Forward Guidance and Heterogeneous Beliefs*

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Abstract

We analyze the effects of forward guidance policy when agents have heterogeneous beliefs about its macroeconomic impact. Using survey expectations, we first document that forward guidance lowered disagreement about future short term interest rates to historically low levels while it did not impact disagreement about future inflation and consumption growth. We introduce in an otherwise standard New-Keynesian model the possibility that agents have heterogenous beliefs on policies and fundamentals. We show that agreement on the future path of interest rates is consistent with disagreement on the length of the trap when agents also disagree on the nature - *delphic* or *odyssean* - of forward guidance. We also show that such type of heterogeneous beliefs can strongly alter the optimal forward guidance policy compared to the predictions of an equivalent model with homogenous beliefs in the central bank's commitment. In some cases, forward guidance can even be detrimental compared to *status-quo*.

Keywords: Monetary policy, forward guidance, communication, heterogeneous beliefs, disagreement.

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1 Introduction

> The FOMC has not been clear about the purpose of its forward guidance. Is it purely a transparency device, or is it a way to commit to a more accommodating future policy stance to add more accommodation today?
>
> Charles I. Plosser, March 6, 2014.

When facing a Zero Lower Bound (ZLB) on its nominal policy rate, a central bank can still affect current allocations by committing to future monetary stimulus, as emphasized by Krugman (1998) and Eggertsson and Woodford (2003). In the aftermaths of the Great Recession, several central banks started to implement such forward guidance policies. They typically provided information on the period during which they expect the policy rate will be kept at or near zero. Such policies have been effective in lowering expected future short-term interest rates as illustrated by e.g. Swansson and Williams (2014). However, this impact on future interest rates does not guarantee that forward guidance policies were effectively expansionary. As stressed by Campbell et al. (2012), such reaction of expected future interest rates can be associated with two different types of forward guidance. It can convey information on commitment to future expansionary policy – forward guidance is then perceived as *odyssean*; but it can also provide information on the macroeconomic fundamentals – forward guidance being then perceived as *delphic*. Plosser's quote above suggests that the forward guidance of the Fed could be interpreted both ways. However, to date there is scant evidence whether private agents understood forward guidance to be odyssean or delphic.

This paper makes three contributions. First, we uncover that forward guidance announcements coincide with an historical evolution of disagreement among professional forecasters on future short-term interest rates, inflation and consumption which implies that agents had different interpretation of the nature of such policy. Second, we replicate this fact in a New-Keynesian model with heterogeneous beliefs about the commitment type of the central bank and fundamentals, so that an odyssean and delphic interpretation of the same path of the policy rate can coexist. Third, we use the model to analyze the efficiency and the optimal length of the forward guidance. We show that with too few odyssean believers, forward guidance policy can be detrimental.

We start by documenting new facts from the US Survey of Professional Forecasters.[1] There is a break in the disagreement about future macroeconomic outcomes across forecasters. Disagreement about future short-term interest rates dropped to historically low levels after

[1]Our analysis mainly focuses on the US experience but we also provide comparable evidence for the euro area and the UK in the paper.

the Fed strengthened its forward guidance with fixed date commitments in 2011Q3. This evidence is consistent with forward guidance coordinating private agents' opinion about future monetary policy. However, disagreement about future consumption growth and inflation rates decreased an order of magnitude less than the one about future interest rates. In sum, at the time the Fed (and other central banks who adopted forward guidance policies) started to make explicit forward guidance announcements, private agents agreed about the path of future short term interest rates, but disagreed on what it would mean for inflation and consumption. These facts would be hard to obtain in any model in which a monetary policy rule (either the normal times one or systematic deviations from such normal times rule) relates interest rates to inflation and activity. How can agents agree on future interest rates while they do not agree on future inflation and activity?

We then build a New-Keynesian model of the ZLB where agents rationally agree to disagree on the nature of forward guidance policy as the commitment ability of the monetary authority is not observable. In this setup, households face a common preference shock pushing the economy towards the ZLB. Private agents observe the current preference shock and the resulting current allocation. Still, they do not know the number of periods this shock will last. This information is not available until the economy reaches the actual end of the trap. We first show that, in this framework, agents can agree on the path of nominal interest rates, without agreeing on the length of the trap, as long as they disagree on the commitment ability of the authority. Some agents anticipate a shorter liquidity trap associated with an accommodative stance of monetary policy after the trap (odyssean forward guidance) while others only expect the trap to be longer (delphic forward guidance). This disagreement about the possibility of future accommodation leads agents to have different views about the medium-run effects of monetary policy on aggregate demand and inflation. So, similarly to the pattern of SPF forecasts, the model generates disagreement about future consumption and inflation in spite of their consensus about future interest rates.

Finally, we use the model to investigate how heterogeneity in agents' beliefs affects the efficiency of forward guidance. Agents who interpret the policy as odyssean consume more in anticipation of future higher inflation and consumption, while agents who interpret it as delphic consume less anticipating less inflation. It follows that when a high enough proportion of agents take forward guidance as *delphic*, the implementation of an extended period of low-interest rates may be inefficient, and even detrimental compared with no forward guidance *status quo*. This is because these latter agents drag current aggregate consumption down. In contrast, forward guidance can stimulate consumption and raise inflation expectations if a high enough proportion of agents believe it is *odyssean*.

Related literature Gürkaynak et al. (2005) show that FOMC expansionary announcements have strong impact on asset prices and in particular lower expected future policy rates. Campbell et al. (2012) confirm such results in a sample that includes the Great Recession. These latter authors stress that such a decrease in expected future policy rates is associated with a worsening of macroeconomic prospects, namely unemployment and inflation forecasts. This is consistent with market participants interpreting FOMC's announcements as being *delphic* rather than *odyssean*. They also find that the forward guidance policy adopted since 2009 did not overturn such results. Our analysis complements their empirical exercise by analyzing the dispersion, rather than the average, of individuals' macroeconomic forecasts. We show that forecasters interpreted forward guidance announcements differently and provide an explanation why this is so.

Our paper is also related to the literature on the effectiveness of forward guidance. Carlstrom et al. (2012) and Del Negro et al. (2013) underline that standard DSGE models predict incredibly high positive impacts of forward guidance policies on future inflation and activity. According to these models, announcements such as those made by the Fed should have led to a boom in demand much greater than what has been observed, a result Del Negro et al. (2013) dubbed the "forward guidance puzzle". These papers consider that announcements were unambiguously perceived as sequences of deviations from the normal times reaction function of the central bank. We show that agents had different forecasts of such deviations and we analyse how this heterogeneity of interpretation reduces the aggregate impact of such policies.

In addition, and in contrast to frequent policy discussions (e.g. Filardo and Hofmann, 2014), our results underline that gauging the efficiency of forward guidance's announcements by looking at the mere reaction of expected future policy rates can be misleading as agents may disagree on the meaning of such low future interest rate path. As Woodford (2012) emphasizes, for forward guidance to be effective, private agents should not only believe that interest will remain low in the future but they also should understand that the reason *why* they will is that the central bank will temporarily allow for more inflation than in normal times.

Krugman (1998), Eggertsson and Woodford (2003) and Werning (2012) study the optimal policy at the ZLB in an infinite horizon model, emphasizing the associated commitment problem. We extend their analyses to a setup where beliefs about the commitment type of the central bank need not be identical across agents. Bodenstein et al. (2012) quantitatively investigate how imperfect credibility of future policy rate announcements in Sweden and in the US lowered the impact of FG policies. Levin et al. (2010) compare the efficiency of forward guidance across economies that enter the ZLB following shocks of different persistence and intensity. The policy conclusion of these papers is that the period of low interest rate should be extended – and future monetary stimulus increased – for forward guidance to remain fully efficient. In contrast, we show that the impact of forward guidance may not only be muted compared to the full commitment case, but that it can even be detrimental when the credibility of commitment is not enough broadly shared among agents.

Finally, Wiederholt (2014) also considers heterogeneous expectations at the ZLB. More precisely, he analyzes the effect of dispersed information about the size of current aggregate shock generating the liquidity trap on the dynamics of the economy at the ZLB. In his model, agents have imperfect information, hence dispersed beliefs, about the current shock because they cannot perfectly observe market variables. Kiley (2014) and Gaballo (forth.) illustrate how imperfect information can reduce FG efficiency in NK models. In our model agents perfectly observe market variables, but they disagree about the unobserved length of the trap. In contrast to our setup, such modeling frameworks cannot replicate situations where agents agree on future interest rates but disagree about future activity and inflation. Neither can the modeling approach of McKay et al. (2015) who account for the "forward guidance puzzle" by relying on heterogeneous agents and borrowing constraints.

2 Stylized facts

Did private agents have the same understanding of the type of forward guidance policy implemented by central bankers? We address this question by looking at the evolution of the cross-section dispersion – or disagreement – in surveys of macroeconomic forecasts. We first layout a simple theoretical setup where agents view the short-term interest rate as predominantly determined by a central bank's reaction function. In that case the disagreement about future short-term interest rates should be related to the disagreement about variables such as inflation or activity that are inputs to usual monetary policy rules.

We then present and discuss the main finding of this section which is that forward guidance strongly coordinated beliefs about future short-term interest rates up to 1Y and 2Y ahead but did not have a comparable impact for variables that should impact future monetary decisions such as inflation or consumption growth. We argue that it implies that disagreement about the deviation from the normal times reaction function of the central bank spiraled up precisely when the Fed reinforced its forward guidance by making a clear statement about the length of the period during which it expected to keep its policy rate at zero.

We finally provide further evidence in support of our main empirical results. First, in normal times, disagreement about future interest rates is significantly correlated with disagreement about inflation and consumption or activity. This is consistent with the assumption that private agents use standard monetary policy rules to forecast future short-term interest rates. In addition, the pattern of disagreement observed in the US starting with the explicit date based forward guidance policy has never been observed over the past three decades. In particular, it is not related to the fact that the US economy hit the ZLB. Moreover, similar patterns are also observed in other economies like the euro area and the UK at the time when they engage into forward guidance. Such pattern therefore appear to be specific to forward guidance.

2.1 Empirical framework

We assume that private agents consider that the central bank monetary policy decisions can be described by the following rule

$$r = f \cdot \Omega + \epsilon,$$

where r is the short-term interest rate, Ω is the set of variables that are relevant to the central bank decisions, f is the (linear) reaction function of the central bank and ϵ is a non-systematic deviation from such a reaction function.

Let $E_t^i(\cdot)$ denotes the expectation of an individual i conditional on its information set available at date t. At date t, individual i's forecast of future interest rate is given by $E_t^i(r) = E_t^i(f \cdot \Omega) + E_t^i(\epsilon)$. Assuming that every agent i agrees on the reaction function f, we get that:

$$E_t^i(r) = f \cdot [E_t^i(\Omega)] + E_t^i(\epsilon). \tag{1}$$

This expression makes clear that the disparity of opinions about future interest rates is driven by two sources: disagreement about future fundamentals Ω, and disagreement about future deviations from the usual reaction function ϵ. Andrade et al. (2013) provide evidence that, over the past 30 years, disagreement about future short-term interest rate can be explained by forecasters agreeing on the Fed's reaction function but disagreeing about two fundamentals, namely future inflation and future growth rate.[2]

Equations (1) has important implications for the joint evolution of disagreement about future interest rates and future fundamentals. For a given disagreement about future deviations from the policy reaction function ϵ, changes in the disagreement about future policy r should be coincident with changes in future fundamentals Ω. Alternatively, observing a

[2]Notre that if agents also disagree about the reaction function f, one can rewrite this type of disagreement as disagreement about future deviations η from an average rule as $E_t^i(r) = f \cdot [E_t^i(\Omega)] + E_t^i(\eta)$ with $E_t^i(\eta) = (f_i - f) \cdot [E_t^i(\Omega)] + E_t^i(\epsilon)$.

change in the disagreement about future values of r that differs from the one implied by the rule and the disagreement about future fundamentals Ω means that the disagreement about future deviations from the rule ϵ as changed.

