Optimal Policy Rules in HANK

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Inequality & stabilization policy

Does **inequality** change optimal **stabilization policy**? If so, how?

- Recently: increased policy interest & fast-growing academic literature.
  E.g.: Bhandari et al. (2021), Acharya et al. (2022), LeGrand et al. (2022), Davila-Schaab (2023), ...
  a) **Transmission**: how do instruments affect any given target? (e.g., output & inflation)
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- **This paper**: linear-quadratic approximation to HANK model
  - Derive **optimal policy rules** as forecast target criteria, applicable for all shocks
  - Main benefits of our approach:
    1. Separate role of inequality through transmission vs. objectives
    2. Sufficient statistics for optimal rules

Main results

a) **Dual mandate** $[\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{\lambda_\pi \widehat{\pi}_t^2 + \lambda_y \widehat{y}_t^2\}]$

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b) **Ramsey policy** $[\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{\lambda_\pi \widehat{\pi}_t^2 + \lambda_y \widehat{y}_t^2 + \text{inequality term}\}]$

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- Find: implications of inequality for opt. policy depend on distributional incidence of policy
  E.g.: MP is progressive in Bhandari et al. (2022) vs. distributionally neutral in Werning (2015).
- Our strategy: infer distributional incidence from rich quantitative model.
  (i) Monetary policy is close to neutral w.r.t. distribution.
  $\Rightarrow$ Optimal policy close to dual-mandate policy.
  (ii) Stimulus checks have strong distributional effects.
  $\Rightarrow$ Complementary to monetary policy.

Model Environment

- We study perfect foresight transitions.
- Optimal stochastic linear-quadratic regulator features certainty equivalence.

Households

Unit continuum of ex-ante identical households $i \in [0, 1]$

- **Consumption-savings problem**
  - Standard preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_{it}) - \nu\left(\ell_{it}\right)\right]$$

  - Idiosyncratic earnings—allows for unequal exposure to business cycle:

$$e_{it} = \Phi(\zeta_{it}, m_t, e_t), \quad \int_0^1 e_{it} di = e_t$$

    where m_t is an "inequality shock" (= demand shock) & e_t is aggregate labor income

  - Budget constraint [a_{it} is value of portfolio]:

$$c_{it} + [\text{cost of asset purchases}] = a_{it} + (1 - \tau_y + \tau_{e,t})e_{it} + \boldsymbol{\tau_{x,t}}, \quad a_{it} \geq \underline{a}$$

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- **Labor supply**: intermediated by labor unions

Production & wage-setting

- **Supply side structure**

  a) **Production**

  - Intermediate goods are produced using capital and labor: $y_{jt} = Ak_{jt}^{\alpha}\ell_{jt}^{1-\alpha}$
  - Subject to nominal rigidities. Pay labor & capital, and earn pure profits. A share $1-\alpha$ of profits goes to labor. Hard-wiring constant labor share.
  - Aggregate capital is fixed at $\bar{k}$

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  b) **Wage-setting**

  - Unions use rep.-agent MRS for wage-setting [= MRS at average, not average MRS]
  - Assume uniform labor rationing—everyone works the same amount
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- Standard **NKPC**: $\widehat{\pi}_t = \kappa\widehat{y}_t + \beta\widehat{\pi}_{t+1} + \psi\varepsilon_t$, where ε_t is a cost-push shock.

Assets

- Households can trade **three assets**:
  1. Capital
  2. Short-term nominal bonds
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- Don't need to model portfolio choice (all assets pay same return for $t = 1, 2, \cdots$)
- Only need existing date-0 portfolios when asset prices respond to news
  - We will impute these using data on portfolio composition across net worth levels
    Related to approach in Auclert-Rognlie (2020)

Government & eq'm characterization

- Policymaker sets two **policy instruments**
  1. Short-term nominal rate $\boldsymbol{i_t}$
  2. Uniform lump-sum transfers $\boldsymbol{\tau_{x,t}}$

  Background: taxes/transfers $\tau_{e,t}$ adjust to keep long-term budget balance.

- **Perfect-foresight eq'm** [notation: boldface = time paths]

Equilibrium

Given paths of shocks $\{m_t, \varepsilon_t\}_{t=0}^{\infty}$ *and government policy instruments* $\{i_t, \tau_{x,t}\}_{t=0}^{\infty}$*, paths of aggregate output and inflation* $\{y_t, \pi_t\}_{t=0}^{\infty}$ *are part of a linearized equilibrium if and only if*

$$\widehat{\boldsymbol{\pi}} = \kappa \widehat{\boldsymbol{y}} + \beta \widehat{\boldsymbol{\pi}}_{+1} + \psi \widehat{\boldsymbol{\varepsilon}} \quad \text{(NKPC)}$$

$$\widehat{\boldsymbol{y}} = \tilde{\mathcal{C}}_y \widehat{\boldsymbol{y}} + \tilde{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}} + \tilde{\mathcal{C}}_i \widehat{\boldsymbol{i}} + \tilde{\mathcal{C}}_\tau \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_m \widehat{\boldsymbol{m}} \quad \text{(IS*)}$$

Dual Mandate

Dual mandate optimal policy problem

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$$\mathcal{L}^{DM} \equiv \sum_{t=0}^{\infty} \beta^t \left[\lambda_\pi \widehat{\pi}_t^2 + \lambda_y \widehat{y}_t^2\right] = \lambda_\pi \widehat{\boldsymbol{\pi}}' W \widehat{\boldsymbol{\pi}} + \lambda_y \widehat{\boldsymbol{y}}' W \widehat{\boldsymbol{y}} \quad (1)$$

where $W = \text{diag}(1, \beta, \beta^2, \dots)$

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    where $W = \text{diag}(1, \beta, \beta^2, \dots)$
  - **Constraint set** [follows from eq'm characterization]

    $$\widehat{\boldsymbol{\pi}} = \kappa \widehat{\boldsymbol{y}} + \beta \widehat{\boldsymbol{\pi}}_{+1} + \psi \widehat{\boldsymbol{\varepsilon}} \tag{NKPC}$$

    $$\widehat{\boldsymbol{y}} = \tilde{\mathcal{C}}_y \widehat{\boldsymbol{y}} + \tilde{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}} + \tilde{\mathcal{C}}_i \widehat{\boldsymbol{i}} + \tilde{\mathcal{C}}_x \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_m \widehat{\boldsymbol{m}} \tag{IS*}$$

