Long run inflation expectations

Jonas Fisher, Leonardo Melosi and Sebastian Rast

Mark W. Watson: Remarks

March 31, 2023

π **process:**

$$\pi_t = \overline{\pi}_t + \pi_t^{Stat}$$

- $\overline{\pi}_t$ is a random walk with RW stochastic volatility
- π_t^{Stat} is a TV ARMA(1,1) also with stochastic volatility

Forecaster i

- Information

$$s_t(i) = \begin{bmatrix} s_{1,t} = \pi_t = \overline{\pi}_t + \pi_t^{Stat} \\ s_{2,t}(i) = \overline{\pi}_t + \alpha(i)v_{c,t} \\ s_{3,t}(i) = \overline{\pi}_t + v_t(i) \end{bmatrix}$$

- Goal

$$\overline{\pi}_{t|t}(i) = E(\overline{\pi}_t | \{s_k(i)\}_{k=1}^t, \theta(i))$$

where θ are the (time invariant) parameters indexing the stochastic process.

Data:

$$\pi_t, \overline{\pi}_{t|t}(i) \text{ for } t = 1, ..., T \text{ and } i = 1, ..., N$$

Interesting objects:

$$N^{-1} \sum_i \overline{\pi}_{t|t}(i) - E(\overline{\pi}_t | \{\pi_k\}_{k=1}^T)$$

$$\frac{\partial E(\overline{\pi}_t | \{s_k(i)\}_{k=0}^t)}{\partial (shocks)} \qquad \text{('Impulse Responses')}$$

$$N^{-1} \sum_i \overline{\pi}_{t|t}(i) = f(shocks_{1:t}) \qquad \text{('Historical Decompositions')}$$

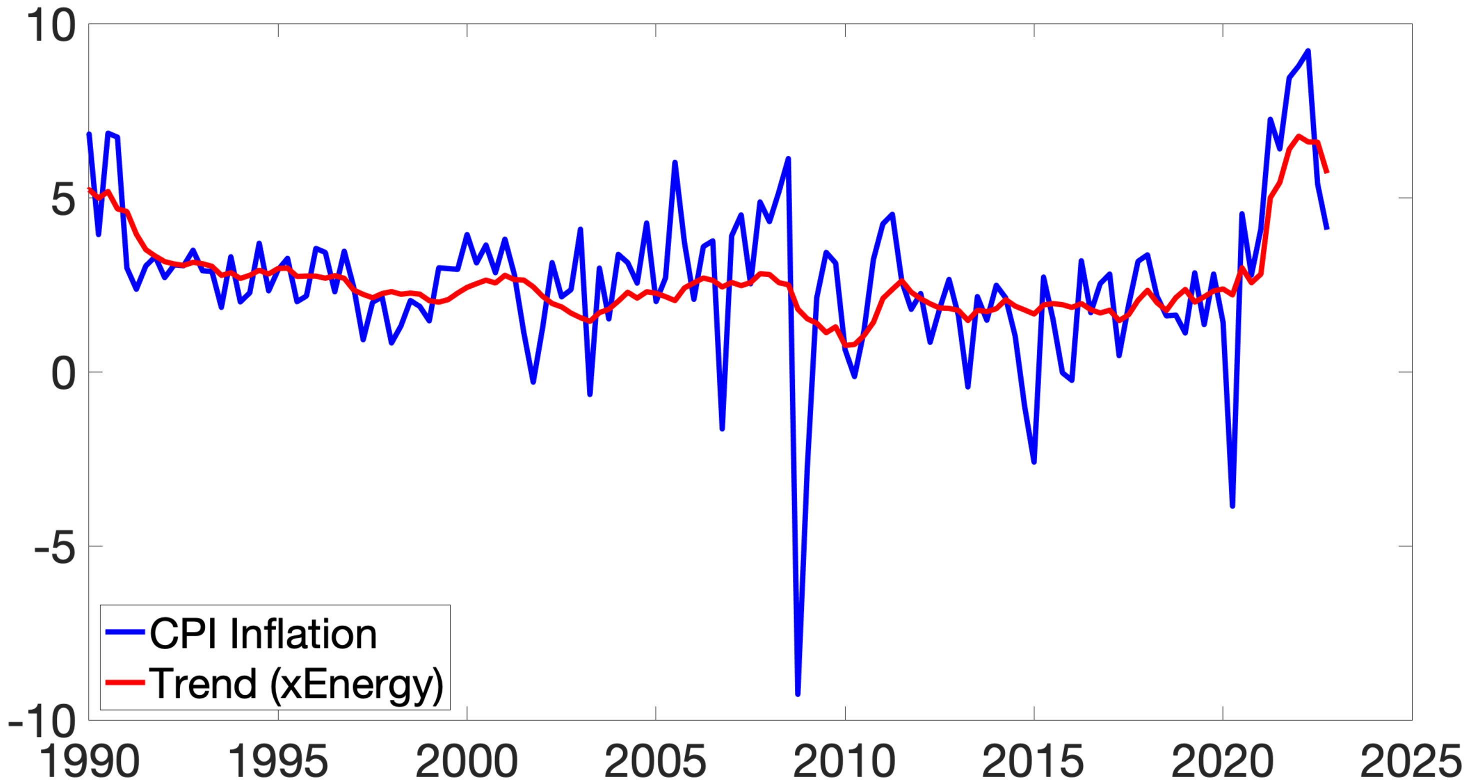
Paper's Contributions:

