The Central Bank's Balance Sheet as an Instrument of Monetary Policy
Technical Appendix*

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This technical appendix describes the key equations needed for equilibrium determination in section 1. In section 2 we describe the optimal policy problem and first order conditions. Section 3 describes the optimal steady state, and section 4 describes the numerical calibration.

1 Equilibrium Conditions

This section describes the complete model of credit frictions.[1] Section 1.1 contains all variables in the model. Section 1.2 defines the functional forms used. Section 1.3 lists all the non-linear equations needed for equilibrium determination, aside from the interest rate and central bank lending policy rules. Section 1.4 describes some auxiliary definitions and functionals referred to in the paper and other sections of the paper. Section 1.5 describes the welfare objective.

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

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[1] For details on the derivations please refer to Cúrdia and Woodford (2009) and its technical appendix.

1.1 Variables

Endogenous variables:

$$\left\{i_t^d, \omega_t, \lambda_t^b, \lambda_t^s, \Pi_t, Y_t, K_t, F_t, \Delta_t, b_t, L_t^{cb}, \Xi_t\right\} \tag{1.1}$$

with:

- i_t^d : deposit/policy rate
- ω_t : spread between borrowing and deposit rates
- λ_t^b : marginal utility of expenditure of borrowers
- λ_t^s : marginal utility of expenditure of savers
- Π_t : gross inflation rate
- Y_t : real output
- K_t : artificial variable used in recursive version of inflation dynamics
- F_t : artificial variable used in recursive version of inflation dynamics
- Δ_t : measure of price dispersion
- b_t : total borrowing in the economy
- L_t^{cb} : central bank lending
- Ξ_t : Total intermediation resource costs, including both private and central bank.

Exogenous variables

$$\xi_t = \left\{A_t, \bar{C}_t^b, \bar{C}_t^s, G_t, \mu_t^w, \tau_t, b_t^g, \bar{H}_t, \tilde{\Xi}_t, \tilde{\Xi}_t^+, \tilde{\chi}_t, \tilde{\chi}_t^+\right\} \tag{1.2}$$

with:

- A_t : productivity shock
- $\bar{C}_t^b$: shock to utility of borrowers
- $\bar{C}_t^s$: shock to utility of savers

- G_t : government consumption
- μ_t^w : wage markup
- τ_t : marginal tax rate
- b_t^g : government debt
- $\bar{H}_t$: labor disutility shock
- $\tilde{\Xi}_t$: multiplicative private intermediation resource cost shock
- $\tilde{\Xi}_t^+$: additive private intermediation resource cost shock
- $\tilde{\chi}_t$: multiplicative shock to default rate
- $\tilde{\chi}_t^+$: additive shock to default rate

In the text we also refer to a vector Z_t used in the inflation dynamics, which is defined as

$$Z_t = \begin{bmatrix} K_t \\ F_t \end{bmatrix}. \tag{1.3}$$

Other auxiliary variables

- c_t^b : consumption of borrowers
- c_t^s : consumption of savers
- $\tilde{\lambda}_t$: weighted average of λ_t^b and λ_t^s
- Λ_t : another weighted average of λ_t^b and λ_t^s
- $\tilde{\Lambda}_t$: another weighted average of λ_t^b and λ_t^s
- Ω_t : ratio of marginal utilities of expenditure: $\Omega_t \equiv \lambda_t^b/\lambda_t^s$

1.2 Financial Intermediation Functional Forms

The private intermediaries resource cost, given the satiation of reserves, is given by:

$$\Xi^p\left(L;\xi_t\right)=\tilde{\Xi}_t L^\eta+\tilde{\Xi}_t^+ L, \tag{1.4}$$

where L is the amount of privately intermediated credit, and $\eta \geq 1$. The losses from fraudulent credit are

$$\chi\left(L;\xi_t\right)=\tilde{\chi}_t L^{1+\varkappa}+\tilde{\chi}_t^+ L, \tag{1.5}$$

where $\varkappa \geq 0$.

Central bank lending resource cost

$$\Xi^{cb}\left(L^{cb}\right)=\tilde{\Xi}^{cb} L^{\eta_{cb}}, \tag{1.6}$$

with $\eta_{cb} \geq 1$. But in the numerical analysis we consider $\eta_{cb}=1$.

1.3 Non-linear equations

The equations describing the economy are summarized below:

$$\lambda_t^b=\left(1+i_t^d\right)\left(1+\omega_t\right)\beta E_t\left[\left[\delta+(1-\delta)\pi_b\right]\frac{\lambda_{t+1}^b}{\Pi_{t+1}}+(1-\delta)\left(1-\pi_b\right)\frac{\lambda_{t+1}^s}{\Pi_{t+1}}\right], \tag{1.7}$$

$$\lambda_t^s=\left(1+i_t^d\right)\beta E_t\left[(1-\delta)\pi_b\frac{\lambda_{t+1}^b}{\Pi_{t+1}}+\left[\delta+(1-\delta)\left(1-\pi_b\right)\right]\frac{\lambda_{t+1}^s}{\Pi_{t+1}}\right], \tag{1.8}$$

$$K_t=\Lambda\left(\lambda_t^b,\lambda_t^s\right)\mu^p\left(1+\omega_y\right)\psi\mu_t^w\tilde{\lambda}\left(\lambda_t^b,\lambda_t^s\right)^{-1}\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}+\alpha\beta E_t\left[\Pi_{t+1}^{\theta\left(1+\omega_y\right)}K_{t+1}\right], \tag{1.9}$$

$$F_t=\Lambda\left(\lambda_t^b,\lambda_t^s\right)\left(1-\tau_t\right)Y_t+\alpha\beta E_t\left[\Pi_{t+1}^{\theta-1}F_{t+1}\right], \tag{1.10}$$

$$\begin{aligned}\left(1+\pi_b\omega_t\right)b_t &= \pi_b\pi_s B\left(\lambda_t^b,\lambda_t^s,Y_t,\Delta_t;\xi_t\right)-\pi_b b_t^g \\ &\quad +\delta\left[b_{t-1}\left(1+\omega_{t-1}\right)+\pi_b b_{t-1}^g\right]\frac{1+i_{t-1}^d}{\Pi_t},\end{aligned} \tag{1.11}$$

