Optimal Monetary Policy in Production Networks

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In the canonical New Keynesian model,

- optimal policy: stabilize the aggregate price level
- why? price stability preserves productive efficiency and implements the first best
- "Divine Coincidence" Blanchard and Gali (2007)
  - price stability minimizes both inflation and the "output gap"
- target is straightforward in the model: aggregate price level = average price across firms

But the real world is much more complex.

- multiple, heterogeneous sectors that interact in a network of intermediate good trade
- how should the aggregate price index depend on:
  - whether sectors produce final goods or intermediate inputs? e.g. CPI vs. PPI?
  - the relative position of sectors in the input-output network?
  - differences in the relative price flexibility of sectors?
  - changes in the relative size of sectors? e.g. healthcare and services

Our Question

How does the multi-sector, input-output structure of the economy

affect the optimal conduct of monetary policy?

Our Framework

- multi-sector, input-output model, à la Long and Plosser (1983), Acemoglu et al (2012)
  - input-output network of intermediate good trade across sectors
  - sectoral productivity shocks $\rightarrow$ underlying flex-price economy is efficient
- firms face nominal rigidities
  - must set nominal prices before observing demand
  - informational friction, à la Woodford (2003), Mankiw Reis (2002), Angeletos La'O (2020)

Our Results

- Divine Coincidence is non-generic
  - efficient allocation cannot be implemented under sticky prices
- Optimal policy stabilizes an optimal price index with greater weight on:
  - larger sectors (as measured by Domar weights, i.e. sales shares of GDP)
  - stickier sectors
  - more upstream sectors, sectors with stickier customers, sectors with more flexible suppliers
- Quantitative welfare improvements from adopting the optimal policy
  - we calibrate the model: BEA US input-output tables + data on price stickiness
  - CPI stabilization $\rightarrow$ optimal policy $\approx$ welfare gain of .5 percentage point of quarterly consumption

Related Literature

- production networks
  - efficient economies: Long and Plosser (1983), Acemoglu et al (2012), Baqaee and Farhi (2019)
  - markups and misallocation: Jones (2013), Bigio and La'O (2020), Baqaee and Farhi (2020)
  - nominal rigidities: Pasten, Schoenle, and Weber (2019), Ozdagli and Weber (2019), Rubbo (2020)

- monetary policy in multi-sector New Keynesian models
  - two-sector: Erceg, Henderson, Levin (1999), Aoki (2001), Woodford (2003, 2010), Benigno (2004)
  - multi-sector: Mankiw and Reis (2003), Eusepi, Hobijn, Tambalotti (2011)
  - w/intermediate good trade: Basu (1995), Huang and Liu (2005)

- informational frictions as nominal rigidities
  - Lucas (1972), Woodford (2003), Mankiw and Reis (2002), Adam (2007), Nimark (2008), Mackowiak and Wiederholt (2009), Lorenzoni (2010), Paciello and Wiederholt (2014), Angeletos and La'O (2016, 2020),...

The Environment

The Environment

- static environment
- production: n sectors indexed by $i \in I \equiv \{1, \ldots, n\}$
  - input-output network of intermediate good trade across sectors
- continuum of identical firms within a sector, indexed by $k \in [0, 1]$
  - firms produce differentiated goods $\rightarrow$ monopolistic competitors
  - firm managers make nominal pricing decision under incomplete info

Technology

- CRS production function of firm k in sector i

$$y_{ik} = z_i F_i(\ell_{ik}, x_{i1,k}, \ldots, x_{in,k}) = z_i \ell_{ik}^{\alpha_i} \prod_{j \in I} x_{ij,k}^{a_{ij}}$$

  - input-output matrix $A = [a_{ij}]$
- nominal profits

$$\pi_{ik} = (1 - \tau_i) p_{ik} y_{ik} - w \ell_{ik} - \sum_{j=1}^{n} p_j x_{ij,k}$$

- for every $i \in I$, perfectly-competitive CES aggregator firm

$$y_i = \left(\int_0^1 y_{ik}^{\frac{\theta_i - 1}{\theta_i}} \, dk \right)^{\frac{\theta_i}{\theta_i - 1}}$$

