The Risks of Safe Assets

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US government bonds exhibit many characteristics often attributed to safe assets. Indeed, a growing literature documents significant convenience yields, perhaps due to liquidity, in US Treasuries, suggesting that rising Treasury supply and government debt comes with a declining liquidity premium and a fall in firms' relative cost of debt financing. In this paper, we empirically document and theoretically evaluate a dual role for government debt. Through a *liquidity channel* an increase in government debt improves liquidity and lowers liquidity premia by facilitating debt rollover, thereby reducing credit spreads. Through an *uncertainty channel*, however, rising government debt creates policy uncertainty, raising credit spreads and default risk premia. We interpret and quantitatively evaluate these two channels through the lens of a general equilibrium asset pricing model with risk-sensitive agents subject to liquidity shocks, in which firms issue defaultable bonds and the government issues tax-financed bonds that endogenously enjoy liquidity benefits. The calibrated model generates quantitatively realistic liquidity spreads and default risk premia,and suggests that while rising government debt reduces liquidity spreads, it not only crowds out corporate debt financing, and therefore, investment, but also creates uncertainty reflected in endogenous tax volatility, credit spreads, and risk premia, and ultimately consumption volatility. Therefore, increasing safe asset supply can be risky.

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1 Introduction

US government bonds exhibit many characteristics often attributed safe assets: They are very liquid and lenders readily accept them as collateral. Indeed, as pointed out in the recent literature, for example, by Longstaff (2003), Bansal, Coleman, and Lundblad (2012), Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2016), Jiang, Krishnamurthy, and Lustig (2018) or Jiang, Krishnamurthy, and Lustig (2019) Treasuries exhibit many money-like features so that investors attach a 'liquidity premium' or 'convenience yield' to holding these assets. Arguably, therefore, by issuing Treasuries and raising debt, the US government can provide liquidity services to investors and facilitate transactions in the economy. Such an increase in the supply of safe assets comes with a declining liquidity premium and thus a fall in firms' relative cost of debt financing, providing further stimulus to the economy.

In this paper, we empirically document and theoretically evaluate a dual role of government debt for credit markets. While indeed, through a *liquidity channel* an increase in government debt improves liquidity and lowers liquidity premia by facilitating debt rollover, thereby reducing credit spreads. Through an *uncertainty channel*, however, rising government debt creates policy uncertainty, raising credit spreads, default risk premia, and expected corporate bond excess returns and eventually leads to rising relative costs of firms' debt financing. Under such a dual view, we identify a novel fiscal risk channel associated with rising US government debt. Ultimately therefore, increasing safe asset supply can be risky.

Our analysis starts from the empirical observation that the government debt to GDP ratio has a dual role in predicting the costs of debt financing. While, indeed, it exhibits significantly negative predictive power for money market instruments, confirming its negative impact on liquidity premia, we provide novel empirical evidence that it significantly positively predicts future corporate bond credit spreads and excess returns. More formally, we also present econometric evidence from estimating a VAR that suggests that liquidity premia and credit risk premia respond to innovations to the debt to GDP ratio with opposite sign. Taken together, this evidence suggests that a rising debt to GDP ratio is correlated with future risks in the economy reflected in rising risk premia[1].

To interpret and quantify the dual role of government debt for credit markets, and to understand the sources and effects of fiscal risks, we introduce a novel general equilibrium asset pricing model

[1] See Liu (2018) and Croce, Nguyen, Raymond, and Schmid (2019) for further evidence that measures of government debt predict positive excess returns in various asset classes.

that can rationalize our evidence. In the model, risk-sensitive agents with Epstein-Zin preferences invest in government bonds, corporate bonds as well as stocks to smooth their consumption. The government finances debt by levying taxes on wages and corporate income, while corporations issue defaultable bonds to finance investment according to their advantageous tax treatment, in line with the US tax code. Households are subject to sporadic liquidity shocks that create funding needs. While households can always liquidate their asset holdings to cover their funding needs, they can also trade their asset positions subject to transaction costs in the market place. In our model, different asset classes endogenously provide differential liquidity benefits to investors across time and states reflected in endogenously time-varying liquidity premia, liquidity risk premia and trading volumes.

Increasing the supply of government bonds facilitates covering liquidity needs in the market place and thus endogenously leads to a decline in liquidity premia on safe assets in the model, in line with the empirical evidence. However, issuing debt also raises the government's future funding needs through the government budget constraint. In particular, critically, higher debt does not only lead to higher average future tax obligations going forward, but it also renders them more *volatile*. Intuitively, the present value of future tax commitments becomes more sensitive to shocks as government debt grows[2]. A rising supply of safe assets, therefore, does not only facilitate transactions in the economy, but it also gives rise to policy uncertainty and therefore constitutes a source of fiscal risk. In the presence of elevated tax uncertainty, firms in our model exploit the tax advantage of debt financing more cautiously, which raises their overall costs of financing. This effect depresses corporate investment, so that there is not just 'financial crowding out' of private debt through government activity, but in our production economy, also 'real crowding out'. Ultimately, we show that in our model the dual role of government debt in terms of enhanced liquidity services and elevated policy uncertainty is reflected in rising consumption volatility.

Quantitatively, our dual mechanism of liquidity provision versus policy uncertainty provides a realistic account of the empirical evidence. It rationalizes liquidity premia declining with safe asset supply, and credit spreads rising with the latter, in line with the data. The model also endogenously generates time-varying risk premia in that the conditional volatility of the stochastic discount factor reflects the tax volatility that endogenously moves with the supply of government debt. The latter feature of the model also gives rise to a realistic description of credit spreads, in

[2]A similar mechanism is at work in, for example, Croce, Nguyen, and Schmid (2012) and Croce, Nguyen, Raymond, and Schmid (2019).

that a sizeable component of spreads is a credit risk premium, accounted for not just by expected losses in default, but by the observation that losses in default tend to occur in downturns, which bondholders require compensation for. Finally, our model with endogenous leverage also delivers a sizeable equity premium, while generating realistic macroeconomic risks.

1.1 Related Literature

Our work is related to and links several strands of literature. We build on the empirical observation, well-known e.g. from Longstaff (2003), Bansal, Coleman, and Lundblad (2012), Krishnamurthy and Vissing-Jorgensen (2011), Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2016), Du, Im, and Schreger (2018), Jiang, Krishnamurthy, and Lustig (2018) or Jiang, Krishnamurthy, and Lustig (2019), that U.S. Treasuries are arguably among the worlds safest and most liquid financial assets and investors attach a 'liquidity premium' or 'convenience yield' to holding these assets. We connect this stylized fact to the recent, and growing, evidence in Liu (2018) and Croce, Nguyen, Raymond, and Schmid (2019) that a rising supply of Treasuries, while lowering liquidity premia, significantly predicts rising excess returns on a variety of asset classes. While we present novel evidence in the context of credit, the papers cited provide further evidence across asset classes.

Our model embeds defaultable corporate debt along the lines of Gomes, Jermann, and Schmid (2016), Miao and Wang (2010), and Corhay (2015) into a general equilibrium asset pricing model with a rich fiscal sector, similar to Croce, Kung, Nguyen, and Schmid (2012) or Gomes, Michaelides, and Polkovnichenko (2013). We follow Croce, Kung, Nguyen, and Schmid (2012) in assuming that taxes have a negative long-run effect on productivity growth, so that effectively fiscal policy provides a source of 'long-run productivity risk' as specified in Croce (2014) and micro-founded in a setting with endogenous growth in Croce, Nguyen, and Schmid (2012), Kung and Schmid (2015), or Corhay, Kung, and Schmid (2017).

Our work also contributes to the literature on equilibrium models of corporate bond pricing, motivated by the observation, often referred to as the 'credit spread puzzle' that credit spreads tend to be high relative to the average losses bondholders have to expect in default. Our model gives a general equilibrium perspective on the recent literature that attributes a large component of credit spreads to a default risk premium compensating bondholders for incurring losses in high marginal utility episodes, as spearheaded by Chen, Collin Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010b), Bhamra, Kuehn, and Strebulaev (2010a), and Chen

(2010). While we abstract from significant cross-sectional heterogeneity as in Gomes and Schmid (2019), we emphasize the liquidity attributes of bonds similar to Chen, Cui, He, and Milbradt (2018) and He and Milbradt (2014). In that respect, our model of liquidity attributes builds on and generalizes the work of Amihud and Mendelson (1986) and, more recently, He and Xiong (2012). In particular, our work contributes to the literature on liquidity premia by integrating a model of differential liquidity attributes across assets into a quantitative general equilibrium asset pricing model.

More broadly, our work contributes to the literature on production-based asset pricing in general equilibrium models, along the lines of Jermann (1998), Kaltenbrunner and Lochstoer (2010), Kuehn (2007), or Kuehn, Petrosky-Nadeau, and Zhang (2014). Relative to that work, our model also implies that part of the resolution to the low risk-free rate puzzle embedded in the equity premium, may stem from the liquidity services or the convenience yields that safe assets provide, similar to Bansal and Coleman (1996).

2 Empirical Motivation

We start by collecting and documenting some stylized facts regarding the link between safe asset supply and liquidity and default premiums, respectively. Similar, and richer, results have been reported previously in the literature. Therefore, the objective of this section is to set the stage and provide some context on the empirical patterns our model is meant to capture.

Table 1 provides some first regression evidence. We focus on the GZ spread in Gilchrist and Zakrajŝek (2012) as the relevant corporate bond spread, and the spreads between general collateral repo rate (Repo) and treasury bill rate as a measure of the liquidity premium[3]. Panels A and B in the table document that government debt, as measured by the log debt-to-GDP ratio, is significantly positively related to default premia , while significantly negatively so to liquidity premia. This holds both in levels as well as in first differences. Moreover, the results get stronger when controliing for another well-known determinant of both liquidity and default premia, namely realized stock return volatility. In terms of predictive regressions, panel C documents that government debt predicts significantly higher expected bond excess going forward.

Together, these results provide suggestive evidence that increasing the supply of government

[3]See Liu (2018) for rich alternative specifications, and robustness

bonds does indeed lubricate the economy by lowering liquidity premiums, as suggested previously in the literature, but at the same time, raises default risk and default premia in the corporate sector.

