Reach for Yield by U.S. Public Pension Funds[1]

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Abstract

We study public pension funds' risk-taking theoretically and empirically. The theory relates risk-taking to risk-free rates, plan underfunding, sponsors' fiscal condition, and sponsors' incentives to shift risk to debt holders. Our empirics create a new methodology to infer funds' risk from limited data on returns and portfolio weights. We adjust underfunding measures to better reflect the risk of funds' liabilities. We find funds took more risk when risk-free rates and funding ratios were lower, and when sponsors had weaker public finances, in line with risk-shifting. One-third of funds' total risk was related to underfunding and low rates in recent years.

Keywords: U.S. public pension funds, reach for yield, Value at Risk, underfunding, duration-matched discount rates, state public debt.

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1. Introduction

How do low interest rates affect the investment behavior of institutional investors? How is this behavior influenced by investors' financial condition, and what are its potential consequences? This paper studies these questions in the context of state and municipal U.S. public pension funds (henceforth PPFs, funds, or plans). Specifically, we investigate whether PPFs reach for yield (RFY) by holding riskier investment portfolios to increase their expected returns when interest rates on relatively safe assets are low. In addition, we study how the extent of funds' underfunding and the fund sponsors' fiscal condition affect the funds' risk-taking behavior, and how such behavior in turn may affect the funds and their sponsors. To study these relationships, we first present a simple theoretical model relating funds' risk-taking to the level of risk-free rates, to their underfunding, and to the fiscal condition of their state sponsors. The theory identifies two distinct channels through which interest rates and other factors may affect risk-taking: funding ratios and risk premia. The theory also shows that the effect of state finances on funds' risk-taking depends on states' risk-shifting incentives. We use the theory to interpret our empirical findings. To study the determinants of funds' risk-taking behavior, we create a new methodology for measuring funds' asset portfolio risk, and we use improved measures of plan underfunding based on liabilities that are discounted using risk-free discount rates following the approach in Rauh (2017). Using these improved measures, we perform a panel regression analysis to assess how funds' asset risk is related to risk-free rates, to plan underfunding, and to states' fiscal condition. In addition, we study the implications of our results for state finances.

A public pension plan is underfunded if the present value of its assets is less than the net present value of liability payments to its pension holders. When a PPF is underfunded, state sponsors are limited in their ability to close the funding gap by reducing promised pension benefits because in most states public employee retirement benefits are either guaranteed by state constitutions or constitute a contractual obligation between the sponsor and plan members.[2] Sponsors do, however, have many other choices: their funds can invest in assets with higher expected returns—but also risk—hoping this will close the gap; they can require greater contributions from future pension beneficiaries; or sponsors can provide higher contributions to the plan,

[2] Munnell and Quinby (2012) provide an analysis of the restrictions on the reduction of pension benefits to public sector employees and retirees.

which they would fund by current or future taxation, by borrowing, or by cutting expenditures on other governmental programs and prerogatives. The choice that is ultimately made is a political decision.[3,4]

Our theoretical analysis models the political decision in a stylized setting. Specifically, we model the asset portfolio choice of a public pension plan that is acting on behalf of its sponsors and can invest in risky and risk-free assets. The model captures the tradeoff that plan sponsors face when choosing between their constituents paying higher future taxes to support pension beneficiaries, or by the plan taking more risk in the hopes that the risky assets perform well and reduce the amount of taxes that need to paid to support beneficiaries. In the model, funds' incentives to take risk operate through two main channels. The first channel operates through funding ratios, defined as the ratio of the present value of funds' assets relative to their liabilities. When funding ratios are lower for any reason—including but not limited to lower assets, higher future liabilities, or lower interest rates—funds may choose to take more risk in the hopes of "catching up". This is the reach-for-yield channel in our model. The second is a risk-premium channel that operates if lower risk-free interest rates alter risk premia and hence plans' incentives to take risk. The risk-premium channel is conceptually separate from reach-for-yield. The model also captures the possibility that some sponsors may choose to default on their non-pension debt in order to more easily make required payments to pension fund beneficiaries. Theoretically and empirically we examine how the possibility of default, the level of interest rates, the funding ratio, the amount of non-pension debt relative to state income, and their interactions jointly determine funds' risk-taking.

Our empirical findings show that lower funding ratios and lower interest rates on safe assets caused PPFs to increase portfolio risk. Interpreted through our model, we find evidence for both

[3] Political considerations can also affect the PPFs' investment behavior because governmental sponsors of PPFs have discretion regarding the level of contributions to the fund and the setting of funds' target asset return. Government accounting standards require that plan sponsors develop a plan to fully fund public pension plans over a period no greater than thirty years. The plan requirements are not binding. Many plan sponsors do not adhere to the funding schedules specified in the plan. Moreover, standards governing public sector pension plans provide sponsors with considerable discretion in the choice of accounting assumptions. Naughton, Petacchi and Weber (2015) provide evidence that plan sponsors use this discretion to reduce reported levels of underfunding and contributions. Kelley (2014) finds that political factors have a significant influence on plan funding levels.

[4] A large share of the public pension plan board members consists of political appointees and elected officials. Return objectives of state public sector pension plans are often set by state legislatures in the budgeting process. Similar processes are used by local government pension plan sponsors. For details see Andonov et al (2017).

a reach-for-yield channel acting through funding ratios as well as a channel capturing how interest rates affect risk-premia. Second, our findings show an interaction between underfunding and low interest rates, i.e. the effect of a lower funding ratio on risk-taking behavior was more pronounced when interest rates were relatively low, such as during the last five years of our sample (2012-2016). Third, PPFs affiliated with state or municipal sponsors with weaker public finances—as reflected by higher levels of public debt or worse credit ratings—also took more risk. In line with our model implications, we find a notable interaction between public finances and interest rates, as PPFs from states in worse fiscal condition took more risk especially during periods of low interest rates.

Our modeling of state finances suggests that if states can default on their non-pension debt, states with high debt-to-income ratios may choose to take higher risk in their pension funds because they can shift the risk of poor fund performance away from taxpayers and toward state debt holders. On the other hand, our model also implies that states may choose to take less risk if state debt is high but they cannot default on it, or if the penalties for defaulting are large. Viewed through the model, our empirical analysis of state finances is mostly consistent with higher state debt-to-income ratios leading to higher risk in pension plans' portfolios, and thus with risk-shifting. Therefore, our analysis implies that, because PPFs in states with weaker financial conditions take more risk, they run the risk of further weakening state finances. We quantify that the potential loss to the states if a 1-in-20 years episode of adverse returns had occurred in 2016 would have been on average about 3% of states' personal income, or about 39% of states' debt.

A related theory of why PPFs' risk-taking behavior has increased during the recent low-yield environment is that sponsors may attempt to mask their PPFs' extent of underfunding, and may do so by holding riskier assets with higher returns to reduce the reported value of their liabilities. Under-reporting of liability values can occur because GASB accounting rules allow U.S. PPFs to discount their liabilities based on the expected return on their assets. Andonov et al. (2017)'s cross-country study provides evidence consistent with this theory. However, Boubaker et al. (2018) find that given their asset holdings, PPFs tend to significantly exaggerate the expected returns on their assets; which means they do not necessarily have to hold riskier assets in order to mask a part of their underfunding. This phenomenon is partly illustrated in Figure 1 (panel a), which shows that public pension funds' expected return targets have declined little while the Treasury yields have significantly declined.

This study makes several contributions relative to the existing literature on PPFs' risk-taking behavior. First, we present a stylized theoretical model to help interpret our empirical results. Importantly, the model highlights the role of risk-shifting as a determinant of PPFs' risk-taking behavior. In addition, the theory highlights that risk-taking occurs through separate channels linked to funding ratios and risk premia, and illustrates the importance of distinguishing among them.

Second, we use a new and more flexible approach for measuring funds' asset risk based on the limited data that are publicly available. To do so, we assume that funds' returns in each asset category (e.g., equities, fixed income, alternatives, etc.) consist of the return on an unknown category return index that is common across funds plus a fund-specific component. We estimate the category return indices and their constituents, as well as the funds' residual risks econometrically. Then we use the estimated category return indices measured at high frequency, the funds' portfolio weights, and the fund-specific components to estimate funds' risk. Unlike our approach, some papers in the literature measure funds' risk while assuming that a particular index (such as the S&P 500 for equities) is representative of funds' returns in each asset category. Other papers measure risk using less comprehensive measures, such as the share of equities or risky assets in the funds' investment portfolios, without accounting for all asset classes, or for the time-varying riskiness of each asset class. In contrast, our approach to risk measurement is more comprehensive because it accounts for all asset classes for which data is reported, identifies market indices relevant for each asset class (rather than assuming them), and allows for time-varying correlations of returns across asset classes.

Our third contribution is that we use improved measures of underfunding based on the methodology in Rauh (2017), which provides a method to approximately discount the value of funds' liabilities at risk-free rates. Because of strong legal protections for public pension benefits, we believe PPFs' promised benefits are nearly risk-free and therefore should be valued using risk-free discount rates instead of the rates associated with the GASB reporting standards.[5] The use of more appropriate discount rates reduces error in the measurement of funds' underfunding. Consistent with our measurement error interpretation, the improved measures of underfunding enhance the goodness of fit for regressions relating PPFs' risk to underfunding.

1.1 Literature

Our paper is related to three strands of the literature. The first is the literature on PPFs' risk taking. Boubaker et al. (2018) and Mohan and Zhang (2014) measure funds' risk using aggregate market beta coefficients, with different betas assumed for each asset category. Andonov et al. (2017) measure funds' risk as the share of risky assets in the portfolio. Pennacchi and Rastad (2011) measure funds' risk based on the tracking error volatility between the value of assets and liabilities. The volatility of assets is measured under the assumption that returns in each asset category are determined by specific return indices. Relative to these papers, our approach improves the measurement of risk in two ways. First, measures of risk based on covariation with the market, such as beta, or on the share of risky assets held do not measure the time series aspect of risk. For example, high covariation with the market or large holdings of risky assets in the portfolio increase a PPF's riskiness when the market is expected to be more volatile, but the afore-mentioned measures do not account for such time variation in riskiness. In contrast, our approach based on funds' Value-at-Risk accounts for it, as described in our risk measurement section. Second, we use an approach to measuring the variance of asset portfolios that is similar to Pennacchi and Rastad (2011), but instead of assuming that returns in each category are driven by a particular index, our approach is more flexible. We believe our method of measuring the riskiness of pension funds' portfolios has potential to improve on other methods used when the data are limited.

The second strand of related literature concerns how the risk-taking behavior of financial intermediaries varies with macroeconomic conditions. As is the case with our study, these papers rely on cross-sectional differences between institutions in their response to changes in macroeconomic conditions to identify risk-taking behavior. Becker and Ivashina (2015) examine

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[5] See Novy-Marx and Rauh (2011) for a wide range of considerations in assessing the riskiness of PPF benefits from the perspective of taxpayers.

reach-for-yield behavior among life insurance companies. They find that life insurers tend to assume greater levels of investment risk during economic expansions and that this effect is more pronounced among more poorly capitalized firms. DiMaggio and Kacperczyk (2017) find evidence of greater risk-taking by money funds when interest rates are low. This effect is stronger for independent funds than for funds affiliated with insurance companies, commercial or investment banks. They argue that reputational considerations tend to moderate reach-for-yield behavior by affiliated funds. Studies of commercial banks also find evidence of increased risk-taking in low rate environments, but the effect of financial condition on risk taking is mixed. Jiménez et al. (2014) examine lending activity by Spanish banks. They find that lower overnight rates induce banks to do more risky lending. This effect is stronger among more poorly capitalized institutions. Dell'Ariccia et al. (2017) examine commercial banks in the United States. They also find evidence of greater risk-taking in a low rate environment; however the increase in risk-taking is more prevalent among well-capitalized institutions. Unlike for banks, they posit that financial intermediaries with negative maturity mismatches, such as insurance companies and pension funds, should switch to risky assets in response to monetary easing, and that this behavior should be most pronounced for the least capitalized financial institutions, which we demonstrate in our paper. Closer to our study, Boubaker et al. (2018) study reach-for-yield by PPFs in a framework that models the evolution of PPFs' asset category risk-exposures and monetary policy innovations in a Bayesian VaR framework with Markov regime switching. Consistent with our results, they find that monetary easing is consistent with greater portfolio risk. Because of differences in how riskiness is measured, and our very different modeling methodologies, we view our two approaches as complementary.

The third strand of related literature concerns the effect of PPFs' obligations on state and local finances. Increased risk-taking and reach-for-yield behavior increase the exposure of plan sponsors to large declines in asset values, and hence increases the volatility of contributions necessary to fund pension promises. A growing literature considers the impact of pension costs, underfunding, and investment losses on state and local government borrowing costs (Novy-Marx and Rauh, 2012, and Boyer, 2018). Several academic and policy studies have examined the effect of a decline in asset prices on the required contributions of plan sponsors (Novy-Marx and Rauh, 2014, Boyd and Ying, 2017, and Mennis et. al., 2018). Measures of PPFs' risk-taking presented herein should be useful in future work concerning the vulnerability of PPFs and plan sponsors to adverse shocks in asset prices.

The remainder of the paper proceeds as follows. Section 2 presents our stylized theoretical model. Section 3 present our data and our methodology for measuring PPFs' asset portfolio risk and plan underfunding. Section 4 contains our empirical analysis of how PPFs' risk has changed over time and in the cross section. It examines the relationship between asset portfolio risk, the interest rate environment, plan underfunding, and the financial condition of fund sponsors. Section 5 discusses the implications of our results for states' public finances. Section 6 concludes.

2. Theoretical Model

To guide our thinking about the determinants of risk-taking by public pension plans, and to provide some intuition for how to interpret the findings from our econometric analysis, here we present a very simple "benchmark" two-period model of risky portfolio choice for a state or municipal pension plan. We refer to the plan sponsor as the state throughout.

For our modeling, we assume there is no conflict of interest between the state and the manager of its pension investments, and that therefore the pension plan is managed in accordance with the wishes of plan sponsors. Therefore we model the state as controlling the amount of assets managed by the pension fund, and how those assets are invested. Multi-period treatments of the pension fund's portfolio choice problem are contained in Pennacchi and Rahstad (2011) and D'Arcy et al (1999); following their approach, the state is assumed to choose the pension assets to maximize a utility function that is based on the preferences of its citizens, denoted as the representative citizen RC, hereafter. We interpret the representative citizen as the median voter within the state or municipality associated with a pension plan, but we acknowledge the utility function could have richer interpretations. In particular, it may embed preferences on how the plans' and sponsors' actions affect conflicting special interests such as plan beneficiaries, state taxpayers, and the holders of state debt.[6] Because of these potential conflicts and other potential imperfections, we don't interpret maximization of the utility function as maximization of social welfare, but we regard it as a useful modeling device for our positive analysis. In our modeling below, we rely primarily on a median voter interpretation, but we allow for the possibility that the interests of the median voter and state debt holders may be in conflict.

There are two dates in the model. Date 0, which represents today, and date t, a date t years in the future. The RC is endowed with income Y_0 and Y_t. The income Y_t is assumed to be net of all tax payments other than those that may need to be made to support pension beneficiaries, or to pay off state debt. To simplify the analysis, Y_t and Y_0 are assumed to be known at date 0.[7] In addition, the state has zero coupon debt with face value D_t that must paid at date t and it has a pension liability of L_t that must be paid to its workers at date t. We model the debt D_t in two different cases. In the first the debt is risk-free and the state will pay it out of state income Y_t. In the second the state can choose to default on its debt and will do so if the taxes needed to support its pension plan and state debt are too high. We first focus on the risk-free debt case and later turn to the case with risky debt. At date 0, the state pension plan has assets A_0 to invest on behalf of its pension beneficiaries. At date t, the portfolio grows to value $A_0 R_{p,t}$, where $R_{p,t}$ is the gross return on the portfolio, then the portfolio is liquidated and the full proceeds from liquidation are turned over to workers.[8] The pension's liabilities consist of a single lump sum payment L_t that must be paid to beneficiaries at time t. If the liabilities toward workers exceed the proceeds from asset liquidation, the difference is paid as a transfer from taxpayers to the pension beneficiaries. The taxes to support the pension plan at date t are given by $T_t = Max(L_t - A_0 R_{p,t}, 0)$. The consumption of the RC at date t is then given by income at date t less debt payments and taxes, $C_t = Y_t - D_t - T_t$. Similarly, consumption at date 0 is $C_0 = Y_0 - A_0$.

[6] The preferences could reflect median voters, vocal interest groups or a blend of the median voter and the median voter as in Kelley (2014); or they could represent the preferences of median voters or fund managers as in Pennacchi and Rahstad (2011). Or they could represent the preferences of the median voter in a setting where there are information asymmetries among voters regarding pay structure as in Glaeser et al (2014).

[7] If instead Y_t is stochastic, then the investment choices of the pension fund would be used to hedge against the risk of Y_t as pointed out by Lucas and Zeldes (2009).

[8] The workers are assumed to get all of the cash flows from their asset portfolio even if it exceeds the liabilities. This assumption follows the practice that only workers have access to the cash flows that have been set aside in their pension funds.

The state chooses A_0 and the proportion of its portfolio to invest in risky assets, ω, to maximize the discounted expected utility of consumption of the RC subject to the constraint that the pension liabilities and debt are paid off:

$$Max_{A_0,\omega}\, U_0(C_0) + E_0\delta^t U_t(C_t) \qquad (1)$$

where δ is the instantaneous rate at which the RC discounts the future, and U_0 and U_t are strictly increasing concave functions of utility over consumption in each period. The optimization is also equivalent to minimizing the expected utility loss due to tax payments at period t.[9] In this view of the problem, the utility functions can be interpreted as incorporating the costs of distortionary taxes.[10]

In our theoretical analysis the pension fund can invest in only two assets.[11] There is a risk-free asset with instantaneous net return r_f which we treat as fixed between dates 0 and t. One dollar invested in the risk-free asset at date 0 grows to $e^{r_f t}$ at date t. In addition, the pension fund can invest in a risky asset, which we think of as an equity index whose log-return between dates 0 and t is normally distributed:

$$\ln(R_t) \sim N\left(\left[r_f + \lambda - .5\,\sigma^2\right]t, \sigma^2 t\right)$$

where λ is the market price of risk, which is the reward for exposure to stock-market risk, and σ is the instantaneous standard deviation of the return on the risky asset. This assumption implies the gross return R_t has the functional form:

$$R_t = Exp\big(\,(r_f + \lambda - .5\sigma^2)t + \sigma\sqrt{t}\,\epsilon\,\big), \quad (2)$$

where $\epsilon \sim N(0,1)$.

