Discussion of

The Term Structure of Real Rates and Expected Inflation

by Andrew Ang and Geert Bekaert

Martin Evans
Georgetown University, NBER and
I.E.S. Fellow, Princeton University

Overview

- The paper presents a "No Arbitrage" model of nominal interest rates that incorporates regime switching.
- Model estimates allow us to decompose movements in the nominal term structure into variations in (i) real yields, (ii) expected inflation, and (iii) (inflation) risk premia.
- The model estimates imply:
  - the real term structure is fairly flat ~1.4%,
  - real rates are negatively correlated with inflation (expected and unexpected),
  - the inflation risk premium rises with maturity,
  - expected inflation and inflation risk account for ~80% of the variation in nominal rates.

Main Comment: This is a nice paper with a clear and important goal.

A Nominal Yield Decomposition

$$y_t^k = E_t \pi_{t+k,k} + \hat{y}_t^k + \frac{1}{k} E_t \sum_{i=0}^{k-1} \left\{ \theta_{t+i}^{k-i} - \hat{\theta}_{t+i}^{k-i} + \varphi_{t+i} \right\}$$

where

Nominal Term Premia:	$\theta_t^k \equiv E_t \left[p_{t+1}^{k-1} - p_t^k \right] - y_1$
Real Term Premia:	$\hat{\theta}_t^k \equiv E_t \left[\hat{p}_{t+1}^{k-1} - \hat{p}_t^k \right] - \hat{y}_1$
Inflation Risk Premia:	$\varphi_t \equiv y_t^1 - \hat{y}_t^1 - E_t \pi_{t+1,1}$

Observations:

- The absence of arbitrage opportunities only places weak restrictions on θ_t^k, $\hat{\theta}_t^k$, and φ_t.
- Decomposing movements in y_t^k will depend critically on how $E_t \pi_{t+k,k}$, $\hat{y}_t^k$ and the risk premia are identified.
- Previous work on the nominal term structure suggests that θ_t^k is time-varying.

Alternative Identification/Estimation Strategies

Regression Method (Mishkin 1990): Assume $\theta_{t+i}^{k-i} - \hat{\theta}_{t+i}^{k-i} + \varphi_{t+i}$ constant, and $\pi_{t+k,k} = E_t\pi_{t+k,k} + \eta_{t+k,k}$ where $\eta_{t+k,k}$ is a RE forecast error. Estimates of $\hat{y}_t^k$ (and $E_t\pi_{t+k,k}$) are obtained from projecting $y_t^k - \pi_{t+k,k}$ on variables that span the time t information set.

Model-Based 1 (Evans 2003) Identify θ_{t+i}^{k-i} , $\hat{\theta}_{t+i}^{k-i}$ and φ_{t+i} via "No Arbitrage" + other assumptions. Use UK data on y_t^k and $\hat{y}_t^k$ to estimate $E_t\pi_{t+k,k}$.

Model-Based 2 (A&B): Identify θ_{t+i}^{k-i} , $\hat{\theta}_{t+i}^{k-i}$ and φ_{t+i} via "No Arbitrage" + other assumptions. Use US data on y_t^k and $\pi_{t+k,k}$ + RE to estimate $E_t\pi_{t+k,k}$ and $\hat{y}_t^k$.

A&B's Model

$$1 = \quad E_t\left[\exp\left(\hat{m}_{t+1}\right)\mathcal{R}^i_{t+1}\right]$$

$$\hat{m}_{t+1} = \quad -\delta_0 - \delta_1' X_t - \lambda_{t+1}' \lambda_{t+1} - \lambda_{t+1}' \varepsilon_{t+1}$$

$$\delta_1' = \left[\begin{array}{ccc} 1 & 1 & \delta_\pi \end{array}\right] \ \lambda_{t+1}' = \left[\begin{array}{ccc} \gamma_1 q_t & \lambda_f\left(s^f_{t+1}\right) & 0 \end{array}\right]$$

$$X_{t+1} = \quad \mu(s_{t+1}) + \Phi X_t + \Sigma(s_{t+1})^{1/2}\varepsilon_{t+1}$$

$$X_{t+1}' = \left[\begin{array}{ccc} q_{t+1} & f_{t+1} & \pi_{t+1} \end{array}\right] \ \mu(s_{t+1})' = \left[\begin{array}{ccc} 0 & 0 & \mu_\pi(s^\pi_{t+1}) \end{array}\right]$$

$$diag\left[\Sigma(s_{t+1})\right]' = \left[\begin{array}{ccc} \sigma_q^2 & \sigma_f^2(s^f_{t+1}) & \sigma_\pi^2(s^\pi_{t+1}) \end{array}\right]$$

