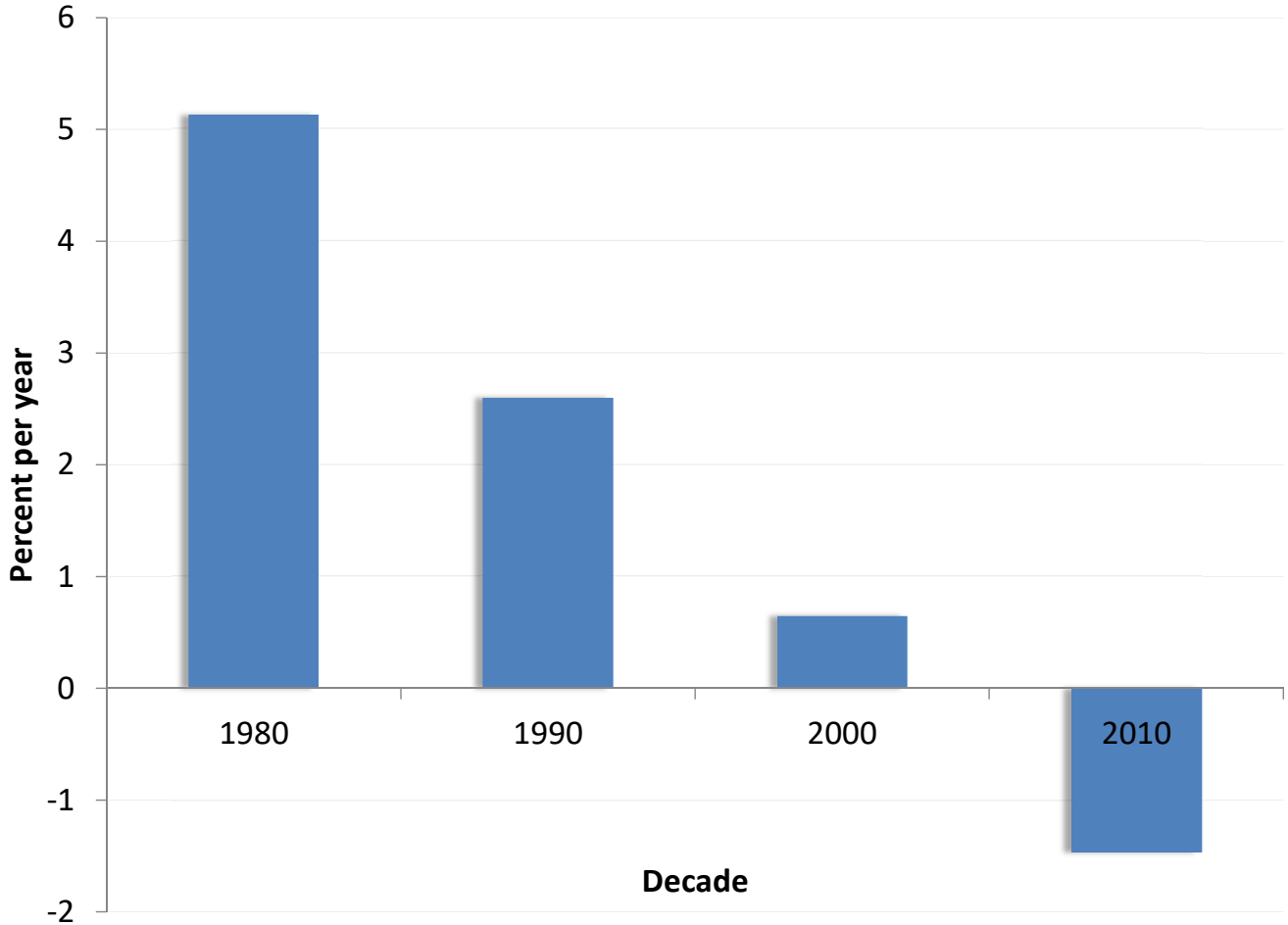
Decade Averages of the Real Interest Rate on 1-Year Treasury Notes



NOTE: Slides are preliminary. Please visit Stanford.edu/~rehall for updated versions.

Model

Product $i = 1$ is immediate delivery of the good with certainty, numeraire, $p_1 = 1$

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Risk-averse similar with U^* and c_i^*

.

Heterogeneous preferences

Different curvature

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Different beliefs about probabilities

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Different beliefs about probabilities

Assume additively separable in states and times:

$$U(c_1, \ldots, c_N) = \sum_{i=1}^{N} \beta^{\tau(i)} \phi_i \frac{c_i^{1-\gamma}}{1-\gamma}$$

and

$$U^*(c_1, \ldots, c_N) = \sum_{i=1}^{N} \beta^{\tau(i)} \phi_i^* \frac{(c_i^*)^{1-\gamma^*}}{1-\gamma^*}$$