The pension fund invests ω percent of its wealth in the risky asset and $1-\omega$ percent in the risk-free asset subject to the constraint that $0 \leq \omega \leq 1$. This constraint rules out the use of leverage

[9] Utility at time t is decreasing and convex in taxes T_t.
[10] Distortionary tax representations of the problem in two period and multi-period settings are contained in Lucas and Zeldes (2009) and Epple and Schipper (1981).
[11] We consider a larger number of investment types in our empirical analysis.

by the fund as well as the use of short sales. The resulting return on the asset portfolio is given by:

$$R_{p,t} = (1-\omega)e^{r_f t} + \omega e^{(r_f+\lambda-.5\sigma^2)t+\sigma\sqrt{t}\,\epsilon}$$

Substituting $R_{p,t}$ into the expression for utility, given a choice of A_0 at time 0, the maximization for the choice of ω, the share of risky assets in the portfolio, after simplification reduces to:

$$Max_{\omega}\, E_0\, U_t\left(Y_t - D_t - Max\left(L_t - A_0\left[(1-\omega)e^{r_f t} + \omega e^{(r_f+\lambda-.5\sigma^2)t+\sigma\sqrt{t}\,\epsilon}\right], 0\right)\right) \quad (3)$$

Our analysis studies how the riskiness of the portfolio depends on risk-free interest rates, plan funding, and state finances. We measure pension underfunding by its funding ratio, which is the present value of the fund's assets divided by the present value of its liabilities. When the funding ratio is 1 or over, a plan is fully funded; when it is less than one, then the plan is underfunded. Because, as discussed further below, payments to beneficiaries are very likely to be paid in full, we treat them as risk-free and discount their value using risk-free rates. Furthermore, we proxy for the debt burden of state finances to the representative citizen by the ratio D_t/Y_t, which is the state debt to income ratio, SDI_t; and L_t/Y_t, which is the pension debt to income ratio, PDI_t. With these transformations, the portfolio choice problem in equation (3) can be rewritten as:

$$\max_{\omega} E_0\, U_t[Y_t \times \left(1 - \frac{D_t}{Y_t} - \frac{L_t}{Y_t} Max\left(1 - \frac{A_0}{\frac{L_t}{e^{r_f t}}}\left[(1-\omega) + \omega e^{(\lambda-.5\sigma^2)t+\sigma\sqrt{t}\epsilon}\right], 0\right)\right)]$$

$$= \max_{\omega} E_0\, U_t\left[Y_t \times \left(1 - SDI_t - PDI_t \times Max\left(1 - FR_0(r_f, A_0, L_t)\right)\left[(1-\omega) + \omega e^{(\lambda-.5\sigma^2)t+\sigma\sqrt{t}\epsilon}\right], 0\right)\right)\right] \quad (4)$$

where $FR_0(r_f, A_0, L_t) = A_0/(\frac{L_t}{e^{r_f t}})$ is the funding ratio, which is the present value of fund assets divided by the present value of fund liabilities.

Equation (4) illustrates the role of the funding ratio in determining fund risk. If the funding ratio is greater than or equal to 1, corresponding to a fully-funded pension plan, equation (4) shows that by investing only in the risk-free asset (by setting $\omega = 0$), the proceeds from the asset portfolio are sufficient to pay off the pension liability. If instead the funding ratio is less than one because of low assets, high future liabilities, low risk-free rates, or for any other reason, then the equation shows the plan's obligations cannot be met by investing in risk-free assets alone. Instead, the plan could attempt to meet its obligations by taking on more risk, i.e. reaching for yield, as described in Rajan (2005) or Yellen (2011), and/or plan sponsors must pay more to pension beneficiaries through taxes at time t.[12]

Equation (4) allows risk-free interest rates to also affect risk-taking by altering the risk-premium. To denote this possibility, we model the risk-premium as a function of the risk-free rate: $\lambda = \lambda(r_f)$, and refer to this as the risk-premium channel. A series of papers tracing back to Campbell (1987) study whether equity risk premia are time varying and predictable from interest rates or other variables.[13] Our reading of the recent literature is that in univariate return predictability regressions, the evidence for predictability is weak, and the regression coefficients are time varying and unstable (Welch and Goyal, 2008, and Paye and Timmermann, 2006). However, when univariate forecasts are combined, forecastability improves, especially near recessions (Rapach et al., 2010). In addition, in univariate forecasting regressions, imposing restrictions from economic theory on the regressions improves predictability (Petenuzzo et al., 2014). For illustrative purposes, in our theoretical analysis we rely on an estimate of how Treasury-bill rates affect the equity premium in a univariate regression with economic restrictions based on Pettenuzzo et al (2014). They estimated the following relationship towards the end of their data sample:[14]

$$\lambda(r_f) = .004 - .007 \times\ r_f, \qquad (5)$$

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[12] Rajan (2009) discusses the need for private insurance companies to increase portfolio risk when rates fall: "Insurance companies may have entered into fixed rate commitments. When interest rates fall, they may have no alternative but to seek out riskier investments – if they stay with low return but safe investments, they are likely to default for sure on their commitments, while if they take riskier but higher return investments, they have some chance of survival." Similarly, Yellen (2011) states "[I]mportant classes of generally unlevered investors (for example, pension funds) are reportedly finding it difficult in the present low rate environment to meet nominal return targets and may be reaching for yield by assuming greater interest-rate and credit risk in their portfolios."

[13] For reviews of this literature see Rapach et al (2013), and Timmermann (2018).

[14] The estimates reported in the published version of Pettenuzzo et al (2014) end in the mid-1980s. We thank the authors for providing us with estimates from December 2010, the end of their sample, and the middle of our sample. We use their constrained estimates above. Their estimates for the unconstrained equation are $\lambda = .008 - .09 \times r_f$ where r_f is the yield on 3-month Treasury bills.

where r_f is the yield on 3-month U.S. Treasury bills, which we will interpret as the risk-free rate. Inserting this expression for λ inside equation (4) shows that the short-term risk-free rate affects risk-taking through the funding ratio and the equity premium.

Because our model (see equation 4) shows funding ratios are determined by the value of funds' assets, the face value of funds liabilities, and risk-free rates, plans' funding ratios can vary independently from risk-free rates.[15] Therefore, in our theoretical analysis we treat funding ratios and risk-free rates as separate determinants of funds' risk-taking. Moreover, the reach-for-yield channel operates through the funding ratio; the risk-premium channel operates through interest rates, and the two interact in determining risk-taking.

As a first cut for providing intuition on how the funding ratio and interest rates affect risk-taking, we assume the RC has a utility function for time t that has a power utility form:

$$U_t(C_t) = -(C_t)^{-k} \text{ for } k \geq 1.$$

We focus on the case of k=10 in all of numerical analysis below, and for now focus on the case of the State Debt-to-Income Ratio = 3%, and we assume that state debt is risk-free and will not be defaulted upon. We then numerically solve for the optimal portfolio choices and risk-taking as a function of the funding ratio and the risk-free rate. The main results from the numerical analysis are presented in Figure 2. The figure shows that risk, here measured by the proportion of the portfolio invested in risky assets, increases in pension underfunding. This finding is consistent with reach for yield behavior because risk increases as the funding ratio decreases. In addition, holding underfunding fixed, lower risk-free rates are associated with more risk-taking through the risk-premium channel from equation (5). Figure 2 also shows there is an interaction effect: The marginal effect of underfunding on risk-taking is larger when interest rates are lower, and the marginal effect that interest rates have on risk-taking is more pronounced when underfunding is higher. This shows the reach-for-yield and risk premium channels interact, and one should account for the interaction that these different channels have on risk taking. These

[15] For example, during the 2007-09 crisis, many funds' risky assets performed poorly, reducing funding ratios by amounts that varied based on plans' exposures to risky assets.

findings provide justification for our empirical specification that studies how funding ratios, interest rates, and their interactions affect risk taking.

Equation (4) also shows state finances measured by debt to be paid at date t as fraction of state income SDI_t also affects risk-taking. To investigate the role of state finances on risk-taking, we consider two circumstances, the first being when the state will not default on its debt. We model this case with our assumption that its debt will be covered by taxes on income at date t. Under our assumptions, Figure 3 shows that a lower funding ratio increases risk for a range of levels of state debt. This is consistent with the reaching-for-yield implication in Figure 2. The figure also shows that greater state debt relative to income leads to reduced risk-taking by the state's pension fund. The intuition for this result is when the state is more indebted, then the taxes it faces if the pension fund performs poorly have a much greater effect on the utility of the representative citizen than it does if the state is less indebted. To avoid the more severe consequences, the pension fund takes less risk if the taxpayers have to make up the shortfall for large losses by the pension fund.

A more general model of state finances would account for the possibility that some states might actually default on and/or renegotiate their debt, and that all else equal larger pension liabilities increase the risk of default or renegotiation (see Boyer, 2018). To model this in a simple way, we assume that when a state defaults on its debt, it defaults on all of its debt, and when it does so it incurs a penalty measured in utility terms that is proportional to the amount of its debt. Specifically, the representative citizen receives the following utility when it does not and does default at date t:

$$U_t(.) = \begin{cases} U_t(Y_t - D_t - T_t) & No\ Default \\ U_t(Y_t - T_t) - \gamma \times D_t & Default \end{cases}$$

where $\gamma \times D_t$ $(\gamma > 0)$ is the penalty for defaulting, and T_t represents the taxes if any that need to be paid to pension beneficiaries at date t. The state will choose to default on its debt if doing so raises its utility. Analysis below shows the state will choose to default on its debt when its pension fund assets perform poorly enough. The option for the state to default shifts some of the downside risk of the pension fund's performance from state taxpayers to the holders of the state's debt. The ability to shift some of the downside risk of the pension fund to debt holders affects PPFs' incentives to take risk. Risk shifting is only valuable for the state if it sometimes chooses to default on its debt. Therefore, to understand incentives to shift risk, it is necessary to understand when default is valuable. Algebra shows that to a second-order approximation the state will choose to optimally default when the taxes required to support the pension plan satisfy the condition:

$$\gamma \leq U_t'(Y_t - T_t) - 0.5 * U_t''(Y_t - T_t) * D_t.$$

Rearrangement shows default is optimal when:

$$\gamma \leq U_t'(Y_t - T_t) \times \left(1 + .5 \times CRRA \times \frac{SDI_t}{1 - \frac{T_t}{Y_t}} \right),$$

where $CRRA$ is the coefficient of relative risk aversion. Because utility is increasing and concave, the condition shows for a given risky asset portfolio, the default condition is more likely to be satisfied when γ and income Y_t are smaller, or when taxes T_t, relative risk aversion $CRRA_t$, or the state debt to income ratio SDI_t are greater. In addition, the determinants of T_t such as the funding ratio and risk-free rates also affect the default condition. Because these variables affect the decision to default, they also affect incentives to shift risk.[16] To further examine how the possibility of shifting risk through default affects the pension funds' investment decisions, we numerically solve for the pension funds' optimal investments when the state can default on its debt. The results are presented in Figures 4 and 5. Figure 4 shows that for low funding ratios, as the amount of debt to income increases from 0, risk-taking goes down, just as it did in Figure 3. But, unlike Figure 3, when the debt-to-income ratio increases enough, risk-taking suddenly becomes much higher. This non-monotonicity is due to the risk-shifting effect. Moreover, Figure 4 shows an interaction between state debt-to-income and pension

[16] A related but mechanism for risk-shifting to occur comes from recalling that pension claims are low risk to beneficiaries because they are very senior in the state's debt structure. As noted by Ivanov and Zimmerman (2018) when states are more indebted, they tend to issue more senior debt (bank debt) to increase their debt capacity, and the borrowers that do this are high risk. Because pension liabilities are very senior, paying a greater share of workers' salary through pension benefits represents another channel to shift risk from state taxpayers to state bond holders.

underfunding: risk-shifting occurs for higher levels of state debt when a pension plan is more underfunded. In Figure 5, an additional interaction effect exists between state debt and the risk-free rate. More specifically, the effect that risk-free rates have on risk-taking through the risk premium channel can depend on the state debt-to-income ratio. This interaction is present when the state cannot default on its debt (not shown) and also in some circumstances when the state can default on its debt. In particular, Figure 5 shows that when the state debt-to-income ratio is high enough, a decline in the risk-free rate leads to a non-monotonic increase in risk-taking.

In our empirical examination of the role of state finances as a driver of pension fund risk-taking, we study whether higher state debt to income is associated with higher pension fund risk-taking as would be consistent with the state pension funds shifting risk to debt holders. In addition, we examine if there is an interaction effect between the ratio of debt-to-income and interest rates.

In summary, five main results emerge from our theoretical analysis:

1. The effect that the funding ratio has on risk-taking captures the effect of reach for yield. In our regression analysis we interpret the coefficient on the funding ratio as capturing reach-for-yield effects.
2. After controlling for funding ratios, interest rates may also affect risk-taking because they affect risk-premia. We interpret the coefficient on interest rates in our regression analysis as the risk-premium effect.
3. The reach-for-yield and risk-premium effects interact in theory. We allow for their interaction in our empirical specification.
4. How state finances affect risk-taking depends on whether states can shift risk to their debt holders. If they do shift risk, then state debt-to-income ratios that are large enough lead to higher risk-taking, especially for underfunded pension plans. If states don't shift risk, then greater state debt is predicted to lead to lower risk.
5. The effect that state finances have on risk-taking also interacts with the risk premium channel of risk-free rates. If states can default on their debt, then for state debt-to-income ratios that are high enough, lower risk-free rates lead to higher risk-taking.

The appendix provides further comparative statics results for the case when state debt is risk free.

3. Data and Measurement of Risk, Underfunding

This section describes our data and the methodology we use to measure pensions funds' risk as well as their funding ratio.

3.1 Data

Publicly available data on PPFs' investment performance, risk-taking, and the value of liabilities is limited and incomplete. In this section we describe our data on PPFs and the methods we use to measure the PPFs' risk and underfunding despite the data limitations. Our main data set on state and local public pension plans is the Public Plans Database (PPD) from the Center for Retirement Research at Boston College.[17] The PPD currently contains plan-level annual data from 2001 through 2016 for 170 public pension plans: 114 administered by states and 56 administered locally. This sample covers 95 percent of public pension plan membership and fund assets nationwide.[18] The data set includes annual (by fiscal year) observations on the returns on each fund's assets, the percentage of the fund's portfolio invested in six main asset categories (equities, fixed income, real estate, cash, alternatives, and other), the market (fair) value of funds' assets, and the actuarial value of funds' liabilities.[19]

Descriptive statistics on our PPD data are contained in Table 1. On average across time and funds, the largest asset holdings were equities and fixed income (54 and 27 percent of total assets, respectively), followed by alternatives and real estate (10 and 5 percent, respectively). The value of assets represented only 80 percent of the actuarial value of liabilities, pointing to substantial underfunding. Figure 1 also shows the evolution of these variables over time: As shown in panel (b), the ratio of actuarial assets to liabilities (henceforth funding ratio, or FR) declined from about 100 percent in 2001 to little more than 70 percent in 2016. In panel (c), the

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[17] The PPF data is available at: http://publicplansdata.org/public-plans-database/download-full-data-set/.

[18] The sample of plans is a carry-over from the Public Fund Survey (PFS), which was constructed with an eye toward the largest state-administered plans in each state, but also includes some large local plans such as New York City ERS and Chicago Teachers.

[19] The data on holdings in some asset categories are further subdivided into foreign and domestic subcategories. Because the subcategories are not well populated, we combine like subcategories into the six broader categories noted above.

shares of equities and fixed income assets in funds' portfolios declined, while the share of alternative assets rose steeply from 5 percent in 2001 to almost 20 percent in 2016.

The data on funds' risk exposures is coarse. Therefore, to infer funds' risk, in the next subsection we bring in additional information by assuming that funds' returns in each asset category can be decomposed into the return on an asset-category index and fund-specific risk. Furthermore, we assume that each category index returns is spanned by a linear combination of returns on tradable market indices and estimate the linear combination for each category. Although we could choose a wider set of tradeable indices for our analysis, for now we rely on the 17 tradable indices, which are detailed in Table 2.

To measure underfunding, we would like to compare the market value of funds' assets with the market value of funds' liabilities. However, as noted in the introduction, the actuarial value of liabilities is measured using GASB standards that discount liability cash flows based on the properties of the funds' assets, not their liabilities. This approach to liability valuation is inconsistent with finance theory and has been widely criticized (see for example Brown and Wilcox, 2009). We adjust the value of plan liabilities using a discount rate that is reflective of the risk and timing of payments of PPF benefit obligations. In particular, following Rauh's (2017) methodology, based on the duration of each funds liabilities we infer the discount rates that funds should have used to discount them from the U.S. Treasury zero coupon yield curve.[20] We use information on funds' interest rate sensitivities to compute the duration and convexity of their liabilities. The information on the sensitivity of funds' liabilities to interest rate changes comes from GASB Statement 67 or from funds' Comprehensive Annual Financial Report (CAFR). This sensitivity information is only available starting in 2014, when GASB 67 required funds to report interest rate sensitivities.[21] GASB 67 data was available for 108 of the 170 funds

[20] The zero coupon Treasury yield curve is computed using the methodology in Gurkaynak et al (2006). This data is updated daily, and is provided by the Federal Reserve Board at: https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html.

[21] GASB Statement 67 disclosures require plans to disclose their Net Pension Liability (NPL) under alternative assumptions of the discount rate being 1 percentage higher and 1 percentage lower.

in the Boston College dataset.[22] As a robustness check, following Lucas (2017), we also revalue the liabilities with discount factors based on a high-quality corporate bond yield curve.[23]

We provide further information on our data in the discussion of results. The following two subsections discuss how we measure funds' risk and how we rediscount (i.e. revalue) their liabilities.

3.2 Risk Measurement

There are many possible measures of funds' portfolio risk that could conceivably be used in our analysis. We have chosen to focus on funds' 5% annual value-at-risk (VaR), which measures the minimum potential loss that a fund could sustain over a one-year horizon with 5% probability. For example, if the probability a fund could lose 12% or more over the next year is 5%, then the funds' 5% value-at-risk is 12%. Put differently, an annual loss of 12% or more is expected to occur in one out of every 20 years for the fund. An advantage of using VaR to measure portfolio risk is that it is comprehensive: it depends on the joint distribution of the returns on all of the assets in the fund's portfolio. In addition, VaR changes through time as the joint distribution of asset returns changes.