In the next sub-section, we document that the joint evolution of the cross-section dispersion of individual forecasts of the short-term interest rate and of macroeconomic determinants of future monetary policy discussion observed precisely when the Fed started to conduct fixed-date forward guidance policy implies an increase in the disparity of opinions about future deviations from the monetary policy reaction ϵ.

2.2 Main Results

We rely on the US quarterly Survey of Professional Forecasters (SPF). We focus on several forecast horizons ranging from, depending on the variable, 1 quarter to 2 years ahead, and on three macroeconomic variables, the short-term interest rate, the inflation rate and the consumption growth rate.[3] The measure of disagreement is the interquantile range in the distribution of individual forecasts, i.e. the difference between the 75th quantile and the 25th quantile in the cross-section distribution of individual forecasts for a given quarter. We have data for a 1982Q1-2014Q4 sample.

The data reveal three facts specific to the period when forward guidance was implemented: (i) disagreement about future short-term interest rates decreased to historically low levels; this is true for both short and medium term forecast horizons; (ii) disagreement about future consumption growth and inflation rates decreased but stayed in their usual ranges both for short- and medium-term horizons; and consequently (iii) the link between disagreement about future short-term interest rates and disagreement about future consumption growth and the inflation rates dramatically changed.

Forward guidance reduced disagreement about future interest rates to historically low levels Figure 1 reports the evolution of disagreement about 1Q, 1Y, and 2Y ahead forecast of US short-term interest rates (for the latter two horizons data are available only for a subsample at the end of the period). Campbell et al. (2012) and Swansson and Williams (2014) illustrate that US forward guidance announcements lowered expected future interest rates. Looking at Figure 1 reveals that, in addition, forward guidance was associated with a sharp reduction in the heterogeneity of forecasts about future US short-term interest rates.

[3] More specifically, the survey collects each quarter individual forecasts for the 3-Month T-Bill rate, the headline CPI rate, and the private consumption growth rate. 1Q and 1Y ahead forecasts are quarterly averages. For further horizons, the survey report annual averages.

In particular "fixed-date" statements (which started in August 2011) and "state-contingent" statements (which started in December 2012) were associated with a strong coordination of opinions of both short (1Q) and medium term (1Y/2Y) interest rate forecasts. In contrast, the so-called "open-date" announcements (which started in January 2009) were associated with a drop in the mere one-quarter ahead interest rate forecasts.

Importantly, Figure 1 also illustrates that disagreement on future interest rates 1 year and 2 years ahead did not decline when the economy reached the ZLB, that is in 2008Q4. Such disagreements declined markedly with the reinforcements of forward guidance which occurred in Summer 2011 and at the end of 2012. Such clear forward guidance statements were much more efficient in coordinating opinions about future interest rate for longer horizons than the "open-date" announcements used from 2008Q4 to 2011Q2 onwards.[4]

Disagreement about future consumption growth and inflation also dropped but much less than for future interest rates How did disagreement about future variables that should impact future monetary policy, like inflation and demand, evolved when forward guidance was implemented? Figure 2 displays the evolution of disagreement about 1Q, 1Y and 2Y ahead forecast of US consumption growth and inflation rates. It shows that forecasters' disagreement about these two variables also decreases substantially starting 2009, and in particular for short horizon forecasts. However, for both variables, the disagreement did not substantially fall below their previous historical minima. This stands in contrast with what is observed for the disagreement about future interest rates.

Figure 3 presents the evolution of the disagreements of 1-year and 2-year ahead expected consumption and inflation scaled by the disagreement about the 1-year and 2-year ahead short-term interest rate forecasts. These two graphs illustrate that these relative disagreements spiked up to unprecedented values concomitantly with the reinforcement of forward guidance policy in Summer 2011. While disagreement about future short-term interest rates and future consumption growth and inflation rates usually evolve in a way so that they fluctuate around a constant ratio, that normal-times relationship has been broken when the Fed strengthened its forward guidance. Notice that again, such striking evolution did not happen when the economy hit the ZLB.

[4]Forward guidance statements were also combined with quantitative easing operations which may have had an impact on future interest rate forecasts through a signalling channel. In this paper, we do not make the distinction between this signalling effects of QE and forward guidance policies.

Forward guidance increased disagreement about deviations from the normal times reaction function of the central bank The previous discussion underlines that at the time the Fed reinforced its forward guidance, disagreement about future short term interest rates reached unprecedented low levels and became disconnected from disagreement about future fundamentals such as consumption and inflation.

Disagreement about future consumption and future inflation can be linked to disagreement about future shocks that affect these two variables or to model disagreement leading to different perceptions of future effects of the same current shock. However, no matter the source of disagreement about future consumption and inflation, when the central bank follow its reaction function, it should also lead to disagreement about the monetary policy reaction. Indeed, according to equation (1), disagreement on fundamentals Ω – which determine policy rates through the mapping of the reaction function f – should lead to disagreement about future interest rates r, unless agents disagree about the sequence of future monetary shocks ϵ in a compensating manner. So, letting DIS[$\cdot$] denote disagreement measured by the interquantile range, the previous stylized facts imply that $\text{DIS}[E_t^i(r)] = 0$ and $\text{DIS}[f \cdot E_t^i[(\Omega)] = -\text{DIS}[E_t^i(\epsilon)]$.

In the next section, we present a model in which agents share the same view about future policy rates but (rationally) disagree about the type (odyssean or delphic) of forward guidance that is implemented. To put it differently, agents interpret differently the same policy announcement about future r, but it conveys different degree of future monetary policy accommodation, that is different deviations from the normal time policy reaction function ϵ.

2.3 Additional Evidence

Does disagreement about future interest rates correlates with disagreement about fundamentals in normal times? We assume that, in normal times, private agents use standard monetary policy rules to forecast future short-term interest rates. If the assumption is valid, according to equation (1), disagreement on future interest rates forecasts should be related to disagreement on future fundamentals. If not, then disagreement about future interest rates always evolve independently from such fundamentals and what we observe in Figure 3 might just be a pure random event.

Table 1 displays the regression results of the (log) disagreement about 1-year ahead interest rates on the (log) disagreement on a set of such potential fundamentals, namely 1-year ahead inflation, output growth and consumption growth, and 1-quarter ahead interest rates (which captures policy smoothing in the reaction function). We contrast a pre-forward guidance sample (1982Q1-2011Q2) with a forward guidance sample (2011Q3-2014Q4). Two conclusions can be made. First, consistent with our assumption of agents relying on policy rules, in regular times, disagreement on future interest rate is significantly correlated with disagreement on future fundamentals. Second, that link disappears over the period when forward guidance started to be implemented. Again, this is consistent with our interpretation that, in the forward guidance period, the link between disagreement about future fundamentals influencing monetary policy and disagreement about future interest rates significantly changed, because of expected future deviations from the normal times rule.

Figure 4 provides a visual illustration of these previous results. It displays the observed (log) disagreement on 1-year ahead inflation forecasts with the one that would correspond to the observed (log) disagreement on 1-year ahead future interest rate had the link between the two variables remained constant after 2008. The results make clear that, starting the "fixed-date" forward guidance period disagreement about future inflation should have been significantly lower than the one that has been observed. Notice also that this does not occur at the ZLB and usually did not occur on a longer sample period.

Are the facts specific to forward guidance? The regularities previously discussed are coincident with the "fixed-date" forward guidance announcement of Summer 2011. But are they specific to periods of forward guidance? We provide further evidence suggesting that this is indeed the case.

First let us reiterate that such regularities appear later than the date when the US economy enter a trap. So there are not a mere result of the ZLB constraint. Figure 5 goes further by showing the evolution of disagreement about US consumption and inflation 1 year ahead relative to disagreement about future interest rates 1 year ahead over the whole sample of data available. The forward guidance episode stands out as a clear outlier within more than 30 years of US economic history. So what was observed when the Fed implemented forward guidance has never been observed in recent history.

Second, Figure 6 displays the evolution of disagreement about inflation 1 year ahead relative to disagreement about future interest rates 1 year ahead in two other economies which implemented forward guidance policy, namely the Euro Area and the UK. The same pattern of a sharp increase in this relative disagreement. So the fact is not specific to the US, but can be found in other economies at the time they clearly engage into forward guidance.[5]

[5] Further investigation also underlines that the communication of the Fed was more efficient in coordinating private beliefs about future interest rates than the ones of other central banks. See Appendix A.

3 Theory: Does agreeing on the interest rate path imply agreeing on policy?

The aim of this section is to clarify what we can learn from observed agreement on the interest rate path exploiting all equilibrium restrictions implied by a standard New Keynesian model. To this purpose, we build a NK economy allowing for heterogeneous beliefs, we present the benchmark forward guidance analysis and extend it to the case of heterogeneous beliefs on the commitment ability of the central bank. We will conclude that an ambiguity remains to which extent agreement on future interest rates reflects a more accommodative future policy rather than a more severe recession.

3.1 NK-economy with heterogeneous beliefs

To illuminate the importance of heterogeneous beliefs - even in the presence of risk sharing incentives - we introduce a household family in an otherwise standard NK economy. The household family endogenously[6] produce heterogeneous consumption-saving paths as long as agents maintain different views about the conduct of the monetary policy. In particular, agents can anticipate that in the long run they will equalize their wealth, which is essential to the existence of a unique steady state equilibrium for each individual.[7]

Household. The household family is constituted by a continuum of agents of mass one indexed by $i \in [0, 1]$. Each agent decides how much to work, consume and save in order to maximally contribute to the household welfare

$$U_\tau = \int_0^1 \sum_{t=\tau}^{\infty} \beta^t e^{\xi_t} \left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) di, \tag{2}$$

where $C_{i,t}$ and $L_{i,t}$ are respectively consumption and labor supply of agent i in period t. The parameter $\beta \in (0, 1)$ is a discount factor, the parameter $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution, and the parameter $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply. The variable ξ_t is a preference shock discussed below.

Each agent manages a portfolio representing a fraction of the household wealth. Between

[6]Previous works on heterogeneous agents in the New-Keynesian model (see Curdia and Woodford, 2010, among others) need ah-hoc assumption to ensure this condition.

[7]In Appendix B.2, we show that the household family model is first-order equivalent to both a proper decentralized insurance market and to an economy with a representative agent uncertain about the type of monetary policy announced.

periods t and $t+1$, agent i deals with the following flow budget constraint:

$$B_{i,t} = R_{t-1}B_{i,t-1} + W_t L_{i,t} + D_t - P_t C_{i,t} + Z_{i,t}, \tag{3}$$

where $B_{i,t}$ are bond holdings of the agent between periods $t-1$ and t, R_{t-1} is the gross nominal interest rate on bond holdings between periods $t-1$ and t, W_t is the nominal wage rate in period t, D_t is the difference between nominal profits received and nominal lump-sum taxes paid, by each agent in period t (we assume here diffuse ownership), and P_t is the price of the final good in period t. The agent can borrow (formally, bond holdings can be negative), but the household is not allowed to run a Ponzi scheme. Finally, the term $Z_{i,t}$ denotes a nominal intra-household transfer voluntarily received or carried out by agent i.

Intra-Household risk sharing. Each period is divided in three stages. In the first stage, current shocks hit and agents observe them. At this stage agents form their beliefs on the state of the world. In the second stage, agents can implement a feasible transfer plan $\{Z_{i,t}\}_0^1$ such that

$$\int_0^1 Z_{i,t} di = 0. \tag{4}$$

only if every agents agree on it. When no agreement is reached, then no transfers are made; in such a case each agent owns the wealth resulting from her own portfolio management. Finally, agents cannot commit on future transfers. Let us therefore introduce the following formal definition.