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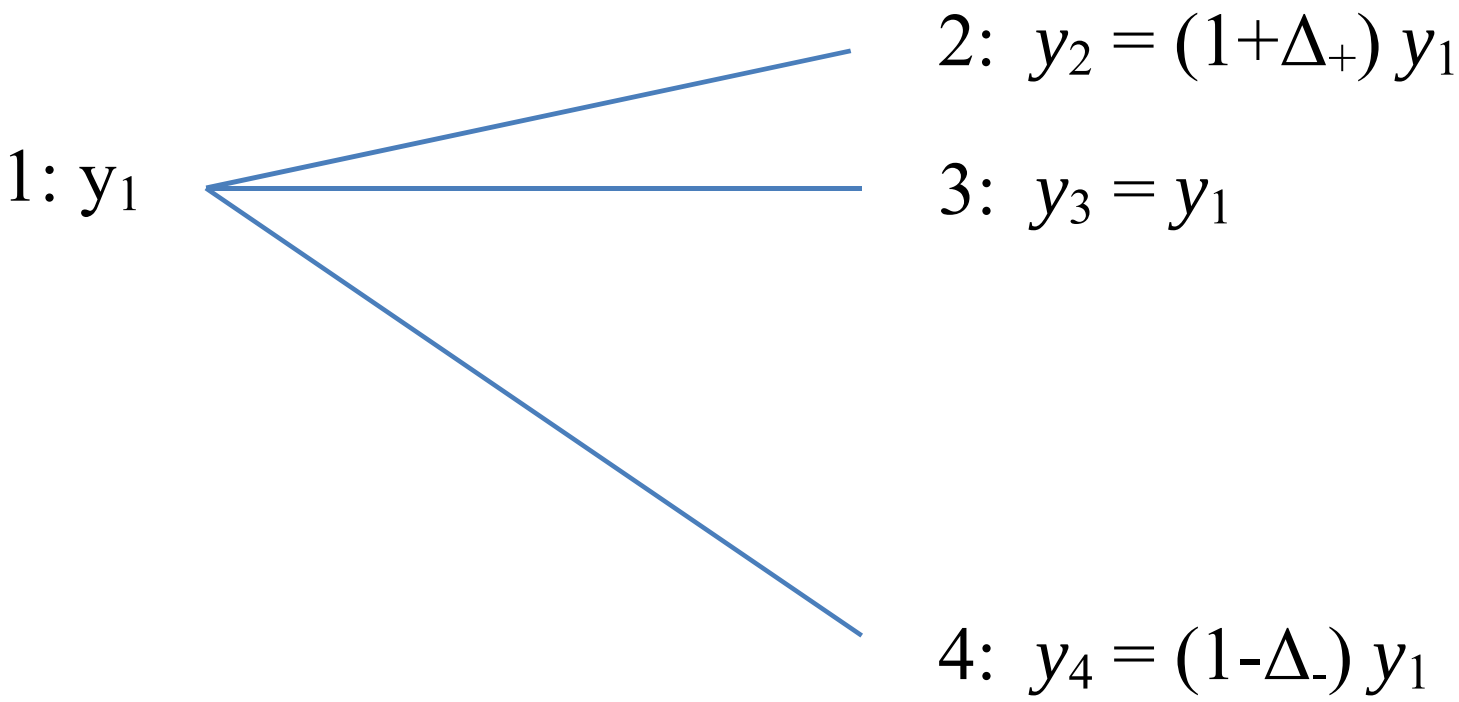
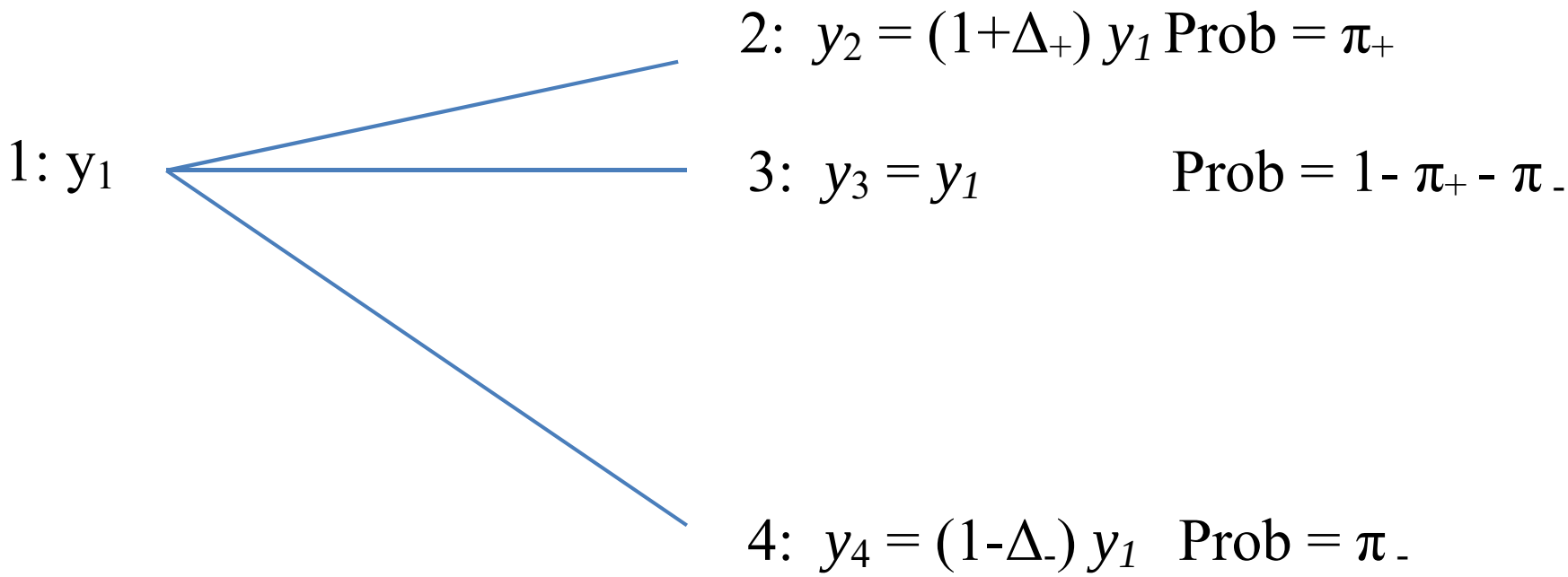
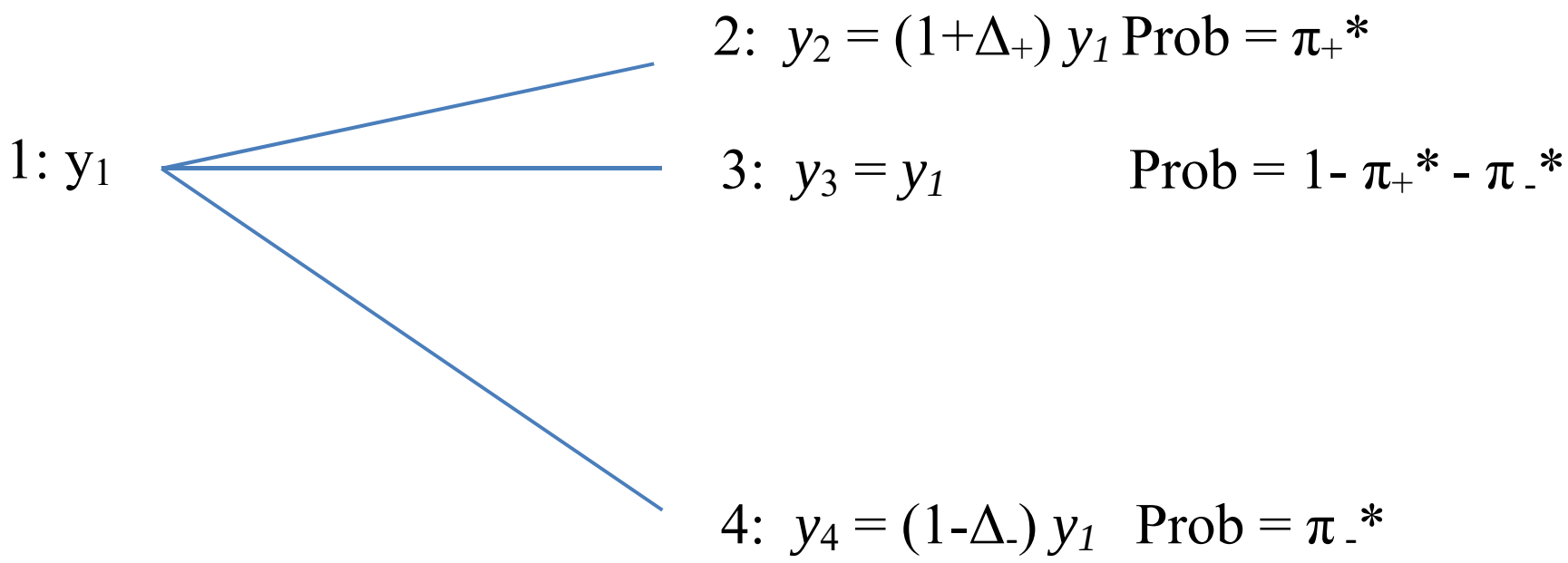
Physical environment