By contrast, some of the other risk measures that have been used in the literature on pension funds' risk-taking are less comprehensive or do not capture time variation in risk. For example, some papers in the literature have measured risk as the share of equities in a fund's portfolio. This measure is not comprehensive because it does not consider all portfolio assets. In addition, it does not capture time variation in the risk of portfolio assets. For example, two portfolios that have the same equity shares have different value at risk in 2006 and 2009 because market conditions in 2009 were much more volatile than in 2006. A risk measure that only focused on the share of equity holdings would be insensitive to this difference.

[22] We gathered CAFRs from public pension fund websites. In many instances, GASB 67 information was not included in the materials posted on the website.
[23] We use the return on the Citigroup Treasury Model Curve, which is created through a multistep process, starting with the Citigroup Corporate Index and taking corporate bonds rate AA-, AA, and AA+ by S&P. The data includes yields ranging from half a year to 30 years, reported monthly.

In fairness, an important part of the reason VaR has rarely been used in the academic literature on pension fund risk is because computing funds' VaR requires data on both funds' risk exposures and on the returns of the assets the funds are exposed to. The data available are much more limited; we know funds' total annual return as well as portfolio weights for six asset categories: equities, fixed income, real estate, cash, alternatives, and other. Another complicating factor is the data is not time-synchronous because returns and weights are both measured at the end of each fund's fiscal year.

An important contribution of our paper is that we develop a methodology to measure funds' VaR despite the data limitations. To do so, we make the following assumptions:

Assumption 1: Each fund i's return for asset category c at time t, $r_{c,i,t}$, can be expressed as the projection of the return onto a risk-category index $r_{c,t}$ that is common across funds plus a fund-specific residual return $\epsilon_{c,i,t}$ that is not correlated with any of the category return indices:

$$r_{c,i,t} = \alpha_{c,i} + r_{c,t} + \epsilon_{c,i,t} \qquad (7)$$

Assumption 2: The return of each risk-category index $r_{c,t}$ is spanned by the return of publicly available asset return indices indexed by j:

$$r_{c,t} = \alpha_c + \sum_j r_{j,t}\theta_{c,j} \qquad (8)$$

Assumption 1 is strong. It assumes within each risk category, fund returns for that category can be decomposed into a category index that is common across funds, plus a fund specific component that is uncorrelated with all of the category return indices.[24] Assumption 2 is weaker. It assumes the returns for each category index can be spanned by the returns of publicly available indices. Assumption 2 should be satisfied if the category return indices are well diversified, so they only depend on pervasive risk-factors (i.e., risk factors that affect a significant part of the

[24] Our analysis admits a slightly more general framework in which funds return within each category have the form:

$$r_{c,i,t} = \alpha_{c,i} + \beta_c r_{c,t} + \epsilon_{c,i,t},$$

with β_c common across funds. Without loss of generality, we have normalized β_c to 1 in our analysis.

economy), if the publicly-available indices are well diversified, and if the same factors that drive the returns on the category indices also drive the returns of the publicly-available indices.

Using the arbitrage pricing theory from finance, the intercepts in equations (7) and (8) should be nonzero only if they represent compensation for non-diversifiable risks that are priced by the market but not included as regressors in the equation; i.e. it is compensation for the risks in the residuals of the equations. Because the residuals in equation (7) are by assumption fund-specific and not captured by the category return indices, they should receive a zero price, hence $\alpha_{c,i} = 0$ for all c and i. Relatedly, in equation (8) because there are no residuals by assumption, α_c= 0 for all c. The implications of this reasoning are summarized in the following assumption:

Assumption 3: $\alpha_c = 0$ for all c, and $\alpha_{c,i} = 0$ for all c and i.

To illustrate how these assumptions make it possible to compute fund risk, note that each fund i's asset return can be written as the sum of its portfolio weights times its asset return in each category:

$$r_{i,t} = \sum_c w_{i,c,t} r_{c,i,t}.$$

Substituting in for each fund's category return from Assumption 1 and the decomposition of the category return from Assumption 2, this equation can be rewritten as a regression equation in which the right-hand side variables are an intercept term and funds' portfolio weights interacted with the returns on the publicly-available indices:

$$r_{i,t} = \sum_c w_{i,c,t}(\alpha_{c,i} + r_{c,t} + \epsilon_{c,i,t})$$

$$= \sum_c w_{i,c,t}\left[\alpha_{c,i} + \left(\alpha_c + \sum_j r_{j,t}\theta_{c,j}\right) + \epsilon_{c,i,t}\right]$$

$$= \sum_c w_{i,c,t}\left[\alpha_{c,i} + \alpha_c\right] + \sum_c \sum_j w_{i,c,t}\, r_{j,t}\theta_{c,j} + \sum_c w_{i,c,t}\,\epsilon_{c,i,t}$$

$$= \alpha_{i,t} + \sum_c \sum_j w_{i,c,t}\, r_{j,t} \theta_{c,j} + \epsilon_{i,t}$$

$$= \alpha + \sum_{c=1}^{C} \sum_{j=1}^{J} w_{i,c,t}\, r_{j,t} \theta_{c,j} + u_{i,t}, \qquad (9)$$

where

$$u_{i,t} = \epsilon_{i,t} + \left(\alpha_{i,t} - \overline{\alpha_{i,t}}\,\right), \qquad (10)$$

and $\alpha = \overline{\alpha_{i,t}}$ is the average value of $\alpha_{i,t}$ when averaged over all pension funds and time periods.

Equation (9) is a regression equation with a fairly large number of regressors. In particular, in what follows we estimate the intercept in equation (9) and the slope coefficients $\theta_{c,j}$ corresponding to 17 publicly traded indices (J=17) interacted with the portfolio weights for 6 categories of assets (C=6), for a total number of J×C=102 slope coefficients.

The estimated $\theta_{c,j}$ coefficients identify the stochastic part of the category return indices denoted by $\widetilde{r_c}$, whose time t realization is given by $\widetilde{r_{c,t}} = \sum_j r_{j,t} \theta_{c,j}$. The regression residuals $u_{i,t}$ that are recovered from estimation of the regression consist of two pieces. The first piece is $\epsilon_{i,t}$, which is the stochastic part of each fund's returns that is not explained by the category return indices. Additionally, $\epsilon_{i,t}$ is a component of the risk of funds' investments. The second piece is $\left(\alpha_{i,t} - \overline{\alpha_{i,t}}\,\right)$, which is a function of funds' portfolio weights and of the regression intercepts in equations (7) and (8). Importantly, the second piece of the residual is not a source of investment risk for the funds, hence if the residual has this component, then the funds' risk will be slightly overstated because of it. It turns out the second piece of the residual will be uniformly equal to zero if the regression intercepts in equations (7) and (8) are zero, as assumed using finance theory in Assumption 3. This implies that under Assumptions 1 – 3, the regression residuals in equation (9) will only contain the stochastic part of funds' investment risks that are not explained by the category return indices. In summary, our methodology can estimate the stochastic part of the funds' category returns and residual risks. Given these estimates, we can compute funds' value at risk through time. In what follows we describe how we estimate funds' value at risk and how we estimate the regression parameters in equation (9).

To estimate funds' value at risk, we make the following assumptions for the return of the category return indices and the idiosyncratic risks.

Assumption 4: Let $R_{c,t}$ denote the C×1 vector of category return indices in year t and let $\epsilon_{i,t}$ denote the residual return on fund i's investment portfolio in year t. Then,

$$R_{c,t} \sim N(\mu_t, \Sigma_t)$$

$$\epsilon_{i,t} \sim N(0, \sigma_\epsilon^2)$$

$$Cov(R_{c,t}, \epsilon_{i,t}) = 0.$$

Note that the category return indices and the residual returns are modeled as Gaussian for simplicity. Additionally, for simplicity the residual returns are for now modeled as independently and identically distributed (i.i.d.) across funds and time, and by Assumption 1 their covariance with the category indices is 0. There is scope to relax the conditions in Assumption 4 if needed.

In this paper we have chosen to measure funds' value-at-risk as the 5th percentile of the unexpected component of a fund's return distribution, and then express this quantity as a loss. This is best illustrated using an example. If fund $i's$ return has distribution $r_i \sim N(\mu_i, \sigma_i^2)$, then the unexpected component of the fund's return is the return less its expected value $r_i - \mu_i$. Furthermore the 5th percentile of the funds unexpected return distribution is $\Phi^{-1}(.05)\sigma_i = -1.65\sigma_i$. Expressed as a loss, $VaR(.05) = 1.65\ \sigma_i$.

Using analogous reasoning, a fund with portfolio weights $w_{i,t}$ at the beginning of time *t* has annual 5 percent value at risk given by

$$VaR_{i,t}(5\%) = 1.65\sqrt{w_{i,t}'\Sigma_t w_{i,t} + \sigma_\epsilon^2} \quad (11)$$

In order to compute VaR, we need measures of Σ_t and σ_ϵ^2. Because our data on pension fund returns is annual and has less than 20 annual time series observations, it would not be possible to estimate a time-varying Σ_t matrix using the short span of annual data on publicly-available indices that is used to estimate equation (9). To overcome this problem, we estimate Σ_t using daily data on the public indices returns. In particular, using the estimated coefficients for $\theta_{c,j}$ and daily data on public indices returns, we construct daily series of the returns on the category indices. Let $R_{c,d_1,t-1}, \ldots R_{c,d_N,t-1}$ denote the vector of estimated daily returns on the category indices for each trading day of the calendar year $t-1$. Our estimate of the annual variance-covariance matrix in calendar year t conditional on daily returns in year $t-1$ is:

$$\Sigma_t = 250 \times \frac{1}{N} \sum_{k=1}^{N} R_{c,d_k,t-1} * R'_{c,d_k,t-1}. \qquad (12)$$

Thus, Σ_t is equal to the conditional variance-covariance of daily category index returns scaled up by 250, the number of business days per year, to make it a variance-covariance matrix of annualized category index returns.[25]

Although we can estimate Σ_t using daily returns for the estimated category indices, we cannot estimate σ_ϵ^2 using daily data because we only observe pension fund returns annually, and hence can only observe annual residuals from equation (9). Therefore, to estimate σ_ϵ^2 we simply rely on the estimated residuals from equation (9) and estimate σ_ϵ^2 as the sample variance of the residuals.

As a robustness check for our analysis, we also estimate an unconditional version of Σ that is equal to the variance-covariance matrix of monthly returns scaled up to represent the variance-covariance matrix of annual returns. This matrix is unconditional because it is based on the average variance-covariance matrix of the monthly returns on the category return indices, and does not change over time.

Our approach for estimating each fund's value at risk has many advantages. The first and most important is we estimate the category return indices that best explain funds' annual returns and portfolio weights. This improves on other approaches that do not measure time variation in risk or assume that the returns in different asset categories are the returns of a particular traded index.

[25] In estimating Σ_t we made an assumption that expected daily returns are equal to 0. This assumption approximates the reality that expected returns at a daily frequency are close to 0. Because high frequency estimation of expected returns is very noisy, when estimating the variance-covariance matrix of high frequency returns it is better to set expected returns to 0 rather than trying to estimate them.

The second advantage is by using daily data we are able to overcome some of the limitations in estimating funds' risk on the basis of annual data. In particular, our approach produces estimates of funds' risk that vary through time because of changes in the variance-covariance matrix, and because of changes in funds' asset composition.

The third advantage of our approach is that we can infer the riskiness of funds by taking into account their different definitions of fiscal years. Because the fiscal years of different pension funds can end on different dates, their reported total annual returns span different intervals of a calendar year. Therefore, in the regression described by equation (9), it is important to match the total annual returns with annual market index returns computed in a manner consistent with the fiscal year definition for each fund. We then compute the VaR measure for calendar years to allow for a consistent comparison of riskiness across funds.

To construct our risk measures, it is necessary to estimate the $\theta_{c,j}$ coefficients from equation (9). This task involves the estimation of a large number of parameters with a relatively small sample of data. Therefore, we follow a method that avoids statistical problems associated with overfitting. Based on the fact that to some extent some of the 17 indices are correlated with each other, and given that there might be some common factors driving these indices, we assume that the dependent variable in equation (9) can be closely approximated by using a small subset of these indices for each asset category, which is the approximate sparsity assumption in our paper.[26] This assumption allows us to use a penalized estimation method to estimate the model parameters.

We use a two-step procedure to estimate the model in equation (9). First, we use a penalized regression method to select the most relevant subset of indices for each asset category. Second, we proceed with the estimation based on the selected indices only. In the first step, we use

[26] As discussed by Hastie et al. (2015), there are two settings of the sparsity condition. One is the so-called hard sparsity, in which only a small number of the true coefficient parameters are nonzero. This assumption is overly restrictive, so they also consider the other one, which is the so-called weak sparsity where the true coefficient parameters can be closely approximated by vectors with few nonzero entries, in order words, coefficients can be estimated based on a subset of the explanatory variables and letting the coefficients of the rest explanatory variables being zero. This weakly sparsity is more general and has been widely used in the literature of Lasso-type penalized methods. The approximate sparsity condition used in both this paper and Belloni and Chernozhukov (2013) is the weakly sparse condition.

LASSO regression for equation (9) (i.e., Least Absolute Shrinkage and Selection Operator, see Frank and Friedman, 1993, Tibshirani, 1996; and James et al., 2013) to select the most relevant indices for each asset class and shrink the coefficients on the other variables to 0, essentially eliminating them from the regression.[27,28] After using LASSO, the number of relevant explanatory variables shrinks considerably. Then in the second step, we apply OLS estimation with only the selected indices used as explanatory variables to obtain the estimates of $\theta_{c,j}$.

This two-step procedure estimator, the OLS post-LASSO estimator, is well known in the literature on high-dimensional sparse models. It has been shown that the OLS post-LASSO estimator performs at least as well as the LASSO estimator in terms of the rate of convergence, and has the advantage of a smaller bias (see Belloni and Chernozhukov, 2013).[29] This desirable characteristic holds even if LASSO may omit some components: as long as these components have relatively small coefficients, the OLS post-LASSO estimator still benefits from a high rate of convergence and smaller bias. Regarding the penalized selection method for the first stage, LASSO is a popular and powerful approach but not the only one. Researchers can choose different penalized methods from a variety of choices, for example, threshold LASSO, the Dantzig selector, etc.[30]

Finally, once we obtain the estimates of coefficients $\theta_{c,j}$ with the OLS post-LASSO estimator (reported in Table 3), Assumption 2 and the data on returns of publicly-traded indices allow us to compute the daily returns of category indices $R_{c,d_1,t-1}, \ldots R_{c,d_N,t-1}$ for each asset class c and calendar year $t-1$. Moreover, we identify each fund's residual risk, then use the estimated joint dynamics of category indices and the residual risk to compute funds' VaR as described above.

Figure 6 shows funds' VaR through time when the variance-covariance matrix is estimated on either a conditional or unconditional basis. The figures show that both the conditional and

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[27] In James et al (2013), the main description of LASSO can be found on pages 219 through 227. Useful introductory notes can also be found at: https://onlinecourses.science.psu.edu/stat857/node/158/

[28] Lasso has been used and its properties have been researched in many papers, for instance, Bickel et al. (2009), Meinshausen and Yu (2009), Van de Geer (2008), Zhang and Huang (2008), and so on.

[29] Belloni and Chernozhukov (2013) investigate the properties of OLS post-LASSO in the mean regression problem; Belloni and Chernozhukov (2011) also studies the post-penalized procedures, but different problem of median regression.

[30] Belloni and Chernozhukov (2013) also consider using the threshold LASSO for the first stage. For more details about the Dantzig selector, readers can refer to Bickel et al. (2009), Candes and Tao (2007), and among many others.

unconditional VaRs change over time, but the time variation of the conditional VaR is substantially larger. The reason is that conditional VaR changes through time due to changes in both market conditions and portfolio weights, while unconditional VaR only changes due to the funds' portfolio weights.

3.3 Measurement of Underfunding

In this subsection, we use the approach in Rauh (2017) and Novy-Marx and Rauh (2011) to revalue the PPFs' liabilities using a discount rate that is more appropriate for the riskiness of liabilities and the timing of cash flows. We then use the rediscounted liabilities to better measure the extent of fund's underfunding. Rauh refers to his approach of revaluing the funds' liabilities as rediscounting. Because PPFs' cash flows to liability holders are nearly risk-free, Rauh (2017) rediscounts PPFs' liabilities using the yield of a zero-coupon U.S. Treasury bond whose duration matches the duration of the funds' liabilities. This is the correct discount factor to use if all of the beneficiaries' cash flows occurred at the duration of the liabilities, and is otherwise a reasonable approximation of the correct discount factor. The adjusted value of liabilities is found by using the sensitivity of PPFs' liabilities to changes in interest rates, and then revaluing the liabilities by changing the interest rate used for discounting as discussed below.

The extent of each pension plan's underfunding is measured using its funding ratio, which is an estimate of the present value of its assets over its liabilities:

$$Funding\ Ratio\ (FR) = \frac{Actuarial\ Assets\ (AA)}{Total\ Pension\ Liabilities\ (TPL)}. \qquad (13)$$

In this expression, a lower ratio reflects greater underfunding. We use two measures of total pension liabilities (TPL) to compute funding ratios. One is the amount of TPL_r reported by the PPFs themselves, which are discounted based on their reported expected rates of return r on their asset portfolios. The other measure is the one we obtain following the approach in Rauh (2017) denoted as $TPL_{r'}$ in equation (14), which is an approximation of what *TPL* should be if discounted at the correct duration-matched, risk-free, zero-coupon rate r'. Rauh computes $TPL_{r'}$ using a second-order Taylor series of how TPL_r should change if discounting takes place at rate r' instead of r. Expressed in terms of duration and convexity, the second-order Taylor series has form:

$$TPL_{r'} = TPL_r - TPL_r * Duration * \Delta r + 0.5 * TPL_r * Convexity * (\Delta r)^2 \quad (14)$$

where $\Delta r = r' - r$, with r' denoting the duration-matched Treasury yield[31] and r the funds' original discount rate.[32] To compute the second-order approximation, estimates of duration and convexity are required. GASB 67 provides information on the value of funds' liabilities when valued at its reported discount rate r as well as when discount rates increase or decrease by one percentage point. With this information, the duration and convexity are approximated as:

$$Duration = -\frac{TPL_{\{r+0.01\}} - TPL_{\{r-0.01\}}}{0.02 * TPL_r}, \quad (15)$$

$$Convexity = \frac{TPL_{\{r+0.01\}} + TPL_{\{r-0.01\}} - 2 * TPL_r}{(0.01)^2 * TPL_r}. \quad (16)$$

The PPFs have only recently started reporting the sensitivity of the value of their liabilities to interest rate changes under GASB 67. We use data on PPFs' funding status for the year 2015 to illustrate the importance of rediscounting the PPFs' liabilities using appropriate discount rates. In Figure 7 (left panel), the chart plots funds' reported liabilities ("Total Pension Liabilities") against their adjusted liabilities obtained after discounting them at the correct risk-free rates ("TPL rediscounted"). If rediscounting made no difference, all observations would line up along the 45 degrees line. However, most observations are situated above the 45 degree line, showing that rediscounting at the more realistic, lower rates boosts the present value of liabilities to almost double the reported amounts. Also in Figure 7 (right panel), the chart shows a similar relationship between the funding ratios measured using reported and rediscounted liabilities. In this case, the observations fall below the 45 degrees line, showing that rediscounting boosts liabilities and, as a result, reduces the funding ratios by almost half. Notably, the plot observations do not suggest a linear relationship between the original and adjusted funding ratios. Put differently, re-computing funding ratios with rediscounted liabilities is not equivalent to a linear rescaling of the original funding ratios. Instead, rediscounting liabilities with appropriate risk-free discount rates applies a consistent across funds pricing methodology that follows from sound economic pricing principles. This methodology also corrects the measurement error that occurs if liabilities are discounted at expected rates of return that differ

[31] r' is the rate on the Treasury yield curve that matches the duration of each PPFs' liabilities.
[32] The discount rate is available in the GASB statement 67 reports as "Current Discount Rate."

across funds (because of differences in funds' asset portfolios), and are not risk-free, as we discuss below.

