Extracting Information from Inflation Markets: The Role of TIPS Liquidity

The TIPS Liquidity Premium

by

Martin M. Andreasen

Jens H. E. Christensen

Simon Riddell

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Motivation

- Differences between nominal and real yields, known as breakeven inflation (BEI), are widely used as timely indicators of economic agents' inflation expectations.
- There are two problems with their use for that purpose:
  - BEI contains an inflation risk premium;
  - BEI is biased by liquidity differentials between nominal and real yields.
- We address these challenges by using TIPS prices in combination with a novel arbitrage-free dynamic term structure model that includes a unique factor to capture the liquidity premium in real yields.

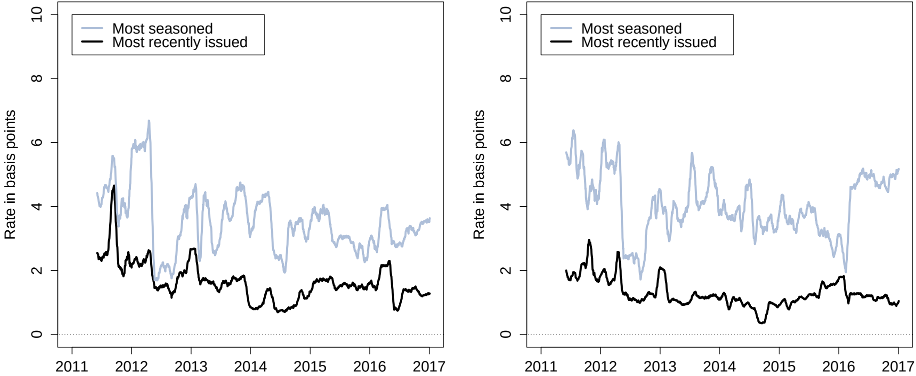
Overview of Findings

- Using TIPS data from July 1997 through the end of 2013, the average estimated TIPS liquidity premium is 38 bps.
- It has notable spikes in the early 2000s and at the peak of the financial crisis.
- Estimates are robust across specifications and sample choices.
- TIPS liquidity premiums are determined by measures of economic uncertainty and limits to arbitrage capital.
- By adjusting for TIPS liquidity premiums, improved inflation forecasts can be obtained.
- In short, the novel model framework looks promising and is likely to be useful in settings where liquidity is an issue.

Outline

1. Model framework and data
2. Estimation results
3. Inflation forecasts
4. Conclusion and update

Structure of TIPS Bid-Ask Spreads



- Shown are bid-ask spreads of the most recently issued and most seasoned TIPS with at least two years to maturity (left: 5-yr TIPS, right: 10-yr TIPS).
- Pervasive pattern:

  Liquidity of a TIPS varies and is *expected* to decline ⇒ Rational investors are aware of these dynamics!

Contribution

- Our key innovation is to assume that the discounting of the cash flow of a given TIPS indexed i is performed with a bond-specific function:
$$\bar{r}_t^{R,i} = r_t^R + \beta^i(1 - e^{-\lambda^{L,i}(t-t_0^i)})X_t^{liq}.$$
- Time since issuance, t-t_0^i, is a proxy for the notion that, as time passes, an increasing fraction of a given security is held by buy-and-hold investors and not available for trading.

- Forward-looking investors factor this into their trading strategies, which determines X_t^{liq} and the TIPS liquidity premiums.

Note: This can be combined with any existing model of r_t^R.

- We focus on joint models of nominal and real yields to be able to account for the value of TIPS deflation options.

CLR Model Framework (1)

Christensen, Lopez, and Rudebusch (2010, CLR) introduce a four-factor arbitrage-free model of nominal and real Treasury yields centered around the arbitrage-free Nelson-Siegel (AFNS) models introduced in Christensen et al. (2011).

The CLR model has four factors $X_t = (L_t^N, S_t, C_t, L_t^R)$.

The instantaneous nominal and real risk-free rates are defined as

$$\begin{aligned} r_t^N &= L_t^N + S_t, \\ r_t^R &= L_t^R + \alpha^R S_t, \end{aligned}$$

while the risk-neutral factor dynamics are assumed given by

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \left(\begin{pmatrix} \theta_{L^N}^Q \\ \theta_S^Q \\ \theta_C^Q \\ \theta_{L^R}^Q \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t \\ C_t \\ L_t^R \end{pmatrix} \right) dt + \Sigma dW_t^Q.$$

For identification, and without loss of generality, we fix $\theta^Q = 0$.