Sequence-space representation of dual-mandate policy

- FOC for choice of i_t

$$\frac{\partial \mathcal{L}^{DM}}{\partial \widehat{\boldsymbol{\pi}}} \quad \frac{\partial \widehat{\boldsymbol{\pi}}}{\partial i_t} + \frac{\partial \mathcal{L}^{DM}}{\partial \widehat{\boldsymbol{y}}} \quad \frac{\partial \widehat{\boldsymbol{y}}}{\partial i_t} = 0$$

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$$(\lambda_\pi W \widehat{\boldsymbol{\pi}})' \ \frac{\partial \widehat{\boldsymbol{\pi}}}{\partial i_t} \ + \ (\lambda_y W \widehat{\boldsymbol{y}})' \ \frac{\partial \widehat{\boldsymbol{y}}}{\partial i_t} = 0$$

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- Same holds at all dates. Transpose and stack equations for all t:

$$\Theta'_{\pi,i}\lambda_\pi W\widehat{\boldsymbol{\pi}} \ + \ \Theta'_{yi}\lambda_y W\widehat{\boldsymbol{y}} = 0$$

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- Forecast targeting rule: adjust expected policy path so condition above holds.

Optimal policy rule

Proposition

The optimal monetary policy rule for a dual mandate policymaker can be written as the ***forecast target criterion***

$$\widehat{\pi}_t + \frac{\lambda_y}{\lambda_\pi \kappa}\left(\widehat{y}_t - \widehat{y}_{t-1}\right) = 0, \quad \forall t = 0, 1, \ldots \tag{2}$$

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- **Same target criterion as in RANK model**
  - Inequality does not affect *optimal* $\{y, \pi\}$ paths in response to *any* non-policy shock
  - Demand block only matters *residually* for sequence of interest rates needed to achieve the optimal $\{y, \pi\}$ paths?

▸ Example

Ramsey Problem

Social welfare function

- We consider a **social welfare function** with Pareto weights

$$\mathcal{V}^{HA} = \sum_{t=0}^{\infty} \beta^t \int \varphi(\zeta) \left[u\big(\omega_t(\zeta) c_t\big) - \nu\left(\ell_t\right) \right] d\Gamma(\zeta) \tag{3}$$

  - ζ is the idiosyncratic history of a household, $\varphi(\zeta)$ is a Pareto weight on the utility of households with history ζ, and $\omega_t(\zeta)$ is the time-t consumption share of such households

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- Our approach: ensure **efficient steady state** [as in Woodford (2003)] ▸ Formal Discussion

  - Assumptions: production subsidy + back out weights $\varphi(\zeta)$

  - Our SWF will capture cyclical insurance motive, not long-run redistribution

Full Ramsey problem

The problem then fits into **linear-quadratic form**:

- **Loss function:** to second order, social welfare function $\mathcal{V}^{HA}$ is proportional to $-\mathcal{L}^{HA}$

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- **Computation:** stabilizing consumption distribution = stabilizing prices

Ramsey Problem

Optimal Monetary Policy

Optimal Ramsey monetary policy rule

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The optimal monetary policy rule for a Ramsey planner with loss $\mathcal{L}^{HA}$ can be written as the ***forecast target criterion***

$$\underbrace{\Theta'_{\pi,i}\lambda_\pi W\widehat{\boldsymbol{\pi}} + \Theta'_{y,i}\lambda_y W\widehat{\boldsymbol{y}}}_{\textit{dual-mandate criterion}} \quad + \quad \underbrace{\int \Theta'_{\omega(\zeta),i}\lambda_{\omega(\zeta)} W\widehat{\boldsymbol{\omega}}(\zeta)d\Gamma(\zeta)}_{\textit{effects of instrument on consumption shares}} \quad = \quad \mathbf{0}$$

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- So does **inequality** affect the optimal policy rule?
  - No iff policy does not affect consumption shares ($\Theta_{\omega(\zeta),i} = 0$) [e.g. as in Werning (2015)]
  - Yes in prior work: large distributional effects that can offset effects of business-cycle shocks
    Bhandari et al. (2021): rate cut offsets distributional effects of cost-push shock.

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- So does **inequality** affect the optimal policy rule?
  - What do we know about $\Theta_{\omega(\zeta),i}$?

⇒ **Our strategy**: use data on household balance sheets to discipline distributional effects.

Key calibration points

1. Income cyclicality: labor income more cyclical for low-income workers
   $e_{it} = \Phi(\zeta_{it}, m_t, e_t)$ calibrated as in Guvenen et al. (2022) ▸ Figure

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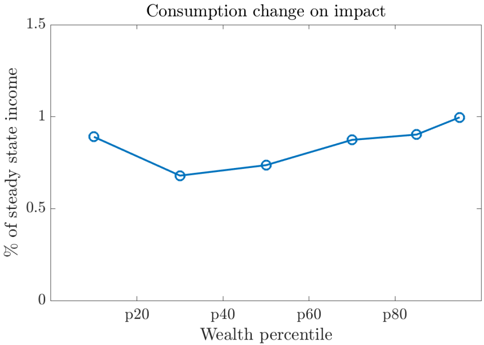
2. Portfolios: using SCF (2019), map household balance sheets into capital, long-term bond, short-term bond. ▸ Table
   - e.g. typical middle-class household is long capital (housing) and short bonds (mortgage)

Consumption effects of monetary policy



▸ More Calibration Details ▸ Income & wealth distributions

- **Model**
  - Simulate expansionary monetary shock,
  - plot initial cons. change by wealth