Definition 1. *An implementable transfer plan at time t is a feasible transfer plan $\{\hat{Z}_{i,t}\}_0^1$ such that*

$$E_t^i[U_t|\{\hat{Z}_{i,t}\}_0^1] \geq E_t^i[U_t|\{Z_{i,t}\}_0^1],$$

for each $i \in [0,1]$ and each feasible transfer plan $\{Z_{i,t}\}_0^1$.

In the last stage, once intra-household wealth transfers are carried out, each agent decides on her own labor supply and consumption, based on their own individual beliefs and taking other agents' decisions as given. All the mechanism is common knowledge.

It is important to remark straightaway that, as a direct consequence of concavity of utility functions, with homogeneous beliefs the only sequence of implementable plans is the one that insures equal wealth to each member of the family. In other words, with homogeneous beliefs the family agree to provide full insurance against idiosyncratic risk to their members: this situation features the case of the canonical representative agent model.

Firms. Production is implemented in the context of a standard monopolistic competition environment.

The final good is produced by competitive firms using the technology: $Y_t = (\int Y_{j,t}^{(\theta-1)/\theta} dj)^{\theta/(\theta-1)}$. Here Y_t denotes output of the final good and $Y_{j,t}$ denotes input of intermediate good j. The parameter is the elasticity of substitution between intermediate goods. Final good firms have perfect information and fully flexible prices. Profit maximization of firms producing final goods implies the following demand function for intermediate good j: $Y_{j,t} = P_{j,t}^{\theta} P_t^{-\theta} Y_t$, where $P_{j,t}$ is the price of intermediate good j and P_t is the price of the final good. Furthermore, the zero profit condition of firms producing final goods implies $P_t = (\int P_{j,t}^{1-\theta} dj)^{1/(1-\theta)}$. Each intermediate good j is produced by a monopolist using the technology $Y_{j,t} = L_{j,t}$ where $Y_{j,t}$ is output and $L_{j,t}$ is labor input of this monopolist. Monopolists producing intermediate goods are subject to a price-setting friction as in Calvo (1983). Each monopolist can optimize its price with probability $1-\chi$ in any given period. With probability χ the monopolist producing good j sets the price $P_{j,t} = P_{j,t-1}$. As we said, each agent own an equal share of each firm and so the firms choose the price $P_{j,t}$ so as to maximize

$$E_{t,j} \sum_{k=0}^{\infty} \chi^k Q_{t,t+k} \left(P_{j,t} Y_{j,t+k} - W_{t+k} L_{j,t+k} \right)$$

where $Q_{t,t+k}$ is a discount factor from time t to time $t+k$. Finally, the price level satisfies $P_t = \left[(1-\chi) P_{t,*}^{1-\theta} + \chi P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}$ which implies that prices are sticky, i.e. they cannot adjust instantaneously, as P_t is generally different from $P_{t,*}$ as long as χ is different from 0. To simplify our exposition we will assume that the information sets of the producers is isomorphic to the one of the household members.

Shock. The preference shock ξ_t takes value zero in normal times and $-\xi$ in crisis times. Once a crisis occurs at time t, it will last for a number of periods $T \in \mathbb{N}$ before switching to normal. We will refer to T as the *length of the trap*. The beliefs of agent i about the number of crisis periods $\{1, ..., T\}$ is denoted by $E_{i,t}[T]$ where $E_{i,t}[\cdot]$ represents the expectation of agent i conditional to the information set and priors of agent i at time t. In practice, $E_{i,t}[T]$ implies $E_{i,t}[\xi_\tau] = -\xi$ for $\tau = t, ..., t+T$. Is is worth to note that the end of the trap has the feature of a "news" on a future shock. As such it can be only assessed ex-post, once its realization occurs. From here onward we will focus on the effects of a single (large-enough) negative preference shock hitting a time $t = 0$.

Monetary policy. The central bank's monetary policy is to set a path of interest rate $\{R_t\}_{t\geq 0}$ in order to maximize the household's utility (2). Yet, this policy faces a zero lower bound (ZLB), that is $R_t \geq 1$, which constraints the policy action. Without loss of generality, we restrict our attention on the following representation of the policy action:

$$R_t = \max\{R\Delta_t \Pi_t^{\phi}(Y_t/Y_t^n)^{\theta}, 1\}. \tag{5}$$

where $R = (1/\beta)$ is the nominal interest rate in the non-stochastic steady state with zero inflation, $\Pi_t = (P_t/P_{t-1})$ denotes the inflation rate, Y_t^n is the natural output (i.e. the counterfactual flexible price outcome) and $\phi > 1$ and $\theta \geq 0$ the monetary authority's systematic response to inflation and output gap respectively.

Finally, $\Delta_t \in \Re$ denotes a deviation from a strict application of a Taylor rule, which instead holds setting $\Delta_t = 1$ at all times. Whatever policy different from a strict application of a Taylor rule can be mapped into the representation above through a sequence of Δ_t different from 1. The policy choice of the authority can be summarized by a path $\{\Delta_0, ...\}$. In this respect, we consider two alternative types of central banker: the type $\mathcal{C}$ can *commit* to set Δ_t in advance, whereas the type $\neg\mathcal{C}$ cannot. In the latter case, the central bank will *ex post* re-optimize its policy each time. Importantly, the type $\varrho \in \{\mathcal{C}, \neg\mathcal{C}\}$ of the monetary authority is not directly observed by agents, but can be eventually inferred from policy actions.

Fiscal policy. Concerning fiscal policy, we assume that the government implements a constant proportional tax on sales proceeds as in Woodford (2003), whose revenues are transferred in a lump sum to households. This ensures that the monetary authority has no inflation bias. In addition, there is no public spending and just note that Ricardian equivalence holds.

Equilibrium. We are now ready to define an equilibrium, where, for given agents' beliefs about the length of the trap and a given type of the monetary authority, agents optimize their consumption choices and the central bank set interest rates to maximize welfare.

Definition 2. *For a given sequence of shocks $\{\xi_0, \xi_1, ...\}$, an equilibrium is:*

i) given a sequence of policy deviations $\{\Delta_0, ...\}$ and a set of beliefs about the end of the trap and the type of the authority $\{E_{i,0}[T], E_{i,0}[\varrho]\}_{i\in[0,1]}$,

$$\{C_{i,t}, L_{i,t}, B_{i,t}, D_t, R_t, W_t, Z_{i,t}, P_t\}_{i\in[0,1],t\geq 0}$$

solves household's and firms' problems, satisfies the monetary policy rule (5) and so that markets clear;

ii) given a type of the central bank and given agents' optimal reaction, $\{\Delta_0, ...\}$ solves the central bank's problem;

iii) agents beliefs $\{E_{i,0}[T], E_{i,0}[\varrho]\}_{i\in[0,1]}$ *are consistent with the current allocation.*

Condition (iii) establishes that agents' beliefs must be rational expectations in the sense that any available observable produced in equilibrium will be used by agents to restrict their beliefs about the length of the trap. In this respect, *we do not assume any informational friction or ad-hoc asymmetry.* As said, the only two elements that are not directly observable to agents are the length of the trap and the commitment-type of the authority.

3.2 FG-benchmark: agreement on *both* interest rates and policy

This section presents the analysis with homogeneous beliefs on the commitment-type of the central bank. In this case, if agents have the same expectations about the policy path, they also share the same views about the length of the trap. In the end, they perfectly share risks and the economy behaves as if there was a representative agent.

Inflation targeting in normal times. Given individual beliefs and wealth distribution, agents' first order conditions yield the consumption Euler equation in any period $t \geq 0$:

$$c_{i,t} = -\gamma^{-1}(E_{i,t}[\xi_{t+1}] - \xi_t + r_t - E_{i,t}[\pi_{t+1}]) + E_{i,t}[c_{i,t+1}], \tag{6}$$

that we express here with small case denoting log-linear deviations from steady state. Current consumption increases as the current interest rate decreases or future inflation or consumption increase. The labor-decision equation instead entails a static relation, $-\gamma c_{i,t} = w_t - p_t + \psi l_{i,t}$, meaning that, for the same real wage, a lower consumption increases labor supply. This determines a unique equilibrium as stated below. Manipulating the equilibrium relations as shown in appendix B.1, we can recover the well-known New-Keynesian Phillips curve

$$\pi_t = \kappa c_t + \beta \int_0^1 E_{i,t}[\pi_{t+1}]di, \tag{7}$$

linking current inflation to current aggregate consumption and the average expectation of future inflation.

In the absence of the ZLB constraint, the central bank would be able to perfectly smooth preference shocks. In normal times, the first best allocation can be implemented both by the commitment- and the no-commitment type central banks by setting the interest rate to a level given by the rule (5) with $\Delta_t = 1$ and $\theta = 0$. The result $\theta = 0$ follows directly from the observation that in presence of shocks to the discount factor only, there is no trade-off between inflation and output gap so that, a strong response to inflation only (i.e. a ϕ chosen sufficiently high with $\theta = 0$) is sufficient to fully stabilize the economy. The resulting allocation is the steady state $\{c_{i,t}, \pi_t, r_t\} = \{0, 0, 0\}$ at any t for each i.

Inflation targeting at the ZLB. Yet, when the preference shock is too large, the downward limit on interest rates, $R_t \geq 1$, may prevent such stabilization. Suppose agents have homogeneous beliefs that the trap will last $E_{i,0}[T]$ and that the authority will take interest rates at zero until $E_{i,0}[T]$, then the resulting expected and current consumption is given for each i by,

$$c_{i,t} = \gamma^{-1}(\log R + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}] \text{ for } t \in [0, E_{i,0}[T]-1], \tag{8a}$$

$$E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R + \xi) + E_{i,0}[c_{i,t+1}], \text{ for } t = E_{i,0}[T], \tag{8b}$$

$$E_{i,0}[c_{i,t}] = c \text{ for } t \geq E_{i,0}[T]+1, \tag{8c}$$

where the inflation path expected is determined in accordance with the Phillips curve (7). The ZLB on the interest rate imposes $\log R$ as an upper bound to the stimulative impulse that monetary olicy can give under the restriction $\Delta_t = 1$. In this case, the typical path of consumption is the plotted in green in figure 7, which will be commented below.

Note that when the authority follows an inflation targeting rule, then the number of periods at which the interest will stay at zero is *equal to* the length of the trap. We then define the following.

Definition 3. *When the authority sets $\Delta_t = 1$ at any t, then, the policy has a Delphic nature, i.e. for a given T it will be $R_t = 1$ for $t \in \{0, ..., T\}$ and $R_t = R$ for $t > T$.*

We use the term Delphic consistently with Campbell et al. (2012), with the meaning that, beliefs about the number of periods at the ZLB shall correspond exactly to beliefs about the length of the trap. Under this restriction, an update in one dimension implies an identical update in the other.

FG-benchmark. As shown by Krugman (1998), Eggertsson and Woodford (2003) and Werning (2012), when the ZLB binds, the second-best policy prescribes policy rates at zero for longer than required by an otherwise optimal inflation targeting. In fact, the authority can stimulate current consumption promising lower short-term rates once the trap ends. This case can be characterized by $\Delta_t = 0$, that is $r_t = 0$, for an optimal number of periods $T_{cb} \geq T$ and irrespective of the course of inflation.