We next elaborate on this link by providing some more formal econometric evidence on the dynamic relationship between government debt and yield spreads. We do by analyzing the impulse response functions in a vector autoregression framework. In particular, we estimate a seven-variable VAR of the following form

$$Z_t = \Phi Z_{t-1} + u_t$$

$$Z_t = [ffr_t,\ \Delta ip_t,\ r_t^{ex},\ by_t,\ GZ_t,\ GZp_t,\ repobill_t]$$

The VAR includes fed funds rate (ffr_t), industrial production growth (Δip_t), corporate bond excess return (r_t^{ex}), debt-to-GDP ratio (by_t), corporate bond spread (GZ_t) and premium (GZp_t) in Gilchrist and Zakrajšek (2012), and the spreads between general collateral repo rate and treasury bill rate ($repobill_t$). We use an identification strategy that recursively orders the variables as above. We identify the fourth shock as a non-discretionary increase of government debt. This shock increases the debt-to-GDP ratio but is orthogonal to feds fund rate, IP and corporate bond excess return contemporaneously. Therefore, the shock is not driven by conditions in the macroeconomy, monetary policy, treasury market, and corporate bond market. We estimate the impulse response of yield spreads to the debt shock. Figure 1 shows the impulse response of the spreads in corporate bond market where credit risks are important. The debt shock significantly increases GZ spreads and premiums in the corporate bond market while decreasing Repo/Bill spreads in the money market where credit risks are of second order. These results confirm that government debt has differential effects on different markets.

Figure 2 illustrates the dynamic relationship of liquidity and default premiums in our sample by plotting the demeaned corporate bond spread in Gilchrist and Zakrajŝek (2012) and the spreads between general collateral repo rate (Repo) and treasury bill rate. While naturally default premiums jump up during recessions (indicated by the shaded bars), when government debt tends to rise, liquidity premiums tend to fall. This pattern is especially pronounced in more recent recessions, such as the great recession following the financial crisis. In the recessions at the beginning of our sample, the pattern is somewhat weaker, but nevertheless, there tends to be downward pressure on liquidity premiums in the earlier stages of the downturns.

These observations prompt us to develop a formal model to examine, and to quantify, the role of government debt supply, for liquidity and default premiums, and the macroeconomy.

3 Model

We develop a general equilibrium asset pricing model with endogenous liquidity and default premiums. There is a consumer sector with risk-sensitive households, a production sector in which firms finance investment with equity and defaultable bonds, and a government which finances expenditures by levying corporate taxes and issuing bonds. Households face stochastic liquidity needs which they can cover by selling off financial assets, subject to transaction costs. We start by describing the household, production, and government sectors, and then detail the pricing of financial assets.

3.1 Households

The economy is populated by a continuum of households of measure one. Households have Epstein-Zin recursive preferences preferences defined over a composite of aggregate consumption, C_t, and labor, L_t, defined as $\tilde{C}_t = C_t^{\vartheta}(1-L_t)^{1-\vartheta}$, so that

$$U_t = [(1-\delta)\tilde{C}_t^{\frac{1-\gamma}{\theta}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}},$$

where δ is the time discount factor, γ is the relative risk aversion, ψ denotes the intertemporal elasticity of substitution (IES), and $\theta \equiv \frac{1-\gamma}{1-/\psi}$. We assume that $\psi > \frac{1}{\gamma}$, so that the agent has a preference for early resolution of uncertainty following the long-run risks literature.

Households maximize utility by supplying labor and by participating in financial markets. Specifically, the household can take positions in the stock market, S_t, in corporate bonds, B_t, and in government bond markets, B_t^g. Let the values of stocks, corporate bonds, and government bonds be denoted $V_{e,t} = P_t^e S_t, V_{c,t} = Q_t B_{t+1}, V_{g,t} = Q_t^g B_{t+1}^g$ respectively. Here, P_t denotes the price per share of equity, Q_t is the price of a corporate bond, and Q_t^g is the price of a government bond. These prices will be determined endogenously below. Participating in financial markets exposes households to liquidity needs in the magnitude ξ_{t^+} with probability λ_t, covering which involves trading in financial assets that is associated with costs $\lambda_t \nu_t(V_{g,t}, V_{c,t}, \xi_{t^+})$ that we endogenize below.

Moreover, wage bills are subject to income taxes $\tau_{l,t}$. Accordingly, households' budget constraint becomes

$$
\begin{aligned}
&C_t + V_{g,t} + V_{c,t} + V_{e,t} + \lambda_t \nu_t(V_{g,t}, V_{c,t}, \xi_{t^+}) \\
=&V_{g,t-1}R_{g,t} + V_{c,t-1}R_{c,t} + V_{e,t-1}R_{e,t} + w_t L_t(1-\tau_{l,t}),
\end{aligned}
$$

so that the stochastic discount factor is given, in a standard manner, by

$$
M_{t+1} = \delta(\frac{C_{t+1}}{C_t})^{-1}(\frac{\tilde{C}_{t+1}}{\tilde{C}_t})^{1-1/\psi}(\frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}]})^{1-1/\theta}.
$$

3.1.1 Endogenous Liquidity

A critical feature of our model is that all financial assets endogenously exhibit different liquidity attributes, and thus, liquidity premiums. We now describe how we embed a model of endogenous liquidity in our equilibrium asset pricing model. Our market structure is similar to Amihud and Mendelson (1986) and He and Xiong (2012). The key innovation is that our agents can choose between several different assets to sell when they are hit by a liquidity shock, while agents are forced to sell one asset in the early literature. This feature generates interdependence of liquidity across different markets. To that end, we assume that every period t contains an intra-period t^+ in which agents in each household serve distinct roles as workers, firm managers, asset managers, and intermediaries, respectively.

Timeline We start by detailing the timeline.

- Time t.

Household take their asset allocation decisions. Holdings of government bonds and corporate bonds are V_g, and V_c, respectively.

- Time t^+. The intra-period.

Within each household, each asset manager is hit by a liquidity shock with probability λ_t with size ξ_{t^+}. We assume that liquidity shocks follow a log-normal distribution, so that $\xi \sim \log N(\mu_\xi,\ \sigma_\xi^2)$. While, assuming so, we are implicitly effectively attributing all corporate bond trading to liquidity trades, we can, realistically perhaps, interpret the liquidity shocks in our model as capturing funding shocks, and portfolio rebalancing needs, shocks to individual beliefs, or idiosyncratic preference shocks, more broadly. Liquidity shocks bring about liquidity needs, which asset managers can choose to cover either by selling off an amount of ξ_{t^+} of total assets to the competitive intermediaries, or by liquidiating subject to liquidation costs φ_l. Liquidated assets are returned in form of cash to workers who deposit the proceeds with intermediaries. The intermediaries buy these assets using deposits. The intermediaries, thus, essentially provide a technology of liquidity transformation.

To liquidate the assets, asset managers have to sell them off at a price lower than the equilibrium price at time t and incur liquidation costs φ. We assume that government bond and corporate bond come with different transaction costs in that $\varphi_g < \varphi_c$. Intermediaries are competitive and use households' stochastic discount factor to value assets, so that $Q_{t^+} = Q_t$. They bid at $Q_{t^+}(1-\varphi)$, so that they make the profits $Q_{t^+}\varphi$, which they return back to the household.

- Time $t+1$.

Workers, firm managers, asset managers, and intermediaries all convene back at the household and make consumption decisions. We have perfect consumption risk sharing in the households.

Household's Liquidation Problem When hit by a liquidity shock, households need to decide how to optimally cover their liquidity needs by either selling off government bond holdings of size u_g and corporate bond holdings of size u_c, or to liquidate some of their positions. We assume that households choose u_g and u_c to minimize liquidation costs. This problem is static. More formally, therefore, their liquidation choices satisfy

$$\min_{u_g, u_c} \varphi_l \max[\xi - (u_g + u_c), 0] + \varphi_g u_g + \varphi_c u_c$$

In our setup, households' liquidation problem has a straightforward solution. In particular, because the liquidation cost exceeds the transaction cost in that $\varphi_g < \varphi_c < \varphi_l$, the solution follows a pecking order:

$$\begin{cases} u_g = \xi & \xi < V_g \\ u_g = V_g,\ u_c = \xi - V_g & V_g < \xi < V_g + V_c \\ u_g = V_g,\ u_c = V_c & \xi > V_g + V_c \end{cases}$$

In other words households find it optimal to first sell off government bonds, then cover the remaining liquidity needs by selling corporate bonds, and only liquidate assets in case liquidity needs exceed joint government and corporate bond holdings.

Liquidity Premium In our model, liquidity premiums will arise endogenously from the marginal savings of liquidation costs given some government and corporate bonds holdings. We can determine expected liquidation costs by forming expectations over ξ. In other words, formally, we have

$$\begin{aligned}\nu(V_g, V_c, \xi) = &\int_0^{V_g} \varphi_g \xi d\Phi_\xi + \int_{V_g}^{V_g+V_c} \left[\varphi_g V_g + \varphi_c(\xi - V_g)\right] d\Phi_\xi \\ &+ \int_{V_g+V_c}^{\infty} \left[\varphi_g V_g + \varphi_c V_c + \varphi_l(\xi - V_g - V_c)\right] d\Phi_\xi\end{aligned}$$

Given our assumption that ξ has a continuous cumulative distribution function Φ_ξ, it follows that $\nu(V_g, V_c, \xi)$ is differentiable, so that we can determine the marginal benefits of a government bond as

$$\frac{\partial}{\partial V_g}\nu(V_g, V_c, \xi) = -(\varphi_c - \varphi_g)\int_{V_g}^{V_g+V_c} d\Phi_\xi - (\varphi_l - \varphi_g)\int_{V_g+V_c}^{\infty} d\Phi_\xi$$

Accordingly, the benefits of having an additional unit of government bond stem from saving liquidation costs if households either sell corporate bonds (first term) or sell everything (second term). Similarly, the marginal benefits of corporate bond holdings are

$$\frac{\partial}{\partial V_c}\nu(V_g, V_c, \xi) = -(\varphi_l - \varphi_c)\int_{V_g+V_c}^{\infty} d\Phi_\xi$$

so that the benefits of holding an additional corporate bond stem from saving liquidation costs if households have to sell everything. Overall, the liquidation costs that emerge endogenously in our model share many properties with common reduced-form specifications of transaction costs, in that, formally, we have that that $\nu > 0, \nu' < 0, \lim \nu' \to 0$, and $\nu'' > 0$. Moreover, a number of important economic properties of our liquidation costs are straightforward to establish. To begin, increasing the supply of government bonds decreases the liquidity benefits, in that it renders government bonds less useful assets to buffer liquidity shocks, because

$$\frac{\partial^2}{\partial V_g^2}\nu(V_g, V_c, \xi) = -(\varphi_c - \varphi_g)[\phi_\xi(V_g + V_c) - \phi_\xi(V_g)] + (\varphi_l - \varphi_g)\phi_\xi(V_g + V_c)$$
$$= (\varphi_l - \varphi_c - (\varphi_c - \varphi_g))\phi_\xi(V_g + V_c) + (\varphi_c - \varphi_g)\phi_\xi(V_g)$$

a higher government bond supply reduces the its benefit in buffering liquidity shocks when households sell corporate bonds (first term) and everything (second term). Formally then, because $\varphi_l \gg \varphi_c$, we have that $\varphi_l - \varphi_c - (\varphi_c - \varphi_g) > 0$. Thus, $\frac{\partial^2}{\partial V_g^2}\nu(V_g, V_c, \xi) > 0$, in our setting.