$\Rightarrow$ **find rather small distr. effects**

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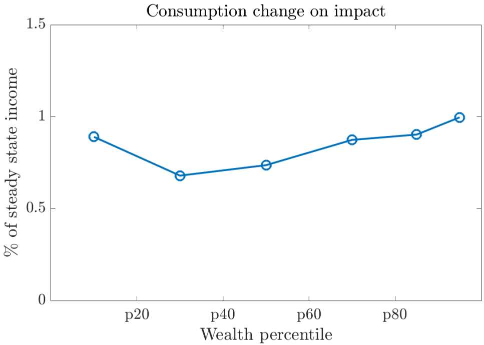

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- **Model**
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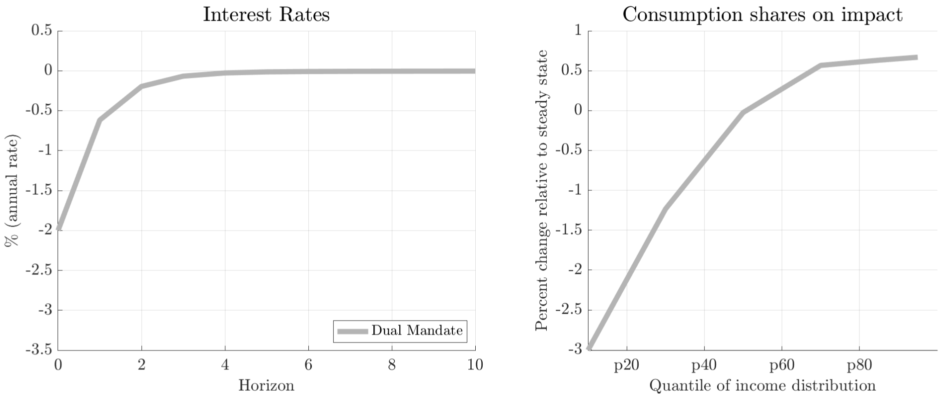
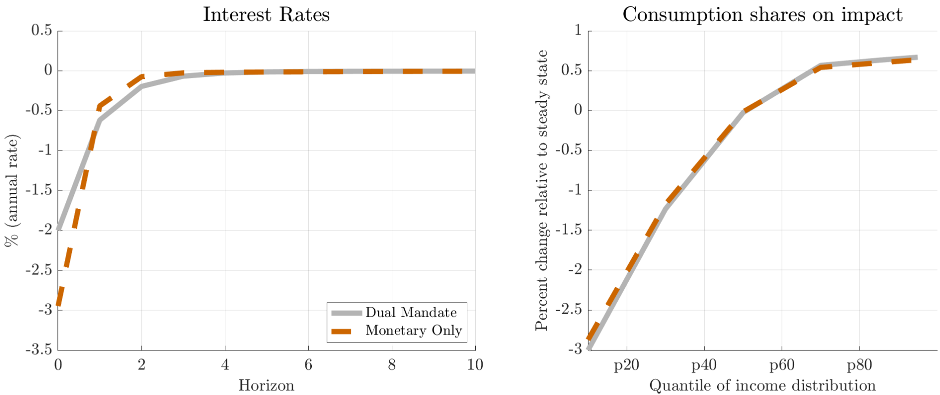
- **Empirical evidence**
  - Holm et al. ('21): U-shaped effect
  - Coibion et al. ('17): progressive effect
  - Chang & Schorfheide ('22): regressive effect

Application: distributional shock



- **Dual mandate**: cut rates to perfectly stabilize aggregate demand and so $\{y, \pi\}$

Application: distributional shock



- **Ramsey policy**: similar, since monetary policy is ill-suited to offset the distr. incidence
  - $\Rightarrow$ stabilizing consumption at the bottom would imply large overshooting of y and π ▸ Details

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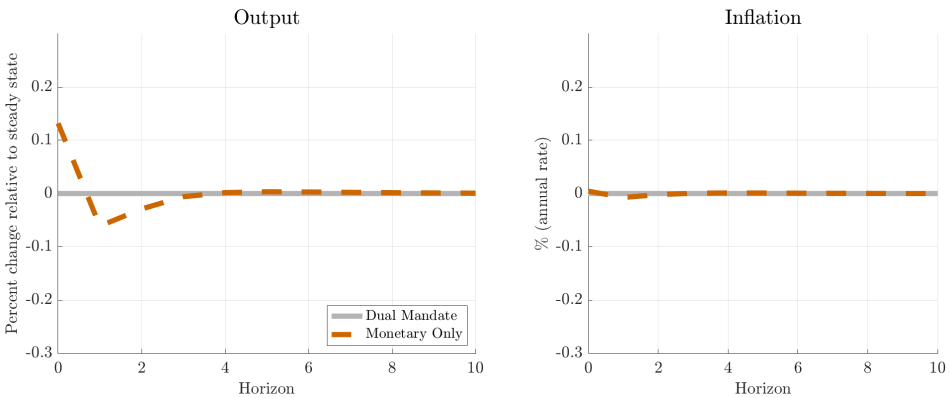

