Maintaining Central-Bank Solvency under New-Style Central Banking *

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Abstract

Since 2008, the central banks of advanced countries have borrowed trillions of dollars from their commercial banks in the form of reserves and invested the proceeds in portfolios of risky assets. They are paying interest on those reserves. We investigate how this new style of central banking affects central banks' solvency. We find that a central bank that pays dividends equal to net income will always be solvent, but if the central bank is not recapitalized when net income is negative, or other precautions are put in place, then reserves will tend to grow more in crises than they shrink in normal times. We compute measures of the financial strength for the Federal Reserve and the European Central Bank and find that the main risk to their solvency comes in the recovery from the crisis, when they suffer capital losses on their large bond portfolios. For both central banks, we conclude that the risks to their solvency as of 2013 are remote.

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The Bank of Japan in the 1990s, followed by the Federal Reserve, the European Central Bank, and the Bank of England in the 2000s, made dramatic changes in the conduct of monetary policy in response to financial crises. They borrowed trillions of dollars from commercial banks by expanding reserves, using the proceeds to enlarge their holdings of government bonds and other forms of debt. At the same time, those central banks paid interest on reserves and used the rate on reserves as a policy instrument to stabilize inflation, or plan to do so once their economies return to normal. Critics of the new style of central-bank policy worry that, once market interest rates begin to rise as the crisis recedes, these central banks will come under stress. Might the double burden of capital losses on central banks' portfolios and the rising cost of paying market interest on trillions of dollars of reserves threaten the banks' solvency?

Previous authors have studied central bank solvency—see Milton and Sinclair (2010), for a survey—but none have answered this question directly. Leone (1994), Ize and Oulidi (2009), and Dalton and Dziobek (2005) focus on central bank losses that result from changes in the domestic value of their foreign reserves, or from ill-advised direct domestic loans, neither of which are the relevant issues for advanced central banks today. Stella (1997), Stella (2007), and Klüh and Stella (2002) investigate whether measures of central bank accounting capital constrain monetary policy actions. Rudebusch (2011) provides an informal treatment of the current risks facing the Federal Reserve and the ECB. Carpenter, Ihrig, Klee, Quinn and Boote (2013) and Greenlaw, Hamilton, Hooper and Mishkin (2013) present statistical projections for components of the balance sheets of the Federal Reserve and the ECB.

In this paper, we develop a formal model of central-bank risk, founded in modern finance theory, to describe the evolution of a central bank's financial position. We focus on the path of reserves, because central banks are unique among financial institutions in their power to compel banks to make loans to the central bank in the form of reserves. We derive measures of solvency from the model. We compare alternative measures of solvency risk and their implications for the Federal Reserve and the ECB in the aftermath of the financial crisis. While most work in this literature focuses on the peculiar accounting rules of central banks and on the items in their balance sheet, we focus instead on the economics behind the resource constraint that the central bank faces, following Reis (2013).

Our approach starts from the observation that a policy of maintaining large volumes of reserves and paying interest on them is fully consistent with effective price stabilization.

Section 1 reviews the literature that has established that the interest rate paid on reserves is an effective tool for monetary policy. Section 2 develops measures of central-bank solvency risk. By solvency, we mean the ability of a central bank to obey the dividend rule laid out in its statutes. We present a dynamic model of the evolution of reserves needed to satisfy the bank's resource constraint, given the rule prescribing dividends paid to the government. A central bank with a large bond portfolio faces a risk of losses, either from defaults on the bonds or from declines in their market values when interest rates rise. A negative dividend is, in effect, a recapitalization of the central bank. If the central bank always pays dividends equal to net income, including when income is negative, reserves are stationary. But, if the dividend rule rules out negative dividends, the central bank may resort to an inflationary policy—issuing reserves without paying high enough interest to maintain stable prices—to satisfy its resource constraint. We measure the excessive reserves that result as well as the potential expected increase in inflation and propose these as metrics of the financial strength of a central bank.

We apply these measures in section 3 to appraise the current financial strength of the Federal Reserve. Our measures confirm that the Fed faces two risks to its solvency. First, the huge expansion of the Fed's portfolio and the inclusion of some bonds with default risk exposes the Fed to capital losses when interest rates rise. Second, the new obligation to pay rising interest rates as the economy returns to normal will expand the Fed's reserves. In spite of these two dangers, we find that in almost all conditions the Fed would be able not only to pay a reserve interest rate high enough to keep inflation on target but also to continue paying a substantial positive dividend to the Treasury. The exception is early in the recovery, when capital losses on the bond portfolio cannot be covered by immediate earnings, but could easily be covered by later earnings.

We perform similar calculations for the ECB in section 4 and find that it faces the same dangers as the Federal Reserve. An important consideration for the ECB is what would happen under its repurchase agreements with banks in times of financial stress. The repos are term debt obligations of banks collateralized by government debt and some other types of securities. The ECB suffers capital losses only if the banks fail to repay the debt *and* the collateral loses value by more than the repo haircut. Only in the case of a comprehensive financial meltdown would the ECB's solvency be compromised.

We conclude that an economic recovery that brings back normal interest rates is a minor threat to both the Fed and even less to the ECB, easily managed by a temporary buildup of reserves. The only serious threat is a loss of substantial value of the government securities and other asset holdings of the central banks, which could only occur as part of a collapse of the U.S. government in the case of the Fed or euro-area governments and euro-area banks in the case of the ECB.

1 Interest on Reserves and Inflation in Central-Bank Policy

A central bank issues reserves, which are one-period debt claims on the central bank held by commercial banks. We assume that the central bank does not default on reserves. Moreover, reserves are the economy's unit of value. One unit of output costs p units of reserves; p is the price level. The real value of a unit of reserves is $1/p$.

1.1 Using the interest rate on reserves to control inflation

The central bank has complete control over the interest rate it pays banks for holding reserves, given that the banking system has no choice but to hold the volume of reserves the central bank chooses. A central bank can use either the quantity of reserves or their interest rate as the instrument of monetary policy. The former was the worldwide rule until recently. Section 3 of chapter 1 of Woodford (2003), titled "Monetary Policy without Control of a Monetary Aggregate" states a modern consensus that a rule on the interest rate in reserves is effective at controlling the price level, and many central banks are shifting toward the reserve-rate instrument. The intuition seems clear: By making reserves more attractive, the central bank has the same contractionary effect on the economy as cutting the quantity of reserves. An increase in demand is equivalent to a cut in supply. The price of reserves—in the sense of the holding cost—is the nominal interest rate less the rate that the bank pays on reserves. The quantity and the price are substitutes as policy instruments. To head off incipient inflation, the central bank can cut the quantity of reserves or raise the rate it pays on reserves. Thus the principles of modern inflation economics apply just as well in an economy where the reserve rate is the instrument of monetary policy as one where the quantity of reserves plays that role, as emphasized in Hall (1997).

A powerful argument favors making the reserve rate the primary policy instrument. The central bank can achieve the Friedman optimum with respect to reserves by choosing a large quantity, thereby reducing the implicit tax on reserves to a low level, and then paying close to the market rate on reserves. In an economy saturated with reserves, control of the reserve rate controls all safe short-term nominal interest rates in the same currency. Reserves and, for example, Treasury bills, are close substitutes. They must command essentially equal returns in an economy saturated with reserves. Over the period when the Fed has been paying interest on reserves, the rate has generally been within 10 basis points of the both the interbank rate and the market rate on short-term Treasury bills. The central bank can anchor the inflation rate by setting the short-term nominal rate according to a suitable Taylor rule, as laid out in Woodford (2003).

1.2 Financial environment

We use the scalar s to denote the state of the economy, which obeys a Markov process in the set $\{1, \cdots, S\}$, and has a transition probability $\omega_{s,s'}$, where a prime denotes next period's value. We use the terminology in the rest of the paper that a variable is *state-dependent* if it is a function of s and possibly s' as well. The standard assumption is that policy sets the nominal interest rate paid on holders of reserves between this period and the next, using a state-dependent nominal rate i_s.

With reserve saturation, the rate on reserves is essentially the same as the market nominal rate on safe short-term debt. We let $m_{s,s'}$ denote the stochastic discount factor for a real payoff. Then, the real present value of a random real payoff one period in the future, $y_{s'}$, is

$$\mathbb{V}(y_{s'}) = \sum_{s,s'} \omega_{s,s'} m_{s,s'} y_{s'}. \tag{1}$$

Therefore, the safe market nominal rate is equal to:

$$\frac{1}{1+i_s} = \mathbb{V}\left(\frac{p}{p'}\right). \tag{2}$$

We also assume that the result of the policy is a state-dependent inflation rate:

$$\pi_s = \frac{p'}{p} - 1. \tag{3}$$

We do not deal here with the important issue of how inflation is kept on this target. We cannot do justice here to the modern literature on price-level determination except to say that its canon is Woodford (2003). We note that equation (3) assumes that the central bank commits in the preceding period to achieve a future price p'. As a result, the safe real interest rate r_s is:

$$1 + r_s = \frac{1}{\mathbb{V}(1)} = (1 + \pi_s)(1 + i_s). \tag{4}$$

This assumption makes the real interest rate not depend on the subsequent state s', which is a minor convenience in the analysis. None of our results depend on this assumption in a significant way.