As before, suppose agents have homogeneous beliefs that the trap will last $E_{i,0}[T]$ and that the authority will keep interest rates at zero until $E_{i,0}[T_{cb}] \geq E_{i,0}[T]$, then the resulting expected and current consumption is given for each i by,

$$c_{i,t} = \gamma^{-1}(\log R + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}] \text{ for } t \in [0, E_{i,t}[T] - 1], \quad (9a)$$

$$E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R + \xi + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}], \text{ for } t = E_{i,0}[T], \quad (9b)$$

$$E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}] \text{ for } t \in [E_{i,0}[T] + 1, E_{i,0}[T_{cb}] - 1], \quad (9c)$$

$$E_{i,0}[c_{i,t}] = \gamma^{-1} \log R + E_{i,0}[c_{i,t+1}] \text{ for } t = E_{i,0}[T_{cb}], \quad (9d)$$

$$E_{i,0}[c_{i,t}] = c \text{ for } t \geq E_{i,0}[T_{cb}] + 1, \quad (9e)$$

where the inflation path expected is determined in accordance with the Phillips curve (7). Note that this policy generates an expansionary stimulus after the end trap, that boosts inflation and consumption in the future, which is partly anticipated in current consumption. Therefore, this policy will deliver higher welfare than inflation targeting for an optimal choice of T_{cb}. In this case, the typical path of consumption is the plotted in blue in Figure 7.

When the authority implements this second-best policy, then the number of periods at which the interest will stay at zero is *longer than* the length of the trap. We then define the following.

Definition 4. *When the authority sets $\Delta_t = 0$ for $T_{cb} > T$ periods and $\Delta_t = 1$ after, then, the policy has a Odyssean nature, i.e. for a given T it will be $R_t = 1$ for $t \in \{0, ..., T, ..., T_{cb}\}$ and $R_t = R$ for $t > T_{cb}$.*

Again, we borrow the term Odyssean from Campbell et al. (2012), with the meaning that, beliefs about the number of periods at the ZLB do not directly match the length of the trap, but rather they are induced by the optimal policy choice T_{cb}. It is important to note that the authority *must* commit to this Odyssean policy - it tightens her hands as Odysseus in front of the mermaids. Let us clarify this point in the following.

Time-inconsistency of FG. Once the recovery occurs, inflation is no longer socially desirable, and the authority would be tempted to renege her promise and set $\Delta_t = 1$ from T onward, which corresponds to the time-consistent solution with perfect stabilization at steady state, after the end of the trap.

Therefore, the implementation of the first-best policy at the ZLB generates a time-consistency problem so that the possibility to implement it relies on the central bank being of the commitment type ($\varrho = \mathcal{C}$). When instead the authority cannot commit ($\varrho = \neg\mathcal{C}$), then the optimal (second best) policy corresponds to the one implemented in normal times as soon as the economy exit from the trap. This is common knowledge among agents and pins down their beliefs.

Proposition 1. *Suppose agents have homogeneous beliefs about both the number of periods with interest rates at the ZLB, T_{zlb}, and the type of authority, ϱ, then they have the same expectation on the length of the trap, in particular:*

- *if $E_{i,0}[T_{zlb}] = \hat{T}$ and $E_{i,0}[\varrho] = \neg\mathcal{C}$ for each i, then policy is Delphic so that*

$$\hat{T} = E_{i,0}[T] \text{ for each } i,$$

and $c_0 = \underline{c}$;

- *if $E_{i,0}[T_{zlb}] = \hat{T}$ and $E_{i,0}[\varrho] = \mathcal{C}$ for each i, then there exists an optimal Odyssean policy $T_{cb}(T)$ such that*

$$\hat{T} = E_{i,0}[T_{cb}(T)] > E_{i,0}[T] \text{ for each } i,$$

and $c_0 = \bar{c} > \underline{c}$.

This proposition is the consequence of the results established by Eggertsson and Woodford (2003) and Werning (2012). This scenario represents the benchmark against which we will contrast the effect of heterogeneous beliefs. The proposition stresses that if agents agree on a path of interest rates and on the commitment ability of the central bank, then they should also agree on the length of the trap and (obviously) the current equilibrium allocation.

Figure 7 plots the reaction of consumption and inflation to the sequence of shocks with homogeneous beliefs in the two scenarios with and without believed commitment ability. Odyssean policy (in blue) requires to keep interest rates at the ZLB during 4 additional quarters with respect to Delphic policy. Odyssean policy allows to avoid a major recession, while consumption and inflation would drop if monetary policy does not provide additional support and follows a standard inflation targeting rule.

Unfortunately, we neither observe beliefs about the type of the monetary authority nor on the length of the trap so we cannot conclude that agreement on the interest rate path implies agreement on one of this two dimensions. As we will see, although agents have homogeneous beliefs about the interest rate path and observe the current equilibrium allocation, they can still disagree on the length of the trap and the commitment ability. Let us discuss this scenario in the next subsection.

3.3 Agreement on interest rates with disagreement on policy

In this subsection, we maintain homogeneous beliefs on the path of interest rates and on the (observable) current allocation and show that this is compatible with heterogeneous beliefs jointly on the commitment ability of the authority and the length of the trap.

Optimists and Pessimists. Agents can agree on the path of interest rates for different reasons. Let us consider the case in which agents agree that the interest rate will be at the ZLB for T_{zlb} periods. Agents can be of two types depending on their priors on the commitment ability of the authority: some agents think that the central bank can commit ($\varrho = \mathcal{C}$), while others not think so ($\varrho = \neg\mathcal{C}$). The former hold a belief $E_{o,0}[T_{cb}] = T_{zlb}$, i.e. they expect the trap to be shorter than the period at the ZLB ($E_{o,0}[T] < T_{zlb}$) - we label them *optimists*. Conversely, the latter, who do not believe in the commitment, will hold a belief $E_{p,0}[T] = T_{zlb}$, i.e. they expect the trap to be as long as the period at the ZLB ($E_{o,0}[T] = T_{zlb}$) - we label them *pessimists*. Finally, we denote by $\alpha \in [0, 1]$ the fraction of agents that are pessimists whereas the rest are optimists.

Both optimists and pessimists agree on the same path of nominal interest rates until date T_{zlb}, but they expect different evolutions of the economy, in particular a different path of the real interest rate. Their disagreement can coexist because there is no realization at time 0 that can unfold the truth about the type of the monetary authority or the length of the trap. The current allocation itself, although determined by the distribution of types in the population, does not aggregate any "truth", but only beliefs. In this sense, agents can agree to disagree.

However, it is common knowledge that, at the optimistic date for the end of the trap ($E_{o,t}[T]$), only one of the two types will be right, as optimists can be either confuted or confirmed by the observable lasting or end of the trap. At that point, heterogeneity of beliefs will not be sustained any longer.

Risk-sharing with disagreement. Disagreement has major consequences on the dynamics of intra-household transfers. At the second stage of each period, agents need to decide on the wealth transfers. In the absence of disagreement, this would optimally result in an even distribution of wealth.

Yet, the type of the authority will unfold only once the date $E_{o,t}[T]$ is reached, as said in the previous paragraph. Before that date, agents have different opinions on which transfer plan maximizes the family welfare, even though they anticipate that they will share wealth in the future. This implies that no transfer plans can be implemented before date $E_{o,t}[T]$.

The following proposition states this formally.

Proposition 2. *Consider the case of heterogeneous beliefs on the end of the trap, namely* $E_{o,0}[T] < E_{p,0}[T]$*, then the only equilibrium sequence of implementable plans of transfers*

$\{\{Z_{i,t}^*\}_0^1\}_{t=0}^{\infty}$ *is the one providing for* $\{Z_{i,t}^*\}_0^1 = 0$ *at each* $t \neq E_{o,0}[T]$ *and* $\{Z_{i,E_{o,0}[T]}^*\}_0^1$ *such that*

$$U_c(C_{i,T}) = U_c(C_{j,T}) \text{ for } t = E_{o,0}[T], \tag{10}$$

namely, the marginal utility of consumption is equal between types at the time where the truth unfolds, which implies $B_{i,t} = B_{j,t}, \forall (i,j)$ *for* $t \geq E_{o,0}[T]$.

Proof. See Appendix. □

As no transfers are made, during the trap, the two types then consume according to their beliefs managing the share of wealth that they hold at the beginning of the trap.

Allocation with heterogeneous beliefs. The effect of disagreement on the current and expected allocation is established by the following proposition.

Proposition 3. *Suppose agents have homogeneous beliefs about the number of periods with interest rates at the ZLB,* T_{zlb}*, and there exists a fraction* $1-\alpha$ *of optimists anticipating a trap of length* $E_{o,0}[T] \leq T_{zlb}$*, then the unique path of each own individual expected consumption is given respectively by (9) with* $i = o$ *for the optimists, and (8) with* $i = p$ *for the pessimists, where the inflation path is expected by each type in accordance with the Phillips curve:*

$$E_{i,0}[\pi_t] = \kappa E_{i,0}[\alpha c_{p,t} + (1-\alpha)c_{o,t}] + \beta E_{i,0}[\alpha E_{p,t}[\pi_{t+1}] + (1-\alpha)E_{o,t}[\pi_{t+1}]], \tag{11}$$

where

$$E_{i,0}[c_{j,t}] = E_{j,0}[c_{j,t}] \text{ and } E_{i,0}[E_{j,t}[\pi_{t+1}]] = E_{j,0}[\pi_{t+1}] \text{ for } t \in [0, E_{o,0}[T]], \tag{12a}$$

$$E_{i,0}[c_{j,t}] = E_{i,0}[c_{i,t}] \text{ and } E_{i,0}[E_{j,t}[\pi_{t+1}]] = E_{i,0}[\pi_{t+1}] \text{ for } t > E_{o,0}[T], \tag{12b}$$

with $(i,j) \in \{o,p\}^2$.

Proof. See Appendix. □

The interpretation of (3) is intuitive. Each type understands that there is no evidence that can let the other type changes her beliefs until the date $E_{o,0}[T]$ comes. After that date, each type expects that the other will conform to her own expectations. In the short run, agents agree on the path of both inflation and consumption and they only disagree for periods after the optimistic end of the trap ($E_{o,0}[T]$). After that date, optimists believe that monetary policy will engineer a boom resulting into higher inflation and higher consumption, and that pessimists will finally share their views. Conversely, pessimists expect that the economy will be still experiencing the negative shock and that optimists will finally share their views as well.

In sum, disagreement on the commitment ability of the authority tends to yield disagreement over medium-term inflation and consumption expectations, whereas will have no impact over short-term expectations.

Figure 8 plots the reaction of consumption and inflation when beliefs about the nature of policy are heterogeneous. More precisely, one fourth of households are convinced that the monetary policy will be odyssean and the other half believes that the policy is delphic ($\alpha = .25$).

With this fraction of optimists and pessimists, the central bank is still better off implementing odyssean guidance. Yet, to compensate the presence of pessimists, she has to keep interest rates low for more periods (6 instead of 5). This results into a larger and longer boom in the end of the trap.

Figure 8 also illustrates how heterogeneity matters. Although agents agree on current and short-term future allocations, their heterogeneous beliefs about future effects of policy contemporaneously result in heterogeneous actions: optimists consume more in the short run as they expect higher inflation in the medium run than pessimists.

In addition, optimists expect pessimists to consume less than them in the short run as they know that pessimists do not share their beliefs, but they expect pessimists to upwardly revise their beliefs at date T and, then, to consume more in the future, catching up with optimists. This expected learning by pessimists will contribute to the optimists' anticipation of a boom.[8]

In the end, heterogeneity in actions and in beliefs are asynchronous. Agents have different consumption paths in the short run - optimists consume more than pessimists - but they agree on both short-run consumption and inflation. In the medium term, the opposite occurs: agents expect that they will consume the same, but they have different views about the state of the economy. In the long run, agents agree on the long run, as they know that they economy will go back to steady state.