1. Model
2. Sophisticated Implementation
3. Empirical Results

Model:

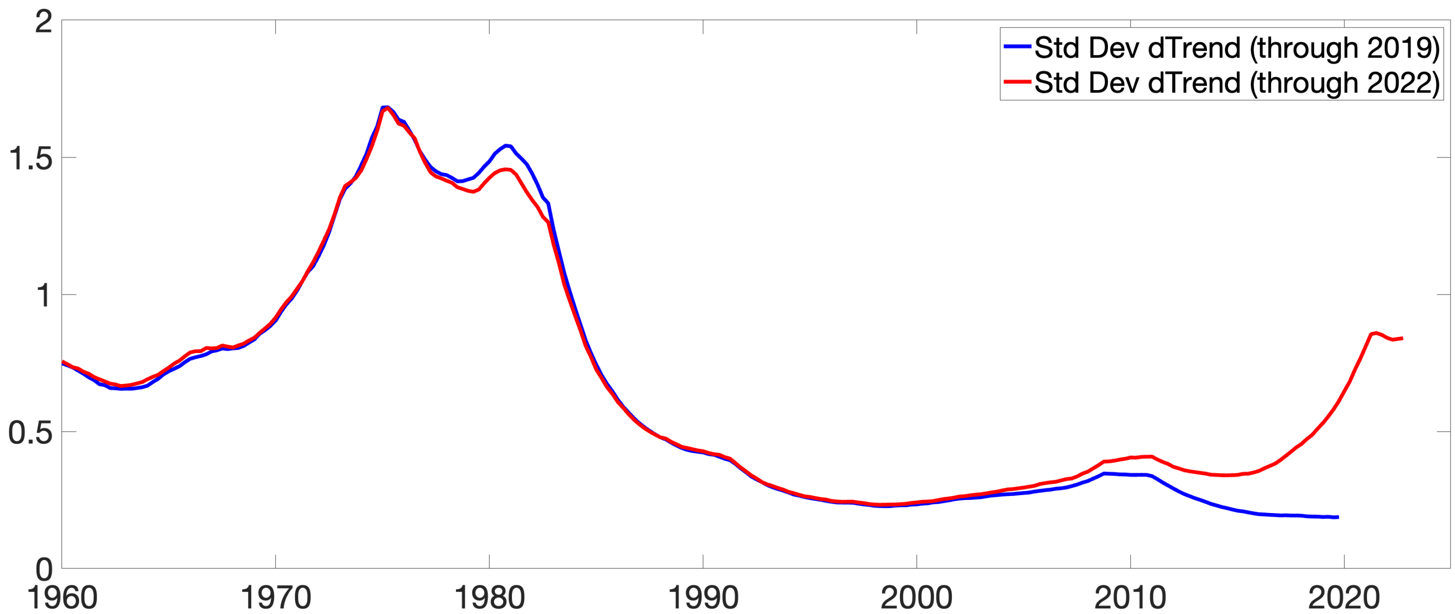
$$\pi_t = \overline{\pi}_t + \pi_t^{Stat}$$