Similarly, our specification implies that

$$\frac{\partial^2}{\partial V_c \partial V_g}\nu(V_g, V_c, \xi)$$
$$= (\varphi_l - \varphi_c)\phi_\xi(V_g + V_c)$$

a higher government bond supply reduces the relevance of corporate bond holdings in buffering liquidity shocks, and, finally, because

$$\frac{\partial^2}{\partial V_g \partial V_c}\nu(V_g, V_c, \xi)$$
$$= (\varphi_l - \varphi_c - (\varphi_c - \varphi_g))\phi_\xi(V_g + V_c),$$

higher corporate bond holdings render government bonds less attractive securities to buffer liquidity shocks.

Trading Volume Our model also has implications for the endogenous trading volumes of government and corporate bonds. In particular, the expected trading volume of governments is straightforward to determine as

$$E_t^\lambda(u_{g,t^+}) = \int_0^{V_g} \xi d\Phi_\xi + V_g \int_{V_g}^\infty \phi(\xi) d\Phi_\xi,$$

while we find that the expected trading volume of corporate binds satisfies

$$E_t^\lambda(u_{c,t^+}) = \int_{V_g}^{V_g+V_c} (\xi - V_g) d\Phi_\xi + V_c \int_{V_g+V_c}^\infty \phi(\xi) d\Phi_\xi.$$

3.2 Firms

There is a continuum of ex ante identical firms. Firms invest, hire labor, and produce according to a constant returns to scale technology. Given advantageous tax treatment in line with the US tax code, firms issue debt as well as equity to finance expenditures. Ex post, firms are subject to an iid cash flow shock, which may be potentially large, and can lead firms to declare bankruptcy. The trade-off between tax advantages and default costs determines firms capital structure decisions.

3.2.1 Production

Firms use capital, K_t and labor, L_t, to produce according to the constant returns to scale production technology

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha},$$

where A_t is a stochastic productivity process, whose evolution is given as

$$\Delta a_{t+1} = \mu + x_t - \phi_\tau(\tau_t - \tau_{ss}) + \sigma_a \eta_{a,t+1}.$$

Here x_t is persistent long-run productivity, with $x_{t+1} = \rho_x x_t + \sigma_x \eta_{x,t+1}$ and τ_t is the prevailing tax rate, which will be pinned down endogenously below through the government's budget constraint. This specification captures the long-run effects of elevated taxation on economic growth in a parsimonious and tractable way. While the notion that rising tax rates exert a negative effect on productivity growth is consistent with the empirical evidence, such as that documented in Jaimovich and Rebelo (2017), we specify that link directly here. However, as shown for example in Croce, Nguyen, and Schmid (2012), it is easily endogenized in the context of a model with endogenous growth.

After solving the static labor choice problem, we can define firms' profit function in a straightforward manner as follows

$$\Pi(K_t) = \alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha}.$$

To introduce firm heterogeneity in a meaningful and tractable manner, we assume that firms are subject to additive idiosyncratic shocks on their cash flow $-(1-\tau_t) z_{i,t} q_{k,t} K_t$. The shock is scaled by capital price and capital. The scaling is important for aggregation. We assume that these shocks are i.i.d. across firms and time, and follow a normal distribution, so $z_{i,t} \sim N(0, \sigma^2_{z,t})$. Moreover, we specify idioyncratic volatility as countercyclical in that $\sigma_{z,t} = \sigma_{z,0} \exp(-\phi_{\sigma,a}(\Delta a_t - \mu))$.

We think of these as direct shocks to firms operating income and not necessarily output. They summarize the overall firm specific component of their business risk. Although they average to zero in the cross section, they can potentially be very large for any individual firm.

3.2.2 Investment and Financing

We assume that capital adjustment is costly in that investment is subject to convex adjustment cost. Firm level capital accumulation is thus given by

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t$$

where Φ denotes the adjustment cost function.

Given the advantageous tax treatment of debt in the tax code, firms fund investment by issuing both equity and defaultable debt. For tractability, we assume that debt comes in form of one-period securities and refer to the stock of outstanding defaultable debt at the beginning of period as B_t. In addition to the principal, the firm is also required to pay a coupon C per unit of outstanding debt. Let Q_t denote the price of a new bond issue that comes due at time $t+1$. We will determine the bond pricing function endogenously below.

With this notation at hand, we can, taking into account investment expenses and net debt outlays, write firms' equity distributions as

$$D_{i,t} = (1-\tau_t)(\Pi(K_t) - z_{i,t}q_{k,t}K_t) - I_{i,t} + Q_t B_{t+1} - (1 + (1-\tau_t)C)B_t.$$

The last term reflects the fact that interest payments are tax deductible, in line with the tax code.

3.2.3 Firms' Problem

Firms' objective is to maximize equity value, that is, $V_{i,t}(K_t, B_t, z_{i,t}; S_t)$. The individual state variables are capital K_t, bond B_t, and idiosyncratic shock $z_{i,t}$. We denote the aggregate state variables as S_t, which contains the long-run productivity x_t and fiscal policies specified below. If a firm does not default, it invests, issues new debt, and pays dividends. We can therefore write the equity value function as

$$V_{i,t}(K_t, B_t, z_{i,t}; S_t) = \max_{I_{i,t}, K_{t+1}, B_{t+1}} D_{i,t} + E_t[M_{t+1} \int^{z^*_{t+1}} V_{i,t+1}(K_{t+1}, B_{t+1}, z_{i,t+1}; S_{t+1}) dF]$$

The truncation of the integral reflects the possibility of default: a sufficiently severe cash flow shock implies an equity value of zero. In this case, equity holders are unwilling to inject further capital in the firm, and are better off defaulting. In our setup, default occurs whenever cash flow shocks exceed an endogenous cutoff level of z^*_t, which is implicitly defined by the condition $V_{i,t}(K_t, B_t, z^*_t; S_t) = 0$.

We note that given our assumption of iid cash flow shocks, outside default, all firms make identical investment and financing decisions.

Optimality Conditions Denoting the capital price by $q_{k,t}$, corporate policies satisfy the Euler conditions

$$q_{k,t} \Phi'_t(\frac{I_t}{K_t}) = 1,$$

and

$$q_{k,t} = \frac{\partial Q_t}{\partial K_{t+1}} B_{t+1} + E_t \left[M_{t+1} \int^{z^*_{t+1}} \frac{\partial V_{i,t+1}}{\partial K_{t+1}} dF \right],$$

so that at the optimum, the cost of investment is offset by the increase of the bond price $\frac{\partial Q_t}{\partial K_{t+1}} B_{t+1}$ and the increase in future equity value $E_t \left[M_{t+1} \int^{z^*_{t+1}} \frac{\partial V_{i,t+1}}{\partial K_{t+1}} dF \right]$). Similarly, we have

$$\frac{\partial Q_t}{\partial B_{t+1}} B_{t+1} + Q_t + E_t \left[M_{t+1} \int^{z^*_{t+1}} \frac{\partial V_{i,t+1}}{\partial B_{t+1}} dF \right] = 0,$$

so that the fall in bond prices $\frac{\partial Q_t}{\partial B_{t+1}} B_{t+1}$ and future equity values $E_t \left[M_{t+1} \int^{z^*_{t+1}} \frac{\partial V_{i,t+1}}{\partial B_{t+1}} dF \right]$) is offset by the increasing debt financing in the magnitude of Q_t.

Defining the capital return to be $R_{k,t} = \frac{1}{q_{k,t-1}} \left[(1-\tau_t)\Pi_{K,t} + q_{k,t}((1-\delta) - \Phi'_t \frac{I_t}{K_t} + \Phi_t) \right]$, we can write the envelope conditions compactly as

$$\frac{\partial V_{i,t+1}}{\partial K_{t+1}} = q_{k,t-1} R_{k,t} - (1-\tau_t) z_{i,t} q_{k,t}, \qquad \text{and} \qquad \frac{\partial V_{i,t+1}}{\partial B_{t+1}} = -(1 + (1-\tau_t)C).$$

The default boundary z_t^* satisfies $V_{i,t}(K_t B_t, z_t^*; S_t) = 0$, so that

$$V_{i,t}(K_t B_t, z_t^*; S_t) = q_{k,t-1} R_{k,t} K_t - q_{k,t} K_{t+1} - (1-\tau_t) z_{i,t} K_t + Q_t B_{t+1} - (1 + (1-\tau_t) C) B_t + V_t^{ex} = 0,$$

and we can solve for

$$z_t^* = \frac{q_{k,t-1} R_{k,t} K_t - q_{k,t} K_{t+1} + Q_t B_{t+1} - (1 + (1-\tau_t) C) B_t + V_t^{ex}}{(1-\tau_t) q_{k,t} K_t}.$$

3.3 Government

We assume that the government faces an exogenous and stochastic expenditure stream that evolves as follows

$$\frac{G_t}{Y_t} = \mu_g + \rho_g \frac{G_{t-1}}{Y_t} + \sigma_g \varepsilon_{b,t}.$$

Moreover, the government also faces an exogenous and stochastic stream of transfers that we specify as follows

$$\frac{TR_t}{A_t} = \mu_{tr} + \sigma_b \varepsilon_{b,t}.$$

Both spending and transfers are required outlays that the government needs to finance by issuing debt or raising taxes, at an endogenous, and possibly, time-varying tax rate τ_t. Spending and transfers exhibit some relevant economic differences.[4] Spending affects the resource constraint so that it raises aggregate demand and has to be met by a higher supply of goods in equilibrium. However, transfers *within* representative households do not affect aggregate demand directly, so that these shocks purely affect government outlays.