CLR Model Framework (2)

Nominal Treasury zero-coupon yields take the functional form

$$y_t^N(\tau) = L_t^N + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A^N(\tau)}{\tau}.$$

The real TIPS zero-coupon yields are given by

$$y_t^R(\tau) = L_t^R + \boxed{\alpha^R} \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \boxed{\alpha^R} \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A^R(\tau)}{\tau}.$$

Note: The Nelson-Siegel factor loading structure is preserved for both yield curves.

In our model, these two equations are interpreted as the ***frictionless*** nominal and real yield curves, respectively.

Risk-Neutral Dynamics in the CLR-L Model

We refer to the CLR model augmented with the liquidity risk factor as the CLR-L model.

Its five state variables, $X_t = (L_t^N, S_t, C_t, L_t^R, X_t^{liq})$, have risk-neutral dynamics given by

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \\ dX_t^{liq} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \kappa_{liq}^Q \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \theta_{liq}^Q \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t \\ C_t \\ L_t^R \\ X_t^{liq} \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L^N,Q} \\ dW_t^{S,Q} \\ dW_t^{C,Q} \\ dW_t^{L^R,Q} \\ dW_t^{liq,Q} \end{pmatrix}.$$

<u>Note:</u> The liquidity risk factor, X_t^{liq}, is modeled as an independent Vasiček (1977) process under the Q-measure.

TIPS Pricing

Now, consider the value of the TIPS issued at time t_0 with maturity at $t+\tau$ that pays an annual coupon C semi-annually and has accrued inflation compensation equal to Π_t/Π_0.

Its price is

$$\begin{aligned}\overline{P}_t(t_0,\tau) &= \frac{C}{2}\frac{(t_1-t)}{1/2}E^Q\left[e^{-\int_t^{t_1}\overline{r}^R(s,t_0)ds}\right]+\sum_{j=2}^{N}\frac{C}{2}E^Q\left[e^{-\int_t^{t_j}\overline{r}^R(s,t_0)ds}\right]\\ &\quad +E^Q\left[e^{-\int_t^{t+\tau}\overline{r}^R(s,t_0)ds}\right]+DOV_t\left(\tau,\frac{\Pi_t}{\Pi_0}\right).\end{aligned}$$

Note: Deflation option values are calculated using the frictionless nominal and real short rates, r_t^N and r_t^R, based on formulas provided in Christensen et al. (2012).

Minor omission: We do not account for lag in infl. indexation, but effect likely small, see Grishchenko and Huang (2013).

Market Prices of Risk

To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee (2002).

This implies that the risk premiums Γ_t are state-dependent

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

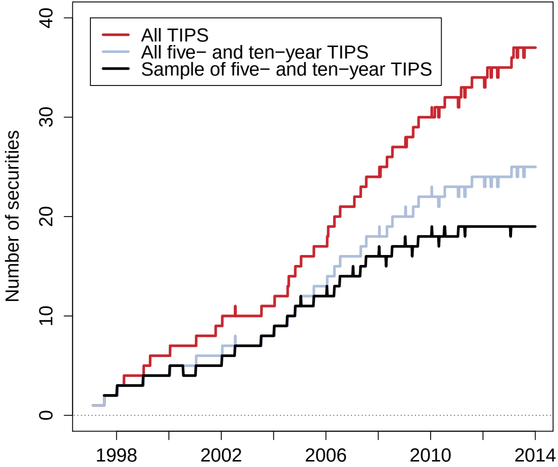
where $\gamma^0 \in \mathbf{R}^5$ and $\gamma^1 \in \mathbf{R}^{5\times 5}$ are unrestricted.

Thus, the unrestricted CLR-L model has P-dynamics

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \\ dX_t^{liq} \end{pmatrix} = \begin{pmatrix} \kappa_{11}^P & \kappa_{12}^P & \kappa_{13}^P & \kappa_{14}^P & \kappa_{15}^P \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P & \kappa_{24}^P & \kappa_{25}^P \\ \kappa_{31}^P & \kappa_{32}^P & \kappa_{33}^P & \kappa_{34}^P & \kappa_{35}^P \\ \kappa_{41}^P & \kappa_{42}^P & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P \\ \kappa_{51}^P & \kappa_{52}^P & \kappa_{53}^P & \kappa_{54}^P & \kappa_{55}^P \end{pmatrix} \left(\begin{pmatrix} \theta_1^P \\ \theta_2^P \\ \theta_3^P \\ \theta_4^P \\ \theta_5^P \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t \\ C_t \\ L_t^R \\ X_t^{liq} \end{pmatrix} \right) dt + \Sigma dW_t^P.$$

This is the transition equation in the Kalman filter estimation.