Finally, no theory of inflation is complete without a statement of how fiscal policy interacts with monetary policy. To focus on central-bank solvency, we must take a stand on the solvency of the overall government. We portray the central bank as an arm of a government that is committed to a fiscal policy that satisfies the intertemporal budget constraint by adjusting taxes and spending under all possible realizations of random events, without relying on inflationary finance from the central bank. See Hall (2013) for evidence that the U.S. satisfied that hypothesis prior to the financial crisis starting in 2008.

2 The Financial Stability of a Central Bank

The traditional measure of a corporation's financial standing is the accounting measure variously termed capital, equity, or net worth. There are typically three reasons to measure it: to calculate the residual winding up value of the corporation, to assess its market value, and to ascertain the weight of equity versus credit in the firm's funding.

None of these reasons applies to a central bank. Central banks cannot be liquidated because their creditors cannot demand to convert their credit into anything but what they already hold, currency and reserves. Therefore, there is no meaning to the residual value of a central bank because the central bank cannot be wound up. Central banks also do not have a meaningful market value, since their goal is not profits, and shares in the bank are typically not traded. Finally, governments own the central bank, they often deposit funds with the central bank, and most of the assets of the central bank are government liabilities. The traditional distinction between equity-holders and credit-holders is blurry and confusing for a central bank.

Nonetheless, central banks may run into financial difficulties. So far, actual insolvency of the central bank has occurred in some countries but never in an advanced county. Klüh and Stella (2002) discuss many of these insolvencies of the past thirty years. The main focus of our analysis in this section is on the central bank's dividend rule, specifying the amount the bank owes the government. An important issue is what happens when the rule calls for negative dividends, that is, for the government to recapitalize the central bank. The bank may substitute expansion of reserves, but repeated episodes of covering negative dividends with further borrowing will result in upward drift of the volume of reserves.

2.1 The central bank's resource constraint

Central banks owe commercial banks an amount V in units of output—this amount is the real value of reserves. On the asset side, they typically hold debt instruments of different maturities. Before the 2008 financial crisis, the Federal Reserve mostly held Treasury bills and bonds, while the ECB invested in foreign bonds and on the reverse side of the repo market, holding claims on commercial banks collateralized by short-term government debt. With the crisis, both central banks borrowed extensively from commercial banks to increase their holdings of debt instruments of longer maturities and with higher default risks.

We lump all central-bank assets into a single category called bonds, and denote by $B_s \geq 0$ the total number of bonds held by the central bank. In the model, a bond is an instrument that pays a real coupon c_s next period, a fraction $1-\delta$ in its second period, $(1-\delta)^2$ in its third period, and so on. We add up bonds in terms of the amount of output they will pay in the current period. Each period, the bonds inherited from the previous period shrink by the factor $1-\delta$. The bonds, which sell for price q_s, then give a payoff equal to the difference between the coupon payments and the repayment of the principal by a factor δ. This approach gives a convenient, flexible formulation to capture the maturity of a bond portfolio, as in Woodford (2001) and others.

The central bank's outstanding real reserves follow the law of motion:

$$V' = (1+r_s)V + q_{s'}[B_{s'} - (1-\delta)B_s] - c_s B_s - n_{ss'} + d_{s'}. \tag{5}$$

In a period when the economy is in state s' and was previously in state s, the central bank's new level of reserves covers the following: (1) the previous level of reserves times their real payment $(1+r_s)V$, (2) net bond purchases at real cost $q_{s'}[B_{s'} - (1-\delta)B_s]$, (3) receipts of bond interest this period on last period's holdings and promised coupon rate $c_s B_s$ units of output, (4) receipts of seignorage $n_{s,s'}$, and (5) payment of dividends to the government $d_{s'}$.

Equation (5) ignores the operating expenses of the central bank because these are usually small relative to its assets. The same applies to the revenue from direct loans to banks and to the mandatory statutory dividends that central banks are often committed to pay to the providers of their paid-in capital. We also ignore the use of resources for quasi-fiscal operations such as bailing out the banking system. These are common in developing countries—see Dalton and Dziobek (2005)—but not in advanced countries, which are the focus of our analysis. More significant is the omission of gold, special drawing rights, and foreign exchange reserves, which are substantial for most central banks, with the exception of the Federal Reserve. Considering the portfolio risk from exchange rate movements raises more issues than we can address in this paper, but it is a high priority for future research.

The resource constraint of the central bank holds regardless of its monetary policy. The simplest case is the helicopter-drop model of a central bank that issues no reserves ($V = V' = 0$) and holds no bonds ($B_s = B_{s'} = 0$). In this case, equation (5) boils down to:

$$d_{s'} = n_{s,s'}. \tag{6}$$

Every period, the central bank transfers its seignorage revenue to the Treasury, as routinely assumed in simple models of monetary policy.

With outstanding reserves that pay interest, and a portfolio of bonds that fluctuates in value, the resource constraint of the central bank becomes more interesting. We separately discuss the three elements affecting the evolution of reserves in equation (5): bonds, seignorage and dividends.

2.2 Bonds and their risks

Central bankers pride themselves on holding conservative portfolios and, as a result, the solvency of the central bank is often taken for granted. Yet, the asset holdings of the central bank pose at least three distinct risks, which are especially acute during financial crises. We use the highest-numbered state S to capture a financial crisis.

First, there is the risk that the bonds will not pay their full coupon, a rare but possible occurrence. We assume that the coupon on the bonds is one in all states apart from the crisis state S, when bonds suffer an impairment captured by $c_S \leq 1$. This possibility is real in Europe, where Greece recently restructured its debt, reducing coupon payments on the Greek debt held by the ECB.

The second risk comes with changes in asset prices. In our model, the price of the bonds q_s satisfies the standard no-arbitrage asset-pricing recursion:

$$q_s = \mathbb{V}(c_s + (1 - \delta)q_{s'}). \tag{7}$$

Changes in coupons as well as in real interest rates will cause q_s to change across states, especially as the economy enters and exits the crisis state.

Finally, during a crisis, the central bank's policy may require an expansion in its bond holdings. This will require an expansion in reserves to finance the move to a high value of B_S. During the recent financial crisis, both the Federal Reserve and the ECB altered their holdings of bonds to intervene in selected financial markets. Instead of trying to model all of these reasons, we will take the choices of B_s as given. We take an empirical approach, choosing B_s to mimic the data. Our only assumption, implicit in appending the s subscript, is that the choice of B_s is state-dependent.

2.3 Seignorage and inflation

Modern central banks keep the values of reserves and currency at exact parity by trading one for the other. Therefore, the volume of currency outstanding is not a policy variable. There is nothing to the idea that central banks can expand their portfolios by "printing money." Rather, they can issue reserves, so V is the direct policy variable, not the issuance of currency. Moreover, the volume of currency tends to be stable in advanced countries. Central banks expand their portfolios to high levels almost entirely by borrowing from commercial banks at market interest rates.

We let N_s be the public's real holdings of currency in state s. The central bank's real seignorage revenue from expanding the stock of outstanding currency is:

$$n_{s,s'} = \frac{p' N_{s'} - p N_s}{p'}. \tag{8}$$

Although a central bank does not control seignorage directly, by using its policy rule to keep prices on target, it affects the demand for currency and thus the amount of seignorage. Real seignorage increases with inflation until it reaches a maximum and then declines. Seignorage is a form of taxation on the real currency held by the public, where inflation acts as the implicit tax, and there is a point beyond which higher taxes reduce revenue. Cagan (1956) is the classic derivation of this function starting from the properties of the desire for real money balances. We only assume that the seignorage function is state-dependent, and measure the relation between a higher inflation target and extra seignorage empirically.

As we stated before, we assume that the central bank keeps inflation π_s on a state-dependent target. An alternative central bank policy would be a state-dependent price-level policy, so that we could write the price level as a function of the state of the economy, p_s. We do not pursue this approach because a state-dependent price level implies essentially zero inflation—every time the economy returns to a given state, the change in the price level since the first time it was in that state is exactly zero. No central bank has an inflation target of zero. The announced targets of central banks in advanced countries cluster around two percent per year. It is therefore more realistic to assume a state-dependent rate of inflation with all of the π_ss non-negative and presumably near two percent.

2.4 Net income and dividends

Central banks pay dividends d_s to their governments. If the central bank and the Treasury were perfectly integrated, the resource constraint in equation (5) would have little bite. The dividends d_s would just record the transfers between a particular branch and the overall government, but these would all be subsumed within the operations of the government as a whole. The individual accounts of most government agencies do not merit any special attention.

However, central banks are different from government agencies. In most advanced countries, they are a separately organized entity, analogous to a wholly owned subsidiary of a corporation, with independence to pursue its mandate. There are many definitions and measures of how independent is a central bank, but a minimal one is that it need not ask the government for resources—see Cukierman's (2008) survey. That is, a central bank is independent as long as its dividends do not deviate from the rule that was set out in the charters that gave it its independence. Its financial strength is its ability to stick to that rule. Unlike accounting measures, this definition is economically motivated and, as we will proceed to show, it carries implications for what the central bank can achieve while still being implementable.