$s^i_{t+1} = \{1, 2\}$, Independent, Markov Chains

Observation 1: The model restricts the inflation risk premium. In particular

$$Cov_t \left[\exp\left(\hat{m}_{t+1}\right), \exp\left(\pi_{t+1,1}\right)\right] = 0,$$

so

$$\varphi_t = -\ln\left[E_t \exp\left(-\pi_{t+1,1}\right)\right] - E_t\pi_{t+1,1} \leq 0.$$

Observation 2: The real and nominal term premia can vary within a regime. For example, suppose there is no switching. Then.

$$\begin{aligned}\hat{\theta}_{2,t} &= Cov_t\left(\hat{m}_{t+1}, r_{t+1}\right) - \frac{1}{2}Var_t\left(r_{t+1}\right), \\ &= \lambda_f\sigma_f + \lambda_1\sigma_q q_t - \frac{1}{2}\delta_1'\Sigma\delta_1.\end{aligned}$$

A&B's estimates $\Rightarrow q_t$ is very persistent. (NB λ_{t+1} is not the price of risk when switching is present).

Observation 3: The model ignores reporting lags in the CPI. There is variable reporting lag in the CPI of approximately 2 weeks, which might be economically significant when inflation is high and variable.

Observation 4: Nominal Yields and Inflation display the same persistence across regimes:

$$
\begin{aligned}
y_t^k &= -\frac{1}{k}\left(A_k(s_t) + B_k X_t\right) \\
\pi_{t,1} &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X_t \\
X_{t+1} &= \mu(s_{t+1}) + \Phi X_t + \Sigma(s_{t+1})^{1/2}\varepsilon_{t+1}
\end{aligned}
$$

- Evidence on state-dependent mean-reversion in short rates is reported by Gray (1996), Bekaert et al (2001) and Ang and Bekaert (2002).
- Evidence on state-dependent mean-reversion in inflation is reported in Evans and Wachtel (1993), and Evans and Lewis (1995).

Comment 1: Consider a representative consumer model with CRRA utility. With conditional log normality and no switching

$$\begin{aligned}\psi_t &= Cov_t\left(m_{t+1}, \pi_{t+1,1}\right) - \frac{1}{2}Var_t\left(\pi_{t+1,1}\right) \\ &= -\gamma Cov_t\left(\Delta c_{t+1}, \pi_{t+1,1}\right) - \frac{1}{2}Var_t\left(\pi_{t+1,1}\right)\end{aligned}$$

How big is $Cov_t\left(\Delta c_{t+1}, \pi_{t+1,1}\right)$? We can estimate $Cov\left(\Delta c_{t+1}, \pi_{t+1,1} | \Omega_t\right)$ where Ω_t denotes the information set available to the researcher, but

$$\begin{aligned}Cov\left(\Delta c_{t+1}, \pi_{t+1,1} | \Omega_t\right) &= E\left[Cov_t\left(\Delta c_{t+1}, \pi_{t+1,1}\right) | \Omega_t\right] \\ &\quad + Cov\left[E_t \Delta c_{t+1}, E_t \pi_{t+1,1} | \Omega_t\right]\end{aligned}$$

$\Rightarrow Cov\left(\Delta c_{t+1}, \pi_{t+1,1} | \Omega_t\right)$ may be biased.

