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Abstract

Long-term nominal interest rates are known to be highly sensitive to high-frequency (daily or monthly) movements in short-term rates. We find that, since 2000, this high-frequency sensitivity has grown even stronger in U.S. data. By contrast, the association between low-frequency changes (at six- or twelve-month horizons) in short- and long-term rates, which was also strong before 2000, has weakened substantially. We show that this puzzling post-2000 combination of high-frequency "excess sensitivity" and low-frequency "decoupling" of short- and long-term rates arises because increases in short rates temporarily raise the term premium on long-term bonds, leading long rates to temporarily overreact to changes in short rates. The post-2000 frequency-dependent sensitivity of long-term rates can be understood using a model in which (i) declines in short rates lead to outward shifts in the demand for long-term bonds (for example, because some investors "reach for yield") and (ii) the arbitrage response to these demand shifts is slow. We discuss the implications of our findings for the transmission of monetary policy and the validity of the event-study methodologies.

Key words: interest rates, conundrum, monetary policy transmission

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The sensitivity of long-term interest rates to movements in short-term rates is a central feature of the term structure and is thought to play a crucial role in the transmission of monetary policy to the real economy. Short-term nominal interest rates are determined by current monetary policy and its near-term expected path. Shocks to monetary policy and the macroeconomy are generally seen as being short-lived, so long-term rates should not be highly sensitive to changes in short rates if the expectations hypothesis holds (Shiller, 1979). However, a large literature demonstrates that long-term nominal rates are more sensitive to high-frequency changes in short rates than is predicted by this standard view combining fast mean-reversion in short rates with the expectations hypothesis (Shiller et al., 1983; Cochrane and Piazzesi, 2002; Gürkaynak et al., 2005; Giglio and Kelly, 2018), implying either a failure of the expectations hypothesis and/or surprisingly persistent shocks to short rates. Despite its importance for monetary policy, the deeper forces underpinning this puzzling degree of high-frequency sensitivity—and the extent to which this sensitivity has evolved over time—remain poorly understood.

In this paper, we document an important and previously unrecognized fact about the term structure of nominal interest rates: the association between high-frequency changes (at daily or 1-month horizons) in short- and long-term rates has strengthened considerably since 2000. In contrast, the relationship between low-frequency changes (at 6- or 12-month horizons) in short- and long-term interest rates, which was also quite strong before 2000, has weakened substantially in recent years. Prior to 2000, the sensitivity of long-term rates to changes in short-term rates was similar at both high and low frequencies. Between 1971 and 1999, a daily regression of changes in 10-year U.S. Treasury yields on changes in 1-year yields delivers a coefficient of 0.56; and the analogous regression using 12-month changes gives the same coefficient of 0.56.[1] But strikingly, between 2000 and 2017, the coefficient from the daily regression jumps to 0.86, while the coefficient from the corresponding 12-month regression drops to just 0.20. As a result, the sensitivity of long-term rates to changes in short-term rates has become highly frequency-dependent since 2000.[2] Figure 1 shows sensitivity of 10-year yields to changes in 1-year yield as a function of horizon in the pre-2000 and post-2000 subsamples. We find similar patterns for Canada, Germany, and the U.K.

We can also use a simple back-of-the-envelope calculation to put the sensitivities shown in Figure 1 into perspective. Assuming that the expectations hypothesis holds, the pre-2000 daily sensitivity of 10-year yields of 0.56 suggests that short rate shocks have a half-life of 5.3 years.[3] This degree of implied

[1]Because of the importance of communication about the near-term path of monetary policy, we take the short rate to be the 1-year nominal Treasury yield rather than the overnight federal funds rate targeted by the Federal Reserve.

[2]As we show below, this finding is not driven by distortions stemming from the 2009–2015 period when overnight nominal rates were stuck at the zero lower bound in the U.S.

[3]These calculations assume that the short-rate follows an AR(1) with a monthly autocorrelation coefficient of ρ_i. If the

persistence is high, but it is not too dissimilar from the persistence of short rates observed in the pre-2000 data, where the monthly autocorrelation coefficient of one-year yields is 0.974.By contrast, the post-2000 daily sensitivity 10-year yields of 0.86 would imply short-rate shocks with a half-life of 21.4 years. Such a persistent short-rate process would seem to strain credulity. Furthermore, this elevated sensitivity of long-term rates is present a high frequencies, but not at low frequencies.

What explains the puzzling post-2000 tendency of short- and long-term rates to move together at high frequencies but not at low frequencies? As a matter of statistical description, we show that this pattern arises because, all else equal, past increases in short rates predict a subsequent flattening of the yield curve—and subsequent declines in long-term yields and forward rates—in the post-2000 data. Furthermore, these predictable reversals in long-term rates are associated with a *transitory* rise in the expected returns on long-term bonds relative to those on short-term bonds: since 2000 term premia on long-term bonds appear to be *temporarily* elevated following past increases in short rates. Thus, relative to an expectations-hypothesis baseline, long rates *temporarily overreact* to movements in short rates, exhibiting a form of what Mankiw and Summers (1984) have dubbed "excess sensitivity." Concretely, in the post-2000 data, we estimate that 10-year yields rise by 64 basis points (bps) in response to a 100 bps monthly increase in 1-year yields. Over the next 6 months, 10-year yields are expected to fall by 42 bps, all else equal, reversing roughly two-thirds of the initial response. But, while this predictable reversion of long-term rates is a robust feature of the post-2000 data, this pattern is not present before 2000.

What deeper forces have led to the shifting relationship between changes in long- and short-term yields? Gürkaynak et al. (2005) note that the strong sensitivity of long-term nominal rates could be consistent with the expectations hypothesis if one adopts the view that long-run inflation expectations are unanchored and are continuously being updated in light of incoming news. In other words, they argue that the strong sensitivity of long-term rates to short rates could work through a expectations-hypothesis channel once one allows for persistent shocks to inflation expectations. We argue that the narrative in Gürkaynak et al. (2005) is a good explanation for the high degree of sensitivity observed prior to 2000. Indeed, consistent with the expectations-hypothesis logic of their explanation, in the pre-2000 data, we find no evidence that the reaction of long yields to movements in short rates tends to reverse predictably.

However, as shown by Beechey and Wright (2009), Hanson and Stein (2015), Abrahams et al. (2016), and Nakamura and Steinsson (2018), in the post-2000 period, the strong high-frequency sensitivity of long-term nominal rates primarily reflects the sensitivity of long-term *real* rates to short-term nominal

expectations hypothesis holds, the sensitivity of 10-year zero-coupon yields would be $\left[\left(1-\rho_i^{120}\right)/\left(1-\rho_i\right)\right]/120$. So a daily sensitivity of 0.56 would imply $\rho_i = 0.989$ or short-rate shocks with a half-life of $5.3 = \ln(0.5)/\ln(\rho_i^{12})$ years.

rates, rather than the sensitivity of long-term break-even inflation. To the extent that one shares the widespread view that expected future real rates at distant horizons should not fluctuate meaningfully at high frequencies (see Gürkaynak et al. (2005)), this casts doubt on an expectations hypothesis-based explanation of the strong high-frequency sensitivity of long rates in the post-2000 data. To resolve this puzzle, Hanson and Stein (2015) argue that excess sensitivity works through term premia on long-term bonds: shocks to short rates temporarily move term premia in the same direction. Consistent with Hanson and Stein (2015), in the post-2000 data, we find strong evidence that the reaction of long-term yields to movements in short rates tends to predictably reverse, giving rise to short-lived shifts in the expected returns to holding long-term bonds.

We construct a simple model to help understand the shifting relationship between movements in long- and short-term yields, especially the puzzling post-2000 combination of high-frequency excess sensitivity and low-frequency decoupling. In our model, risk-averse arbitrageurs can either invest in short- or long-term nominal bonds. While monetary policy pins down the rate on short-term nominal bonds, long-term bonds are available in a net supply that varies randomly over time. (The net supply is the gross supply of long-term bonds net of the amount inelastically demanded by other investors.) Since shocks to the supply and demand for long-term bonds must be absorbed by risk-averse arbitrageurs, shifts in net supply affect term premia on long-term bonds as in Vayanos and Vila (2009). The key friction in these models—which have proved invaluable in understanding the effect of Quantitative Easing policies—stems from the limited risk-bearing capacity of the specialized fixed-income arbitrageurs who must absorb shocks to the supply and demand for long-term bonds.

In the pre-2000 period, we assume there was a large persistent component of short-term nominal rates, reflecting shocks to trend inflation as in Stock and Watson (2007). The existence of this highly persistent component in combination with expectations-hypothesis logic, explains the strong sensitivity of long rates at both high and low frequencies before 2000. In the post-2000 period, the volatility of this persistent component of short rates has dropped sharply. From an expectations-hypothesis perspective, this should have reduced the sensitivity of long rates at all frequencies. In the data, this occurs at low frequencies, but we see greater-than-ever sensitivity at high frequencies.

The contribution of our model is to explain how such frequency-dependent sensitivity may arise in the post-2000 data. Our explanation rests on two key ingredients: (i) shifts in the supply and demand for long-term bonds that move term premia in the same direction as short-term rates and (ii) and slow-moving capital. The first key ingredient is our assumption that shocks to the net supply of long-term bonds are positively correlated with shocks to short rates. This implies that increases in short rates are associated with increases in the term premium component of long-term rates, generating "excess sensitivity" relative to the expectations hypothesis. The simplest interpretation of our assumption follows Hanson and Stein (2015) and appeals to inelastic shifts in the demand for long-term bonds from "yield-oriented" investors. These investors target a certain level of portfolio yield and "reach for yield," inelastically demanding more long-term bonds, as short rates decline. However, our assumption can be seen as a reduced-form for several distinct supply-and-demand-based amplification mechanisms that have grown in importance since 2000, including shifts in supply of long-term bonds due to mortgage refinancing waves (Hanson, 2014; Malkhozov et al., 2016) and shifts in the demand for long-term bonds from biased investors who over-extrapolate short-term interest rates (Cieslak, 2018; Giglio and Kelly, 2018).[4] The second key ingredient is that capital is slow-moving as in Duffie (2010): these supply and demand shocks encounter a short-run demand curve that is steeper than the long-run demand curve, generating a short-run supply-and-demand imbalance in the market for long-term bonds. This slow-moving capital dynamic implies that the shifts in bond term premia triggered by movements in short rates are transitory. As a result, the excess sensitivity of long rates is greatest when measured at high frequencies.

In summary, this combination of reaching-for-yield and slow-moving capital enables our model to match the frequency-dependent sensitivity of long rates observed since 2000. And, our model explains the shift in the relationship between long- and short-term rates that occurred around 2000 as stemming from (1) a decline in the volatility of the persistent component of short rates and (2) the growing importance of the kinds of supply-and-demand-based amplification mechanisms noted above.

Our findings have important implications for how one should interpret event-study evidence based on high-frequency changes in long-term bond yields. Macroeconomic news—including news about monetary policy—comes out in a lumpy manner, and the short-run change in long-term yields around news announcements is often used as a convenient and unconfounded measure of the longer-run impact of news shocks. Gertler and Karadi (2015) and Nakamura and Steinsson (2018) are two prominent recent examples of this increasingly popular high-frequency approach to identification in macroeconomics.[5] However, if, as we show, some of the impact of a news shock on long-term rates tends to wear off quickly over time, then a shock's short- and long-run impact will be quite different. And, the event-study approach will

[4]The idea that supply-and-demand effects can have important consequences for long-term rates features prominently in many other recent papers, especially in analyses of Quantitative Easing (Gagnon et al., 2011; Hamilton and Wu, 2012; Krishnamurthy and Vissing-Jorgensen, 2011, 2012).

[5]Earlier papers examining the high-frequency response of long-term rates to news about monetary policy include Evans and Marshall (1998), Kuttner (2001), and Cochrane and Piazzesi (2002).

necessarily capture only the short-run impact. Indeed, it is common for news announcements to cause large jumps in 10-year forward rates, but we show that a large portion of these jumps are due to *transient* shifts in term premia. As a result, event-study methodologies are likely to provide biased estimated of the longer-run impact of news on long-term yields. Thus, macroeconomists face an important bias-variance trade-off: high-frequency event studies provide precise estimates of the short-run impact of news surprises on long-term yields, but these are likely to be systematically biased estimates of the longer-run impact which is often of greatest interest.

Our findings also have implications for monetary policy transmission. In the textbook New Keynesian view (Gali, 2008), the central bank adjusts short-term nominal rates. This affects long-term rates via the expectations hypothesis, which in turn influences aggregate demand. Stein (2013) points out that the excess sensitivity of long-term yields—whereby shocks to short rates move term premia in the same direction—should strengthen the effects of monetary policy relative to the textbook view. Stein (2013) refers to this as the "recruitment" channel of monetary transmission. We find that the behavior of interest rates does not conform to the textbook view in which term premia are constant. Nonetheless, our findings suggest that the recruitment channel may not be as strong as Stein (2013) speculates since a portion of the resulting shifts in term premia are transitory and, thus, likely to have only modest effects on aggregate demand. To be clear, we do not argue that there is no recruitment channel, just that it is smaller than one might conclude based on the high-frequency response of term premia to policy shocks documented in Hanson and Stein (2015), Gertler and Karadi (2015), and Gilchrist et al. (2015).

Finally, our findings help explain the rising prevalence of episodes like the one that former Federal Reserve Chairman Greenspan famously called the "conundrum"—the period after June 2004 when the Fed raised short-term rates, but longer-term yields declined. Specifically, we show that the fact that past changes in the level of rates increasingly predict a future flattening of the yield curve helps explain several noteworthy "conundrum" episodes where short- and long-term rates have moved in opposite directions.

The plan for the paper is as follows. In Section 1, we document our key stylized facts about the changing high- and low-frequency sensitivity of long-term interest rates. In Section 2, we show that past increases in short rates predict a future flattening of the yield curve in the post-2000 data, reflecting a new form of bond return predictability. Section 3 develops the economic model that we use to interpret our findings. Section 4 discusses the implications of our findings for identification strategies exploiting high-frequency responses of long-term yields, for the transmission of monetary policy, for bond market "conundrums," and for affine term structure models. Section 5 concludes.

1 Main findings

This section presents our main findings. Prior to 2000, the sensitivity of long-term rates to changes in short-term rates was similarly strong at both high- and low-frequencies. Since 2000, the association between high-frequency changes in short- and long-term interest rates has grown even stronger. By contrast, the association between low-frequency changes in short- and long-term interest rates has weakened substantially. As a result, the sensitivity of long-term rates has become surprisingly frequency-dependent since 2000. We first document these basic facts for the U.S. We then show that broadly similar results hold for Canada, Germany, and the U.K.

1.1 U.S. evidence

We obtain historical data on the nominal and real U.S. Treasury yield curve from Gürkaynak et al. (2007) and Gürkaynak et al. (2010). We focus on continuously compounded 10-year zero-coupon yields and 10-year instantaneous forward rates. We also decompose nominal yields into real yields and inflation compensation, defined as the difference between nominal and real yields derived from Treasury Inflation-Protected Securities (TIPS). Our sample begins in 1971, which is when reliable data on 10-year nominal yields first become available, and ends in 2017. For real yields and inflation compensation, we only study the post-2000 sample, since data on TIPS are not available until 1999. All data are measured as of the end of the relevant period—e.g., the last trading day of each month.

In standard monetary economics models, the central bank sets overnight nominal interest rates, and other interest rates are influenced by the expected path of overnight rates. A large literature argues that central banks in the U.S. and abroad have increasingly relied on communication—implicit or explicit signaling about the future path of overnight rates—as an active policy instrument (Gurkaynak et al., 2005; Lucca and Trebbi, 2009). To capture news about the near-term path of monetary policy that would not impact the current overnight rate, we take the short rate to be the 1-year nominal Treasury rate which follows approaches in the recent literature (Campbell et al., 2012; Gertler and Karadi, 2015; Gilchrist et al., 2015; Hanson and Stein, 2015).

To illustrate our key stylized fact, we begin by regressing changes in 10-year yields or forward rates on changes in 1-year nominal yields. Specifically, we estimate regressions of the form:

$$y_{t+h}^{(10)} - y_t^{(10)} = \alpha_h + \beta_h (y_{t+h}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+h} \tag{1.1}$$

and

$$f_{t+h}^{(10)} - f_t^{(10)} = \alpha_h + \beta_h(y_{t+h}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+h}, \tag{1.2}$$

where $y_t^{(n)}$ is the n-year zero-coupon yield in period t and $f_t^{(n)}$ is the n-year-ahead instantaneous forward rate. Panel A in Table 1 reports estimated coefficients β_h in equation (1.1) for zero-coupon nominal yields, real yields, and inflation compensation using daily data and using end-of-month data with $h = 1, 3, 6, 12$—i.e., we report coefficients for daily, monthly, quarterly, semi-annual, and annual changes in yields.[6] The results are shown for the pre-2000 and post-2000 subsamples separately. We base this sample split on a number of break-date tests that we will discuss shortly. Figure 1 plots the estimated coefficients β_h in equation (1.1) for nominal yields versus monthly horizon h for the pre-2000 and post-2000 subsamples. Panel B in Table 1 reports the corresponding β_h coefficients in equation (1.2) using changes in instantaneous forwards as the dependent variable.[7]

At a daily frequency, the regression coefficients have risen between the pre-2000 and post-2000 subsamples. The increasing sensitivity of long-term rates at high frequencies is our first finding. Specifically, the daily coefficient for 10-year yields in Panel A has increased from $\beta_{day} = 0.56$ in the pre-2000 subsample to $\beta_{day} = 0.86$ in the post-2000 subsample with the increase being statistically significant (p-val< 0.001). Similarly, from Panel B, the coefficient for daily changes in 10-year forward rates is $\beta_{day} = 0.39$ pre-2000 and $\beta_{day} = 0.47$ post-2000. Table 1 shows a second fact that has not been previously noted: the coefficients at lower frequencies are much smaller after 2000. For example, the coefficient for $h = 12$-month changes in 10-year yields is $\beta_{12} = 0.56$ before 2000 but only $\beta_{12} = 0.20$ in the post-2000 sample and this difference is statistically significant (p-val < 0.001). Similarly, for 10-year forward rates, the coefficient at a 12-month horizon is $\beta_{12} = 0.39$ in the pre-2000 sample but $\beta_{12} = -0.17$ after 2000.

Combining these two findings, Figure 1 shows our main finding: in the post-2000 sample, the coefficient β_h is a steeply declining function of the horizon h over which yield changes are calculated. By contrast, β_h is a relatively constant function of horizon h in the pre-2000 sample. In other words, Table 1 and Figure 1 show that, prior to 2000, there was a strong tendency for short- and long-term interest rates to rise and fall together at both high- and low-frequencies. While the high-frequency relationship has grown even stronger since 2000, the low-frequency relationship has weakened significantly. In terms of

[6] Bond maturities are in years and time periods are in months, except when we estimate regressions at a daily frequency.

[7] Since we use overlapping changes in equation (1.1) when $h > 1$, we report Newey and West (1987) standard errors using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; when $h = 1$, we report heteroskedasticity-robust standard errors. To address the tendency for statistical tests based on Newey-West standard errors to over-reject in finite samples, we compute p-values using the asymptotic theory of Kiefer and Vogelsang (2005) which gives more conservative p-values and has better finite-sample properties than traditional Gaussian asymptotic theory.

a decomposition between real yields and inflation compensation, Table 1 shows that the majority of the decline in β_h as a function of h during the post-2000 sample is accounted for by the real component.

In the Internet Appendix, we show that very similar results obtain when we use long-term private yields as the dependent variable in equation (1.1). Specifically, we examine long-term corporate bond yields with Moody's ratings of Aaa and Baa, the 10-year swap yield, and the yield on Fannie Mae mortgage-backed-securities. For of all these long-term yields, the sensitivity to changes in 1-year Treasury rates was similar irrespective of frequency before 2000. After 2000, the sensitivity at high frequencies increases while the sensitivity at low frequencies declines significantly. The Internet Appendix also shows that similar results hold using using different proxies for the short-term rate—i.e., using changes in 3-month, 6-month, or 2-year Treasury yields as the independent variable in equation (1.1).

We use two approaches to determine the timing of the break as being around 2000. First, we estimate equations (1.1) and (1.2) using 10-year rolling windows. The estimated coefficients for $h = 12$-month changes are shown in Figure 2 for 10-year yields and forwards. These β_{12} coefficients decline substantially in more recent windows.[8] The second approach is to test for a structural break in equations (1.1) and (1.2) for $h = 12$-month changes, allowing for an unknown break date. We use the test of Andrews (1993) who conducts a Chow (1960) test at all possible break dates, and then takes the maximum of the Wald test statistics. Figure 3 plots the Wald test statistic for each possible break date in equations (1.1) and (1.2) along with the Cho and Vogelsang (2017) critical values for a null of no structural break. The strongest evidence for a break is in 1999 or 2000 in both equations (1.1) and (1.2) and the break is highly statistically significant. In the Internet Appendix, we use a similar Andrews (1993) test based on differences between the monthly (β_1) and annual (β_{12}) coefficients to date the emergence of the *frequency-dependent* sensitivity of long-term rates. This alternate test dates the break to 1999 or 2000.

Finally, one might wonder if our dating of this break is driven by distortions stemming from the 2009–2015 period when overnight nominal rates were stuck at the zero lower bound in the U.S. Our use of 1-year rates as the independent variables in equations (1.1) and (1.2) limits any potential distortions since 1-year nominal yields continued to fluctuate from 2009 to 2015 (Swanson and Williams, 2014). Indeed, even if we end our sample period in 2008, we still detect a break around 2000. Specifically, if the post-2000 sample ends in December 2008, we find a daily $\beta_{day} = 0.77$ and a yearly $\beta_{12} = 0.20$, which are essentially indistinguishable from the numbers in Table 1.

[8] There may be breaks in the regression coefficients for higher frequency changes at other dates, and for example, Thornton (2018) argues that there is a break in the relationship between monthly changes in 10-year yields and monthly changes in the federal funds rate somewhat earlier in the sample.

1.2 International evidence

Our focus is on the U.S., but it is useful to consider whether these same patterns are also observed in other large, highly-developed economies. In Table 2, we briefly explore evidence for the U.K., Germany, and Canada. Panel A of Table 2 shows estimates of equation (1.1) for the U.K., where data is available beginning in 1985. For the U.K., the estimates are broken out into real yields and inflation compensation. The evidence for the U.K. is remarkably similar to the U.S. evidence in Table 1. Before 2000, the daily coefficient ($\beta_{day} = 0.44$) and the yearly coefficient ($\beta_{12} = 0.38$) are similar in the U.K. After 2000, the daily sensitivity increases ($\beta_{day} = 0.86$), and the yearly sensitivity declines ($\beta_{12} = 0.29$). Because we have data on real yields prior to 2000 in the U.K., we can decompose the change in β_h into its real and inflation compensation components. As shown in Table 2, the inflation compensation component of β_h is stable across sample periods and frequency h. Thus, most of the changes in β_h are accounted for by changes in the real component of nominal yields.

Panel B of Table 2 shows estimates of equation (1.1) for Germany and Canada. For Germany, monthly data is available beginning in 1972 and daily data is available starting in 2000. For Canada, monthly and daily data are available beginning in 1986. Again, we observe similar patterns to those in the U.S. In the pre-2000 sample, β_h is stable across frequencies in Germany and Canada. After 2000, we observe greater sensitivity at high frequencies and less sensitivity at lower frequencies.

2 Yield-curve dynamics and bond return predictability

In this section, we first pinpoint the term structure dynamics that account for the greater high-frequency sensitivity and smaller low-frequency sensitivity of long rates to short rates in the post-2000 data. Specifically, we demonstrate that this frequency-dependent sensitivity of long-term rates arises because, all else equal, past increases in short rates predict a subsequent flattening of the yield curve—and a subsequent decline in long-term yields and forwards—in the post-2000 data. Statistically, this means that post-2000 yield curve dynamics are "path-dependent" or non-Markovian: it is not enough to know the current shape of the yield curve. Instead, to form the best forecast of future bond yields and returns, one also needs to know how the yield curve has shifted in recent months.

Second, we show that these non-Markovian dynamics are closely linked to a transient rise in the expected returns on long-term bonds over to those on short-term bonds. Specifically, since 2000, term premia on long-term bonds are temporarily elevated following past increases in short rates. Thus, relative to an expectations-hypothesis baseline, long rates exhibit excess sensitivity at high frequencies and *temporarily overreact* to changes in short rates.

2.1 Non-Markovian yield-curve dynamics

To begin, we note that one would not expect β_h to vary strongly with the horizon h as in the post-2000 data. In a standard term-structure model with a single factor, we have $y_t^{(10)} = \alpha + \beta \times y_t^{(1)}$ for some $\beta \in (0, 1)$, implying that $\beta_h = \beta$ for all h, regardless of whether or not the expectations hypothesis holds. More generally, even if they include multiple risk factors, term premia only fluctuate at business-cycle frequencies in conventional asset-pricing models, implying that β_h should be quite stable across monthly horizons h. Indeed, as discussed in Section 3, if there are both persistent and transient shocks to short rates, the expectations hypothesis actually suggests that β_h should be slightly *increasing* in h.

In this section, we show that strong horizon-dependence of β_h in the post-2000 period arises because yield curve dynamics have become non-Markovian: to form the best forecast of future yields, one needs to know the current shape of the yield curve and how short rates have changed recently.

2.1.1 Predicting level and slope

When examining term structure dynamics, it is useful and customary to study the dynamics of yield-curve factors, especially level and slope factors (Litterman and Scheinkman, 1991). We define the level factor as the 1-year yield ($L_t \equiv y_t^{(1)}$) and the slope factor as the 10-year yield less the 1-year yield ($S_t \equiv y_t^{(10)} - y_t^{(1)}$).[9] Most term structure models are Markovian with respect to current yield curve factors, meaning that the conditional mean of future yields depends only on today's yield-curve factors. However, our key finding—the post-2000 horizon-dependence of the relationship between long- and short-term yields—suggests that it may be useful to include lagged factors when forecasting yields. This idea has proven useful in several other contexts, including in Cochrane and Piazzesi (2005), Duffee (2013), Feunou and Fontaine (2014) and Feunou and Fontaine (2018). Specifically, we consider the following system of predictive monthly regressions:

$$L_{t+1} = \delta_{0L} + \delta_{1L} L_t + \delta_{2L} S_t + \delta_{3L}(L_t - L_{t-6}) + \delta_{4L}(S_t - S_{t-6}) + \varepsilon_{L,t+1} \tag{2.1a}$$

$$S_{t+1} = \delta_{0S} + \delta_{1S} L_t + \delta_{2S} S_t + \delta_{3S}(L_t - L_{t-6}) + \delta_{4S}(S_t - S_{t-6}) + \varepsilon_{S,t+1}, \tag{2.1b}$$

[9] The level and slope factors are sometimes defined as the first two principal components of a set of yields. For simplicity, we have defined the level and slope factors using fixed maturities on the yield curve. However, this choice makes little difference: we find similar results if we examine the first two principal components.

These regressions include level and slope as well as their changes over the prior six months.