Most central banks have dividend rules based on a well-defined concept of net income. A bank must pay all of its net income to the Treasury as a dividend. A modern central bank earns a return on its bond portfolio, pays interest on the reserves it issues, and receives an inflow of seignorage. If its net income is calculated the way an economist would, in real terms with its portfolio marked to market, the dividend would obey the *real mark-to-market dividend rule*:

$$d_{s'} = (c_s + q_{s'} - q_s - \delta q_{s'})B_s + n_{s,s'} - r_s V. \tag{9}$$

The dividend is the sum of (1) the return on the bond portfolio, the coupon payment c_s plus the capital gain $q_{s'} - q_s$ less the "depreciation" $\delta q_{s'}$, all applied to the number of bonds; (2) the seignorage revenue from issuance of currency, $n_{s,s'}$; and (3) the interest paid on reserves.

In this case, a striking result follows:

Proposition 1 *Under the real mark-to-market dividend rule in equation* (9)*, the law of motion of reserves is:*

$$V_s = q_s B_s + V_0 - q_0 B_0. \tag{10}$$

Here $V_0 - q_0 B_0$ is an initial condition. Reserves are a function V_s of the current state.

Proof: Substitute equation (9) for $d_{s'}$ in equation (5). The result simplifies to

$$V' - q_{s'} B_{s'} = V - q_s B_s. \tag{11}$$

Therefore, $V - q_s B_s$ is constant across periods. Letting the initial condition be $V_0 - q_0 B_0$ gives the result. □

During a crisis in state S, a central bank whose policy assigns a high value to B_S will expand its bond holdings funded with added reserves. When the crisis is over and the bond-holding rule B_s assigns a normal, smaller level of bond holdings, reserves will contract as the bank sells bonds and uses the proceeds to pay off its debt to commercial banks. As the proposition shows, reserves are state dependent and the central bank is always able to honor its debts whatever is the payment on reserves that it chooses. The dividend rule may, in exceptional conditions, call for negative dividends, that is, a government recapitalization of the bank.

The formula in equation (9) is not how a central bank's dividends are determined. Rather, dividends focus on net worth,

$$W = q_s B_s - V - N_s, \tag{12}$$

with currency N_s treated as a liability. Net income is defined as the change in nominal net worth that would occur if dividends were zero. Then, by definition, nominal net worth pW is constant under a rule that sets dividends equal to that concept of net income. This dividend rule also meshes well with the notion that central banks engage in open-market operations, where either reserves or currency—the monetary base—move one-to-one with the purchase or sale of bonds. Carpenter et al. (2013) describe the peculiarities of central bank accounting for the Federal Reserve and Vergote, Studener, Efthymiadis and Merriman (2010) for the ECB.

If the central bank pursued a state-dependent price-level policy, we could derive results parallel to the ones for the real mark-to-market dividend rule—real reserves would be state dependent and have a constant additive term interpreted as an initial condition. With the more realistic state-dependent inflation policy, where all of the π_ss are positive, there are important differences. Combining the nominal payment-on-reserves rule, with the law of motion for reserves in equation (5) and the condition that $p'W' = pW$ with equation (12) gives the *nominal mark-to-market dividend rule*:

$$d_{s'} = \left(c_s + q_{s'} - \frac{q_s}{1+\pi_s} - \delta q_{s'} \right) B_s - \frac{i_s V}{1+\pi_s}. \tag{13}$$

The first distinction from equation (9) is that net income is calculated in nominal terms, so that higher inflation raises measured capital gains $q_{s'} - q_s/(1+\pi_s)$. The second distinction is that central-bank accounting treats the growth of currency and resulting increase in bond holdings as exactly offsetting, because currency is treated as a liability, even though the present value of its future cash burden is zero. Given the accounting convention that the income statement is the first difference of the balance sheet, nothing makes its way to the income statement to reveal the income from seignorage.

The constancy of nominal net worth implies

Proposition 2 *Under the nominal mark-to-market dividend rule with $W_0 = 0$, reserves follow the state-dependent law of motion,*

$$V_s = q_s B_s - N_s. \tag{14}$$

If $W_0 > 0$, beyond this state-dependent component, there is a time-dependent component which dies away if inflation is always positive and is equal to a constant if inflation is always zero.

Proof: The constancy of nominal net worth implies that, in time-dependent notation:

$$q_t B_t - V_t - N_t = \frac{p_0}{p_t} W_0, \tag{15}$$

so

$$V_t = q_t B_t - N_t - \prod_{\tau=0}^{t-1} \left(\frac{1}{1+\pi_\tau} \right) W_0 \tag{16}$$

With $W_0 = 0$ we obtain equation (14), the state-dependent component. The last term on the right-hand side is: zero if $W_0 = 0$, depends on time and dies away if $W_0 > 0$ and $\pi_t > 0$, and is constant if $W_0 > 0$ and $\pi_t = 0$ for all t. $\square$

In general, from an arbitrary starting point of real net worth W_0, reserves are not state dependent. Rather, they are the sum of a state-dependent component $q_t B_t - N_t$ and a negative component associated with the initial level of real net worth that dies away with time. The dying away would be at a constant rate if the rate of inflation were constant across all states.

Under this dividend rule, reserves fluctuate not only because of changes in bond holdings but also as the public varies its demand for currency. As before, the central bank is able to pay its reserves at whatever level it desires, and reserves during a crisis, V_S, may be high but they are sustainable. Again, the rule may call for negative dividends.

2.5 When is net income negative?

Paying interest on reserves and holding long-term bonds opens the possibility of negative dividends. Substituting the law of motion for reserves, equation (24), into the real mark-to-market dividend rule in equation (9) describes dividends in terms of exogenous variables and the initial condition:

$$d_{s'} = n_{s,s'} - r_s(V_0 - q_0 B_0) + (c_s - \delta q_{s'} - r_s q_s)B_s + (q_{s'} - q_s)B_s. \tag{17}$$

The first component of dividends is $n_{s,s'}$, the central bank's real seignorage income. If the central bank does not pay interest on reserves and holds only short-term bonds, this component is the only non-zero one. As long as seignorage is positive, the central bank will pay a positive dividend to the Treasury.

The second component, $r_s(V_0 - q_0 B_0)$, is the payment on the real value of reserves in excess of the value of the bonds. Recall from proposition 1 that the amount in brackets is constant over time. In times when real interest rates rise, the central bank may find itself driven towards negative dividends if, as assumed here, it is committed to stabilizing inflation using the payment-on-reserves rule. The third component is the sum of (1) $(c_s - \delta q_{s'} - r_s q_s)B_s$, the difference between the coupon payment adjusted for bond depreciation less the real rate on reserves, and (2) $(q_{s'} - q_s)B_s$, the capital gain on the bond portfolio. If $\delta = 1$ this second term is exactly zero. But for a central bank that holds long-term bonds, net income is more likely to be negative when the bond repayment is impaired or when there is a capital loss in the bond portfolio. Therefore, the most likely sources of negative net income are defaults on bonds held by the central bank and the decline in the market values of bonds as the economy transits to a higher interest rate.

Using instead the nominal dividend rule in equation (13), we find, with the same substitutions:

$$d_{s'} = \frac{i_s}{1+\pi_s}(N_s + W_0) + \left[c_s + (1-\delta)q_{s'} - \left(\frac{1+i_s}{1+\pi_s}\right) q_s\right] B_s \tag{18}$$

The first term is the inflation tax on holders of currency, who pay it by holding banknotes instead of interest-bearing assets. The quantity within the square brackets is the difference between the actual and the expected payment on the bonds. It is zero on average, but it may well be negative if coupons or future prices fall below average. Again, impaired coupons, falls in bond prices, or rises in real interest rates all make it possible for net income to be negative.

Equation (18) holds whether interest is paid on reserves or not. Paying interest on reserves requires resources, but because it also lowers net income in the dividend rule, it has no effect on the evolution of reserves or on equilibrium dividends paid. What distinguished old-style from new-style central banking is instead that longer maturity bonds are held, increasing the risk from changes in real interest rates, coupons, and bond prices. If $\delta = 1$ and only one-period bonds were held, then the term within square brackets is identically zero, and the central bank always has positive dividends equal to the inflation tax.

The Federal Reserve has never had negative net income in its history. But, with the new style of central banking, negative income is more likely to occur.

2.6 Can the central bank pay a negative dividend?

Paying negative dividends requires receiving a capital contribution from the Treasury. The appropriation of government funds would have to be approved by fiscal authorities, subject to the political process underlying it. This opens the door to political interference in monetary policy decisions. A central bank that relies on frequent recapitalizations from the Treasury will not be financially independent and will likely also not be able to determine monetary policy independently.