Comment 2: The introduction of switching has more potential than the model allows. For example, we could have

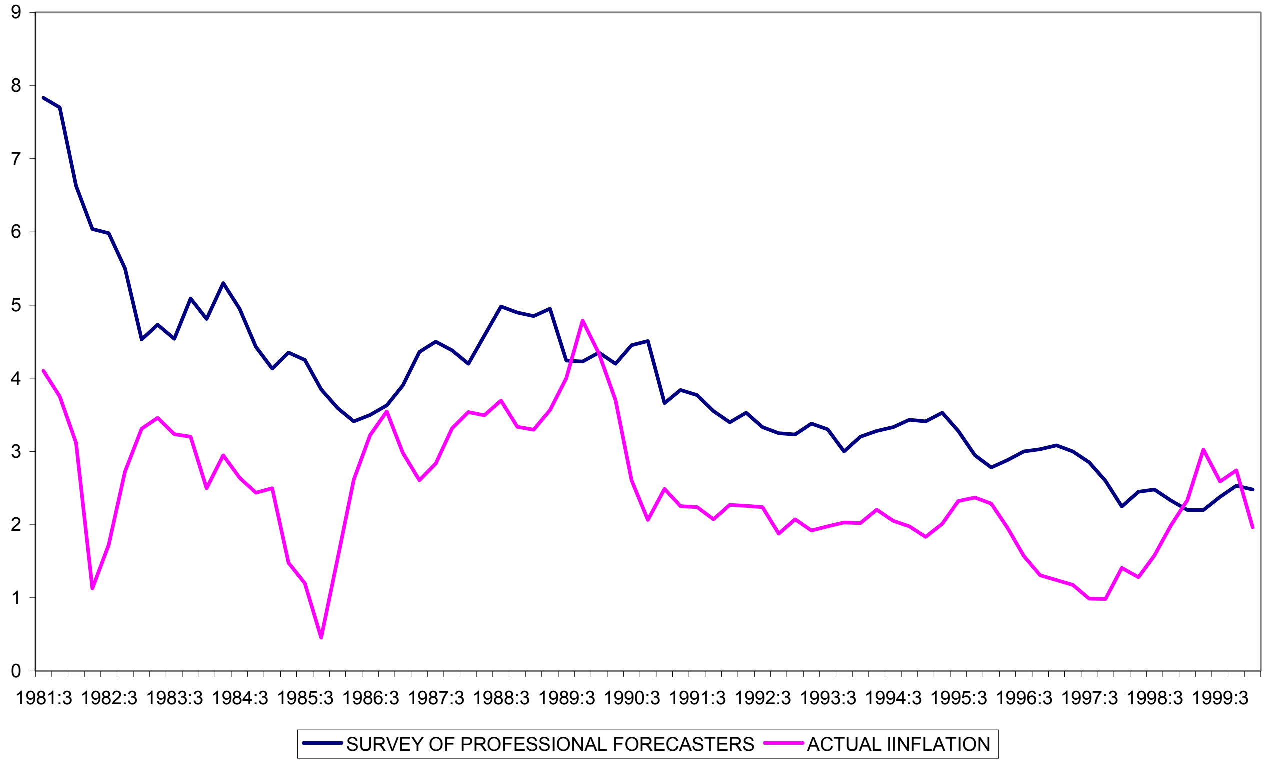
$$X_{t+1} = \mu(s_t) + \Phi(s_t)X_t + \Sigma(s_t)^{1/2}\varepsilon_{t+1}$$

A&B chose not to go this route because no closed form solution for yields is available with λ_{t+1} a function of q_t. However, if we eliminated q_t from λ_{t+1} so that

$$\hat{m}_{t+1} = -\delta_0 - \delta_1' X_t - \lambda(s_t)\lambda_t(s_t) - \lambda_t(s_t)\varepsilon_{t+1}$$

we could solve for yields.