Table 3 reports estimates of equations (2.1a) and (2.1b) for both the pre-2000 and post-2000 subsamples. We include specifications omitting all lagged changes (imposing $\delta_3 = \delta_4 = 0$), omitting lagged changes in slope (imposing $\delta_4 = 0$), and including all predictors. Based on the AIC or BIC, the model in column (1) with no lagged changes is chosen in the pre-2000 subsample, while the model in column (5) with lagged changes in level is selected in the post-2000 subsample. As shown in the bottom panel, in the post-2000 subsample, the lagged change in level is a highly significant negative predictor of the slope—i.e., increases in the level of yields predict subsequent yield curve flattening. For example, as shown in column (5), a 100 basis point increase in the level over the prior 6-months is associated with a 12 basis *per-month* decline in slope in the post-2000 sample (p-val < 0.001). By contrast, as shown in column (2), the coefficient on $L_t - L_{t-6}$ in the pre-2000 sample is zero. And, we can easily reject the hypothesis that the coefficients on $L_t - L_{t-6}$ in the pre- and post-2000 samples are equal (p-val < 0.001).[10]

The model in equations (2.1a)-(2.1b) can match the puzzling post-2000 horizon-dependent behavior of β_h that we documented above. This model can be written as a restricted vector autoregression (VAR) in $\mathbf{y}_t = (L_t, S_t)'$ of the form:

$$\mathbf{y}_{t+1} = \mu + \mathbf{A}_1 \mathbf{y}_t + \mathbf{A}_2 \mathbf{y}_{t-6} + \varepsilon_{t+1}. \tag{2.2}$$

Let $\Gamma_{ij}(h)$ denote the ijth element of the autocovariance of $\mathbf{y}_t$ at a lag of h months—i.e., the the ijth element of $\mathbf{\Gamma}(h) = E[(\mathbf{y}_t - E[\mathbf{y}_t])(\mathbf{y}_{t-h} - E[\mathbf{y}_{t-h}])']$. Given the estimated parameters from equations (2.1a) and (2.1b), we can work out $\Gamma_{ij}(h)$ to obtain the VAR-implied values of β_h in equation (1.1):

$$\beta_h = \frac{Var(L_t - L_{t-h}) + Cov(S_t - S_{t-h}, L_t - L_{t-h})}{Var(L_t - L_{t-h})} = 1 + \frac{2\Gamma_{12}(0) - \Gamma_{12}(h) - \Gamma_{12}(-h)}{2(\Gamma_{11}(0) - \Gamma_{11}(h))}. \tag{2.3}$$

In the pre-2000 sample, Table 1 reported estimates of $\beta_1 = 0.46$ and $\beta_{12} = 0.56$. In the post-2000 sample, the estimates are $\beta_1 = 0.64$ and $\beta_{12} = 0.20$. The last two rows of Table 3 show the VAR-implied values of β_1 and β_{12} from equation (2.3). In the pre-2000 data, all of the VAR models can roughly match both β_1 and β_{12}. In the post-2000 sample, all models can match β_1, but only the models that include lagged changes in level—i.e., models that allow for non-Markovian dynamics—can match the sharp drop in β_{12}. Specifically, if the post-2000 VAR does not include lagged changes ($\delta_3 = \delta_4 = 0$) as in column (4), the VAR-implied values of β_{12} would be 0.57 and would be nowhere near what we observe in the data.

[10] In unreported results, we also find that past changes in the level factor are associated with declines in the slope factor over the following month when the change in the level is computed over the prior 3 or 12 months.

2.1.2 Predictable reversals in long-term rates

We next show that these non-Markovian dynamics imply that, in the post-2000 data, there are predictable reversals in long-term rates following past increases in short-term rates—i.e., long-term rates temporarily over-react to changes in short-terms rates. To see this explicitly, in Table 4 we estimate impulse-response functions that are reminiscent of Jorda (2005) local projections. Specifically, we predict the future changes in 10-year yields and forwards from month t to $t+h$ using the current level (L_t) and slope (S_t) of the yield curve as well as the prior month's change in level ($L_t - L_{t-1}$) and slope ($S_t - S_{t-1}$):

$$z_{t+h} - z_t = \delta_0^{(h)} + \delta_1^{(h)} L_t + \delta_2^{(h)} S_t + \delta_3^{(h)} (L_t - L_{t-1}) + \delta_4^{(h)} (S_t - S_{t-1}) + \varepsilon_{t \to t+h}, \tag{2.4}$$

Table 4 reports estimates of equation (2.4) for $z_t = y_t^{(10)}$ and $f_t^{(10)}$ in the pre- and post-2000 samples for h = 3-, 6-, 9-, and 12- month changes. In Figure 4, we plot the coefficients $\delta_3^{(h)}$ on $L_t - L_{t-1}$ for $h = 1, 2, ..., 12$, effectively tracing out the predictable future change in z_t from month t to $t+h$ in response to an unexpected change in the level of rates between $t-1$ and t.[11]

Figure 4 plots the coefficients $\delta_3^{(h)}$ on $L_t - L_{t-1}$ versus monthly horizon h for both 10-year yields and 10-year forward rates. Figure 4 shows that, in the post-2000 data, there are predictable reversals in both 10-year yields and forwards following a past increase in short-term rates. However, there is no such reversal in the pre-2000 data. For instance, for 10-year yields, Table 4 reports that $\delta_3^{(6)} = -0.42$ (p-val $= 0.03$) after h = 6-months in the post-2000 data. (The difference between $\delta_3^{(6)}$ in the pre- and post-2000 data is significant with a p-value of 0.02.) Since 2000, Table 1 showed that a +100 bps increase in short-rates in month t is associated with a +64 bps contemporaneous rise of long-term yields. Strikingly, Table 4 suggests that 42 bps—or roughly two-thirds—of this initial response is expected to reverse within 6 months. As in Table 1, the post-2000 reversion in 10-year forwards is even larger in magnitude and is statistically stronger. For 10-year forwards, we have $\delta_3^{(6)} = -0.58$ (p-val < 0.01) and the difference between $\delta_3^{(6)}$ in the pre- and post-2000 data is highly significant (p-val < 0.01). In summary, Table 4 and Figure 4 show that long-term rates appear to temporarily overreact to changes in short-term rates in the post-2000 data, but there was no such tendency before 2000.

To better understand these results, we decompose 10-year yields into the sum of a level component

[11] The inclusion of L_t and S_t as controls means that the $\delta_3^{(h)}$ and $\delta_4^{(h)}$ coeffiecients can be interpretted as reflecting the response of z_t to an unexpected changes in the level and slope between $t-1$ and t. The estimated $\delta_3^{(h)}$ coeffiecients are similar if we omit $S_t - S_{t-1}$ from the regression, thus imposing $\delta_4^{(h)} = 0$. And, the estimates are also quite similar, albeit with slightly larger standard errors, if we omit the controls altogether.

and a slope component as in Table 3—i.e., $y_t^{(10)} = L_t + S_t$—and plot the coefficients $\delta_3^{(h)}$ versus h for both level ($z_t = L_t$) and slope ($z_t = S_t$). Consistent with Table 3, Table 4 and Figure 4 show that the predictable reversals in long-term yields reflects the juxtaposition of two opposing forces in the post-2000. First, past increases in short-term rates predict subsequent increases in short-term rates in the post-2000 data, perhaps owing to the Fed's growing desire to adjust the overnight rate gradually over time (Stein and Sunderam, 2018). However, past increase in short-term rates strongly predict a subsequent flattening of the yield curve since 2000. Since the latter effect outweighs the former, we see predictable reversals in long-term rates post-2000.

2.2 Predicting bond returns

In this subsection, we recast our main finding—the fact that, in recent years, β_h is so large at high frequencies and then declines rapidly as a function of horizon h —as a result about bond return predictability. Specifically, we show that this result arises because past increases in the level of rates lead to *temporary* rise in the expected return on long-term bonds relative to those on short-term bonds—i.e., a temporary rise in bond term premia. Thus, our findings reflect a new form of bond return predictability.

Results for 10-year bonds: Recall that the k-month log excess return on n-year bonds over the riskless return on k-month bills, $(k/12)\, y_t^{(k/12)}$, is:

$$rx_{t\to t+k}^{(n)} \equiv (k/12)\,(y_t^{(n)} - y_t^{(k/12)}) - (n - k/12)(y_{t+k}^{(n-k/12)} - y_t^{(n)}). \tag{2.5}$$

We first forecast the excess return on $n = 10$-year zero-coupon bonds using level, slope, and the 6-month past changes in these two yield-curve factors:

$$rx_{t\to t+k}^{(10)} = \delta_0 + \delta_1 L_t + \delta_2 S_t + \delta_3(L_t - L_{t-6}) + \delta_4(S_t - S_{t-6}) + \varepsilon_{t\to t+k}. \tag{2.6}$$

In Table 5, we report the results from estimating these predictive regressions for $k = 1$, 3, and 6-month returns. Panel A reports the results for the pre-2000 sample and Panel B shows the post-2000 results.[12,13]

[12]In general, we take the yield on k-month Treasury bills, $y_t^{(k/12)}$, from the yield curve estimates in Gürkaynak et al. (2007). But this yield curve is based on only coupon securities with at least three months to maturity, and so does not fit the very short end of the yield curve well in the pre-2000 data. In light of this, we take the 1-month bill yield from Ken French's website for the pre-2000 sample.

[13]We obtain broadly similar results in Table 5 if we forecast returns using 3- or 12-month past changes in level and slope. And, the return predictability associated with past changes in level remains similar if, instead of controlling for level and slope, we control for the first five forward rates as in Cochrane and Piazzesi (2005).

In the post-2000 data, Table 5 shows the past change in the level of rates is a robust predictor of the returns on long-term bonds. However, there is no such predictability in the pre-2000 data. For instance, in column (6) of Panel A, we see that, all else equal, a +100 bps increase in short-term rates over the prior 6 monthly is associated with a $\delta_3 = +202$ bps (p-val < 0.01) increase in expected 3-month bond returns and the difference between δ_3 in the pre- and post-2000 data is statistically significant (p-val < 0.01). Table 5 also shows that the post-2000 return predictability associated with past increases in the level of rates is short-lived and generally dissipates after $k = 6$ months. In other words, past increases in the level of rates lead to a temporary increase in the risk premia on long-term bonds.

To draw out the connection to the predictable curve flattening discussed above, we show that these results for 10-year returns are related to predictability of the returns on what we refer to as "level-mimicking" and "slope-mimicking" portfolios. Specifically, we follow Joslin et al. (2014) and construct bond portfolios that locally mimic changes in the level and slope factors. Consider a factor-mimicking portfolio that places weight w_n on zero-coupon bonds with n years to maturity. The k-month excess return on this portfolio from t to $t+k$ is $rx^{P}_{t\to t+k} = (\sum_n w_n \times rx^{(n)}_{t\to t+k})/\,|\sum_n w_n|$. The level-mimicking portfolio has a weight -1 on 1-year bonds and no weight on any other bonds. For small k, we have $rx^{(10)}_{t\to t+k} \approx -10 \times (\Delta_k L_{t+k} + \Delta_k S_{t+k})$ and $rx^{(1)}_{t\to t+k} \approx -1 \times \Delta_k L_{t+k}$. Thus, the level- mimicking portfolio has a k-month excess return of $rx^{LEVEL}_{t\to t+k} = -1 \times rx^{(1)}_{t\to t+k} \approx \Delta_k L_{t+k}$. The slope-mimicking portfolio has a weight 1 on 1-year bonds and -0.1 on 10-year bonds, so $rx^{SLOPE}_{t\to t+k} = (1 \times rx^{(1)}_{t\to t+k} - 0.1 \times rx^{(10)}_{t\to t+k})/0.9 \approx \Delta_k S_{t+k}/0.9$.[14] Finally, we note that the excess returns on 10-year bonds are just a linear combination of the excess returns on the level- and slope-mimicking portfolios: $rx^{(10)}_{t\to t+k} = -9 \times rx^{SLOPE}_{t\to t+k} - 10 \times rx^{LEVEL}_{t\to t+k}$.

In Panels B and C of Table 5, we estimate equation (2.6) using $rx^{LEVEL}_{t\to t+k}$ and $rx^{SLOPE}_{t\to t+k}$ as the dependent variable. In the post-2000 sample, the excess returns on the slope-mimicking portfolio depend negatively on $L_t - L_{t-6}$, but the excess returns on the level-mimicking portfolio depend positively on $L_t - L_{t-6}$.[15] While the two effects partially cancel when predicting 10-year excess returns, the net effect is positive and statistically significant in the post-2000 data. Furthermore, the results in Panels B and C of Table 5 where we forecast $rx^{LEVEL}_{t\to t+k}$ and $rx^{SLOPE}_{t\to t+k}$ are entirely consistent with those in Table 3.

In summary, we find that, since 2000, term premia on long-term bonds are *temporarily* elevated following past increases in short rates. This implies that, relative to an expectations-hypothesis baseline,

[14] The slope-mimicking portfolio is approximately hedged against parallel shifts in the yield curve. Thus, $rx^{SLOPE}_{t\to t+k}$ corresponds to the returns on a "steepener" trade that will profit if the yield curve steepens.

[15] The latter fact is consistent with Piazzesi et al. (2015) and Cieslak (2018), who account for it either with expectational errors or time-varying risk premia. Brooks et al. (2017) also show that the federal funds rate displays short-term momentum.

long-term rates *temporarily overreact* to movements in short rates, exhibiting what Mankiw and Summers (1984) called "excess sensitivity" at high frequencies.

Results for other bond maturities: Finally, we examine the predictability for other bond maturities. If, as we argue, past increases in short rates temporarily raise the net supply of long-term bonds that investors must hold, thereby raising the compensation investors require for bearing interest-rate risk, this should have a larger impact on the expected returns of long-term bonds than intermediate bonds. This is because the returns on long-term bonds are more sensitive to shifts in yields than those on intermediate bonds (Vayanos and Vila, 2009; Greenwood and Vayanos, 2014). We explore this prediction in Figure 5. We separately forecast the 3-month returns on bonds with different maturities n, estimating:

$$rx_{t\to t+3}^{(n)} = \delta_0^{(n)} + \delta_1^{(n)} L_t + \delta_2^{(n)} S_t + \delta_3^{(n)} (L_t - L_{t-6}) + \delta_4^{(n)} (S_t - S_{t-6}) + \varepsilon_{t\to t+3}^{(n)} \tag{2.7}$$

separately for $n = 1, ...20$-year bonds. We then plot the coefficients $\delta_3^{(n)}$ on the past change in level from estimating equation (2.7) versus bond maturity n for the pre-2000 and post-2000 samples. (For the pre-2000 sample, the longest available maturity is $n = 15$ years). Consistent with the idea that past increases in short rates temporarily raise the compensation for bearing interest-rate risk, the coefficients $\delta_3^{(n)}$ are monotonically increasing in bond maturity n in the post-2000 sample. By contrast, there is no such predictability in the pre-2000 sample.

This temporary rise in the compensation for bearing interest rate risk impacts the yield and forward rate curves. As explained in Greenwood and Vayanos (2014), a short-lived rise in the compensation for bearing interest rate risk may have relatively constant or even a hump-shaped effect on the yield and forward curves as opposed to the monotonically increasing effect shown above for returns. The intuition is that the impact on bond yields equals the effect on a bond's average expected returns over its lifetime. As a result, a *temporary* rise in the compensation for bearing interest rate risk can have a greater impact intermediate-term yields than on long-term yields. Thus, we plot the slope coefficients $\delta_3^{(n)}$ versus maturity n from estimating:

$$f_{t+3}^{(n-3/12)} - f_t^{(n)} = \delta_0^{(n)} + \delta_1^{(n)} L_t + \delta_2^{(n)} S_t + \delta_3^{(n)} (L_t - L_{t-6}) + \delta_4^{(n)} (S_t - S_{t-6}) + \varepsilon_{t\to t+3}^{(n)}, \tag{2.8}$$

and

$$y_{t+3}^{(n-3/12)} - y_t^{(n)} = \delta_0^{(n)} + \delta_1^{(1)} L_t + \delta_2^{(1)} S_t + \delta_3^{(3)} (L_t - L_{t-6}) + \delta_4^{(n)} (S_t - S_{t-6}) + \varepsilon_{t\to t+3}^{(n)}. \tag{2.9}$$

for $n = 1, 2, ..., 20$ years for both the pre-2000 and post-2000 samples.[16] After 2000, the second plot in Figure 5 shows that a past increase in short-term rates has a slight humped-shaped effect on the evolution of the forward curve with the peak impact at 4 years. Turning to yields, while past increases in level of short rates forecast a future flattening of the yield curve in the post-2000 data, the bottom plot in Figure 5 shows that the expected changes in long-term yields is relatively constant beyond 5 years.

In summary, the results for different maturities support the view that past increases in short-term rates *temporarily* raise the compensation that investors earn for bearing interest-rate risk.

2.3 Interpreting the evidence

Before developing our economic model in the next section, we pause to take stock and to interpret our results. We note that our key findings are all consistent with the view that, in recent years, the term premium on long-term bonds is increasing in the recent change in short-term rates, other things being equal. This single non-Markovian assumption can match the facts that, in the post-2000 data (i) β_h declines with horizon h and (ii) that, controlling for current yield curve factors, past changes in short-term interest rates predict (a) future yield-curve flattening, (b) future declines in long-term rates, and (c) high future excess returns on long-term bonds.

To develop these ideas, take the long-term yield y_t and break it into an expectations-hypothesis piece eh_t that reflects expected future short-term rates and a term premium piece tp_t that reflects future bond risk premia: $y_t = eh_t + tp_t$. Let i_t denote the short-term interest rate. Thus, by definition, β_h—the total sensitivity of long-term yields at horizon h—is the sum of an expectations-hypothesis piece β_h^{eh} and a term premium piece β_h^{tp}:

$$\overbrace{\frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]}}^{\beta_h} = \overbrace{\frac{Cov\left[eh_{t+h} - eh_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]}}^{\beta_h^{eh}} + \overbrace{\frac{Cov\left[tp_{t+h} - tp_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]}}^{\beta_h^{tp}}. \tag{2.10}$$

First, consider the expectations-hypothesis piece, β_h^{eh}. For now, assume that the short-rate follows a univariate AR(1) process, implying that $eh_t = \alpha^{eh} + \beta^{eh} \cdot i_t$ and $\beta_h^{eh} = \beta^{eh}$ for all h. Next, consider the term premium piece. With conventional asset pricing theories, term premia only vary at business cycle

[16] In equation (2.8), $f_t^{(n)} \equiv ny_t^{(n)} - (n-1)\,y_t^{(n)}$ is the 1-year rate $(n-1)$ years forward as opposed to the instantaneous forward n years forward. Since $f_{t+k}^{(n-k/12)} - f_t^{(n)} = -(rx_{t\to t+k}^{(n)} - rx_{t\to t+k}^{(n-1)})$, there is a tight connection between the coefficients in equations (2.8) and (2.7). Specifically, defining $f_{t+k}^{(1-k/12)} - f_t^{(1)} = -rx_{t\to t+k}^{(1)}$, we have $rx_{t\to t+k}^{(n)} = \sum_{m=1}^{n}(f_t^{(m)} - f_{t+k}^{(m-k/12)})$. Thus, the coefficients in equation (2.7) for maturity n can be recovered by summing up the -1 times coefficients in equation (2.8) for all maturities $m \leq n$. Similarly, the coefficients from equation (2.9) for maturity n are approximately the average of the coefficients in equation (2.8) for all maturities $m \leq n$.

frequencies, so one would not expect β_h^{tp} to vary strongly with monthly horizon h. Thus, roughly speaking, conventional theories suggest that $tp_t \approx \alpha^{tp} + \beta^{tp} \cdot i_t$, implying that $\beta_h^{tp} = \beta^{tp}$ and $\beta_h = \left(\beta^{eh} + \beta^{tp}\right)$ for all h. In other words, it is difficult for conventional asset pricing theories to match the strong horizon-dependence of β_h that is so evident in the post-2000 data.

To generate horizon-dependent sensitivity, we make the following non-Markovian assumption:

$$tp_t = \alpha^{tp} + \beta^{tp} \cdot i_t + \delta^{tp} \cdot (i_t - i_{t-1}), \tag{2.11}$$

where $\delta^{tp} > 0$. In other words, we assume that term premia depend on the current level of short-term rates and the recent change in short rates. Under this assumption, one can show that:

$$\beta_h = \beta^{eh} + \beta^{tp} + \delta^{tp} \cdot (1 - \gamma_h) \text{ where } \gamma_h \equiv \frac{Cov\left[i_{t+h-1} - i_{t-1}, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]}. \tag{2.12}$$

The key is then to note that γ_h—the coefficient from a regression of $(i_{t+h-1} - i_{t-1})$ on $(i_{t+h} - i_t)$—will be an increasing function of h. When $\delta^{tp} > 0$, this in turn explains why β_h^{tp} is decreasing in h.[17] Furthermore, when $\delta^{tp} > 0$, controlling for current level of short-term rates, past changes in short-rates will predict future yield curve flattening, declines in long-term yields, and high excess returns on long-term bonds.

In summary, our main findings can all be seen as consequences of fact that, in recent years, the term premia on long-term bonds is increasing in the recent change in short-term rates—i.e., of the fact that $\delta^{tp} > 0$. However, it follows that we can conclude little about the sign and magnitude of β^{tp} based on our findings. For instance, the fact that $\beta_1 - \beta_h = \delta^{tp} \times (\gamma_h - \gamma_1)$ rises rapidly with h in the post-2000 data, primarily tells us about the sign and magnitude of δ^{tp}—i.e., about the way that term premia depend on past increases in short rates. By contrast, this finding tells us little about the low-frequency relationship between term premia and the level of short rates—i.e., about β^{tp}.

3 Model

In this section, we construct a simple model that is useful for explaining our key finding: since 2000, the sensitivity of long-term rates to changes in short-term rates is steeply declining in the horizon over which these changes are computed.

In our model, time is discrete and infinite. Risk-averse arbitrageurs can either hold long-term, perpet-

[17] For instance, if i_t follows an AR(1) of the form $i_{t+1} - \bar{\imath} = \rho_i (i_t - \bar{\imath}) + \varepsilon_{i,t+1}$, then $\gamma_h = \left(2\rho_i - \rho_i^{h-1} - \rho_i^{h+1}\right) / \left(2 - 2\rho_i^h\right)$. We have $\gamma_1 = -(1-\rho_i)^2 / (2 - 2\rho_i) < 0$ and $\lim_{h\to\infty} \gamma_h = \rho_i > 0$. And, treating γ_h as continous in h, we have $\partial\gamma_h/\partial h > 0$.

ual nominal bonds or short-term nominal bonds. The nominal interest rate on short-term bonds from t to $t+1$, denoted i_t, follows an exogenous stochastic process. Long-term bonds are available in a given net supply that must be absorbed by the arbitrageurs in our model. The net supply, s_t, is the gross supply of long-term bonds outstanding net of the amount purchased by other investors—e.g., "preferred habitat investors"—who have inelastic demands for long-term bonds. Since arbitrageurs' risk-bearing capacity is limited, shifts in the supply and demand for long-term bonds impact the term premium on long-term bonds as in Vayanos and Vila (2009) and Greenwood and Vayanos (2014).

The first key assumption is that shocks to the net supply of long-term bonds are positively correlated with shocks to short rates: increases in short rates are either associated with increases in the gross supply of long-term bonds or with reductions in the inelastic demands of preferred habitat investors. While we discuss several amplification mechanisms that give rise to this reduced form, one simple interpretation is that there is a growing set of investors who "reach for yield" when short rates decline (Hanson and Stein, 2015). This assumption implies that increases in short rates are associated with increases in term premia, generating "excess sensitivity" of long-term yields beyond that implied by the expectations hypothesis.

The second key assumption, following Duffie (2010), is that arbitrage capital is slow-moving: these supply and demand shocks walk down a short-run demand curve that is steeper than the long-run demand curve. This slow-moving capital dynamic implies that an increase in short-term interest rates leads to a short-lived supply-and-demand imbalance in the market for long-term bonds and, thus, a short-lived increase in bond term premia. As a result, the excess sensitivity of long-term yields is greatest when measured at short horizons. Thus, by combining reaching-for-yield and slow-moving capital, our model helps us understand why long-term yield may temporarily overreact to movements in short-term rates. Formally, our model is a close cousin of the model in Greenwood et al. (2018), who incorporate slow-moving capital effects into a model of the term structure.

Our model can match our key findings—β_h has fallen for large h and risen for small h post-2000—if (i) shocks to short-term nominal rates have become less persistent and (ii) the kinds of supply-and-demand-based amplification mechanisms we emphasize have grown in importance. We argue that (i) is justified since there is strong evidence that shocks to the persistent component of inflation have become less volatile since the mid-1990s (Stock and Watson, 2007). Similarly, we argue that (ii) is justified since many of these amplification mechanisms appear to have become more powerful.

3.1 Model setting

Short- and long-term nominal bonds: Short-term nominal bonds are available in perfectly elastic supply. At time t, investors learn that short-term bonds will earn a riskless log return of i_t in nominal terms between time t and $t+1$. One can think of the short-term nominal interest rate as being determined outside the model by monetary policy.

Long-term nominal bonds are available in a given net supply s_t that must be absorbed by the arbitrageurs in our model. The long-term nominal bond is a perpetuity that pays a coupon of $K > 0$ each period. Let P_t denote the price of this bond at time t, so the return on long-term bonds from t to $t+1$ is $1 + R_{t+1} = (P_{t+1} + K) / P_t$. To generate a tractable linear model, we use the well-known Campbell and Shiller (1988) log-linear approximation to the return on this perpetuity. Defining $\theta \equiv 1/(1+K) < 1$, the log excess return on long-term bonds over short-term bonds from t to $t+1$ is approximately:

$$rx_{t+1} \equiv \ln(1 + R_{t+1}) - i_t \approx \overbrace{\frac{1}{1-\theta}}^{D} y_t - \overbrace{\frac{\theta}{1-\theta}}^{D-1} y_{t+1} - i_t, \tag{3.1}$$

where y_t is the log yield-to-maturity on long-term bonds at time t and $D = 1/(1-\theta) = (K+1)/K$ is the Macaulay duration when the long-term bond is trading at par.[18] Iterating equation (3.1) forward and taking expectations, the yield on long-term bonds is:

$$y_t = (1-\theta) \textstyle\sum_{j=0}^{\infty} \theta^j E_t [i_{t+j} + rx_{t+j+1}]. \tag{3.2}$$

The long-term yield is the sum of (i) an expectations hypothesis component $eh_t = (1-\theta) \sum_{j=0}^{\infty} \theta^j E_t [i_{t+j}]$ that reflects expected future short rates and (ii) a term premium component $tp_t = (1-\theta) \sum_{j=0}^{\infty} \theta^j E_t [rx_{t+j+1}]$ that reflects expected future excess returns on long-term bonds over short-term bonds.

Market participants: There are two groups of risk-averse arbitrageurs in the model, each with identical risk tolerance τ, who differ solely in the frequency with which they can rebalance their bond portfolios.