The government issues one-period zero-coupon bonds with price Q_t^g. We assume that the government conducts fiscal policy by sticking to a debt rule. In particular, we assume that the market value $Q_t^g B_{t+1}^g$ (detrended by Y_t) follows the law of motion

$$\frac{Q_t^g B_{t+1}^g}{Y_t} = \mu_b + \rho_b \frac{B_t^g}{Y_t} + \kappa_\tau (\sigma_g + \sigma_b) \varepsilon_{b,t}.$$

Here, κ_τ captures the tax smoothing policy in that a part of the spending and transfer shock $\varepsilon_{b,t}$ is financed by issuing debt. The rest will be financed by taxes, implied by the government

[4] We assume that spending and transfers are driven by the same shock $\varepsilon_{b,t}$. Otherwise, we need to introduce another state variable to an already large model.

budget constraint. In particular, the government is subject to a standard budget constraint of the form

$$Q_t^g B_{t+1}^g = B_t^g + G_t + TR_t - T_t.$$

Given our specification of the spending, transfer and debt issuance policies, the tax receipts are endogenously determined by the government budget constraint. Formally, we have

$$\frac{T_t}{Y_t} = \mu_g + \mu_{tr} - \mu_b + (1-\rho_b)\frac{Y_{t-1}}{Y_t}\frac{B_t^g}{Y_{t-1}} + \rho_g \frac{G_{t-1}}{Y_t} + (1-\kappa_\tau)\kappa_\tau(\sigma_g + \sigma_b)\varepsilon_{b,t}.$$

The tax base is the sum of capital and labor income subtracting the corporate tax-deductible interest payments, so that $T_t = \tau_t(\Pi(K_t) + w_t L_t - CB_t)$. Given the tax receipts and the tax base, we can compute the corresponding equilibrium tax rate.

Although the tax rate is endogenous and depends on the state of the economy and other policy choices, it follows some intuitive dynamics. First, tax rate increases with the spending and transfer shocks, though the increases are not one-for-one. Second, tax rate increases with the existing government debt as a form of fiscal consolidation. Indeed, from the term, $(1-\rho_b)\frac{Y_{t-1}}{Y_t}\frac{B_t^g}{Y_{t-1}}$, high debt implies high tax in that

$$(1-\rho_b)\frac{Y_{t-1}}{Y_t} > 0.$$

Third, the volatility of tax rate also increases with the existing government debt, in that $(1-\rho_b)^2(\frac{B_t^g}{Y_{t-1}})^2 Var_t(\frac{Y_{t-1}}{Y_t})$. Given the same tax base, a large stock of debt amplifies the shocks so that tax rates have to be more responsive. We will illustrate these dynamics in the numerical solution.

3.4 Equilibrium and Asset Prices

To complete the model, we require the goods markets to clear. We assume that the liquidation costs effectively are the profits of the intermediaries. On the other hand, losses in default are absorbed as profits of the law firms. These profits are also part of output. Given these assumptions, the aggregate resource constraint takes the standard form

$$Y_t = C_t + I_t + G_t.$$

This specification embeds the extreme assumption that government spending is effectively entirely waste.

A critical feature of our model is the interplay between securities liquidity benefits and their default risk. We now turn to a detailed examination of the endogenous linkages that emerge in our setup.

Government bonds We can use households' optimality conditions to determine their valuations of a government bond, and find that its price Q_t^g satisfies

$$Q_t^g(1 + \lambda_t \nu_{g,t}) = E_t\left[M_{t+1}\right],$$

where $\nu_{g,t} \equiv \frac{\partial}{\partial V_g}\nu(V_g, V_c, \xi)$ denotes the marginal value of government bonds' liquidity services. The expression shows that households do not only value government bonds because of their future payments, but also because they are valuable in covering households' liquidity needs in case they are hit by a liquidity shock of size λ_t.

Corporate bonds Corporate bond values Q_t depend on default probabilities and costs of default, as well as on the liquidity benefits they provide to households. Regarding default costs ζ_t, we assume that they are countercyclical in line with the evidence in Chen (2010). Indeed, we specify

$$\zeta_t = \zeta_0 \exp(-\phi_{\zeta,a}(\Delta a_t - \mu)).$$

Accordingly, corporate bond prices satisfy

$$Q_t B_{t+1}(1 + \lambda_t \nu_{c,t}) = E_t\left[M_{t+1}\left(\int^{z_{t+1}^*}(1 + C)B_{t+1}dF + (1 - \zeta_t)\int_{z_{t+1}^*}(V_{i,t+1} + (1 + C)B_{t+1})dF\right)\right],$$

where $\nu_{c,t} \equiv \frac{\partial}{\partial V_c}\nu(V_g, V_c, \xi)$ denotes the marginal liquidity services that corporate bonds offer to households. The first term on the right hand side denotes debt service outside default, while the second term shows that bondholders recover firm value net of default costs ζ_t after a sufficiently adverse cash flow shock.

More compactly, we can thus write the corporate bond pricing equation as

$$Q_t(1 + \lambda_t \nu_{c,t}) = E_t\left[M_{t+1}\left((1 + C)F(z_{t+1}^*) + Q_t R_{rec,t+1}\right)\right],$$

where $R_{rec,t+1}$ denotes the recovery value.

$$R_{rec,t+1} = (1-\zeta_t)\frac{\int_{z^*_{t+1}}(V_{i,t+1} + (1+C)B_{t+1})dF}{Q_t B_{t+1}}$$

Corporate yield spread To determine credit spreads, we first note that coporate bond yields can be computed as

$$\frac{1+C}{Q_t} = \frac{1+\lambda_t \nu_{c,t} - E_t[M_{t+1}R_{rec,t+1}]}{E_t[M_{t+1}F(z^*_{t+1})]}$$

so that comparing with the yield on a government bond yield with the same coupon, that is $\frac{1+\lambda_t \nu_{g,t}}{E_t[M_{t+1}]}$ gives

$$y^c_t - y^g_t = E_t[M_{t+1}]E_t[\frac{1-F(z^*_{t+1})}{E_t[M_{t+1}]} - R_{rec,t+1}] + Cov_t[M_{t+1}, \frac{1-F(z^*_{t+1})}{E_t[M_{t+1}]} - R_{rec,t+1}] + \lambda_t \nu_{c,t} - \lambda_t \nu_{g,t}$$

The first term captures expected losses in default, while the second term is a credit risk premium in that it captures to what extent losses arise in high marginal utility states. Finally, the last term captures the differential liquidity services that government, and corporate bonds, respectively, provide to households. This liquidity spread increases with the probability of liquidity shocks λ_t and the liquidity advantage of government bond over corporate bond, measured by the differential of the marginal values of liquidity services $\nu_{c,t} - \nu_{g,t}$.

4 Quantitative Analysis

Most of our quantitative analysis is based on model simulations. We use a global approximation technique to solve for the model policy functions. We describe our numerical approach in the next section, along with our parameter choices.

4.1 Computation and Calibration

The possibility of default induces strong nonlinearities in payoffs, discount factor, and policies. Therefore, we use a global, nonlinear solution method. More specifically, we solve the model globally using a collocation approach. Since we face multiple state variables and the curse of dimensionality, we use Smolyak polynomials as the basis functions to approximate the policy functions.

Briefly, we approximate N_c control variables as functions of N_s state variables using N_p Smolyak polynomials and $N_p \times N_c$ coefficients β. There are $N_s = 6$ state variables $K_t,\ x_t,\ B_t,\ B_t^g,\ \varepsilon_{b,t},\ G_t/Y_t$, $N_c = 5$ control variables $L_t,\ U_t,\ B_{t+1},\ Q_t,\ Q_t^G$, and $N_p = 85$ Smolyak polynomials. We choose the approximation level of the Smolyak method to be 2 so that the highest order polynomial of each state variable is 5. We compute the system of N_c equilibrium conditions over a grid of $N_p \times N_c$. We solve the system of equations and obtain β. This process involves projecting state variable one period forward, computing the implied approximation errors, and minimizing these with respect to β. We then simulate the model and compute the approximation arrors in the state space, and repeat the process until convergence. A more detailed description of the algorithm is provided in the appendix.

The model is calibrated at quarterly frequency. We report our parameter choices in table 2. Regarding preference and technology parameters, such as risk aversion γ, intertemporal elasticity of substitution ψ, time discount β, leisure parameter ϑ, capital share α, and depreciation δ, we pick standard values in line with the literature. Our parameter choices for preferences imply that households have a preference for early resolution of uncertainty, so that they are concerned about shocks to long-run growth prospects. The adjustment cost function has the form $\Phi(\frac{I}{K}) = [\frac{a_1}{1-1/\xi_k}(\frac{I}{K})^{1-1/\xi_k} + a_2]$, where the coefficients a_1 and a_2 are chosen such that $\Phi(\frac{I}{K}) = 0$ and $\Phi'(\frac{I}{K}) = 0$ at the steady state. The coupon rate on corporate bonds is set at 1.5%.

The quarterly growth rate of productivity μ is 0.45%. The volatility of the productivity shock σ_a is set to match the volatility of consumption growth. The long run productivity has a persistence ρ_a of 0.965 and its shock volatility σ_x is 5% of the short run shock volatility. We pick the idiosyncratic shock volatility $\sigma_{z,0}$ to match the average default rate. The default loss ζ is set at 0.3 of the total asset value. Both idiosyncratic volatility and default losses exhibit mild countercyclicality governed by the parameters $\phi_{\sigma,a}$ and $\phi_{\zeta,a}$, consistent with the evidence in Chen (2010) for example.