Distribution of TIPS



- Shown is the universe of TIPS outstanding since 1997.
- To facilitate implementation, we focus on the universe of five- and ten-year TIPS.
- Due to data quality issues, we drop the last two years of trading for each TIPS, see Gürkaynak et al. (2010).

TIPS and Treasury Data

- For identification of the TIPS-related factors (L_t^R, X_t^{liq}), we need a minimum of two TIPS to be trading.
- This determines the start date, July 11, 1997, while the sample ends on December 27, 2013.
- To balance the number of observations versus the computational burden, we use weekly (Fridays) data.
- We combine TIPS mid-market prices from Bloomberg with a standard sample of off-the-run Treasury yields from Gürkaynak et al. (2007) with 12 maturities:

  3-m, 6-m, 1-yr, 2-yr, ... , 10-yr.

- All TIPS have the same measurement error distribution (ε_t^R). The same holds for all the Treasury yields (ε_t^N). Finally, all errors are assumed to be *i.i.d.*

Measurement Equation and Model Estimation

- The measurement equation for the Treasury yields is

$$y_t^N(\tau) = \widehat{y}_t^N(\tau) + \varepsilon_t^N.$$

- To facilitate the empirical implementation, we fit the model to TIPS prices instead of TIPS yields-to-maturity.

- To make the fitted TIPS pricing errors comparable across maturities and time, we scale each TIPS price by its duration

$$\frac{\overline{P}_t(t_0,\tau)}{D_t(t_0,\tau)} = \frac{\widehat{P}_t(t_0,\tau)}{D_t(t_0,\tau)} + \varepsilon_t,$$

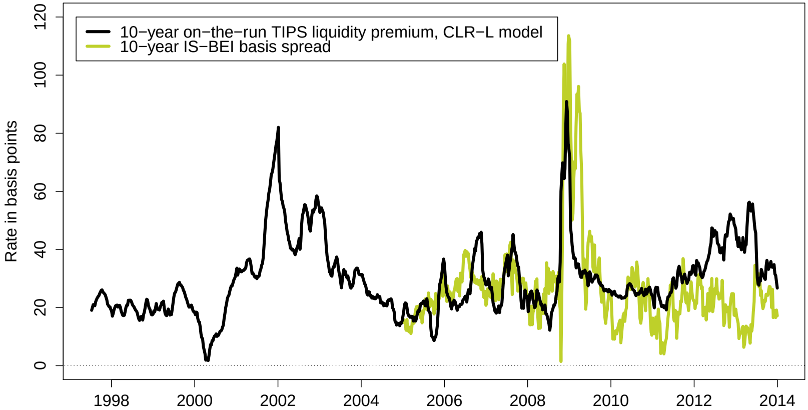
  where $\widehat{P}_t(t_0,\tau)$ is the model-implied TIPS price and $D_t(t_0,\tau)$ is its duration, which is fixed and calculated before estimation.

- Due to the nonlinear measurement, the model is estimated with the extended Kalman filter, see Kim and Singleton (2012).

- For identification, the first TIPS issued (1/15/2007 3.375% 10-yr TIPS) has unit loading on the liquidity factor, i.e., $\beta^1 = 1$.

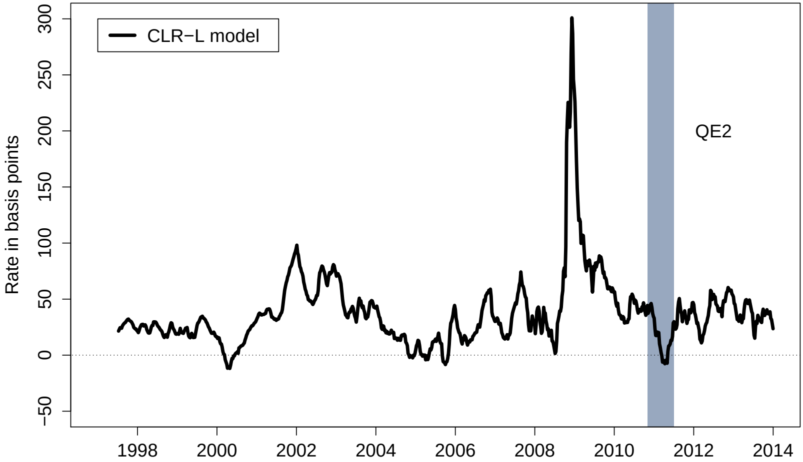
Outline

Ten-Year On-The-Run TIPS Liquidity Premium



- Shown is the estimated liquidity premium of the ten-year on-the-run TIPS at each point in time.
- Also shown is the difference between the ten-year IS rate and the ten-year BEI constructed from Gürkaynak et al. (2007, 2010), see Christensen and Gillan (2017).
- Note the high correlation of these 2 measures of frictions.