4. Empirical Analysis of Risk-Taking Behavior

In this section we empirically study how the PPFs' asset risk is related to plan underfunding, to risk-free interest rates, and to state finances. We use three separate frameworks for our empirical analysis. First, we study the cross-sectional relation between PPFs' riskiness and lagged funding ratios across funds at one point in time (the year 2016), for which the best data is available to measure duration and convexity, rediscount liabilities, and measure plan underfunding.[33] Second, we estimate the cross-sectional relation between risk-taking and funding ratios for each year in the sample. Third, in a panel data context, we study the importance of funding ratios, risk-free rates, and the fiscal condition of the funds' state sponsors as determinants of risk-taking, while allowing for interactions among them. We use our theoretical analysis from Section 2 to interpret our theoretical findings.

4.1 Cross-Sectional Results for 2016: Risk vs. Underfunding

Figure 8 illustrates the cross-sectional relation between the VaR-based measure of risk for 2016 on the vertical axis and the one-year lagged funding ratios on the horizontal axis, using either the original TPL_r reported by funds (left panel) or the $TPL_{r'}$ obtained with rediscounted liabilities (right panel) to compute funding ratios.

Several conclusions emerge from the comparison of the two panels. First, the link between funds' riskiness and the one year-lagged funding ratios is negative and statistically significant in both cases, i.e., funds with ex-ante lower funding ratios had asset portfolios with higher risk. This is consistent with funds' reaching for yield, as described in Section 2. Second, rediscounting liabilities shifts the entire distribution of funding ratios to the left, as discussed earlier. Third, the approach with rediscounted liabilities results in a steeper slope coefficient and higher statistical significance for the link between riskiness and funding ratios, as well as a

[33] We use a recent year for analysis because we need to rely on GASB 67 data, a relatively new reporting, in order to rediscount reported liabilities.

higher regression R-squared (right panel). The increased statistical significance and higher R-squared are consistent with an interpretation that funding ratios, when computed using liabilities discounted with risk-free rates, are related to funds' risk-taking. Furthermore, it supports the idea that funding ratios based on liabilities discounted with expected returns on assets are measured with error, since classical measurement error reduces both goodness of fit and statistical significance.

4.2 Cross-Sectional Results over Time: Risk vs. Underfunding

When examining the relation between PPFs' riskiness and funding ratios over the entire sample period, data availability constrains our ability to rediscount liabilities going back in time because the duration and convexity data needed to perform rediscounting (based on a new GASB accounting requirement) was not available before 2014. To overcome this important data limitation, we adopt two strategies to be able to use funding ratios with rediscounted liabilities for the entire panel.

First, we use the funding ratio with rediscounted liabilities for 2015 computed like in equation (5) as a time-invariant proxy for the funds' underfunding status over the entire sample period. Despite obvious shortcomings, this approach receives some validation from evidence that the funds' relative underfunding has been persistent over the sample period: From looking at reported funding ratios, funds that were relatively more underfunded prior to the 2008 global financial crisis tended to remain more underfunded in the post-crisis period.[34] This approach also avoids the potential measurement error that could result if the duration and convexity from the most recent years are used to rediscount liabilities from the distant past. Measurement error could also occur if the duration and convexity reported specifically for Total Pension Liabilities (available only since 2014) were applied to Actuarial Liabilities (available for the entire sample period).

[34] Using the PPFs' own reported funding ratios computed as Actuarial Assets divided by Actuarial Liabilities, we find that the average funding ratios for 2014-16 are highly informative about the average funding ratios for 2001-06 in the cross-section of funds. Regressing the latter on the former, we obtain a slope coefficient close to unit (0.83), statistically significant at the 1 percent level, and an R-squared value of 0.43.

Second, as an alternative to the approach with fixed funding ratios, we make a number of strong assumptions to rediscount past liabilities and compute time-varying funding ratios for the entire panel interval. Specifically, we assume that the duration and convexity of TPLs observed for the most recent years are informative about the past, and adjust them by the evolution of fund- and state-level demographics over time.[35] Then we use the adjusted duration and convexity along with the duration-matched, zero-coupon Treasury yields to rediscount Actuarial Liabilities (instead of TPLs), and thus to recompute funding ratios as Actuarial Assets divided by Actuarial Liabilities going back to 2001 (Figure 9).

While neither of the two approaches above is without criticism, each provides insight into the role of funding ratios and risk-free rates as determinants of funds' risk-taking behavior over time. Thus, using the original and rediscounted funding ratios obtained under each of the two approaches, Table 4 shows the cross-sectional link between PPFs' riskiness and funding ratios for each year during 2002-2016. In panels (a) and (b), using the funding ratios fixed at their 2015 values, the results show a negative link between PPFs' riskiness and funding ratios. Similarly, using the time-varying funding ratios in panels (c) and (d), the results show a negative link between riskiness and lagged funding ratios. Notably, the link is statistically significant for the interval 2012-2016, which largely coincides with the post-crisis period of low risk-free rates.

4.3 Panel Data Results: Risk vs. Underfunding, Interest Rates, and State Finances

For our panel analysis, we examine the link between funds' riskiness as the dependent variable and the following explanatory variables: (i) funding ratios computed as in Section 4.2; (ii) proxies for the level of risk-free rates, including the one- and 10-year Treasury yields as well as an indicator variable for the post-crisis period of low interest rates; and (iii) measures of the

[35] To adjust duration and convexity for trends in fund characteristics and state-level demographics, we first take the mean of duration and convexity over the period 2014-2016. Then we estimate cross-sectional regressions for duration and convexity on the funds' beneficiaries-to-members ratio and the states' life expectancy at birth in 2015. For both duration and convexity, we find a negative and statistically significant relation with the beneficiaries-to-members ratio, and a positive and significant relation with life expectancy (i.e., fewer beneficiaries relative to members at the fund level and relatively higher life expectancy at the state level were associated with higher duration). Finally, we use the cross-sectional estimates from 2015 and historical fund- and state-level data to obtain time series for duration and convexity over 2001-2015.

sponsor states' public finances, such as debt-to-income ratios and state bond ratings. These variables mimic the determinants of risk-taking behavior considered in the model in Section 2.

We use both approaches discussed in Section 4.2 to overcome data limitations and compute rediscounted funding ratios for the entire sample period. For the approach with fixed funding ratios, we use the following specification:

$$VaR_{pt} = \alpha + \beta * FR_p + \gamma * TrYield_t + \delta * FR_p * TrYield_t + \epsilon_{pt} \quad (17)$$

where *p* denotes a fund, *t* denotes the year, and VaR_{pt} is the conditional measure of risk defined in Section 3.2. Among the explanatory variables, FR_p is the funding ratio computed as actuarial assets divided by actuarial liabilities (both original and rediscounted). Since the funding ratio is time-invariant, the effect of FR_p would be fully absorbed by fund fixed effects. We therefore do not include fund fixed effects in equation (17). In different specifications, $TrYield_t$ is the 1-year and the 10-year Treasury yield, or alternatively is replaced by a post-crisis dummy variable for the 2009-2016 period of low interest rates. To avoid the type of reverse causation potentially driven by economic downturns, when financial market volatility coincides with monetary policy easing, we exclude years 2002, 2003, and 2009 from the sample. We cluster the standard errors at both the fund and year level.

The results for equation (17) are presented in Table 5. In all columns, the coefficients on funding ratios are negative and statistically significant, showing that underfunding is associated with more risk-taking. Turning to the risk-free rates, the coefficients on the 1-year Treasury yield are negative and statistically significant (columns 1-2), showing that funds engaged in more risk-taking in the low-interest rate environment. Similarly, the coefficients on 10-year Treasury yield are negative (although not statistically significant, columns 3-4), while the coefficient on the post-crisis dummy variable are positive (and borderline significant, columns 5-6). Importantly, the coefficients on interaction terms between Treasury yields and funding ratios are positive and statistically significant (columns 2, 3, and 4), while those on interactions between the post-crisis dummy and funding ratios are negative and statistically significant (columns 5-6). These results suggest that the more underfunded funds took more risk especially during episodes of low interest rates. Notably, the slope coefficients on funding ratios—both individual and interacted with risk-free rates—are larger in magnitude and gain statistical significance when the funding ratios are based on rediscounted liabilities (columns 2, 4, and 6). The result suggests that rediscounting provides a more consistent comparison of the extent of true underfunding across funds, which in turn acts as a driver of risk-taking.

Interpreted through the model of Section 2, these results provide empirical support for the role of underfunding and risk-free rates as drivers of risk-taking through the reach-for-yield and risk-premium channels, both individually and interacted with each other, as illustrated in Figure 2.

For the approach with time-varying funding ratios, the specification is:

$$VaR_{pt} = \alpha + \beta * FR_{pt-1} + \gamma * TrYield_t + \delta * FR_{pt-1} * TrYield_t + \mu_p + \epsilon_{pt} \qquad (18)$$

which is similar to equation (17) in most respects, except that FR_{pt-1} is the time-variant funding ratio computed as Actuarial Assets divided by Actuarial Liabilities, with liabilities taken either as reported or rediscounted. Due to multicollinearity between funding ratios and interest rates, it is infeasible to identify their effects on risk-taking separately. To overcome this problem, we demean the funding ratios by the cross-sectional mean of each year to remove the time-series variation associated with interest rates.[36] We cluster the standard errors at both the fund and year levels. In some specifications, we also include the fund fixed effect μ_p.

Table 6 shows similar results for equation (18). The coefficients on lagged funding ratios are always negative and statistically significant at the 1 percent level in the case without fund fixed effects (columns 1-4). They remain negative but their statistical significance decreases in the presence of fund fixed effects (columns 5-8). Given the persistence in underfunding status across funds over time, i.e., funds that started by being underfunded remained underfunded, it is not surprising that the fund fixed effects decrease the explanatory power of funding ratios. Therefore, to avoid the role of funding ratios being obscured by fixed effects, we report regression results with and without fund fixed effects. Regarding the role of risk-free rates, the coefficients on Treasury yields are negative and statistically significant (columns 1, 2, 5, and 6), while those on the post-crisis dummy are positive (columns 4 and 8), indicating that PPFs took

[36] The Variance Inflation Factor (VIF) for the original funding ratio is 10.9, which exceeds the threshold level of 10 and thus suggests a high degree of multicollinearity. After time-demeaning, the VIF for the funding ratio drops to 1.

more risk when the risk-free rates were lower. The interactions between Treasury yields and funding ratios are positive and statistically significant (columns 1, 2, 3, 6, and 7), while those between the post-crisis dummy and funding ratios are negative and statistically significant (column 6), showing that funds with relatively lower funding ratios engaged in more risk-taking especially during periods with low risk-free rates. Notably, the coefficients on interacted terms remain statistically significant in the presence of fund fixed effects. Like before, the coefficients on interacted terms are greater in magnitude and gain statistical significance when funding ratios are based on rediscounted liabilities (columns 1 vs. 2 and 5 vs. 6), highlighting the importance of measuring underfunding consistently across funds. Once again, interpreted through the model, these results show lower funding ratios and risk-free rates, as well as their interaction are determinants of risk-taking as discussed in Section 2 and illustrated in Figure 2.

To examine the role of sponsor states' public finances as an additional determinant of funds' risk-taking behavior, we add relevant measures of state finances in equations (17) and (18). Specifically, $State_{pt}$ is measured as either the states' debt-to-income ratios or state bond ratings. For state bond ratings, higher values imply worse ratings. The variables are demeaned relative to the sample average for each year. They enter the regressions both in levels and interacted with the funding ratios and risk-free rates, as follows:

$$VaR_{pt} = \alpha + \beta * State_{pt} + \gamma * TrYield_t + \delta * FR_p + \eta * State_{pt} * TrYield_t + \epsilon_{pt} \qquad (19)$$

$$VaR_{pt} = \alpha + \beta * State_{pt} + \gamma * TrYield_t + \delta * FR_p + \eta * State_{pt} * FR_p + \epsilon_{pt} \qquad (20)$$

The results are shown in Table 7. On the first two rows, the coefficients for state debt-to-income ratios and state bond ratings are positive and statistically significant in most specifications. They show that funds sponsored by states with higher debt-to-income ratios or worse bond ratings engaged in more risk-taking behavior. Like before, the coefficients on risk-free rates and funding ratios taken individually are negative and statistically significant in most specifications. These results are consistent with funds shifting risk to state debt holders as shown in the model and illustrated in Figures 3, 4, and 5.

The results for state finances interacted with Treasury yields as in equation (19) are presented in columns 1 and 4. The negative coefficients on the interaction terms suggest that during periods of low risk-free interest rates, it is especially the funds with state sponsors with higher debt-to-income ratios and worse bond ratings that take more risk. These results support the model implications with risky state debt and risk-shifting discussed in Section 2 and illustrated in Figure 5. Specifically, for a level of state debt that is large enough, lower risk-free rates provide an incentive for funds to take more risk, as their sponsor states shift the risk to debtholders. The interaction between state finances and funding ratios as in equation (20) has a positive coefficient; this finding departs from the predictions associated with risk-shifting. Overall, the majority of empirical results support the role of state public finances as a determinant of risk-taking behavior through risk-shifting.

4.4 Robustness Checks

We perform a set of robustness tests for the results presented in Sections 4.1 and 4.3. Our tests involve using alternative measures of risk as the dependent variable, such as the conditional VaR computed with portfolio shares that abstract from valuation changes,[37] or the unconditional VaR measure. They also involve alternative explanatory variables, such as funding ratios obtained by rediscounting liabilities with duration-matched, high-quality corporate bond yields instead of Treasury yields. Our robustness tests provide strong support for the earlier results.

For the cross-sectional relation between the PPFs' riskiness and lagged funding ratios in 2016, the negative link is robust to using the conditional VaR with portfolio shares that abstract from valuation changes (Figure 10, panel a), the unconditional VaR (panel b), or funding ratios rediscounted with corporate bond yields (panel c).

For the panel data analysis, the robustness checks in Table 8 support our earlier results. The negative coefficients on funding ratios and positive coefficients on the interactions with Treasury yields are preserved when the conditional VaR is computed with portfolio shares that abstract

[37] We decompose the change in portfolio weights Δw_{pat} (fund *p*' portfolio share of asset type *a* held at time *t*) into two parts: (1) a more passive change component driven by valuation changes, and (2) a residual change component that more closely resembles active portfolio reallocations that involve trading. The valuation-driven weight change is defined as: $ValuationChange_{pat} = w_{\{p,a,t-1\}} * \frac{1+R_{at}}{\sum_{j=1}^{6} w_{\{p,j,t-1\}} * (1+R_{jt})} - w_{\{p,a,t-1\}}$, where R_{at} is the net return of asset class a. The reallocation change is defined as: $Reallocation_{pat} = w_{pat} - ValuationChange_{pat}$. Subsequently, we cumulate the reallocation changes to obtain the weights that abstract from valuation changes. See Ahmed et al. (2016) for similar decompositions applied to international portfolio flows.

from valuation changes, when the unconditional VaR measure is used, and also when funding ratios are based on liabilities rediscounted with corporate bond yields. This is the case for both fixed and time-varying funding ratios, and for their interactions with Treasury yields. The 1-year Treasury yield has a negative and statistically significant impact on risk-taking in most specifications, with the exception of the unconditional VaR (columns 2 and 5). The coefficient on the 10-year Treasury yield is negative but not statistically significant in any specification (columns 7-12).

We also consider using a more traditional measure of risk as the dependent variable, such as the share of risky assets in the composition of PPF portfolios. In Table 9, we find some evidence of risk-taking behavior with the share of alternatives as the measure of risk, albeit considerably weaker than with the conditional VaR measure of risk. In this case, only the negative coefficients on 1-year and 10-year Treasury yields, as well as the positive coefficients on the post-crisis dummy variable preserve statistical significance, which is consistent with the rising share of alternatives in funds' portfolios over time. However, the coefficients on funding ratios and state debt-to-income ratios are not statistically significant, and neither are the interacted terms between funding ratios, risk-free rates, and state finances.

5. Economic Significance

This section quantifies the contributions of underfunding and low risk-free rates to the funds' risk-taking behavior, and also infers their potential implications for state public finances.

5.1 Magnitude of Risk-Taking Behavior due to Underfunding and Low Interest Rates

To quantify the contributions of various determinants to funds' risk-taking behavior, we use our cross-section and panel regression results from Sections 4.1 and 4.3 to decompose the conditional VaR measure into two components: one that is related to the most recent levels of underfunding and risk-free rates, and a residual that abstracts from these two factors. We follow two approaches to perform this decomposition.