  - output may be either consumed or used as an intermediate good

Representative Household

- preferences

$$U(C) - V(L)$$

$$C = \mathcal{C}(c_1, \ldots, c_n) = \prod_{i \in I} (c_i / \beta_i)^{\beta_i}$$

- budget set

$$\sum_{i \in I} p_i c_i \leq wL + \sum_{i \in I} \int_0^1 \pi_{ik} dk + T$$

The Government and Market Clearing

- government has full commitment, fiscal budget set

$$T = \sum_{i \in I} \tau_i \int_0^1 p_{ik} y_{ik} dk$$

- monetary authority controls aggregate nominal demand

$$m = PC = \sum_{i \in I} p_i c_i$$

- market clearing

$$y_j = c_j + \sum_{i \in I} \int x_{ij,k} dk \quad \forall j \in I, \qquad \text{and} \qquad L = \sum_{i \in I} \int \ell_{ik} dk$$

Nominal Rigidity = Informational Friction

- sectoral technology shocks

$$\log z_i \sim \mathcal{N}\left(0, \delta^2 \sigma_z^2\right) \qquad \text{i.i.d.}$$

- Gaussian information set: vector of signals about technology shocks

$$\omega_{ik} = (\omega_{i1,k}, \ldots, \omega_{in,k})$$

$$\omega_{ij,k} = \log z_j + \varepsilon_{ij,k}, \qquad \text{with} \qquad \varepsilon_{ij,k} \sim N\left(0, \delta^2 \sigma_i^2\right)$$

- aggregate state

$$s = (z, \omega) \in S$$

  - vector of sectoral productivities $z = (z_1, \ldots, z_n)$
  - entire distribution of information sets ω

Nominal Rigidity = Informational Friction

1. Firms' nominal pricing decisions made under incomplete info

$$p_{ik}(\omega_{ik})$$

   - nominal rigidity = measurability constraint on the nominal price

2. All other market outcomes, allocations adjust to the aggregate state

   - household chooses consumption
   - inputs must adjust so that supply = demand (but input mix chosen optimally)

$$y_{ik}(s), \ell_{ik}(s), x_{ij,k}(s)$$

   - monetary policy contingent on s, but sectoral taxes are non-contingent

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First Best

Proposition

The first-best allocation ξ^* is the unique feasible allocation which satisfies

$$V'(L(s)) = U'(C(s))\frac{dC(s)}{dc_i} z_i(s)\frac{dF_i(s)}{d\ell_i}, \quad \forall i,k,s$$

$$\frac{dC(s)}{dc_j} = \frac{dC(s)}{dc_i} z_i(s)\frac{dF_i(s)}{dx_{ij}}, \qquad \forall i,j,k,s$$

- efficiency requires zero dispersion in quantities within sectors

$$\ell_i(s) = \ell_{ik}(s), \quad x_{ij}(s) = x_{ij,k}(s), \quad y_i(s) = y_{ik}(s), \qquad \forall k \in [0,1]$$

but movement in relative quantities across sectors

Equilibrium

Equilibrium Definition

Definition

A sticky price equilibrium is a set of allocations, prices, and policies such that:

(i) prices $p_{ik}(\omega_{ik})$ maximize the firm's expected real value of profits given information set ω_{ik};

(ii) firms optimally choose inputs to meet realized demand;

(iii) the representative household maximizes her utility;

(iv) the government budget constraint is satisfied; and

(v) markets clear.

Definition

A flexible price equilibrium is a set of allocations, prices, and policies such that:
same as above, but

$$p_{ik}(s)$$

Proposition

A feasible allocation is implementable as a flexible-price equilibrium iff

$$V'(L(s)) = \chi_i U'(C(s)) \frac{dC(s)}{dc_i} z_i(s) \frac{dF_i(s)}{d\ell_i}, \quad \forall i,k,s$$

$$\frac{dC(s)}{dc_j} = \chi_i \frac{dC(s)}{dc_i} z_i(s) \frac{dF_i(s)}{dx_{ij}}, \qquad \forall i,j,k,s$$

where $\chi_i \equiv (1-\tau_i)\left(\frac{\theta_i - 1}{\theta_i}\right)$.

Proposition

The first best allocation ξ^* *can be implemented under flexible prices with* $\chi_i = 1$, $\forall i$.