With risk-tolerant beliefs



With risk-averse beliefs



Restrictions on beliefs

Both types believe in zero growth of the endowment, to avoid growth effects:

$$\pi_- \Delta_- = \pi_+ \Delta_+$$

and similarly for risk-averse

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With non-negativity included, the space of beliefs has one dimension:

$$0 \leq \pi_- \leq \frac{\Delta_+}{\Delta_+ + \Delta_-}$$

and

$$\pi_+ = \frac{\Delta_-}{\Delta_+} \pi_-$$

and similarly for π^*

.

Equilibrium

$p_2, \ldots, p_N$ and $x_1, \ldots, x_N$ such that

$$\beta^{\tau(i)} \phi_i c_i^{-\gamma} = p_i c_1^{-\gamma},$$

$$c_i = \alpha y_i - x_i,$$

$$\beta^{\tau(i)} \phi_i^* (c_i^*)^{-\gamma^*} = p_i (c_1^*)^{-\gamma^*},$$

$$c_i^* = (1-\alpha) y_i + x_i,$$

and

$$\sum_i p_i x_i = 0$$

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One-period debt

$$P = p_2 + p_3 + p_4$$

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Interest rate:

$$r = P^{-1} - 1$$

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Parameter Values of the Physical Environment

		Base value	*Alternative value*
Δ_+	Increment to endowment	0.04	
Δ_-	Decrement to endowment	0.60	
ω	Share of endowment owned by risk-tolerant consumers	0.75	0.25

Parameter Values of Preferences Including Beliefs about Probabilities

	Parameter	*Risk tolerant*		*Risk averse*	
		Base value	*Alternative value*	*Base value*	*Alternative value*
γ	Coefficient of relative risk aversion	2.00	1.74	2.00	2.30
π_-	Belief about probability of bad decrement to endowment	0.0100	0.0071	0.0100	0.0140
π_+	Belief about probability of good increment to endowment	0.150	0.107	0.150	0.210

Latent Prices with No Heterogeneity among Investors

	Endowment, y	*Trade, x*	*Consumption/ endowment*		*A-D price, p*	*Probability, π*	*SDF, p/π*
			Risk-tolerant	*Risk-averse*			
Initial	1	0.0000	1.000	1.000	1	1	1
After one year	0.4	0.000	1.000	1.000	0.058	0.010	5.81
	1	0.000	1.000	1.000	0.781	0.840	0.93
	1.04	0.000	1.000	1.000	0.129	0.150	0.86
Utility discount factor, β			0.930				
Expected consumption growth			0.0000	0.0000			
Price with certainty			0.968				
Annual interest			3.27				
Gross trade			0.0000				