$$Y_t=\pi_b\bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b}+\pi_s\bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s}+G_t+\Xi_t, \tag{1.12}$$

$$\Delta_t = \alpha \Delta_{t-1} \Pi_t^{\theta(1+\omega_y)} + (1-\alpha) \left(\frac{1-\alpha \Pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{\theta(1+\omega_y)}{\theta-1}}, \tag{1.13}$$

$$\frac{1-\alpha \Pi_t^{\theta-1}}{1-\alpha} = \left(\frac{F_t}{K_t} \right)^{\frac{\theta-1}{1+\omega_y \theta}}, \tag{1.14}$$

$$\omega_t = (1+\varkappa) \tilde{\chi}_t \left(b_t - L_t^{cb} \right)^{\varkappa} + \tilde{\chi}_t^{+} + \eta \tilde{\Xi}_t \left(b_t - L_t^{cb} \right)^{\eta-1} + \tilde{\Xi}_t^{+}, \tag{1.15}$$

$$\Xi_t = \tilde{\Xi}_t \left(b_t - L_t^{cb} \right)^{\eta} + \tilde{\Xi}_t^{+} \left(b_t - L_t^{cb} \right) + \Xi^{cb} \left(L_t^{cb} \right). \tag{1.16}$$

1.4 Auxiliary Definitions

The ratio of marginal utilities is given by

$$\Omega_t \equiv \frac{\lambda_t^b}{\lambda_t^s}. \tag{1.17}$$

At different stages we consider three alternative weighted averages of the marginal utility of expenditures of the two types:

$$\begin{aligned}
\tilde{\lambda}\left(\lambda_t^b, \lambda_t^s\right) &\equiv \psi \left[\pi_b \left(\frac{\lambda_t^b}{\psi_b} \right)^{\frac{1}{\nu}} + \pi_s \left(\frac{\lambda_t^s}{\psi_s} \right)^{\frac{1}{\nu}} \right]^{\nu}, && (1.18) \\
\Lambda\left(\lambda_t^b, \lambda_t^s\right) &\equiv \pi_b \lambda_t^b + \pi_s \lambda_t^s, && (1.19) \\
\tilde{\Lambda}\left(\lambda_t^b, \lambda_t^s\right) &\equiv \psi^{\frac{1}{1+\nu}} \left[\pi_b \psi_b^{-\frac{1}{\nu}} \left(\lambda_t^b\right)^{\frac{1+\nu}{\nu}} + \pi_s \psi_s^{-\frac{1}{\nu}} \left(\lambda_t^s\right)^{\frac{1+\nu}{\nu}} \right]^{\frac{\nu}{1+\nu}}. && (1.20)
\end{aligned}$$

with

$$\psi^{-\frac{1}{\nu}} \equiv \pi_b \psi_b^{-\frac{1}{\nu}} + (1-\pi_b) \psi_s^{-\frac{1}{\nu}}. \tag{1.21}$$

Equation (1.14) defines inflation as a function of K_t and F_t. A more parsimonious way of writing it, used in the main text, is

$$\Pi_t = \Pi(Z_t), \tag{1.22}$$

with

$$\Pi(Z_t) \equiv \left[\frac{1-(1-\alpha)\left(\frac{F_t}{K_t}\right)^{\frac{\theta-1}{1+\omega_y \theta}}}{\alpha} \right]^{\frac{1}{\theta-1}}. \tag{1.23}$$

In the main text we combine equations (1.9) and (1.10) into a single expression:

$$Z_t = z\left(Y_t, \lambda_t^b, \lambda_t^s; \xi_t\right) + E_t\left[\Phi\left(Z_{t+1}\right)\right], \tag{1.24}$$

with

$$z\left(Y_t, \lambda_t^b, \lambda_t^s; \xi_t\right) \equiv \begin{bmatrix} \Lambda\left(\lambda_t^b, \lambda_t^s\right)\mu^p\left(1+\omega_y\right)\psi\mu_t^w\tilde{\lambda}\left(\lambda_t^b, \lambda_t^s\right)^{-1}\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y} \\ \Lambda\left(\lambda_t^b, \lambda_t^s\right)\left(1-\tau_t\right)Y_t \end{bmatrix}, \tag{1.25}$$

$$\Phi\left(Z_{t+1}\right) \equiv \begin{bmatrix} \alpha\beta\Pi\left(Z_{t+1}\right)^{\theta(1+\omega_y)}K_{t+1} \\ \alpha\beta\Pi\left(Z_{t+1}\right)^{\theta-1}F_{t+1} \end{bmatrix}. \tag{1.26}$$

Equation (1.13) defines the law of motion of price dispersion which can be recast as

$$\Delta_t = h\left(\Delta_{t-1}, \Pi_t\right), \tag{1.27}$$

as presented in the main text, where

$$h\left(\Delta_{t-1}, \Pi_t\right) \equiv \alpha\Delta_{t-1}\Pi_t^{\theta(1+\omega_y)} + (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(1+\omega_y)}{\theta-1}}. \tag{1.28}$$

In the law of motion of debt, equation (1.11), we use

$$\begin{aligned} B\left(\lambda_t^b, \lambda_t^s, Y_t, \Delta_t; \xi_t\right) \equiv\ & \bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b} - \bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s} \\ & -\left[\left(\frac{\lambda_t^b}{\psi_b}\right)^{\frac{1}{\nu}} - \left(\frac{\lambda_t^s}{\psi_s}\right)^{\frac{1}{\nu}}\right]\left(\frac{\tilde{\lambda}_t}{\psi}\right)^{-\frac{1+\nu}{\nu}}\mu_t^w\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}\Delta_t. \end{aligned} \tag{1.29}$$

Definition of consumption of borrowers and savers:

$$c_t^b = \bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b}, \tag{1.30}$$

$$c_t^s = \bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s}. \tag{1.31}$$