Individual consumption and financial positions. As optimists consume more more than pessimists, they also save less than pessimists. As a consequence, pessimists accumulate positive net positions against optimists. The difference between the budget constraints of the two types is:

$$B_{o,t} - B_{p,t} = R_t(B_{o,t-1} - B_{p,t-1}) + W_t(L_{o,t} - L_{p,t}) - P_t(C_{o,t} - C_{p,t}),$$

[8]Symmetrically, pessimists expect optimists to consume more than them in the short run, but they also expect them to revise their expectations downward at date T, forcing the economy to a new recession.

A higher consumption level by optimists imply they have a lower marginal utility of consumption and, as a consequence, they provide lower working hours. This leads to the following proposition.

Proposition 4. *Optimists have a lower stock of bonds then pessimists all along the trap* ($B_{o,t} \leq B_{p,t}$ *for* $t \leq E_{o,0}[T]$)*.*

Such a difference in financial wealth has also a tendency to grow at the prevailing interest rate, as long as agents do not agree to share resources again.

3.4 Optimal policy response with heterogeneous beliefs

In this subsection, we determine how the presence of optimists and pessimists affects the design of odyssean forward guidance. This amounts to endogenize the additional stimulus T_{cb} by a commitment-type central bank facing a fraction α of pessimists in a trap of length T.

The central bank's problem is to maximize utility (2) over the period of zero interest rates T^{CB}, such that monetary policy is described by (5) and given agents' optimal consumption, pricing decisions and beliefs.

Let then solve the optimal problem of the monetary authority, for given length of the trap T and the existence of a fraction α of pessimists. Our main result, as described by the following proposition, is a non-monotonic response of the central bank with respect to the share of pessimists.

Proposition 5. *There exists two values* $\underline{\alpha} >$ *and* $\bar{\alpha}$ *where* $\underline{\alpha} > \bar{\alpha}$ *such that, for a given* T*, the policy map* $T_{cb}(\alpha, T)$ *is:*

- *increasing in* α*, i.e.* $T_{cb}(0, T) < T_{cb}(\alpha, T)$*, for* $\alpha < \underline{\alpha}$*;*

- *decreasing in* α*, i.e.* $T_{cb}(\alpha', T) > T_{cb}(\alpha'', T)$*, for* $\underline{\alpha} < \alpha' < \alpha'' < \bar{\alpha}$*;*

- *equal to* T*, i.e.* $T_{cb}(0, T) > T_{cb}(\alpha, T) = T$*, for* $\alpha > \bar{\alpha}$*.*

The proof is simple and intuitive. When only a small share of agents misunderstand the odyssean forward guidance, the central bank is better reinforcing this policy: a stronger expected monetary stimulus leads to a further drop in pessimists' consumption - as they wrongly interpret the additional periods of low interest rate as the sign of a longer trap - but, as optimists consume sufficiently more, aggregate consumption also increases. In the end, the central bank is marginally better off to increase the period of low interest rate T_{zlb}.

Yet, Proposition 5 suggests that a coordination on an extended period of low interest rates can also be detrimental when sufficiently misunderstood. Indeed, misinterpretation can then have more dramatic effects than just mitigating the effect of forward guidance, but can exacerbate the consequences of the ZLB. As a result the central bank can be better off not implementing an odyssean forward guidance policy, no matter whether she is willing and able to commit to it.

Formation of optimistic beliefs. This proposition allows to complete the description of an equilibrium as we can recover the way optimists are forming beliefs about the length of the trap for a given share of pessimists α:

Corollary 6. *For a given α, the expected length of the trap by optimists $E_{o,0}[T]$ solves*

$$T_{cb}(\alpha, E_{o,0}[T]) = E_{o,0}[T_{zlb}].$$

The non-monotonicity of the optimal length of the period of zero interest rates (T_{zlb}) may result into multiple equilibria depending on the information structure. Yet, as long as agents observe the current allocation, they can infer the value of α, pinning down a unique value for $E_{o,0}[T]$.

Numerical illustration. The decision to implement forward guidance depends on the relative impact of optimists and pessimists, and thus, on the relative share of the former and the latter. The resulting hump-shaped policy as described by Proposition 5 is numerically illustrated by Figure 9: the presence of pessimists forces the central bank to extend its monetary stimulus, until the contractionary effects that are growing with the share of pessimist outweigh the benefits of additional stimulus. Then, the central bank starts to reduce the length of its stimulus and, ultimately, prefers not to implement odyssean forward guidance.

This figure also illustrates some comparative statics with respect to parameters. The threshold value of pessimists after which the central bank prefers not to intervene decreases with the size of the shock as the contractionary effects of pessimists increase with their numbers. Conversely, the weight of output gap in the central bank's objective function does not affect this threshold value but only the length of the additional stimulus: as we are considering a demand shock, there is no trade-off between inflation and output that may reverse the sign of welfare for a given policy rate path.

Forward Guidance Puzzle. Moreover, the quantitative effect of odyssean forward guidance is usually found to be much lower than what the theory predicts (see Carlstrom et al., 2012; Del Negro et al., 2013, among others). Our model can lead to such a low effect for odyssean forward guidance. Indeed, whatever the effect of odyssean forward guidance when fully understood, when α is close to $\bar{\alpha}$, odyssean forward guidance has almost no effect.

Identifying odyssean forward guidance. Section 2 documents the presence of additional disagreement on aggregate consumption and inflation. The following corollary interprets this evidence in light of our model:

Corollary 7. *Disagreement on aggregate consumption and inflation arise in the short run if and only if* $\alpha \in (0, \bar{\alpha})$*, in which case the central bank bank implements a odyssean forward guidance.*

In particular, the presence of some disagreement related to forward guidance allows to identify that some agents understood the odyssean nature of forward guidance announcements. This gives an alternative identification scheme compared with Gürkaynak et al. (2005) or Campbell et al. (2012) to elicit information contained in monetary policy announcements.

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Dependent variable: DIS(r)								
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
DIS(π)	0.506***	0.836	0.374**	0.975	0.407***	1.261	0.344***	0.183
	0.136	0.751	0.145	0.727	0.138	0.793	0.104	0.949
DIS(y)			0.252*	-.479				
			0.131	0.845				
DIS(c)					0.282**	-1.334	0.107	-1.138
					0.121	1.081	0.112	0.883
DIS(r_{-1})							0.334***	0.692*
							0.051	0.317
R2	0.156	0.073	0.196	0.095	0.203	0.194	0.411	0.365
Start	1982Q1	2011Q3	1982Q1	2011Q3	1982Q1	2011Q3	1982Q1	2011Q3
End	2011Q2	2014Q4	2011Q2	2014Q4	2011Q2	2014Q4	2011Q2	2014Q4

Table 1: The link between disagreement on future interest rates and on future fundamentals

Disagreement is measured by the 75/25 inter-quantile range in the distribution of 1-year ahead individual mean point forecasts for the 3-month T-Bill interest rate (DIS(r)), CPI inflation (DIS(π)), real GDP growth (DIS(y)), real consumption growth (DIS(c)) and 1-quarter ahead individual mean point forecasts for the 3-month T-Bill interest rate (DIS(r_{-1})). Regressions are in logs. The sample covers 1982Q1-2014Q4. Standard errors are obtained via a HAC Newey-West procedure. ***, **, * indicate significance at resp. the 1%, 5% and 10% level.

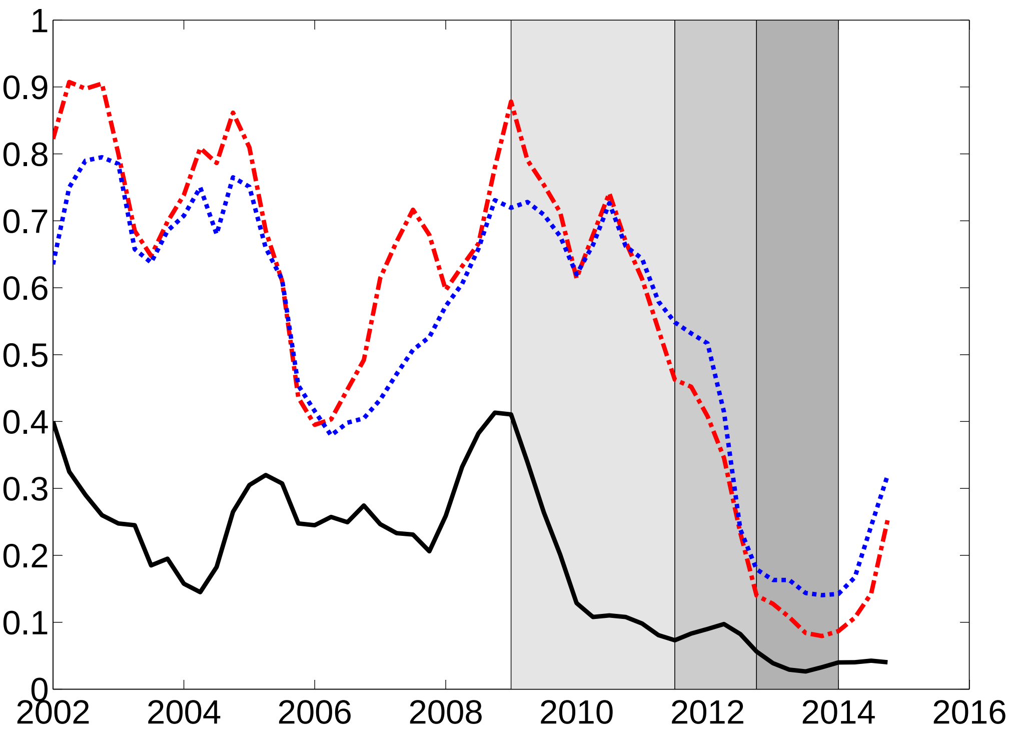
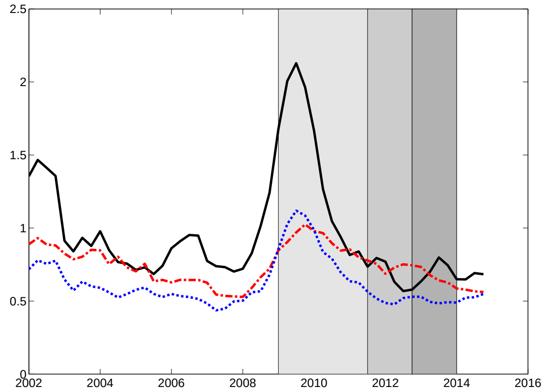


Figure 1: Disagreement about future short-term interest rates.

The chart displays the evolution of a moving average over the last 4 quarters of the 75/25 inter-quantile range in the distribution of 1-quarter (black line), 1-year (red line), and 2-year (blue line) ahead individual mean point forecasts for 3-month T-Bill interest rate. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "fixed-date" forward guidance and the "state-contingent" forward guidance.

(a) Consumption



(b) Inflation

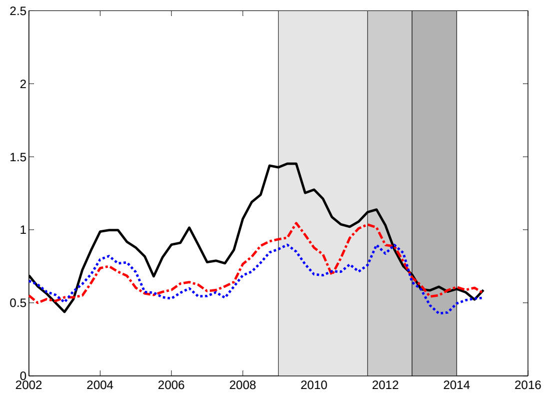
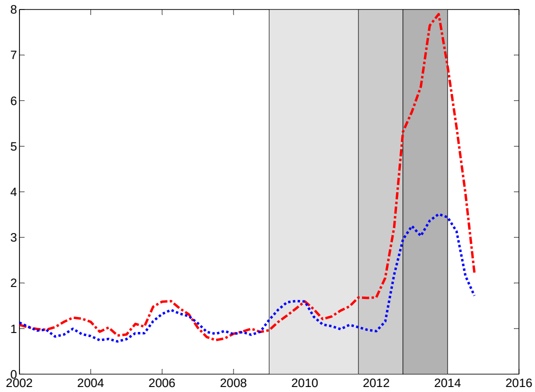


Figure 2: Disagreement about future consumption growth and inflation.