The long run productivity growth effects of taxation ϕ_τ are set at 0.05, consistent with the estimates in Croce, Kung, Nguyen, and Schmid (2012). This parameter choice captures the notion that raising taxes has detrimental effects on productivity growth in the long run. The processes for the government debt and the spending are chosen to match their data counterparts. The parameter κ_τ determines the degree of tax smoothing and is set to match the tax rate persistence and volatility.

We calibrate the liquidation cost φ_g and φ_c to match bid ask spreads of government and corporate bonds. The liquidity shock probability λ is calibrated to the the absolute deviation of the money market mutual fund flow relative to the fund size. This data moment, estimated to be 0.12, captures that around 12 pecent of the momey market mutual fund flows in and out on a quarterly basis. The distribution of the liquidity shock determines the turnover of government and corporate bonds. We discipline μ_ξ and σ_ξ by matching the relative turnover of treasury and corporate bonds, and the liquidity premium on treasury bills, measured by the Repo/Bill spread.

4.2 Quantitative Results

We start by assessing the overall quantitative relevance of our model for liquidity and credit spreads by inspecting a wide range of credit market statistics, along with macroeconomic moments. We then illustrate the basic intuition and discuss the economic mechanisms more succinctly by evaluating the relevant equilibrium policies.

4.2.1 Moments

Table 3 reports basic moments from model simulations regarding some of the main building blocks of our model. To illustrate the quantitative relevance of our liquidity model, we report statistics obtained from a model specification in which we abstract from liquidity considerations alongside, labeled 'Default Only'. Panel A shows that our calibrated model is consistent with relevant aspects of the dynamics of fiscal variables. While overall government debt dynamics are targeted through our specification of the fiscal rule, the levels and dynamics of taxes are endogenously determined through the government's budget constraint. In particular, levels, volatilities, and persistence of taxes are matched quite well in our model. A quantitatively relevant account of tax dynamics is critical in our context, as taxes emerge as an endogenous source of long-run productivity risk in our model, priced in equity and credit markets.

Panel B reports statistics regarding default risk in corporate credit markets. While at around forty percent, recovery rates in default on corporate bonds are slightly low relative to the roughly forty to fifty percent in the data, default rates in the model are low, and very close to their empirical counterparts. In spite of this low default risk, leverage ratios are well matched in the model, at around forty percent. This joint observation is often labeled as the 'low leverage puzzle' in the empirical literature,referring to the question why in the presence of significant tax advantages of debt, and low default probabilities, firms use leverage rather moderately. Our explanation in this model is related to the one in the recent literature on the 'credit spread puzzle' literature (see, for example, Chen, Collin Dufresne, and Goldstein (2009), Chen (2010), or Bhamra, Kuehn, and Strebulaev (2010b)), which observes that credit spreads are relatively high in spite of low expected losses in default, although with a twist. In our risk-sensitive model, in which households have recursive preferences and are subject to long-run productivity risks, defaults tend to cluster in downturns, so that bondholders incur losses precisely when their marginal valuations are highest. Through that mechanism, credit spreads in the model are realistically matched at around a hundred basis points, because a substantial fraction of spreads, namely about twenty percent, is made up by a default premium that investors require as compensation for countercyclical losses. In contrast to the credit spread puzzle literature, this default premium here emerges in a fully fledged general equilibrium production economy. As panel B shows, moreover, the component of credit spreads that compensates investors for average losses is about fifty percent. Our model also gives rise to a novel twist in determining credit spreads, in that corporate bonds provide less valuable liquidity services to investors than do government bonds. This differential liquidity benefit is priced into corporate bonds and contributes a quantitatively significant amount to spreads. Indeed, in our calibrated setting it makes up for about one fourth of the overall credit spread.

In panel C, we turn to a more detailed investigation of the quantitative implications of the model for liquidity premia, and the dual role of safe asset supply for liquidity and risk premia more specifically. We first document a liquidity premium on government bonds of about 0.3, in line with the empirical estimates obtained in the recent literature. This number suggests, therefore, that yields on traded government bonds are significantly below to the equilibrium risk free rate due to the liquidity services they provide. The table also shows that corporate bonds also enjoy some liquidity benefits in spite of their inherent default risk. That liquidity premium, however, is, at 0.06, substantially smaller than the one in government bonds. Intuitively, in the context of the model, they trade with higher transaction costs in the market for liquidity services. While our model falls short of matching the turnover ratios in both government and corporate bond markets, it captures their relative magnitudes. In particular, it is quantitatively consistent with the intuitive observation that turnover in Treasury markets is substantially higher than in corporate bond markets.

Next, we evaluate the potential of our model to capture some of the stylized facts about the differential effects of safe asset supply on liquidity and default spreads, respectively. As demonstraded in section 2, a rising government debt supply lowers liquidity premia, but does raise default premia on corporate bonds at the same time. Our model is consistent with that observation. In particular, as reported in panel C, the regression coefficients on default spreads and liquidity spreads on the debt to GDP ratio in simulated data have positive and negative signs, respectively. Moreover, the magnitudes are roughly in range of the empirical counterparts. In the next section, we discuss the economic mechanism underlying this quantitative result in more detail.

In panel D, we provide some quantitative evidence on the magnitude of 'crowding out' of corporate debt through the issuance of government debt. Indeed, in the data the regression coefficient of the aggregate market value of corporate debt on the government debt-to-GDP ratio is negative, suggesting that government debt crowds out some of public debt market activity. A similar pattern obtains in the model. Quantitatively, the model overstates the magnitude somewhat, but it is broadly in line with the data.

Beyond credit market statistics, our model is quantitatively broadly consistent with a wide range of stylized facts about aggregate fluctuations and stock returns, as table 4 shows. In particular, in spite of a realistically moderate amount of aggregate consumption risk, our model produces a significant equity premium of about five percent annually, and annual return volatility of close to ten percent, thus giving rise to a realistic Sharpe ratio in the range of 0.5. While this is to a large extent due to the exogenous and tax-based endogenous movements in long-run productivity risk, it obtains in spite of realistically low corporate leverage.

4.2.2 Equilibrium Policies

We now illustrate and examine the basic model mechanisms by means of the equilibrium policy functions in the following figures. Figure 3 illustrates the policy functions of various key macroeconomic variables with respect to government debt, holding the other state variables fixed. Not surprisingly, overall tax pressure endogenously rises when government debt is increasing, as the top left panel shows. Critically, moreover, not only does our model predict rising tax levels, but also higher tax volatility. This is because given the same tax base, a large stock of debt amplifies the shocks so that tax rates have to be more responsive. While we have presented this result analytically from the government budget constraint, the top middle panel demonstrates that the implied tax volatility effects are quantitatively significant.

The remaining panels of figure 3 show that rising tax risk spills over into the remaining macroeconomic variables. The top right panel shows that it comes with higher consumption risk. Such higher consumption risk is reflected in increasing volatility of the stochastic discount factor, which is further amplified through the endogenous effects of tax dynamics on productivity growth. Indeed, as a source of endogenous long-run productivity risks, tax volatility amplifies these low frequency risks that are priced in the presence of Epstein-Zin preferences. Overall, therefore, the market price of risk is therefore endogenusly increasing with government debt in our model. Lastly, the bottom right panel shows that higher tax volatility is also reflected in falling aggregate investment rates. While higher aggregate risk is naturally reflected in low investment given a precautionary motive, in our model, this effect is amplified given firms' cost of debt financing, as we show next.

Figure 4 illustrates various credit market variables with respect to movements in government debt. In line with the stylized empirical evidence presented earlier, default spreads, and default premia, rise with government debt, while liquidity premia fall. While it is intuitive that given a higher supply of government bonds, liquidity shocks are more easily absorbed leading to falling liquidity premia, the effects on default are related to the endogenous tax dynamics illustrated in figure 3. Rising tax volatility creates not only higher aggregate risk leading to higher default probabilities and therefore default spreads, but also to higher long-run productivity risk. This is because in our model taxes are an endogenous source of long-run productivity risk. As illustrated above, this additional source of priced risk leads to higher risk premia in our setup. The required compensation for the additional default risk therefore goes up, and leads to a higher default risk premium, as shown in the top middle panel.

A higher supply of safe assets has further implications for government and corporate bond markets. First, as shown in the bottom middle panel, government bond turnover falls, as liquidity shocks are more easily absorbed. Second, as documented in the bottom right panel, corporate leverage falls. In this sense, rising government debt indeed crowds out corporate debt. Importantly, in our setup with financial frictions, this decline in corporate leverage has real effects. Rising costs of debt financing increase firms' overall cost of capital, leading to a decline in corporate investment, as illustrated above. In this sense, our model also predicts a sort of 'real crowding out'.

Figure 5 shows that the equilibrium policies give rise to empirically plausible impulse response functions, in line with those obtained in the data and documented earlier in figure 1. We show the responses of expected corporate bond excess returns, credit spreads, the default risk premium, as well as the Treasury liquidity premium, to a one standard deviation government debt shock ε_b. Qualitatively, the model reproduces the evidence quite well in that expected excess returns, credit spreads, and the default risk premium rise, while liquidity premia decline with an increasing supply of safe assets.

4.2.3 Time-Varying Liquidity and Liquidity Crises

Thus far, in the model, liquidity premia on both government and corporate bonds were time-varying as they depended on households' asset positions, that were in turn dependent on the state of the economy. On the other hand, probabilities and distributions of liquidity shocks were assumed constant. Plausibly and realistically, investors' liquidity needs change over time for alternative reasons as well, such as in an aggregate liquidity crisis. We now extend our baseline model to account for that possibility.

We introduce time-varying liquidity needs into our model by specifying the different constituents of our liquidity model as persistent stochastic processes. We consider different specifications. In particular, we specify, one at a time, as autoregressive process i) the probability of being hit by a liquidity shock, λ_t , ii) the mean size, $\mu_{\xi,t}$, iii) the volatility of the size $\sigma_{\xi,t}$, thereby constituting a liquidity uncertainty shock, as well as iv) the transaction costs. We can think of extremely adverse realizations of the processes as representations of liquidity crises in our model. Our nonlinear solution technique allows to accommodate these features.