If the government resists paying into the central bank, on the grounds that it amounts to a bailout, then the central bank cannot sustain the rules in propositions 1 or 2. To see this, consider the scenario in which the central bank pays positive dividends equal to net income but never receives a government appropriation when net income is negative:

$$d_s = \max(y_s, 0), \tag{19}$$

where y_s is the measure of net income otherwise used to set dividends. In this case, the central bank issues additional reserves to compensate for the foregone payment from the government, in the amount of the shortfall:

$$z_s = d_s - y_s. \tag{20}$$

Consider first a central bank subject to the real mark-to-market dividend rule. An increase in reserves resulting from a positive shortfall z has the same permanent effect that we analyzed in connection with the initial condition in proposition 1. We let Z denote the cumulative sum of the zs, so $Z' = Z + z_s$. It is not a function of the current state alone. Rather, since equation (19) and equation (20) imply that z_s is always non-negative, as long as there is a positive probability of negative net income, then Z is a weakly increasing sequence over time that will tend to infinity. Retracing the steps that led to proposition 1 gives:

Proposition 3 *If the central bank pays real mark-to-market dividends according to the rule in equation* (19)*, with net income given by equation* (9)*, reserves are:*

$$V_t = q_t B_t + (V_0 - q_0 B_0) + Z_t, \tag{21}$$

so they drift upward without limit and are non-stationary.

At some point along this path where reserves run off to infinity, the central bank would likely be dissolved and a new currency introduced. From a different perspective, as reserves increase without bound, so does the interest expenses of the central bank, driving net income to unbounded negative realizations. The rule in equation (19) is unsustainable as it violates a no-Ponzi-scheme condition on central bank reserves.

The situation is somewhat different in the more realistic case of a central bank subject to a nominal mark-to-market rule and positive state-dependent inflation. Combining the rule in equation (13) with the constraint that dividends are non-negative in equation (20) into the law of motion for reserves in equation (5),

$$W' = \frac{W}{1+\pi_s} - z'. \tag{22}$$

Whenever dividends are higher than net income, so z is positive, then net worth falls. In nominal terms, $p'z'$ gives the the decline in nominal net worth this period.

Let pZ now denote the shortfall of nominal net worth from its initial level $p_0 W_0$. With zero inflation, this is still the cumulative sum of the zs, but with positive inflation,

$$Z' = \left(\frac{1}{1+\pi_s}\right) Z + z_s. \tag{23}$$

Inflation makes the real shortfall in net worth fall over time if the π_ss are positive. In this case, Z obeys a stationary autoregressive time-series process.

Given the law of motion for net worth, the same steps that led to proposition 2 now show the following result:

Proposition 4 *If the central bank pays nominal mark-to-market dividends according to the rule in equation* (19)*, with net income given by equation* (13)*, reserves are:*

$$V_t = q_t B_t - N_t - \prod_{\tau=0}^{t-1} \left(\frac{1}{1+\pi_\tau} \right) W_0 + Z_t. \tag{24}$$

The term $q_t B_t - N_t$ is state-dependent. If inflation is positive, the term in W_0 dies away and Z_t is stationary, so reserves V are stationary. If inflation is always zero, the term in W_0 is constant, and Z_t drifts upwards without limit, so reserves V are non-stationary.

A corollary of the proposition is that nominal reserves pV are non-stationary and track pZ. Inflation therefore plays an important role with a nominal mark-to-market dividends rule. If inflation is zero or, similarly, if the price-level is state-dependent, then reserves again shoot to infinity as a result of the Treasury not paying into the central bank. With positive inflation though, real reserves are stationary. A policy that tries to keep nominal net worth constant leads to falling real net worth when inflation is positive. As a result, when the central bank fails to receive funds from the Treasury, inflation over time erodes this real shortfall and keeps real reserves stationary.

2.7 Deferred assets and retaining earnings

Most advanced central banks have some discretion over their dividends—they are able to recover from the issuance of reserves in lieu of recapitalization by paying subsequent dividends that are less than net income. At the same time, the spirit of the rules in the charter of most central banks prevents the central bank from accumulating a large surplus on the government, and gives the Treasury discretion to reclaim this surplus through statutory transfers.

We model the relationship by hypothesizing an account balance D that measures the backlog of negative net income realizations that the Treasury did not cover by recapitalizing the central bank. This is a deferred asset capturing the claim that the central bank has on its own future remittances to the Treasury. According to the accounting rules of the Federal Reserve it would appear as a negative liability. When D is positive, the central bank can draw it down by paying a dividend d' less than net income y'. On the other hand, when y' is negative, the balance D rises by $-y'$. The dividend rule is

$$d' = \max(y' - D, 0). \tag{25}$$

We also assume that an upper limit $\bar{D}$ applies to the balance. Central-bank charters are vague about this limit but they allow the Treasury to, at its discretion, reclaim the surplus accumulated by the central bank. It is quite plausible that if the balance in D was high the Treasury would perceive the payments associated with this debt to the central bank as an effective recapitalization. A balance above $\bar{D}$ would put in question the independence of the central bank in the same way that we argued before when we set this limit to zero.

Balances in the D account decline with inflation, matching the decline that occurs in the extra component of reserves, Z. Putting all of these elements together, we get the law of motion,

$$D' = \min\left(\bar{D}, \frac{1}{1+\pi_s}(D - \max(y' - d', 0) + \max(-y', 0))\right). \tag{26}$$

As before, the law of motion of the bulge in reserves caused by missed recapitalizations, Z, is

$$Z' = \frac{1}{1+\pi_s} Z + d' - y'. \tag{27}$$

This provision, with a reasonably generous value of $\bar{D}$, will cancel a bulge of reserve issuance following an episode of negative net income by cutting subsequent dividends and using the funds to pay off the bulge of reserves. The degree of protection depends on $\bar{D}$ and the frequency and magnitude of negative incomes compared to offsetting positive incomes in normal times. But in an infinite lifetime, the buildup of reserves Z will rise above any finite level with positive probability at some time. Allowing the central bank to pay low dividends to recover from a buildup lowers the probability that reserves will reach a high level but does not prevent it.

The message from these results is that, whatever the dividend rule or the inflation policy, negative net income poses a challenge. If the Treasury does not recapitalize the central bank, reserves will have to rise. In all cases, reserves could with positive probability become arbitrarily high, and in some cases may even become non-stationary.

2.8 Quantitative measures of the financial strength of a central bank

Facing a shortfall in its capital caused by negative net income, a central bank can only obtain funds to prevent reserves from jumping to high levels from two sources: the Treasury or seignorage. Either of them puts in question the central bank's ability to pursue its mandate, the first by jeopardizing its independence, and the second by requiring inflation above target. These two sources of the resources needed by the central bank provide two different measures of its financial strength.

The first measure is the value of Z. This is the shortfall in its capital at a date in time, and also the resulting excess reserves outstanding. After this amount is paid into the central bank, reserves can revert to their state-dependent levels in propositions 1 and 2. A central bank relying on a large discretionary intervention of the Treasury is financially weak, and it is weaker the larger is the required amount Z.

Instead of relying on the Treasury, the central bank can raise seignorage by increasing its inflation target. This generates the resources to raise net income to either bring Z down or prevent negative net income from occurring in the first place. A second measure of the financial weakness of a central bank is the extra inflation it needs to bring about in order to maintain the stationarity of reserves.

3 The Federal Reserve

The main lesson from the previous section is that the solvency of a central bank depends crucially on what happens to its dividends when net income is negative. Any answer to the question of how the federal government would deal with a situation calling for recapitalization of the Fed is strictly conjectural. Prior to 2008, the Fed held mostly safe short-term Treasury debt with small potential capital losses and it never paid interest on reserves. As a result, net income was always positive. So far, during and after the crisis, the Fed has functioned as a highly profitable hedge fund, borrowing at low rates in the short-term market to fund holdings of higher-yield longer-term debt generating substantial positive net income.

The solvency of the Fed is at stake when the recovery occurs. At that time, interest rates increase, which both raises the payment on reserves and lower the value of the Fed's portfolio of longer-term bonds. In this section, we simulate what would happen to reserves and dividends under different scenarios for bonds prices and interest rates. But first we describe how our stylized model approximates the Federal Reserve and how we measure the relevant variables and calibrate the model.

From the start, it is important to understand the nature of the exercise. We take as given inflation, the value and maturity of bonds held by the Fed, and the real interest rate, and we calculate whether and when the solvency of the Fed might be at risk. If the Fed runs into trouble, it will likely affect at least one of these variables in response. Modeling this response, and especially how it would conflict with the goals of monetary policy, we leave for future work. The starting point to understand these trade-offs should be to understand when is solvency is at risk. This is what our exercises provide.

3.1 The Fed and the model

The Federal Reserve does not mark its bond portfolio to market, so unrealized capital gains do not enter into its calculations of net income and are not paid out as dividends. Moreover, the Fed does not hold delta bonds, but rather Treasury bills, notes, and bonds of various maturities. All actual Treasury instruments pay off face value at maturity, rather than melting away like a delta bond, and notes and bonds make periodic coupon payments as well. There is no computational obstacle to building a model that keeps track of the historical path of purchases and sales and record gains and losses only upon sale. But the complexity of the model would stand in the way of the points we want to make in this paper.