The first group are "fast-moving arbitrageurs" who are free to adjust their holdings of long-term and short-term bonds each period. Fast-moving arbitrageurs are present in mass q and we denote their demand for long-term bonds at time t by b_t. Fast-moving arbitrageurs have mean-variance preferences

[18] We review the derivation of equation (3.1) in the Internet Appendix.

over 1-period portfolio log returns. Thus, their demand for long-term bonds at time t is:

$$b_t = \tau \frac{E_t [rx_{t+1}]}{Var_t [rx_{t+1}]}. \tag{3.3}$$

The second group are "slow-moving arbitrageurs" who can only adjust their holdings of long-term and short-term bonds every k periods. Slow-moving arbitrageurs are present in mass $1 - q$. A fraction $1/k$ of slow-moving arbitrageurs is active each period and can reallocate their portfolios, but they must maintain this same allocation for the next k periods. As in Duffie (2010), this is a reduced-form way to model the forces—whether due to institutional frictions or to limited attention—that may limit the speed of arbitrage capital flows. Since they only rebalance every k periods, slow-moving arbitrageurs have mean-variance preferences over their k-period *cumulative* portfolio excess return. Thus, the demand for long-term bonds from the subset of slow-moving arbitrageurs who are active at time t is:

$$d_t = \tau \frac{E_t[\sum_{j=1}^{k} rx_{t+j}]}{Var_t[\sum_{j=1}^{k} rx_{t+j}]}. \tag{3.4}$$

Risk factors: Holders of long-term bonds face two different types of risk. First, they are exposed to *interest rate risk:* they will suffer a capital loss on their long-term bond holdings if short-term rates unexpectedly rise. Second, they are exposed to *supply risk*: there are shocks to the net supply of long-term bonds that impact long-term bond yields, holding fixed the expected future path of short-term interest rates. In other words, these supply shocks impact the term premium on long-term bonds. We make the following assumptions about the evolution of these two risk factors.

Short-term nominal interest rates: The short-term nominal interest rate is the sum of a highly persistent component $i_{P,t}$ and a more transient component $i_{T,t}$:

$$i_t = i_{P,t} + i_{T,t}. \tag{3.5}$$

A natural interpretation is that the persistent component reflects long-run inflation expectations and the transient component reflects cyclical variation in short-term real rates and expected inflation. The persistent component $i_{P,t}$ follows an exogenous AR(1) process:

$$i_{P,t+1} = \bar{\imath} + \rho_P (i_{P,t} - \bar{\imath}) + \varepsilon_{P,t+1}, \tag{3.6}$$

where $0 < \rho_P < 1$ and $Var_t[\varepsilon_{P,t+1}] = \sigma_P^2$. The transient component $i_{T,t}$ also follows an exogenous AR(1):

$$i_{T,t+1} = \rho_T i_{T,t} + \varepsilon_{T,t+1}, \tag{3.7}$$

where $0 < \rho_T \leq \rho_P < 1$ and $Var_t[\varepsilon_{T,t+1}] = \sigma_T^2$.

If $\rho_T < \rho_P$ and σ_P is large relative to σ_T, then short-term nominal rates will be highly persistent. As a result, long-term nominal rates will be highly sensitive to movements in short-term nominal rates due to standard expectations-hypothesis logic. Indeed, a large value of σ_P is a good explanation for the high sensitivity of long-term rates observed in the 1970s, 1980s, and the 1990s when long-run inflation expectations were not well-anchored (Gürkaynak et al., 2005). However, long-run inflation expectations have become firmly anchored in recent decades and there is strong evidence that shocks to the persistent component of nominal inflation have become far less volatile since the mid-1990s (Stock and Watson, 2007). Thus, we posit that σ_P has declined since the late 1990s.

Supply of long-term bonds: The long-term nominal bond is available in an exogenous, time-varying *net supply* s_t that must be held in equilibrium by fast arbitrageurs and slow-moving arbitrageurs. This net supply equals the *gross supply* of long-term bonds *minus the demand* from other investors outside the model who have inelastic demands. Formally, we assume that s_t follows an AR(1) process:

$$s_{t+1} = \overline{s} + \rho_s (s_t - \overline{s}) + \varepsilon_{s,t+1} + C\varepsilon_{P,t+1} + C\varepsilon_{T,t+1}, \tag{3.8}$$

where $0 < \rho_s \leq \rho_T < 1$, $C \geq 0$, and $Var_t[\varepsilon_{s,t+1}] = \sigma_s^2$. When $C > 0$, shocks to short rates are positively associated with shocks to the net supply of long-term bonds. The $\varepsilon_{s,t+1}$ shocks capture other forces that impact the net supply of long-term bonds. The model can be solved for any arbitrary correlation structure among the three underlying shocks $\varepsilon_{P,t+1}$, $\varepsilon_{T,t+1}$, and $\varepsilon_{s,t+1}$. However, for simplicity, we that the three shocks are mutually orthogonal.

3.2 Shocks to bond supply and the short rate

Suppose that $C > 0$ in equation (3.8). In the Internet Appendix, we show that:

$$\begin{aligned} s_t &= \overline{s} + C[(i_{P,t} - \overline{\imath}) - (\rho_P - \rho_s) \textstyle\sum_{j=0}^{\infty} \rho_s^j (i_{P,t-j-1} - \overline{\imath})] \\ &\quad + C[i_{T,t} - (\rho_T - \rho_s) \textstyle\sum_{j=0}^{\infty} \rho_s^j i_{T,t-j-1}] + [\textstyle\sum_{j=0}^{\infty} \rho_s^j \varepsilon_{s,t-j}]. \end{aligned} \tag{3.9}$$

Thus, when $\rho_s < \rho_T$, net bond supply is increasing in the differences between the current level of each component of the short rate and a geometric moving-average of past values of that component. As a result, net bond supply s_t will be high when short rates have recently risen.

The specification in equation (3.9) is a reduced-form way of capturing several different supply-and-demand mechanisms that help explain why negative shocks to short-term rates are associated with declines in the term premium on long-term bonds. While the precise mix of these mechanisms and their combined strength may vary over time and across countries, there is a growing consensus that these mechanisms play an increasingly important role in fixed-income markets.

The "reaching-for-yield" channel: The simplest interpretation of our assumption that $C > 0$ in the post-2000 data, is that there is a growing set of investors who "reach for yield" when short rates decline. Specifically, according to the reaching-for-yield channel (Hanson and Stein, 2015), negative shocks to short rates boost the demand for long-term bonds from "yield-oriented investors." The idea is that, for either frictional or behavioral reasons, these investors care about the *current yield* on their portfolios over and above their expected portfolio returns. Because expected mean reversion in short rates implies that the yield curve is steep when short rates are low, yield-oriented investors' demand for long-term bonds is greater when short rates are low. Thus, holding fixed the gross supply, the *net supply* of long-term bonds that must be held by fast- and slow-moving arbitrageurs declines when short rates fall.[19]

Lian et al. (2017) provide experimental evidence that there is a non-linear relationship between reaching-for-yield behavior and the level of rates. Specifically, building on Prospect Theory (Kahneman and Tversky, 1979), they argue that reaching for yield becomes more pronounced as rates fall further below some reference level that investors are accustomed to based on past experience, just as in equation (3.9). Relatedly, they argue that, because people tend to think in proportions as opposed to in differences—a well-known tendency known as Weber's law—small return differentials loom larger in investors' minds when short rates are lower (see Bordalo et al. (2013)). For instance a 1% pick-up in yield from buying long-term bonds instead short-term bonds seems like "a better deal" when short rates are 1% than when they are 5%, because it doubles yield rather than increasing it by a factor of 1.2. Thus, their evidence suggests that the reaching-for-yield channel has grown stronger in recent years as interest rates have reached historically low levels. In the language of our model, this suggests that C has risen.

[19] More generally, low short rates may increase investors' risk appetites through a variety of channels, thereby depressing term premia (Maddaloni and Peydró, 2011; Di Maggio and Kacperczyk, 2017; Drechsler et al., 2014).

The mortgage refinancing channel: According to the mortgage refinancing channel (Hanson, 2014; Malkhozov et al., 2016), negative shocks to short-term rates induce mortgage refinancing waves that lead to temporary declines in the duration of outstanding fixed-rate mortgages—i.e., a temporary reduction in the gross supply of long-term bonds. As a result, declines in short rates are associated with temporary declines in term premia. And, due to these mortgage refinancing dynamics, the current gross supply of long-term bonds depends on the difference between current interest rates and a moving-average of past rates as in equation (3.9). The mortgage refinancing channel is only relevant in countries like the U.S. where fixed-rate mortgages with an embedded prepayment option—i.e., mortgages that are "negatively convex"—are an important source of financing.

Hanson (2014) shows that the strength of this channel grew during the 1990s as mortgage-backed securities became a larger component of the U.S. bond market. Specifically, Hanson (2014) presents evidence that long-term rates are more sensitive to short rates when the aggregate negative convexity of the mortgage market is larger relative to the broader U.S. bond market, and that aggregate negative convexity has trended upwards. In our framework, this means than C has risen over time.

Asset and liability management by insurers and pensions: Domanski et al. (2017) and Shin (2017) point to a related convexity-based amplification mechanism stemming from the desire of insurers and pensions to match the duration of their assets and liabilities. They argue that the convexity of insurers' and pensions' liabilities is greater than the convexity of their assets. Thus, as interest rates decline, the duration of their liabilities increases more than the duration of their assets, and insurers and pensions increase their demand for long-term bonds to match asset and liability duration. Holding fixed the *gross supply* of long-term bonds, this means that the *net supply* of long-term bonds that must be held by arbitrageurs is lower when short rates are low. As a result, term premia on long-term bonds fall when short rates decline. This mechanism is arguably quite important in European bond markets where insurers and pensions play an especially important role. And this dynamic may have grown in recent years as regulators have pushed insurers to more prudently manage their interest rate exposures.

A behavioral over-extrapolation mechanism: According to this channel hinted at by Giglio and Kelly (2018) and extensively documented in Cieslak (2018), there is a set of biased investors who over-estimate the persistence of short-term rates, perhaps, because they have "diagnostic expectations" (Bordalo et al., 2017) and form their expectations of future short rates using the representativeness heuristic (Kahneman and Tversky, 1972). As a result, negative shocks to short rates lead these biased investors to demand more long-term bonds relative to rational investors who properly estimate the persistence of short rates. This means that the net supply of long-term bonds that must be held by unbiased arbitrageurs declines when short rates are low, leading to a decline in term premium. In the simplest telling, there is little reason to expect that this extrapolative tendency should have increased since 2000. However, in a more complicated telling, the amount of over-extrapolation might have risen if some investors are using an "outdated" model that features a larger fraction of persistent short rate shocks than there has been in recent years. This might be because some investors were slow to learn about the decline in the volatility of trend inflation documented in Stock and Watson (2007). This more complicated version of the over-extrapolation story is again consistent with the idea that C has risen since 2000.

3.3 Equilibrium yields

At time t, there is a mass q of fast-moving arbitrageurs, each with demand b_t, and a mass $(1-q)\,k^{-1}$ of active slow-moving arbitrageurs who rebalance their portfolios, each with demand d_t. These arbitrageurs must accommodate the *active supply*, which is the total net supply s_t of long-term bonds less any supply held off the market by inactive slow-moving arbitrageurs who do not rebalance the portfolios, $(1-q)\,k^{-1}\sum_{j=1}^{k-1} d_{t-j}$. Thus, the market-clearing condition for long-term bonds at time t is:

$$\overbrace{qb_t}^{\text{Fast demand}} + \overbrace{(1-q)k^{-1}d_t}^{\text{Active slow demand}} = \overbrace{s_t}^{\text{Total supply}} - \overbrace{(1-q)(k^{-1}\textstyle\sum_{j=1}^{k-1} d_{t-j})}^{\text{Inactive slow holdings}}. \tag{3.10}$$

We conjecture that equilibrium yields y_t and the demands of active slow-moving arbitrageurs d_t are linear functions of a state vector, $\mathbf{x}_t$, that includes the steady-state deviations of both components of short-term nominal interest rates, the net supply of bonds, and holdings of bonds by inactive slow-moving arbitrageurs. Formally, we conjecture that the yield on long-term bonds is $y_t = \alpha_0 + \alpha_1'\mathbf{x}_t$ and that slow-moving arbitrageurs' demand for long-term bonds is $d_t = \delta_0 + \delta_1'\mathbf{x}_t$, where the $(k+2)\times 1$ dimensional state vector, $\mathbf{x}_t$, is given by $\mathbf{x}_t = [i_{P,t} - \bar{\imath},\ i_{T,t},\ s_t - \bar{s}, d_{t-1} - \delta_0, \cdots, d_{t-(k-1)} - \delta_0]'$. These assumptions imply that the state vector follows a VAR(1) process $\mathbf{x}_{t+1} = \mathbf{\Gamma}\mathbf{x}_t + \epsilon_{t+1}$, where $\mathbf{\Gamma}$ depends on the parameters δ_1 governing slow-moving arbitrageurs' demand.

In the Internet Appendix, we show how to solve for equilibrium yields in this setting. A rational expectations equilibrium of our model is a fixed point of a specific operator involving the "price-impact" coefficients, (α_1'), which show how the state variables impact bond yields, and the "demand-impact" coefficients, (δ_1'), which show these variables impact the demand of active slow-moving investors. Specifically, let $\omega = (\alpha_1', \delta_1')'$ and consider the operator $\mathbf{f}(\omega_0)$ which gives (i) the price-impact coefficients that will clear the market for long-term bonds and (ii) the demand-impact coefficients consistent with optimization on the part of active slow-moving investors when agents conjecture that $\omega = \omega_0$ at all future dates. A rational expectations equilibrium of our model is a fixed point $\omega^* = \mathbf{f}(\omega^*)$. Solving the model involves numerically finding a solution to a system of $2k$ non-linear equations in $2k$ unknowns.

An equilibrium solution only exists if arbitrageurs are sufficiently risk tolerant (i.e., for τ sufficiently large). When an equilibrium exists, there can be multiple equilibria. Equilibrium non-existence and multiplicity of this sort are common in overlapping-generations, rational-expectations models such as ours where risk-averse arbitrageurs with finite investment horizons trade an infinitely-lived asset that is subject to supply shocks. Different equilibria correspond to different self-fulfilling beliefs that arbitrageurs can hold about the price-impact of supply shocks and, hence, the risks of holding long-term bonds. However, we always find a unique equilibrium that is stable in the sense that equilibrium is robust to a small perturbation in arbitrageurs' beliefs regarding the equilibrium that will prevail in the future. Consistent with the "correspondence principle" of Samuelson (1947), this unique stable equilibrium has comparative statics that accord with standard economic intuition. We focus on this unique stable equilibrium in our numerical illustrations. See Greenwood et al. (2018) for an extensive discussion of these issues.

3.4 The sensitivity of long-term yields

In this section, we explain the factors that shape the sensitivity of long-term rates in our model and how this sensitivity depends on horizon. Consider the model-implied counterpart of the empirical regression coefficient in equation (1.1). In the model, the coefficient β_h from a regression of $y_{t+h} - y_t$ on $i_{t+h} - i_t$ is:

$$\beta_h = \frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]} = \frac{\alpha_1'(2\mathbf{V} - \mathbf{\Gamma}^h\mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h)\mathbf{e}}{\mathbf{e}'(2\mathbf{V} - \mathbf{\Gamma}^h\mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h)\mathbf{e}}, \tag{3.11}$$

where $\mathbf{V} = Var\left[\mathbf{x}_t\right]$ denotes the variance of the state vector $\mathbf{x}_t$ and $\mathbf{e}$ denotes the $(k+2) \times 1$ vector with ones in the first and second positions and zeros elsewhere.[20,21]

To begin, suppose that $C = 0$ in equation (3.8), so shifts in the supply of long-term bonds are unrelated to movements in short-term rates. As a result, changes in term premia are unrelated to shifts in short-term rates and long-term yields will not exhibit excess sensitivity relative to the expectations

[20] To derive this expression, note that $y_{t+h} - y_t = \alpha_1'(\mathbf{x}_{t+h} - \mathbf{x}_t)$ and $i_{t+h} - i_t = \mathbf{e}'(\mathbf{x}_{t+h} - \mathbf{x}_t)$. Since the state-vector $\mathbf{x}_t$ follows a VAR(1) process $\mathbf{x}_{t+1} = \mathbf{\Gamma}\mathbf{x}_t + \epsilon_{t+1}$ with $\Sigma = Var\left[\epsilon_{t+1}\right]$, we have $vec(\mathbf{V}) = (I - \mathbf{\Gamma} \otimes \mathbf{\Gamma})^{-1} vec(\mathbf{\Sigma})$. Noting that $Cov\left[\mathbf{x}_{t+j}, \mathbf{x}_t'\right] = \mathbf{\Gamma}^j\mathbf{V}$ and $Cov[\mathbf{x}_t, \mathbf{x}_{t+j}'] = \mathbf{V}(\mathbf{\Gamma}')^j$, we have $Var\left[\mathbf{x}_{t+h} - \mathbf{x}_t\right] = 2\mathbf{V} - \mathbf{\Gamma}^h\mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h$ and the result follows.

[21] To help build intuitions about the behavior of β_h in the model, the Internet Appendix explores the special case in which there is no slow-moving capital (i.e., if either $q = 1$ or $k = 1$). In this case, the model can be solved in closed form.

hypothesis. When $C = 0$, we have the following results:

- **If $C = 0$ and $\rho_T = \rho_P$, then β_h is a *constant* that is independent of h.** In this benchmark case, all shocks to short rates have the same persistence and the sensitivity of long rates to short rates is the same at all horizons.

- **If $C = 0$ and $\rho_T < \rho_P$, then β_h is a mildly *increasing* function of h.** When there are both transient and persistent short rates shocks , β_h rises with h because (i) movements in the persistent short rate component are associated with larger movements in long-term yields by the expectations hypothesis and (ii) the persistent component dominates changes in short rates at longer horizons.

- **If $C = 0$ and $\rho_T < \rho_P$, then the *level* of β_h is increasing in σ_P for all h.** An increase in σ_P raises the fraction of total short-rate variation at all horizons that is due to the persistent component. Since shocks to the persistent component of short rates have larger impact on long-term yields, an increase in σ_P raises the level of β_h at all horizons.

Based on these results, the pre-2000 data—when β_h was sizable, but largely independent of horizon h—are consistent with a regime where σ_P was large and C was negligible. If σ_P declined in the post-2000 data, this should have reduced β_h at all horizons. But how can one then understand the facts that β_h is so large for small h and declines so steeply with h in the post-2000 data? To begin, one might assume that $C > 0$, so that there is excess sensitivity—so term premia respond positively to shifts in short rates. However, if the resulting shifts in term premia are as persistent as the underlying shifts in short rates, then β_h will still be a constant function of h. Specifically, we have the following result:

- **Suppose $C > 0$, $\rho_s = \rho_T = \rho_P$, and that all capital is fast-moving. Then β_h is a *constant* that is independent of h.** In this case, there is excess sensitivity relative to the expectations hypothesis. However, this excess sensitivity is the same irrespective of horizon because (i) the relevant supply shocks are just as persistent as short-rate shocks ($\rho_s = \rho_T = \rho_P$) and (ii) there is no slow-moving capital. For instance, a simple version of the "reaching-for-yield" channel (Hanson and Stein, 2015) would suggest that $\rho_s = \rho_T = \rho_P$, implying that $s_t - \bar{s} = C \times (i_t - \bar{\imath})$ by equation (3.9). Thus, in the absence of slow-moving capital, such a reaching-for-yield channel would not generate greater excess sensitivity at high frequencies.

Thus, as noted in 2.3, to explain why β_h is a steeply decreasing function of h as in the post-2000 data, shifts in short rates must give rise to *transient* movements in term premia. In our model, transient movements in term premia can arise from transient supply shocks, slow-moving capital, or some combination of the two. Specifically, we have the following result:

- **Suppose $C > 0$ and $\rho_s \leq \rho_T = \rho_P$. If either (i) supply shocks are transient ($\rho_s < \rho_T$) or (ii) capital is slow-moving (i.e., $q < 1$ and $k > 1$), then β_h is decreasing in h.** Under these conditions, long-term yields will exhibit excess sensitivity that declines with horizon h.
  - **When $C > 0$ and $\rho_s < \rho_T = \rho_P$, β_h is decreasing in h even in the absence of slow-moving capital.** If the supply shocks induced by shifts in short rates are more transient than the underlying short-rate shocks ($\rho_s < \rho_T = \rho_P$), term premia will react more in the short run even in the absence of slow-moving capital. For instance, if supply shocks are due to mortgage refinancing, the resulting supply shocks may be quite transient since Hanson (2014) shows that even persistent declines in rates only induce short-lived mortgage refinancing waves.
  - **When $C > 0$, $\rho_s = \rho_T = \rho_P$, and capital is slow-moving, then β_h is decreasing in h.** Suppose $\rho_s = \rho_T = \rho_P$, so $s_t - \overline{s} = C \times (i_t - \overline{\imath})$, consistent with a simple "reaching-for-yield" story. When capital is slow-moving (i.e., when $k > 1$ and $q < 1$), these supply shocks will walk along a short-run demand curve that is steeper than the long-run demand curve. As a result, β_h will decline with h even though supply shocks are as persistent short-rate shocks.

In summary, in order for β_h to be a steeply declining function of h as in the post-2000 data and to match our associated return forecasting results, we need (i) $C > 0$ and either (ii.a) transitory supply shocks ($\rho_s < \rho_T$) or (ii.b) slow-moving arbitrageurs. Under these conditions, shifts in short-term rates give rise to transitory movements in term premia, leading long-term yields to temporarily over-react to short rates. In practice, we believe both transitory supply shocks and slow-moving capital likely play some role in explaining why β_h declines steeply with h in the recent data. Furthermore, these two mechanisms reinforce one another: it is easiest to match the steep decline in β_h as a function of h using calibrations, such as our illustrative calibration below, that feature both mechanisms.

3.5 Model calibration

Our main empirical findings are that β_h has declined in the post-2000 sample at low frequencies (high h) but has risen at high frequencies (low h), leading β_h to decline steeply with horizon h in the post-2000 data. In this section, we argue that our model can match these surprising patterns if two underlying parameters shifted in the post-2000 period:

1. σ_P **has fallen**: Shocks to the persistent component of short-term nominal rates have become less volatile in the post-2000 period.

2. C **has risen**: The kinds of supply-and-demand-based amplification mechanisms that we emphasize have grown in importance.

Specifically, we consider an illustrative calibration of the model in which each time period is a month. We assume the following parameters were the same in both the pre-2000 and post-2000 periods:

- **Persistence:** $\rho_P = 0.995$, $\rho_T = 0.96$, and $\rho_s = 0.80$. This implies that shocks to the persistent short rate component have a half-life of 11.5 years, shocks to the transient component have a half-life of 1.4 years, and shocks to the net supply of long-term bond have a half-life of 3 months. The short half-life of supply shocks is consistent with the mortgage refinancing channel.

- **Slow-moving capital:** $q = 30\%$ and $k = 12$. Thus, $1 - q = 70\%$ of the arbitrageurs are slow-moving and only rebalance their bond portfolios every 12 months. These assumptions capture the idea that many large institutional investors only rebalance their portfolios annually.

- **Volatility of the transient component of short rates:** $\sigma_T^2 = 0.15\%$.

- **No independent supply shocks:** $\sigma_s^2 = 0$. We make this assumption for simplicity only. Thus, the supply shocks induced by shocks to short rates are the only reason term premia vary.

- **Other parameters:** $\tau = 0.5$ and $\theta = 119/120$, so the duration of the perpetuity is $D = 1/(1-\theta) = 120$ months—i.e., 10 years.

Supported by the previous discussion, we assume that two model parameters, C and σ_P, changed between the pre-2000 and the post-2000 periods. For the pre-2000 period, we assume that:

- **Large persistent component of short rates:** $\sigma_P^2 = 0.15\%$. The implied standard deviation of the short rate is 4.12% which compares with a pre-2000 volatility of 1-year yields of 2.63%.

- **No supply shocks induced by short rate shocks:** $C = 0$. Thus, we assume that there is no excess sensitivity in the early period.

By contrast, for the post-2000 period, we assume that:

- **Small persistent component of short rates:** $\sigma_P^2 = 0.012\%$. The implied standard deviation of the short rate is 1.77% which is similar to the post-2000 volatility of 1-year yields of 1.85%.

- **Supply shocks induced by short rate shocks:** $C = 0.55 > 0$.

3.5.1 Model-implied regression coefficients

The first graph in Figure 6 plots the model-implied coefficients β_h in equation (3.11) against the monthly horizon h for the pre-and post-2000 calibrations. In the pre-2000 calibration where σ_P is large and $C = 0$, β_h is high and is largely independent of horizon h. In fact, β_h rises gradually with h in the pre-2000 calibration—as it does in the pre-2000 data—because the more persistent component of short rates dominates changes in short rates at longer horizons. By contrast, in the post-2000 calibration where σ_P is smaller and C is large, β_h declines steeply with h. And, since σ_P is lower, β_h eventually reaches a lower level for large h than in the pre-2000 calibration.

β_h declines with h in the post-2000 calibration because short-rate rate shocks give rise to transient shocks to the supply of long-term bonds ($C > 0$ and $\rho_s < \rho_T$) that encounter a short-run demand curve that is far steeper than the long-run demand curve due to slow-moving capital ($q < 1$ and $k > 1$). However, the second graph in Figure 6 shows that β_h only declines moderately with h in our post-2000 calibration if we assume there are transitory supply shocks, but drop the slow-moving capital assumption. Thus, from a quantitative perspective, slow-moving capital appears to be essential for matching the fact that β_h declines steeply with h in the post-2000 data.

Figure 7 shows the model-implied impulse response functions in the post-2000 calibration following a +100 bp shock to short rates that lands in month $t = 12$. (We assume there is a +50 bp shock to both the persistent and transient components of the short rate.) The long-term yield is the sum of an expectations-hypothesis component and a term premium component: $y_t = eh_t + tp_t$. Thus, the term spread is $y_t - i_t = (eh_t - i_t) + tp_t$. We show the impulse responses for short-term rates (i_t), long-term yields (y_t), the term spread ($y_t - i_t$), and the term premium (tp_t) in Figure 7.

The initial shock to short rates leads to a rise in term premia. Thus, relative to the expectations-hypothesis, long-term rates are excessively sensitive to short rates. However, the rise in term premia wears off quickly, explaining our key finding that β_h declines sharply with horizon h. Nonetheless, the rise in short rates causes the yield curve to flatten on impact just as in the data. This is because $(eh_t - i_t)$ falls on impact and this flattening due to the expectations hypothesis outweighs the steepening due to the rise in term premia. However, the initial rise in short rates is predicts additional yield curve flattening—

and predictable reversals in long-term yields—over the following months. Indeed, raising C or increasing the degree of slow-moving capital raises $Corr(\Delta i_t, \Delta tp_t) > 0$ in the model, giving rise to greater high-frequency excess sensitivity. These same forces lower $Corr(\Delta i_t, \Delta tp_{t+j}) < 0$ for $j > 0$, heightening the predictable reversals in long-term rates.