5 Conclusion

We empirically and theoretically examine the impact of safe asset supply through government bonds on credit markets, and firms' cost of debt financing. Our results emphasize a dual role of government debt in credit market activity. Through a *liquidity* channel an increase in government debt improves liquidity and lowers liquidity premia by facilitating debt rollover, thereby reducing credit spreads. Through an *uncertainty* channel, however, rising government debt creates policy uncertainty,credit spreads, default risk premia, and expected corporate bond excess returns and eventually leads to rising relative costs of firms' debt financing. We first present empirical evidence regarding the dual role of government debt in credit markets, and interpret it through the lens of a novel general equilibrium asset pricing model with endogenous credit markets and a rich role for the government in setting fiscal policy. We use the model to identify and quantify a novel fiscal risk channel associated with rising US government debt. Under such a dual view, ultimately therefore, increasing safe asset supply can be risky.

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Table 1: Regression Analysis

	by	(t-stat)	R^2	by	(t-stat)	vol	(t-stat)	R^2
A. Level								
GZ Spread	0.84	(2.40)	0.10	0.93	(4.36)	0.68	(3.42)	0.46
Repo/Tbill	-0.77	(-4.30)	0.29	-0.76	(-4.42)	0.10	(1.46)	0.32
B. First Diff								
GZ Spread	3.41	(2.12)	0.10	3.44	(2.32)	0.08	(2.57)	0.17
Repo/Tbill	-0.81	(-1.72)	0.00	-0.80	(-1.70)	0.02	(1.17)	0.01
C. Predictive								
Excess Return	0.01	(2.39)	0.02	0.01	(2.40)	0.00	(0.03)	0.02

The table reports estimates from OLS regressions of yield spreads and corporate bond excess returns on log debt-to-GDP ratio and stock market realized vol.
In Panel A, $spread_t = \beta_0 + \beta_1 by_t + \beta_2 vol_t + u_t$
In Panel B, $\Delta spread_t = \beta_0 + \beta_1 \Delta by_t + \beta_2 \Delta vol_t + u_t$
In Panel C, $r_{corp,t+1} - r_{f,t} = \beta_0 + \beta_1 by_t + \beta_2 vol_t + u_{t+1}$
GZ spread is the corporate bond spread in Gilchrist and Zakrajŝek (2012). Repo/Bill is the spreads between general collateral repo rate (Repo) and treasury bill rate. $r_{corp,t+1} - r_{f,t}$ is the excess corporate bond return. by is the log debt-to-GDP ratio. vol is the stock return realized volatility. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors. The sample is from 1973:1 to 2014:12.

Table 2: Parameter Values

Parameter	Description	Value
A. Preferences		
β	Subjective discount factor	0.997
ψ	Elasticity of intertemporal substitution	2.00
γ	Risk aversion	10
θ	Labor elasticity	0.23
B. Production		
α	Capital share	0.345
δ	Depreciation rate of capital stock	0.15
ξ_k	Adjustment cost parameter	0.8
μ	Mean productivity growth	0.0045
σ_a	Conditional volatility	0.0113
ρ_x	Long-run persistence	0.965
σ_x	Long-run conditional volatility	0.00056
$\sigma_{z,0}$	Idiosyncratic volatility	0.28
$\phi_{\sigma,a}$	Idiosyncratic Cylicality	20
C. Financing		
C	Corporate coupon rate	0.015
ζ_0	Default loss mean	0.3
$\phi_{\zeta,a}$	Default loss cyclicality	20
ϕ_l	Liquidation cost	0.05
ϕ_g	Treasury transaction cost	0.001
ϕ_c	Corporate transaction cost	0.01
λ	Liquidity shock mean arrival rate	0.4
μ_ξ	Liquidity shock size mean	0.25
σ_ξ	Liquidity shock size std	0.45
D. Fiscal Policy		
ϕ_τ	Long-run tax effects	0.05
μ_g	Spending constant	0.008
ρ_b	Spending persistence	0.96
σ_g	Spending volatility	0.0028
μ_{tr}	Transfer constant	0.186
σ_g	Transfer volatility	0.14
μ_b	Debt constant	0.064
ρ_b	Debt persistence	0.96
κ_τ	Tax smoothing	0.93

This table summarizes the parameter values used in the benchmark calibration of the model. The table is divided into four categories: Preferences, Production, Financing, and Fiscal Policy.

Table 3: Moments I

Variable	Model	Default Only	Data	Comments
A. Fiscal				
$Q^g B^g/Y$	0.37	0.37	0.37	government debt-to-GDP ratio
$sd(Q^g B^g/Y)$	0.11	0.11	0.15	
$AR1(Q^g B^g/Y)$	0.96	0.96	0.98	
τ	0.30	0.30	0.33	corporate tax rate
$sd(\tau)$	3.45	3.45	8.90	
$AR1(\tau)$	0.95	0.95	0.95	
B. Credit				
r_{rec}	0.39	0.39	0.50	recovery rate of corporate bond
$F(z)$	1.05	1.03	1.00	default rate
$Q_t B_{t+1}/(Q_t B_{t+1} + V_t^{ex})$	0.35	0.35	0.40	leverage
yield spread	1.13	0.82	1.00	
spread: default	0.83	0.82		
spread: default loss	0.61	0.60		
spread: default premium	0.21	0.21		
spread: liquidity	0.30	0		$\nu_c - \nu_g$
C. Liquidity				
ν_g	-0.36	0	-0.30	risk-free rate/treasury bill spread
ν_c	-0.06	0		liquidity premium of corporate bond
$u_{g,t+\epsilon}/Q_t^g B_{t+1}^g$	0.46	0	19.75	government bond turnover
$u_{c,t+\epsilon}/Q_t B_{t+1}$	0.15	0	0.85	treasury bond turnover
turnover ratio	23.24		23	
QB/Y	0.30	0.30	0.30	corporate debt-to-GDP ratio
β_{csd}	1.09	1.31	0.84	ols of spread default on $Q^g B^g/Y$
β_{csl}	-1.12	0	-0.77	ols of spread liquidity on $Q^g B^g/Y$
D. Crowding out				
β_{Vc}	-0.25	-0.21	-0.08	ols of QB on $Q^g B^g/Y$

This table summarizes the main statistics obtained from model simulations. We report moments of fiscal variables in panel A, credit variables in panel B, and liquidity moments in panel C.

Table 4: Moments II

Variable	Model	Comments
Δc	2.27	consumption growth
r_k	3.07	capital return
r_d	5.06	equity return
r_f	0.15	risk-free rate
r_b	5.94	government bond return
$r_k - r_f$	2.91	
$r_d - r_f$	4.90	
$sd(\Delta c)$	2.75	
$sd(r_k)$	5.38	
$sd(r_d)$	8.82	
$sd(r_f)$	0.35	
$sd(r_b)$	0.35	

This table summarizes the main statistics obtained from model simulations. We report moments of macroeconomic and return variables.

Table 5: Moments III

Variable	Bench	High λ	Low λ	High μ_ξ	Low μ_ξ	High σ_ξ	Low σ_ξ	High $\varphi's$	Low $\varphi's$
A. Credit									
r_{rec}	0.39	0.40	0.39	0.42	0.39	0.40	0.39	0.40	0.39
$F(z)$	1.05	1.07	1.04	1.56	1.02	1.13	1.02	1.07	1.04
leverage	0.35	0.35	0.35	0.39	0.34	0.36	0.34	0.35	0.35
yield spread	1.13	1.46	0.97	1.58	0.98	1.17	1.12	1.46	0.97
spread: default	0.83	0.84	0.82	1.15	0.81	0.88	0.81	0.84	0.82
default loss	0.61	0.62	0.61	0.87	0.60	0.65	0.60	0.62	0.61
default premium	0.21	0.22	0.21	0.28	0.21	0.23	0.21	0.22	0.21
spread: liquidity	0.30	0.62	0.15	0.43	0.17	0.29	0.31	0.62	0.15
B. Liquidity									
ν_g	-0.36	-0.72	-0.18	-1.63	-0.17	-0.52	-0.32	-0.72	-0.18
ν_c	-0.06	-0.10	-0.03	-1.20	0.00	-0.23	-0.01	-0.10	-0.03
$u_{g,t+\epsilon}/Q_t^g B_{t+1}^g$	0.46	0.92	0.23	0.48	0.41	0.44	0.46	0.46	0.46
$u_{c,t+\epsilon}/Q_t B_{t+1}$	0.15	0.30	0.07	0.43	0.05	0.17	0.14	0.15	0.14
turnover ratio	23.24	21.33	24.27	1.14	8536	3.41	6.8×10^7	21.33	24.27
QB/Y	0.30	0.31	0.30	0.36	0.30	0.31	0.30	0.31	0.30
β_{csd}	1.09	0.91	1.19	0.52	1.32	0.96	1.31	0.91	1.19
β_{csl}	-1.12	-2.18	-0.56	-0.04	-1.30	-0.90	-1.23	-2.18	-0.56

This table summarizes the main statistics obtained from model simulations. We report moments of credit variables in panel A, and liquidity moments in panel B. In the "Bench" column, we use our benchmark calibration. In the "High" columns, we set the parameter to be twice the value in the benchmark. In the other "Low" columns, we set the parameter to be half of the value in the benchmark. In the "$\varphi's$" columns, we change all the three transaction cost parameters φ_g, φ_c and φ_l.