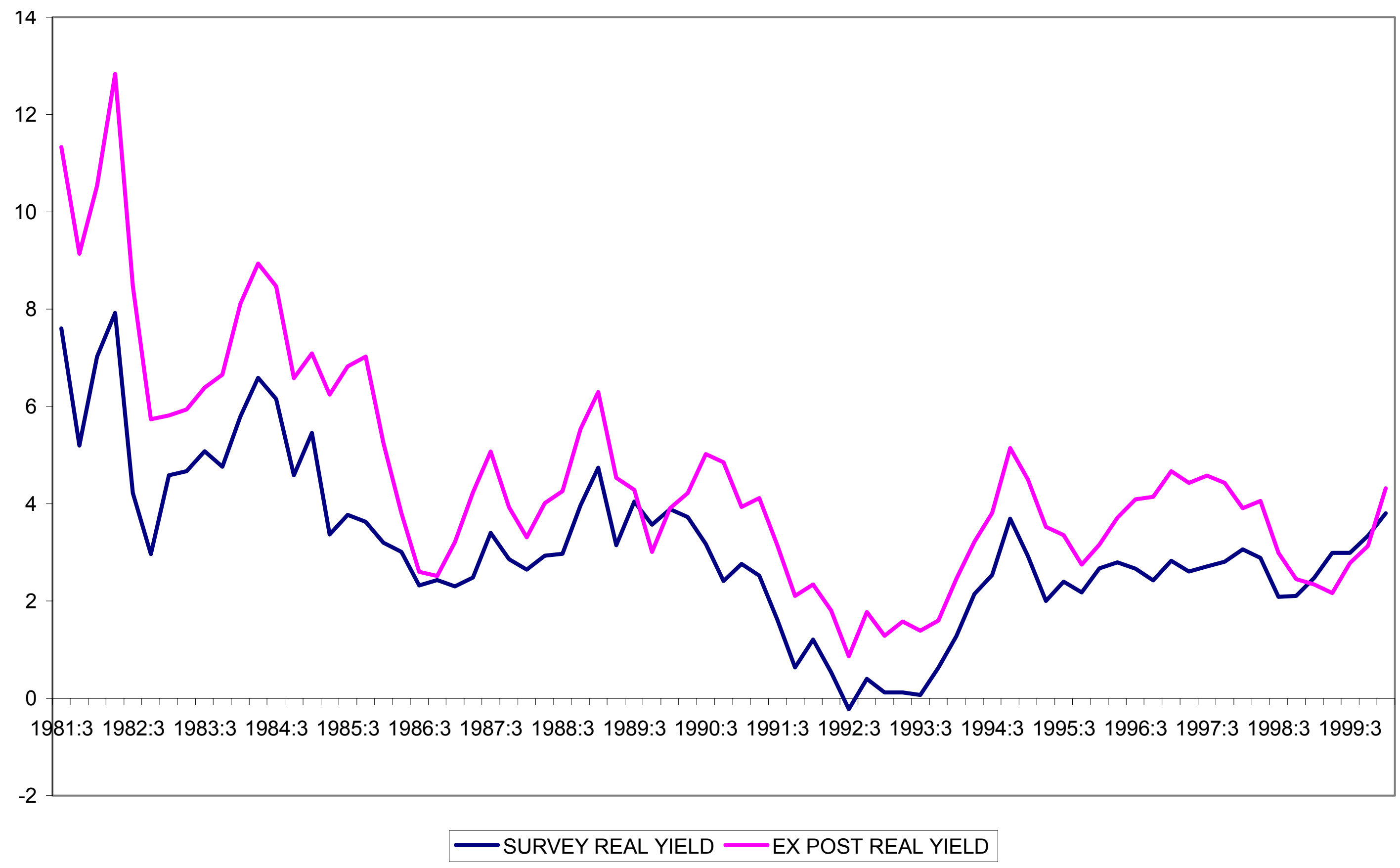
Comment 3: Switching makes "choosing the right specification" tricky. A&B favor version IV of their model because the inflation forecast errors are not serially correlated. This is certainly a large sample property of the model. But.....



- Clearly the these forecast errors are serially correlated (beyond the forecast horizon).

Are the forecasters irrational, or was there a peso/learning problem?

The answer matters: Consider the alternative estimates of one-year real yields.



Summary

- Making inferences about the source of nominal term structure movements is HARD. The "No Arbitrage" model helps, but does not make up for the lack of data on real yields/inflation expectations/an economic model.
- Introducing regime-switching is a good idea because it has the potential to deliver a good deal of model flexibility (c.f. changing monetary policy as in Cogley and Sargent 2002).
- My preference would be to drop within-regime variation in the risk premia, and allow for state-dependent mean reversion (as in Evans 2003).
- Whatever the approach, relying on just nominal yields and realized inflation is not enough to establish "stylized facts". We need:
  - to account for the data on inflation expectations, and/or,
  - a GOOD model for the discount factor $\hat{m}_{t+1}$.