3.5.2 Matching related findings

In addition to matching the fact that the β_h coefficients decline steeply with h in the post-2000 period, our model is also capable of matching the related facts documented above. First, the model is consistent with our return forecasting evidence: in the post-2000 calibration where $C > 0$, bond risk premia $E_t[rx_{t+1}]$ will be elevated when short-term rates have recently risen. To see this, note that risk premia are:

$$E_t[rx_{t+1}] = \tau^{-1} V^{(1)} \times b_t = (\tau q)^{-1} V^{(1)} \times (s_t - (1-q)k^{-1} \textstyle\sum_{j=0}^{k-1} d_{t-j}). \tag{3.12}$$

The idea is that, when $C > 0$, fast-moving arbitrageurs will be bearing greater interest-rate risk when short-rates have recently risen—i.e., $b_t = q^{-1}(s_t - (1-q)k^{-1} \sum_{j=0}^{k-1} d_{t-j})$ will be higher—and they will require compensation for bearing this extra risk. Again, when $C > 0$, there are two reasons why increases in short rates lead to increases in b_t and $E_t[rx_{t+1}]$. First, even if there are no slow-moving arbitrageurs (i.e., if $q = 1$, so $b_t = s_t$), when supply shocks are less persistent than short rates (i.e., $\rho_s < \rho_T \leq \rho_P$), equation (3.9) shows that supply s_t is likely to be high when short rates have recently risen. Second, even if supply shocks are as persistent as short-rate shocks (i.e., $\rho_s = \rho_T = \rho_P$), when there is slow-moving capital, b_t will be high when short rates have recently risen since some slow-moving arbitrageurs will not have rebalanced their portfolios in response to the related supply shock.

Second, let $L_t = i_t$ and $S_t = y_t - i_t$ denote the model-implied level and slope factors. If we estimate equation (2.1b) in data simulated from the model, we find that past increases in the level of rates predict a flattening of the yield curve in the post-2000 calibration but not in the pre-2000 calibration. When $C > 0$ as in the post-2000 calibration, past increases in the level of rates are associated with a higher current risk premium on long-term bonds. Since the risk premium is $E_t[rx_{t+1}] = S_t - \theta(1-\theta)^{-1}(E_t[\Delta S_{t+1}] + E_t[\Delta L_{t+1}])$, all else equal, $E_t[\Delta S_{t+1}]$ is lower short rate have recently risen. Thus, the model generates the non-Markovian dynamics emphasized in Section 2.1.

4 Implications

4.1 High-frequency identification

First, our findings have clear implications for identification approaches based on the high-frequency response of long-term rates to macroeconomic news announcements. The short-run change in long-term yields around news announcements is increasingly used as an unconfounded measure of the longer-run impact of news shocks—see e.g., Gertler and Karadi (2015) and Nakamura and Steinsson (2018). However, if, as we argue, some of the impact of a news shock on long-term rates reflects transient shifts in term premia that quickly revert, then a shock's short- and long-run impact on long-term rates will be quite different. As a result, identification based on the high-frequency responses of long-term rates are likely to provide biased estimates of the longer-run impact of announcements. In this way, our results suggest that economists face an important bias-variance trade-off: high-frequency identification allows for precise estimates of the short-run impact of news on long-term yields, but these are likely to be biased estimates of the longer-run impact that is often of greatest interest.

Still, it is conceivable that changes in 1-year yields that are associated with macroeconomic news announcements are different, and do not cause transient changes in term premia, as argued by Nakamura and Steinsson (2018) and Hördahl et al. (2015). To get some direct evidence on this question, we form an "economic news index" for month t, $News_t$, by cumulating daily changes in 1-year yields within month t on days with important macroeconomic news announcements. Our data on the timing of macroeconomic news announcements comes from Money Market Services/Action Economics and begins in 1980. The announcements we consider are: (1) FOMC announcement, (2) CPI, (3) PPI, (4) durable goods orders, (5) new and existing home sales, (6) housing starts, (7) the employment report, and (8) retail sales. We then estimate the following predictive regression for the subsequent change in 10-year forward rates:

$$f_{t+h}^{(10)} - f_t^{(10)} = \delta_0 + \delta_1 L_t + \delta_2 S_t + \delta_3 (L_t - L_{t-1}) + \delta_4 (S_t - S_{t-1}) + \phi \times News_t + \varepsilon_{t+h}, \tag{4.1}$$

where L_t and S_t denote the level and slope of the yield curve at the end of month t. In other words, equation (4.1) simply adds $News_t$ to the Jorda (2005) impulse-response functions we estimated in equation (2.4). Table 6 shows the results for both pre- and post-2000 samples and for $h =$ 3-, 6-, 9-, and 12- month future changes in forward rates.

In Panel A, we omit $News_t$, so the estimates are the essentially same as those in Table 4.[22] As

[22] Since the pre-2000 sample in Table 6 runs from 1980-1999, the pre-2000 results in Table 6 differ slightly from those in

previously shown, past increases in short-term rates are associated with predictable future declines in long-term forward rates in the post-2000 data, but there is no such tendency in the pre-2000 data. In Panel B, we add $News_t$, but omit the prior change in level and slope. We see that positive values of the news index predict subsequent declines in long-term forwards in the post-2000 data. Indeed, the coefficients on $News_t$ in Panel B are similar to those on $L_t - L_{t-1}$ in Panel A.

In Panel C, we include the total change in level of short rates in month t, $L_t - L_{t-1}$, as an independent variable. The goal is to see if shifts in short-term rates on announcement and non-announcement days have different implications for the expected future change in long-term forward rates. Once we control for the total change in short rates in month t, we find that the coefficient on $News_t$ is small and insignificant, indicating that shifts in short-term rates on announcement and non-announcement days have similar effects on subsequent changes in forwards. Specifically, movements in long-term forwards rates on announcement days appear to be just as likely to reverse as movements on non-announcement days.

In Panel D, we break $News_t$ into two pieces—one reflecting changes in short-term rates on FOMC announcement days ($News_{t,FOMC}$) and one for all other news announcements ($News_{t,Other}$)—to see if FOMC announcements differ from other news announcements. We exclude the 1980-81 monetary targeting regime and thus FOMC announcement dates begin in 1982. As in Panel C, we include $L_t - L_{t-1}$ as an independent variable. If anything, the results in Panel D suggest that changes in short rates on FOMC announcement days are *more* likely to be followed by subsequent reversals in forward rates than changes on non-announcement days since 2000.

In summary, we conclude that since 2000, the high-frequency response of long-term rates to economic news appears to dissipate at lower frequencies, posing challenges to interpreting high-frequency yield curve responses in more recent data.

4.2 Monetary policy transmission

Second, our results have important implications for the transmission of monetary policy. Central banks conduct conventional monetary policy by adjusting short-term nominal rates. According to the standard New Keynesian view (Gali, 2008), changes in short-term nominal rates affect short-term real rates because of nominal rigidities. And, the resulting shifts in short-term real rates affects long-term real rates via the expectations hypothesis, which in turn influence household spending and firm investment. Stein (2013) points out that the excess sensitivity of long-term yields—whereby shocks to short rates move term premia

Table 4 where the data begins in 1971. However, the post-2000 results are identical in Tables 4 and 6.

on long-term bonds in the same direction—should strengthen the effects of monetary policy relative to the canonical view. Stein (2013) refers to this as the "recruitment" channel of monetary transmission.

In our theoretical framework, the strength of this recruitment channel at medium-run or business-cycle frequencies (e.g., over a 1 to 3-year horizon) depends on (i) the relative strength of the relevant supply-and-demand-based amplification mechanisms (i.e., the size of C relative to investor risk tolerance τ) and (ii) the persistence of the associated supply and demand shocks. Specifically, when ρ_s is well below ρ_T as under the mortgage-convexity interpretation of C, the associated shifts in term premia would be quite transient and would likely have only modest effects on investment and spending. By contrast, when $\rho_s \approx \rho_T$ as under the reaching-for-yield interpretation of C, the shifts in term premia would be more persistent and likely to have larger effects on aggregate demand.

Our empirical results do not allow us to speak directly to the strength of this non-standard channel at business cycle frequencies—i.e., to assess the extent to which monetary policy influences term and other risk premia over the course of a monetary policy cycle. Instead, what we can confidently say is that, some of the influence of short-term rates on term premia is quite transitory. However, regardless of the medium-run strength of the recruitment channel, our model shows that, when capital is slow-moving, the short-run effect of shifts in short rates on term premia will exceed the medium-run effect. Thus, our findings suggest that recruitment channel is smaller than one would conclude based on a simplistic extrapolation of the high-frequency response of term premia to policy shocks documented by Hanson and Stein (2015), Gertler and Karadi (2015), and Gilchrist et al. (2015).

More generally, our findings hint that, in the presence of slow-moving capital, central banks should care about the way policy impacts financial conditions at business-cycle frequencies, but should focus less on the immediate market response to announcements since much of the latter may be quite transitory. In this way, our findings lend support to the argument in Stein and Sunderam (2018) that the Federal Reserve has become too focused on high-frequency asset price movements.[23]

4.3 Bond market "conundrums"

Third, our findings can help explain the rising prevalence of bond market episodes like the one that former Federal Reserve Chairman Greenspan famously called the "conundrum"—the period after June 2004 when the Fed raised short-term rates, but longer-term yields declined. This "conundrum" was first noted in Greenspan (2005) and has been explored in many papers, including Backus and Wright (2007).

[23] These ideas are also related to the argument that the large declines in bond yields on the days of large-scale asset purchase announcements by central banks may have been somewhat transitory (Wright, 2012; Greenlaw et al., 2018).

Consistent with the weaker low-frequency sensitivity of long-term rates in recent years, "conundrum" episodes—defined as 6-month periods where short- and long-term rates move in *opposite* directions—have grown increasingly common. Specifically, since 2000, 1- and 10-year nominal Treasury yields have moved in the opposite direction in 38% of all 6-month periods. By contrast, from 1971 to 1999, the corresponding figure was 18%, and the difference is statistically significant (p-val < 0.001).[24]

Here we show that the non-Markovian yield-curve dynamics documented in Section 2—i.e., the fact that past changes in the level of rates increasingly predict a future flattening of the yield curve—help explain several noteworthy "conundrum" episodes. Figure 8 plots 1-year and 10-year Treasury rates around three widely discussed "conundrums": Greenspan's original 2004 "conundrum", 2008 which was a "conundrum in reverse," and the 2017 "conundrum." In all three cases, 1-year and 10-year yields moved in opposite directions.

Consider Greenspan's original 2004 "conundrum." To draw the link between non-Markovian yield-curve dynamics and this "conundrum," we use the system of predictive equations for level and slope from Table 3. Starting in May 2004, we simulate the counterfactual path of 10-year yields that would have prevailed if, in the post-2000 sample, the slope of the yield curve had not responded to past changes in the level. To do so, we take the unrestricted estimates of the predictive equation (2.1b) for slope from column (6) in Table 3 and the restricted estimates from column (4) which constrain past changes to have no effect ($\delta_{3S} = \delta_{4S} = 0$). Starting in May 2004, we generate the counterfactual path of 10-year yields that would have obtained if $\delta_{3S} = \delta_{4S} = 0$. We hold the level factor at its actual value and use the residuals from the unrestricted regression in column (6), but set the parameters for the slope equation to their estimated values from the *restricted* regression in column (6).

The top panel of Figure 8 plots the actual 1- and 10-year yields over this 2004 conundrum period along with the 10-year yield under this alternate counterfactual scenario. Had the slope not responded to lagged changes in the level of the yield curve, Figure 8 shows that, instead of falling, 10-year yields would have risen in 2004. The next two panels of Figure 8 repeat this exercise for the 2008 "conundrum in reverse" (beginning in December 2007) and the 2017 "conundrum" (beginning in November 2016). If the slope had not responded to past changes in level, 10-year yields would have moved in the same direction as 1-year yields in both cases.

[24] Greenspan's "conundrum" moniker is somewhat confusing. Specifically, unless long-term inflation expectations are highly unstable, the puzzle from an expectations hypothesis perspective is not the weak relationship between long- and short-term rates observed recently at low frequencies. Instead, the puzzle is why this relationship was previously so strong at all frequencies and why it has become stronger at high frequencies.

4.4 Affine term structure models

Finally, we explore the implications of our results for affine term structure models which are a widely-used, reduced-form tools for understanding the term structure of bond yields (Duffee, 2002; Duffie and Kan, 1996). In these models, the n-year zero coupon yield, $y_t^{(n)}$, takes the affine form: $y_t^{(n)} = \alpha_{0(n)} + \alpha'_{1(n)}\mathbf{x}_t$, where $\mathbf{x}_t$ is a vector of state variables and the $\alpha_{0(n)}$ and $\alpha_{1(n)}$ satisfy a set of recursive equations. In the Internet Appendix, we apply the estimation methodology of Adrian et al. (2013) and fit affine term structure models using the first K principal components of 1- to 10-year yields as the state variables $\mathbf{x}_t$. We show that standard affine models—models that are Markovian with respect to these current yield-curve factors—cannot fit our key finding that the sensitivity of long rates to short rates β_h declines so strongly with horizon h in the post-2000 data. Furthermore, we show that this remains so even if we estimate models that include many (e.g., $K = 5$) current yield-curve factors as state variables.

However, we show that our key finding is consistent with non-Markovian term structure models in which past lags of the yield-curve factors are treated as "unspanned state variables." In standard affine models, if the true model is known, one can recover the full set of state variables $\mathbf{x}_t$ by inverting an appropriate set of yields—i.e., the state variables are "spanned" by current yields. An unspanned state variable is a variable that is useful for forecasting future bond yields and returns but that has no impact on the current yield curve. This non-Markovian model allows us to parsimoniously capture our result that past changes in the level of rates are useful for forecasting future bond yields and returns. And, similar models have been considered in Joslin et al. (2013). To be clear, we do not argue that the past increase in the level of rates is *literally* unspanned. Instead, we think this variable is *close* to being unspanned.[25]

Finally, we use a bootstrap procedure to test the hypothesis that each affine model is correctly specified, using the ratio of yearly to monthly coefficients from equation (1.1) as the test statistic. The test rejects if the observed value of β_{12}/β_1 is too high or low to have been generated by that model. This test is in the spirit of Giglio and Kelly (2018), who test the hypothesis an affine model is correctly specified by checking whether the comovement of yields at different points on the curve is consistent with the model. Using this bootstrap procedure, we conclude that, in the post-2000 sample, the Markovian

[25] Specifically, like any factor that has a short-lived impact on bond risk premium, past increases in the level of rates should have only a small effect on the current yield curve. Thus, in practice, it may be quite difficult to recover information about this variable from current yields—e.g., because yields are measured with a tiny amount of error or because the true data-generating model evolves over time. As a result, conditioning on lagged yields will add information beyond that readily revealed by current yields. Indeed the model in Section 3, does not feature unspanned variables. Specifically, we can recast the model, which only has one class perpetual long-term bonds, to have a set of zero-coupon bonds with different maturities. Using the resulting affine model, if one knew the true process generating yields, one could recover the full $(k+2)$-dimensional state vector $\mathbf{x}_t$ from any set of $(k+2)$ yields. However, many of these state variables would be close to unspanned—they would have only minimal effects on yields—and, in practice, it would be difficult to extract them from yields.

models are decisively rejected: if these standard models were correctly specified it would be highly unlikely to observe a value of β_{12}/β_1 as small as we do in the data. However, the non-Markovian models are not rejected post-2000. Thus, we conclude that affine models need to include lagged yield-curve factors to match the fact that the sensitivity of long rates declines so sharply with horizon post-2000.

5 Conclusion

The strong sensitivity of long-term interest rates to changes in short rates is a long-standing puzzle. In this paper, we have shown that since 2000 this sensitivity has become even stronger at high frequencies. By contrast, this sensitivity has fallen significantly when looking at low-frequency changes. As a result, in the post-2000 data, the sensitivity of long-term rates to changes in short-term rates declines steeply with the horizon over which these changes are computed.

Before 2000, long-term interest rates were quite sensitive to short-term interest rates because unstable inflation expectations made short rates very persistent. Since 2000, the sensitivity is horizon-dependent and arises because past increases in short-term rates temporarily raise the term premium component of long-term yields. Consistent with this view, we show that, controlling for current yields, past changes in short-rates predict (i) future yield-curve flattening, (ii) future declines in long-term yields and forwards, and (iii) high future excess returns on long-term bonds in the post-2000 data.

We have proposed a simple economic model that can explain these puzzling facts. In our model, the rising excess sensitivity of long rates observed at high frequencies since 2000 is explained by the combination of (i) a growing set of investors who tend to reach for yield when short rates decline and (ii) a gradual arbitrage response to these demand shifts.

Our findings have important implications for the transmission of monetary policy and event-studies. The excess sensitivity of long-term yields reinforces the effects of monetary policy (Stein, 2013), but in recent years this channel of policy has been far more short-lived than one might conclude based on a simplistic reading of high-frequency evidence. More broadly, part of the high-frequency response of long rates to shocks to short rates represents term premium movements that tend to wear off quickly. Consequently, it is important to remember that event-study approaches only measure high-frequency responses to macroeconomic news and that the impact may often be more muted at the lower frequencies that are often of greatest interest to macroeconomists and policymakers.

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Table 1: Regressions of changes in long-term rates on short-term rates. This table reports the estimated regression coefficients from equations (1.1) and (1.2) for each reported sample. The dependent variable is the change in the 10-year U.S. Treasury yield or forward rate, either nominal, real or their difference (IC, or inflation compensation). The independent variable is the change in the 1-year nominal U.S. Treasury yield in all cases. Changes are considered with daily data, and with monthly data using monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) horizons. In the 1971-1999 monthly sample, time t runs from 1971m8 to 1999m12 and the number of monthly observations is 341 irrespective of h. In the 2000-2017 monthly sample, t runs from 2000m1 to 2017m12, so the number of monthly observations runs 215 from for $h = 1$ to 204 for $h = 12$. For $h > 1$, we report Newey-West (1987) standard errors are in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; for $h = 1$, we report heteroskedasticity robust standard errors. Significance: $^{*}p < 0.1$, ** $p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

Panel A: 10-year zero coupon yields and IC

	(1) Nominal	(2) Nominal	(3) Real	(4) IC
Daily	0.56***	0.86***	0.55***	0.31***
	[0.02]	[0.03]	[0.03]	[0.02]
Monthly	0.46***	0.64***	0.37***	0.26***
	[0.04]	[0.11]	[0.09]	[0.10]
Quarterly	0.48***	0.42***	0.21*	0.22
	[0.04]	[0.07]	[0.11]	[0.13]
Semi-annual	0.50***	0.31***	0.20**	0.12
	[0.04]	[0.07]	[0.08]	[0.10]
Yearly	0.56***	0.20***	0.13*	0.07
	[0.05]	[0.04]	[0.06]	[0.05]
Sample	1971-1999	2000-2017	2000-2017	2000-2017

Panel B: 10-year instantaneous forward yields and IC

	(1) Nominal	(2) Nominal	(3) Real	(4) IC
Daily	0.39***	0.47***	0.31***	0.17***
	[0.03]	[0.04]	[0.03]	[0.03]
Monthly	0.29***	0.22	0.17**	0.06
	[0.04]	[0.14]	[0.08]	[0.09]
Quarterly	0.31***	0.03	0.08	-0.04
	[0.05]	[0.09]	[0.05]	[0.05]
Semi-annual	0.33***	-0.06	0.03	-0.09**
	[0.06]	[0.07]	[0.04]	[0.04]
Yearly	0.39***	-0.17***	-0.03	-0.14***
	[0.07]	[0.05]	[0.05]	[0.03]
Sample	1971-1999	2000-2017	2000-2017	2000-2017

Table 2: Regressions of changes in long-term international rates on short-term rates. This table reports the estimated regression coefficients from equation (1.1) for the United Kingdom (UK), Germany (DE), and Canada (CAN) on each reported sample. We obtain data on each country's zero-coupon government bond yield curve from each country's central bank website. The dependent variable is the change in the 10-year zero-coupon yield, either nominal, real, or their difference—i.e., inflation compensation (IC). The independent variable is the change in the 1-year nominal yield in all cases. Changes are considered with daily data, and with monthly data using monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) horizons. For $h > 1$, we report Newey-West (1987) standard errors in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; for $h = 1$, we report heteroskedasticity robust standard errors. Significance: $^{*}p < 0.1$, ** $p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

Panel A: UK 10-year zero-coupon yields

	(1) Nominal	(2) Nominal	(3) Real	(4) Real	(5) IC	(6) IC
Daily	0.44***	0.86***	0.14***	0.63***	0.29***	0.23***
	[0.04]	[0.03]	[0.01]	[0.03]	[0.04]	[0.02]
Monthly	0.47***	0.55***	0.19***	0.12	0.28***	0.43***
	[0.06]	[0.13]	[0.04]	[0.23]	[0.08]	[0.14]
Quarterly	0.49***	0.43***	0.23***	0.04	0.26**	0.39***
	[0.08]	[0.10]	[0.04]	[0.17]	[0.10]	[0.09]
Semi-annual	0.45***	0.39***	0.22***	0.07	0.23**	0.32***
	[0.09]	[0.08]	[0.05]	[0.11]	[0.11]	[0.06]
Yearly	0.38***	0.29***	0.16**	0.05	0.22**	0.24***
	[0.06]	[0.06]	[0.06]	[0.08]	[0.08]	[0.03]
Sample	1985-1999	2000-2017	1985-1999	2000-2017	1985-1999	2000-2017

Panel B: German and Canadian 10-year nominal zero-coupon yields

	(1) DE	(2) DE	(3) CAN	(4) CAN
Daily		0.65***	0.42***	0.71***
		[0.03]	[0.03]	[0.03]
Monthly	0.34***	0.50***	0.46***	0.51***
	[0.05]	[0.10]	[0.05]	[0.08]
Quarterly	0.41***	0.44***	0.51***	0.38***
	[0.04]	[0.07]	[0.05]	[0.05]
Semi-annual	0.41***	0.41***	0.50***	0.26***
	[0.04]	[0.08]	[0.07]	[0.05]
Yearly	0.43***	0.33***	0.43***	0.12*
	[0.04]	[0.10]	[0.08]	[0.06]
Sample	1972-1999	2000-2017	1986-1999	2000-2017

Table 3: Estimates of predictive equations for level and slope. This table reports the estimated regression coefficients from monthly predictive equations (2.1a) and (2.1b) for the 1971m8–1999m12 and 2000m1–2017m12 subsamples. Dependent variables are the level ($L_t \equiv y_t^{(1)}$) and slope ($S_t \equiv y_t^{(10)} - y_t^{(1)}$) of the U.S. Treasury zero-coupon yield curve. Heteroskedasticity robust standard errors are in brackets. Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. The table also shows AIC and BIC values (to be minimized) for each possible specification of the system of two equations. Lastly, the implied β_1 and β_{12} coefficients from equation (2.3) for each possible specification of the system are reported.

	Pre-2000			Post-2000		
	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable: L_{t+1}						
L_t	0.98***	0.97***	0.96***	0.97***	0.97***	0.98***
	[0.02]	[0.02]	[0.02]	[0.01]	[0.01]	[0.01]
S_t	0.00	-0.01	-0.02	-0.03*	-0.01	-0.00
	[0.04]	[0.04]	[0.04]	[0.02]	[0.02]	[0.02]
$L_t - L_{t-6}$		-0.01	0.05		0.08***	0.06*
		[0.04]	[0.05]		[0.03]	[0.03]
$S_t - S_{t-6}$			0.13*			-0.04*
			[0.07]			[0.02]
Dependent variable: S_{t+1}						
L_t	0.01	0.01	0.01	-0.00	-0.01	-0.01
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
S_t	0.96***	0.96***	0.97***	0.97***	0.94***	0.94***
	[0.03]	[0.02]	[0.03]	[0.02]	[0.02]	[0.02]
$L_t - L_{t-6}$		0.00	-0.03		-0.12***	-0.13***
		[0.02]	[0.03]		[0.03]	[0.03]
$S_t - S_{t-6}$			-0.08			-0.01
			[0.05]			[0.03]
N	341	335	335	215	215	215
Implied β_1	0.46	0.46	0.46	0.65	0.67	0.67
Implied β_{12}	0.52	0.51	0.55	0.57	0.34	0.28
AIC	-5720.3	-5607.9	-5608.7	-4048.3	-4081.2	-4079.8
BIC	-5697.3	-5577.4	-5570.5	-4028.1	-4054.2	-4046.1
Sample	1971-1999	1972-1999	1972-1999	2000-2017	2000-2017	2000-2017

Table 4: Predictable yield-curve dynamics following an impulse to short-term interest rates. This table reports the estimated regression coefficients in equation (2.4) for the 1971m8–1999m12 and 2000m1–2017m12 subsamples. For $h = 3$, 6, 9, and, 12-months changes, we show results for 10-year yields ($z_t = y_t^{(10)}$), 10-year forward rates ($z_t = f_t^{(10)}$), level ($z_t = L_t$), and slope ($z_t = S_t$). We report Newey-West standard errors in brackets using a lag truncation parameter of $\lceil 1.5 \times h \rceil$. Significance: $^{*}p < 0.1$, $^{**}\ p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	Pre-2000				Post-2000			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dep. var with h	3	6	9	12	3	6	9	12
Dependent variable: $y_{t+h}^{(10)} - y_t^{(10)}$								
L_t	-0.06*	-0.11*	-0.17*	-0.23**	-0.09***	-0.16***	-0.21***	-0.24***
	[0.03]	[0.06]	[0.08]	[0.10]	[0.03]	[0.05]	[0.06]	[0.06]
S_t	-0.13*	-0.27**	-0.45***	-0.62***	-0.14**	-0.27***	-0.35***	-0.42***
	[0.07]	[0.11]	[0.16]	[0.19]	[0.05]	[0.09]	[0.10]	[0.12]
$L_t - L_{t-1}$	-0.03	0.10	0.04	0.23*	-0.16	-0.42**	-0.44**	-0.15
	[0.12]	[0.13]	[0.11]	[0.12]	[0.12]	[0.18]	[0.20]	[0.19]
$S_t - S_{t-1}$	0.03	0.30	0.34*	0.55**	-0.08	-0.15	-0.11	-0.14
	[0.18]	[0.19]	[0.20]	[0.26]	[0.15]	[0.17]	[0.15]	[0.20]
Adj.R^2	0.03	0.08	0.14	0.20	0.07	0.16	0.21	0.24
Dependent variable: $f_{t+h}^{(10)} - f_t^{(10)}$								
L_t	-0.04	-0.09	-0.14*	-0.19**	-0.04	-0.07	-0.09	-0.09
	[0.03]	[0.05]	[0.08]	[0.09]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.16**	-0.31***	-0.48***	-0.66***	-0.13**	-0.25***	-0.35***	-0.43***
	[0.06]	[0.11]	[0.15]	[0.19]	[0.05]	[0.08]	[0.11]	[0.13]
$L_t - L_{t-1}$	-0.08	0.09	-0.05	0.12	-0.36***	-0.58***	-0.75***	-0.71***
	[0.12]	[0.11]	[0.10]	[0.12]	[0.10]	[0.18]	[0.13]	[0.15]
$S_t - S_{t-1}$	0.01	0.18	0.16	0.34	-0.14	-0.10	-0.03	-0.16
	[0.18]	[0.19]	[0.20]	[0.22]	[0.16]	[0.20]	[0.19]	[0.21]
Adj.R^2	0.04	0.10	0.17	0.24	0.06	0.14	0.24	0.30
Dependent variable: $L_{t+h} - L_t$								
L_t	-0.09*	-0.18**	-0.26**	-0.36***	-0.08***	-0.18***	-0.25**	-0.33**
	[0.05]	[0.08]	[0.11]	[0.12]	[0.03]	[0.06]	[0.10]	[0.13]
S_t	-0.00	-0.09	-0.24	-0.33	-0.04	-0.06	-0.00	0.07
	[0.11]	[0.15]	[0.20]	[0.25]	[0.04]	[0.10]	[0.16]	[0.21]
$L_t - L_{t-1}$	0.07	0.26	0.18	0.60***	0.63***	0.78*	1.23**	1.61**
	[0.24]	[0.21]	[0.20]	[0.19]	[0.17]	[0.38]	[0.54]	[0.63]
$S_t - S_{t-1}$	0.13	0.64**	0.57*	1.07**	-0.23**	-0.40*	-0.62*	-0.61
	[0.30]	[0.30]	[0.31]	[0.49]	[0.11]	[0.21]	[0.29]	[0.39]
Adj.R^2	0.03	0.08	0.12	0.17	0.20	0.25	0.35	0.42
Dependent variable: $S_{t+h} - S_t$								
L_t	0.03	0.07**	0.10**	0.13***	-0.00	0.02	0.05	0.09
	[0.03]	[0.03]	[0.04]	[0.04]	[0.03]	[0.07]	[0.09]	[0.13]
S_t	-0.12*	-0.18*	-0.21*	-0.29**	-0.10**	-0.21**	-0.35**	-0.48**
	[0.07]	[0.10]	[0.11]	[0.13]	[0.05]	[0.09]	[0.13]	[0.19]
$L_t - L_{t-1}$	-0.10	-0.16	-0.14	-0.37***	-0.79***	-1.20***	-1.67***	-1.76***
	[0.16]	[0.12]	[0.12]	[0.11]	[0.17]	[0.35]	[0.41]	[0.53]
$S_t - S_{t-1}$	-0.10	-0.34*	-0.23	-0.51	0.15	0.25	0.51*	0.46
	[0.17]	[0.18]	[0.16]	[0.30]	[0.12]	[0.22]	[0.27]	[0.32]
Adj.R^2	0.08	0.16	0.20	0.28	0.18	0.26	0.40	0.46
N	340	340	340	340	213	210	207	204
Sample	1971-1999	1971-1999	1971-1999	1971-1999	2000-2017	2000-2017	2000-2017	2000-2016