- $\overline{\pi}_t$ is a random walk with RW stochastic volatility; π_t^{Stat} is a TV **white noise** also with stochastic volatility



$$\pi_t = \overline{\pi}_t + \pi_t^{Stat}$$

$$StdDev(\Delta\overline{\pi}_t) = \sigma_{\Delta\overline{\pi}}$$



$\sigma_{\Delta\overline{\pi}}$ and Long-Run Expectations (Beveridge-Nelson Arithmetic in time-invariant linear models)

- $\Delta\pi_t$: Change in inflation
- z_t vector of other variables that might be informative about future values of π
- Suppose

$$\begin{bmatrix} \Delta\pi_t \\ z_t \end{bmatrix} = C(L)\varepsilon_t$$

- Let

$$\pi_t^{LR} = \pi_t^{BN} = \lim_{k\to\infty} \pi_{t+k|t}$$

  Then

  - π_t^{BN} is a martingale

$$\Delta\pi_t^{LR} = e_1'C(1)\varepsilon_t$$

    where $e_1 = [1, 0, 0, ..., 0]'$.

  - And

$$var(\Delta\pi_t^{LR}) = e_1'C(1)\Sigma_\varepsilon C(1)'e_1 = var^{LR}(\Delta\pi_t) \overset{model}{=} var(\Delta\overline{\pi}_t)$$

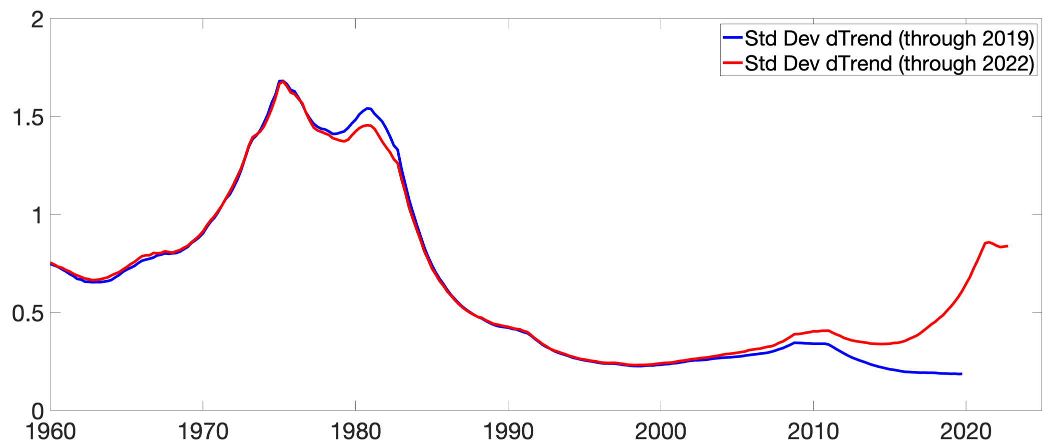
- Thus: Forecasters with different z's will have different long-run forecasts ($\pi_t^{LR} = \lim_{k\to\infty} \pi_{t+k|t}$) ... but the variances of the change in forecasts will be identical.

2 Implications:

1. We can learn about $var^{LR}(\Delta\pi_t) \overset{model}{=} var(\Delta\overline{\pi}_t)$ from $var(\Delta\pi_t^{LR})$

2. Differences across time or forecaster in $var(\Delta\pi_t^{LR})$ are associated with diffences in $var^{LR}(\Delta\pi_t) \overset{model}{=} var(\Delta\overline{\pi}_t)$.