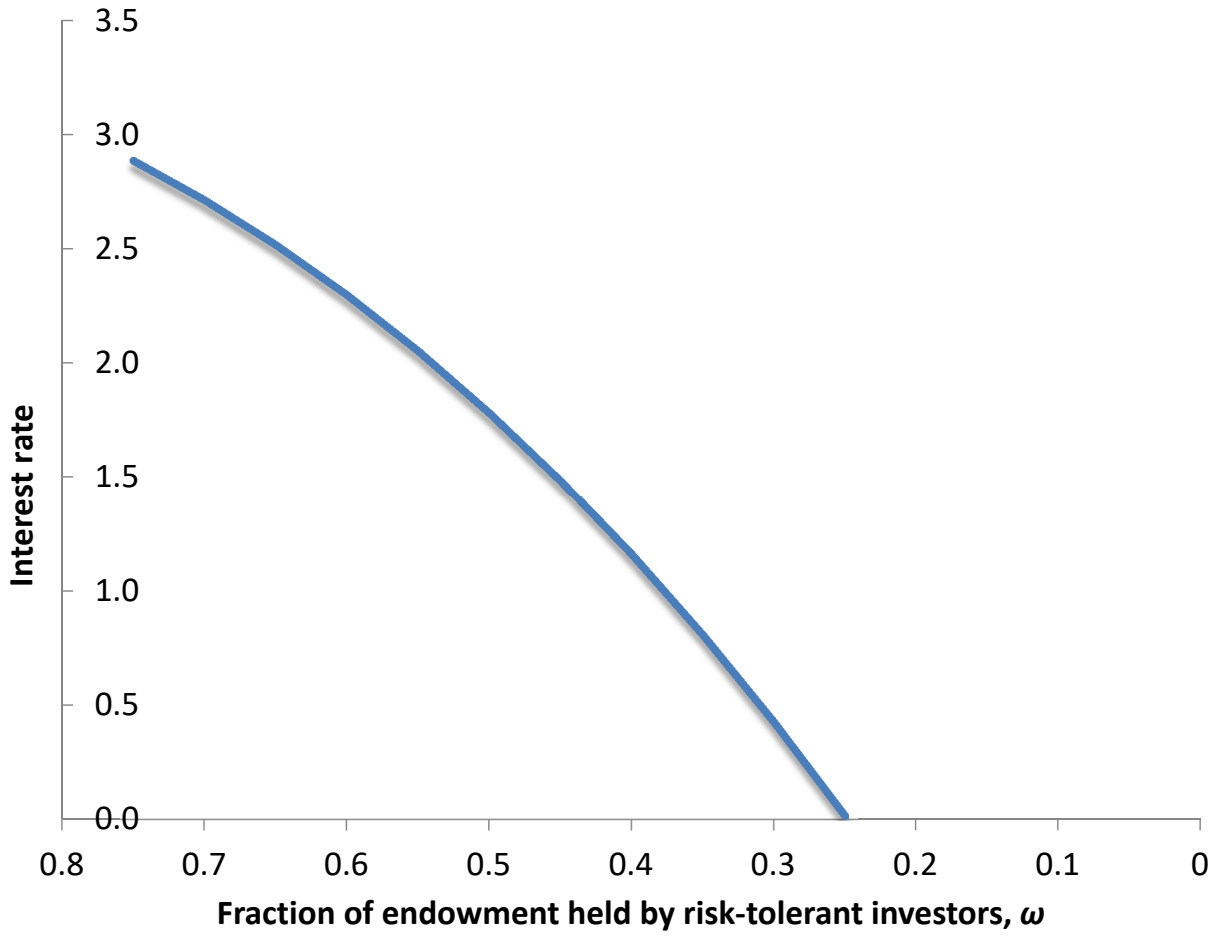
Heterogeneity in Risk Aversion

	Endow-ment, y	*Trade, x*	*Consumption/ endowment*		*A-D price, p*	*Proba-bility, π*	*SDF, p/π*
			Risk-tolerant	*Risk-averse*			
Initial	1	-0.0004	1.000	0.999	1	1	1
After one year	0.4	0.019	0.937	1.188	0.052	0.010	5.15
	1	0.000	1.000	0.999	0.785	0.840	0.94
	1.04	-0.002	1.003	0.991	0.130	0.150	0.87
Utility discount factor, β			0.935				
Expected consumption growth			0.0001	-0.0004			
Price with certainty			0.967				
Annual interest			3.37				
Gross trade			0.0042				

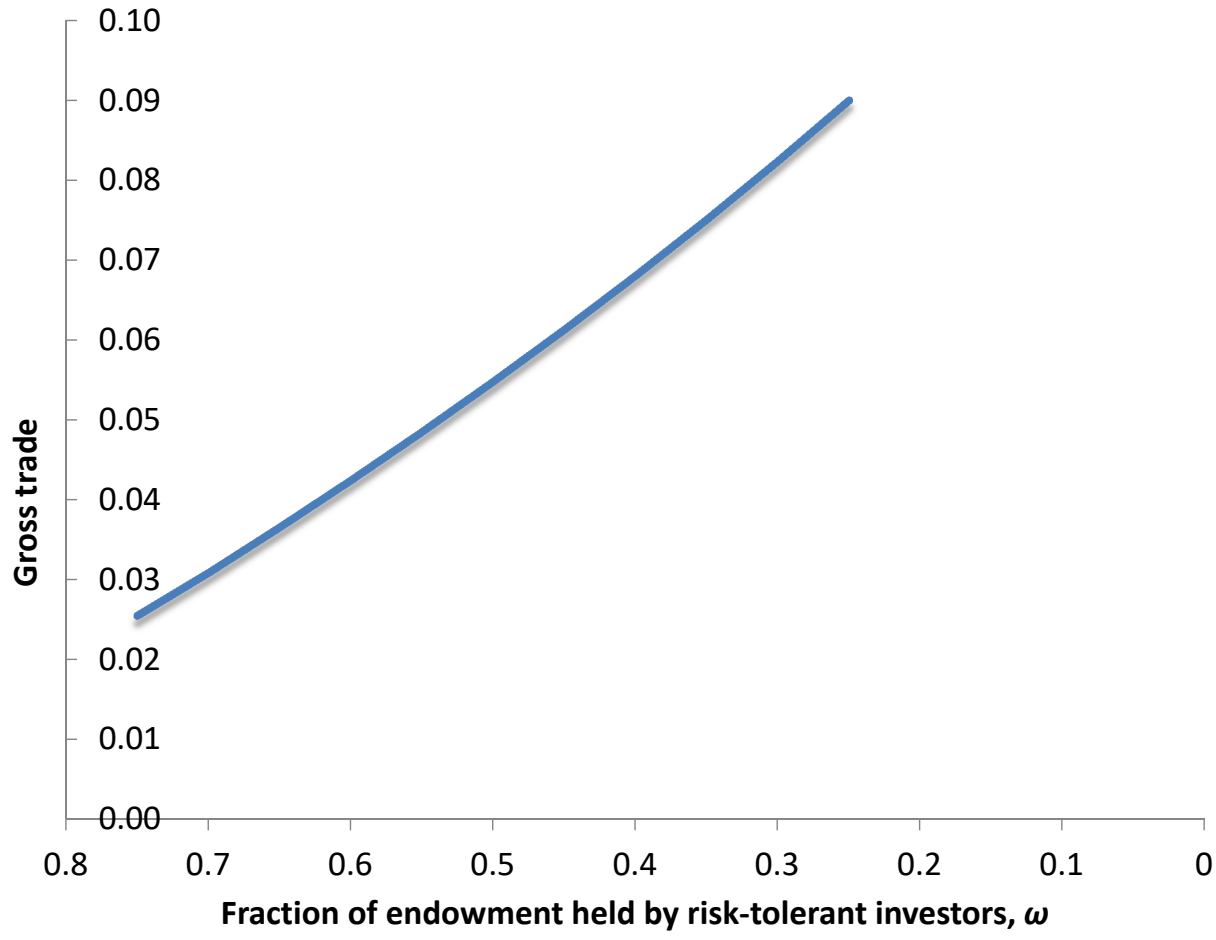
Heterogeneity in Beliefs about Probabilities

	Endow-ment, y	*Trade, x*	*Consumption/ endowment*		*A-D price, p*	*Proba-bility, π*	*SDF, p/π*
			Risk-tolerant	*Risk-averse*			
Initial	1	0.0002	1.000	1.001	1	1	1
After one year	0.4	0.027	0.909	1.274	0.051	0.007	7.12
	1	-0.012	1.016	0.952	0.807	0.886	0.91
	1.04	0.071	0.909	1.274	0.113	0.107	1.05
Utility discount factor, β			0.941				
Expected consumption growth			0.0040	0.0232			
Price with certainty			0.971				
Annual interest			3.03				
Gross trade			0.0260				

The Interest Rate as a Function of the Endowment Share of Risk-Tolerant Investors, Both Types of Heterogeneity



Gross Trade as a Function of the Endowment Share of Risk-Tolerant Investors, Both Types of Heterogeneity



Importance of improbable highly adverse outcomes

All results based on $\Delta_- = 0.6$, which implies that investors believe that every 100 or 50 years, the endowment drops by 60 percent

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Barro-Mollerus survey that literature and discuss the issue in connection with the demand of risk-averse investors for safe debt-type investments.