Table 5: Estimates of predictive equations for bond excess returns in the pre-2000 sample This table reports the estimated regression coefficients in equation (2.6) using monthly data from the 1971m8–1999m12 and 2000m1–2017m12 subsamples. We report results various return forecast horizon (k). Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. For $k = 1$-month returns, we report heteroskedasticity robust standard errors are in brackets. For $k = 3$ and 6-month returns, we report Newey and West (1987) standard errors in brackets, using a lag truncation parameter of 5 and 9 months, respectively. In this case, p-values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

Panel A: Pre-2000 sample

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dep. var for k	1	1	1	3	3	3	6	6	6
Dependent Variable: $rx^{(10)}_{t\to t+k}$									
L_t	0.17	0.18	0.22*	0.53*	0.54	0.65*	0.99*	1.01	1.25**
	[0.13]	[0.13]	[0.12]	[0.31]	[0.33]	[0.34]	[0.57]	[0.62]	[0.60]
S_t	0.55**	0.61**	0.66***	1.57**	1.91***	2.07***	3.17***	3.62***	3.98***
	[0.24]	[0.24]	[0.24]	[0.64]	[0.65]	[0.64]	[1.04]	[1.13]	[1.09]
$L_t - L_{t-6}$		0.10	-0.18		0.59	-0.22		0.79	-1.00
		[0.19]	[0.29]		[0.49]	[0.54]		[0.69]	[0.90]
$S_t - S_{t-6}$			-0.57			-1.67**			-3.67***
			[0.44]			[0.81]			[1.30]
Adj.R^2	0.02	0.01	0.02	0.05	0.06	0.07	0.11	0.12	0.17
Dependent Variable: $rx^{LEVEL}_{t\to t+k}$									
L_t	-0.04*	-0.04*	-0.04**	-0.08*	-0.09*	-0.10**	-0.11**	-0.11**	-0.13**
	[0.02]	[0.02]	[0.02]	[0.04]	[0.05]	[0.05]	[0.05]	[0.05]	[0.05]
S_t	-0.06	-0.07*	-0.08**	-0.15*	-0.21**	-0.24***	-0.23***	-0.26***	-0.29***
	[0.04]	[0.03]	[0.04]	[0.09]	[0.08]	[0.08]	[0.09]	[0.09]	[0.09]
$L_t - L_{t-6}$		-0.01	0.04		-0.10	0.03		-0.04	0.09
		[0.03]	[0.04]		[0.08]	[0.08]		[0.09]	[0.09]
$S_t - S_{t-6}$			0.11*			0.26**			0.27*
			[0.06]			[0.11]			[0.14]
Adj.R^2	0.01	0.01	0.02	0.04	0.06	0.08	0.09	0.10	0.12
Dependent Variable: $rx^{SLOPE}_{t\to t+k}$									
L_t	0.02	0.02	0.03**	0.03	0.04	0.04	0.01	0.01	0.01
	[0.01]	[0.01]	[0.01]	[0.02]	[0.03]	[0.03]	[0.03]	[0.03]	[0.03]
S_t	0.00	0.01	0.01	-0.01	0.02	0.03	-0.09	-0.11	-0.12
	[0.03]	[0.02]	[0.03]	[0.06]	[0.05]	[0.06]	[0.08]	[0.08]	[0.08]
$L_t - L_{t-6}$		0.00	-0.03		0.04	-0.00		-0.04	0.01
		[0.02]	[0.03]		[0.05]	[0.06]		[0.04]	[0.05]
$S_t - S_{t-6}$			-0.06			-0.10			0.11
			[0.05]			[0.09]			[0.10]
Adj.R^2	0.01	0.01	0.01	0.02	0.03	0.03	0.04	0.05	0.06
N	341	335	335	341	335	335	341	335	335
Sample	1971-1999	1972-1999	1972-1999	1971-1999	1972-1999	1972-1999	1971-1999	1972-1999	1972-1999

Panel B: Post-2000 sample

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dep. var for k	1	1	1	3	3	3	6	6	6
Dependent Variable: $rx^{(10)}_{t\to t+k}$									
L_t	0.34***	0.39***	0.34***	0.86***	0.98***	0.86***	1.57***	1.72***	1.54***
	[0.12]	[0.12]	[0.12]	[0.28]	[0.28]	[0.31]	[0.45]	[0.46]	[0.51]
S_t	0.69***	0.80***	0.70***	1.78***	2.11***	1.84***	3.32***	3.74***	3.35***
	[0.24]	[0.23]	[0.24]	[0.51]	[0.52]	[0.60]	[0.78]	[0.85]	[0.89]
$L_t - L_{t-6}$		0.42*	0.74**		1.24**	2.02***		1.57**	2.70**
		[0.25]	[0.35]		[0.47]	[0.61]		[0.74]	[1.09]
$S_t - S_{t-6}$			0.50			1.24			1.79
			[0.36]			[0.75]			[1.27]
Adj.R^2	0.03	0.04	0.05	0.10	0.14	0.16	0.20	0.24	0.26
Dependent Variable: $rx^{LEVEL}_{t\to t+k}$									
L_t	-0.04***	-0.03***	-0.03***	-0.09***	-0.08***	-0.07***	-0.12***	-0.11***	-0.10***
	[0.01]	[0.01]	[0.01]	[0.02]	[0.02]	[0.02]	[0.03]	[0.03]	[0.03]
S_t	-0.05***	-0.04**	-0.03**	-0.11***	-0.08**	-0.06	-0.13**	-0.08*	-0.06
	[0.01]	[0.02]	[0.02]	[0.04]	[0.04]	[0.04]	[0.05]	[0.04]	[0.04]
$L_t - L_{t-6}$		0.05*	0.02		0.11*	0.05		0.19**	0.12
		[0.03]	[0.03]		[0.06]	[0.06]		[0.07]	[0.08]
$S_t - S_{t-6}$			-0.03**			-0.10**			-0.11**
			[0.02]			[0.04]			[0.05]
Adj.R^2	0.07	0.10	0.11	0.14	0.20	0.22	0.21	0.36	0.38
Dependent Variable: $rx^{SLOPE}_{t\to t+k}$									
L_t	0.00	-0.01	-0.00	0.01	-0.02	-0.01	-0.04	-0.07	-0.06
	[0.01]	[0.01]	[0.01]	[0.04]	[0.03]	[0.03]	[0.06]	[0.04]	[0.05]
S_t	-0.02	-0.05**	-0.04*	-0.07	-0.14***	-0.13**	-0.22**	-0.32***	-0.31***
	[0.02]	[0.02]	[0.02]	[0.05]	[0.05]	[0.05]	[0.09]	[0.08]	[0.08]
$L_t - L_{t-6}$		-0.10***	-0.11***		-0.26***	-0.28***		-0.38***	-0.43***
		[0.03]	[0.04]		[0.05]	[0.07]		[0.07]	[0.11]
$S_t - S_{t-6}$			-0.02			-0.03			-0.08
			[0.03]			[0.07]			[0.13]
Adj.R^2	0.00	0.07	0.07	0.04	0.22	0.22	0.12	0.34	0.34
N	215	215	215	213	213	213	210	210	210
Sample	2000-2017	2000-2017	2000-2017	2000-2017	2000-2017	2000-2017	2000-2017	2000-2017	2000-2017

Table 6: Economic news and subsequent changes in forward rates. This table reports the regression coefficents in equation (4.1) using monthly data from the 1971m8–1999m12 and 2000m1–2017m12 subsamples. Newey-West (1987) standard errors are in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$. Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	Pre-2000				Post-2000			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$f_{t+h}^{(10)} - f_t^{(10)}$ with h	3	6	9	12	3	6	9	12
Panel A								
L_t	-0.04	-0.08	-0.13*	-0.18**	-0.04	-0.07	-0.09	-0.09
	[0.03]	[0.06]	[0.08]	[0.09]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.11	-0.24*	-0.37**	-0.52**	-0.13**	-0.25***	-0.35***	-0.43***
	[0.07]	[0.13]	[0.17]	[0.21]	[0.05]	[0.08]	[0.10]	[0.13]
$L_t - L_{t-1}$	-0.09	0.11	-0.04	0.12	-0.36***	-0.58***	-0.75***	-0.71***
	[0.12]	[0.12]	[0.12]	[0.13]	[0.10]	[0.18]	[0.16]	[0.15]
$S_t - S_{t-1}$	0.03	0.29	0.19	0.35	-0.14	-0.10	-0.03	-0.16
	[0.19]	[0.20]	[0.22]	[0.24]	[0.16]	[0.20]	[0.20]	[0.21]
Adj.R^2	0.02	0.05	0.09	0.14	0.06	0.14	0.24	0.30
Panel B								
L_t	-0.04	-0.08	-0.12	-0.17*	-0.05*	-0.08	-0.09	-0.09
	[0.03]	[0.06]	[0.08]	[0.09]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.10	-0.23*	-0.37**	-0.52**	-0.13***	-0.25***	-0.34***	-0.43***
	[0.07]	[0.12]	[0.17]	[0.20]	[0.05]	[0.08]	[0.10]	[0.13]
$News_t$	-0.29	-0.12	-0.80***	-0.46	-0.45**	-0.62*	-0.80***	-0.61*
	[0.21]	[0.30]	[0.24]	[0.26]	[0.19]	[0.36]	[0.27]	[0.32]
Adj.R^2	0.02	0.05	0.11	0.14	0.06	0.12	0.21	0.28
Panel C								
L_t	-0.04	-0.08	-0.12	-0.17*	-0.05	-0.07	-0.08	-0.08
	[0.03]	[0.06]	[0.08]	[0.09]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.11	-0.24*	-0.37**	-0.52**	-0.13**	-0.25***	-0.35***	-0.43***
	[0.07]	[0.13]	[0.17]	[0.21]	[0.05]	[0.08]	[0.11]	[0.13]
$L_t - L_{t-1}$	-0.06	0.14	0.19	0.29*	-0.29*	-0.63**	-0.84***	-0.88***
	[0.16]	[0.15]	[0.16]	[0.17]	[0.17]	[0.26]	[0.22]	[0.18]
$S_t - S_{t-1}$	0.03	0.29	0.23	0.38	-0.15	-0.09	-0.02	-0.15
	[0.19]	[0.20]	[0.20]	[0.23]	[0.16]	[0.20]	[0.18]	[0.21]
$News_t$	-0.11	-0.11	-0.99**	-0.72**	-0.16	0.12	0.25	0.42
	[0.34]	[0.35]	[0.42]	[0.29]	[0.30]	[0.50]	[0.39]	[0.31]
Adj.R^2	0.01	0.05	0.10	0.14	0.06	0.14	0.24	0.30
N	240	240	240	240	213	210	207	204
Sample	1980-1999	1980-1999	1980-1999	1980-1999	2000-2017	2000-2017	2000-2017	2000-2016
Panel D								
L_t	-0.07*	-0.13**	-0.18**	-0.20**	-0.05	-0.07	-0.09	-0.08
	[0.03]	[0.06]	[0.09]	[0.10]	[0.03]	[0.05]	[0.06]	[0.07]
S_t	-0.07	-0.17	-0.28	-0.42**	-0.13***	-0.25***	-0.36***	-0.43***
	[0.07]	[0.14]	[0.19]	[0.21]	[0.05]	[0.08]	[0.11]	[0.14]
$L_t - L_{t-1}$	0.09	0.03	0.03	-0.00	-0.30*	-0.64**	-0.86***	-0.88***
	[0.18]	[0.21]	[0.24]	[0.27]	[0.16]	[0.25]	[0.21]	[0.18]
$S_t - S_{t-1}$	0.20	0.30	0.33	0.55**	-0.16	-0.11	-0.05	-0.16
	[0.24]	[0.27]	[0.23]	[0.25]	[0.16]	[0.20]	[0.18]	[0.21]
$News_{t,\mathrm{FOMC}}$	0.11	-0.01	-1.89	-0.44	-0.87*	-0.59	-1.10	0.07
	[0.77]	[1.04]	[1.56]	[1.56]	[0.46]	[0.39]	[0.71]	[0.54]
$News_{t,\mathrm{Other}}$	-0.25	-0.17	-0.45	-0.32	0.11	0.39	0.77*	0.56
	[0.33]	[0.45]	[0.48]	[0.43]	[0.29]	[0.59]	[0.44]	[0.44]
Adj.R^2	0.03	0.08	0.14	0.16	0.07	0.14	0.25	0.29
N	216	216	216	216	213	210	207	204
Sample	1982-1999	1982-1999	1982-1999	1982-1999	2000-2017	2000-2017	2000-2017	2000-2016

Figure 1: Regressions of changes in long-term yields on short-term rates. This figure plots the estimated regression coefficients β_h from equation (1.1) versus horizon (h) for the pre-2000 and post-2000 sample: $y^{(10)}_{t+h} - y^{(10)}_t = \alpha_h + \beta_h (y^{(1)}_{t+h} - y^{(1)}_t) + \varepsilon_{t,t+h}$. The dependent variable is the h-month change in the 10-year nominal U.S. Treasury yield and the independent variable is the h-month change in the 1-year nominal U.S. Treasury yield. Changes are considered with daily data (plotted as $h = 0$ in the figure) and with monthly data using $h = 1$, ..., 12-month changes.

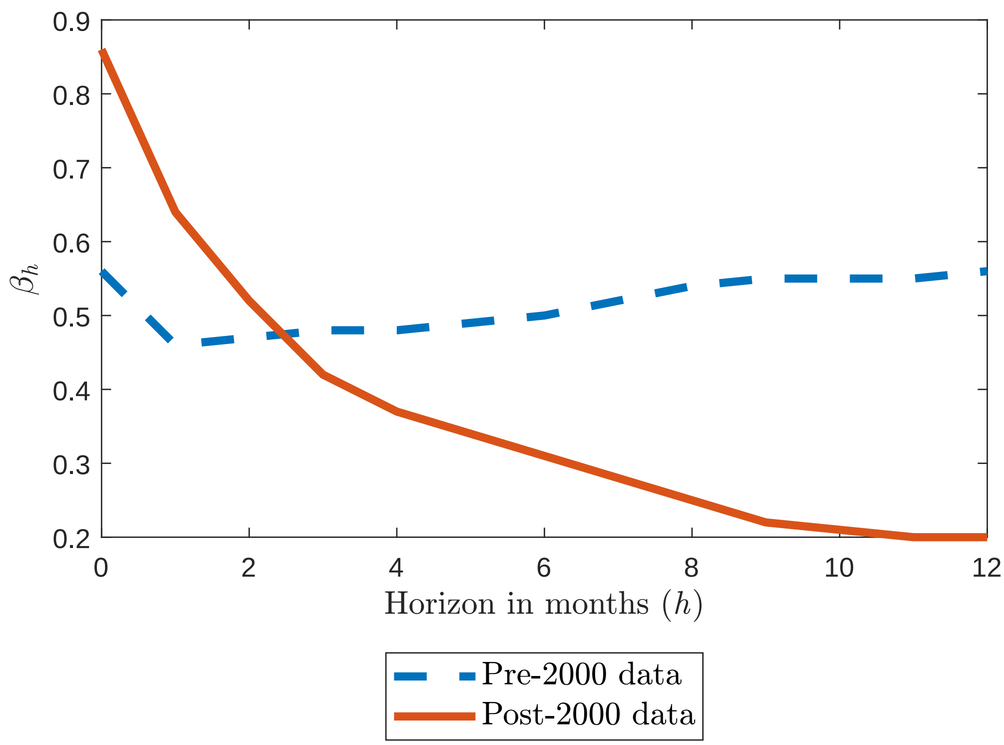


Figure 2: Rolling regression estimates of equations (1.1) and (1.2) This figure plots rolling estimates of the slope coefficients in equations (1.1) and (1.2) with $h = 12$-month changes using 10-year rolling windows for estimation. Results are plotted against the midpoint of the 10-year rolling window. 95% confidence intervals are included (shaded areas), formed using Newey-West standard errors with a lag truncation parameter of 18 and 95% critical values from the asymptotic theory of Kiefer and Vogelsang (2005). Specifically, the 95% confidence interval is ± 2.41 times the estimated standard errors as opposed to ± 1.96 under traditional asymptotic theory.

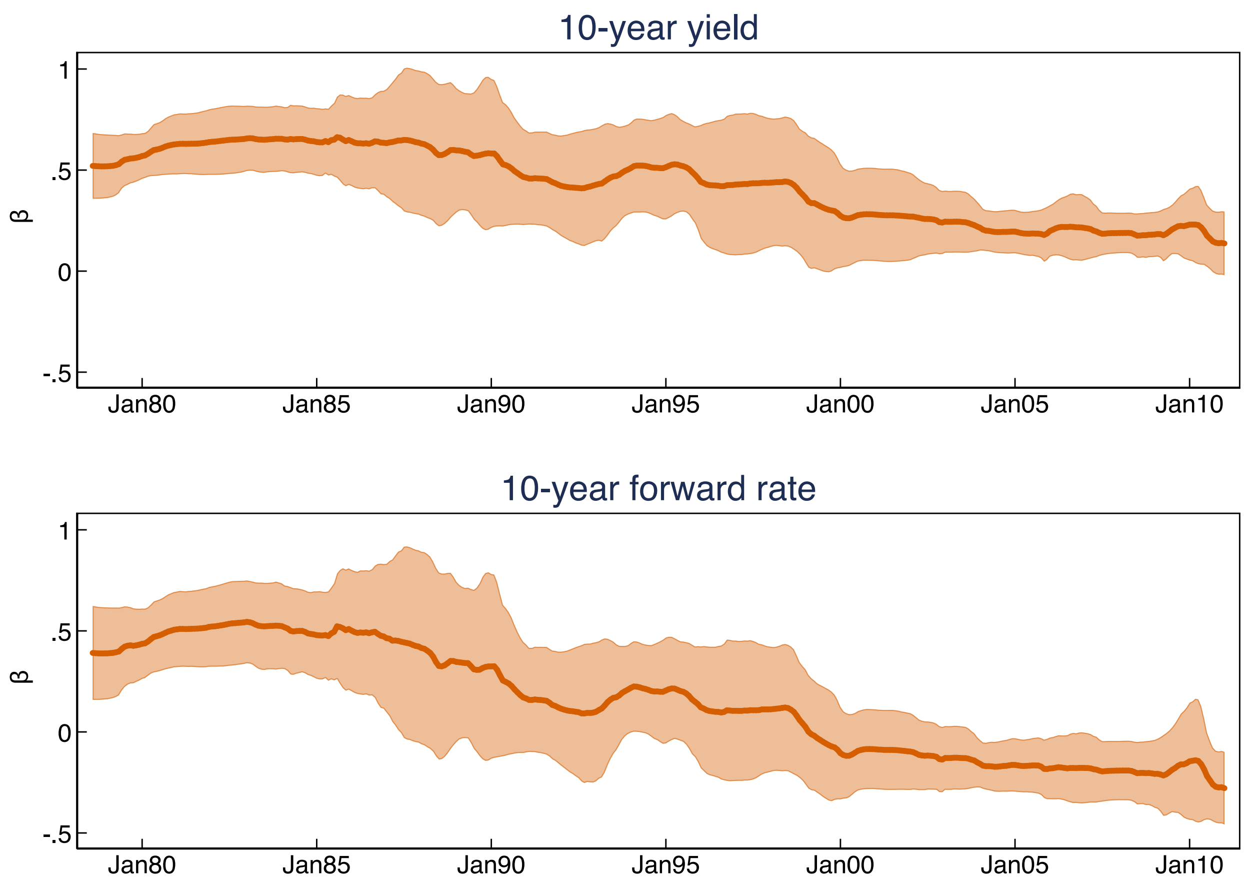


Figure 3: Break tests for equations (1.1) and (1.2) This figure plots the Wald test statistic for each possible break date in equations (1.1) and (1.2) with h = 12-month changes from a fraction 15% of the way through the sample to 85% of the way through the sample. The horizontal red dashed lines denote 10%, 5% and 1% critical values for the maximum of these Wald statistics as in Andrews (1993). Our Wald tests use a Newey and West (1987) variance matrix with a lag truncation parameter of 18. To address the tendency for tests based on the Newey-West variance estimator to over-reject in finite samples, we use the Cho and Vogelsang (2017) critical values for a null of no structural break. The Cho and Vogelsang (2017) critical values are based on the asymptotic theory of Kiefer and Vogelsang (2005) and are slightly larger than the traditional critical values from Andrews (1993).

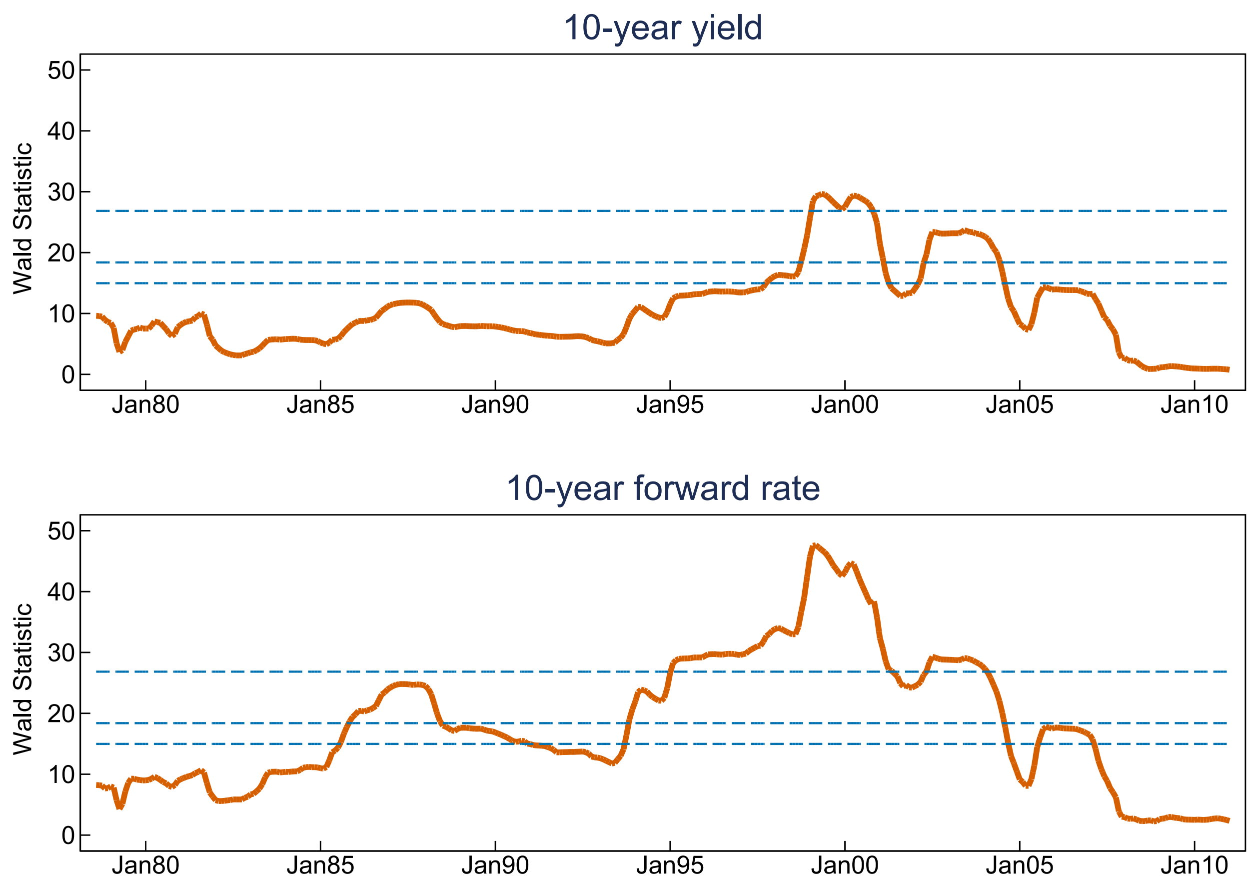


Figure 4: Predictable yield-curve dynamics following an impulse to short-term interest rates. The figures plot the coefficients $\delta_3^{(h)}$ versus horizon h from estimating equations (2.4) for various horizons $h = 1..., 12$-months in the pre-2000 and post-2000 subsamples. We show results for 10-year yields ($z_t = y_t^{(10)}$), 10-year forward rates ($z_t = f_t^{(10)}$), level ($z_t = L_t$) and slope ($z_t = S_t$). 95% confidence intervals are shown as dashed lines, formed using Newey-West standard errors and 95% critical values from the asymptotic theory of Kiefer and Vogelsang (2005). We use a Newey-West lag truncation parameter of 0 for $h = 1$ and $\lceil 1.5 \times h \rceil$ for $h > 1$.