Our first simplification is the assumption that the Fed holds only delta bonds and that the parameter δ remains constant from normal times to crises. We know the assumption is false, because the Fed has lengthened the average maturity of its portfolios substantially since 2008. Using the data in the annual report of the Federal Reserve on the value and maturity of the securities it holds, we calculate the value-weighted average maturity of the Fed's financial assets for every year. The parameter δ is the reciprocal of the average maturity, and our series for δ is stable around a value of 0.23 before 2007. Between 2009 and 2011, δ is on average 0.15. Variations in δ by state, though feasible, would much complicate the model. Our procedure is to use the post-crisis value of δ for the simulations, even when the economy is outside of a crisis. This procedure is reasonably accurate, because the central risk to the Fed occurs upon exiting the crisis state, when the portfolio is large. Prior to the crisis, the portfolio is smaller, so our exaggeration of the capital gain at the beginning of the crisis is not large.

From state number	Transition probabilities to state					Stationary probabilities
	1	*2*	*3*	*4*	*5*	
1	0.40	0.27	0.20	0.07	0.07	0.244
2	0.38	0.31	0.31	0.00	0.00	0.197
3	0.15	0.23	0.31	0.31	0.00	0.232
4	0.14	0.07	0.14	0.64	0.00	0.246
5	0.00	0.00	0.20	0.00	0.80	0.081

Table 1: Transition Matrix and Stationary Distribution of States of the Economy

Our second simplification is to describe the Fed's financial stability under the hypothesis that it pays dividends according to the nominal mark-to-market rule discussed in the previous section. The Fed is more likely to have trouble under that rule than under its actual dividend rule, because it earns capital gains on its portfolio upon entering the crisis state, which it would pay out immediately to the Treasury. In actuality, the Fed retains those funds unless it sells the bonds. When the economy exits the crisis state, the Fed's dividend does not fall into negative territory. The actual policy has the effect of smoothing the Fed's dividend relative to our assumption of marking to market. Actual practice avoids a big payment at the beginning of the crisis followed by a recapitalization by the Treasury upon recovery. We discuss alternatives in section 3.6.

3.2 The data and states of the economy

Our annual data cover the period from 1954 through 2011. To define the states of the U.S. economy, we use the realized real T-bill rates, measured as the nominal rate on one-year bills minus the increase in the Consumer Price Index. In our model, it is movements in asset prices, which we take as exogenous, that trigger a financial crisis and threaten the Fed's stability. We allow for 5 states. The estimated transition matrix appears in Table 1, together with the implied stationary distribution across states.

The top four states have about the same stationary probabilities and correspond to the behavior of the interest rate through 2008. The real rates for these states are the medians within the quartiles of the distribution of the rate over that period. Since then, the economy has been in state 5, the crisis state, with a real rate essentially minus the inflation rate,

State number	Model inputs: Safe rate, *r*	Bond holdings, *B*	Currency, *N*	Other data: Inflation, *p'/p-1*	Reported income, *y*
1	0.039	0.0079	0.0504	0.028	0.0038
2	0.021	0.0089	0.0574	0.023	0.0026
3	0.010	0.0089	0.0547	0.034	0.0028
4	-0.009	0.0091	0.0548	0.057	0.0031
5	-0.021	0.0243	0.0661	0.017	0.0050

Table 2: Inputs from Data

because the nominal rate has been close to zero. The transition into state 5 from 2007 to 2008 occurred out of state 1. Our estimates imply that the economy has a financial crisis with a probability of $0.07 \times 0.242 = 0.016$, or once every 62 years.

Because the economy has yet to transit out of the crisis state, we took the exit rate to be 20 percent per year as a rough estimate, and assumed the transition was into state 3, the median state. We therefore assumed that a crisis has an expected life of 5 years. We investigate the robustness of the results to this assumption later in section 3.7.

We use data from the annual financial statements of the Federal Reserve System to fill in the variables in the model. Table 2 shows each of these variables at each state of the economy. We measure the holdings of bonds B_s as the total U.S. Treasury and agency securities held by the Federal Reserve system, and y_s as its current income. To calculate currency holdings N_s we use currency in circulation. Finally, we divide all nominal variables by GDP to express them in real terms as a percentage of GDP, and use the GDP deflator as our measure of inflation π_s. Note that in the crisis state, the bond portfolio of the Fed is more than twice as large in the other four states.

Table 2 also shows two variables of interest that are not direct inputs to the model, the Fed's reported income, divided by GDP, and the rate of increase of the GDP price index. Reported income is substantially higher in the crisis state than in the normal states, reflecting the success of the Fed as a hedge fund taking a risky long position on bonds funded with short-term borrowing. The rate of inflation has been somewhat above the presumed target of two percent prior to the crisis and somewhat below it during the crisis.

3.3 Calibrating asset prices and capital gains

Recall that the price of delta-bonds is given by

$$q_s = \mathbb{V}(c_s + (1-\delta)q_{s'}).$$

We use this delta-bond price to account for capital gains and losses on the portfolio of the Fed. To apply the $\mathbb{V}$ operator, we need to have the stochastic discount factor $m_{s,s'}$. A reasonable class of SDFs satisfies

$$m_{s,s'} = \beta \frac{\mu_{s'}}{\mu_s}. \tag{28}$$

For example, in the consumption CAPM with time-separable expected utility, μ_s is marginal utility. Under this assumption, the definition for the real interest rate gives a condition that the SDF must satisfy:

$$\beta(1+r_s)\sum_{s'} \omega_{s,s'}\mu_{s'} - \mu_s = 0. \tag{29}$$

With the normalization $\mu_1 = 1$, this is a system of N equations in N unknowns, $\beta, \mu_2, \ldots, \mu_N$ that takes as inputs the data on r_s from Table 2 . Though nonlinear, the system solves easily by standard methods. Bond prices then solve the linear pricing recursion.

The other input into the bond-pricing formula is the set of coupon payments. In the four non-crisis states, the coupon rate is 1, while in the fifth state it is a possibly lower value c_5. The Fed suffers a default loss of $1 - c_5$ of its coupon expectation every period that it remains the financial crisis state. Note that the bond-pricing equation assumes that all bonds resume paying full coupon rates once the economy exits state 5—the bank does not lose a fraction $1 - c_5$ of the value of its bond holdings. We calculate a counterfactual where $c_5 < 1$ and see whether they compromise the Fed's solvency.

Within our sample, $c_5 = 1$, as there have been no defaults in the Fed's portfolio. Combining this coupon with the real interest rates in the data gives the results in Table 3. Notice that marginal utility is quite low in the crisis state, 5, corresponding to a high level of consumption in that state. To rationalize the low real interest rate in the crisis state in terms of the consumption Euler equation, it must be the case that consumption is expected to fall when in the crisis. Accordingly, consumption in the crisis must be unusually high. Obviously aggregate consumption does not behave this way across our states. Hall (2011) discusses

State	*Safe rate, r*	*Marginal utility, μ*	*Coupon, c*	*Bond price, q*
1	3.92	1.000	1.00	5.99
2	2.09	0.995	1.00	6.05
3	1.02	0.963	1.00	6.22
4	-0.88	0.928	1.00	6.43
5	-2.10	0.821	1.00	6.88

Table 3: Real Interest Rate, Marginal Utility, and Delta-Bond Price

how low real rates in the crisis economy might be consistent with the consumption paths of a limited group of wealthier consumers who participate in securities markets. Most consumers, in that view, are at a corner in their intertemporal consumption problems, so the slope of their consumption profiles does not contribute to asset pricing. He points to data from the Survey of Consumer Finances to support this hypothesis.

3.4 The Fed in crisis and recovery

We begin with the case where the Treasury recapitalizes the Fed whenever net income is negative, so reserves are state-dependent (we assume any initial effect W_0 has long since melted away on account of positive chronic inflation). We track the Fed's finances as the economy moves over a 10-year period from state 1 to a crisis in state 5 for five years and a recovery in state 3 for four years. Throughout, we assume a 2-percent inflation target normalizing the price level to one in year 1.

Figure 1 shows the nominal interest rate and the nominal bond price over the period. The interest rate starts at a fairly high level prior to the crisis, plunges to zero for the crisis years, and then resumes a moderate positive level for the remaining time in state 3. The bond price follows the price level on a trend, but rises at the beginning of the crisis and falls when the crisis ends. The figure shows that the Fed faces significant risk during the recovery, when the fall in bond prices causes capital losses in its portfolio, and the rising interest rate raise the payment on reserves.