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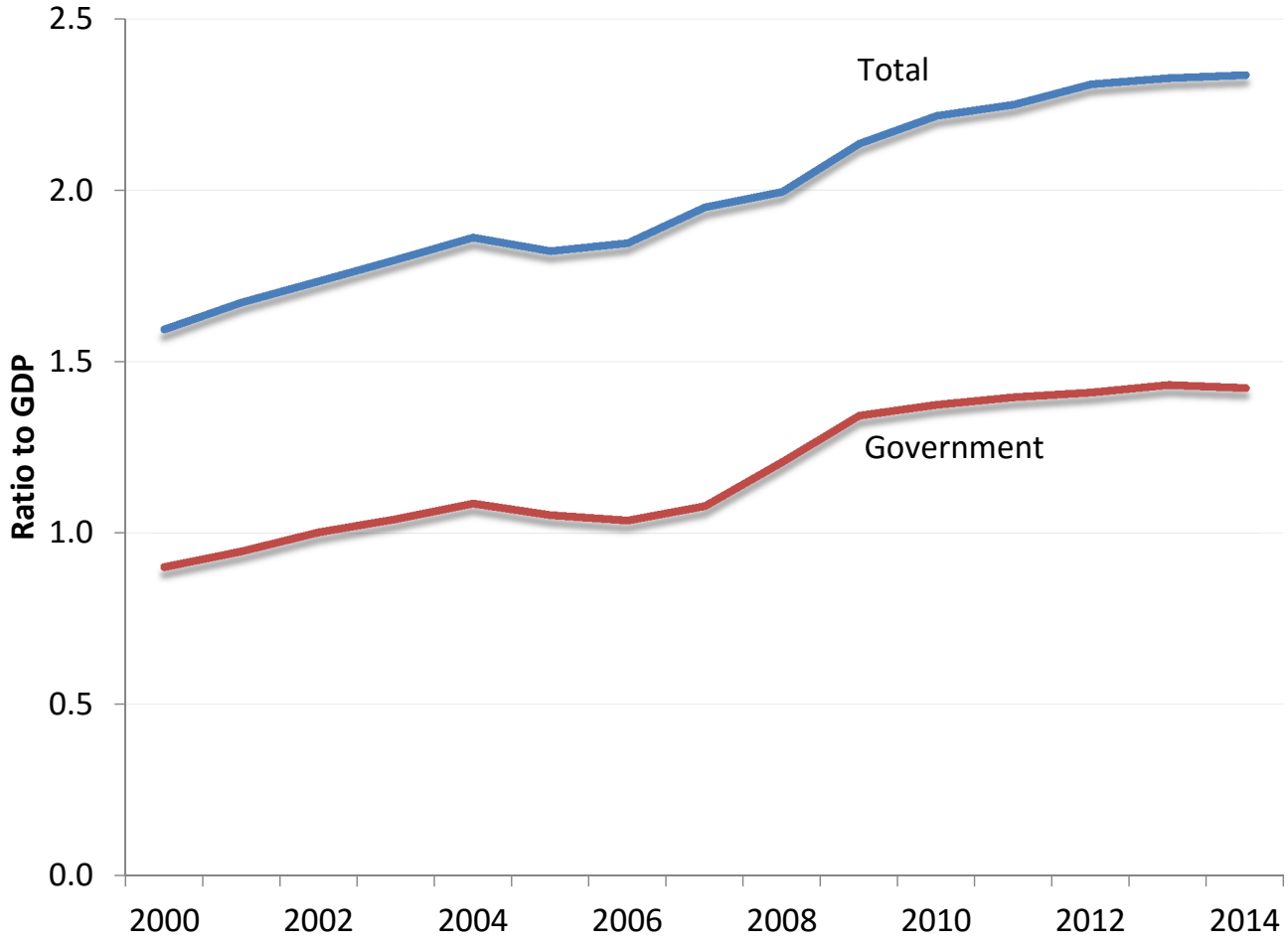
Debt of U.S. Investors in 2015, in Trillions of Dollars and as Ratios to GDP

	$ trillions	*Ratio to GDP*
Federal government debt	15.2	0.85
Federally guaranteed GSE debt and guaranteed mortgages	8.1	0.45
State and local government debt	3.0	0.17
Non-financial business, bonds and loans	12.8	0.71
Non-guaranteed household mortgages	1.4	0.08
Other debt of households	4.7	0.26
Total	45.1	2.52