First, the cross-sectional results in Figure 8 (panel b) imply the following relation between risk-taking and the funding ratio: $VaR_{p,2016} = \alpha - 0.0319 * FR_{p,2015}$. Since a lower funding ratio is associated with higher risk, we define the risk component associated with underfunding for each fund p relative to the fully-funded counterfactual:

$$VaR^{UF}{}_{p,2016} = 0.0319 * \left(1 - FR_{p,2015}\right),$$

where the brackets capture the gap between the fully-funded counterfactual status (i.e., FR=1) and each fund's actual funding ratio ($FR_{p,2015}$). The residual is the risk component that abstracts from underfunding:

$$VaR^{RES}{}_{p,2016} = VaR_{p,2016} - VaR^{UF}{}_{p,2016}.$$

To convert our measure of risk into dollar losses that each fund would have suffered if a severe stress event were to materialize in 2016 (i.e., an event with returns in the bottom 5 percentile of realizations), we multiply each fund' actuarial assets at the end of 2015 by its 5-percentile VaR.[38] We then aggregate the dollar losses across funds at the state level and express them as a fraction of state income for scaling purposes.[39] Using this metric in Figure 11 (panel a), we illustrate the risk component related to underfunding (the red portion of the bars) and the residual component (the yellow bars). On average across states, the risk component associated with underfunding represented about 12 percent of the total risk in 2016.

Second, we repeat the exercise using the panel results from Table 5 (column 2). We isolate the portion of total risk driven by both (i) underfunding defined relative to the fully-funded counterfactual and (ii) the low level of the 1-year Treasury yield in 2016 relative to its pre-crisis average. The residual is the risk component that abstracts from both underfunding and the low

[38] The magnitude of projected losses should be interpreted cautiously, because the variance-covariance of daily returns may differ during crisis vs. normal years.

[39] We rescale VaR losses to take into account the fact that, in some states, the data on duration and convexity necessary to rescale liabilities is only available for a sub-set of funds. In such cases, we scale up the predicted 5-percent VaR losses by the inverse of the fraction of fund liabilities from each state for which duration and convexity data are available.

risk-free rates in 2016.[40] Like before, we translate risk into dollar losses, which we aggregate at the state level and normalize by state income. In Figure 11 (panel b), the risk component associated with underfunding and low risk-free rates (the red portions of the bars) represented on average about 32 percent of the total risk in 2016.

Overall, we find that the funds' risk-taking behavior related to underfunding was responsible for about 12 percent of total risk, and the risk-taking behavior related to both underfunding and low risk-free rates accounted for about one-third of their total risk at the end of our sample period.

5.2 Implications for State Finances

The results of our study have implications for the state and local-level public finances. The shift of funds into riskier investments raises concerns regarding the potential impact of sharp declines in asset values or low returns on state and local finances.[41] Sponsors of these plans typically have little ability to alter benefit levels or the terms of retirement plans.[42] As a result, most of the downside risks associated with a decline in asset values or lower investment returns is likely to be borne by the taxpayers of the jurisdiction sponsoring the plan. In light of the results in Section 4.3, the burden of possible pension fund losses on state finances could be sizeable: As shown in Figure 11, the potential loss implied by the 5-percentile VaR risk, which corresponds to a severe stress event occurring once in 20 years, would have represented about 3 percent of state income on average across the U.S. states in 2016. Compared to the average state debt-to-income ratio of 7.7 percent, the potential loss associated with the 5-percent VaR would have represented about 39 percent of the state-level public debt, with up to 1/3 driven by risk-taking behavior attributed with underfunding and low risk-free rates in recent years.

In addition, the impact of funds' risk-taking behavior on state finances is likely to be skewed, with states with weaker finances likely to be hit more. In Section 4.3, we find that funds from the more financially-constrained states are more likely to assume additional levels of risk. That

[40] Given the results in Table 5, column 2, the risk component associated with both underfunding and lower rates in 2016 relative to the pre-crisis period is: ${VaR^{UFLR}}_{p,2016} = -0.035 * (FR_{p,2015} - 1) - 0.018 * (TrYield_{2016} - TrYield_{2001-07}) + 0.0057 * (TrYield_{2016} * FR_{p,2015} - TrYield_{2001-07} * 1)$, where $1yrTrYield_{2016} = 0.667$ and $1yrTrYield_{2001-07} = 3.092$.
[41] Boyd and Yin (2017).
[42] Munnell and Quimby (2012)

is, risk-taking behavior is most pronounced among funds with sponsors with the least ability to bear additional risk.

6. Conclusion

This paper examined the determinants of risk-taking behavior by U.S. public pension funds. To motivate and interpret our empirical analysis, we developed a simple theoretical model that relates funds' risk-taking to their underfunding (the reach-for-yield channel), to interest rates (the risk-premium channel), and to the condition of state public finances. To measure risk, we developed a new methodology for inferring funds' asset exposures and compute their Value-at-Risk on the basis of limited public data. In addition, to create meaningful measures of underfunding, we rediscount their liabilities with discount rates that better match the riskiness of liabilities. We find evidence consistent with both the reach-for-yield and risk-premia channels of risk-taking behavior, as funds take more risk in response to underfunding and low interest rates on safe assets. The effects of low interest rates on risk-taking are especially pronounced for funds that are more underfunded or are affiliated with states with weaker public finances.

Our measure of risk also allow us to compute the losses that would be suffered by public pension funds under a severely adverse economic scenario, which would place an additional burden on the public finances of sponsoring states. Based on our results, we infer that the potential loss transferable to the states if a 1-in-20 years adverse return event were to occur in 2016 would have been on average about 3% of states' income, or about 39% of states' debt. We attribute between 12% and 32% of the losses to the component of funds' risk-taking behavior driven by underfunding and low risk-free rates.

References

Ahmed, Shaghil, Stephenie Curcuru, Frank Warnock, and Andrei Zlate, 2016. Decomposing international portfolio flows. mimeo, Federal Reserve Bank of Boston, Federal Reserve Board, and the University of Virginia.

Andonov, Alexander, Rob M. M. J. Bauer and K. J. Martijn Cremers, 2017, Pension fund asset allocation and liability discount rates, *Review of Financial Studies*, 30., 2555-2595.

Becker, Bo and Victoria Ivashina, 2015, Reaching for yield in the bond market, *Journal of Finance*, 70, 1863-1901.

Belloni, Alexanrdre. and Victor Chernozhukov, V., 2011, l_1-penalized quantile regression in high-dimensional sparse models, *Annals of Statistics*, 39, 82-130.

Belloni, A. and Chernochukov, V., 2013, Least squares after model selection in high-dimensional sparse models, *Bernoulli*, 19(2), 521-547.

Bickel, Peter.J., Ya'acov Ritov, and Alexandre B. Tsybakov, 2009, Simultaneous analysis of lasso and Dantzig selector, *Annals of Statistics*, 37, 1705-1732.

Boubaker, Sabri, Dimitrios Gounopoulos, Duc Khuong Nguyen and Nikos Paltaldis, 2018, Assessing the effects of unconventional monetary policy and low interest rates on pension fund risk incentives, *Journal of Banking and Finance*, v. 92, pp. 340-357.

Boyd, Don and Yimeng Yin, 2017, Investment risk-taking by public pension plans: potential consequences for pension funds, State and Local Governments, and Stakeholders in Government, working paper, The Rockefeller Institute of Government, State University of New York.

Boyer, Chuck, 2018, Public pensions, political economy and state government borrowing costs, working paper, University of Chicago.

Brown, Jeffrey R. and David W. Wilcox, 2009, Discounting state and local pension liabilities, *American Economic Review: Papers and Proceedings*, v. 99, n. 2, pp. 538-542.

Campbell, John Y., 1987, Stock Returns and the Term Structure, *Journal of Financial Economics*, 18 (2), 373-399.

Candes, Emmanuel and Terrence Tao, 2007, The Dantzig selector: statistical estimation when p is much larger than n. *Annals of Statistics*, 35, 2313-2351.

D'Arcy, Stephen P., Dulebohn, James H., and Oh, Pyungsuk, 1999, "Optimal Funding of State Employee Pension Systems," *Journal of Risk and Insurance*, 66, (3): 345-380.

Dell'Ariccia, Giovanni, Luc Laeven and Gustavo Suarez, 2017, Bank leverage and monetary policy's risk taking channel: evidence from the United States, *Journal of Finance*, 72, 613-654.

Di Maggio, Marco and Marcin Kacperczyk, 2017, The unintended consequences of the zero lower bound policy, *Journal of Financial Economics*, 123, 59-80.

Epple, Dennis and Katherine Schipper, 1981, "Municipal Pension Funding: A Theory and Some Evidence," *Public Choice,* 37 (1), Carnegie Papers on Political Economy, Volume 1: Proceedings of the Carnegie Conference on Political Economy (1981), 141-178.

Frank, Ildiko .E. and Jerome H. Friedman, 1993, A statistical view of some chemometrics regression tools, *Technometrics*, 35, 109-135.

Gurkaynak, Refet S, Sack, Brian, and Jonathan H. Wright, 2006, "The U.S. Treasury Yield Curve, 1961 to the Present," Finance and Economics Discussion Series, 2006-28.

Hastie, Trevor, Tibshirani, Robert, and Martin Wainwright, 2015, Statistical Learning with Sparsity: The Lasso and Generalizations, Boca Raton FL: CRC Press.

Ivanov, Ivan T., and Tom Zimmermann, 2018, "The 'Privatization' of Municipal Debt," Hutchins Center Working Paper #45, Hutchins Center on Fiscal and Monetary Policy at Brookings.

James, Gareth, Daniela Witten, Trevor Hastie, and Robert Tibshirani, 2013. An introduction to statistical learning, v. 112, New York: Springer.

Jiménez, Gabriel, Steven Ongena, José-Luis Peydró and Jesús Saurina, 2014, Hazardous times for monetary policy: what do twenty-three million bank loans say about the effects of monetary policy on credit risk taking?, *Econometrica*, 82, 463-505.

Glaeser, Edward L, and Ponzetto, Giacomo A.M., 2014, "Shrouded costs of government: The political economy of state and local pension plans," *Journal of Public Economics*, 116, 89-105.

Kelley, Dashle, 2014, The Political Economy of Unfunded Public Pension Liabilities, *Public Choice*, v. 158, no 1/2, pp. 21-38.

Lucas, Deborah, 2017, "Towards Fair Value Accounting for Public Pensions: The Case for Delinking Disclosure and Funding Requirements," Prepared for Harvard Kennedy School Conference: "Gathering Storm: The Risks of State Pension Underfunding," October 19-20, 2017.

Lucas, Deborah J., and Stephen P. Zeldes, 2009 "How Should Public Pension Plans Invest?," *American Economic Review*, 99 (2) May: 527-32.

Meinshausen, Nicolai and Bin Yu, 2009, Lasso-type recovery of sparse representations for high-dimensional data, *Annals of Statistics*, 37, 246-270.

Mennis, Greg, Susan Banta and David Draine, 2018, Assessing the risk of fiscal distress for public pensions: Mossavar-Rahmani Center for Business and Government, no 92.

Mohan, Nancy and Ting Zhang, 2014, An analysis of risk taking behavior for public defined benefit pension plans, *Journal of Banking and Finance*, v. 40, n. 20, pp. 403-419.

Munnell, Alicia H. and Laura Quinby, 2012, Legal constraints on changes in state and local pensions, Center for Retirement Research at Boston College.

Naughton, James, Reining Petacchi and Joseph Weber, 2015, Public pension accounting rules and economic outcomes, *Journal of Accounting and Economics*, v. 59, pp. 221-241.

Norcross, Eileen, and Olivia Gonzalez, 2016, “Ranking the States Fiscal Condition: 2016 Edition,” Mercatus Research, Mercatus Center at George Mason University, June.

Novy-Marx, Robert, and Joshua D. Rauh, 2011, Public Pension Promises: How Big Are They and What are They Worth? *Journal of Finance* 66(4), pp. 1211-1249.

Novy-Marx, Robert, and Joshua D. Rauh, 2012, Fiscal imbalances and borrowing costs: evidence from state investment losses, *American Economic Journal: Economic Policy*, v. 4, pp. 182-213.

Novy-Marx, Robert, and Joshua D. Rauh, 2014, The revenue demands of public employee pension promises, *American Economic Journal: Economic Policy*, v. 6, pp. 193-229.

Paye, Bradley P., and Allan Timmermann, 2006, Instability of Return Prediction Models, *Journal of Empirical Finance*, 13: 274-315.

Pennacchi, George and Mahdi Rastad, 2011, Portfolio allocation for public pension funds, *Journal of Pension Economics and Finance*, v. 10, pp. 221.245.

Pettenuzzo, Davide, Timmermann, Allan, and Rossen Valkanov, 2014, Forecasting Stock Returns Under Economic Constraints, *Journal of Financial Economics*, 114: 517-553.

Rajan, Raghuram G, 2005, Has Financial Development Made the World Riskier? NBER Working Paper 11728.

Rapach, David E., and Guofu Zhou, 2013, “Forecasting Stock Returns,” in Elliot, G., Timmermann, A., (Eds.), Handbook of Economic Forecasting, vol 2., North-Holland.

Rapach, David E., Strauss, Jack K., and Guofu Zhou, 2010, Out-of-Sample Equity Premium Prediction: Combination Forecasts and Links to the Real Economy, *Review of Financial Studies,* 23 (2): 821-862.

Rauh, Joshua D., 2017, Hidden debt, hidden deficits: 2017 Edition, Hoover Institution.

Tibshirani, Robert. 1996. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*, 267-288.

Timmermann, Allan, 2018, Forecasting Methods in Finance, *Annual Reviews of Financial Economics*, 10: 449-479.

Van de Geer, Sara.A., 2008, High-dimensional generalized linear models and the lasso, *Annals of Statistics*, 36, 614-645.

Welch, Ivo, and Amit Goyal, 2008, "A Comprehensive Look at the Empirical Performance of Equity Premium Prediction," *Review of Financial Studies,* 21: 1455-508.

Yellen, Janet, "Assessing potential financial imbalances in an era of accommodative monetary policy, Bank of Japan, 2011, International Conference: Real and Financial Linkage and Monetary Policy, Tokyo Japan, June 2, 2011.

Zhang, Cun-Hui. and Jian Huang, 2008, The sparsity and bias of the lasso selection in high-dimensional linear regression. *Annals of Statistics*, 36, 1567-1594.

Table 1: Summary Statistics for Public Pension Fund Variables

Variable	Mean	Median	St Dev
Returns (Annual)	.062	.085	.109
% Equities	.536	.551	.111
% Fixed Income	.273	.262	.090
% Real Estate	.054	.053	.045
% Cash	.024	.013	.037
% Alternatives	.098	.073	.104
% Other	.012	0	.030
Actuarial assets ($ mil)	15.949	6.682	28.045
Actuarial liabilities ($ mil)	20.044	9.094	33.876
Funding ratio, original	.800	.808	.197
Funding ratio, rediscounted	.507	.492	.1537
State debt-to-income ratio	.083	.079	.039
State bond rating	2.852	3	1.558

Notes: Table 1 is mostly based on the Public Plans Data dataset from the Center for Retirement Research (CRR) at Boston College, and the Center for State and Local Government Excellence (SLGE). The data are publicly available at www.publicplansdata.org. *Returns (Annual)* is the InvestmentReturns_1yr variable in the dataset, which reports each fund's returns in a given fiscal year. *%Asset Class* shows the percentage allocation to funds' portfolios a particular asset class. The dataset provides a breakdown for six asset classes: equities, fixed income, real estate, cash, alternatives, and other. Their allocation shares are found as equities_tot, FixedIncome_tot, RealEstate, CashAndShortTerm, alternatives, and other (note capitalization) respectively in the original dataset. *Actuarial assets* and *Actuarial liabilities* are found under ActAssets_GASB and ActLiabilities_GASB in thousands of dollars, which we convert to millions. *Funding ratio, original* is given by ActFundedRatio_GASB, which is ActAssets_GASB divided by ActLiabilities_GASB in the dataset. *Funding ratio, rediscounted* is the time-varying funding ratio computed as ActAssets_GASB divided by rediscounted ActLiabilities_GASB, where the rediscounting uses the duration and convexity computed as follows. We compute each fund's duration and convexity during 2014-2016 as in equations (15) and (16), based on GASB data available only for these years; we take averages for duration and convexity over 2014-2016, then extrapolate them for the interval 2001-2016 adjusted for demographics as discussed in Section 4.2. We build *State debt-to-income ratio* with state income from the Bureau of Economic Analysis and state debt from the United States Census Bureau. Finally, *State bond rating* is the Standard and Poor's rating by state-year, coded numerically as AAA = 1 through BBB = 8, i.e., with higher values indicating worse ratings.

Table 2: List of Publicly-Traded Market Indices

Index name	Symbol
HFRX Global	HFRXGlobal
Bloomberg Commodities	BCOM
Bloomberg Commodities total returns	BCOMTR
Thomson Reuters/CoreCommodity Index	CRY
Thomson Reuters/CoreCommodity Index total returns	CRYTR
Credit Suisse Hedge Index	HEDGNAV
Barclays Hedge Fund	BGHSHEDG
ICE USD LIBOR 3 Mon	ICELIBOR3Mon
SIFMA Minu Swap Index	MUNIPSA
S&P 500 Index	SPX
Russel 3000	Russel3000
FTSE All World Excluding US	FTAW02
Dow Jones Global Index	W1DOW
ICE BoAML US Broad Market Ind	US00
ICE BofAML Global Broad Market	GBXD
Citi World Government Bond Ind	SBWGU
FTSE NAREIT All Equity REITS I	FNER

Notes: The table provides information on the set of publicly traded indices that are used to estimate funds' category-index risk exposures. See Section 3.2 of the text for further details.

Table 3. Coefficient Estimates for $\boldsymbol{\theta_{c,j}}$ from OLS post-LASSO Estimation

Asset Category	Index	OLS Coefficient	St. Err.
Equities	Russel 3000	1.02***	(0.13)
Equities	ICE LIBOR 3 Mon	-0.042	(0.035)
Equities	W1DOW	-1.11***	(0.29)
Equities	FTAW02	0.86***	(0.17)
Equities	SBWGU	0.019	(0.033)
Equities	US00	-0.47***	(0.067)
Fixed Income	Russel 3000	0.30***	(0.10)
Fixed Income	FTSE	0.12***	(0.039)
Fixed Income	SBWGU	-0.27***	(0.088)
Fixed Income	BCOMTR	0.27***	(0.10)
Fixed Income	CRYTR	-0.11	(0.082)
Real Estate	Russel 3000	-3.29***	(0.98)
Real Estate	FTSE	0.45***	(0.12)
Real Estate	W1DOW	7.13***	(2.29)
Real Estate	SBWGU	-3.30**	(1.29)
Real Estate	US00	-0.70***	(0.14)
Real Estate	BCOMTR	0.19	(0.11)
Cash	Russel 3000	0.23	(0.18)
Cash	FTSE	0.063	(0.17)
Alternatives	Russel 3000	0.11	(0.56)
Alternatives	FTSE	0.25***	(0.057)
Alternatives	W1DOW	-0.32	(1.38)
Alternatives	SBWGU	0.60	(0.80)
Alternatives	US00	-0.38***	(0.070)
Other	FTSE	0.33**	(0.13)
Other	BCOMTR	-1.62*	(0.85)
Other	CRY	-2.70	(2.46)
Other	CRYTR	4.14	(2.56)
		-0.26***	(0.040)
Observations		2,473	
R-squared		0.897	

Robust standard errors in parentheses, with *** p<0.01, ** p<0.05, * p<0.1

Notes: The table provides the coefficient estimates for $\theta_{c,j}$ from the OLS post-LASSO estimation of equation (9), for each asset category c and selected market index j. See Section 3.2 of the text for further details.