Figure 2 shows the Fed's bond holdings and reserves outstanding, scaled to approximate the U.S. economy. The difference between the two is close to the value of the currency outstanding, as the Fed's accounting net worth is relatively small. At the outset of the crisis, the Fed borrows about \$1.53 trillion from the banking system to buy bonds. High

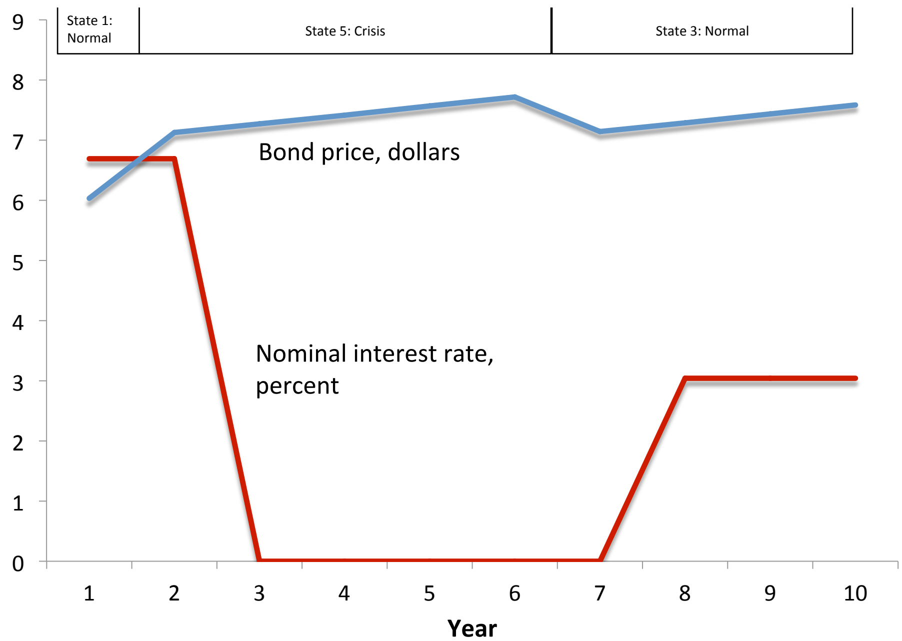

Figure 1: Interest Rate and Bond Price

levels of bond holdings financed by high levels of reserve borrowing continue until the end of the crisis, when the economy shifts to state 3. The Fed sells about $1.8 trillion in bonds and uses the funds to retire a similar volume of reserves, which decline to almost their initial level. Reserves are elevated during the crisis states, but are otherwise low and stable during the the other states.

Figure 3 shows the Fed's dividend to the Treasury and the flows that determine it. The least important determinant is the payment on reserves, which appears negatively because it is a deduction from the dividend. During the crisis the interest rate paid is zero, and outside of the crisis reserves are small so the interest paid has a barely visible negative effect. The coupon earnings from the bond portfolio less depreciation are likewise of little importance. Deducting depreciation is the equivalent of not counting the return of principal as part of the earnings from a bond.

The most important determinant of dividends is the capital gain on the bonds. The Fed records a capital gain upon entering the crisis state, with negative real interest, and pays a corresponding dividend. Upon exiting the crisis state, the Fed suffers a capital loss. The loss is larger than the gain because it applies to the large portfolio acquired during the crisis.

Figure 4 shows the flows that raise or reduce reserves. Interest on reserves is a minor factor contributing to growth and seignorage is a minor factor contributing to shrinkage. The

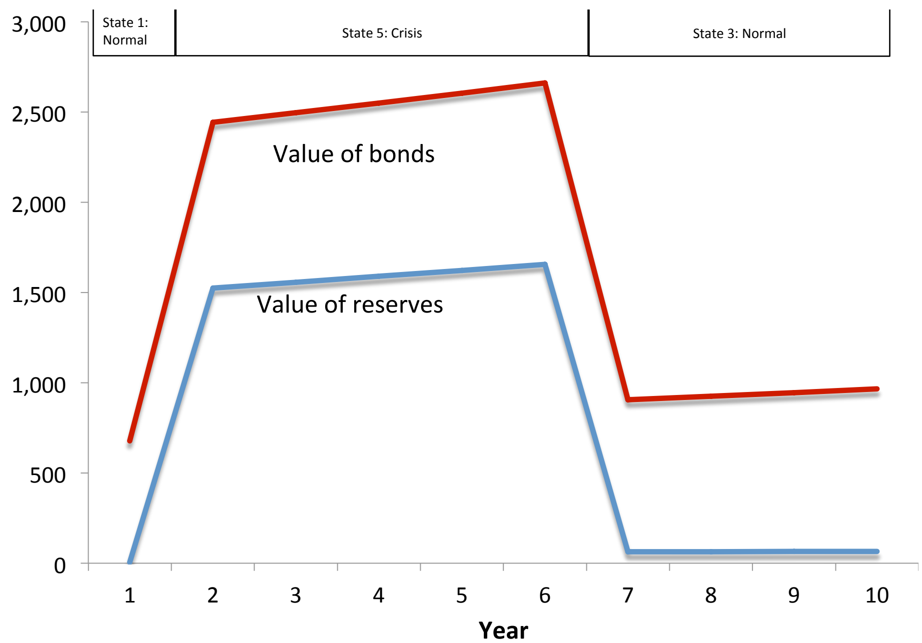

Figure 2: The Values of the Fed's Bond Holdings and Reserves Outstanding, Billions of Dollars

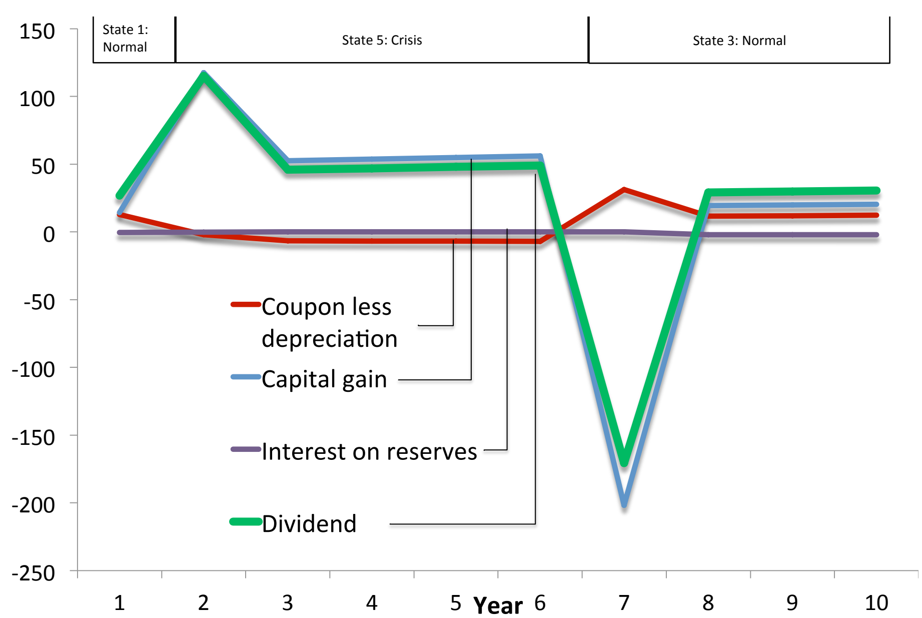

Figure 3: Components of the Fed's Dividend to the Treasury

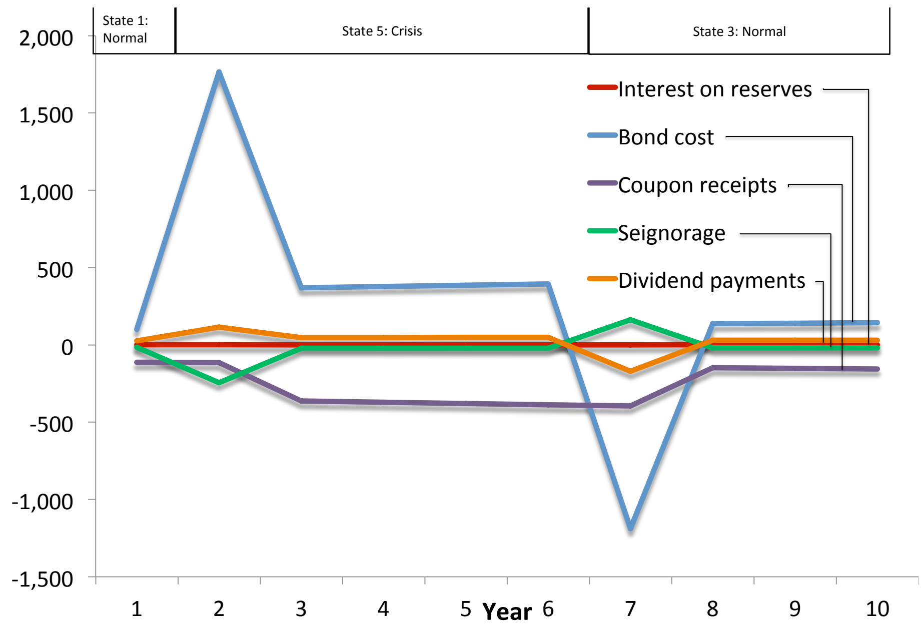

Figure 4: Flows Into and Out of Reserves

big factor is purchasing and selling bonds. When the crisis strikes, the Fed expands reserves to buy bonds; when it ends, the Fed sells a large volume of bonds and pays down reserves. Dividend payments, which add to reserves, also have noticeable roles at the beginning of the crisis—when capital gains from the lower interest rate accrue and are paid to the Treasury—and at the end of the crisis—when capital losses from the higher interest rate accrue and the Treasury bails the Fed out. The negative dividend payment at that time is effectively a recapitalization of the Fed.