The figure shows the evolution of a moving average over the last 4 quarters of the 75/25 inter-quantile range in the distribution of 1-quarter (black line), 1-year (red line), and 2-year (blue line) ahead individual mean point forecasts for real consumption growth and CPI inflation. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "fixed-date" forward guidance and the "state-contingent" forward guidance.

(a) Consumption



(b) Inflation

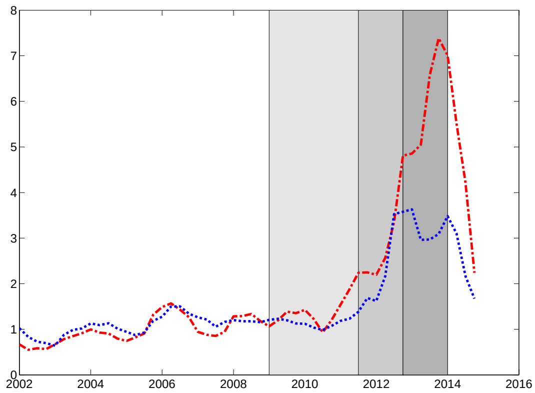


Figure 3: Disagreement about future consumption growth and inflation relative to disagreement about future short-term interest rates.

The figure provides the ratio of disagreement on 1-year (red line) and 2-year (blue line) ahead consumption growth and inflation to disagreement on 1-year and 2-year the short-term interest rates. Disagreements are measured as a moving average over the last 4 quarters of the 75/25 inter-quantile range in the distribution of corresponding individual mean point forecasts. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "fixed-date" forward guidance and the "state-contingent" forward guidance.

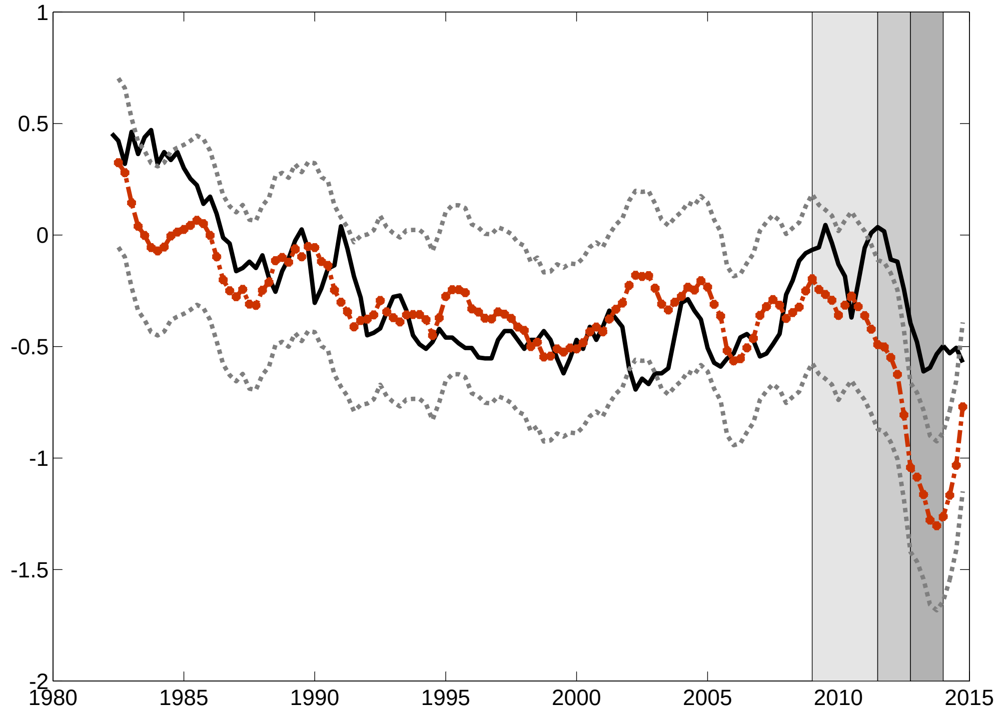
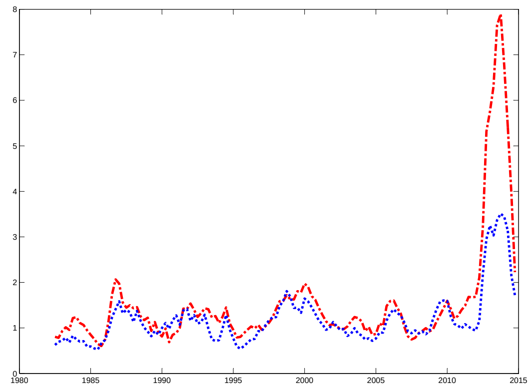


Figure 4: Disagreement on inflation forecasts: observed vs. predicted

The Figure plots the (log) disagreement on 1-year ahead inflation forecasts (black line) observed in the SPF data together with the (log) disagreement predicted (red line) by a regression of such a variable on the disagreement about 1-year ahead short-term interest rate and output growth forecasts estimated on a pre-crisis sample (1982Q1-2008Q4). Dotted light grey lines give 90% confidence bands. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "fixed-date" forward guidance and the "state-contingent" forward guidance.

(a) Consumption



(b) Inflation

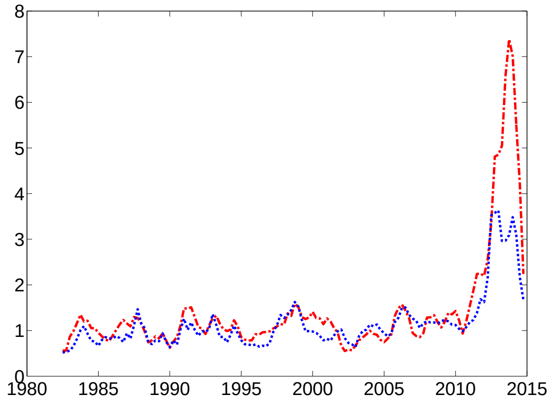


Figure 5: Disagreement about future consumption growth and inflation relative to disagreement about future short-term interest rates – Long sample

The figure provides the ratio of disagreement on 1-year (red line) and 2-year (blue line) ahead consumption growth and inflation to disagreement on 1-year and 2-year the short-term interest rates on an extended smaple (1982-2014). Disagreements are measured as a moving average over the last 4 quarters of the 75/25 inter-quantile range in the distribution of corresponding individual mean point forecasts.

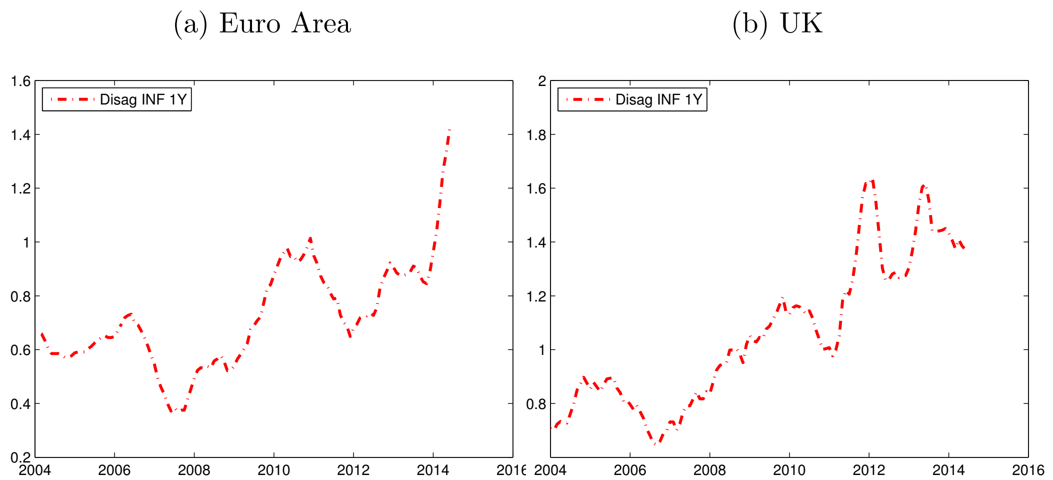


Figure 6: Disagreement about next year inflation scaled by disagreement about one year ahead short-term interest rate.

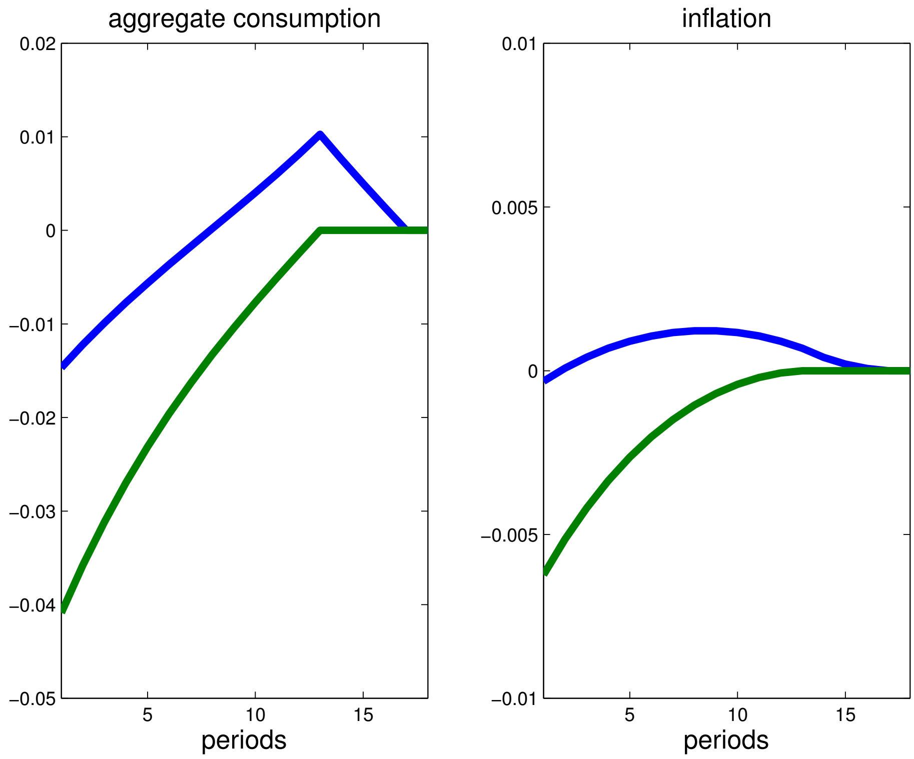


Figure 7: The effect of Delphic (green) and Odyssean (blue) policies.

We consider a shock ($\xi = -0.01$) on the discount rate that lasts 12 quarters and implies a drop of consumption of 4% at impact in the absence of odyssean forward guidance, which provides for 4 extra quarters of accommodation. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.