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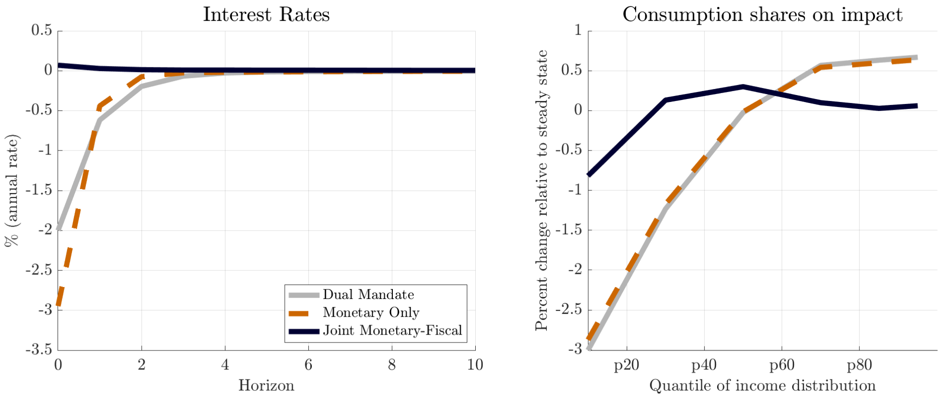
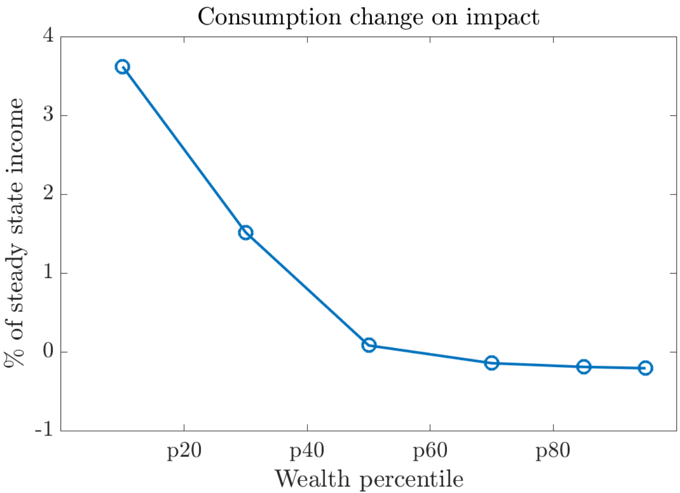


- **Joint fiscal-monetary**: stimulus checks provide agg. & cross-sectional stabilization

  ⇒ monetary policy at the Ramsey optimum barely responds

Stimulus check incidence



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  b) **Ramsey policy**

     - Deviate from dual mandate policy iff monetary policy has substantial distributional effects
     - Our reading of evidence & model: exposure to MP is fairly uniform
     - Fiscal policy is much better-suited for cyclical insurance

Appendix

Production block

- Unit continuum of unions k, demand ℓ_{ikt} units from household i. Total union labor supply is $\ell_{kt} \equiv \int_0^1 e_{it}\ell_{ikt}di$.
- Total output is

$$y_t = \left(\int_k \ell_{kt}^{\frac{\varepsilon_t - 1}{\varepsilon_t}} dk\right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$$

- The price index of the labor aggregate is

$$w_t = \left(\int w_{kt}^{1-\varepsilon_t} dk\right)^{1/(1-\varepsilon_t)}$$

and demand for labor from union k is

$$\ell_{kt} = \left(\frac{w_{kt}}{w_t}\right)^{-\varepsilon_t} y_t.$$

back

Production block

- Union problem: choose the reset wage w^* and ℓ_{kt} to maximize

$$\sum_{s\geq 0}\beta^s\theta^s\left[u_c(c_{t+s})(1-\tau_y)\frac{\bar{\varepsilon}\Xi}{(\bar{\varepsilon}-1)(1-\tau_y)}\frac{w^*}{p_{t+s}}\ell_{kt}-\nu_\ell\left(\ell_{t+s}\right)\ell_{kt}\right]$$

subject to labor demand constraint

Ξ is subsidy-related steady-state wedge, see loss function proof.

- This gives

$$\widehat{\pi}_t=\kappa\widehat{y}_t+\beta\widehat{\pi}_{t+1}+\psi\widehat{\varepsilon}_t$$

where $\kappa\equiv(\phi+\gamma)\frac{(1-\theta)(1-\beta\theta)}{\theta}$, $\phi\equiv\frac{\nu_{\ell\ell}(\bar{\ell})\bar{\ell}}{\nu_\ell(\bar{\ell})}$ and $\psi\equiv-\frac{\kappa}{(\phi+\gamma)(\epsilon-1)}$

- Aggregating production gives $y_t=\frac{\ell_t}{d_t}$ where $\ell_t\equiv\int_0^1\int_0^1 e_{it}\ell_{ikt}\,di\,dk$ and d_t captures efficiency losses

back

Equilibrium characterization

- NKPC is as in original optimality condition. Proof combines all other optimality and market-clearing conditions to get (IS*)

- Consumption-savings problem gives aggregate consumption function. Using output market-clearing, $e_{it} w_t \ell_{it} = e_{it} y_t$, we get

$$\widehat{\boldsymbol{y}} = \mathcal{C}_y \widehat{\boldsymbol{y}} + \mathcal{C}_r \widehat{\boldsymbol{r}} + \mathcal{C}_x \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_e \widehat{\boldsymbol{\tau}}_e + \mathcal{C}_m \widehat{\boldsymbol{m}}$$

- Write relationships between asset prices and rates of return as

$$\widehat{r}_0 = r_0(\widehat{\pi}_0, \widehat{y}_0, \widehat{q}_0), \;\; \widehat{\boldsymbol{r}}_{+1} = r_{+1}(\widehat{\boldsymbol{i}}, \widehat{\boldsymbol{\pi}}), \;\; \widehat{\boldsymbol{q}} = q(\widehat{\boldsymbol{\pi}}_{+1}, \widehat{\boldsymbol{y}}_{+1}, \widehat{\boldsymbol{r}}_{+1})$$