1.5 Welfare Objective

The welfare objective is:

$$U_t = \pi_b \frac{\left(c_t^b\right)^{1-\sigma_b^{-1}} \left(\bar{C}_t^b\right)^{\sigma_b^{-1}}}{1-\sigma_b^{-1}} + \pi_s \frac{\left(c_t^s\right)^{1-\sigma_s^{-1}} \left(\bar{C}_t^s\right)^{\sigma_s^{-1}}}{1-\sigma_s^{-1}} - \frac{\psi}{1+\nu} \left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu} \left(\frac{Y_t}{A_t}\right)^{1+\omega_y} \Delta_t,$$

and using equations (1.30) and (1.31), we can write the welfare objective in terms of the marginal utilities,

$$U_t = \pi_b \frac{\left(\lambda_t^b\right)^{1-\sigma_b} \bar{C}_t^b}{1-\sigma_b^{-1}} + \pi_s \frac{\left(\lambda_t^s\right)^{1-\sigma_s} \bar{C}_t^s}{1-\sigma_s^{-1}} - \frac{\psi}{1+\nu} \left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu} \left(\frac{Y_t}{A_t}\right)^{1+\omega_y} \Delta_t. \tag{1.32}$$

We can further express the two marginal utilities of consumption as

$$\lambda_t^b \equiv \lambda^b \left(Y_t, \Omega_t, \Xi_t; \xi_t\right), \tag{1.33}$$

$$\lambda_t^s \equiv \lambda^s \left(Y_t, \Omega_t, \Xi_t; \xi_t\right), \tag{1.34}$$

where these functions are implicitely defined as the solutions to equations (1.12) and (1.17). The weighted utility of consumption of the two types (the first two terms in (1.32)) can be recast as

$$\tilde{u} \left(Y_t, \Omega_t, \Xi_t; \xi_t\right), \tag{1.35}$$

and the disutility of of supplying Y_t (the last term in (1.32)) can be written as

$$\Lambda\left(\Omega_t\right) \tilde{v}\left(Y_t; \xi_t\right) \Delta_t, \tag{1.36}$$

where $\Lambda\left(\Omega_t\right)$ and $\tilde{v}\left(Y_t; \xi_t\right)$ are defined as:

$$\Lambda\left(\lambda_t^b, \lambda_t^s\right) \equiv \frac{1}{1+\nu} \left(\frac{\left[\pi_b \psi_b^{-1/\nu} \Omega_t^{\frac{1}{\nu}} + \pi_s \psi_s^{-1/\nu}\right]^{\nu}}{\left[\pi_b \psi_b^{-1/\nu} \Omega_t^{\frac{1+\nu}{\nu}} + \pi_s \psi_s^{-1/\nu}\right]^{\frac{\nu}{1+\nu}}} \right)^{-\frac{1+\nu}{\nu}}, \tag{1.37}$$

$$\tilde{v}\left(Y_t; \xi_t\right) \equiv \bar{H}_t^{-\nu} \left(\frac{Y_t}{A_t}\right)^{1+\omega_y}. \tag{1.38}$$

We can thus recast the welfare criterion as

$$U_t = U\left(Y_t, \Omega_t, \Xi_t, \Delta_t; \xi_t\right), \tag{1.39}$$

which is the functional described in the main text, with

$$U\left(Y_t, \Omega_t, \Xi_t, \Delta_t; \xi_t\right) \equiv \tilde{u}\left(Y_t, \Omega_t, \Xi_t; \xi_t\right) - \Lambda\left(\Omega_t\right) \tilde{v}\left(Y_t; \xi_t\right) \Delta_t. \tag{1.40}$$

2 Optimal policy

The optimal policy problem is to maximize welfare (1.32) with respect to the variables listed in (1.1) subject to the laws of motion of the economy (1.7) - (1.16), with multipliers φ_t^i for $i = 1, ..., 10$, respectively, and two additional constraints to enforce non-negative central bank lending

$$L_t^{cb} \geq 0, \tag{2.1}$$

and non-negative interest rate

$$i_t^d \geq 0, \tag{2.2}$$

and the multipliers for the additional constraints, ζ_t and Υ_t. We present each first order condition (FOC) below.

FOC w.r.t. i_t^d

$$0 = \varphi_t^1 \lambda_t^b + \varphi_t^2 \lambda_t^s + \delta\beta E_t \left[\varphi_{t+1}^5 \left[b_t\left(1+\omega_t\right) + \pi_b b_t^g\right] \frac{1+i_t^d}{\Pi_{t+1}}\right] + \Upsilon_t. \tag{2.3}$$

Complementary slackness for i_t^d is given by the following three conditions,

$$\begin{aligned} 0 &= \Upsilon_t i_t^d, & (2.4)\\ i_t^d &\geq 0, & (2.5)\\ \Upsilon_t &\geq 0. & (2.6) \end{aligned}$$

FOC w.r.t. ω_t

$$0 = \varphi_t^1 \frac{\lambda_t^b}{1+\omega_t} - \varphi_t^5 \pi_b b_t + \delta\beta E_t \left[\varphi_{t+1}^5 \frac{1+i_t^d}{\Pi_{t+1}} b_t\right] + \varphi_t^9. \tag{2.7}$$

FOC w.r.t. L_t^{cb}

$$0 = \varphi_t^{10}\left[\Xi_t^{p\prime}\left(b_t - L_t^{cb}\right) - \Xi^{cb\prime}\left(L_t^{cb}\right)\right] + \varphi_t^{9}\left[\chi_t^{\prime\prime}\left(b_t - L_t^{cb}\right) + \Xi_t^{p\prime\prime}\left(b_t - L_t^{cb}\right)\right] + \zeta_t. \tag{2.8}$$

and under the current functional forms this is equivalent to

$$\begin{aligned} 0 &= \varphi_t^{10}\left[\eta\tilde{\Xi}_t\left(b_t - L_t^{cb}\right)^{\eta-1} + \tilde{\Xi}_t^{+} - \eta_{cb}\tilde{\Xi}^{cb}\left(L_t^{cb}\right)^{\eta_{cb}-1}\right] \\ &\quad +\varphi_t^{9}\left[\varkappa\left(1+\varkappa\right)\tilde{\chi}_t\left(b_t - L_t^{cb}\right)^{\varkappa-1} + \eta\left(\eta-1\right)\tilde{\Xi}_t\left(b_t - L_t^{cb}\right)^{\eta-2}\right] \\ &\quad +\zeta_t. \end{aligned} \tag{2.9}$$