Table 4. Risk-taking vs. underfunding, cross-sectional relation over time

(a) Fixed funding ratio, original

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Year:	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Dependent variable:	Conditional VaR														
FR original	-0.0089	-0.0083	-0.012	-0.0061	-0.0054*	-0.0039	-0.0052	-0.019	-0.0064	-0.0070	-0.015	-0.0056	-0.0060*	-0.0057	-0.014***
	(0.012)	(0.014)	(0.0083)	(0.0043)	(0.0031)	(0.0037)	(0.0062)	(0.015)	(0.013)	(0.0077)	(0.0098)	(0.0051)	(0.0034)	(0.0038)	(0.0050)
Observations	108	108	108	108	108	108	108	108	108	108	108	108	108	108	108
R-squared	0.005	0.003	0.018	0.019	0.029	0.010	0.007	0.014	0.002	0.008	0.023	0.011	0.028	0.021	0.065

(b) Fixed funding ratio, rediscounted

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Year:	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Dependent variable:	Conditional VaR														
FR rediscounted	-0.013	-0.0098	-0.012	-0.010	-0.010*	-0.013*	-0.024**	-0.080***	-0.057**	-0.038***	-0.054***	-0.025***	-0.018***	-0.016**	-0.032***
	(0.023)	(0.025)	(0.016)	(0.0081)	(0.0057)	(0.0069)	(0.011)	(0.028)	(0.023)	(0.014)	(0.018)	(0.0093)	(0.0062)	(0.0071)	(0.0093)
Observations	108	108	108	108	108	108	108	108	108	108	108	108	108	108	108
R-squared	0.003	0.001	0.006	0.015	0.029	0.030	0.039	0.071	0.054	0.065	0.079	0.062	0.071	0.047	0.100

(c) Time-varying funding ratio, original

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Year:	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Dependent variable:	Conditional VaR														
FR original	0.0077	0.0044	-0.0012	0.00088	-0.00041	0.0019	0.0062	0.0052	-0.00060	-0.011	-0.029**	-0.016***	-0.014***	-0.0088*	-0.017***
	(0.012)	(0.013)	(0.0086)	(0.0047)	(0.0034)	(0.0041)	(0.0071)	(0.017)	(0.013)	(0.0086)	(0.011)	(0.0062)	(0.0041)	(0.0045)	(0.0061)
Observations	96	102	101	104	108	109	109	109	108	110	110	110	108	108	104
R-squared	0.004	0.001	0.000	0.000	0.000	0.002	0.007	0.001	0.000	0.014	0.057	0.060	0.094	0.035	0.070

(d) Time-varying funding ratio, rediscounted

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Year:	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Dependent variable:	Conditional VaR														
FR rediscounted	-0.00050	-0.0052	-0.0072	-0.0042	-0.0034	-0.0020	-0.00066	-0.0015	-0.0018	-0.012	-0.044**	-0.028**	-0.022***	-0.013*	-0.028**
	(0.016)	(0.016)	(0.013)	(0.0063)	(0.0050)	(0.0054)	(0.0093)	(0.023)	(0.020)	(0.014)	(0.017)	(0.011)	(0.0068)	(0.0076)	(0.011)
Observations	96	102	101	104	108	109	109	109	108	110	110	110	108	108	104
R-squared	0.000	0.001	0.003	0.004	0.004	0.001	0.000	0.000	0.000	0.007	0.056	0.053	0.087	0.027	0.062

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Notes: The table shows the link between PPFs' risk-taking and underfunding based on univariate cross-sectional regressions for each year. The constant term coefficients are not reported. In each regression, the dependent variable is the conditional VaR measure of portfolio risk. The explanatory variable is the lagged funding ratio. Panels (a) and (b) use the funding ratios fixed at their 2015 levels, obtained as the ratio of actuarial assets to total pension liabilities. Liabilities are measured either as reported by PPFs (panel a) or rediscounted using duration-matched, zero-coupon Treasury yields as in Section 3.3 (panel b). Panels (c) and (d) use time-varying funding ratios computed with actuarial liabilities measured either as reported by PPFs (panel c) or rediscounted as in Sections 3.3 and 4.2 (panel d).

Table 5. Drivers of risk-taking: underfunding and risk-free rates, using fixed funding ratios

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR
FR	-0.0089***	-0.035***	-0.013***	-0.052***	-0.0064***	-0.014***
	(0.00026)	(0.0026)	(0.0014)	(0.0050)	(0.00035)	(0.00096)
1 yr Tr Yield	-0.018**	-0.018**				
	(0.0069)	(0.0069)				
1 yr Tr Yield * FR	0.00077	0.0057***				
	(0.00052)	(0.0013)				
10 yr Tr Yield			-0.015	-0.015		
			(0.011)	(0.011)		
10 yr Tr Yield * FR			0.0016***	0.0077***		
			(0.00034)	(0.0011)		
Post-crisis					0.054*	0.054*
					(0.030)	(0.030)
Post-crisis * FR					-0.0021***	-0.020***
					(0.00024)	(0.0027)
Constant	0.19***	0.19***	0.21***	0.21***	0.13***	0.13***
	(0.027)	(0.027)	(0.046)	(0.046)	(0.011)	(0.011)
FR rediscounted	No	Yes	No	Yes	No	Yes
Fund fixed effects	No	No	No	No	No	No
Observations	1,296	1,296	1,296	1,296	1,296	1,296
Number of funds	108	108	108	108	108	108
R-squared	0.237	0.238	0.069	0.070	0.182	0.183

Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Notes: The regressions examine the link between PPFs' risk-taking, underfunding, and risk-free rates, using the following panel specification: $VaR_{pt} = \alpha + \beta * FR_p + \gamma * TrYield_t + \delta * FR_p * TrYield_t + \epsilon_{pt}$. The data are annual. The dependent variable is the conditional VaR measure of portfolio risk. The funding ratios are fixed at their 2015 levels and computed with total pension liabilities measured either as reported by PPFs (columns 1, 3, and 5) or rediscounted as in Section 3.3 (columns 2, 4, and 6). They are demeaned relative to the sample mean. The measure of risk-free rates is given by the 1-year Treasury yield (columns 1 and 2) or the 10-year Treasury yield (columns 3 and 4). In columns 5 and 6, the Treasury yield is replaced by a post-crisis dummy variable that takes the value of 1 for years 2010-2016 and zero otherwise. The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Table 6. Drivers of risk-taking: underfunding and risk-free rates, using time-varying funding ratios

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable:	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR
FR (t-1)	-0.014***	-0.035***	-0.083***	-0.0052***	-0.0079	-0.060	-0.13**	-0.037
	(0.0019)	(0.00079)	(0.0059)	(0.00034)	(0.030)	(0.045)	(0.056)	(0.030)
1 yr Tr Yield	-0.017**	-0.017**			-0.017**	-0.017**		
	(0.0068)	(0.0068)			(0.0067)	(0.0067)		
1 yr Tr Yield * FR (t-1)	0.0038***	0.0085***			0.0028	0.0099**		
	(0.0011)	(0.0015)			(0.0025)	(0.0038)		
10 yr Tr Yield			-0.015				-0.015	
			(0.011)				(0.011)	
10 yr Tr Yield * FR (t-1)			0.018***				0.022***	
			(0.0016)				(0.0046)	
Post-crisis				0.053*				0.053*
				(0.029)				(0.029)
Post-crisis * FR (t-1)				-0.034***				-0.045**
				(0.0011)				(0.018)
Constant	0.19***	0.19***	0.21***	0.13***	0.19***	0.19***	0.21***	0.13***
	(0.026)	(0.026)	(0.046)	(0.011)	(0.026)	(0.026)	(0.045)	(0.011)
FR rediscounted	No	Yes	Yes	Yes	No	Yes	Yes	Yes
Fund fixed effects	No	No	No	No	Yes	Yes	Yes	Yes
Observations	1,289	1,289	1,289	1,289	1,289	1,289	1,289	1,289
Number of funds	111	111	111	111	111	111	111	111
R-squared	0.238	0.239	0.072	0.184	0.254	0.255	0.089	0.201

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: The regressions examine the link between PPFs' risk-taking, underfunding, and risk-free rates, using the following panel specification: $VaR_{pt} = \alpha + \beta * FR_{pt-1} + \gamma * TrYield_t + \delta * FR_{pt-1} * TrYield_t + \mu_p + \epsilon_{pt}$. Columns 1-4 show results from regressions without fixed effects, while columns 5-8 show results with fund fixed effects. The data are annual. The dependent variable is the conditional VaR measure of portfolio risk. The time-varying funding ratios are based on actuarial liabilities measured as reported by PPFs (columns 1 and 5) or rediscounted as in Sections 3.3 and 4.4 (in all other columns). The funding ratios are demeaned by the cross-sectional mean of each year. The measure of risk-free rates is given by the 1-year Treasury yield (columns 1, 2, 5, and 6) or the 10-year Treasury yield (columns 3 and 7). In columns 4 and 8, the Treasury yield is replaced by a post-crisis dummy variable that takes the value of 1 for years 2010-2016 and zero otherwise. The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Table 7. Drivers of risk-taking: state public finances, underfunding, and risk-free rates

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR	Conditional VaR
State debt/income	0.027***	0.0095***	0.0054			
	(0.0011)	(0.0011)	(0.017)			
State bond rating				0.0015***	0.0012***	0.0013**
				(0.00020)	(0.000056)	(0.00049)
1 yr Tr yield	-0.018**	-0.018**	-0.018**	-0.018**	-0.018**	-0.018**
	(0.0074)	(0.0074)	(0.0076)	(0.0068)	(0.0068)	(0.0067)
1 yr Tr Yield * State debt/income	-0.010**					
	(0.0033)					
1 yr Tr Yield * State bond rating				-0.00039***		
				(0.000090)		
FR fixed	-0.021***	-0.030***		-0.017***	-0.018***	
	(0.0017)	(0.0016)		(0.0013)	(0.0014)	
FR fixed * State debt/income		0.77***				
		(0.00013)				
FR fixed * State bond rating					0.021***	
					(0.0038)	
FR (t-1)			-0.017*			-0.020***
			(0.0078)			(0.0024)
FR (t-1) * State debt/income			0.20			
			(0.18)			
FR (t-1) * State bond rating						0.028***
						(0.0070)
Constant	0.19***	0.19***	0.19***	0.19***	0.19***	0.19***
	(0.029)	(0.029)	(0.030)	(0.026)	(0.027)	(0.026)
FR rediscounted	Yes	Yes	Yes	Yes	Yes	Yes
Fund fixed effects	No	No	No	No	No	No
Observations	1,089	1,089	1,089	1,185	1,185	1,185
Number of funds	102	102	102	102	102	102
R-squared	0.253	0.254	0.253	0.240	0.242	0.244

Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Notes: The regressions examine the link between PPFs' risk-taking, state finances, underfunding, and risk-free rates. State finances are measured as either the state debt to income ratios (columns 1-3) or state bond ratings (columns 4-6); they are demeaned by the cross-sectional mean of each year. For state bond ratings, higher values reflect worse ratings. The regressions allow for interactions between state finances and risk-free rates (columns 1 and 4) or state finances and funding ratios (columns 2, 3, 5, and 6). Funding ratios are either fixed (columns 1, 2, 4, and 5) or time-varying (columns 3 and 6), and are obtained with rediscounted liabilities as in Tables 5 and 6. The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Table 8. Drivers of risk-taking, robustness with VaR-based measures of PPF risk

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Dependent variable:	Conditional VaR (active shares)	Uncond VaR	Conditional VaR	Conditional VaR (active shares)	Uncond VaR	Conditional VaR	Conditional VaR (active shares)	Uncond VaR	Conditional VaR	Uncond VaR (active shares)	Uncond VaR	Conditional VaR
FR fixed	-0.043***	-0.034***	-0.0078***				-0.069***	-0.050***	-0.013***			
	(0.0029)	(0.0082)	(0.0016)				(0.0047)	(0.011)	(0.00013)			
FR (t-1)				-0.033***	-0.023**	-0.036***				-0.081***	-0.050***	-0.076***
				(0.00049)	(0.0089)	(0.0023)				(0.0047)	(0.015)	(0.0062)
1 yr Tr Yield	-0.019**	0.000039	-0.018**	-0.019**	0.00016	-0.017**						
	(0.0074)	(0.00032)	(0.0069)	(0.0072)	(0.00030)	(0.0068)						
1 yr Tr Yield * FR fixed	0.0080***	0.0038**	-0.00035									
	(0.0014)	(0.0014)	(0.00068)									
1 yr Tr Yield * FR (t-1)				0.0079***	0.0045*	0.0088***						
				(0.0013)	(0.0021)	(0.0016)						
10 yr Tr Yield							-0.018	-0.00065	-0.015	-0.018	-0.00048	-0.015
							(0.013)	(0.00069)	(0.011)	(0.013)	(0.00068)	(0.011)
10 yr Tr Yield * FR fixed							0.012***	0.0065**	0.0013***			
							(0.0011)	(0.0028)	(0.00028)			
10 yr Tr Yield * FR (t-1)										0.018***	0.010**	0.015***
										(0.0013)	(0.0039)	(0.0015)
Constant	0.20***	0.17***	0.19***	0.20***	0.17***	0.19***	0.23***	0.17***	0.21***	0.23***	0.17***	0.21***
	(0.029)	(0.0012)	(0.027)	(0.028)	(0.0013)	(0.026)	(0.051)	(0.0021)	(0.046)	(0.051)	(0.0022)	(0.046)
FR rediscounted	Yes	Yes	Corporate	Yes	Yes	Corporate	Yes	Yes	Corporate	Yes	Yes	Corporate
Fund fixed effects	No	No	No	No	No	No	No	No	No	No	No	No
Observations	1,296	1,296	1,296	1,289	1,289	1,289	1,296	1,296	1,296	1,289	1,289	1,289
Number of funds	108	108	108	111	111	111	108	108	108	111	111	111
R-squared	0.252	0.046	0.237	0.251	0.018	0.240	0.084	0.050	0.069	0.084	0.023	0.073

Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Notes: The table shows robustness checks for the results from Tables 5 and 6. They use the conditional VaR measure of risk computed with portfolio shares that abstract from valuation changes as the dependent variable (marked as "active shares" in columns 1, 4, 7, and 10); use the unconditional VaR measure of risk as the dependent variable (columns 2, 5, 8, and 11); and use funding ratios with liabilities rediscounted with duration-matched, high-quality corporate bond yields instead of Treasury yields as the explanatory variable (columns 3, 6, 9, and 12). The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Table 9. Drivers of risk-taking, robustness with traditional measures of PPF risk

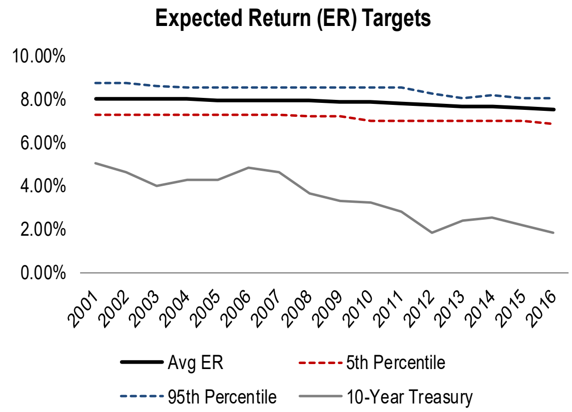
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dependent variable:					Share of alternatives				
FR fixed	-0.13	-0.26	0.012				-0.068	-0.092	
	(0.11)	(0.19)	(0.044)				(0.080)	(0.091)	
FR (t-1)				0.017	0.0084	-0.012			0.020
				(0.078)	(0.17)	(0.038)			(0.055)
1 yr Tr Yield	-0.029***			-0.026***			-0.026***	-0.026***	-0.025***
	(0.0048)			(0.0043)			(0.0045)	(0.0045)	(0.0043)
1 yr Tr Yield * FR fixed	0.037								
	(0.023)								
1 yr Tr Yield * FR (t-1)				-0.0081					
				(0.018)					
10 yr Tr Yield		-0.048***			-0.048***				
		(0.0055)			(0.0064)				
10 yr Tr Yield * FR fixed		0.057							
		(0.037)							
10 yr Tr Yield * FR (t-1)					-0.0037				
					(0.038)				
Post-crisis			0.11***			0.10***			
			(0.015)			(0.015)			
Post-crisis * FR fixed			-0.14						
			(0.085)						
Post-crisis * FR (t-1)						0.018			
						(0.077)			
State debt/income							0.44	0.39*	0.31
							(0.28)	(0.18)	(0.20)
1 yr Tr Yield * State debt/income							-0.032		
							(0.061)		
FR fixed * State debt/income								1.82	
								(2.24)	
FR (t-1) * State debt/income									-0.84
									(1.64)
Constant	0.16***	0.28***	0.056***	0.16***	0.28***	0.058***	0.16***	0.16***	0.15***
	(0.014)	(0.024)	(0.011)	(0.014)	(0.026)	(0.011)	(0.014)	(0.014)	(0.014)
FR rediscounted	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Fund fixed effects	No	No	No	No	No	No	No	No	No
Observations	1,393	1,393	1,393	1,277	1,277	1,277	1,166	1,166	1,177
Number of funds	108	108	108	111	111	111	106	106	110
R-squared	0.208	0.263	0.250	0.167	0.218	0.206	0.194	0.196	0.180

Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

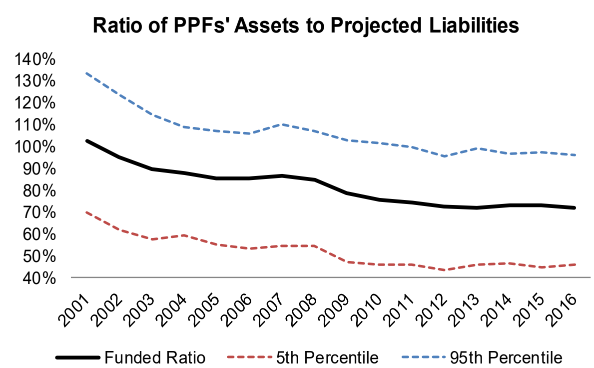
Notes: The table shows robustness checks for the results from Tables 5, 6, and 7. The main difference is the dependent variable, which now consists of the share of alternatives instead of the conditional VaR as the measure of risk. The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Figure 1. PPFs' portfolio allocation and expected return targets

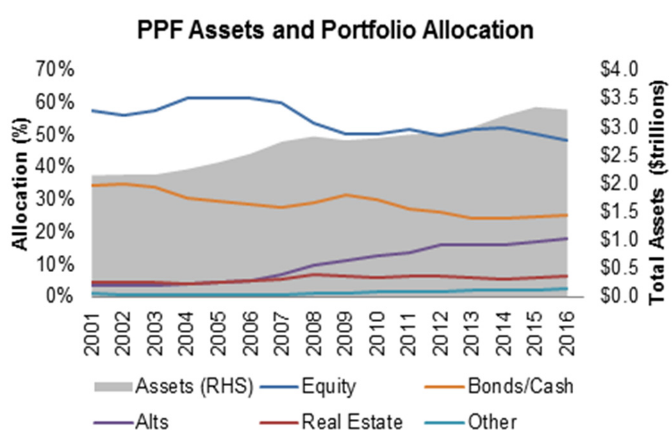
Panel (a)



Panel (b)

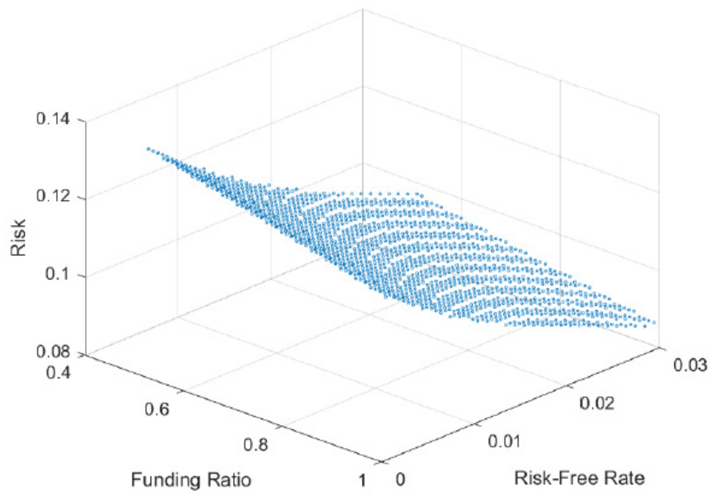


Panel (c)



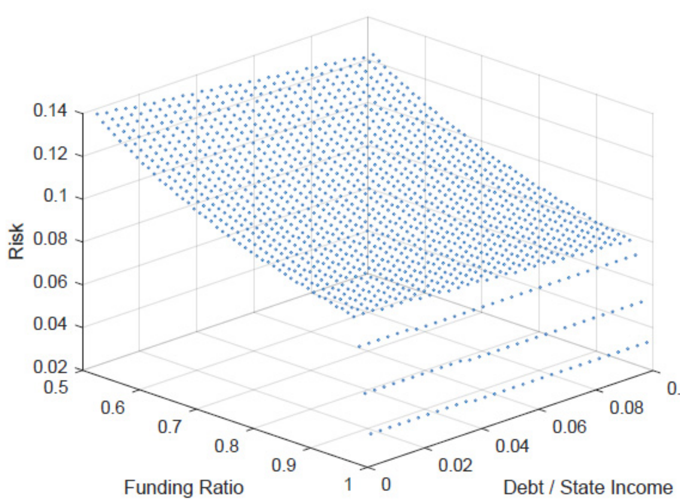
Notes: The figure presents summary information on PPFs' expected return targets (panel a), the ratio of assets to actuarial liabilities (panel b), and the change aggregate asset allocations over time (panel c).

Figure 2: Risk vs Risk Free Rate and Pension Funding Ratio when State Debt is Risk-Free



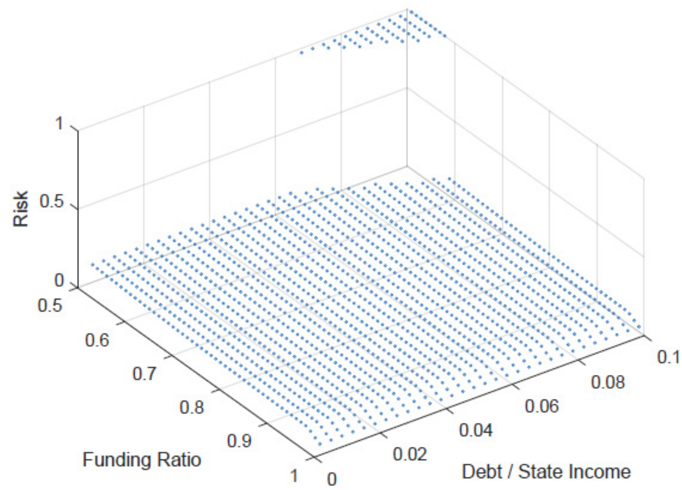
Notes: For the theoretical model in Section 2, when states are unable to default on their debt, the Figure presents the relationship between pension fund asset risk, the plans funding ratio, and the risk-free interest rate. Risk in the figure is measured as the proportion of risky asset in the fund's asset portfolio. The funding ratio is the present value of fund liabilities discounted by the risk-free rate. For further details see Section 2 of the text.

Figure 3: Risk as a Function of Debt to State Income and Pension Funding Ratio when State Debt is Risk-Free



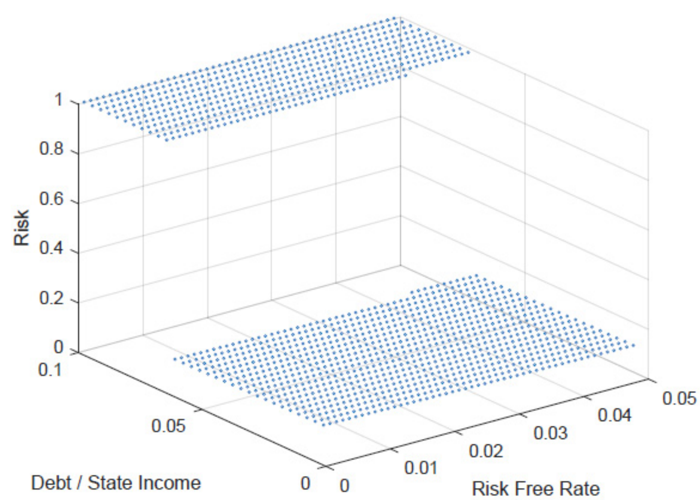
Notes: For the theoretical model in Section 2, when states are unable to default on their debt, the Figure presents the relationship between pension fund asset risk, the plans funding ratio, and the ratio of state debt to state income. Risk in the figure is measured as the proportion of risky asset in the fund's asset portfolio. The funding ratio is the present value of fund liabilities discounted by the risk-free rate. Debt to state income is measured as debt at date t divided by state income at date t. For further details see Section 2 of the text.

Figure 4: Pension Fund Risk vs Pension Funding Ratio and Debt to State Income when State Debt is Risky



Notes: For the theoretical model in Section 2, when states can choose to default on their debt, the Figure presents the relationship between pension fund asset risk, the plans funding ratio, and the ratio of state debt to state income. Risk in the figure is measured as the proportion of risky asset in the fund's asset portfolio. The funding ratio is the present value of fund liabilities discounted by the risk-free rate. Debt to state income is measured as debt at date *t* divided by state income at date *t*. For further details see Section 2 of the text.

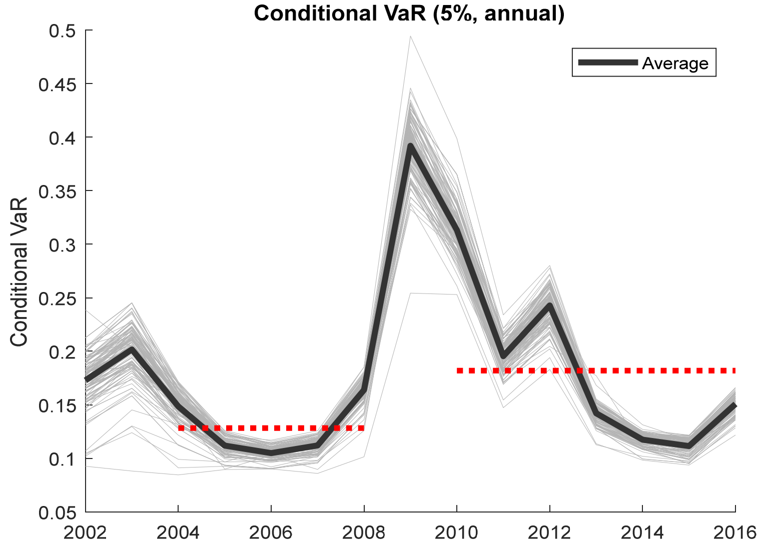
Figure 5: Pension Fund Risk vs Risk-Free Rate and Debt to State Income when State Debt is Risky



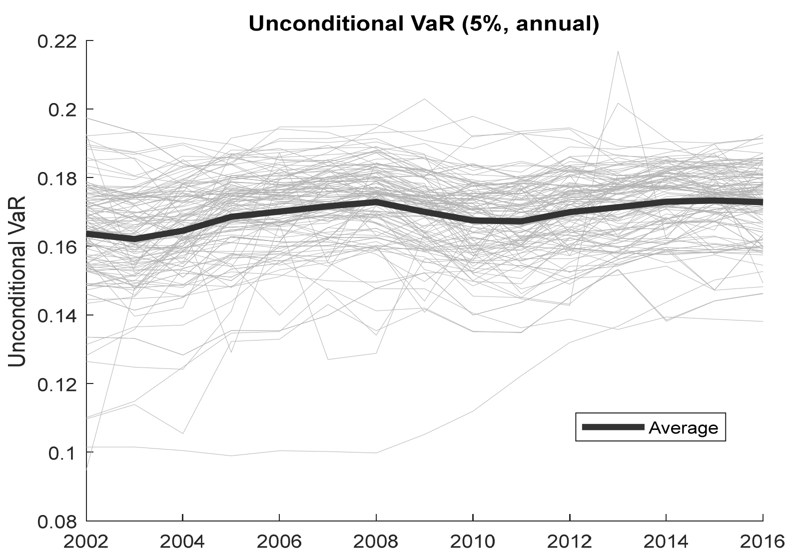
Notes: For the theoretical model in Section 2, when states can choose to default on their debt, the Figure presents the relationship between pension fund asset risk, the risk free rate, and the ratio of state debt to state income. Risk in the figure is measured as the proportion of risky asset in the fund's asset portfolio. Debt to state income is measured as debt at date *t* divided by state income at date *t*. For further details see Section 2 of the text.

Figure 6. VaR-based measures of risk

Panel (a)



Panel (b)



Notes: For the Public Pension Funds in our data sample, Panel (a) present time-series of 5% Value-at-Risk (VaR) for each fund (gray lines), average of funds 5% VaR (black lines) and averages of VaR pre- and post- 2007-09 financial crisis (red lines). Panel (b) presents time-series of 5% unconditional VaR for each fund (gray lines), and the average of funds 5% VaR (black lines).

Figure 7. The impact of rediscounting liabilities on funding ratios in 2015

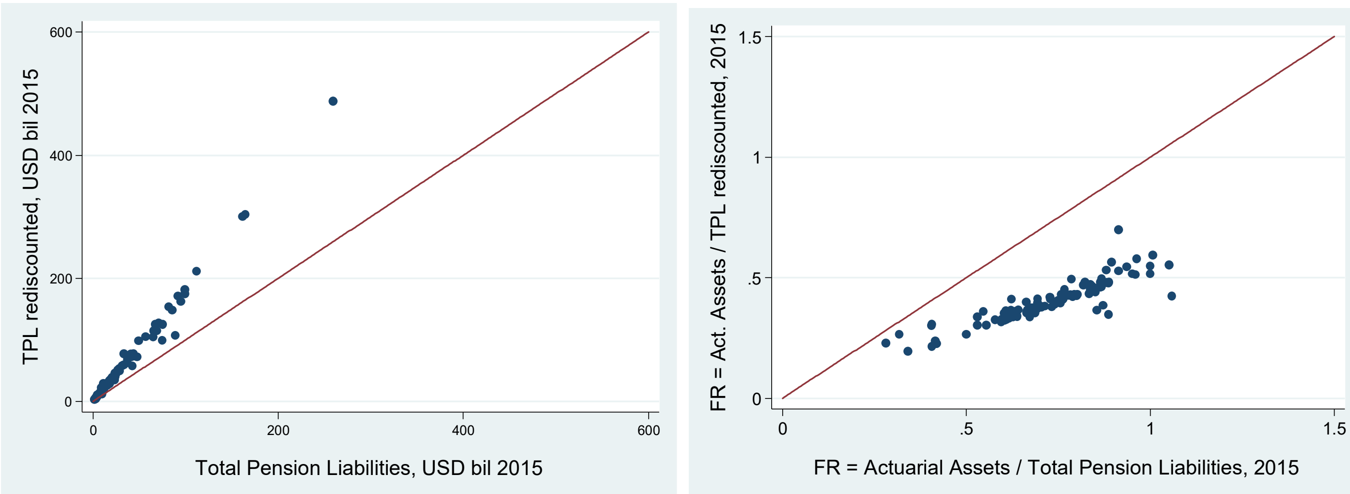


Figure 8. Conditional VaR vs. lagged funding ratio, cross-sectional relation for 2016

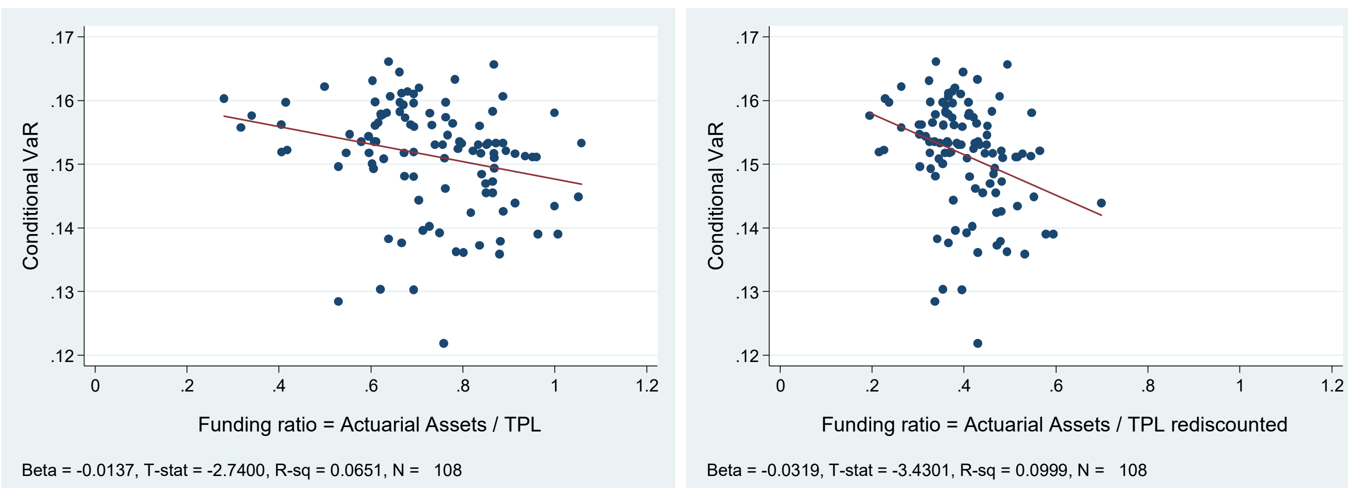


Figure 9. The impact of rediscounting liabilities on funding ratios over time

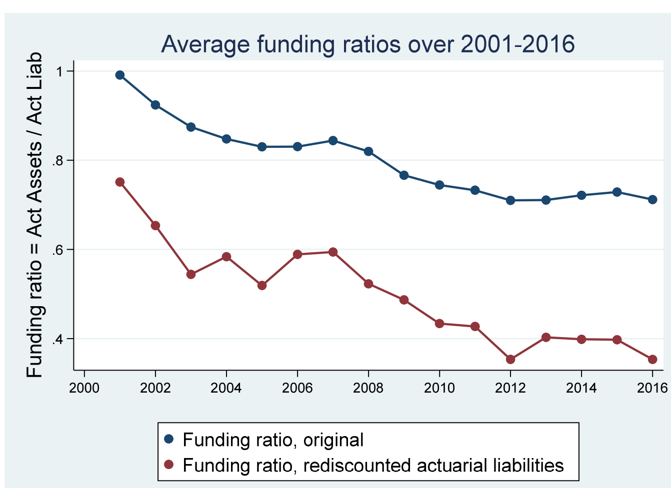
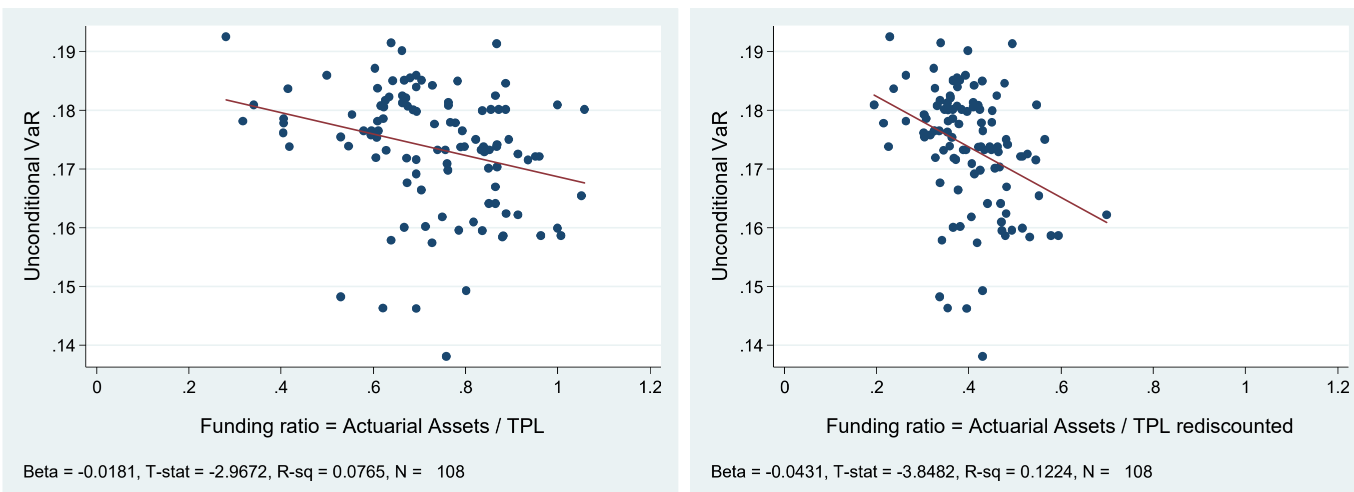