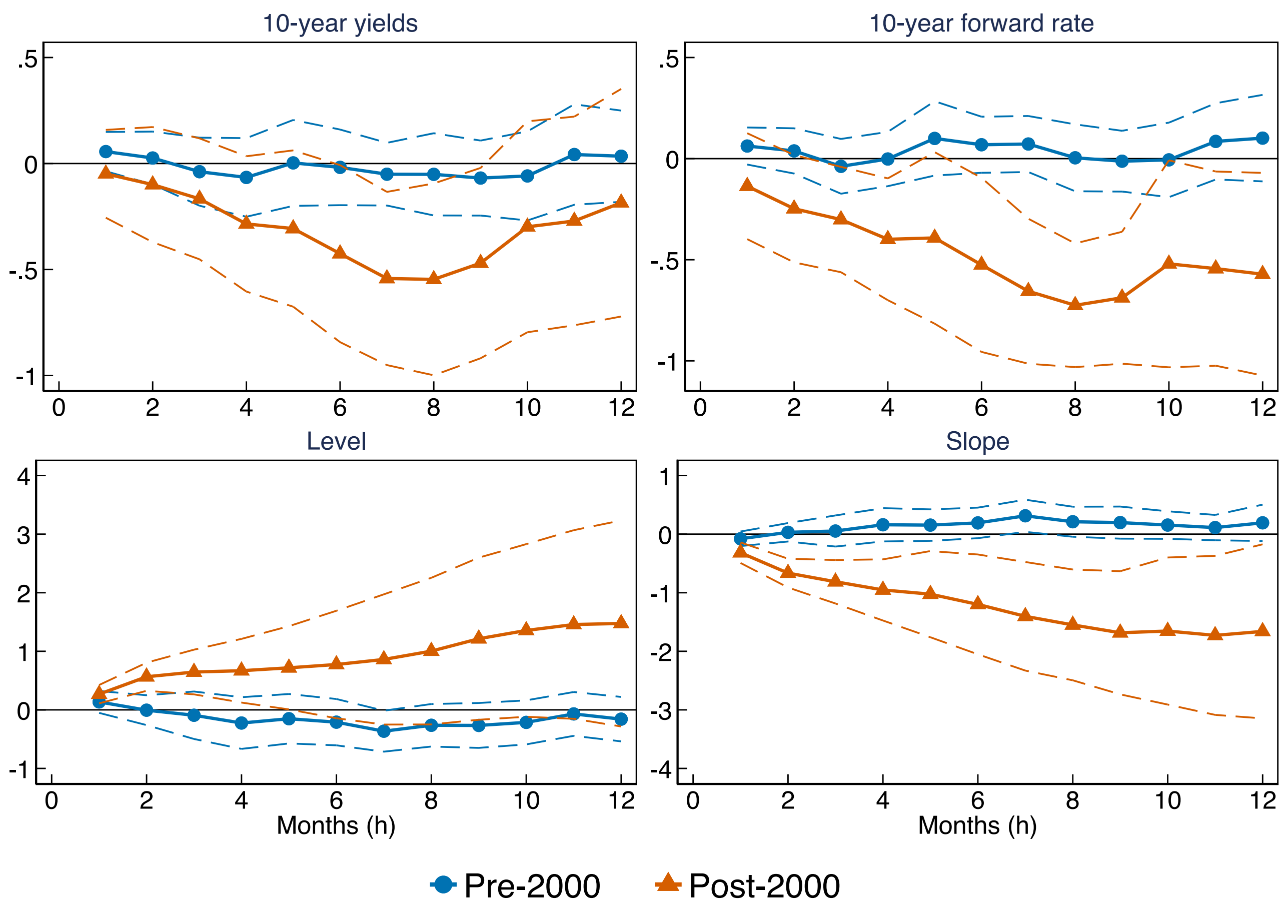


Figure 5: Predicting returns, the changes in forwards, and the change in yields for various bond maturities n. This figure plots the coefficients $\delta_3^{(n)}$ on the past 6-month change in the level factor versus bond maturity n from estimating equation (2.7) for returns, equation (2.8) for the change in forward rates, and equation (2.9) for the change in yields. The results are shown for $k = 3$-month returns or future changes. Due to the use of overlapping data, we plot 95% confidence intervals using Newey-West (1987) standard errors with a lag truncation parameter of 5. The critical values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

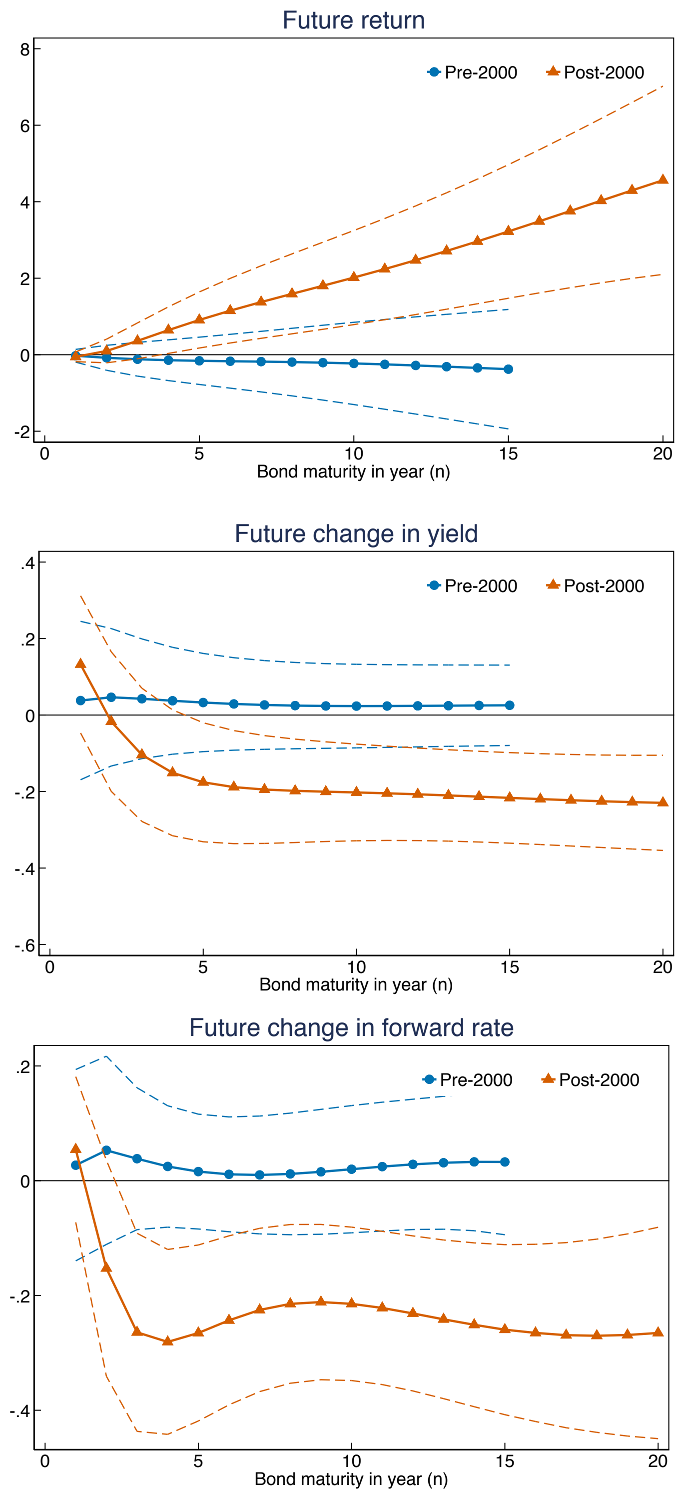


Figure 6: Model-implied coefficients β_h versus horizon (h) in months. The first figure shows the model-implied β_h coefficients from equation (3.11) for the pre-2000 and post-2000 calibrations discussed in the text. The second figure isolates the role of slow-moving capital is the post-2000 calibration, alternately setting $q = 100\%$ ("No slow-moving capital)" and $q = 30\%$ ("With slow-moving capital").

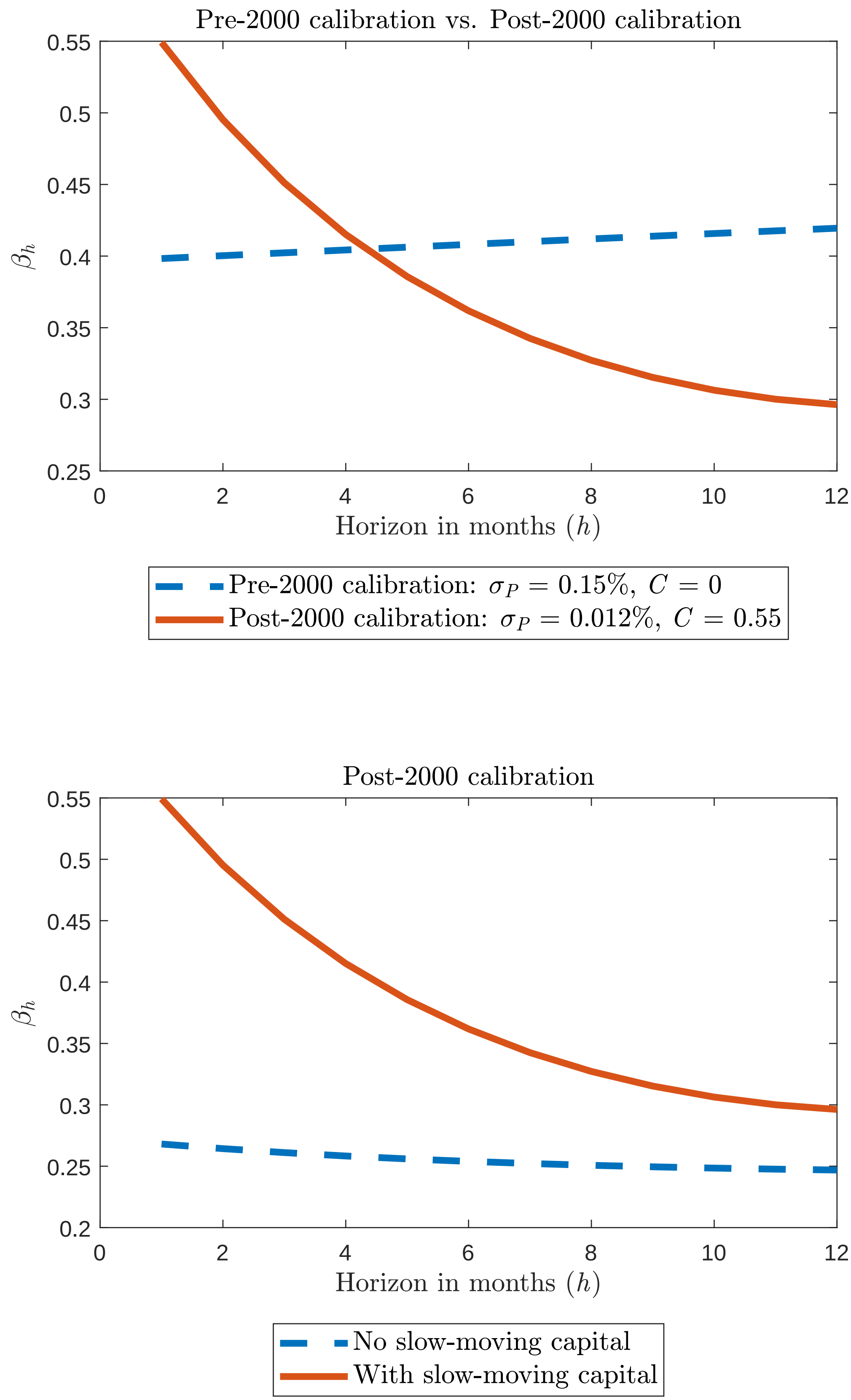


Figure 7: Model-implied impulse response functions for the post-2000 calibration. For the post-2000 calibration, we show the response of short-term and long-term interest rates following a one-time shock to short-term interest rates. We plot short-term nominal interest rates (i_t), long-term nominal yields (y_t), the term spread ($y_t - i_t$), and the term premium (tp_t). Initially, short-term nominal rates are at their steady-state level of $\bar{\imath} = 3\%$ and the term premium on long-term nominal bonds is at a steady-level of 2%. We then assume there is a +50 bp shock to both the persistent and transient components of the short rate that lands at $t = 12$, leading short-term nominal rates to jump from 3% to 4%.

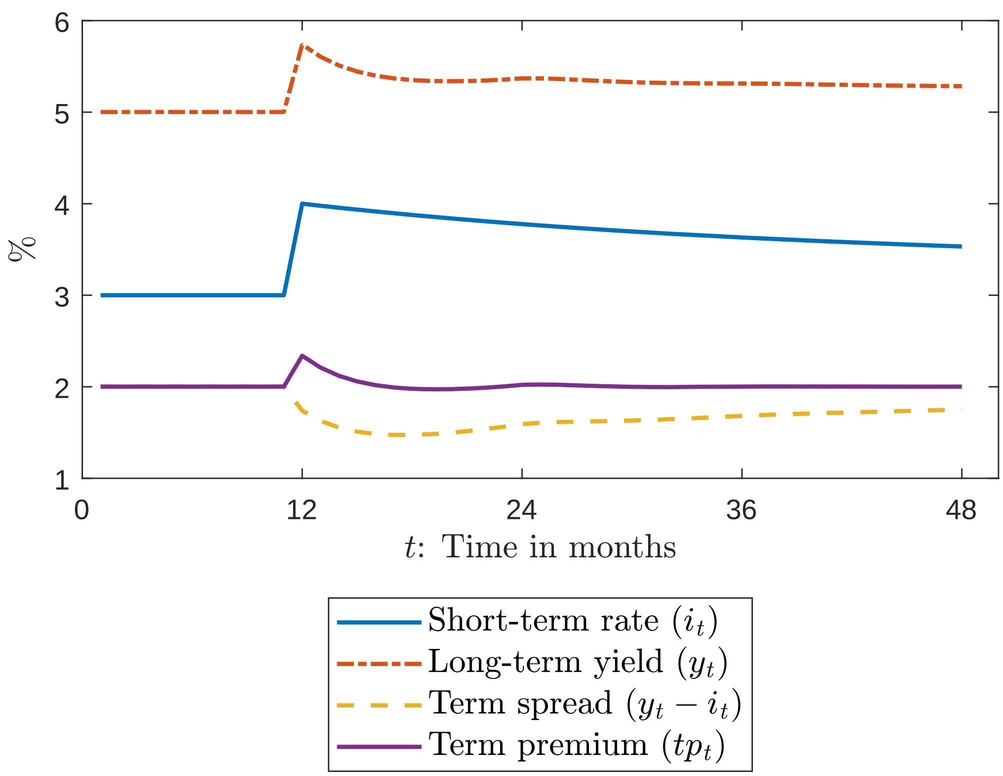
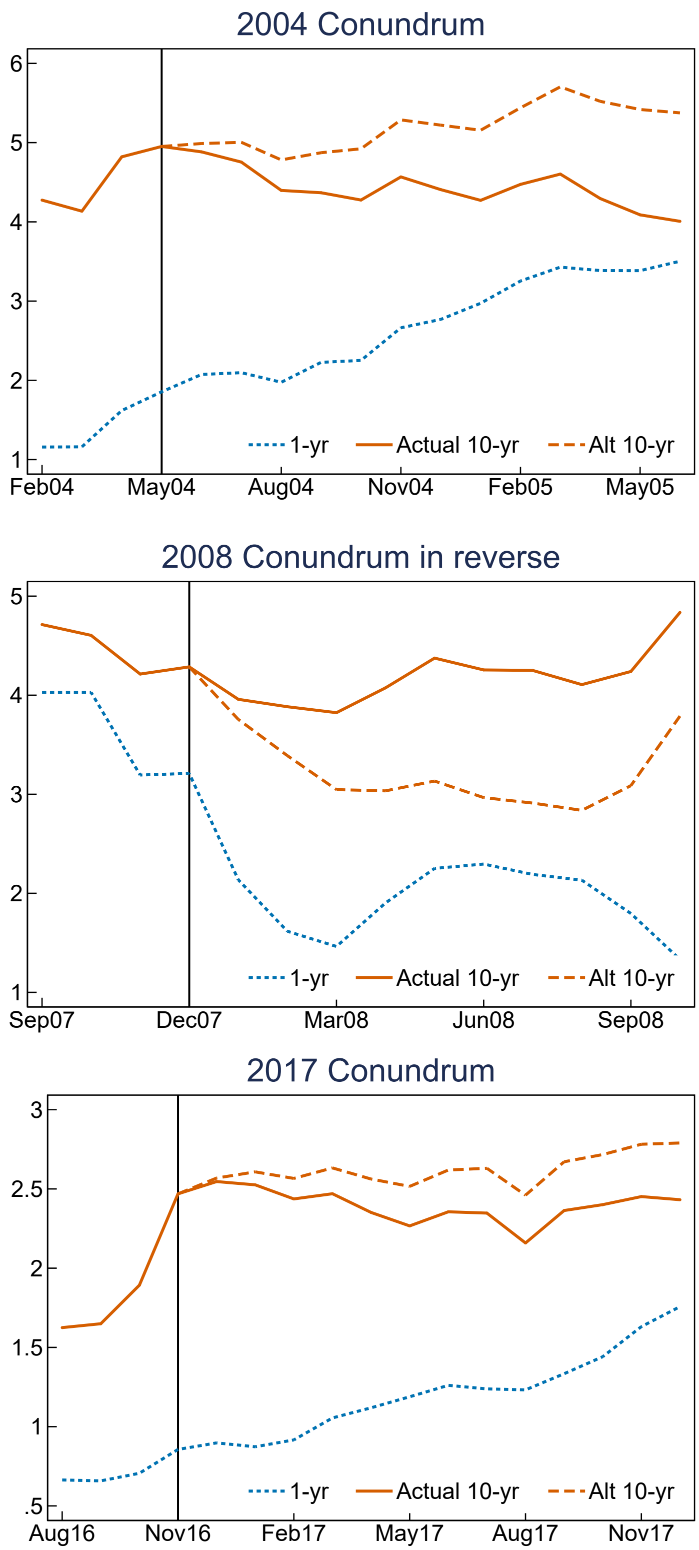


Figure 8: Counterfactual paths of ten-year yields in selected "conundrum" episodes. This figure plots 1- and 10-year yields in the original 2004 "conundrum" episode, the 2008 "conundrum in reverse" episode and the " 2017 conundrum." As described in the text, we also plot counterfactual 10-year yields (Alt 10-yr) generated from restricting the slope to depend on lags of level and slope, but not also on lagged changes in level and slope.



Appendix of supplementary materials for “The Excess Sensitivity of Long-Term Rates: A Tale of Two Frequencies”

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October 15, 2018

A Additional empirical results

This Appendix collects several supplementary empirical results that are mentioned in the main text.

A.1 Different short-term rates

Table A.1 shows that similar results hold using using different proxies for the short rate in equation (1.1) of the main text—i.e., using changes in 3-month, 6-month, or 2-year Treasury yields as the independent variable in equation (1.1). (The results in Table 1 of the text correspond to those reported in Table A.1 for the 1-year Treasury rate.) For all of these short rate proxies, the sensitivity of 10-year yields to changes in short rates was similar irrespective of frequency prior to 2000. After 2000, the sensitivity at high frequencies increases while the sensitivity at low frequencies declines.

A.2 Long-term private yields

Table A.2 shows that we obtain very similar results using a host of long-term private yields as the dependent variable in equation (1.1). We report results for both Aaa and Baa seasoned corporate bond yields from Moodys, the 10-year swap yield, and the yield on current coupon Fannie Mae MBS (FNCL). For of all these long-term private yields, the sensitivity to changes in 1-year Treasury rate is similar at high- and low- frequencies in the pre-2000 sample. Post-2000, the sensitivity at high frequencies increases while the sensitivity at low frequencies declines significantly.

A.3 Dating the break

Here we use an alternate procedure to date the break in the sensitivity of long-term rates to movements in short-term rates. As opposed to focusing simply on the break the low-frequency

Table A.1: Regressions of changes in long-term rates on short-term rates. This table reports the estimated regression coefficients from equation (1.1) in the main text for each reported sample. The dependent variable is the change in the 10-year nominal U.S. Treasury yield or forward rate, either nominal. The independent variable is alternately the change in the 3-month, 6-month, 1-year, and 2-year nominal U.S. Treasury yield. Changes are considered with daily data, and with monthly data using monthly ($h = 1$) and annual ($h = 12$) horizons. In the 1971-1999 monthly sample, time t runs from 1971m8 to 1999m12. In the 2000-2017 monthly sample, t runs from 2000m1 to 2017m12. For $h > 1$, we report Newey-West (1987) standard errors are in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; for $h = 1$, we report heteroskedasticity robust standard errors. Significance: $^{*}p < 0.1$, ** $p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	Pre-2000			Post-2000		
	(1) Daily	(2) Monthly	(3) Annual	(4) Daily	(5) Monthly	(6) Annual
3-month	0.16***	0.26***	0.39***	0.29***	0.25**	0.15***
	[0.02]	[0.03]	[0.05]	[0.02]	[0.10]	[0.04]
6-month	0.41***	0.37***	0.46***	0.65***	0.44***	0.16***
	[0.02]	[0.03]	[0.06]	[0.03]	[0.11]	[0.04]
1-year	0.56***	0.46***	0.56***	0.86***	0.64***	0.20***
	[0.02]	[0.04]	[0.05]	[0.03]	[0.11]	[0.04]
2-year	0.65***	0.57***	0.68***	0.86***	0.76***	0.31***
	[0.01]	[0.04]	[0.05]	[0.02]	[0.09]	[0.06]

sensitivity, β_{12}, here we seek to date the emergence of the *frequency-dependent* sensitivity of long-term rates. Specifically, we consider the following system of regression equations:

$$\begin{aligned} y_{t+1}^{(10)} - y_t^{(10)} &= \alpha_1^{Pre} + \varphi_1^{Post} \cdot Post_t + \beta_1^{Pre} \cdot (y_{t+1}^{(1)} - y_t^{(1)}) \\ &\quad + \Delta_1^{post} \cdot (y_{t+1}^{(1)} - y_t^{(1)}) \times Post_t + \varepsilon_{t,t+1} \end{aligned} \tag{A.1a}$$

$$\begin{aligned} y_{t+12}^{(10)} - y_t^{(10)} &= \alpha_{12}^{Pre} + \varphi_{12}^{Post} \cdot Post_t + \beta_{12}^{Pre} \cdot (y_{t+12}^{(1)} - y_t^{(1)}) \\ &\quad + \Delta_{12}^{post} \cdot (y_{t+12}^{(1)} - y_t^{(1)}) \times Post_t + \varepsilon_{t,t+12}, \end{aligned} \tag{A.1b}$$

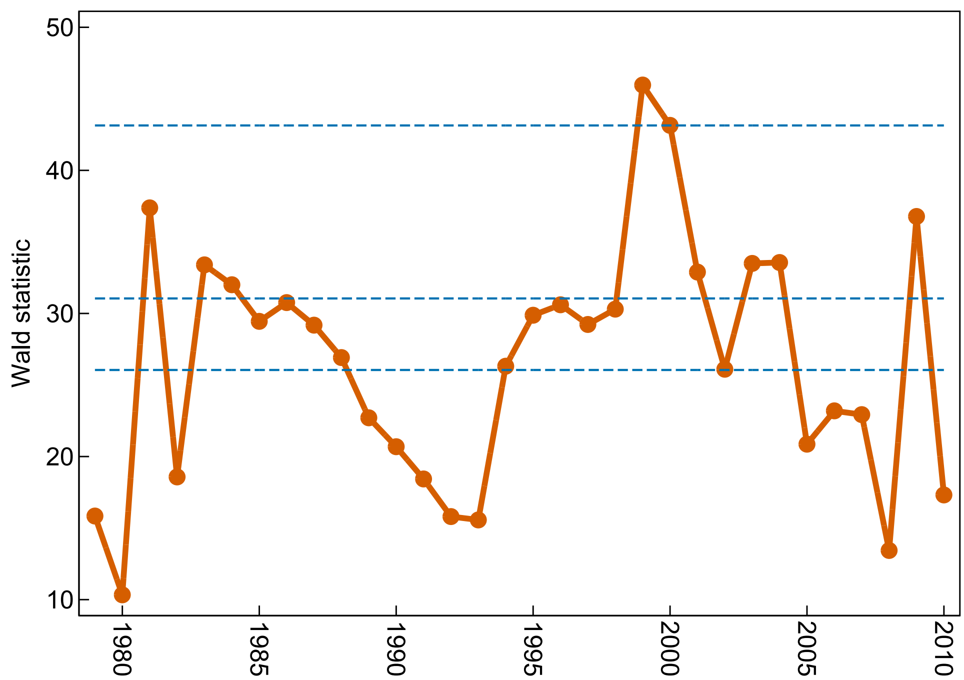
where $Post_t$ is an indicator variable that switches of after some pre-specified break date. Thus, for $h = 1$ and 12-month changes, the pre-break estimate of β_h is β_h^{Pre} and the post-break estimate of β_h is $\beta_h^{Pre} + \Delta_h^{Post}$.

To date the emergence of the *frequency-dependent* sensitivity of long-term rates, we estimate equations (A.1a) and (A.1b) imposing the restriction that $\beta_1^{Pre} = \beta_{12}^{Pre}$. For a given break date, we estimate this system of equations using the generalized method of moments and compute standard errors using a Newey-West variance-covariance matrix with a lag truncation parameter of 18 months. We use a standard two-step GMM estimator that uses 8 moment conditions—namely, the four least-squares normal equations for both equations (A.1a) and (A.1b)—to identify the 7

Table A.2: Regression of changes in corporate bond, swap and secondary mortgage market rates on short-term rates. This table reports the estimated slope coefficients from equation (1.1) in the main text for each reported sample. Specifically, the dependent variables are long-term corporate bond yields with Moody's ratings of Baa and Aaa (labled BAA and AAA), the 10-year swap yield (SWAP10), and the yield on current-coupon Fannie Mae mortgage-backed-securities (FNCL). The independent variable in all regressions is the change in the 1-year nominal Treasury yield. Changes are considered with daily data, and with monthly data using monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) horizons. We report Newey-West (1987) standard errors in brackets, using a lag truncation parameter of $1.5 \times (h - 1)$ (rounded to the nearest integer). Significance: $^{*}p < 0.1$, ** $p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

	(1) BAA	(2) AAA	(3) SWAP10	(4) FNCL	(5) BAA	(6) AAA	(7) SWAP10	(8) FNCL
Daily	0.48***	0.51***	0.74***	0.95***	0.55***	0.57***	0.94***	0.88***
	[0.02]	[0.02]	[0.04]	[0.04]	[0.04]	[0.04]	[0.03]	[0.04]
Monthly	0.52***	0.59***	0.81***	0.92***	0.21	0.34***	0.81***	0.73***
	[0.05]	[0.05]	[0.07]	[0.06]	[0.13]	[0.11]	[0.10]	[0.11]
Quarterly	0.49***	0.57***	0.80***	0.84***	0.04	0.21***	0.61***	0.52***
	[0.06]	[0.06]	[0.07]	[0.07]	[0.12]	[0.07]	[0.09]	[0.09]
Yearly	0.41***	0.55***	0.67***	0.73***	-0.06	0.08	0.36***	0.31***
	[0.08]	[0.09]	[0.08]	[0.10]	[0.12]	[0.06]	[0.05]	[0.06]
Sample	1986-1999	1983-1999	1988-1999	1984-1999	2000-2017	2000-2017	2000-2017	2000-2017

Figure A.1: Break test for the emergence of horizon-dependent sensitivity. This figure plots the Wald test statistic for each possible break date in equations (A.1a) and (A.1b) from a fraction 15% of the way through the sample to 85% of the way through the sample. The figure shows Wald test statistics for a break in equations (A.1a) and (A.1b) jointly, imposing the restriction that $\beta_1^{Pre} = \beta_{12}^{Pre}$. We estimate equations (A.1a) and (A.1b) jointly using a standard two-step efficient GMM estimator. The horizontal red dashed lines denote 10%, 5% and 1% critical values for the maximum of these Wald statistics as in Andrews (1993). Our Wald tests use a Newey and West (1987) variance matrix with a lag truncation parameter of 18. To address the tendency for tests based on the Newey-West variance estimator to over-reject in finite samples, we use the Cho and Vogelsang (2017) critical values for a null of no structural break. The Cho and Vogelsang (2017) critical values are based on the asymptotic theory of Kiefer and Vogelsang (2005) and are slightly larger than the traditional critical values from Andrews (1993).



parameters of the system. For a given break date, we test the null that $\varphi_1^{Post} = \Delta_1^{Post} = \varphi_{12}^{Post} = \Delta_{12}^{Post} = 0$: this test is equivalent to asking whether there is a change in the low-frequency sensitivity ($\Delta_{12}^{Post} \neq 0$) that differs from the change in the high-frequency sensitivity ($\Delta_{12}^{Post} \neq \Delta_1^{Post}$). To date the emergence of horizon-dependent sensitivity, we then use the test of Andrews (1993) who conducts a Chow (1960) test at all possible break dates and then takes the maximum of the Wald test statistics.