3.5 The solvency of the Fed

If the Treasury has a policy of no recapitalization, the Fed must borrow from the banking system by issuing reserves to cover negative income. The adoption of a dividend rule that causes the Fed's debt to the banking system to rise to ever-higher levels in each crisis could compromise the Fed's solvency.

The Treasury allows the Fed to retire the extra reserves through the D account we described earlier. We take the upper limit to be $\bar{D} = 0.02$ or 2 percent of GDP, a limit that is not binding in our scenario. The result is that the balance D is always the same as the extra reserves Z, which is our dollar measure of the financial weakness of the Federal Reserve.

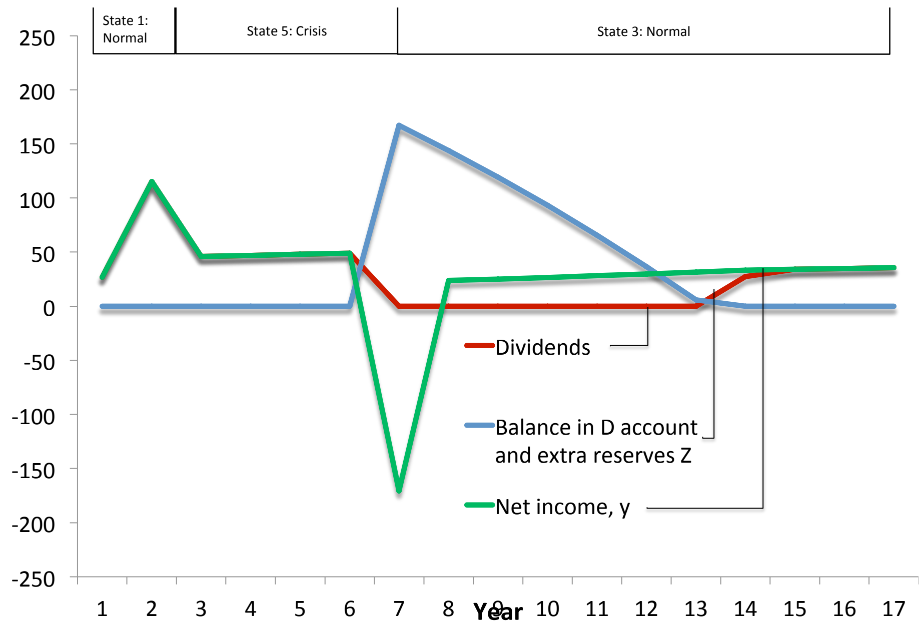

Figure 5: How the D Account Generates a Speedy Elimination of Extra Reserves from a Capital Loss

Figure 5 shows the operation of the D account in our earlier scenario, extended for 7 more years in state 3 to show how the account is gradually worked off. Until the crisis ends, dividends equal net income, resulting in a large payout to the Treasury when the Fed's bond portfolio appreciates at the onset of the crisis. When the economy recovers, net income is negative for a year. The Fed issues extra reserves to cover the absence of recapitalization. The balance in the D account rises to 1 percent of nominal GDP, along with a bulge in reserves of the same amount. For the next 6 years, the Fed pays the Treasury zero dividends and gradually works of the balance in D and the extra reserves that had been issued when the crisis ended. Thus, the Treasury does not make a cash payment to the Fed, but the equivalent happens over time, as the Treasury foregoes dividends in equivalent amount. The following year, the Fed pays a positive dividend but less than net income. In the succeeding years, $D = Z = 0$ and reserves are back to their state-dependent normal values.

3.6 Marking the Fed's portfolio

The scenario in Figure 5 assumes that the Fed's bond holdings are market to market each year in determining net income y. In fact, as we noted earlier, the Fed does not mark a bond to market, but computes net income as if the bond were invariably worth its nominal purchase price, which is usually quite close to the nominal principal returned at maturity (we skip over the intricacies of accounting for a difference between the purchase price and the principal amount). Because most of the bonds the Fed held prior to the crisis would mature in the immediate post-crisis years, the extra income takes the form, not of the immediate capital gain we record, but rather of a stream of coupon payments at rates higher than the current market rate during the crisis years. Thus our scenario is a reasonable approximation to what would happen in reality, except that we record a large bulge of income in one year, whereas in fact the same bulge would be spread over several years.

By the end of the crisis in our scenario, the situation would be similar—the Fed would have paid high dividends summing to a considerable amount. When the crisis ends and the Fed's income is low, both because of low coupons on its holdings of bonds purchase during the crisis years and because of paying now-higher rates on reserves, the Fed would go through a period of quite low dividends. Our scenario mimics all aspects of reality except the timing of the high dividend. In particular, with the addition of the D account, we have created a reasonable facsimile of the danger of insolvency, which would arise in case the upper limit on D, $\bar{D}$, was restrictive and crises frequent. If $\bar{D}$ is 0.005 (half a percent of GDP), the extra reserves from the end of the crisis are much more persistent—10 years after the recovery, 50 percent of the excess is still present.

3.7 The features of a crisis: defaults and exit state

Default risk may also imperil the Fed's solvency. We can incorporate defaults by setting the coupon that bonds pay in crisis years (c_5), to less than its normal value of one. We recompute bond prices for this case since we are describing an economy where bondholders are aware in all states that bonds lose value from shortfalls in coupon payments in the crisis state.

In our scenario, the economy spends 5 consecutive years in the crisis state, which is also the expected duration of crises in general. A shortfall in the coupon of one percent ($c_5 = 0.99$) for one period lowers the value of a bond by 1/6 of a percent, because the bond sells for about 6 units of output. Thus a loss of one percent for 5 consecutive years lowers the value by about $5/6 = 0.83$ of a percent. We consider a loss of 20 percent ($c_5 = 0.8$), corresponding to an immediate shortfall in bond prices at the outset of a crisis, relative to their crisis values with full coupon payments ($c_5 = 1$) of about 9 percent ($q_5 = 6.88$ with

$c_5 = 1$ against $q_5 = 6.23$ with $c_5 = 0.8$). This decline in bond values on account of default is far larger than the Fed's experience so far.

Figure 6 repeats the dividend account in Figure 3, now with bond defaults. The Fed's solvency is less at risk than before, and net income is never negative. On the one hand, as expected the flow of coupon income is lower than before, and no longer covers the depreciation of the bond as before, lowering net income. But, on the other hand, the lower coupon implies that bond prices during the crisis are lower, and therefore closer to their pre and post crisis values. In the model without bond defaults, the bond price rises from $q_1 = 5.99$ to $q_5 = 6.88$, whereas with defaults, the price rises from $q_1 = 5.92$ to only $q_5 = 6.23$. Therefore, there is no longer a spike in net income and dividends when the crisis starts or ends. This confirms our earlier conclusion that the danger to the Fed's solvency comes from the capital losses when exiting the crisis, so anything that lowers the fall in bond prices, including lower coupons during the crisis, reduces the danger of insolvency.

To further confirm this result, we perform a second experiment, where we make the exit from the crisis occur to state 1, rather than 3. The fall in bond prices when exiting the crisis is now higher and so are the capital losses. Figure 7 shows that the amount accumulated in the D account is now larger, showing that capital losses are the key danger to the Fed. Our conclusion that the Federal Reserve is still at little risk is also approximately unchanged. Now, the maximum recapitalization the Fed would need is \$250 billion, relative to \$171 billion before. It only takes two more years of reclaiming dividends from the Treasury to bring this amount to zero.

4 The European Central Bank

The European Central Bank is the coordinating agency of the Eurosystem, comprising the ECB and the national central banks of the euro countries. Throughout our discussion, we use the name ECB, though our law of motion for reserves describes the Eurosystem, while our analysis of solvency relates more closely to the ECB.