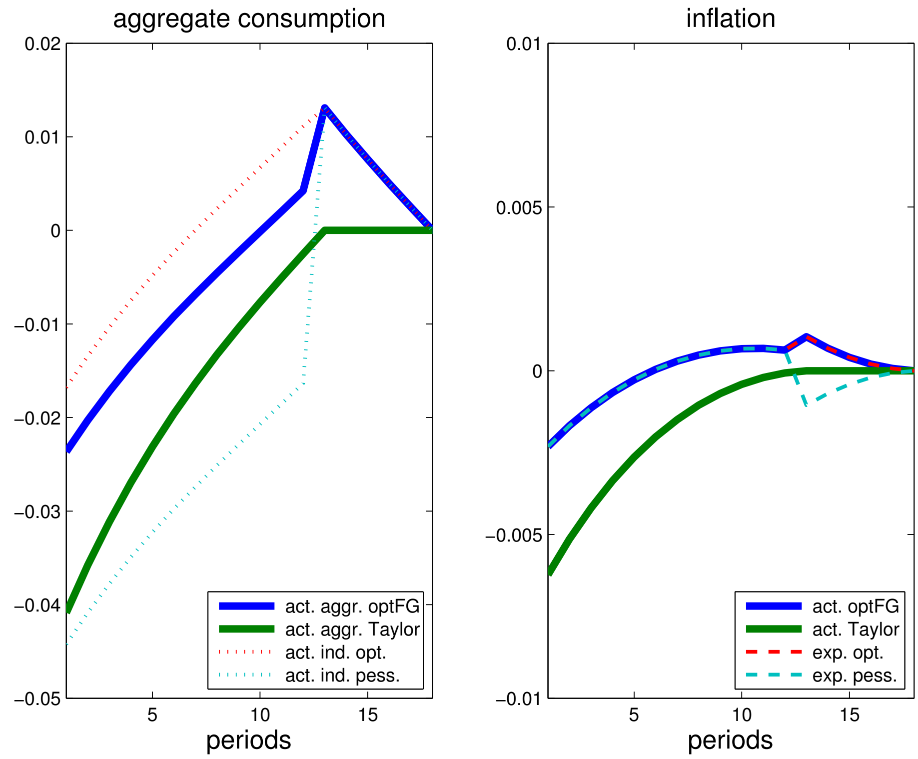


Figure 8: The effect of Odyssean (blue) with a fraction $\alpha = .25$ of pessimists.

We consider a shock ($\xi = -0.01$) on the discount rate that lasts 12 quarters and implies a drop of consumption of 4% at impact in the absence of odyssean forward guidance, which provides for 4 extra quarters of accommodation. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.

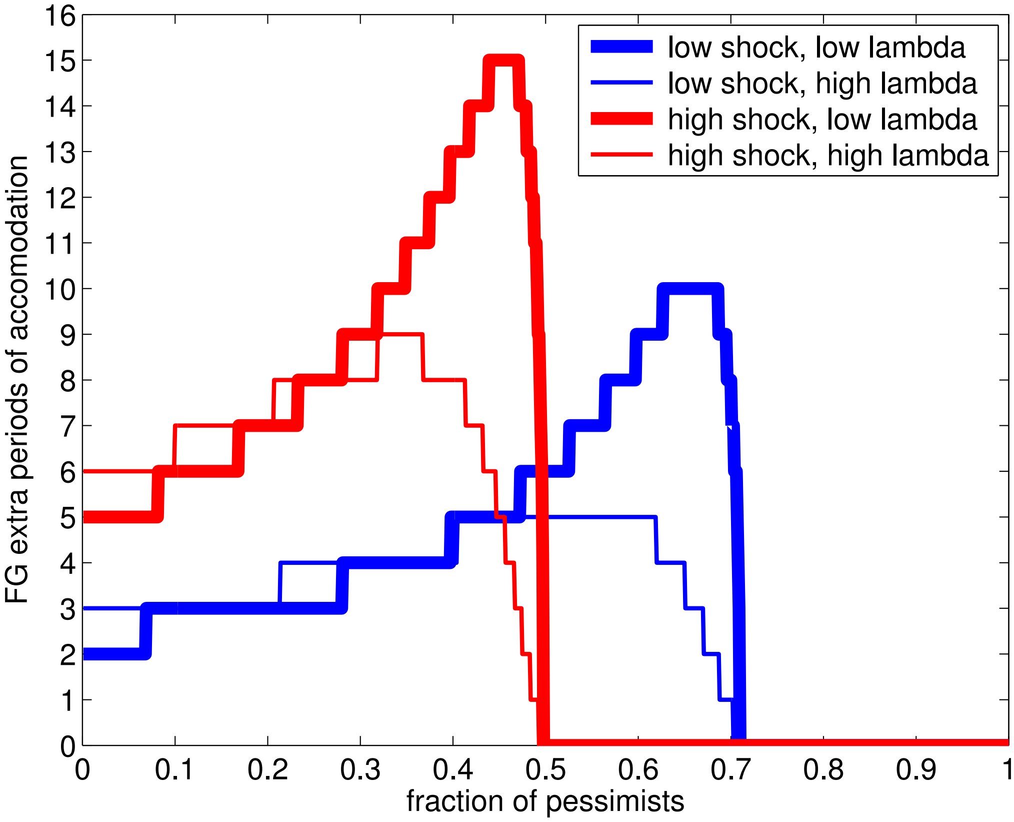
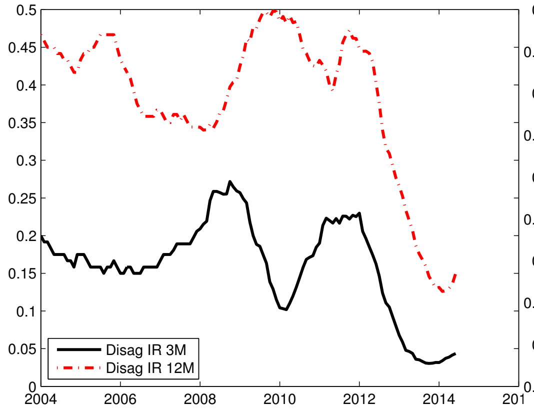


Figure 9: The optimal $T_{cb} - T$ as a function of the fraction of pessimists.

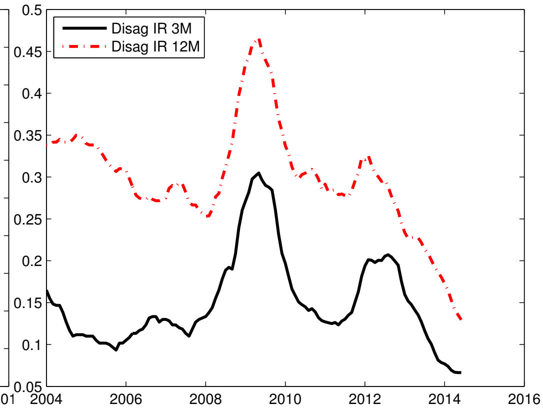
We plot the optimal $T_{cb}(\alpha, 20) - 20$ for $\xi = -.007$ (blue) and $\xi = -.01$ (red) with $\lambda = 0$ (thick) and $\lambda = 50$ (thin), where λ is the weight on output gap in the loss function of the central bank. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.

A Additional evidence in other countries

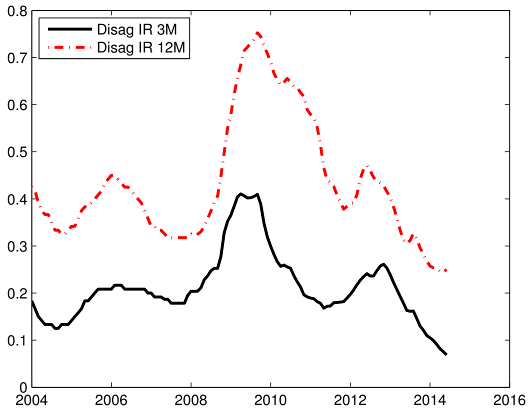
We use survey forecast on Canada, the euro area and the UK from Consensus Inc. Figures 10, 11 display respectively the disagreement about short-term interest rates 1 quarter and 1 year ahead and the ratio of disagreement about next year inflation to disagreement about 1 year ahead short-term interest rates. In Figure 12 in the appendix, we also plot the ratio of disagreement about next year consumption growth rate to disagreement about 1 year ahead short-term interest rates.



(a) Canada



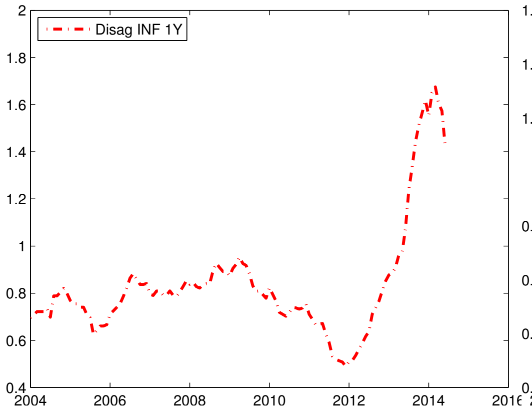
(b) Euro Area



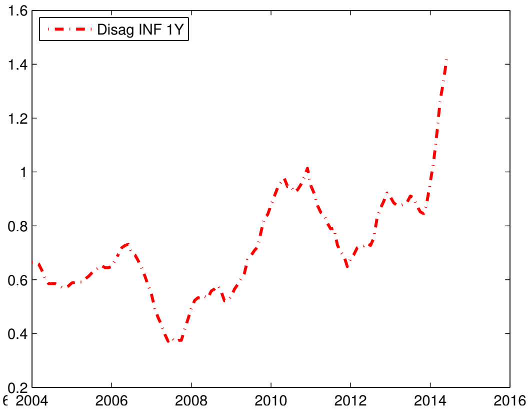
(c) UK

Figure 10: Disagreement about one quarter ahead and one year ahead short term interest rates for various countries.

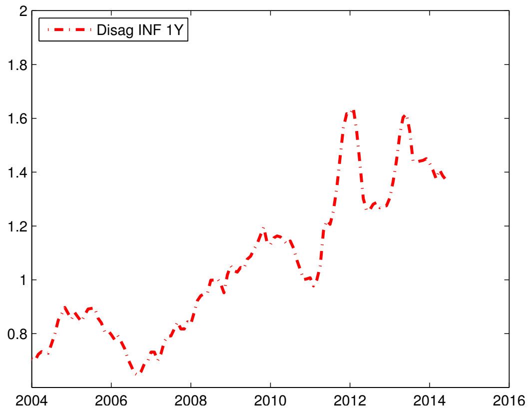
Three main comments emerge. First, some forward guidance are more effective than others: the US forward guidance achieved a coordination of opinions about 1Q ahead but also 1Y ahead short-term interest rates that cannot be found elsewhere. Second, there is a global impact of the US forward guidance: the coordination of opinions about future interest rates observed in the US is also observed in these countries, even in times when they did not conducted a forward guidance policy (e.g. Canada and the euro-area). Third, the US pattern that the ratio of disagreement about future inflation/consumption to the disagreement about future monetary policy increases to unprecedented levels when forward guidance starts to be effective on interest rates also applies to these economies.