Figure A.1 plots the Wald test statistic for each possible annual break date in equations (A.1a) and (A.1b) along with the Cho and Vogelsang (2017) critical values for a null of no structural break. Using this procedure, we find that the strongest evidence for emergence of horizon-dependent sensitivity is in 1999 or 2000.

Here we briefly discuss our GMM estimates of equations (A.1a) and (A.1b) using a break date of 2000 as in the main text. We obtain $\beta_1^{Pre} = \beta_{12}^{Pre} = 0.54^{***}$ [0.05], $\Delta_1^{Post} = 0.11$ [0.14], and $\Delta_{12}^{Post} = -0.33^{***}$ [0.06]. Hansen's J-test does not reject the single over-identifying restriction, yielding $J = 1.07$ (p-val. $= 0.32$). However, the difference between $\beta_1^{Post} = 0.64$ and $\beta_{12}^{Post} = 0.20$ is highly statistically significant (p-val.< 0.001).

Finally, we estimate equations (A.1a) and (A.1b) allowing β_1^{Pre} and β_{12}^{Pre} to differ and assuming a break date in 2000. In this case, our GMM estimates correspond to equation-by-equation OLS estimation of (A.1a) and (A.1b) as in Table 1 of the main text. (This has no effect on β_1^{Post} and β_{12}^{Post} relative to the constrained estimator that imposes $\beta_1^{Pre} = \beta_{12}^{Pre}$, but it does affect the estimates of Δ_1^{Post} and Δ_{12}^{Post}.) Using this unconstrained estimator, we obtain $\beta_1^{Pre} = 0.46^{***}$ [0.05], $\beta_{12}^{Pre} = 0.56^{***}$ [0.05], $\Delta_1^{Post} = 0.18$ [0.14], and $\Delta_{12}^{Post} = -0.36^{***}$ [0.07] just as in Table 1 of the main text. Here the difference between β_1^{Pre} and β_{12}^{Post} is economically small and is not statistically significant (p-val. $= 0.13$). However, the difference between β_1^{Post} and β_{12}^{Post} is economically large and is highly statistically significant (p-val. < 0.001).

A.4 Trading strategies

As another way of assessing the resulting return predictability documented in Section 2.2 of the paper, we consider simple market-timing strategies in which an investor decides to take either a long or short position in the slope-mimicking portfolio—i.e., in a "curve steepener" trade—every month.

Specifically, we consider strategies that take a long (short) position in the slope-mimicking portfolio from month t to month $t+1$ if $L_t < L_{t-h}$ ($L_t > L_{t-h}$). Alternatively, we consider strategies that take a position in the slope-mimicking portfolio from month t to month $t+1$ that is proportional to $-(L_t - L_{t-h})$. Table A.3 computes the annualized Sharpe ratios of these two trading strategies for different choices of h, in the pre- and post-2000 samples.

As shown in Table A.3 the implied annualized Sharpe ratios for these strategies range between about 0.5 to 0.7 in the post-2000 sample but were negligible in the pre-2000 sample.

A.5 Implications for affine term-structure models

Affine term-structure models (ATSMs) are a widely-used, reduced-form tools for understanding the term structure of bond yields. A standard discrete-time affine term-structure model (Duffee, 2002; Duffie and Kan, 1996) starts from the assumption that there is a $m \times 1$ state vector $\mathbf{x}_t$ that follows a VAR(1) under the physical or P-measure:

$$\mathbf{x}_t = \mu + \mathbf{\Phi}\mathbf{x}_{t-1} + \mathbf{\Sigma}\varepsilon_t, \tag{A.2}$$

Table A.3: Sharpe ratios for slope-mimicking portfolios This table reports the annualized Sharpe ratios since 2000 of the strategy of going long (short) the slope-mimicking portfolio if the level fell (rose) over the previous h months and also the strategy of taking a position in the slope-mimicking portfolio that is proportional to $-(L_t - L_{t-h})$, and holding the position from t to $t+1$. The position is rebalanced each month. Annualized Sharpe ratios are computed as the sample average monthly excess returns multiplied by $\sqrt{12}$ and divided by the standard deviation of those monthly excess returns.

Strategy with h:	1	3	6	12
Pre-2000				
$2 \times I(L_t - L_{t-h} < 0) - 1$	0.22	0.03	-0.09	-0.00
$-(L_t - L_{t-h})$	0.08	-0.01	0.12	0.09
Post-2000				
	1	3	6	12
$2 \times I(L_t - L_{t-h} < 0) - 1$	0.52	0.62	0.68	0.64
$-(L_t - L_{t-h})$	0.38	0.62	0.47	0.45

where the error term is Gaussian with mean zero and identity variance-covariance matrix. The short-term riskless interest rate between time t and $t+1$ is an affine function of the state vector: $i_t = \delta_0 + \delta_1' \mathbf{x}_t$. Meanwhile, the pricing kernel or stochastic discount factor is

$$M_{t+1} = \exp(-i_t - \lambda_t' \varepsilon_{t+1} - \frac{1}{2}\lambda_t' \lambda_t), \tag{A.3}$$

where the prices of factor risk, $\lambda_t = \lambda_0 + \mathbf{\Lambda}_1 \mathbf{x}_t$, are also an affine function of the state vector. The price of an n-period zero-coupon bond, $P_t^{(n)}$, satisfies the recursion

$$P_t^{(n)} = E_t^{\mathrm{P}}[M_{t+1} P_{t+1}^{(n-1)}] = \exp(-i_t) E_t^{\mathrm{Q}}[P_{t+1}^{(n-1)}]. \tag{A.4}$$

Here $E_t^{\mathrm{P}}[\cdot]$ denotes expectations under the physical measure or P-measure and $E_t^{\mathrm{Q}}[\cdot]$ denotes expectations under the risk-neutral pricing measure or Q-measure. (For any random variable X_{t+1}, $E_t^{\mathrm{Q}}[X_{t+1}] = E_t^{\mathrm{P}}[M_{t+1} X_{t+1}]/E_t^{\mathrm{P}}[M_{t+1}]$.) Under the Q-measure, the state variables evolve according to

$$\mathbf{x}_t = \mu^* + \mathbf{\Phi}^* \mathbf{x}_{t-1} + \mathbf{\Sigma} \varepsilon_t, \tag{A.5}$$

where $\mu^* = \mu - \mathbf{\Sigma}\lambda_0$ and $\mathbf{\Phi}^* = \mathbf{\Phi} - \mathbf{\Sigma}\mathbf{\Lambda}_1$.

After extensive, but well-known algebra, it follows that

$$P_t^{(n)} = \exp(a_{(n)} + \mathbf{b}_{(n)}' \mathbf{x}_t), \tag{A.6}$$

where $a_{(n)}$ is a scalar and $\mathbf{b}_{(n)}$ is an $m \times 1$ vector that satisfy the recursions:

$$a_{(n+1)} = -\delta_0 + a_{(n)} + \mathbf{b}_{(n)}' \mu^* + \frac{1}{2}\mathbf{b}_{(n)}' \mathbf{\Sigma}\mathbf{\Sigma}' \mathbf{b}_{(n)} \tag{A.7}$$

$$\mathbf{b}_{(n+1)} = \mathbf{\Phi}^{*\prime} \mathbf{b}_{(n)} - \delta_1, \tag{A.8}$$

starting from $a_{(1)} = -\delta_0$ and $\mathbf{b}_1 = -\delta_{(1)}$. The continuously compounded yield on an n-period zero-coupon bond, $y_t^{(n)}$, is in turn given by

$$y_t^{(n)} = -n^{-1}\log(P_t^{(n)}) = -n^{-1}a_{(n)} - n^{-1}\mathbf{b}'_{(n)}\mathbf{x}_t. \tag{A.9}$$

Applying the estimation methodology of Adrian et al. (2013), we fit affine term-structure models with monthly data using the first K principal components of 1- to 10-year yields as the state variables $\mathbf{x}_t$. We do this in the pre-2000 and post-2000 samples separately for $K = 2$ to 5.

We then take the estimated model parameters and work out the model-implied β_h regression coefficients. Specifically, let $\boldsymbol{\Gamma}(j) = E[(\mathbf{x}_{t+j} - E[\mathbf{x}_{t+j}])(\mathbf{x}_t - E[\mathbf{x}_t])']$ denote the autocovariance function of the state vector, which can be obtained from the equations $vec(\boldsymbol{\Gamma}(0)) = (\mathbf{I} - \boldsymbol{\Phi} \otimes \boldsymbol{\Phi})^{-1} vec(\boldsymbol{\Sigma}\boldsymbol{\Sigma}')$ and $\boldsymbol{\Gamma}(j) = \boldsymbol{\Phi}^j \boldsymbol{\Gamma}(0)$ for $j \geq 1$. The population coefficient in a regression of h-month changes in 120-month yields on h-month changes in 12-month yields is then

$$\beta_h = \frac{E[(y_{t+h}^{(120)} - y_t^{(120)})(y_{t+h}^{(12)} - y_t^{(12)})]}{E[(y_{t+h}^{(12)} - y_t^{(12)})^2]} = \frac{1}{10}\frac{\mathbf{b}'_{(120)}[2\boldsymbol{\Gamma}(0) - \boldsymbol{\Gamma}(h) - \boldsymbol{\Gamma}(h)']\mathbf{b}_{(12)}}{\mathbf{b}'_{(12)}[2\boldsymbol{\Gamma}(0) - \boldsymbol{\Gamma}(h) - \boldsymbol{\Gamma}(h)']\mathbf{b}_{(12)}}. \tag{A.10}$$

These model-implied regression coefficients are shown in Panel A of Table A.4. Even if we include a large number of factors as state variables, these standard affine models fail to match the low-frequency decoupling between short- and long-term yields that we observe in the post-2000 data.

We next consider an alternate affine model that augments the state vector $\mathbf{x}_t$ to include not just K principal components of yields, but also $L-1$ additional lags of these principal components. Thus, by construction, the model is non-Markovian with respect to the filtration given by the current principal components. We furthermore treat these lagged principal components as "unspanned state variables." In standard affine models, if the true model is known, one can obtain the full set of state variables by inverting an appropriate set of yields—i.e., the state variables are spanned by current yields. An unspanned state variable is a variable that is useful for forecasting future bond yields and returns but that has no impact on the current yield curve—i.e., it is not "spanned" by current yields—and cannot be recovered in this way. Formally, this means that if the first K elements of the state vector $\mathbf{x}_t$ are the current principal components, all but the first K elements of δ_1 are zero, and the upper right $K \times K(L-1)$ block of $\boldsymbol{\Phi}^*$ is a matrix of zeros. As Duffee (2011) explains, a state variable will be unspanned if it has *perfectly* offsetting effects on the evolution of future short rates and future term premia.

Economically speaking, this is a rather unusual model. However, it allows us to parsimoniously capture our key finding that past changes in the level of rates are useful for forecasting future yields. In addition, similar models have been considered in Joslin et al. (2013). To be clear, we do not believe that the past increase in the level of rates is *literally* unspanned—i.e., that it has no effect on the current yield curve. Instead, we would argue that this variable is *close* to being unspanned. Specifically, like any factor that has a short-lived impact on bond risk premia, past increases in the level of rates should have only a small effect on the yield curve. Thus, in practice it will likely be quite difficult to recover information about this variable from current yields alone—e.g., because yields are measured with a tiny amount of error or because the true data-generating model evolves over time—so conditioning on this variable will add information beyond that revealed by current yields.

Again, the parameters of this augmented model can be estimated, imposing the restriction that the lagged principal components are unspanned factors as in Adrian et al. (2013). The model-implied β_h regression coefficients can again be derived from equation (A.10). These model-implied coefficients are shown in Panel B of Table A.4. The augmented model is able to get reasonably close to matching the empirical regression coefficients at both high- and low-frequencies and in both samples.

Table A.4: Affine Term Structure Model-Implied coefficients in regression of monthly/yearly changes in 10-year yields on changes in 1-year yields This table reports the slope coefficients in equation (A.10) corresponding to the parameters in an affine term structure model estimated as proposed by Adrian et al. (2013) over August 1971-December 2000 and January 2001-December 2017 subsamples at monthly ($h = 1$) and yearly ($h = 12$) frequencies. The term structure model uses K principal components of yields as state variables in panel A, and adds $L - 1$ additional lags of these principal components (for a total of LK state variables) in panel B. p-values are also reported; these are two-sided bootstrap p-values comparing the sample value of β_{12}/β_1 with the bootstrap distribution using that affine model. As memo items the results of the regressions using actual yields are included—these are simply transcribed from Table 1.

		Pre-2000			Post-2000		
		β_1	β_{12}	p-value	β_1	β_{12}	p-value
Panel A: ATSM with K principal components of yields as factors							
$K = 2$		0.42	0.49	0.89	0.83	0.74	0.000
$K = 3$		0.46	0.52	0.65	0.80	0.59	0.006
$K = 4$		0.47	0.52	0.62	0.74	0.55	0.005
$K = 5$		0.47	0.52	0.52	0.71	0.49	0.009
Panel B: ATSM with $L - 1$ lags as additional unspanned factors							
$K = 2$	$L = 6$	0.42	0.50	0.84	0.85	0.32	0.59
$K = 3$	$L = 6$	0.46	0.54	1.00	0.81	0.34	0.42
$K = 2$	$L = 12$	0.42	0.51	0.76	0.90	0.25	0.87
$K = 3$	$L = 12$	0.46	0.55	0.90	0.85	0.29	0.78
Memo: Estimates in data (from Table 1)		0.46	0.56		0.64	0.20	

As another way of looking at this, we use a bootstrap to test the hypothesis that each estimated affine term structure model is correctly specified. Our test uses the ratio of yearly to monthly coefficients (β_{12}/β_1) as the test statistic; the test rejects if this ratio is too low or too high to have been generated by the estimated Q-measure model. This is a test in the spirit of Giglio and Kelly (2018), who test the hypothesis that a given affine model is correctly specified by checking whether the comovement of rates at different points on the term structure is consistent with the estimated Q-measure dynamics.

To implement this test, we simulate the bootstrap distribution of the ratio β_{12}/β_1 for each estimated affine term-structure model. To do this for a given model, we first generate bootstrapped time series of the state vector $\mathbf{x}_t$ using a residual-based bootstrap based on our estimates of the P-measure dynamics in equation (A.2). Using our estimates of $a_{(n)}$ and $\mathbf{b}_{(n)}$ for that same model—which reflect the estimated dynamics under the Q-measure—in combination with a set of bootstrapped draws of the corresponding yield measurement errors, we obtain a bootstrapped time series of yields $y_t^{(n)}$ for $n = 12$ and 120 months. We then compute β_{12}, β_1, and the ratio β_{12}/β_1 for each bootstrapped time series. Repeating this exercise many times, we obtain a bootstrap distribution for the ratio β_{12}/β_1.

For each model, we then compute the two-sided p-value of the observed ratio β_{12}/β_1 with respect to this bootstrap distribution. These p-values are also reported in Table A.4. We find that in the pre-2000 sample, none of the models are rejected. In the post-2000 sample, the Markovian models are decisively rejected: if these standard models were correctly specified it would be highly unlikely to observe a value of β_{12}/β_1 as small as we do in the data. However, the non-Markovian models are not rejected in the post-2000 sample. In this way, we again conclude that a non-Markovian term structure model is required to match the yield-curve dynamics in the post-2000 data.

To summarize, our conclusion is that affine term-structure models need to include lagged yield-curve factors to match the frequency-dependent sensitivity of long-term rates we observe in recent years. A large number of static yield curve factors will not do the job.

B Model solution

This Appendix provides additional details on our economic model. In particular, we provide additional details on how we solve for the rational expectations equilibrium of our model.

B.1 Long-term nominal bonds

In this subsection, we derive the Campbell-Shiller (1988) approximation to the return on a default-free perpetuity.

Consider a perpetual default-free nominal bond which pay a nominal coupon of K each period. Let P_t denote the nominal price of the long-term bond at time t. Thus, the nominal return on the long-term bond from t to $t+1$ is

$$1 + R_{t+1} = \frac{P_{t+1} + K}{P_t}.$$

Defining $\theta \equiv 1/(1+K) < 1$, the one-period log return on the bond from time t to $t+1$ is approximately

$$r_{t+1} \equiv \ln(1+R_{t+1}) \approx \overbrace{\frac{1}{1-\theta}}^{D} y_t - \overbrace{\frac{\theta}{1-\theta}}^{D-1} y_{t+1}, \tag{A.11}$$

where y_t is the log yield-to-maturity at time t and

$$D = \frac{1}{1-\theta} = \frac{K+1}{K} \tag{A.12}$$

is the Macaulay duration when the bond is trading at par. The log-linear approximation for default-free coupon-bearing bonds in equation (A.11) appears in Chapter 10 of Campbell et al. (1996).

To derive this approximation, note that the Campbell-Shiller (1988) approximation of the 1-period log return on the long-term bond is

$$r_{t+1} = \ln(P_{t+1}+K) - p_t \approx \varphi + \theta p_{t+1} + (1-\theta)k - p_t, \tag{A.13}$$

where $p_t = \log(P_t)$ is the log price, $k = \log(K)$ is the log coupon, and where $\theta = 1/(1+\exp(k-\overline{p}))$ and $\varphi = -\log(\theta) - (1-\theta)\log\left(\theta^{-1}-1\right)$ are parameters of the log-linearization. Iterating equation (A.13) forward, we find that the log bond price is

$$p_t = (1-\theta)^{-1}\varphi + k - \textstyle\sum_{i=0}^{\infty}\theta^i E_t\left[r_{t+i+1}\right]. \tag{A.14}$$

Applying this approximation to the *yield-to-maturity*, defined as the *constant return* that equates bond price and the discounted value of promised cashflows, we obtain

$$p_t = (1-\theta)^{-1}\varphi + k - (1-\theta)^{-1} y_t. \tag{A.15}$$

Equation (A.11) then follows by substituting the expression for p_t in equation (A.15) into the Campbell-Shiller return approximation in equation (A.13).

Assuming the steady-state price of the bonds is par ($\overline{p} = 0$), we have $\theta = 1/(1+K)$. Thus, bond duration is $D = -\partial p_t/\partial y_t = (1-\theta)^{-1} = (1+K)/K$. Since $-\partial p_t/\partial y_t = -(\partial P_t/\partial Y_t)((1+Y_t)/P_t) = (Y_t^{\cdot}+1)/Y_t$ this corresponds to Macaulay duration when the bonds are trading at par ($Y_t = K$).

Let i_t denote the interest rate on short-term nominal bonds from t to $t{+}1$ and let $rx_{t+1} \equiv r_{t+1} - i_t$ denote the excess return on long-term nominal bonds over short-term nominal bonds from t to $t{+}1$. Then, iterating equation (A.11) forward and taking expectations, the yield on long-term nominal bonds is given by:

$$y_t = (1-\theta)\textstyle\sum_{j=0}^{\infty}\theta^j E_t\left[i_{t+j} + rx_{t+j+1}\right]. \tag{A.16}$$

B.2 Market participants

There are two types of risk-averse arbitrageurs in the model, each with identical risk tolerance τ, who differ solely in the frequency with which they can rebalance their bond portfolios.

The first group of arbitrageurs are fast-moving arbitrageurs who are free to adjust their holdings of long-term and short-term bonds each period. Fast-moving arbitrageurs are present in mass q and we denote their demand for long-term bonds at time t by b_t. Fast-moving arbitrageurs have mean-variance preferences over 1-period portfolio log returns. Thus, their demand for long-term bonds at time t is given by

$$b_t = \tau \frac{E_t\left[rx_{t+1}\right]}{Var_t\left[rx_{t+1}\right]}, \tag{A.17}$$

where

$$rx_{t+1} \equiv r_{t+1} - i_t = \frac{1}{1-\theta} y_t - \frac{\theta}{1-\theta} y_{t+1} - i_t \tag{A.18}$$

is the excess return on long-term bonds from t to $t+1$.

The second group of arbitrageurs is a set of slow-moving arbitrageurs who can only adjust their holdings of long-term and short-term bonds every k periods. Slow-moving arbitrageurs are present in mass $1-q$. A fraction $1/k$ of these slow-moving arbitrageurs is active each period and can reallocate their portfolios. However, they must then maintain this same portfolio allocation for the next k periods. As in Duffie (2010), this is a reduced-form way to model the frictions that limit the speed of capital flows. Since they only rebalance their portfolios every k periods, slow-moving arbitrageurs have mean-variance preferences over their k-period *cumulative* portfolio excess return. Thus, the demand for long-term bonds from the subset of slow-moving arbitrageurs who are active at time t is given by

$$d_t = \tau \frac{E_t[\sum_{j=1}^{k} rx_{t+j}]}{Var_t[\sum_{j=1}^{k} rx_{t+j}]}. \tag{A.19}$$

B.3 Risk factors

Short-term nominal interest rates: Short-term nominal bonds are available in perfectly elastic supply. At time t, arbitrageurs learn that short-term bonds will earn a riskless log return of i_t in nominal terms between time t and $t+1$. We assume that the short-term nominal interest rate is the sum of a highly persistent component $i_{P,t}$ and a more transient component $i_{T,t}$:

$$i_t = i_{P,t} + i_{T,t}. \tag{A.20}$$

We assume that the persistent component $i_{P,t}$ follows an exogenous AR(1) process:

$$i_{P,t+1} = \bar{\imath} + \rho_P\left(i_{P,t} - \bar{\imath}\right) + \varepsilon_{P,t+1}, \tag{A.21}$$

where $0 < \rho_P < 1$ and $Var_t\left[\varepsilon_{P,t+1}\right] = \sigma_P^2$. Similarly, we assume that the transient component $i_{T,t}$ follows an exogenous AR(1) process:

$$i_{T,t+1} = \rho_T i_{T,t} + \varepsilon_{T,t+1}, \tag{A.22}$$

where $0 < \rho_T \leq \rho_P < 1$ and $Var_t\left[\varepsilon_{T,t+1}\right] = \sigma_T^2$.

Supply of long-term bonds: We assume that the long-term nominal bond is available in an exogenous, time-varying *net supply* s_t that must be held in equilibrium by fast arbitrageurs and slow-moving arbitrageurs. This net supply equals the *gross supply* of long-term bonds *minus the demand* for long-term bonds from any unmodeled agents who have inelastic demand for these bonds. Formally, we assume that s_t follows an AR(1) process:

$$s_{t+1} = \overline{s} + \rho_s\left(s_t - \overline{s}\right) + \varepsilon_{s,t+1} + C\varepsilon_{P,t+1} + C\varepsilon_{T,t+1}, \tag{A.23}$$

where $0 < \rho_s < 1$, $C > 0$, and $Var_t\left[\varepsilon_{s,t+1}\right] = \sigma_s^2$. The $\varepsilon_{s,t+1}$ shocks capture other forces that impact the net supply of long-term bonds. We have made no assumptions about the correlation structure among the three underlying shocks $\varepsilon_{P,t+1}$, $\varepsilon_{T,t+1}$, and $\varepsilon_{s,t+1}$. Indeed, the model can be solved for any arbitrary correlation structure among these shocks.

To understand the implied process for net bond supply, let L denote the time-series lag operator and note

$$\begin{aligned}\left(1-\rho_s L\right)\left(s_t - \overline{s}\right) &= \varepsilon_{s,t} + C\varepsilon_{P,t} + C\varepsilon_{T,t} \\ &= \varepsilon_{s,t} + C\left(1-\rho_P L\right)\left(i_{P,t} - \overline{\imath}\right) + C\left(1-\rho_T L\right) i_{T,t}.\end{aligned} \tag{A.24}$$

Working out the lag polynomial, we see that

$$\begin{aligned}s_t = \overline{s} &+ C[(i_{P,t} - \overline{\imath}) - (\rho_P - \rho_s)\textstyle\sum_{j=0}^{\infty}\rho_s^j\left(i_{P,t-j-1} - \overline{\imath}\right)] \\ &+ C[i_{T,t} - (\rho_T - \rho_s)\textstyle\sum_{j=0}^{\infty}\rho_s^j i_{T,t-j-1}] + [\textstyle\sum_{j=0}^{\infty}\rho_s^j\varepsilon_{s,t-j}].\end{aligned} \tag{A.25}$$

which follows from the fact that

$$\begin{aligned}\left(1-\rho_s L\right)^{-1}\left(1-\rho_x L\right)x_t &= \textstyle\sum_{j=0}^{\infty}\rho_s^j L^j\left(1-\rho_x L\right)x_t \\ &= x_t - \rho_x x_{t-1} + \rho_s x_{t-1} - \rho_s\rho_x x_{t-2} + \rho_s^2 x_{t-2} - \rho_s^3\rho_x x_{t-2} + \cdots \\ &= x_t - \left(\rho_x - \rho_s\right)\textstyle\sum_{j=0}^{\infty}\rho_s^j x_{t-j-1}.\end{aligned} \tag{A.26}$$

B.4 Equilibrium Conjecture

For the sake of concreteness, suppose that $k = 4$. We conjecture that equilibrium yields take the form

$$y_t = \alpha_0 + \alpha_1'\mathbf{x}_t, \tag{A.27}$$

and that the demands of active slow-moving arbitrageurs are of the form

$$d_t = \delta_0 + \delta_1' \mathbf{x}_t, \tag{A.28}$$

where the $k+2$ dimensional state vector is

$$\mathbf{x}_t = \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ s_t - \bar{s} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \end{bmatrix}. \tag{A.29}$$

These assumptions imply that the state vector follows an AR(1) process. Critically, the transition matrix Γ is a function of the parameters δ_1 governing slow-moving arbitrageur demand so we write $\Gamma = \Gamma(\delta_1)$. Specifically, we have

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma(\delta)\, \mathbf{x}_t + \epsilon_{t+1} \qquad \text{(A.30)} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_T & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_s & 0 & 0 & 0 \\ \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ s_t - \bar{s} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ \varepsilon_{s,t+1} + C\varepsilon_{P,t+1} + C\varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

where $\Sigma \equiv Var_t[\epsilon_{t+1}]$. Assuming for simplicity that $\varepsilon_{P,t+1}$, $\varepsilon_{T,t+1}$, and $\varepsilon_{s,t+1}$ are mutually orthogonal, we have

$$\Sigma = \begin{bmatrix} \sigma_P^2 & 0 & C\sigma_P^2 & 0 & 0 & 0 \\ 0 & \sigma_T^2 & C\sigma_\pi^2 & 0 & 0 & 0 \\ C\sigma_P^2 & C\sigma_T^2 & \sigma_s^2 + C^2\sigma_P^2 + C^2\sigma_T^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A.31}$$

We adopt the convention that $\mathbf{e}$ is the vector with a 1 corresponding to $i_{P,t} - \bar{i}$ and $i_{T,t}$ and 0s elsewhere, i.e., $\mathbf{e} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}'$; $\mathbf{e}_s$ is the basis vector with a 1 corresponding to $s_t - \bar{s}$ and 0s elsewhere, i.e., $\mathbf{e}_s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}'$; and $\mathbf{e}_d$ is a 1 corresponding to $d_{t-1} - \delta_0$, $d_{t-2} - \delta_0$, $\cdots$, $d_{t-(k-1)} - \delta_0$ and 0s elsewhere, i.e., $\mathbf{e}_d = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}'$.

