outline for Remainder of Discussion

- What does a prototypical monetary DSGE model have to say about the adjustment?
- Introduce Fed Funds Futures
- Derive a pricing formula, assuming log-normality: conditional covariance of real rates and output growth / consumption growth matter.
- Estimate VAR and calculate adjustment, compare to Monika and Eric's estimates.
- Outlook

A Simple Monetary DSGE Model (I)

- Households maximize:

$$E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\tau} - 1}{1-\tau} + \chi \log \frac{M_s}{P_s} - h_s \right) \right], \tag{8}$$

- Consumption C_t is composed of differentiated products:

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \tag{9}$$

- Budget constraint:

$$C_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} + \frac{T_t}{P_t} = W_t h_t + \frac{M_{t-1}}{P_t} + R_{t-1}\frac{B_{t-1}}{P_t} + Q_{t-1}\frac{F_{t-1} - R_t}{P_t} + D_t. \tag{10}$$

Moreover, households have access to a complete set of state-contingent claims.

A Simple Monetary DSGE Model (II)

- Household has access to Fed Future contracts. Budget constraint:

$$\ldots = \ldots + Q_{t-1}\frac{F_{t-1} - R_t}{P_t} + \ldots$$

- At $t-1$ choose quantity Q_{t-1}, but no payments yet. Contracts are settled in period t. F_{t-1} is Futures rate.

- Production side: monopolistic competition, Calvo-style price rigidities.

- Monetary policy:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi^*}\right)^{\psi_1}\left(\frac{X_t}{X_t^*}\right)^{\psi_2}\right]^{(1-\rho_R)} e^{\epsilon_{R,t}}. \tag{11}$$

A Simple Monetary DSGE Model (II)

- First-order conditions imply

$$\mathbb{E}_{t-1}\left[\left(\frac{C_t}{C_{t-1}}\right)^{-\tau}\frac{F_{t-1}}{\pi_t}\right] = \mathbb{E}_{t-1}\left[\left(\frac{C_t}{C_{t-1}}\right)^{-\tau}\frac{R_t}{\pi_t}\right] \tag{12}$$

- which yields

$$F_{t-1} = \frac{\mathbb{E}_{t-1}\left[\left(\frac{C_t}{C_{t-1}}\right)^{-\tau}\frac{R_t}{\pi_t}\right]}{\mathbb{E}_{t-1}\left[\left(\frac{C_t}{C_{t-1}}\right)^{-\tau}\frac{1}{\pi_t}\right]} \tag{13}$$

- Define $M_t^C = \frac{(C_t/C_{t-1})^{-\tau}}{\pi_t}$, then

$$F_{t-1} = \frac{\mathbb{E}_{t-1}[M_t^C R_t]}{\mathbb{E}_{t-1}[M_t^C]} \tag{14}$$

A Simple Monetary DSGE Model (II)

- Assume M_t^C and R_t are jointly log-normally distributed conditional on time $t-1$ information.
- Use $\widetilde{F}_{t-1}$, $\widetilde{M}_t^C$, and $\widetilde{R}_t$ to denote percentage deviations from steady state.
- Then,

$$\begin{aligned}
\widetilde{F}_{t-1} &= \mathbb{E}_{t-1}[\widetilde{R}_t] + \mathbb{E}_{t-1}[\widetilde{M}_t^C] + \frac{1}{2}\Big(var_{t-1}[\widetilde{R}_t] + var_{t-1}[\widetilde{M}_t^C] + 2cov_{t-1}[\widetilde{R}_t, \widetilde{M}_t^C]\Big) \\
&\quad -\mathbb{E}_{t-1}[\widetilde{M}_t^C] - \frac{1}{2}var_{t-1}[\widetilde{M}_t^C] \\
&= \mathbb{E}_{t-1}[\widetilde{R}_t] + \underbrace{\frac{1}{2}var_{t-1}[\widetilde{R}_t] + cov_{t-1}[\widetilde{R}_t, \widetilde{M}_t^C]}_{\text{Risk-Adjustment, positive according to estimates}} \qquad (15)
\end{aligned}$$

- Roughly speaking, we need negative conditional correlation between real rate and consumption growth.

Risk Adjustments (I)

- Estimate DSGE-VAR (Del Negro and Schorfheide, *International Economic Review*, 2004) to obtain estimates for conditional variances.

- In my simple framework I cannot explain the time variation in the adjustment that Monika and Eric find unless I introduce heteroskedasticity:

  Recession $\Longrightarrow$ stronger negative correlation between consumption growth and real rate.

- Sample period: 1989:I to 2003:III. 4 lags (selected using Bayesian posterior odds),

  – Output growth, inflation (GDP deflator), Fed Funds Rate

  – Consumption growth, inflation (GDP deflator), Fed Funds Rate

  All rates are quarter-to-quarter percentages.

Risk Adjustments (II)

- Stochastic discount factor:

$$\widetilde{M}_t^X = -\tau(\widetilde{X}_t - \widetilde{X}_{t-1}) - \widetilde{\pi}_t \tag{16}$$

  where $\widetilde{X}_t$ is consumption or output.

- Risk aversion: $\tau = 1, 2, 5, 10$.

Posterior for Adjustment (I)

τ	Mean	CI(low)	CI(high)	P & S
1	0.086	-0.009	0.181	0.499
2	0.121	-0.046	0.290	0.499
5	0.226	-0.175	0.621	0.499
10	0.402	-0.387	1.186	0.499

Note: I report posterior mean 90 % probability intervals based on the output growth VAR.

P & S value: Table 1, $n = 3$.

Posterior for Adjustment (II)

τ	Mean	CI(low)	CI(high)	P & S
1	0.109	0.021	0.201	0.499
2	0.173	0.020	0.324	0.499
5	0.363	0.011	0.708	0.499
10	0.681	-0.012	1.351	0.499

Note: I report posterior mean 90 % probability intervals based on the consumption growth VAR. P & S value: Table 1, $n = 3$.

Further Comments and Outlook

- I enjoyed reading the paper...
- it has very interesting empirical results - certainly relevant for VAR studies that identify policy shocks using Futures data.
- Next step: study pricing models for Fed Funds Futures – fit VAR or DSGE model to obtain estimates of conditional moments needed for pricing formula and check whether we can explain magnitude of adjustment and time-variation.