Proposition

A feasible allocation is implementable as a sticky-price equilibrium iff

$$V'(L(s)) = \chi_i \varepsilon_{ik}(\omega_{ik}, s) U'(C(s)) \frac{dC(s)}{dc_i} \left(\frac{y_{ik}(\omega_{ik}, s)}{y_i(s)} \right)^{-1/\theta_i} z_i(s) \frac{dF_i(s)}{d\ell_i}, \quad \forall i, k, s$$

$$\frac{dC(s)}{dc_j} = \chi_i \varepsilon_{ik}(\omega_{ik}, s) \frac{dC(s)}{dc_i} \left(\frac{y_{ik}(\omega_{ik}, s)}{y_i(s)} \right)^{-1/\theta_i} z_i(s) \frac{dF_i(s)}{dx_{ij}}, \quad \forall i, j, k, s,$$

with stochastic wedges (due to pricing errors):

$$\varepsilon_{ik}(\omega_{ik}, s) \equiv \frac{\mathrm{mc}_i(s) \mathbb{E}[v_{ik}(s) | \omega_{ik}]}{\mathbb{E}[v_{ik}(s) \mathrm{mc}_i(s) | \omega_{ik}]},$$

Flexible Price allocations are unattainable

- let X^f denote the entire set of flexible-price allocations
- let X^s denote the entire set of sticky-price allocations

Theorem

The sets X^f and X^s are generically disjoint

$$X^f \cap X^s = \emptyset$$

Divine Coincidence is non-generic

Corollary

The first best allocation cannot generically be implemented under sticky prices:

$$\xi^* \notin X^s$$

- impossible for any monetary policy to simultaneously acheive:
  - productive efficiency within sectors (zero price dispersion within each sector)
  - efficient relative price movement across sectors

When can you implement first best?

Proposition

If there is a single sticky-price industry i*, then*

$$X^f \subset X^s$$

and as a result,

$$\xi^* \in X^s.$$

- nests special cases:
  - canonical NK model
  - Aoki (2001): two-sector model with one flex-price sector, one sticky-price sector
  - Erceg, Henderson, Levin (1999): either wage flexibility or price flexibility

Optimal Monetary Policy

Gaussian Priors and Posteriors

$$\mathbb{E}\left[\log z_j | \omega_{ik}\right] = \phi_i \omega_{ij,k}$$
$$\text{var}[\log z_j | \omega_{ik}] = (1 - \phi_i)\text{var}[\log z_j]$$

- $\phi_i \in [0,1]$ is the *degree of price flexibility* of industry i

$$\phi_i = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_i^2}$$

  - lower ϕ_i is greater "price stickiness"
  - $\phi_i = 1$ is full price flexibility

Welfare Loss Decomposition

Theorem

Let $\mathcal{W}^*$ denote the first-best level of welfare. Up to a second order approximation,

$$\mathcal{W} \propto \mathcal{W}^* \exp\{-\Delta\}$$

Δ *denotes welfare losses from first best:*

$$\Delta \equiv \frac{1}{1/\eta + \gamma}\mathbb{V} + \mathbb{L}_{acr} + \mathbb{L}_{with}$$

- $\mathbb{V}$ *is the volatility of the (endogenous) output gap*
- $\mathbb{L}_{acr}$ *is productive inefficiency: misallocation across sectors*
- $\mathbb{L}_{wi}$ *is productive inefficiency: misallocation within sectors*

Theorem

The optimal monetary policy is a price index stabilization policy:

$$\sum_{i\in I} \psi_i^* \log p_i = 0 \qquad \textit{with} \qquad \sum_{i\in I} \psi_i = 1,$$

with optimal weights $(\psi_1^*, \ldots, \psi_n^*)$ *given by*

$$\psi_i^* \propto \frac{1}{1/\eta + \gamma} \psi_i^{\mathrm{og}} + \psi_i^{wi} + \psi_i^{\mathrm{acr}}$$

- ψ_i^{og} *is the policy that minimizes volatility of the output gap*
- ψ_i^{wi} *is the policy that minimizes within-industry misallocation*
- ψ_i^{acr} *is the policy that minimizes across-industry misallocation*