Complementary slackness for L_t^{cb} is given by the following three conditions:

$$0 = \zeta_t L_t^{cb}, \tag{2.10}$$

$$L_t^{cb} \geq 0, \tag{2.11}$$

$$\zeta_t \geq 0. \tag{2.12}$$

FOC w.r.t. b_t

$$\begin{aligned} 0 &= -\varphi_t^{5}\left(1+\pi_b\omega_t\right) + \delta\beta E_t\left[\varphi_{t+1}^{5}\left(1+\omega_t\right)\frac{1+i_t^d}{\Pi_{t+1}}\right] \\ &\quad -\varphi_t^{6}\Xi_t^{p\prime}\left(b_t - L_t^{cb}\right) - \varphi_t^{9}\left[\chi_t^{\prime\prime}\left(b_t - L_t^{cb}\right) + \Xi_t^{p\prime\prime}\left(b_t - L_t^{cb}\right)\right], \end{aligned} \tag{2.13}$$

and with the current functional forms,

$$\begin{aligned} 0 &= -\varphi_t^{5}\left(1+\pi_b\omega_t\right) + \delta\beta E_t\left[\varphi_{t+1}^{5}\left(1+\omega_t\right)\frac{1+i_t^d}{\Pi_{t+1}}\right] \\ &\quad -\varphi_t^{6}\left[\eta\tilde{\Xi}_t\left(b_t - L_t^{cb}\right)^{\eta-1} + \tilde{\Xi}_t^{+}\right] \\ &\quad -\varphi_t^{9}\left[\varkappa\left(1+\varkappa\right)\tilde{\chi}_t\left(b_t - L_t^{cb}\right)^{\varkappa-1} + \eta\left(\eta-1\right)\tilde{\Xi}_t\left(b_t - L_t^{cb}\right)^{\eta-2}\right]. \end{aligned} \tag{2.14}$$

FOC w.r.t. Ξ_t

$$\varphi_t^{6} = \varphi_t^{10}. \tag{2.15}$$

FOC w.r.t. Π_t

$$\begin{aligned} 0 \;=\;& -\beta^{-1}\varphi^1_{t-1}\left(1+i^d_{t-1}\right)\left(1+\omega_{t-1}\right)\beta\frac{\left[\left[\delta+\left(1-\delta\right)\pi_b\right]\lambda^b_t+\left(1-\delta\right)\left(1-\pi_b\right)\lambda^s_t\right]}{\Pi_t} \\ & -\beta^{-1}\varphi^2_{t-1}\left(1+i^d_{t-1}\right)\beta\frac{\left[\left(1-\delta\right)\pi_b\lambda^b_t+\left[\delta+\left(1-\delta\right)\left(1-\pi_b\right)\right]\lambda^s_t\right]}{\Pi_t} \\ & +\varphi^3_{t-1}\alpha\theta\left(1+\omega_y\right)\Pi_t^{\theta(1+\omega_y)}K_t+\varphi^4_{t-1}\alpha\left(\theta-1\right)\Pi_t^{\theta-1}F_t \\ & -\varphi^5_t\delta\left[b_{t-1}\left(1+\omega_{t-1}\right)+\pi_b b^g_{t-1}\right]\frac{1+i^d_{t-1}}{\Pi_t} \\ & +\varphi^7_t\alpha\theta\left(1+\omega_y\right)\left[\alpha\Delta_{t-1}\Pi_t^{\theta(1+\omega_y)}-\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(1+\omega_y)}{\theta-1}-1}\Pi_t^{\theta-1}\right] \\ & -\varphi^8_t\frac{\alpha}{1-\alpha}\left(\theta-1\right)\Pi_t^{\theta-1}. \end{aligned} \tag{2.16}$$

FOC w.r.t. K_t

$$0=-\varphi^3_t+\varphi^3_{t-1}\alpha\Pi_t^{\theta(1+\omega_y)}+\varphi^8_t\frac{\theta-1}{1+\omega_y\theta}\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}K_t^{-1}. \tag{2.17}$$

FOC w.r.t. F_t

$$0=-\varphi^4_t+\varphi^4_{t-1}\alpha\Pi_t^{\theta-1}-\varphi^8_t\frac{\theta-1}{1+\omega_y\theta}\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}F_t^{-1}. \tag{2.18}$$

FOC w.r.t. Δ_t

$$\begin{aligned} 0 \;=\;& -\frac{\psi}{1+\nu}\left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}}\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}-\varphi^7_t+\alpha\beta E_t\left[\Pi_{t+1}^{\theta(1+\omega_y)}\right] \\ & -\varphi^5_t\pi_b\left(1-\pi_b\right)\left[\left(\frac{\psi}{\psi_b}\frac{\lambda^b_t}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}-\left(\frac{\psi}{\psi_s}\frac{\lambda^s_t}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\right]\tilde{\lambda}_t^{-1}\psi\mu^w_t\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}. \end{aligned} \tag{2.19}$$

FOC w.r.t. Y_t

$$\begin{aligned}
0 &= -\left(1+\omega_y\right)\frac{\psi}{1+\nu}\left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}}\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}\Delta_t+\varphi_t^4\Lambda_t\left(1-\tau_t\right)Y_t+\varphi_t^6 Y_t \\
&\quad+\varphi_t^3\left(1+\omega_y\right)\Lambda_t\mu^p\left(1+\omega_y\right)\psi\mu_t^w\tilde{\lambda}\left(\lambda_t^b,\lambda_t^s\right)^{-1}\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y} \\
&\quad-\varphi_t^5\left(1+\omega_y\right)\pi_b\left(1-\pi_b\right)\left[\left(\frac{\psi}{\psi_b}\frac{\lambda_t^b}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}-\left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\right]\tilde{\lambda}_t^{-1}\psi\mu_t^w\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}\Delta_t.
\end{aligned} \tag{2.20}$$