- From the government budget constraint we get

$$\widehat{\boldsymbol{\tau}}_e = \tau_e(\widehat{\boldsymbol{y}}, \widehat{\boldsymbol{\tau}}_x, \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{q}}).$$

back

Equilibrium characterization

- Plugging the asset pricing and gov't budget relations into the consumption function:

$$\widehat{\boldsymbol{y}} = \mathcal{C}_y \widehat{\boldsymbol{y}} + \mathcal{C}_r \widehat{\boldsymbol{r}}(\widehat{\boldsymbol{y}}, \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{i}}) + \mathcal{C}_x \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_e \widehat{\boldsymbol{\tau}}_e(\widehat{\boldsymbol{y}}, \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{i}}, \widehat{\boldsymbol{\tau}}_x) + \mathcal{C}_m \boldsymbol{m}$$

  and so

$$\widehat{\boldsymbol{y}} = \underbrace{[\mathcal{C}_y + \mathcal{C}_r \mathcal{R}_y + \mathcal{C}_e \mathcal{T}_y]}_{\tilde{\mathcal{C}}_y} \widehat{\boldsymbol{y}} + \underbrace{[\mathcal{C}_r \mathcal{R}_\pi + \mathcal{C}_e \mathcal{T}_\pi]}_{\tilde{\mathcal{C}}_\pi} \widehat{\boldsymbol{\pi}} + \underbrace{[\mathcal{C}_r \mathcal{R}_i + \mathcal{C}_e \mathcal{T}_i]}_{\tilde{\mathcal{C}}_i} \widehat{\boldsymbol{i}} + \underbrace{[\mathcal{C}_x + \mathcal{C}_e \mathcal{T}_x]}_{\tilde{\mathcal{C}}_x} \widehat{\boldsymbol{\tau}}_x + \mathcal{C}_m \boldsymbol{m}$$

- This has verified all eq'm relations, giving sufficiency of (NKPC) and (IS*)

▸ back

Optimal dual mandate rule: proof

- FOCs of optimal policy problem are

$$\begin{aligned} \lambda_\pi W \widehat{\boldsymbol{\pi}} + \Pi_\pi' W \boldsymbol{\varphi}_\pi - \tilde{\mathcal{C}}_\pi' W \boldsymbol{\varphi}_y &= \mathbf{0} \\ \lambda_y W \widehat{\boldsymbol{y}} - \Pi_y' W \boldsymbol{\varphi}_\pi + (I - \tilde{\mathcal{C}}_y') W \boldsymbol{\varphi}_y &= \mathbf{0} \\ -\tilde{\mathcal{C}}_i' W \boldsymbol{\varphi}_y &= \mathbf{0}, \end{aligned}$$

- Guess that $\boldsymbol{\varphi}_y = \mathbf{0}$. Then we get

$$\lambda_\pi \widehat{\boldsymbol{\pi}} + \lambda_y W^{-1} \Pi_\pi' (\Pi_y')^{-1} W \widehat{\boldsymbol{y}} = \mathbf{0}$$

which can re-written to give the stated relation

- Remains to verify the guess that $\boldsymbol{\varphi}_y = \mathbf{0}$

back

Optimal dual mandate rule: proof

- Consider some arbitrary $(\boldsymbol{m}, \boldsymbol{\varepsilon})$, and let $(\widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{\pi}}^*)$ denote the solution of the system (NKPC) + dual mandate rule given $(\boldsymbol{m}, \boldsymbol{\varepsilon})$
- Plugging into the consumption function:

$$\underbrace{\widehat{\boldsymbol{y}}^* - \tilde{\mathcal{C}}_y \widehat{\boldsymbol{y}}^* - \tilde{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}}^* - \mathcal{C}_m \boldsymbol{m}}_{\text{demand target}} = \tilde{\mathcal{C}}_i \widehat{\boldsymbol{i}}$$

- Remains to show that we can find $\widehat{\boldsymbol{i}}^*$ such that this relation holds

▸ back

Optimal dual mandate rule: proof

- Supply term has NPV

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \bar{y}\widehat{y}_t$$

- Aggregating household budget constraints we get that

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \bar{c}\widehat{c}_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \{(1+\bar{r})\bar{a}\widehat{r}_t + (1-\tau_y)\bar{y}\widehat{y}_t + \bar{\tau}_x\widehat{\tau}_{xt} + \bar{\tau}_e\widehat{\tau}_{et}\}$$

  Doing the same for the gov't budget constraint:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^t \{(1+\bar{r})\bar{a}\widehat{r}_t + \bar{\tau}_x\widehat{\tau}_{xt} + \bar{\tau}_e\widehat{\tau}_{et}\} = \sum_{t=0}^{\infty} \tau_y\bar{y}\widehat{y}_t$$

- Thus the two have the same NPV. Then the stated condition is sufficient to ensure implementability.

▸ back

Ramsey loss function

Proposition

To second order, the social welfare function (3) is proportional to $-\mathcal{L}^{HA}$*, given as*

$$\mathcal{L}^{HA} \equiv \sum_{t=0}^{\infty} \beta^t \left[\widehat{\pi}_t^2 + \frac{\kappa}{\bar{\varepsilon}} \widehat{y}_t^2 + \frac{\kappa\gamma}{(\gamma+\phi)\bar{\varepsilon}} \int \frac{\widehat{\omega}_t(\zeta)^2}{\bar{\omega}(\zeta)} d\Gamma(\zeta) \right] \tag{4}$$

where $\widehat{\omega}_t(\zeta) = \omega_t(\zeta) - \bar{\omega}(\zeta)$ *and* $\bar{\omega}(\zeta)$ *is the steady-state consumption share of an individual with history* ζ*.*

back

Ramsey planner loss function: proof

- Write planner per-period utility flow as

$$U_t = \int \varphi(\zeta) \frac{\left(\bar{c} e^{\hat{c}_t} \omega_t(\zeta)\right)^{1-\gamma} - 1}{1-\gamma} d\Gamma(\zeta) - \nu \left(\bar{\ell} e^{\hat{\ell}_t}\right) \tag{5}$$

- Objective: find 2nd-order approximation to U_t that depends only on 2nd-order terms
- Preliminary definitions
  - Steady state needs to equalize marginal utility of consumption across histories:

$$\varphi(\zeta)\bar{c}^{1-\gamma}\bar{\omega}(\zeta)^{-\gamma} = \bar{u}_c \bar{c} \qquad \forall \zeta$$