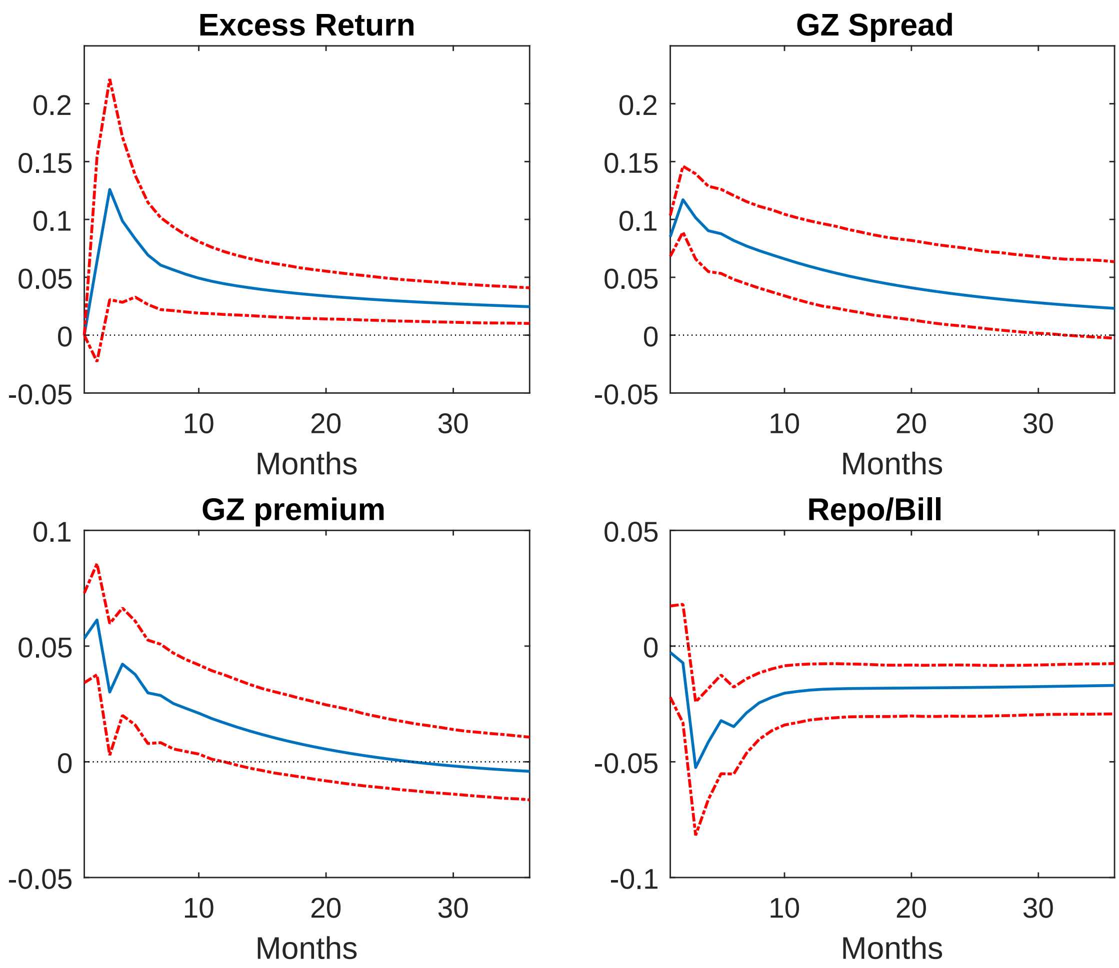

Figure 1: **Impulse Response Functions to a 1 s.d. Shock.** The figure plots the impulse response functions to a shock to debt-to-GDP ratio.

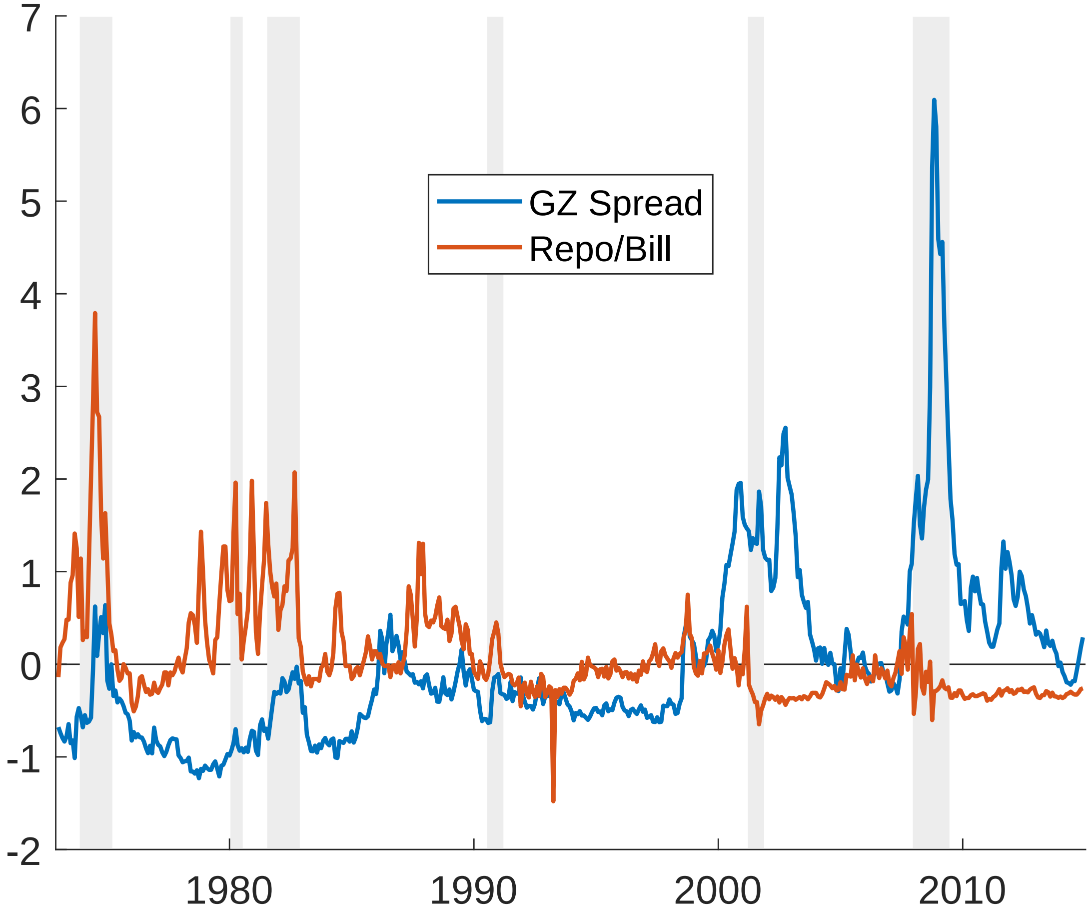

Figure 2: **Debt and Yield.** The figure plots the demeaned corporate bond spread in Gilchrist and Zakrajŝek (2012) and the spreads between general collateral repo rate (Repo) and treasury bill rate.

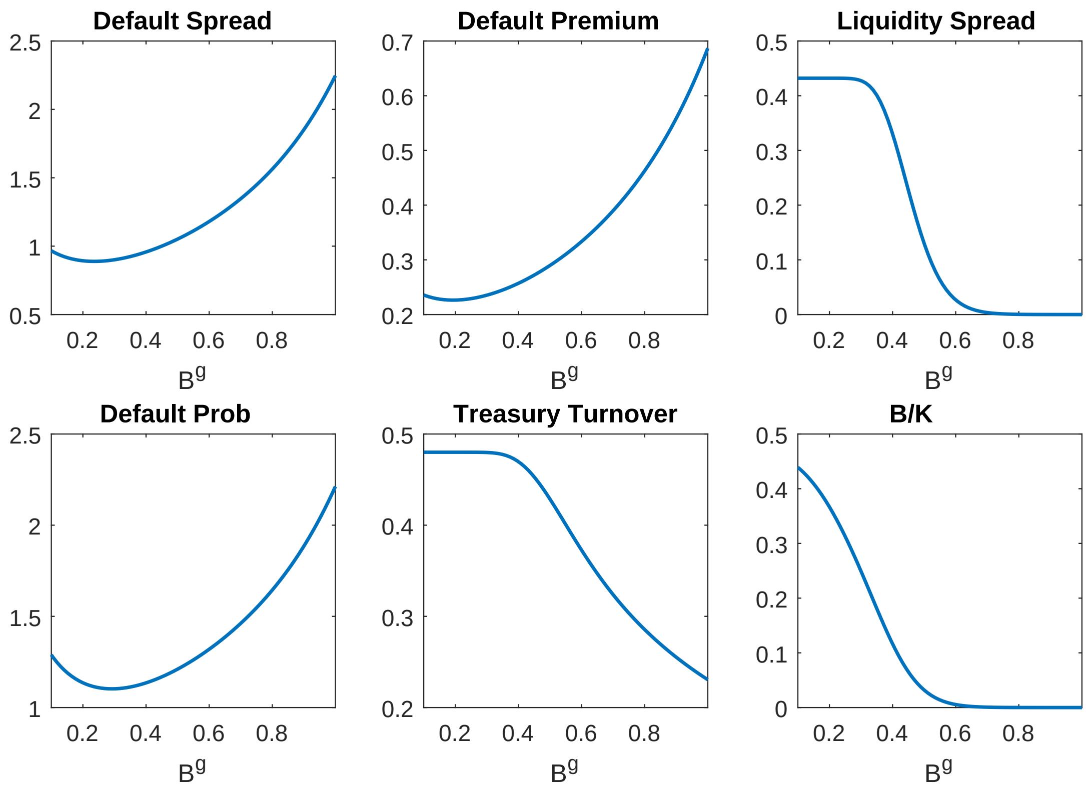

Figure 3: **Policy functions I.** The figure plots the policy functions on government debt, holding other state variables at the mean.