FOC w.r.t. λ_t^b

$$\begin{aligned}
0 &= -\sigma_b\pi_b\bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b} \\
&\quad+\frac{\psi}{\nu}\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}\Delta_t\left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}}\pi_b\left(\frac{\psi}{\psi_b}\frac{\lambda_t^b}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\left[\left(\lambda_t^b\right)^{-1}-\tilde{\lambda}_t^{\frac{1}{\nu}}\right] \\
&\quad-\varphi_t^1+\varphi_{t-1}^1\left[\delta+\left(1-\delta\right)\pi_b\right]\frac{1+i_{t-1}^d}{\Pi_t}\left(1+\omega_{t-1}\right)+\varphi_{t-1}^2\left(1-\delta\right)\pi_b\frac{1+i_{t-1}^d}{\Pi_t} \\
&\quad+\varphi_t^3\pi_b\mu^p\left(1+\omega_y\right)\psi\mu_t^w\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}\tilde{\lambda}_t^{-1}\left[1-\Lambda_t\left(\frac{\psi}{\psi_b}\frac{\lambda_t^b}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\left(\lambda_t^b\right)^{-1}\right] \\
&\quad+\varphi_t^4\pi_b\left(1-\tau_t\right)Y_t-\varphi_t^5\sigma_b\pi_b\left(1-\pi_b\right)\bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b-1} \\
&\quad-\varphi_t^5\frac{1}{\nu}\pi_b\left(1-\pi_b\right)\psi\mu_t^w\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}\Delta_t\times \\
&\quad\times\left(\frac{\psi}{\psi_b}\frac{\lambda_t^b}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\tilde{\lambda}_t^{-1}\left(\lambda_t^b\right)^{-1}\left[\left(1+\nu\right)\left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}-\nu\right] \\
&\quad+\varphi_t^6\sigma_b\pi_b\bar{C}_t^b\left(\lambda_t^b\right)^{-\sigma_b-1}.
\end{aligned} \tag{2.21}$$

FOC w.r.t. λ_t^s

$$
\begin{aligned}
0 \;=\; & -\sigma_s \left(1-\pi_b\right) \bar{C}_t^s \left(\lambda_t^s\right)^{-\sigma_s} \\
& +\frac{\psi}{\nu}\left(\frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t}\right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y} \Delta_t \left(1-\pi_b\right)\left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\left[\left(\lambda_t^s\right)^{-1}-\tilde{\lambda}_t^{\frac{1}{\nu}}\right] \\
& +\varphi_{t-1}^1\left(1-\delta\right)\left(1-\pi_b\right)\frac{1+i_{t-1}^d}{\Pi_t}\left(1+\omega_{t-1}\right) \\
& -\varphi_t^2+\varphi_{t-1}^2\left[\delta+\left(1-\delta\right)\left(1-\pi_b\right)\right]\frac{1+i_{t-1}^d}{\Pi_t} \\
& +\varphi_t^3\left(1-\pi_b\right)\mu^p\left(1+\omega_y\right)\psi\mu_t^w\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}\tilde{\lambda}_t^{-1}\left[1-\Lambda_t\left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\left(\lambda_t^s\right)^{-1}\right] \\
& +\varphi_t^4\left(1-\pi_b\right)\left(1-\tau_t\right)Y_t+\varphi_t^5\pi_b\left(1-\pi_b\right)\sigma_s\bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s-1} \\
& +\varphi_t^5\frac{1}{\nu}\pi_b\left(1-\pi_b\right)\psi\mu_t^w\bar{H}_t^{-\nu}\left(\frac{Y_t}{A_t}\right)^{1+\omega_y}\Delta_t\times \\
& \times\frac{1}{\pi_b}\left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\left(\lambda_t^s\right)^{-1}\tilde{\lambda}_t^{-1}\left[1+\nu\left(1-\pi_b\right)-\left(1+\nu\right)\left(1-\pi_b\right)\left(\frac{\psi}{\psi_s}\frac{\lambda_t^s}{\tilde{\lambda}_t}\right)^{\frac{1}{\nu}}\right] \\
& +\varphi_t^6\sigma_s\left(1-\pi_b\right)\bar{C}_t^s\left(\lambda_t^s\right)^{-\sigma_s-1}.
\end{aligned}
\tag{2.22}
$$

In the text we mention the FOC regarding credit policy in terms of two lagrange multipliers, $\varphi_{\omega,t}$ and $\varphi_{\Xi,t}$,not shown above. These correspond to the lagrange multipliers to the the equations determining the spread (1.15) and the total intermediation resource cost (1.16), so that these are mapped to the above expressions through

$$
\varphi_{\omega,t} = \varphi_t^9, \tag{2.23}
$$

$$
\varphi_{\Xi,t} = \varphi_t^{10}, \tag{2.24}
$$

so that (2.8) is equivalent to

$$
\varphi_{\Xi,t}[\bar{\Xi}_t^{p\prime}(b_t-L_t^{cb})-\Xi^{cb\prime}(L_t^{cb})]+\varphi_{\omega,t}[\bar{\Xi}_t^{p\prime\prime}(b_t-L_t^{cb})+\chi_t^{\prime\prime}(b_t-L_t^{cb})]=-\zeta_t. \tag{2.25}
$$

Because $\zeta_t \geq 0$, then we can write

$$
\varphi_{\Xi,t}[\bar{\Xi}_t^{p\prime}(b_t-L_t^{cb})-\Xi^{cb\prime}(L_t^{cb})]+\varphi_{\omega,t}[\bar{\Xi}_t^{p\prime\prime}(b_t-L_t^{cb})+\chi_t^{\prime\prime}(b_t-L_t^{cb})]\leq 0, \tag{2.26}
$$

as shown in the text.

3 Optimal Steady State

We show that as long as the marginal cost of central-bank lending is large enough (condition (34) in the text), the optimal steady state (i.e., steady-state solution to the above FOCs) involves zero inflation, zero central-bank credit, and a policy rate above zero. We show this by conjecturing a solution of this form, and showing that it is possible to solve for steady-state values of all variables (including the Lagrange multipliers) under this assumption, as long as the marginal cost of central-bank lending is large enough to satisfy condition (34).