(a) Canada

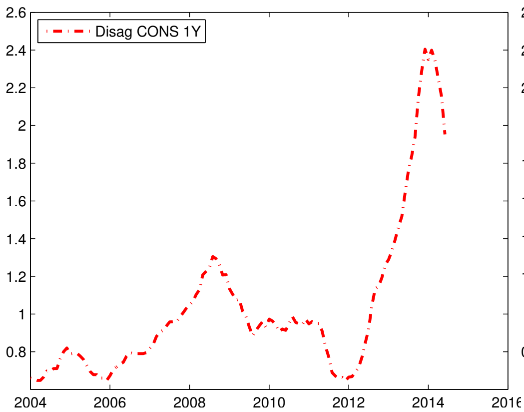


(b) Euro Area

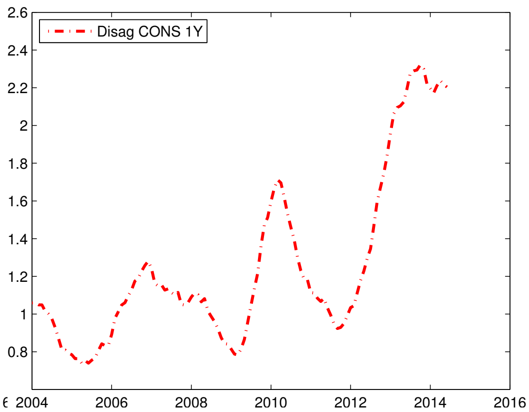


(c) UK

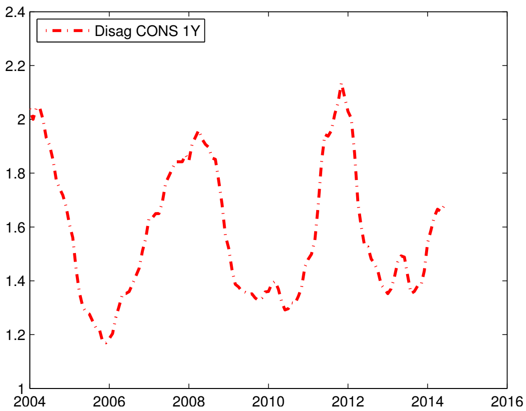
Figure 11: Disagreement about next year inflation scaled by disagreement about one year ahead short-term interest rate.



(a) Canada



(b) Euro Area



(c) UK

Figure 12: Disagreement about next year consumption growth scaled by disagreement about one year ahead short-term interest rate.

B Model derivation and equivalence with the representative agent model

B.1 Model derivation.

Following standard steps, we can write down the log-linearized versions of optimality conditions as:

$$c_{i,t} = -\tfrac{1}{\gamma}\left(E_t^i \xi_{t+1} - \xi_t + r_t - E_t^i \pi_{t+1}\right) + E_t^i c_{i,t+1}, \tag{13}$$

$$\gamma c_{i,t} + \psi l_{i,t} = w_t - p_t \tag{14}$$

Notice that that $\xi_t < 0$ in the trap and $\xi_t = 0$ out of the trap. This means that an exit form the trap, say at time $t+1$, implies $\xi = E_t^i \xi_{t+1} - \xi_t > 0$. So, the term $\xi = E_t^i \xi_{t+1} - \xi_t$ is positive at the time of reverting to normal times and equals 0 otherwise. As a result, the Euler equation (13) implies that consumption decreases at the beginning of the liquidity trap before it gradually increases during the trap.

The optimal price setting for producer j is given by:

$$x_{j,t} \quad = \quad (1-\alpha\beta)\, E_t^j \left[\sum_{\tau=t}^{\infty} (\alpha\beta)^{\tau-t}\, w_\tau\right]$$

as standard in the sticky price literature.

Aggregate behavior. Assuming that ξ can be anticipated a period in advance and by solving forward, we obtain that individual consumption equals:

$$c_{i,t} \quad = \quad -\frac{1}{\gamma} E_{i,t}\left[\sum_{\tau=t}^{\infty} (r_\tau - \pi_{\tau+1} + \xi_{\tau+1} - \xi_\tau)\right]$$

and aggregate consumption equals:

$$c_t \quad = \quad -\frac{1}{\gamma} E_t\left[\sum_{\tau=t}^{\infty} (r_\tau - \pi_{\tau+1})_{\tau+1} + \xi_{\tau+1} - \xi_\tau\right]$$

Notice that as long as agents do not disagree on the size of the shock (this is the case as they observe it), but only on the future date on which it will unfold, it enters as a fix wedge in the IS curve. This wedge will disappear only at the optimistic date when agents will discover the truth.

Aggregating over producers yields:

$$x_t \quad = \quad (1-\alpha\beta)\, w_t + \alpha\beta \int E_{i,t} x_{i,t+1} \mathrm{di}.$$

By noticing that $\pi_t = p_t - p_{t-1}$, we obtain the following new-keynesian Phillips curve:

$$\pi_t \quad = \quad \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(w_t - p_t) + \beta E_t \pi_{t+1}, \tag{15}$$

which is identical to the one under homogeneous beliefs. This result relies on the assumption that producers observe all current variables and only disgree about the future.

We obtain the New-keynesian Phillips Curve in the presence of heterogeneous beliefs as follows. By defining $\Delta_t \equiv \int E_{i,t} x_{i,t+1} \mathrm{di} - E_t x_{t+1}$, we can write x_t recursively as:

$$x_t \quad = \quad (1-\alpha\beta)\, w_t + \alpha\beta E_t x_{t+1} + \alpha\beta \Delta_t$$

At the same time, $x_t = \frac{p_t - \alpha p_{t-1}}{1-\alpha}$ and so, we can write

$$p_t - \alpha p_{t-1} = (1-\alpha)(1-\alpha\beta) w_t + \alpha\beta E_t (p_{t+1} - \alpha p_t) + (1-\alpha)\alpha\beta\Delta_t$$

Thus, we obtain:

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(w_t - p_t) + \beta E_t \pi_{t+1} + (1-\alpha)\beta\Delta_t$$

By definition, $\Delta_t \equiv \int E_{i,t} x_{i,t+1} \mathrm{di} - E_t x_{t+1}$ and $x_{i,t}$ is a function of current and future wages (w_τs). As a result, we can rewrite Δ_t as follows:

$$\Delta_t = (1-\alpha\beta) \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \int E_t^i \left(w_{t+\tau+1} - \int E_{t+1}^i [w_{t+\tau+1}] \mathrm{di} \right) \mathrm{di}$$

which equals 0 in this case, yielding (15).

B.2 Equivalence with the Representative Agent

So far, we consider both uncertain and heterogeneous beliefs about forward guidance. We now investigate how uncertainty differs from heterogeneity. To this purpose, let us consider the following two distributions:

(i) $\lambda_i = 1$ for $i \leq \alpha$ and $\lambda_i = 0$ otherwise. Agents disagree but are certain about their beliefs

(ii) $\lambda_i = \alpha$ for all i. Agents do not disagree but are uncertain, i.e. a representative agent fiction holds.

Given sequence of shocks $\{\xi_0, \xi_1, ...\}$ and a sequence of policy deviations $\{\Delta_0, ...\}$ and a policy announcement, let us consider the equilibrium path associated with the distribution of beliefs (i):

$$\{C_{i,t}^h, L_{i,t}^h, B_{i,t}^h, D_t^h, R_t, W_t^h, Z_{i,t}^h, P_t^h\}_{i\in[0,1],t\geq 0}$$

and the resulting aggregate variables:

$$C_t^h = \int_0^1 C_{i,t}^h di \ , \ L_t^h = \int_0^1 L_{i,t}^h di \text{ and } B_t^h = \int_0^1 B_{i,t}^h di.$$

Let us also consider the equilibrium path associated with the distribution of beliefs (ii):

$$\{C_{i,t}^u, L_{i,t}^u, B_{i,t}^u, D_t^u, R_t, W_t^u, Z_{i,t}^u, P_t^u\}_{i\in[0,1],t\geq 0}$$

and the resulting aggregate variables:

$$C_t^u = \int_0^1 C_{i,t}^u di \ , \ L_t^u = \int_0^1 L_{i,t}^u di \text{ and } B_t^u = \int_0^1 B_{i,t}^u di.$$

Proposition 8. *The aggregate outcome of the two equilibrium paths coincide at the first order, i.e.:*

$$c_t^u = c_t^h + o(||\xi||) \text{ and } l_t^u = l_t^h + o(||\xi||)di.$$

but not to higher-order approximations. More generally, $C_t^u \leq C_t^h$ for all $t \geq 0$.

Similarly, one can show that the household family is equivalent to an economy with self-interested agents that can trade insurance *ex ante* in a complete market setting, when the discount factor is sufficiently close to 1.

C Proofs

C.1 Proof of Proposition 3.

The proof is organized in five steps. First step. Consider an economy with homogeneous agents at the date $T_{cb}+1$ just after the end of the zero-rate period, so that the steady state can be restored. Because of Ricardian equivalence holds, the present value of their life-utility is the same irrespective of the stock of bonds they hold at that time, which is a legacy of the realized states of the words. Therefore, because of the permanent income hypothesis, the level of homogenous individual consumption $C_{T_{cb}+1} = \bar{C}$ is pin down only by the forward evolution of the economy that will remain at steady state. Second step. At time T, as soon as agents become homogeneous, they would agree on a plan of transfers $Z^*_{i,t}\}_0^1$ such that $B_{o,t} = B_{p,t}$, that is, their stock bonds is equalized. In fact, as a consequence, consumption is equalized and so $U_{C_{o,T}} = U_{C_{p,T}}$, that is, social welfare is maximized. After that period, irrespective of whether or not the economy is already at steady state (preference shock does not hit), individual consumption will converge to $C_{T_{cb}+1} = \bar{C}$ because of what argued in the first step. Third Step. Consider now the sequence of transfers $\{\{Z^*_{i,t}\}_0^1\}_{t=0}^{\infty}$, then since step two and three are common knowledge, there is only one equilibrium consumption path associated to each state of the word as decribed in the proposition. Fourth step. Different transfers plans, which modify the path of consumption of the two types, imply, because of the permanent income hypothesis, different level of consumption at steady state. Given that agents anticipate step 2, no plan of this kind can be implemented. In other words, agents anticipate that at time t they will agree to equalize their wealth so that $\bar{C}$ will be their steady state consumption that in turn determines the unique consumption path described at step three. Fifth step. Among all the transfer plans that can engineer an equalization in the stock of bonds at time T onwards, $\{Z^*_{i,t}\}_0^1$ is the only one that is implementable because before time T agents disagree on the actual transfer that will equalize bonds holding at time T as they expect different real interest rates paths, after time T they agree on no transfers.

C.2 Proof of Proposition 4.

Log-linearizing the equation yields:

$$Bb_{i,t} = RB(r_t + b_{i,t-1}) + WL(w_t + l_{i,t}) - PC(p_t + c_{i,t}) + Dd_t,$$

which becomes

$$B(b_{i,t} - b_t) = RB(b_{i,t-1} - b_{t-1}) + WL(l_{i,t} - l_t) - PC(c_{i,t} - c_t),$$

after substituting for $Dd_t = PC(p_t + c_t) - WL(w_t + l_t) + Ss_t$. Given the individual optimal supply of labor we can write

$$l_{i,t} - l_t = -\frac{\gamma}{\psi}(c_{i,t} - c_t),$$

so that finally we have

$$B(b_{i,t} - b_t) = RB(b_{i,t-1} - b_{t-1}) - C(P + W\frac{\gamma}{\psi})(c_{i,t} - c_t),$$

using the steady state relation $L = C$. Finally we can determine the net difference between the real saving position of the two agents as

$$\frac{B}{P}(b_{p,t} - b_{o,t}) = R\frac{B}{P}(b_{p,t-1} - b_{o,t-1}) - C(1 + \frac{W}{P}\frac{\gamma}{\psi})(c_{p,t} - c_{o,t}).$$

Substituting for the steady state consumption relation $C = W/P = 1$ we get

$$\frac{B_{p,t-1} - B_{o,t-1}}{P} = R\frac{B_{p,t-1} - B_{o,t-1}}{P} - (1 + \frac{\gamma}{\psi})(c_{p,t} - c_{o,t}).$$

from which we can easily get

$$\frac{B_{p,t-1} - B_{o,t-1}}{P_t} = \frac{B_{p,t-1} - B_{o,t-1}}{P}(\Pi_1\Pi_2...\Pi_t)^{-1}.$$