  - Imposing that consumption shares integrate to 1 yields

$$\int \varphi(\zeta)^{1/\gamma} d\Gamma(\zeta) = \bar{c}\bar{u}_c^{1/\gamma}$$

back

Ramsey planner loss function: proof

- Preliminary definitions
  - Can recover consumption shares as a function of planner weights:

$$\bar{\omega}(\zeta) = \frac{\varphi(\zeta)^{1/\gamma}}{\int \varphi(\zeta)^{1/\gamma} d\Gamma(\zeta)} \qquad \forall \zeta$$

  - For future reference define

$$\Xi \equiv \left(\int \varphi(\zeta)^{1/\gamma} d\Gamma(\zeta) \right)^{\gamma} = \varphi(\zeta)\bar{\omega}(\zeta)^{-\gamma} \qquad \forall \zeta$$

- Now can begin with first-order terms:
  - For c_t we get

$$\begin{aligned} \frac{\partial U}{\partial \widehat{c}_t} &= \int \varphi(\zeta)(\bar{c}\bar{\omega}(\zeta))^{1-\gamma} d\Gamma(\zeta) \\ &= \bar{c}^{1-\gamma}\Xi \end{aligned}$$

back

Ramsey planner loss function: proof

- Now can begin with first-order terms:
  - For ℓ_t we have
  $$\frac{\partial U}{\partial \widehat{\ell}_t} = -\nu_\ell(\bar{\ell})\bar{\ell}.$$
  Set union subsidy so that $\Xi \bar{c}^{-\gamma} = \nu_\ell$
  - For consumption shares $\omega_t(\zeta)$ we have
  $$\begin{aligned} \frac{\partial U}{\partial \omega_t(\zeta)} &= \varphi(\zeta)\bar{c}^{1-\gamma}\bar{\omega}(\zeta)^{-\gamma} d\Gamma(\zeta) \\ &= \bar{c}^{1-\gamma}\Xi d\Gamma(\zeta) \end{aligned}$$

▸ back

Ramsey planner loss function: proof

- Next consider second-order terms:
  - For level & split of consumption we have

$$\begin{aligned}
\frac{\partial^2 U_t}{\partial \widehat{c}_t^2} &= (1-\gamma)\Xi \bar{c}^{1-\gamma} \\
\frac{\partial U_t}{\partial \omega_t(\zeta)^2} &= -\gamma \bar{c}^{1-\gamma} \frac{\Xi}{\bar{\omega}(\zeta)} d\Gamma(\zeta) \\
\frac{\partial^2 U_t}{\partial \widehat{c}_t \partial \omega_t(\zeta)} &= (1-\gamma)\Xi \bar{c}^{1-\gamma} d\Gamma(\zeta)
\end{aligned}$$

  - For hours worked we have

$$\frac{\partial^2 U}{\partial \widehat{\ell}_t^2} = -\nu_{\ell\ell}(\bar{\ell})\bar{\ell}^2 - \nu_\ell(\bar{\ell})\bar{\ell}$$

back

Ramsey planner loss function: proof

- We can now put everything together:

$$
\begin{aligned}
U_t &\approx \bar{U} + \bar{c}^{1-\gamma}\Xi\widehat{c}_t - \nu_\ell(\bar{\ell})\bar{\ell}\widehat{\ell}_t \\
&+\frac{1}{2}(1-\gamma)\Xi\bar{c}^{1-\gamma}\widehat{c}_t^2 - \frac{1}{2}\left[\nu_{\ell\ell}(\bar{\ell})\bar{\ell}^2 + \nu_\ell(\bar{\ell})\bar{\ell}\right]\widehat{\ell}_t^2 - \frac{1}{2}\gamma\bar{c}^{1-\gamma}\Xi\int\frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)}d\Gamma(\zeta) \\
&+\bar{c}^{1-\gamma}\Xi\int\widehat{\omega}_t(\zeta)d\Gamma(\zeta) + (1-\gamma)\bar{c}^{1-\gamma}\Xi\widehat{c}_t\int\widehat{\omega}_t(\zeta)d\Gamma(\zeta)
\end{aligned}
$$

  Terms in last row are zero.

- Can now write this as

$$
\begin{aligned}
U_t &\approx \bar{U} + \bar{c}^{1-\gamma}\Xi\widehat{c}_t - \nu_\ell(\bar{\ell})\bar{\ell}\left(\widehat{c}_t + \widehat{d}_t\right) \\
&+\frac{1}{2}(1-\gamma)\Xi\bar{c}^{1-\gamma}\widehat{c}_t^2 - \frac{1}{2}(\phi+1)\,\nu_\ell(\bar{\ell})\bar{\ell}(\widehat{c}_t + \widehat{d}_t)^2 - \frac{1}{2}\gamma\bar{c}^{1-\gamma}\Xi\int\frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)}d\Gamma(\zeta)
\end{aligned}
$$

back

Ramsey planner loss function: proof

- Set union subsidy so that the $\widehat{c}_t$ terms cancel. We thus have

$$U_t \approx \bar{U} - \nu_\ell(\bar{\ell})\bar{\ell}\widehat{d}_t - \frac{1}{2}\nu_\ell(\bar{\ell})\bar{\ell}(\gamma+\phi)\widehat{y}_t^2 - \frac{1}{2}\gamma\nu_\ell(\bar{\ell})\bar{\ell}\int \frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)} d\Gamma(\zeta)$$

- Finally follow standard steps to express d_t in terms of the history of inflation. After standard steps we get

$$\begin{aligned} \sum_{t=0}^{\infty}\beta^t U_t &\approx -\nu_\ell(\bar{\ell})\bar{\ell}\sum_{t=0}^{\infty}\beta^t\left[\frac{\theta\bar{\varepsilon}}{2(1-\theta)(1-\beta\theta)}\widehat{\pi}_t^2 + \frac{1}{2}(\gamma+\phi)\widehat{y}_t^2 + \frac{\gamma}{2}\int\frac{\widehat{\omega}(\zeta)^2}{\bar{\omega}(\zeta)}d\Gamma(\zeta)\right] \\ &= -\frac{\nu_\ell(\bar{\ell})\bar{\ell}\theta\bar{\varepsilon}}{2(1-\theta)(1-\beta\theta)}\sum_{t=0}^{\infty}\beta^t\left[\widehat{\pi}_t^2 + \frac{\kappa}{\bar{\varepsilon}}\widehat{y}_t^2 + \frac{\kappa\gamma}{(\gamma+\phi)\bar{\varepsilon}}\int\frac{\widehat{\omega}(\zeta^t)^2}{\bar{\omega}(\zeta^t)}d\Gamma(\zeta^t)\right], \end{aligned}$$