We set, without loss of generality,

$$\bar{Y} = 1, \tag{3.1}$$

$$\psi = 1. \tag{3.2}$$

We further calibrate $\bar{b}/\bar{Y}$, $\bar{b}^g/\bar{Y}$ and the following ratios

$$s_c \equiv \pi_b s_b + \pi_s s_s, \tag{3.3}$$

$$\sigma_{bs} \equiv \sigma_b/\sigma_s, \tag{3.4}$$

$$\bar{\sigma} \equiv \pi_b s_b \sigma_b + \pi_s s_s \sigma_s, \tag{3.5}$$

$$\bar{\tilde{\chi}} = 0, \tag{3.6}$$

$$\bar{\tilde{\chi}}^+ = 0, \tag{3.7}$$

$$\bar{\tilde{\Xi}}^+ = 0. \tag{3.8}$$

Finally, we calibrate the ratio ψ_b/ψ_s to imply equal labor supply in steady state, which implies that

$$\frac{\psi_b}{\psi_s} = \frac{\lambda_b}{\lambda_s}. \tag{3.9}$$

Further consider the following definitions

$$s_g \equiv \bar{G}/\bar{Y}, \tag{3.10}$$

$$s_b \equiv \bar{c}^b/\bar{Y}, \tag{3.11}$$

$$s_s \equiv \bar{c}^s/\bar{Y}, \tag{3.12}$$

$$s_{\Xi^p} \equiv \Xi^p\left(\bar{b}\right)/\bar{Y}. \tag{3.13}$$

For the interest rate we have:

$$1+\bar{r}^d = \frac{(\delta+1)+\bar{\omega}\left[\delta+(1-\delta)\pi_b\right]-\sqrt{\left\{(\delta+1)+\bar{\omega}\left[\delta+(1-\delta)\pi_b\right]\right\}^2-4\delta(1+\bar{\omega})}}{2\delta(1+\bar{\omega})\beta}. \tag{3.14}$$

(Note that if $\bar{\omega}=0$, this reduces to $1+\bar{r}^d=\beta^{-1}$.) We use this steady-state relation to calibrate β, given assumed values for δ, π_b, $\bar{\omega}$ and $\bar{r}^d$.

We can also write

$$1+\bar{\imath}^d = 1+\bar{r}^d. \tag{3.15}$$

The steady state inflation determines the steady state price dispersion:

$$\bar{\Delta} = 1. \tag{3.16}$$

We assume that the steady state spread is due solely to intermediation costs of the convex type, hence

$$\bar{\Xi}^p = \frac{\bar{\omega}}{\eta\bar{b}^{\eta-1}}, \tag{3.17}$$

and the ratio of intermediation costs to output is

$$s_{\Xi^p} = \frac{\bar{\omega}}{\eta}\frac{\bar{b}}{\bar{Y}}. \tag{3.18}$$

The FOC w.r.t. L_t^{cb} implies

$$\bar{\zeta} = \bar{\varphi}^9(\eta-1)\bar{b}^{-1}\bar{\omega}+\bar{\varphi}^6\bar{\omega}-\bar{\varphi}^6\Xi^{cb\prime}(0). \tag{3.19}$$

Furthermore we can write, from one of the Euler equations:

$$\bar{\lambda}^b = \bar{\Omega}\bar{\lambda}^s, \tag{3.20}$$

where

$$\bar{\Omega} \equiv \frac{1-\left(1+\bar{r}^d\right)\beta\left[\delta+(1-\delta)(1-\pi_b)\right]}{(1+\bar{r}^d)\beta(1-\delta)\pi_b}. \tag{3.21}$$

This also implies, from (3.9), that

$$\frac{\psi_b}{\psi_s} = \bar{\Omega}, \tag{3.22}$$

and from the definition of ψ in (1.21), together with the assumption that it is set to 1, we get

$$\begin{aligned}\psi_s &= \left[\pi_b \bar{\Omega}^{-\frac{1}{\nu}} + \pi_s\right]^{\nu}, &(3.23)\\ \psi_b &= \bar{\Omega}\psi_s. &(3.24)\end{aligned}$$

This implies that,

$$\begin{aligned}\Lambda\left(\bar{\lambda}^b, \bar{\lambda}^s\right) &= \left[\pi_b \bar{\Omega} + \pi_s\right]\bar{\lambda}^s, &(3.25)\\ \tilde{\lambda}\left(\bar{\lambda}^b, \bar{\lambda}^s\right) &= \left[\pi_b \bar{\Omega}^{-\frac{1}{\nu}} + \pi_s\right]^{-\nu}\bar{\lambda}^s, &(3.26)\\ \tilde{\Lambda}\left(\bar{\lambda}^b, \bar{\lambda}^s\right) &= \left(\frac{\pi_b \bar{\Omega} + \pi_s}{\pi_b \bar{\Omega}^{-\frac{1}{\nu}} + \pi_s}\right)^{\frac{\nu}{1+\nu}}\bar{\lambda}^s. &(3.27)\end{aligned}$$

Using the inflation equation, which implies $\bar{F} = \bar{K}$,

$$(1-\bar{\tau}) = \mu^p (1+\omega_y)\bar{\mu}^w \tilde{\lambda}\left(\bar{\lambda}^b, \bar{\lambda}^s\right)^{-1} \bar{H}^{-\nu}\frac{\bar{Y}^{\omega_y}}{\bar{A}^{1+\omega_y}},$$

hence

$$\bar{\lambda}^s = \frac{(1+\omega_y)\mu^p \bar{\mu}^w}{(1-\bar{\tau})\left[\pi_b \bar{\Omega}^{-\frac{1}{\nu}} + \pi_s\right]^{-\nu}}\bar{H}^{-\nu}\frac{\bar{Y}^{\omega_y}}{\bar{A}^{1+\omega_y}}, \tag{3.28}$$

and

$$\bar{K} = \bar{F} = \frac{\bar{\Lambda}(1-\bar{\tau})\bar{Y}}{1-\alpha\beta}. \tag{3.29}$$

The resources constraint implies

$$1 - s_c - s_g = s_{\Xi}. \tag{3.30}$$

which determines s_g given s_c and s_{Ξ}.