Figure 10. Risk-taking vs. lagged funding ratio, cross-sectional link for 2016, robustness tests

Panel (a) Conditional VaR measure of risk without valuation changes

Panel (b) Unconditional VaR measure of risk



Panel (c) Liabilities rediscounted with high-quality coprorate bond yields

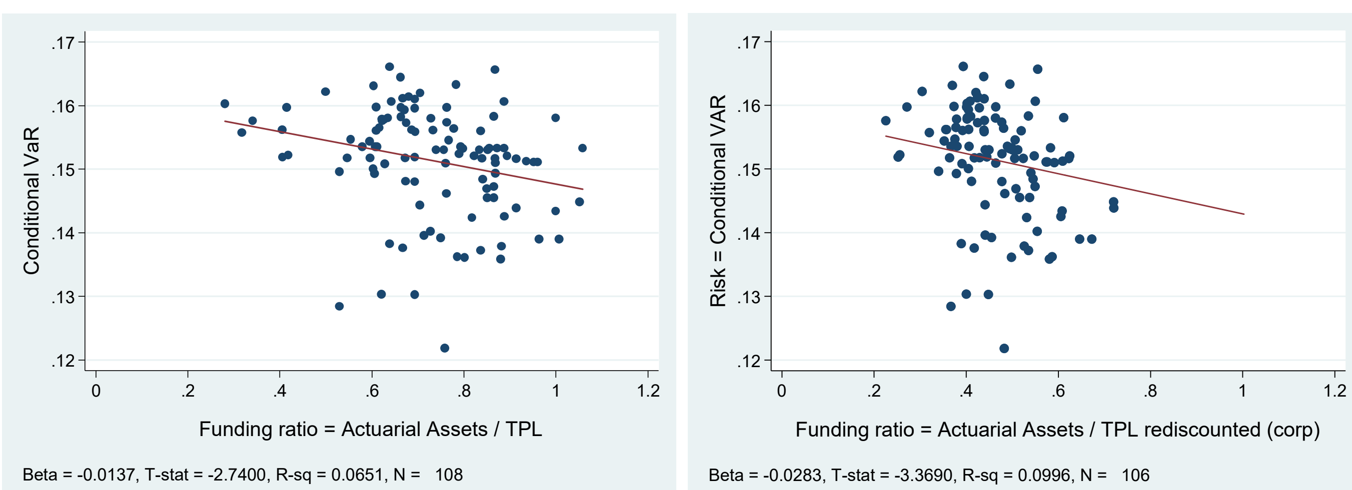
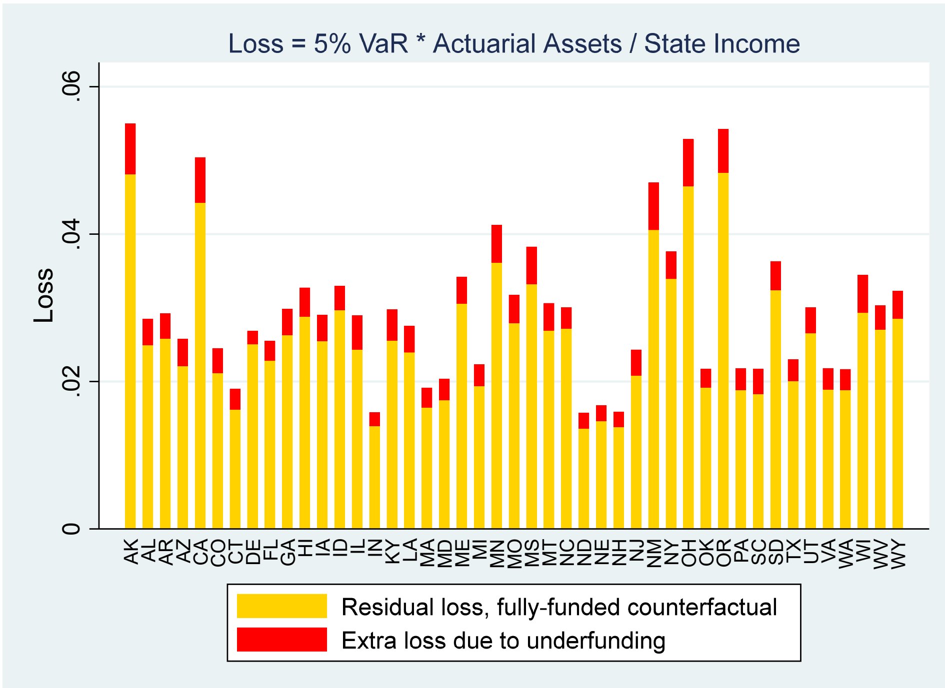
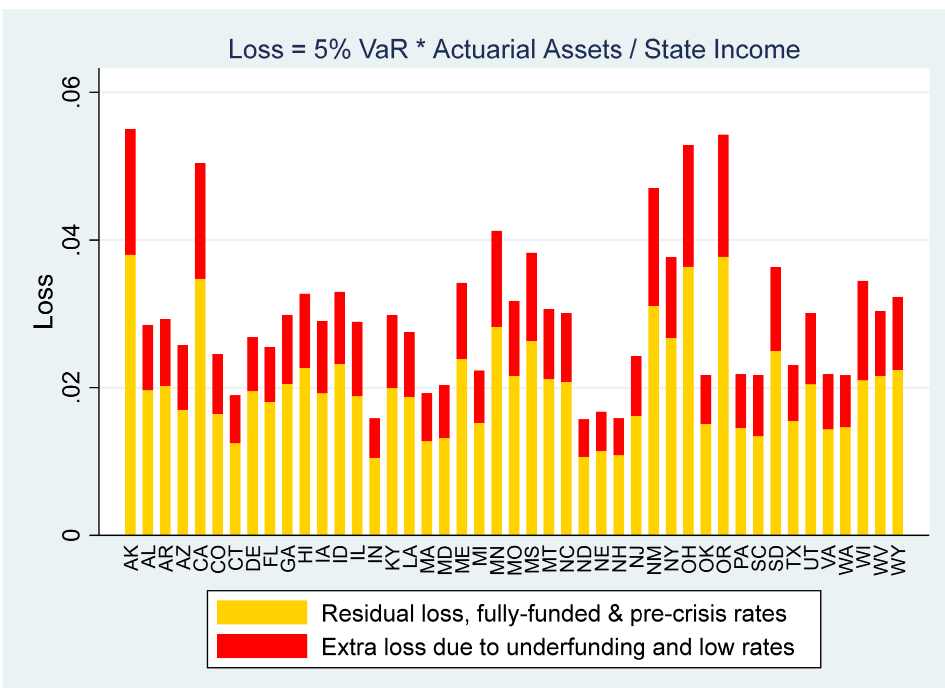


Figure 11. PPFs' risk associated with underfunding and low risk-free rates in 2016

Panel (a) Risk-taking due to underfunding



Panel (b) Risk-taking due to underfunding and low rates



Reach for Yield by U.S. Public Pension Funds[1]

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Internet Appendix

A. Comparative Static Analysis for Theoretical Model.

In this appendix we provide comparative statics analysis for the benchmark theoretical model in section 2. To better understand the generality of our results with power utility, we use comparative static analysis to derive how risk taking is related to funding ratios, state finances, disposable income, and the equity premium when state debt is risk-free. For the comparative statics analysis we assume that the optimal portfolio choice is an interior solution, and we assume the utility function is twice differentiable in wealth. To simplify the comparative statics, we assume that the value of pension fund asset portfolios at date t will be less than pension fund assets with probability 1, and that therefore any difference must be covered by taxes.[2] This assumption implies the terms involving Max(.,0) in the equations above are always positive, which vastly simplifies the analysis. The optimization for choosing the portfolio, after applying the simplification takes the form:

$$Max_{\omega}\ E_0\, U_t[Y_t(1 - SDI_t - \ PDI_t\left(1 - FR_0\ \left(1 + \omega\left(\tilde{R} - 1\right)\right)\right))],$$

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[2] This is equivalent to assuming that the funding ratio is chosen to ensure that assets at date t do not exceed liabilities at date t. This is reasonable since the fund would like to avoid providing beneficiaries with more payments at date t then the beneficiaries are owed. The mathematical form of the assumption is $1 - FR_0(r_f)\left[(1-\omega) + \omega e^{(\lambda - .5\sigma^2)t + \sigma\sqrt{t}\epsilon}\right] \geq 0$.

Where, $\tilde{R} = e^{(\lambda - .5\sigma^2)t + \sigma\sqrt{t}\epsilon}$.

Note: we suppress the dependence of the funding ratio FR_0 and the risk premium λ on the risk-free rate for simplicity since we assume each of these arguments is monotone in the risk-free rate.

The first order condition for choice of ω reduces to the expression

$$E_0\, U_t'[.](\tilde{R} - 1) = 0.$$

Given this first order condition, our comparative statics results use very mild regularity conditions, which require the funding ratio does not exceed 1, and the sum of state debt to income and pension debt to income is less than 1, which appears to be satisfied for all U.S. states.[3] To sign the comparative statics, additional assumptions are required about the representative citizen's preferences. We assumed the representative citizen is strictly risk-averse. This implies $U_t''[.] < 0$. As shown in the proof, the comparative static results also roughly depend on how the absolute risk aversion of the representative citizen at date t (measured by $-U_t''[.]/U_t'[.]$) changes with his consumption. Theory does not pin down how risk preferences change with consumption, therefore we consider both possibilities when reporting the comparative statics results.

Our main finding from the comparative statics is they cannot always be signed, but they can be signed more often when the absolute risk aversion of the representative citizen positively covaries with consumption, which we interpret as the representative citizen wants the state to take less risk as the representative citizen becomes wealthier. In this circumstance, the comparative statics show that risk increases as funding ratios decline, consistent with reach for yield. In addition, risk decreases with the representative citizen's income, and risk decreases with state debt to income. Our findings for the relationship between risk and λ is ambiguous. Intuitively, this is because when the reward for risk-taking increases, it can result in taking less risk to obtain investment objectives or it can result in more taking because it is better rewarded. Because of the ambiguity, our results show it is possible that lower rates can increase risk premia but lead to lower risk taking through the risk premium effect, while at the same time lead to more risk-taking through the reach for yield effect. Which directions these effects go is an empirical question.

The details on our comparative statics results are as follows:

Proposition A1. If $0 < FR_0 < 1,\ 0 < PDI_t < 1,\ Y_t > 0\,, SDI_t + PDI_t < 1$, and the maximizing value of $\omega \in (0,1)$, then

[3] An exception is The Commonwealth of Puerto Rico, where primary "state" debt to income exceeded 1 in 2014 (see Norcross and Gonzalez (2016)).

A. If the absolute risk aversion of the representative citizen is monotone increasing in consumption, then $\frac{d\omega}{dFR_t} < 0$, $\frac{d\omega}{dSDI_t} < 0$, $\frac{d\omega}{dY_t} < 0$, and $\frac{d\omega}{d\,\lambda}$ has ambiguous sign.
B. If the absolute risk aversion of the representative citizen is monotone decreasing in consumption, then $\frac{d\omega}{dSDI_t} > 0$, $\frac{d\omega}{dFR_t}$ has ambiguous sign, $\frac{d\omega}{dY_t}$ has ambiguous sign, and $\frac{d\omega}{d\,\lambda}$ has ambiguous sign.

Proof:

Taking the total differential of the first order condition with respect to $, Y_t, SDI, FR_0$, and λ produces the equation:

$$0 = E_0U_t''[.](\tilde{R}-1)^2 Y_t PDI_t FR_0\ d\omega$$

$$+E_0U_t''[.](\tilde{R}-1)\left[(1-SDI_t - PDI_t\left(1-FR_0\ \left(1+\omega(\tilde{R}-1)\right)\right))\right]dY_t$$

$$+E_0U_t''[.]Y_t(\tilde{R}-1)d\,SDI_t$$

$$+E_0U_t''[.](\tilde{R}-1)[Y_t\,PDI_t(1+\omega(\tilde{R}-1)\,)]dFR_0$$

$$+(E_0U_t'[.]\tilde{R}t + E_0U_t''[.](\tilde{R}-1)Y_t\,PDI_t FR_0\omega\tilde{R}t\,)d\lambda$$

Rearrangement leads to the following comparative static results:

$$\frac{d\omega}{dY_t} = \frac{-E_0U_t''[.](\tilde{R}-1)\left[(1-SDI_t - PDI_t\left(1-FR_0\left(1+\omega(\tilde{R}-1)\right)\right))\right]}{E_0U_t''[.](\tilde{R}-1)^2 Y_t\,PDI_t\,FR_0}$$

$$\frac{d\omega}{dSDI_t} = \frac{-E_0U_t''[.]\,Y_t\,(\tilde{R}-1)}{E_0U_t''[.](\tilde{R}-1)^2 Y_t PDI_t FR_0}$$

$$\frac{d\omega}{dFR_0} = \frac{-E_0U_t''[.](\tilde{R}-1)[Y_t\,PDI_t(1+\omega(\tilde{R}-1)\,)]}{E_0U_t''[.](\tilde{R}-1)^2 Y_t\,PDI_t\,FR_0}$$

$$\frac{d\omega}{d\lambda} = \frac{-(E_0U_t'[.]\tilde{R}t + E_0U_t''[.](\tilde{R}-1)Y_t\,PDI_t FR_0\omega\tilde{R}t\,)}{E_0U_t''[.](\tilde{R}-1)^2 Y_t\,PDI_t\,FR_0}$$

To sign the comparative statics note that the denominator in the expressions for all of the comparative statics is negative. Furthermore, the numerator of the expression for $\frac{d\omega}{dY_t}$, can be written as the sum of the terms $-E_0U_t''[.](\tilde{R}-1)[(1-SDI_t - PDI_t(1-FR_0\,))]$ and $-E_0U_t''[.](\tilde{R}-1)^2 PDI_t\,FR_0\,\omega$. The second of the terms is unambiguously positive. If the first term is also unambiguously positive, it confirms the sign of $\frac{d\omega}{d\,Y_t}$ in case A. In the first term, $[(1 - SDI_t - \;PDI_t(1 - FR_0\,))] > 1 - SDI_t - PDI_t > 0$. Therefore, the first term is greater than zero if $-E_0 U_t''[.]\big[\big(\tilde{R} - 1\big)\big] >\; 0$. This condition can be rewritten as $E_0\,\frac{-U_t''[.]}{U_t'[.]} \times U_t'[.]\big[\big(\tilde{R} - 1\big)\big] > 0$. This can further be written as:

$$E_0\left\{\frac{-U_t''[.]}{U_t'[.]}\right\} \times E_0\big\{\,U_t'[.]\big[\big(\tilde{R} - 1\big)\big]\big\} + Cov\left\{\frac{-U_t''[.]}{U_t'[.]}, U_t'[.]\big[\big(\tilde{R} - 1\big)\big]\right\} > 0 \quad \text{(A.1)}$$

From the first order condition for choice of ω, $E_0\big\{\,U_t'[.]\big[\big(\tilde{R} - 1\big)\big]\big\} = 0$. Therefore, the first term in equation (A.1) above is zero. In the second term above $\frac{-U_t''[.]}{U_t'[.]}$ is absolute risk aversion, and the second term is roughly speaking increasing in consumption since consumption is higher when $\tilde{R}$ increases, and algebra shows the second term is increasing for most plausible values of $\tilde{R}$.[4] Therefore, when absolute risk aversion and consumption are positively correlated, then risk taking is decreasing in income. The proof for $\frac{d\omega}{dFR_t}$ in case A and for $\frac{d\,\omega}{d\,SDI_t}$ are similar in case A.

In case B, there are two terms in the numerator for $\frac{d\omega}{dY_t}$, and they have opposite signs. Hence the sign of $\frac{d\omega}{dY_t}$ is ambiguous. The same is true for $\frac{d\,\omega}{d\,FR_0}$hence it too has an ambiguous sign.

Finally, to prove the results for $\frac{d\omega}{d\,\lambda}$, note that by adding and subtracting $E_0 U_t''[.]\big(\tilde{R} - 1\big)Y_t\; PDI_t FR_0 \omega t$ to the numerator, and rearranging the numerator can be rewritten to have three terms. The first term is $-(E_0 U_t'[.]\tilde{R}t\,)$, which from the first-order condition is equal to $-E_0 U_t'[.]t$, which is less than zero. The second term is $-E_0 U_t''[.]Y_t\; PDI_t FR_0 \omega\big(\tilde{R} - 1\big)^2 t$, which is positive. The third term is $-E_0 U_t''[.]\big(\tilde{R} - 1\big)Y_t\; PDI_t FR_0 \omega t$. This term is also positive in case A as shown above, which makes the sign in case A ambiguous. In case B, the sign of third term is negative, and the sign of the overall expression remains ambiguous. ■

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[4] For the purposes of calibration of the model, the time horizon t is assumed to be one year. For longer horizons it would be necessary to recalibrate risk aversion to the appropriate horizon. $U_t'[.](\tilde{R} - 1)$ is clearly increasing for $\tilde{R} \leq 1$ since the representative citizen is assumed to be strictly risk averse. For $\tilde{R} \geq 1$, differentiation and rearrangement shows the expression is increasing in $\tilde{R}$ if $-\frac{U_t''[.]}{U_t'[.]} Y_t PDI_t\; FR_0\;\omega \leq \frac{1}{\tilde{R}-1}$. If we choose a coefficient of relative risk aversion ($= -\frac{U_t''[.]}{U_t'[.]} Y_t$) of 10 which is plausible for the representative citizen, and a pension debt to income ratio of .7 (the maximum value in Norcross and Gonzalez (2016) and a value of ω = 1, and an FR_0=1, both of which are unrealistically high, then the term is increasing if $\tilde{R} \leq 114$ percent.