Finally, we denote $\mathbf{C}_{[t+i,t+j]} = Cov[\mathbf{x}_{t+i}, \mathbf{x}_{t+j} | \mathbf{x}_t]$ and note that

$$\mathbf{C}_{[t+i,t+j]} = \sum\nolimits_{s=1}^{\min\{i,j\}} [\mathbf{\Gamma}^{i-s}] \mathbf{\Sigma} [\mathbf{\Gamma}^{j-s}]', \tag{A.32}$$

so $\mathbf{C}_{[t+j,t+i]} = \mathbf{C}'_{[t+i,t+j]}$.

B.5 Arbitrageur demands

Fast-moving arbitrageurs' demand: Given this conjecture, we work out fast-moving arbitrageurs' demand for long-term bonds. Given the conjectured form of equilibrium yields, the realized 1-period excess returns on long bonds from t to $t+1$ is

$$\begin{aligned} rx_{t+1} &= \frac{1}{1-\theta}y_t - \frac{\theta}{1-\theta}y_{t+1} - i_t \\ &= \left(\alpha_0 - \bar{i}\right) + \left(\frac{1}{1-\theta}\alpha_1 - \mathbf{e}\right)' \mathbf{x}_t - \left(\frac{\theta}{1-\theta}\alpha_1\right)' \mathbf{x}_{t+1}, \end{aligned} \tag{A.33}$$

which implies

$$E_t\left[rx_{t+1}\right] = \left(\alpha_0 - \bar{r} - \bar{\pi}\right) + \left(\frac{1}{1-\theta}\alpha_1 - \mathbf{e}\right)' \mathbf{x}_t - \left(\frac{\theta}{1-\theta}\alpha_1\right)' \mathbf{\Gamma}\mathbf{x}_t \tag{A.34}$$

and

$$Var_t\left[rx_{t+1}\right] = \left(\frac{\theta}{1-\theta}\right)^2 \alpha_1' \mathbf{\Sigma} \alpha_1 \tag{A.35}$$

Thus, fast-moving arbitrageurs' demand for long-term bonds is

$$b_t = \tau \frac{E_t\left[rx_{t+1}\right]}{Var_t\left[rx_{t+1}\right]} = \left[\tau \frac{\alpha_0 - \bar{i}}{\left(\frac{\theta}{1-\theta}\right)^2 \alpha_1' \mathbf{\Sigma} \alpha_1}\right] + \left[\tau \frac{\left(\frac{1}{1-\theta}\alpha_1 - \mathbf{e}\right)' - \frac{\theta}{1-\theta}\alpha_1' \mathbf{\Gamma}}{\left(\frac{\theta}{1-\theta}\right)^2 \alpha_1' \mathbf{\Sigma} \alpha_1}\right] \mathbf{x}_t. \tag{A.36}$$

Slow-moving arbitrageurs' demand: We next work out slow-moving arbitrageurs' demand for long-term bonds. Given our conjecture, the realized k-period cumulative excess returns on long bonds from t to $t+k$ is

$$\begin{aligned} \textstyle\sum_{j=1}^{k} rx_{t+j} &= \textstyle\sum_{j=0}^{k-1}\left(y_{t+j} - i_{t+j}\right) - \frac{\theta}{1-\theta}\left(y_{t+k} - y_t\right) \\ &= k\left(\alpha_0 - \bar{i}\right) + \left(\alpha_1 - \mathbf{e}\right)' \left(\textstyle\sum_{j=0}^{k-1} \mathbf{x}_{t+j}\right) - \frac{\theta}{1-\theta}\alpha_1' \left(\mathbf{x}_{t+k} - \mathbf{x}_t\right) \end{aligned} \tag{A.37}$$

Thus, expected k-period cumulative returns are

$$E_t[\textstyle\sum_{j=1}^{k} rx_{t+j}] = k\left(\alpha_0 - \bar{i}\right) + \left(\left(\alpha_1 - \mathbf{e}\right)' \left(\mathbf{I} - \mathbf{\Gamma}\right)^{-1} + \frac{\theta}{1-\theta}\alpha_1'\right)\left(\mathbf{I} - \mathbf{\Gamma}^k\right)\mathbf{x}_t, \tag{A.38}$$

and the variance of k-period cumulative excess returns is

$$\begin{aligned} Var_t[\textstyle\sum_{j=1}^{k} rx_{t+j}] &= Var_t[\left(\alpha_1 - \mathbf{e}\right)' \left(\textstyle\sum_{j=1}^{k-1} \mathbf{x}_{t+j}\right) - \left(\frac{\theta}{1-\theta}\right)\alpha_1' \mathbf{x}_{t+k}] \\ &= \left(\alpha_1 - \mathbf{e}\right)' \left(\textstyle\sum_{l=1}^{k-1}\sum_{j=1}^{k-1} \mathbf{C}_{[t+l,t+j]}\right)\left(\alpha_1 - \mathbf{e}\right) + \left(\frac{\theta}{1-\theta}\right)^2 \alpha_1' \mathbf{C}_{[t+k,t+k]}\alpha_1 \\ &\quad - 2\left(\frac{\theta}{1-\theta}\right)\left(\alpha_1 - \mathbf{e}\right)' \textstyle\sum_{j=1}^{k-1} \mathbf{C}_{[t+j,t+k]}\alpha_1. \end{aligned} \tag{A.39}$$

Slow-moving arbitrageurs' demand long long-term bonds is

$$d_t = \tau \frac{E_t[\sum_{j=1}^{k} rx_{t+j}]}{Var_t[\sum_{j=1}^{k} rx_{t+j}]}. \tag{A.40}$$

Thus, given our conjectures, slow-moving arbitrageurs demands will indeed take a linear form. Specifically, we have

$$\delta_0 = \tau \frac{k\left(\alpha_0 - \bar{i}\right)}{V^{(k)}} \tag{A.41}$$

where $V^{(k)} = Var_t[\sum_{j=1}^{k} rx_{t+j}]$ and

$$\delta_1' = \tau \frac{\left((\alpha_1 - \mathbf{e})' (\mathbf{I} - \mathbf{\Gamma})^{-1} + \frac{\theta}{1-\theta}\alpha_1'\right)}{V^{(k)}} \left(\mathbf{I} - \mathbf{\Gamma}^k\right) \tag{A.42}$$

B.6 Equilibrium solution

To solve for the equilibrium, we need to clear the market for bonds in a way that is consistent with optimization on the part of fast-moving arbitrageurs and slow-moving arbitrageurs. The market-clearing condition is

$$\overbrace{(1-q)k^{-1}d_t + qb_t}^{\text{Active demand}} = \overbrace{s_t - (1-q)(k^{-1}\textstyle\sum_{i=1}^{k-1} d_{t-i})}^{\text{Active supply}}. \tag{A.43}$$

Letting $V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\theta}{1-\theta}\right)^2 \alpha_1' \mathbf{\Sigma} \alpha_1$, denote the variance of 1-period excess returns, active demand is

$$\begin{aligned} &(1-q)k^{-1}d_t + qb_t && \text{(A.44)}\\ &= \left[(1-q)k^{-1}\delta_0 + q\tau \frac{\left(\alpha_0 - \bar{i}\right)}{V^{(1)}}\right] + \left[(1-q)k^{-1}\delta_1' + q\tau \frac{\left(\frac{1}{1-\theta}\alpha_1 - \mathbf{e}\right)' - \frac{\theta}{1-\theta}\alpha_1'\mathbf{\Gamma}}{V^{(1)}}\right]\mathbf{x}_t \end{aligned}$$

Active supply is

$$\begin{aligned} &s_t - (1-q)k^{-1}\textstyle\sum_{i=1}^{k-1} d_{t-i} && \text{(A.45)}\\ &= \left[\bar{s} - (1-q)\frac{(k-1)}{k}\delta_0\right] + \left[\left(\mathbf{e}_s - (1-q)k^{-1}\mathbf{e}_d\right)'\right]\mathbf{x}_t. \end{aligned}$$

Matching constants terms, we obtain

$$\alpha_0 = \bar{i} + \frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right) \tag{A.46}$$

Matching slope coefficients, we have

$$\begin{aligned} \alpha_1 &= (1-\theta)\left[\mathbf{I} - \theta\mathbf{\Gamma}'\right]^{-1}\mathbf{e} + (1-\theta)\frac{V^{(1)}}{\tau q}\left[\mathbf{I} - \theta\mathbf{\Gamma}'\right]^{-1}\left[\mathbf{e}_s - k^{-1}(1-q)\left(\mathbf{1}_{(d)} + \delta_1\right)\right] && \text{(A.47)}\\ &= \frac{1-\theta}{1-\theta\rho_P}\mathbf{e}_P + \frac{1-\theta}{1-\theta\rho_T}\mathbf{e}_T + \frac{V^{(1)}}{\tau q}\left[\frac{1-\theta}{1-\theta\rho_s}\mathbf{e}_s - k^{-1}(1-q)(1-\theta)\left[\mathbf{I} - \theta\mathbf{\Gamma}'\right]^{-1}\left(\mathbf{1}_{(d)} + \delta_1\right)\right] \end{aligned}$$

where $\mathbf{e}_P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'$ and $\mathbf{e}_T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}'$.

Thus, equilibrium yields take the form

$$
\begin{aligned}
y_t &= \alpha_0 + \alpha_1' \mathbf{x}_t \qquad\qquad (A.48)\\
&= \overbrace{\bar{i} + \frac{1-\theta}{1-\theta\rho_P}\left(i_{P,t} - \bar{i}\right) + \frac{1-\theta}{1-\theta\rho_T} i_{P,t}}^{\text{Expected future short real rates}} \\
&+ \overbrace{\frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right)}^{\text{Unconditional term premia}} \\
&+ \overbrace{\left[\frac{V^{(1)}}{\tau q}\left(\frac{1-\theta}{1-\theta\rho_s}\left(s_t - \bar{s}\right) - (1-\theta)(1-q)k^{-1}\left(\mathbf{e}_d + \delta_1\right)' \left[\mathbf{I} - \theta\boldsymbol{\Gamma}\right]^{-1} \mathbf{x}_t\right)\right]}^{\text{Conditional term premia}}.
\end{aligned}
$$

Equilibrium excess returns are given by

$$
E_t\left[rx_{t+1}\right] = \frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right) + \frac{V^{(1)}}{\tau q}\left[\left(s_t - \bar{s}\right) - (1-q)k^{-1}\left(\mathbf{1}_{(d)} + \delta_1\right)' \mathbf{x}_t\right] \qquad (A.49)
$$

B.7 Equilibrium existence and uniqueness

A rational expectations equilibrium of our model is a fixed point of a specific operator involving the "price-impact" coefficients, (α_1'), which show how bond supply and inactive slow-moving arbitrageur demand impact bond yields, and the "demand-impact" coefficients, (δ_1'), which show how bond supply and inactive demand impact the demand of active slow-moving arbitrageurs. Specifically, let $\omega = (\alpha_1', \delta_1')'$ and consider the operator $\mathbf{f}(\omega_0)$ which gives (i) the price-impact coefficients that will clear the market for long-term bonds and (ii) the demand-impact coefficients consistent with optimization on the part of active slow-moving arbitrageurs when arbitrageurs conjecture that $\omega = \omega_0$ at all future dates. Thus, a rational expectations equilibrium of our model is a fixed point $\omega^* = \mathbf{f}(\omega^*)$.

Specifically, an equilibrium solves the following system of equations

$$
\alpha_1 = (1-\theta)\left[\mathbf{I} - \theta\boldsymbol{\Gamma}(\delta_1)'\right]^{-1}\left[\mathbf{e} + \frac{V^{(1)}(\alpha_1)}{\tau q}\left(\mathbf{e}_s - k^{-1}(1-q)\left(\mathbf{e}_d + \delta_1\right)\right)\right] \qquad (A.50)
$$

and

$$
\delta_1' = \tau \frac{\left((\alpha_1 - \mathbf{e}_i)'\left(\mathbf{I} - \boldsymbol{\Gamma}(\delta)\right)^{-1} + \frac{\theta}{1-\theta}\alpha_1'\right)}{V^{(k)}(\alpha_1, \delta_1)}\left(\mathbf{I} - \boldsymbol{\Gamma}(\delta_1)^k\right) \qquad (A.51)
$$

where we write $V^{(1)}(\alpha_1)$ to emphasize that the 1-period return variance depends on α_1; $\Gamma(\delta_1)$ to emphasize that the transition matrix depends on δ_1; and $V^{(k)}(\alpha_1, \delta_1)$ to emphasize that the k-period return variance depends on α_1 and δ_1. We can write this system of non-linear equations more compactly as

$$
\alpha_1 = \mathbf{f}_{\alpha_{A1}}(\alpha_1, \delta_1) \text{ and } \delta_1 = \mathbf{f}_{\delta_1}(\alpha_1, \delta_1) \qquad (A.52)
$$

or simply as $\omega = \mathbf{f}(\omega)$ where $\omega = (\alpha_1', \delta_1')'$.

This is a system of $2(k+1)$ equations in $2(k+1)$ unknowns. However, in any rational expectations equilibrium of our model, bond yields always reflect the expected path of future short rates. As a result, equilibrium bond holdings do not depend on future short rates. Formally, it is easy to see that, in any equilibrium, active slow-moving arbitrageur demand does not depend on $i_{P,t}$ and $i_{T,t}$, so the first two elements of δ_1' are zeros and the first two elements are α_1' are $(1-\theta)/(1-\theta\rho_P)$ and $(1-\theta)/(1-\theta\rho_T)$, respectively. This implies that an equilibrium of our model is a solution to a system of $2k$ nonlinear equations in $2k$ unknowns. Specifically, we need to determine how equilibrium yields and active slow-moving demand respond to shifts in the supply of bonds: this generates 2 unknowns and 2 corresponding equations. We also need to determine how equilibrium yields and active slow-moving demand respond to the holdings of inactive slow-moving arbitrageurs: this generates $2(k-1)$ unknowns and $2(k-1)$ corresponding equations.

We solve the relevant system of $2k$ nonlinear equations numerically using the Powell hybrid algorithm which performs a quasi-Newton search to find solutions to a system of nonlinear equations starting from an initial guess.[1] To find all of the solutions, we apply this algorithm by sampling over $10,000$ different initial guesses. Once a solution for α_1 and δ_1 is in hand, we can compute $V^{(1)}$ and $V^{(k)}$ and can then solve for α_0 and δ_0 using

$$\alpha_0 = \bar{i} + \frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right) \text{ and } \delta_0 = \tau\frac{k\left(\alpha_0 - \bar{i}\right)}{V^{(k)}}, \tag{A.53}$$

which yields

$$\alpha_0 = \bar{i} + \frac{\bar{s}}{\tau\left[q\frac{1}{V^{(1)}} + (1-q)\frac{k}{V^{(k)}}\right]} \text{ and } \delta_0 = \frac{\frac{k}{V^{(k)}}}{q\frac{1}{V^{(1)}} + (1-q)\frac{k}{V^{(k)}}} \times \bar{s}. \tag{A.54}$$

When asset supply is stochastic, an equilibrium solution only exists if arbitrageurs are sufficiently risk tolerant (i.e., for τ sufficiently large). When an equilibrium exists, there are multiple equilibrium solutions. Equilibrium non-existence and multiplicity of this sort arise in overlapping-generations, rational-expectations models such as ours where risk-averse arbitrageurs with finite investment horizons trade an infinitely-lived asset that is subject to supply shocks.[2] Different equilibria correspond to different self-fulfilling beliefs that arbitrageurs can hold about the price-impact of supply shocks and, hence, the risks associated with holding long-term bonds. See Greenwood et al. (2018) for an extensive discussion of these issues.

The intuition for equilibrium multiplicity can be understood most clearly in the simple case when there are only fast-moving arbitrageurs. If arbitrageurs are sufficiently risk tolerant there are

[1]Rational expectations models with noisy asset supply can only be solved in closed-form in very simple, special cases. For instance, we have only been able to obtain closed-form solutions to our model if we turn off the key asset pricing friction—namely, slow-moving capital—that is the focus our of paper.

[2]For previous treatments of these issues, see Spiegel (1998), Bacchetta and van Wincoop (2003), Watanabe (2008), Banerjee (2011), Greenwood and Vayanos (2014), Albagli (2015), and Greenwood, Hanson, and Liao (forthcoming).

two equilibria in this special case: a low price impact (or low return volatility) equilibrium and a high price impact (or high return volatility) equilibrium. If arbitrageurs believe that supply shocks will have a large impact on long-term bonds prices, they will perceive bonds as being highly risky. As a result, arbitrageurs will only absorb a positive supply shock if they are compensated by a large decline in bond prices, making the initial belief self-fulfilling. However, if arbitrageurs believe that bond prices will be less sensitive to supply shocks, they will perceive bond as being less risky and will absorb a supply shock even if they are only compensated by a modest decline in bond prices.

Things are slightly more complicated in our general model with slow-moving capital. Specifically, the introduction of slow-moving capital can give rise to additional unstable equilibria. However, we always find a unique equilibrium that is stable in the sense that equilibrium is robust to a small perturbation in arbitrageurs' beliefs regarding the equilibrium that will prevail in the future. Formally, letting $\omega^{(1)} = \omega^* + \xi$ for some small ξ and defining $\omega^{(n)} = \mathbf{f}(\omega^{(n-1)})$, an equilibrium ω^* is stable if $\lim_{n\to\infty} \omega^{(n)} = \omega^*$ and is unstable if $\lim_{n\to\infty} \omega^{(n)} \neq \omega^*$. Let $\{\lambda_i\}$ denote the eigenvalues of the Jacobian $\mathbf{D}_\omega \mathbf{f}(\omega^*)$. If $\max_i |\lambda_i| < 1$, then ω^* is stable; if $\max_i |\lambda_i| > 1$, then ω^* is unstable.

Consistent with Samuelson's (1947) "correspondence principle," which says that the comparative statics of stable equilibria have certain properties, this unique stable equilibrium has comparative statics that accord with standard economic intuition. By contrast, the unstable equilibria have comparative statics that run contrary to standard intuition.[3] We focus on this unique stable equilibrium in our numerical illustrations.

B.8 Solution in the special case without slow-moving capital

In this subsection, we should how to solve the model the special case in which there is no slow-moving capital (i.e., if either $q = 1$ or $k = 1$). For simplicity, we also assume that the three underlying shocks—i.e., $\varepsilon_{P,t+1}$, $\varepsilon_{T,t+1}$, and $\varepsilon_{s,t+1}$—are mutually orthogonal. In this special case, the equilibrium yield on long-term bonds is

$$y_t = \overbrace{\left(\bar{\imath} + \tau^{-1}V^{(1)}\bar{s}\right)}^{\alpha_0} + \overbrace{\frac{1-\theta}{1-\rho_P\theta}}^{\alpha_{1P}}(i_{P,t} - \bar{\imath}) + \overbrace{\frac{1-\theta}{1-\rho_T\theta}}^{\alpha_{1T}} i_{T,t} + \overbrace{\tau^{-1}V^{(1)}\frac{1-\theta}{1-\rho_s\theta}}^{\alpha_{1s}}(s_t - \bar{s}), \tag{A.55}$$

where

$$\begin{aligned} V^{(1)} &= Var_t\left[\frac{\theta}{1-\theta}y_{t+1}\right] \qquad\qquad (A.56)\\ &= Var_t\left[\frac{\theta}{1-\theta}\varepsilon_{P,t+1} + \frac{\theta}{1-\rho_T\theta}\varepsilon_{T,t+1} + \tau^{-1}V^{(1)}\frac{\theta}{1-\rho_s\theta}\left(\varepsilon_{s,t+1} + C\varepsilon_{P,t+1} + C\varepsilon_{T,t+1}\right)\right] \end{aligned}$$

[3]For instance, in the special case where there is no slow-moving capital, the low price-impact equilibrium is stable and the high price-impact equilibrium is unstable. At the stable equilibrium, an increase in the volatility of short rates or the volatility of supply shocks is associated with an increase in the price-impact coefficient and an increase in the volatility of returns. By contrast, these comparitive statics take the opposite sign at the unstable equilibrium.

is the smaller root of the following quadratic equation:

$$0 = \left[\left(\tau^{-1}\frac{\theta}{1-\rho_s\theta}\sigma_s\right)^2 + \left(\tau^{-1}\frac{\theta}{1-\rho_s\theta}C\sigma_P\right)^2 + \left(\tau^{-1}\frac{\theta}{1-\rho_s\theta}C\sigma_T\right)^2\right] \times \left(V^{(1)}\right)^2 \tag{A.57}$$
$$+ \left[2\left(\frac{\theta}{1-\rho_P\theta}\sigma_P\right)\left(\tau^{-1}\frac{\theta}{1-\rho_s\theta}C\sigma_P\right) + 2\left(\frac{\theta}{1-\rho_T\theta}\sigma_T\right)\left(\tau^{-1}\frac{\theta}{1-\rho_s\theta}C\sigma_T\right) - 1\right] \times V^{(1)}$$
$$+ \left[\left(\frac{\theta}{1-\rho_P\theta}\sigma_P\right)^2 + \left(\frac{\theta}{1-\rho_T\theta}\sigma_T\right)^2\right].$$

In this case, the model-implied regression coefficient is

$$\beta_h = \frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]} \tag{A.58}$$
$$= \frac{\alpha_{1P} Var\left[\Delta_h i_{P,t}\right] + \alpha_{1T} Var\left[\Delta_h i_{T,t}\right] + \alpha_{1s}\left(Cov\left[\Delta_h i_{P,t}, \Delta_h s_t\right] + Cov\left[\Delta_h i_{T,t}, \Delta_h s_t\right]\right)}{Var\left[\Delta_h i_{P,t+h}\right] + Var\left[\Delta_h i_{T,t+h}\right]}.$$

where for $X \in \{P, T\}$ we have $Var\left[\Delta_h i_{X,t}\right] = 2\left[(1-\rho_X^h)/\left(1-\rho_X^2\right)\right]\sigma_X^2$ and $Cov\left[\Delta_h i_{X,t}, \Delta_h s_t\right] = C\left[\left(2-\rho_s^h-\rho_X^h\right)/\left(1-\rho_s\rho_X\right)\right]\sigma_X^2$. We assume that throughout that $C \geq 0$ and $\rho_s \leq \rho_T \leq \rho_P$.

Recall that we have assumed that $C \geq 0$ and $\rho_s \leq \rho_T \leq \rho_P$. For simplicity, in the following discussion, we will also assume that $\sigma_s^2 = 0$.[4]

We first consider the *level* of β_h irrespective of horizon h. Inspecting equation (A.58), it is easy to see that:

- **When $C = 0$, the *level* of β_h is increasing in σ_P for all h.** An increase in σ_P raises the fraction of total short-rate variation at all horizons that is due to movements in the more persistent component (i.e., raises $Var\left[\Delta_h i_{P,t+h}\right] / \left(Var\left[\Delta_h i_{P,t+h}\right] + Var\left[\Delta_h i_{T,t+h}\right]\right)$ for all h). Since shocks to the more persistent component of short rates have larger impact on long-term yields via a straightforward expectations hypothesis channel (i.e., since $\alpha_{1P} > \alpha_{1T}$), an increase in σ_P raises the level of β_h at all horizons. Thus, if σ_P declined between the pre-2000 and post-2000 periods as we have argued, this would lead β_h to decline at all horizons h.

We next consider the way β_h behaves as a function of horizon h. Again, using equation (A.58), it is easy to show that:

- **When $C = 0$ and $\rho_T = \rho_P$, β_h is a constant that is independent of h.** These assumptions imply that the expectations hypothesis holds—i.e., there is no excess sensitivity—and that all shocks to short rates have the same persistence. In this benchmark case, $\beta_h = \alpha_{1P} = \alpha_{1T}$ for all h—i.e., the sensitivity of long rates to short rates is the same at all horizons.

[4] This is without loss of generality since σ_s^2 only impacts the level of α_{1s} and does not otherwise affect β_h.

- **When $C = 0$ and $\rho_T < \rho_P$, β_h is an increasing function of h.** These assumptions imply that the expectations hypothesis holds, but there are now transient and persistent shocks to short rates. In this case, β_h rises with h since (i) movements in the more persistent component of short rates are associated with larger movement in long-term yields (i.e., $\alpha_{1P} > \alpha_{1T}$) and (ii) because the persistent component dominates changes in short rates at longer horizons (i.e., $Var\left[\Delta_h i_{P,t+h}\right] / \left(Var\left[\Delta_h i_{P,t+h}\right] + Var\left[\Delta_h i_{T,t+h}\right]\right)$ rises with h when $\rho_T < \rho_P$).

- **When $C > 0$ and $\rho_s = \rho_T = \rho_P$, β_h is a constant that is independent of h.** In this case, there is excess sensitivity—shifts in short rates lead to shifts in the term premium on long-term bonds—but the excess sensitivity is the same irrespective of horizon h. This is because $\Delta_h s_{t+h} = C\Delta_h i_{P,t+h} + C\Delta_h i_{T,t+h}$ when $\rho_s = \rho_T = \rho_P$ (see equation (A.25)) and $Var\left[\Delta_h i_{P,t+h}\right] / \left(Var\left[\Delta_h i_{P,t+h}\right] + Var\left[\Delta_h i_{T,t+h}\right]\right) = \sigma_P^2 / \left(\sigma_P^2 + \sigma_T^2\right)$ when $\rho_T = \rho_P$.

- **When $C > 0$ and $\rho_s < \rho_T = \rho_P$, β_h is a decreasing function of h.** In this case, long-term interest rates exhibit excess sensitivity to movements in short rates that declines with horizon h. Intuitively, if the supply shocks induced by shocks to short rates are more transient than the underlying shocks to short rates, then term premia will react more in the short run than in the long run. Thus, there will be greater excess sensitivity in the short run.

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