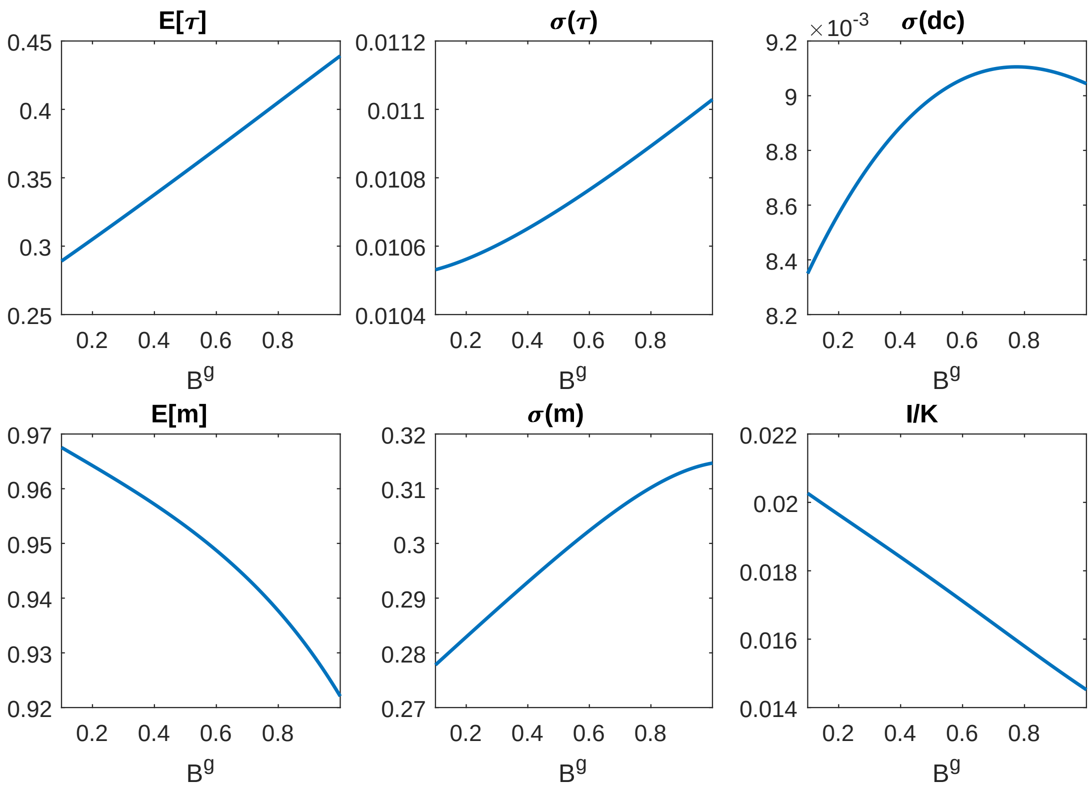

Figure 4: **Policy functions II.** The figure plots the policy functions on government debt, holding other state variables at the mean.

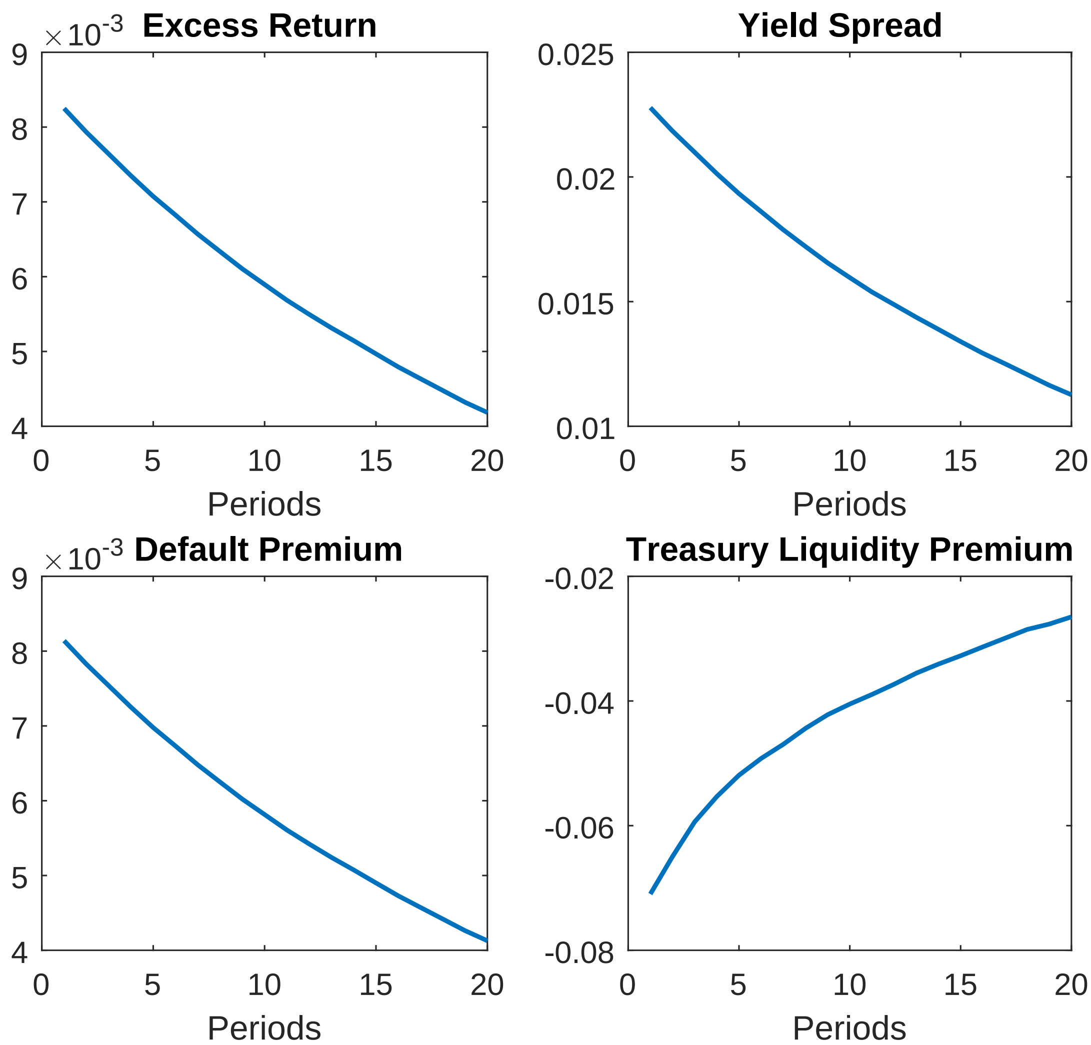

Figure 5: **Impulse response functions.** The figure plots the impluse response functions of expected corporate bond excess return, corporate yield spread, credit default premium, and liquidity premium on treasury bond to one s.d. shock of government debt.

Appendix A. Computational Algorithm

This section presents a brief overview of our computational algorithm. The possibility of default induces strong nonlinearities in both payoffs and the discount factor. Therefore, we use a global, nonlinear solution method. Endogenous variables are approximated using Smolyak polynomials and solved for using projection methods.

A.1 Projection Method

We approximate N_c control variables as functions of N_s state variables using N_p Smolyak polynomials and $N_p \times N_c$ coefficients b. N_p increases with N_s.

We compute the system of N_c equilibrium conditions over a grid of $N_p \times N_c$. We solve the system of equations and obtain b.

In our case,there are 6 state variables $X = [K_t,\ x_t,\ B_t,\ B_t^g,\ \varepsilon_{b,t},\ G_t/Y_t]$ and 5 control variables $L_t,\ U_t,\ B_{t+1},\ Q_t,\ Q_t^G$. $N_s = 6$. $N_c = 5$. $N_p = 85$.

A.2 Algorithm

Step 1. Compute the policy function Given coefficients b and grid X.

Use rescale function $\Phi : R^2 \rightarrow [-1, 1]^{N_s}$ to rescale the state variables. For example,

$$\Phi(K_t) = -1 + 2\frac{K_t - K_{min}}{K_{max} - K_{min}}$$

Use the Smolyak basis functions $\Psi_n(X)$ to compute policy function $\hat{f}(X; b^{(i)}) = \sum_{n=1}^{N_p} b_n \Psi_n(\Phi(X))$

$$[L_t,\ U_t,\ IB_t,\ Q_t,\ Q_t^G]' = \hat{f}(X; b^{(i)}) = \sum_{n=1}^{N_p} b_n \Psi_n(\Phi(X_n))$$

Step 2. Compute the state variables in the next period We use the equilibrium conditions to compute the state variables.

$Y_t,\ F_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$

$C_t,\ F_{L,t} = \frac{(1-\nu)C_t}{\nu(1-L_t)}$

$I_t,\ Y_t = C_t + I_t$

$K_{t+1},\ K_{t+1} = (1-\delta)K_t + \Phi_k(\frac{I_t}{K_t})K_t$

The other state variables follows their law of motions.

Table 6: Number of Polynomials

Dimension	1th Smolyak	2th Smolyak	3th Chebyshev	5th Chebyshev
N	$1+2d$	$1+4d+2d(d-1)$	3^d	5^d
1	3	5	3	5
2	5	13	9	25
3	7	25	27	125
4	9	41	81	625
5	11	61	243	3125
6	13	85	729	15625
7	15	113	2187	78125

Step 3. Compute approximation errors Given the Gaussian quadrature, compute conditional expectation using J integration nodes and weights, $\epsilon_{t+1,j}$ and $\omega_{t,j}$. At each node $X_{t+1,j}$, compute $U_{t+1,j}$

$$E_t[U_{t+1}^{1-\gamma}] = \sum_{j=1}^{J} \omega_{t,j} \left\{ U_{t+1,j}^{1-\gamma} \right\}$$

compute $L_{t+1,j}$, $Y_{t+1,j}$, $C_{t+1,j}$, $I_{t+1,j}$,$M_{t+1,j}$, $q_{k,t+1,j}$,$Q_{t+1,j}$, $K_{t+2,j}$, $B_{t+2,j}$, $V^{ex}_{t+1,j}$, $\int^{z^*_{t+1}} V_{i,t+1,j} dF$, $\frac{\partial Q_{t+1}}{\partial K_{t+1}}$,$R_{k,t+1,j}$, $z^*_{t+1,j}$, ...

Use the variables at $t+1$ and node j to compute all the expectations.

Step 4. Solve system of equations of approximation errors with respect to b

Step 5. Simulate the model and compute approximation errors in the simulated state space

A.3 Smolyak polynomials

Smolyak polynomials are a carefully-selected subset of Chebyshev polynomials. It has an approximation level μ. The maximum order of one dimension is $2^\mu + 1$. For example, the 2th Smoyak polynomials have the highest order of 5, the same the 5th Chebyshv polynomials. However, the number of polynomials are significantly smaller than tensor product.

Appendix B. Data Sources

Our government debt data are from the Federal Reserve Bank of Dallas. We use the FRED database to collect the following data: GDP, Industrial Production, 3-month treasury bill rates and banker's acceptance rate. Returns on corporate bonds are obtained from the investment grade bond return index from Barclay. General collateralized Repo rates are obtained from Bloomberg. We augment the repo rate with banker's acceptance rate before 1991. The GZ spread and credit risk premium are from Simon Gilchrist's website. We collect the total outstanding and annual trading volume of government and corporate debt from the Securities Industry and Financial Markets Association.