▸ back

Getting the Ω's: computational details

- Idea: can obtain fluctuations in consumption shares as a function of fluctuations in a small number of inputs to the consumption-savings problem
- Formally, let $\boldsymbol{x} \equiv (\boldsymbol{r}', \boldsymbol{y}', \boldsymbol{\tau}_x', \boldsymbol{\tau}_e', \boldsymbol{m}')'$ be the stacked sequences of inputs to the household problem. Then can show that there is symmetric matrix Q such that

$$\sum_{t=0}^{\infty} \beta^t \int \frac{\widehat{\omega}_t(\zeta, \boldsymbol{x})^2}{\bar{\omega}(\zeta)} d\Gamma(\zeta) = \widehat{\boldsymbol{x}}' Q \widehat{\boldsymbol{x}} + \mathcal{O}(||\widehat{\boldsymbol{x}}||^3)$$

  - Key step is to show that $\widehat{\omega}_t(\zeta, \boldsymbol{x}) \approx \Omega_t(\zeta)\widehat{\boldsymbol{x}}$ which yields

$$\frac{\widehat{\omega}_t(\zeta^t, \boldsymbol{x})^2}{\bar{\omega}(\zeta^t)} = \widehat{\boldsymbol{x}}' \underbrace{\frac{\Omega_t(\zeta^t)'\Omega_t(\zeta^t)}{\bar{\omega}(\zeta^t)}}_{\equiv Q_t(\zeta^t)} \widehat{\boldsymbol{x}} + \mathcal{O}(||\widehat{\boldsymbol{x}}||^3)$$

  and so

$$\sum_{t=0}^{\infty} \beta^t \int \frac{\widehat{\omega}_t(\zeta^t, \boldsymbol{x})^2}{\bar{\omega}(\zeta^t)} d\Gamma(\zeta^t) = \widehat{\boldsymbol{x}}' \underbrace{\left(\sum_{t=0}^{\infty} \beta^t \int Q_t(\zeta^t) d\Gamma(\zeta^t) \right)}_{\equiv Q} \widehat{\boldsymbol{x}} + \mathcal{O}(||\widehat{\boldsymbol{x}}||^3)$$

back

Getting the Ω's: computational details

- We obtain $\Omega_t(\zeta)$ using sequence-space methods + simulation [see paper for details]
- Given Q, we have a finite-dimensional but non-diagonal LQ problem
  - The objective function can be written as
$$\mathcal{L} \equiv \frac{1}{2}\boldsymbol{x}' P \boldsymbol{x},$$
  - We then get the FOC
$$\Theta'_{xz} P \boldsymbol{x} = 0$$
    and the corresponding optimal instrument path
$$\boldsymbol{z}^* \equiv -\left(\Theta'_{x,z} P \Theta_{x,z}\right)^{-1} \times \left(\Theta'_{x,z} P \Theta_{x,\varepsilon} \cdot \boldsymbol{\varepsilon}\right)$$

back

More on model calibration

γ	CRRA	1.2
ϕ	Frisch elasticity	1
β	Discount factor	0.984
κ	Phillips curve slope	0.022
α	Capital share	36%
δ	Depreciation rate	1%
$\underline{a}/\bar{y}$	Borrowing limit	-0.27
δ	Bond duration	0.025
$\bar{\tau}_x$	Steady state transfer	$0.17 \times GDP$

▸ back

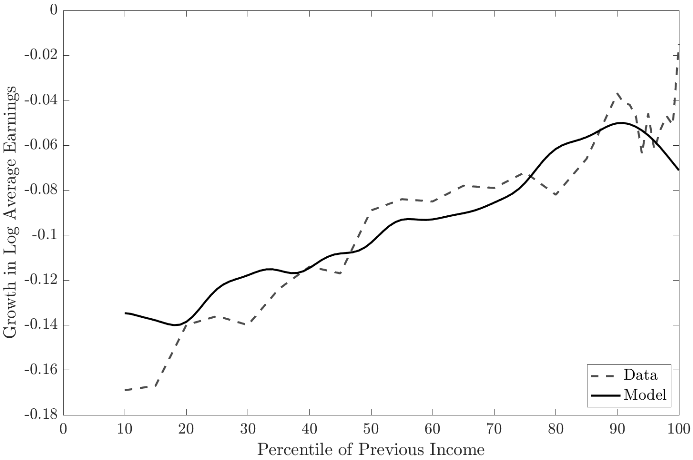
Income and wealth distribution

	Wealth		Income	
	Data	Model	Data	Model
Top 1%	37	27	17	20
Top 5%	65	66	32	32
Top 10%	76	82	43	44
Top 25%	91	96	64	60
Top 50%	99	101	84	77

Table: Shares (%) of wealth and income concentrated in the top x% of the distribution. Data are from the 2019 Survey of Consumer Finance.