Average TIPS Liquidity Premium



- Shown is the average estimated TIPS liquidity premium at each point in time.
- For the entire sample the average is close to 38 basis points.

Interpretation of the Liquidity Factor

Important distinction:

- At the individual level, the amount available for trading (*supply*) should matter for the size of liquidity premiums.
- However, for the collective universe of bonds, the average liquidity premium is a systematic risk influenced by limits to arbitrage capital and investor sentiment (*demand*), which is captured by X_t^{liq}.

Key example:

- At the individual bond level, QE asset purchases should increase liquidity premiums in the targeted securities.
- However, this idiosyncratic effect may be more than offset by effects of QE on the systematic liquidity risk embedded in X_t^{liq}, see Christensen and Gillan (2017).

Determinants of TIPS Liquidity Premiums

	Average TIPS liquidity premium	
Constant	−5.21	−4.30
	(5.36)	(4.05)
VIX	0.85**	0.76**
	(0.20)	(0.21)
HPW	−2.00	–
	(1.08)	
On-the-run spread	0.73*	0.49*
	(0.29)	(0.20)
Ratio of Trading vol	0.0001	–
	(0.06)	
GSW TIPS errors	6.06**	5.53**
	(0.47)	(0.34)
Adjusted R^2	0.769	0.765

Key determinants of TIPS liquidity premiums include:

- General economic uncertainty as captured by the VIX;
- On-the-run premiums in the Treasury market;
- Measures of mispricing in the TIPS market.

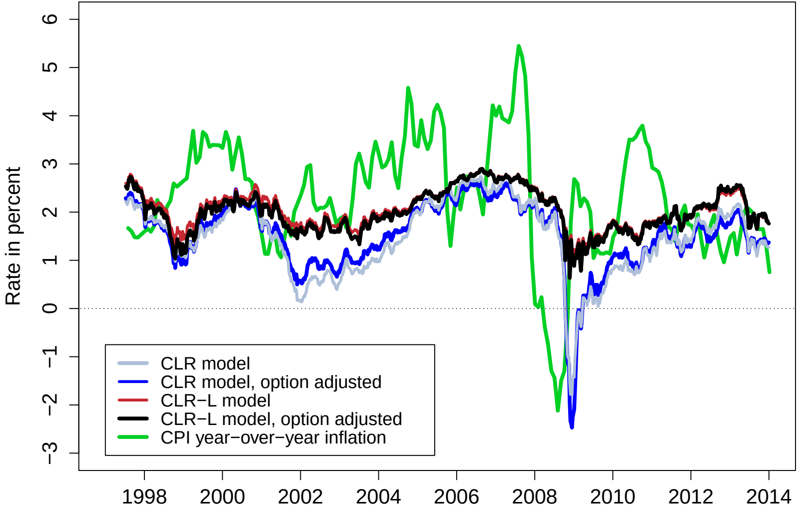
Robustness Checks

The estimated TIPS liquidity premiums are robust to:

- Choice of sample start date.
- Choice of sample frequency.
- Choice of cutoff time for each TIPS.
- Allowing for flexible factor dynamics.
- Including all available TIPS.
- Shadow-rate specification for nominal yields.
- Allowing for more flexible structure than the CLR model.
- Allowing for stochastic yield volatility.

Outline

Inflation Forecasts



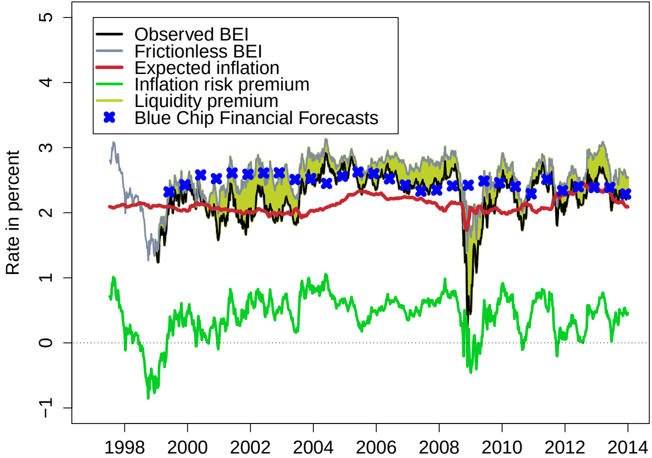
- Shown are one-year inflation forecasts from various models and the subsequent year-over-year realization of CPI inflation.
- Note the higher level when TIPS liquidity is accounted for.
- Caveat: The model forecasts are not real time!