The debt equation is

$$\left[1 + \pi_b \bar{\omega} - \delta(1+\bar{\omega})\left(1+\bar{r}^d\right)\right]\frac{\bar{b}}{\bar{Y}} = \pi_b \pi_s \frac{B\left(\bar{\lambda}^b, \bar{\lambda}^s, \bar{Y}, \bar{\Delta}; 0\right)}{\bar{Y}} - \pi_b \frac{\bar{b}^g}{\bar{Y}}\left[1 - \delta\left(1+\bar{r}^d\right)\right],$$

with

$$\frac{B\left(\bar{\lambda}^b, \bar{\lambda}^s, \bar{Y}, 1; 0\right)}{\bar{Y}} = s_b - s_s,$$

which simplifies due to the assumption of equal labor supply in steady state. This implies that

$$\frac{\bar{b}}{\bar{Y}} = \frac{\pi_b (1-\pi_b)(s_b - s_s) - \pi_b \frac{\bar{b}^g}{\bar{Y}} \left[1 - \delta \left(1 + \bar{r}^d\right)\right]}{1 + \pi_b \bar{\omega} - \delta (1 + \bar{\omega})(1 + \bar{r}^d)}. \tag{3.31}$$

Given that we calibrate $\bar{b}/\bar{Y}$ we can use this equation to determine $s_b - s_s$,

$$s_b - s_s = \frac{\left[1 + \pi_b \bar{\omega} - \delta (1 + \bar{\omega}) \left(1 + \bar{r}^d\right)\right] \frac{\bar{b}}{\bar{Y}} + \pi_b \frac{\bar{b}^g}{\bar{Y}} \left[1 - \delta \left(1 + \bar{r}^d\right)\right]}{\pi_b \pi_s} \tag{3.32}$$

Given our calibration of s_c we can then write

$$s_s = s_c - \pi_b (s_b - s_s), \tag{3.33}$$

and

$$s_b = s_c + \pi_s (s_b - s_s). \tag{3.34}$$

Finally, the steady state levels of the shocks to the marginal utility of consumption of the two types that satisfy the calibration are given by

$$\bar{\bar{C}}^b = s_b \left(\bar{\lambda}^b\right)^{\sigma_b}, \tag{3.35}$$

$$\bar{\bar{C}}^s = s_s \left(\bar{\lambda}^s\right)^{\sigma_s}. \tag{3.36}$$

4 Numerical Calibration

The numerical values for parameters used in our calculations are summarized in Table 1. They are essentially the same as those used in the numerical analysis in Cúrdia and Woodford (2009), where they are discussed in greater detail. The one important difference is that here we calibrate the model so that the steady-state real return on deposits (identified with the real federal funds rates, for purposes of the empirical interpretation of the model) is 3.0 percent per annum. In Cúrdia and Woodford (2009), this rate is calibrated to be 4 percent per annum (1 percent per quarter), for the sake of using a round number (when expressed as a quarterly rate). But because we are interested in the consequences of the zero lower bound on the policy rate in some of the calculations reported below, the question of how far the steady-state policy rate is from the lower bound is of non-trivial import. Hence we have here adopted a value for this target that is closer to the average historical level of the real federal funds rate. Several of the numbers in Table 1 are also slightly different from those reported in the appendix to Cúrdia and Woodford (2009), but all of these are consequences of the change in the calibration target for the steady-state real policy rate, given unchanged numerical values for the other calibration targets.

Table 1: Numerical parameter values used.

π_b	0.5	s_b	0.798	$(\theta-1)^{-1}$	0.15	$\bar{\tau}$	0.2
δ	0.975	s_s	0.602	ϕ^{-1}	0.75	$\bar{\Xi}/\bar{Y}$	0.0003
β	0.990	σ_b	13.6	α	0.66	$\bar{\chi}/\bar{L}$	0
ν	0.105	σ_s	2.72	$\bar{b}^g/\bar{Y}$	0	$1+\bar{\omega}$	$(1.02)^{1/4}$
$\bar{h}^b/\bar{h}^s$	1	$\bar{\lambda}^b/\bar{\lambda}^s$	1.22	$\bar{b}/\bar{Y}$	3.2	η	51.6

Many of the model's parameters are also parameters of the basic New Keynesian model, and in the case of these parameters we assume similar numerical values as in the numerical analysis of that model in Woodford (2003) (Table 5.1.), which in turn are based on the empirical model of Rotemberg and Woodford (1997). Specifically, the values assumed for ν, θ, and ϕ in Table 1 are the same as in Rotemberg and Woodford, and the average of the elasticities σ_τ is chosen so as to imply the same interest-elasticity of aggregate expenditure as in Rotemberg and Woodford.[2]The value assumed for α, the fraction of goods prices that remain unchanged from one quarter to the next, is also the same as in Rotemberg and Woodford. The value assumed for β is also the same (to three decimal places) as the one used by Rotemberg and Woodford, though for a different reason: the value reported in the table is required by our calibration target for the steady-state real policy rate, discussed above.[3]

The new parameters that are needed for the present model are those relating to heterogeneity or to the specification of the credit frictions. The parameters relating to heterogeneity are the fraction π_b of households that are borrowers, the degree of persistence δ of a household's "type", the steady-state expenditure level of borrowers relative to savers, and the interest-elasticity of expenditure of borrowers relative to that of savers, σ_b/σ_s.[4]

[2]To be precise, these are chosen so that the coefficient $\bar{\sigma}$ defined in the Appendix has the same value as the coefficient denoted σ in Table 5.1 of Woodford (2003).

[3]Our calibration target for the steady-state real policy rate is slightly lower than the one assumed by Rotemberg and Woodford, but the credit frictions in our model require a rate of time preference that is slightly higher than the steady-state real policy rate, unlike the model of Rotemberg and Woodford; and the consequences of these two changes for the assumed value of β essentially cancel one another.

[4]Another new parameter that matters as a consequence of heterogeneity is the steady-state level of government debt relative to GDP, $\bar{b}^g/\bar{Y}$; here we assume that $\bar{b}^g = 0$.