back

Factor structure of Volcker recession



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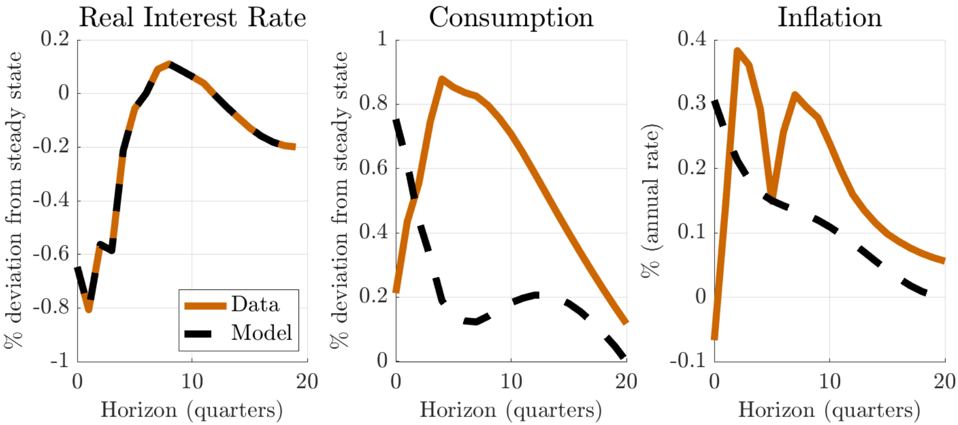
Factor structure of Volcker recession



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Household portfolios

Category	Total	Holdings by net worth group Top 1%	Next 9%	Next 40%	Bottom 50%
Real estate and durables	167	24	48	72	23
Equity and mutual funds	191	101	66	23	2
Currency, deposits, and similar	60	16	23	19	2
Govt. and corp. bonds and similar	29	10	11	7	1
Pension assets	131	6	63	58	4
Mortgage liabilities	49	2	12	24	11
Consumer credit and loans	24	1	2	8	12
Net worth excluding pension assets	374	147	135	89	4
Capital	419	157	135	101	25
Short-term bonds	-12	1	7	-3	-16
Long-term bonds	-33	-11	-8	-9	-5
Total	374	147	135	89	4

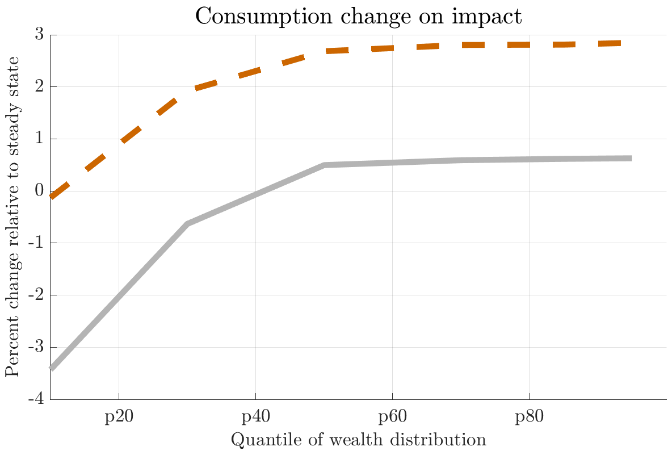
back

Calibration of household portfolios

- **Household portfolios**
  - We classify SCF assets and liabilities into bundles of capital, short-term bonds, and long-term bonds
    - \$1 equity = \$1.32 capital - \$0.20 long-term bonds -\$0.12 short-term bonds
    - \$1 mortgage balance = -\$0.50 long-term bonds -\$0.50 short-term bonds
    - \$1 consumer credit = -\$1 short-term bonds
    - \$1 currency or deposits = \$1 short-term bonds
  - We then impute portfolio for households in our model as a function of their net worth
  - These portfolio positions will matter at date 0, through revaluation effects
- **Pension assets**
  - We treat pensions as part of the government
  - Returns earned on these assets are then paid out slowly through taxes

back

Application: distributional shock



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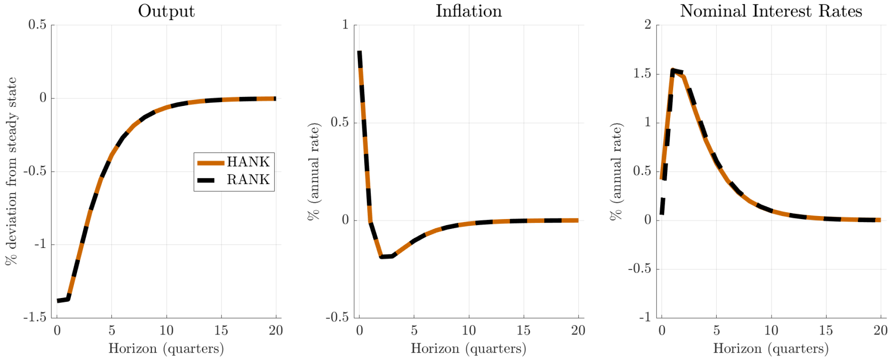
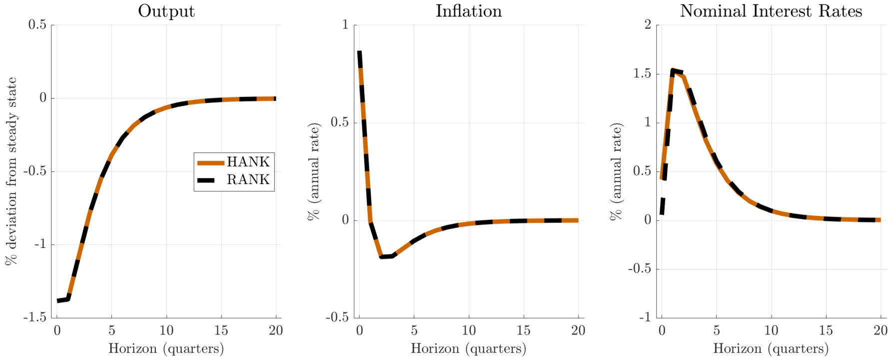
Quantitative illustration: supply shock



- $\{y, \pi\}$ paths agree exactly. What about **interest rates**?

Quantitative illustration: supply shock



- $\{y, \pi\}$ paths agree exactly. What about **interest rates**?
  - Could in principle disagree substantially. But we have emp. evidence on $i \to \{y, \pi\}$
  - Limiting th'm [McKay-Wolf]: optimal i path can in principle be fully characterized using empirical evidence on the propagation of monetary policy shocks