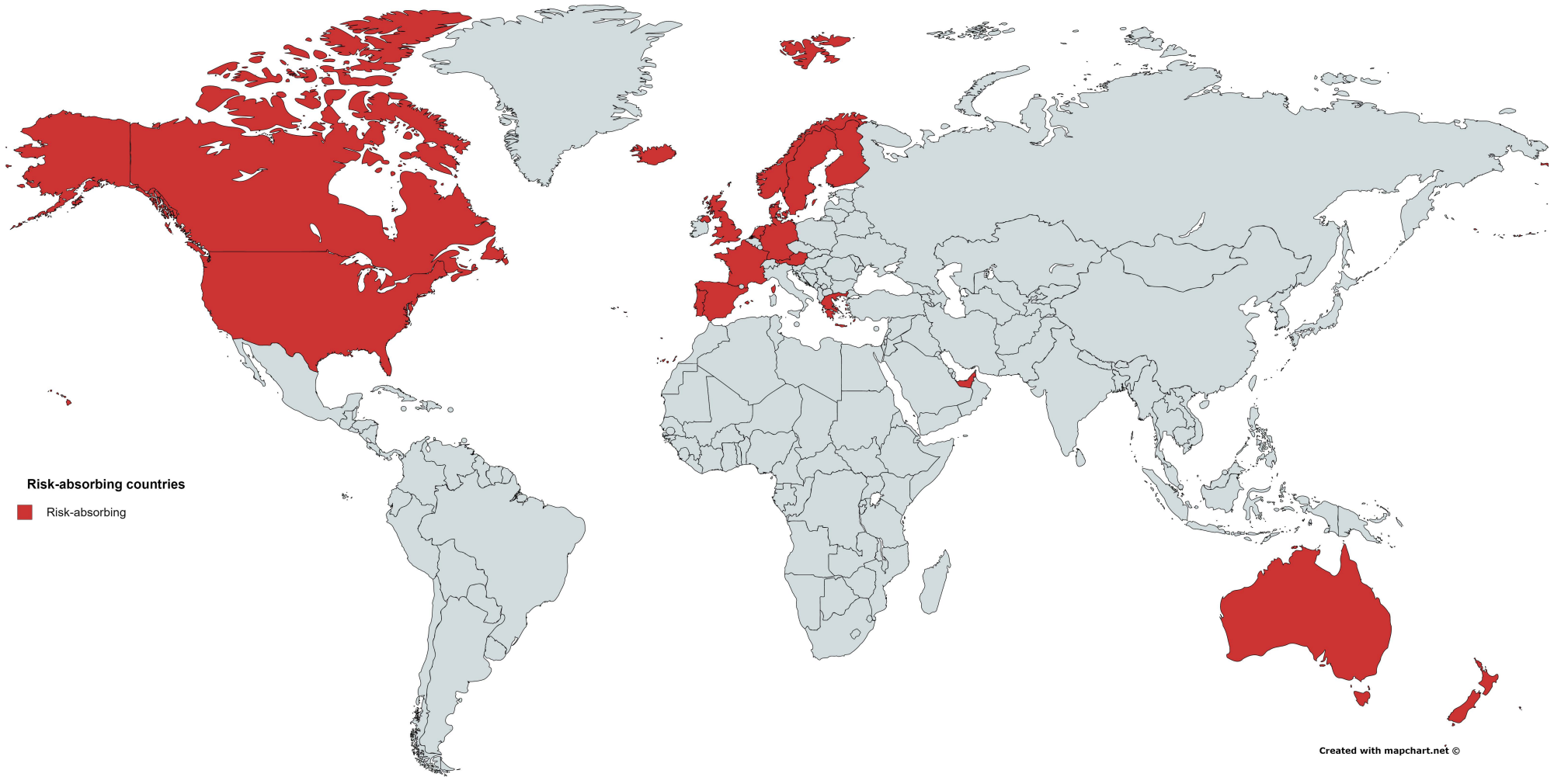
Examples of the Scale of Risk-Splitting Institutions

	Government			*Private*				
Decade	*Consolidated government debt*	*GSE debt*	*GSE guaranteed debt*	*Private equity funds*	*Securitizations*	*Non-financial corporate debt*	*Repos*	*Non-mortgage household debt*
1980s	0.469	0.061	0.091		0.012	0.163	0.103	0.186
1990s	0.611	0.101	0.204		0.086	0.211	0.166	0.204
2000s	0.574	0.203	0.293	0.058	0.233	0.238	0.237	0.239
2010s	0.936	0.126	0.347	0.140	0.109	0.275	0.221	0.251

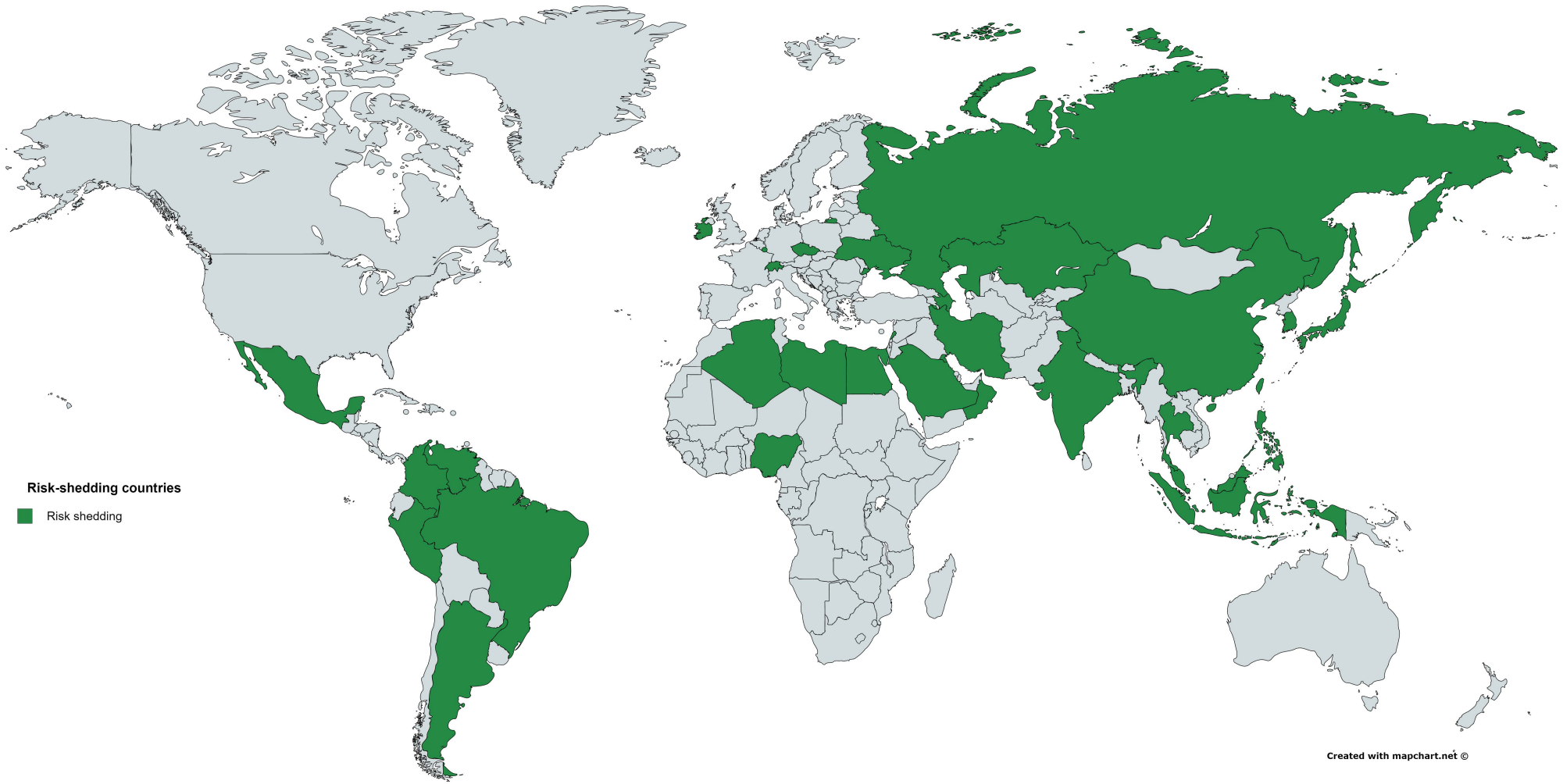
Scale of Risk-Splitting Institutions Relative to GDP