One-Year Ahead CPI Inflation Forecast Evaluation

Model	1997-2004			2005-2013		
	Mean	MAE	RMSE	Mean	MAE	RMSE
Random Walk	20.85	91.50	108.04	-19.16	176.17	230.51
Blue Chip	23.37	73.43	86.44	-0.54	113.28	150.93
Inflation swap rate	n.a.	n.a.	n.a.	44.32	142.24	198.07
CLR	113.54	129.12	147.77	58.09	128.88	162.65
CLR, option adjusted	101.69	118.06	134.67	59.74	127.15	164.91
CLR-L	**49.95**	**85.88**	**99.22**	**1.72**	**109.46**	**141.69**
— frictionless BEI	46.42	81.82	95.92	-2.10	111.28	143.31
CLR-L, option adjusted	**61.38**	**91.12**	**105.30**	**3.58**	**111.17**	**143.01**
— frictionless BEI	57.22	86.57	101.98	1.14	112.45	144.49

- The CLR-L model is systematically better than the CLR model at forecasting CPI inflation one year ahead.
- It is also competitive compared to the random walk, the Blue Chip survey, and the 1-year inflation swap rate.

Ten-Year BEI Decomposition



- Shown is decomposition of ten-year BEI from GSW data based on the CLR-L model with unrestricted K^P matrix.
- Diff. btw. frictionless and observed BEI represents an alternative measure of TIPS liquidity premiums.
- Note the stable inflation expectations and volatile IRP.

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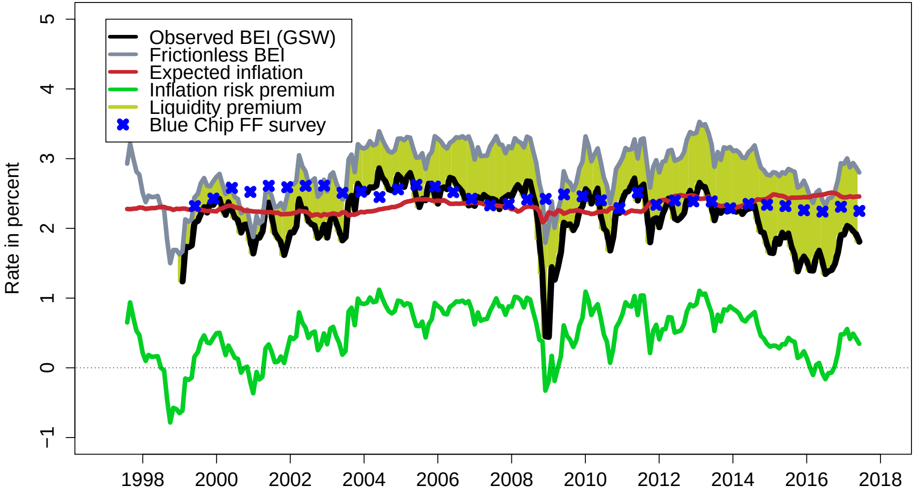
Conclusion

- We modify an existing model of nominal and real yields to account for the liquidity disadvantage of TIPS.
- The model delivers estimated TIPS liquidity premiums that are robust across specifications and sample choices.
- Estimates of expected inflation and risk premiums are sensitive to accounting for TIPS liquidity premiums.
- Our results suggest that accounting for TIPS liquidity premiums may lead to improved CPI inflation forecasts.
- We conclude that the model looks promising and could be useful in a variety of settings where liquidity is an issue.

Update Through May 2017



- Despite declines in ten-year BEI since 2014, long-term inflation expectations have remained anchored at a level consistent with the Fed's target.

Related On-Going TIPS Research

- Andreasen, Christensen, and Rudebusch (2017):

  "**Term Structure Analysis with Big Data.**"

- Christensen, Lopez, and Shultz (2017):

  "**Is There an On-the-Run Premium in TIPS?**"

- Christensen and Rudebusch (2017):

  "**A New Normal for Interest Rates? Evidence from Inflation-Indexed Debt.**"