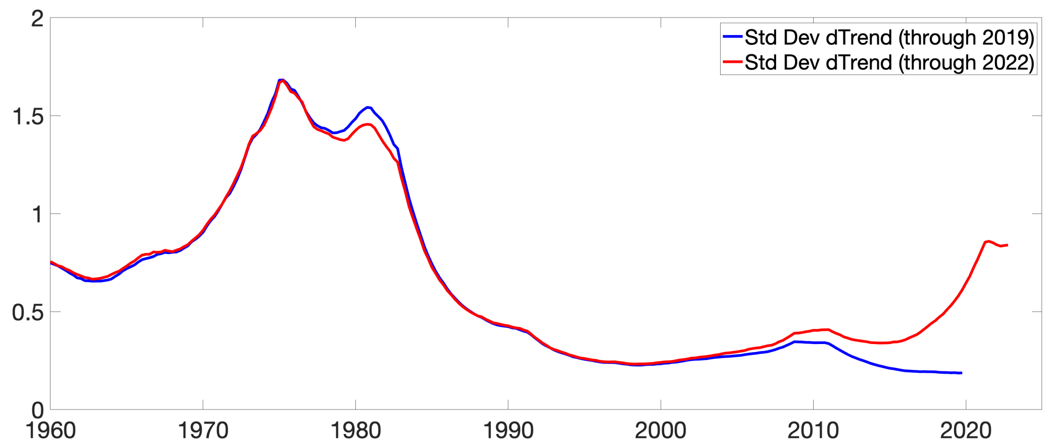
We can we learn about $var^{LR}(\Delta\pi_t) \overset{model}{=} var(\Delta\overline{\pi}_t)$ from $var(\Delta\pi_t^{LR})$



Standard Deviation of $\Delta\pi_{it}^{LR}$ (pooled over forecasters)

Time Period	number of obs	$\hat{\sigma}_{\Delta\pi^{LR}}$
1991-2010	**1714**	**0.27**
2011-2019	**1028**	**0.32**
2020-2022		
2021-2022		

We can we learn about $var^{LR}(\Delta\pi_t) \overset{model}{=} var(\Delta\overline{\pi}_t)$ from $var(\Delta\pi_t^{LR})$



Standard Deviation of $\Delta\pi_{it}^{LR}$ (pooled over forecasters)

Time Period	number of obs	$\hat{\sigma}_{\Delta\pi^{LR}}$
1991-2010	**1714**	**0.27**
2011-2019	**1028**	**0.32**
2020-2022	**219**	**0.30**
2021-2022	**134**	**0.32**

Differences across time and forecaster in $var(\Delta\pi_t^{LR})$

Forecaster i

- Information

$$s_t(i) = \begin{bmatrix} s_{1,t} = \pi_t = \overline{\pi}_t + \pi_t^{Stat} \\ s_{2,t}(i) = \overline{\pi}_t + \alpha(i) v_{c,t} \\ s_{3,t}(i) = \overline{\pi}_t + v_t(i) \end{bmatrix}$$

- Stochastic volatility with different information:

$$\sigma^2_{\Delta\overline{\pi}}(t) = [var(\Delta\overline{\pi}_t)]$$

Forecaster i: $[\sigma^2_{\Delta\overline{\pi}}(t) | s_t(i), s_{t-1}(i), ...]$

- These differences will be reflected in

$$\text{Var}(\Delta\pi_{it}^{LR})$$

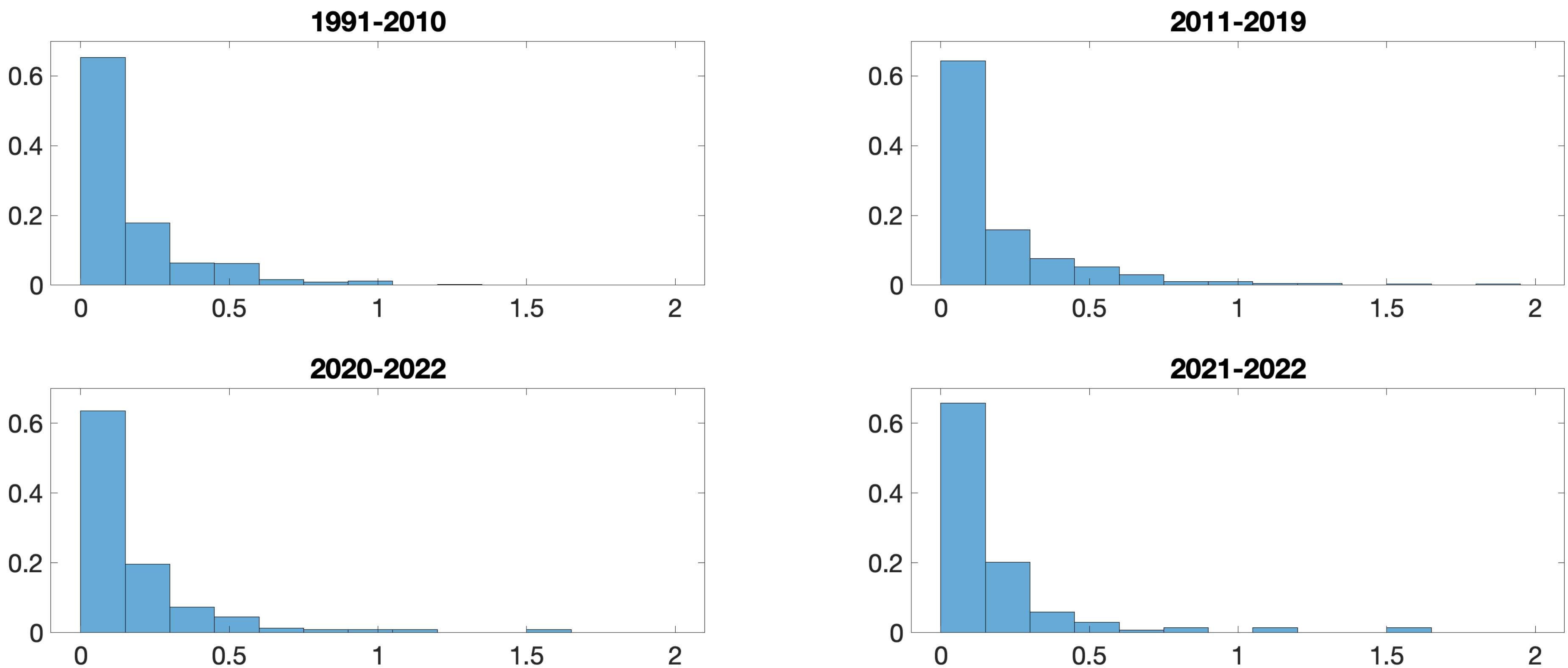
- Are there differences across i in $\text{Var}(\Delta\pi_{it}^{LR})$?

- Have these differences increased recently?

- Problem .. limited data for each forecaster ... observations not independent over i, etc.

One look at data: Histogram of $|\Delta\pi_{it}^{LR}|$ over different time periods. (Disagreement .. fat tails?)

Histograms of $|\Delta\pi_{it}^{LR}|$



Wrapping up:

FMR Inflation model

$$\pi_t = \overline{\pi}_t + \pi_t^{Stat}$$

$$\overline{\pi}_t = \overline{\pi}_{t-1} + \sigma_{\overline{\pi},t}\nu_t$$

$$\ln(\sigma_{\overline{\pi},t}) = \ln(\sigma_{\overline{\pi},t}) + e_t$$

Data

- Inflation Data: significant recent increase in $\sigma_{\overline{\pi},t}$

- Long-run Expectations Data: little change in $\sigma_{\overline{\pi},t}$