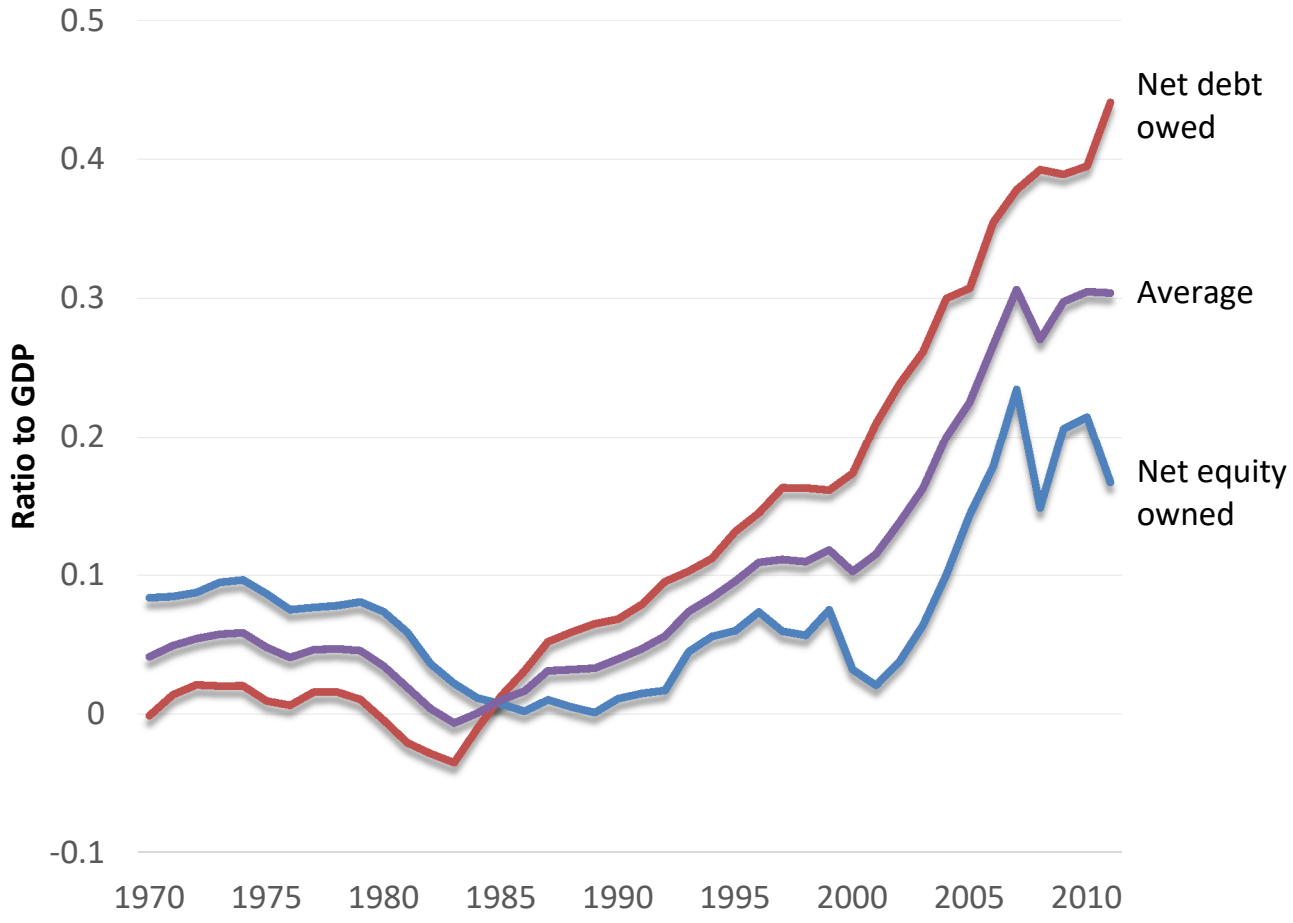
Countries that Absorb Risk by Holding Positive Amounts of Net Foreign Equity or by Borrowing from Foreign Lenders