Optimal Monetary Policy

Theorem

(i) The policy that minimizes volatility of the output gap is given by

$$\psi_i^{\text{og}} \propto \lambda_i(1/\phi_i - 1), \qquad \textit{where} \quad \lambda_i \equiv \frac{p_i y_i}{PC} \quad \textit{is the Domar weight}$$

(ii) The policy that minimizes within-industry misallocation is given by

$$\psi_i^{wi} \propto \lambda_i(1-\phi_i)\theta_i\rho_i, \qquad \textit{where} \qquad \rho_i \equiv \frac{d\log \text{mc}_i(s)}{d\log w(s)}$$

(iii) The policy that minimizes across-industry misallocation is given by

$$\psi_i^{\text{acr}} \propto \lambda_i(1/\phi_i - 1)\left[\rho_0 - \rho_i + \sum_{j\in I}(1-\phi_j)\lambda_j\rho_j\ell_{ji}/\lambda_i\right]$$

General Principles for Monetary Policy

- the optimal price index places greater weight on:
  - larger sectors as measured by Domar weights λ_i
  - stickier sectors (low ϕ_i)
  - more upstream sectors
  - sectors with stickier downstream customers
  - sectors with more flexible upstream suppliers

Quantitative Illustration

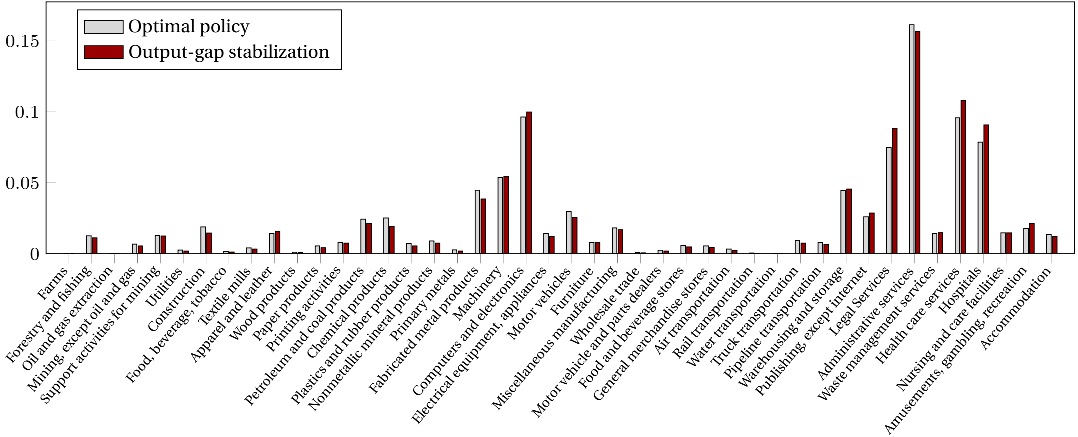
- what would be the welfare gains from adopting the optimal policy?
- we calibrate the model to the U.S. input-output tables and data on price stickiness
- we use model to quantify the welfare gains of the optimal policy relative to CPI stabilization

Welfare Loss relative to the first best

Table 1. Welfare Loss under Various Policies

	optimal policy (1)	output-gap stabilization (2)	CPI targeting (3)	Domar weighted (4)	stickiness weighted (5)
Welfare loss (percent consumption)	2.98	2.99	3.51	3.75	3.22
within-industry misallocation	2.66	2.67	3.00	3.16	2.80
across-industry misallocation	0.32	0.32	0.40	0.42	0.36
output gap volatility	10^{-5}	0	0.11	0.17	0.05
Cosine similarity to optimal policy	1	0.9957	0.5181	0.5929	0.6260

Optimal Weights



Conclusion

- Divine Coincidence is non-generic. In equilibrium, welfare loss arises from:
  - volatility of the output gap
  - misallocation both within and across sectors
- Optimal Policy: price index stabilization with greater weight on:
  - larger (in Domar weights) & stickier sectors
  - more upstream sectors, sectors with stickier customers, sectors with more flexible suppliers
- Quantitative welfare improvements from the adopting optimal policy
  - optimal policy relative to CPI stabilization $\approx$ half percentage point of quarterly consumption
  - output gap stabilization is approximately optimal