In the calculations reported here, we assume that $\pi_b = \pi_s = 0.5$, so that there are an equal number of borrowers and savers. We assume that $\delta = 0.975$, so that the expected time until a household has access to the insurance agency (and its type is drawn again) is 10 years. This means that the expected path of the spread between lending and deposit rates for 10 years or so into the future affects current spending decisions, but that expectations regarding the spread several decades in the future are nearly irrelevant.

We calibrate the model so that private expenditure is 0.7 of total output in steady state, and furthermore calibrate the degree of heterogeneity in the steady-state expenditure of the two types so that the implied steady-state debt $\bar{b}$ is equal to 80 percent of annual steady-state output.[5] This value matches the median ratio of private (non-financial, non-government, non-mortgage) debt to GDP over the period 1986-2008.[6] This requires the values of s_b and s_s shown in the table, where $s_\tau \equiv \bar{c}^\tau/\bar{Y}$ is the steady-state expenditure share for each type τ (using bars to denote the steady-state values of variables).

We assume an average intertemporal elasticity of substitution for the two types that is the same as that of the representative household in the model of Rotemberg and Woodford (1997), as mentioned above, and determine the individual values of σ_τ for the two types on the assumption that σ_b/σ_s is equal to 5. This is an arbitrary choice, though the fact that borrowers are assumed to have a greater willingness to substitute intertemporally is important, as this results in the prediction that an exogenous tightening of monetary policy (a positive intercept shift of a Taylor-type reaction function) results in a reduction in the equilibrium volume of credit b_t (see Cúrdia and Woodford (2009)). This is consistent with the VAR evidence on the effects of an identified monetary policy shock presented in Lown and Morgan (2002).[7]

It is also necessary to specify the unperturbed values of the functions $\omega(b)$ and $\Xi(b)$ that describe the financial frictions, in addition to making clear what kinds of random perturbations of these functions we wish to consider when analyzing the effects of "financial shocks." In the absence of shocks, we assume that $\chi_t(L) = 0$, so that all loans are expected to be repaid, and the credit spread is due purely to the resource costs of intermediation. We assume

[5] In our quarterly model, this means that $\bar{b}/\bar{Y} = 3.2$.

[6] We exclude mortgage debt when calibrating the degree of heterogeneity of preferences in our model, since mortgage debt is incurred in order to acquire an asset, rather than to consume current produced goods in excess of current income.

[7] It is also consistent with the evidence in Den Haan, Summer, and Yamashiro (2004) for the effects of a monetary shock on consumer credit, though commercial and industrial loans are shown to rise. The result for C&I loans may reflect substitution of firms toward bank credit owing to decreased availability of other sources of credit, rather than an actual increase in borrowing; see Bernanke and Gertler (1995) on this point.

an intermediation technology such that when intermediaries hold reserves at or above the satiation level (as occurs in equilibrium under an optimal reserve-supply policy, assumed in all of the numerical exercises reported in this section),

$$\bar{\Xi}^p(L) = \tilde{\Xi}^p L^\eta \tag{4.1}$$

in the absence of shocks. We assume that $\eta > 1$, so that the marginal cost of intermediation is increasing; this corresponds to the idea of a finite lending capacity at a given point in time, due to scarce factors such as intermediary capital and expertise that are here treated as exogenous.[8]

In the numerical results reported here, we assume more specifically a value of η that implies that a one percent increase in private lending implies an increase in the marginal cost of intermediation, and hence in the equilibrium credit spread, of one percentage point (per annum). This implies fairly inelastic credit supply by the private sector, but we believe that this is the case of greatest interest for the exercises here, both because private credit supply is often asserted to be quite inelastic during financial crises, and because this is in any event the assumption most favorable to a potential role for central-bank credit policy, as in this case a substitution of central-bank lending for private lending to even a modest degree can lower the marginal cost of private lending and hence the equilibrium credit spread.[9] We are interested in biasing our results in this direction, not to pre-judge the desirability of central-bank lending, but because our results show in any event that the justification for active credit policy is often fairly modest, even in the case of financial disturbances that increase credit spreads by a significant amount. It is most interesting to observe that this is true even under the assumption of quite a large value for η; in the case of a more modest value of η, then both the shadow value of allowing active credit policy (relaxing the constraint that $L_t^{cb} = 0$) and the optimal scale of central-bank lending in response to shocks will in all cases be substantially smaller than the values reported below.

Finally, we specify the unperturbed value of $\tilde{\Xi}^p$ so that the steady-state credit spread $\bar{\omega}$

[8] The assumption that $\eta > 1$ also allows our model to match the prediction of VAR estimates that an unexpected tightening of monetary policy is associated with a slight reduction in credit spreads (see, *e.g.*, Lown and Morgan (2002), and Gerali, Neri, Sessa, and Signoretti (2008). See Cúrdia and Woodford (2009) for comparison with a model with a linear resource cost function.

[9] In the case that the private intermediary sector has a constant marginal cost, rather than one increasing with the volume of private lending, then central-bank lending will reduce the equilibrium credit spread only to the extent that it *completely* replaces private lending, by lending at a rate that is too low to be profitable for private intermediaries at any positive scale of operation.

equal to 2.0 percentage points per annum, following Mehra, Piguillem, and Prescott (2008). Combined with our assumption that "types" persist for 10 years on average, this implies a steady-state "marginal-utility gap" $\bar{\Omega} \equiv \bar{\lambda}^b/\bar{\lambda}^s = 1.22$, so that there would be a non-trivial welfare gain from transferring further resources from savers to borrowers. Because of the degree of convexity assumed for the intermediation technology, this corresponds to a steady-state resource cost of financial intermediation $\bar{\Xi}$ that is much smaller than 2 percent per year of the steady-state level of private lending, as shown in Table 1;[10] hence in our parameterization, the credit spread represents mainly rents earned by private intermediaries, owing to the scarcity of whatever factor allows only particular firms to engage in this activity.

[10]The value for $\bar{\Xi}$ reported in the table represents the steady-state value of Ξ_t^p, as we assume no central-bank lending to the private sector in the steady state: $\bar{L}^{cb} = 0$.

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