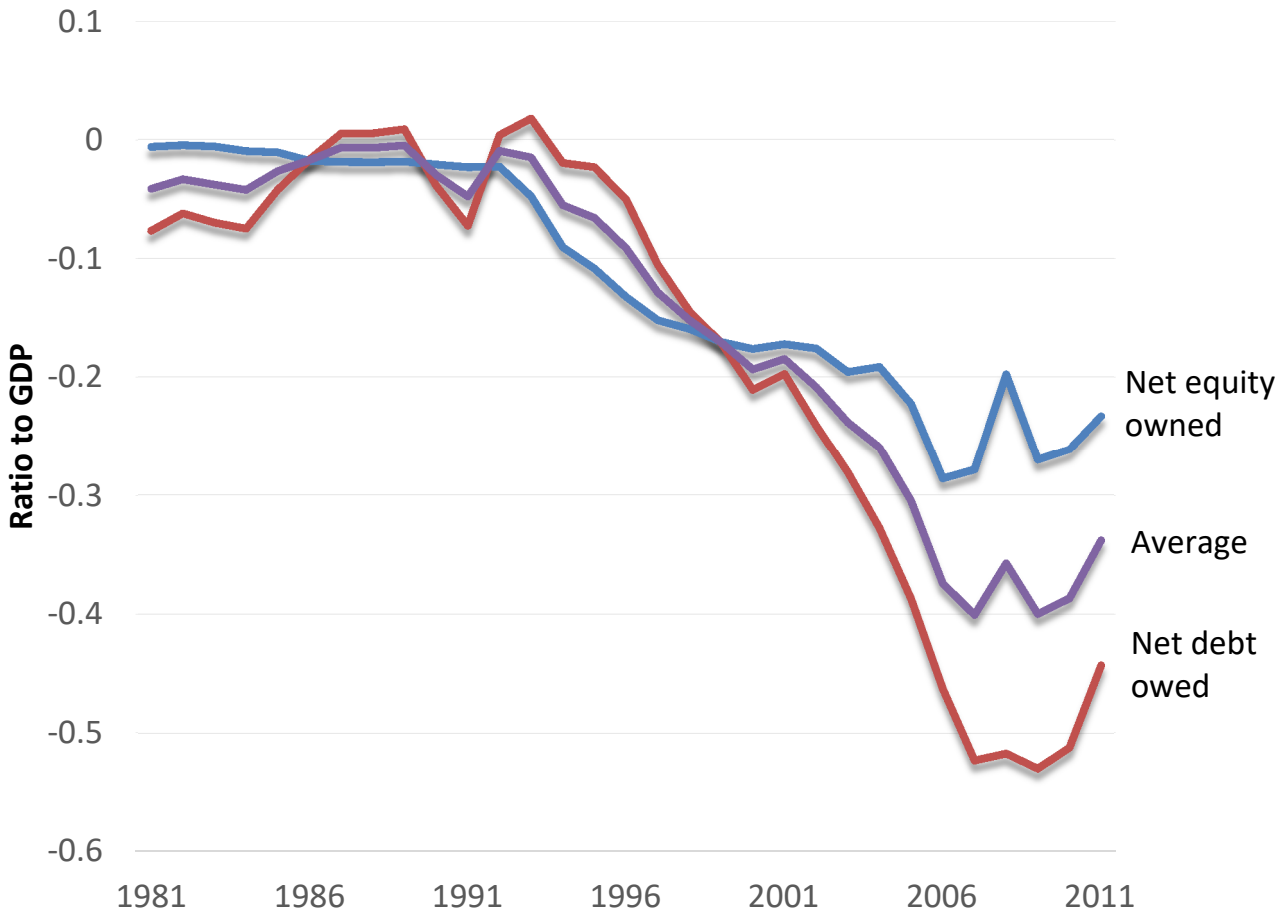
Countries that Shed Risk by Holding Negative Amounts of Net Foreign Equity or by Lending Positive Amounts to Foreign Borrowers



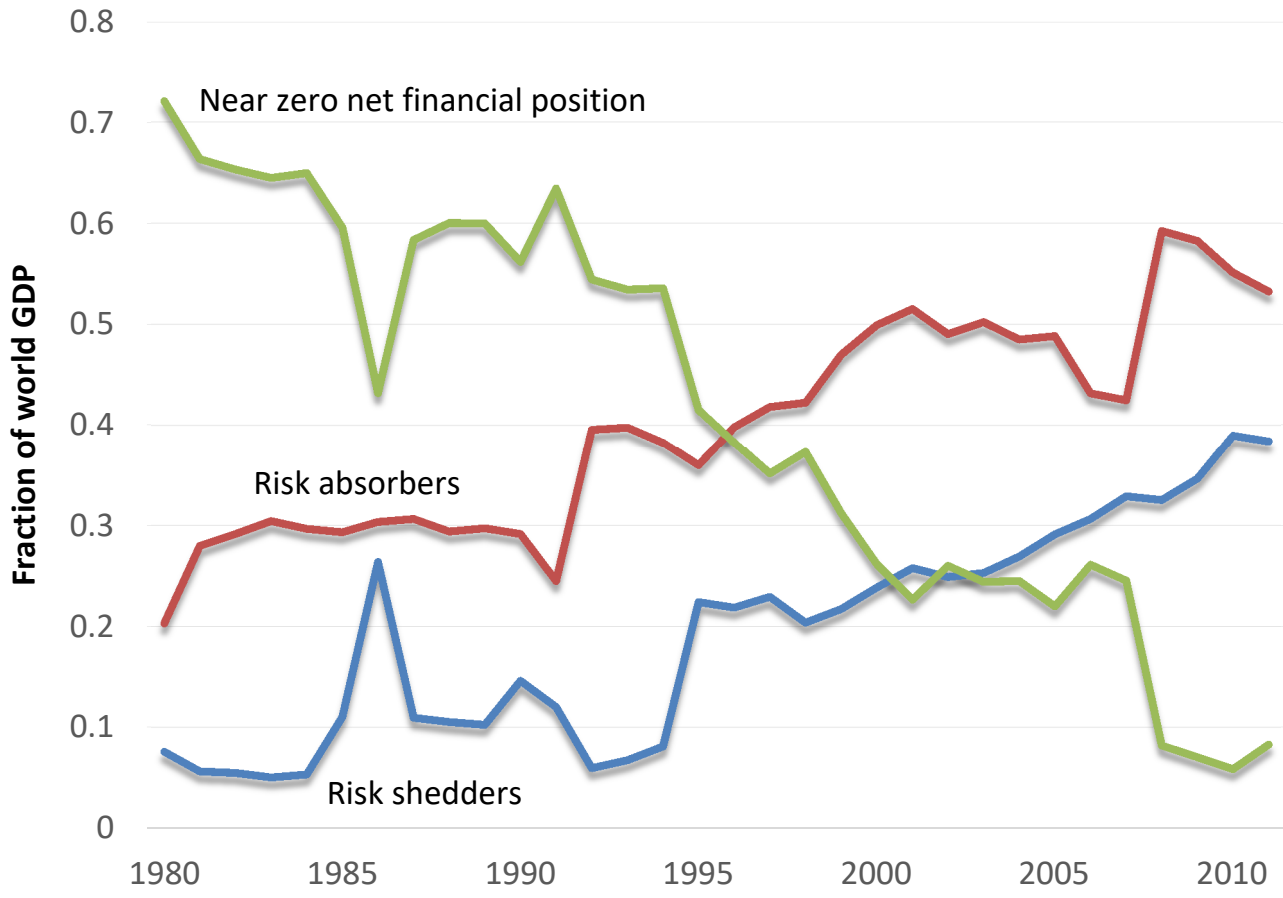
Risk Absorption by the United States, 1970-2011



Risk Shedding by China, 1981-2011



Fractions of World GDP, 1970-2011, for Countries with Positive, Near Zero, or Negative Net Foreign Equity and Debt as Fractions of Their GDP



Share of Globally Integrated GDP Arising from Risk-Absorbing Countries