Since late 2008, the ECB has pursued policies similar to those of the Federal Reserve. Its assets have risen substantially and grown in maturity. The ECB has funded its asset expansion by borrowing from banks by issuing reserves, which pay close-to-market interest rates. Thus our theoretical analysis of central bank solvency in section 2 applies to the ECB. Modeling the ECB requires taking into account some of its idiosyncrasies. After discussing

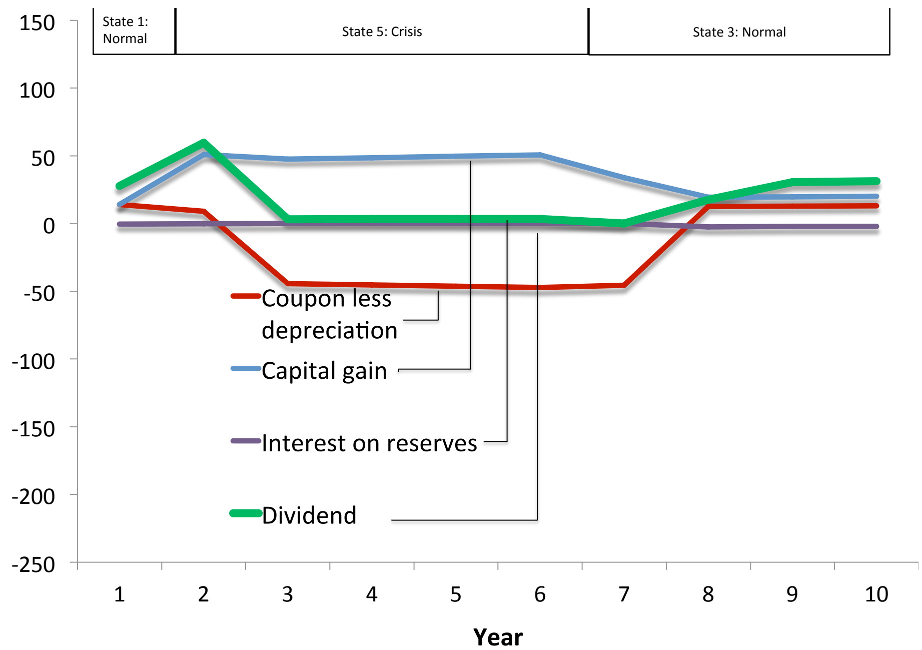

Figure 6: Components of Dividends with Bond Default

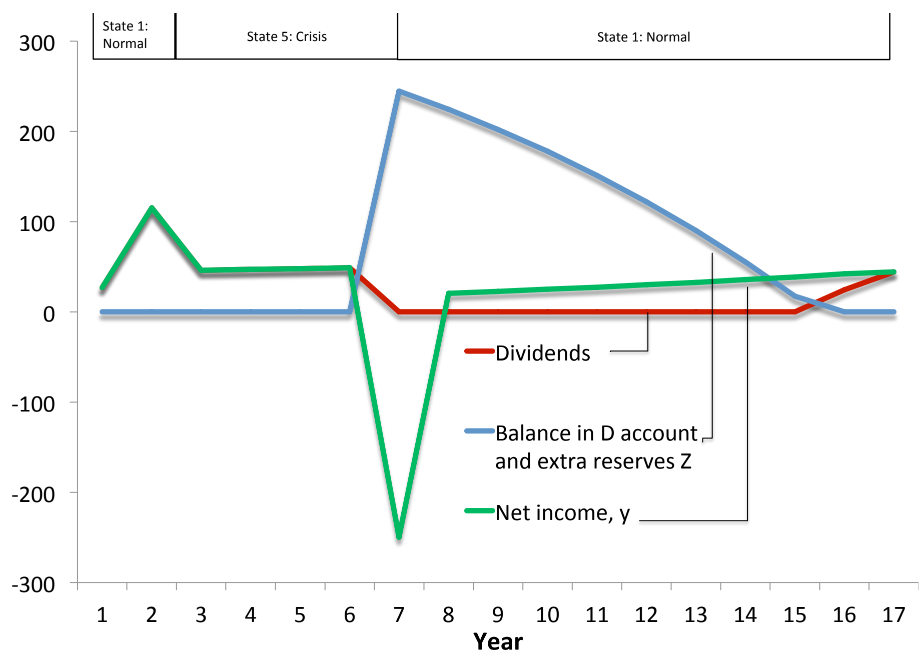

Figure 7: Net income and Dividends with Larger Capital Losses

these, we will present the data, build a crisis scenario for the ECB, and analyze the risks to the its solvency.

4.1 The ECB and the model

The ECB's securities holdings are split between two categories: securities held directly and repurchase agreements with banks. The repos are effectively term lending to commercial banks collateralized by government bonds or government-guaranteed securities. Before 2008, the duration of the repos was mostly one week, with the longest being 3 months. Since then, 6-month and 12-month repos became dominant, and in December 2011 and February 2012, the ECB lent €1 trillion through 3-year long-term refunding operations, effectively three-year repos. These loans have floating rates indexed to the one-week repo rate, so the relevant interest rate is still the short-term safe rate r_s. We let $q_s B_s^d$ denote the real value of the securities held directly and B_s^r denote the real value of repos, so the total assets of the ECB are $B_s = q_s B_s^d + B_s^r$.

The ECB requires collateral in excess of the value of the loan, by as much as 15 percent (the haircut), and frequently revalues its collateral using market prices and requires extra collateral to be posted to lower its credit risk. A credit loss from a repo involves both default by the borrowing bank and an impairment of the value of the collateral in excess of the haircut. Default would result in reorganization of the bank and its exclusion from future borrowing from the ECB. The ECB will suffer credit losses on its repos only in the event of a general financial meltdown in the euro countries.

If a meltdown has low probability, we can treat repos as safe loans, with no risk of capital gains or losses to the ECB. In that case, repos have the same financial character as negative reserves, so the law of motion for reserves becomes:

$$V' - B_{s'}^r = (1 + r_s)(V - B_s^r) + q_{s'}[B_{s'}^d - (1 - \delta)B_s^d] - c_s B_s^d - n_{s,s'} + d_{s'}. \tag{30}$$

Issuing reserves and investing the proceeds in repos has a neutral effect on solvency because this asset and liability earns exactly the same interest rate. All of our analysis and equations from section 2 still applies, as long as we replace B_s with B_s^d and V with $V - B_s^r$.

At the other extreme, with a significant probability of meltdown, the ECB faces the possibility of capital losses, when euro banks become insolvent and the ECB's collateral loses value by more than the haircuts. In this case, we need to take into account the asymmetry

From state number	*To state*		*Stationary probabilities*
	1	*2*	
1	0.983	0.017	0.919
2	0.198	0.802	0.081

Table 4: Transition Matrix and Stationary Distribution for the Euro-area

inherent in ownership of any debt claim, that the owner will never receive more than face value but may receive less. The law of motion for reserves becomes:

$$\begin{aligned} V' &= (1+r_s)V + q_{s'}[B^d_{s'} - (1-\delta)B^d_s] - c_s B^d_s + \mathbb{I} q_{s'}[B^r_{s'} - (1-\delta)B^r_s] - c_s B^r_s \\ &\quad + (1-\mathbb{I})[B^r_{s'} - (1+r_s)B^r_s] - n_{s,s'} + d_{s'}]. \end{aligned} \tag{31}$$

Here $\mathbb{I}$ is an indicator for the bank's failure to pay off. The term preceded by $\mathbb{I}$ says the ECB retains the collateral, taken to be a delta-bond, if the bank fails to pay off the repo. The next term applies when the bank does pay off. The same change applies to the calculation of net income. Consequently, our earlier analysis of the conditions when reserves are stationary continues to apply. But in terms of our simulations that follow, the change when banks fail to pay off on repos on impaired assets can be substantial.

4.2 Data and calibration for the euro area

Our data for Europe cover the period from 2000 through 2011. We take the exogenous real interest rates to be the 1-year Euribor rate minus inflation measured by the euro-area Harmonized Index of Consumer Prices. We define two states, one between 2000 and 2008, and the crisis covering 2009-11. With only one observation of each, it is impossible to estimate a transition matrix. Instead, we take the probabilities associated with a crisis to be the same as for the United States—the euro-area economy enters a financial crisis with probability 0.016 and the stationary probability of a crisis is 0.081.

Table 4 shows the values of the real interest rate, the volume of the repo contracts and direct bond holdings, and seignorage, all as ratios to GDP. Our source for the ECB's finances is its last weekly financial statement published each year. The table also shows the marginal utility and bond prices that result form the same exercise as the one that we undertook for the United States. The crisis comes with a large expansion in assets, and especially in direct bond holdings.

	Model inputs				Other data	
State number	*Safe rate, r*	*Repos, Br*	*Direct bond holdings, Bd*	*Currency, N*	*Coupon, c*	*Bond price, q*
1	0.978	0.049	0.018	0.058	1.000	6.364
2	-1.067	0.077	0.064	0.088	1.000	6.767

Table 5: Inputs

It is hard to estimate the maturity of the bond holdings to calibrate δ because the ECB does not publish data on the maturity of either the securities it holds as collateral or directly. We set $\delta = 0.148$, the value that we estimated for the United States.

4.3 The ECB's solvency risk

To save space, we do not repeat the same set of figures for the ECB that we presented for the Federal Reserve, but they are available from an online appendix. As in the United States, a crisis comes with an increase in bond prices and a fall in nominal interest rates, and an expansion in bond holdings together with reserves. Looking at the components of the dividend paid, they are dominated by capital gains and losses, while the evolution of reserves is likewise dominated by the cost of buying and selling bonds and collecting coupons while suffering depreciation.

Instead, Figure 8 shows the total dividends paid if the ECB is fully recapitalized, in three scenarios. In the first, we treat all of the bonds held by the ECB, directly or as collateral, in the same way we did for the Federal Reserve. In the second scenario, we take the other extreme view that only bonds held directly lead to risk. This scenario corresponds to $\mathbb{I} = 0$ in the law of motion. Finally, in the third scenario, we take the asymmetric case, with $\mathbb{I} = 1$ where the ECB would bear capital losses in excess of haircuts, but not gains, on the bonds that it holds as collateral. If the repurchase agreement of the ECB are all honored, then its net income is always positive and substantial. However, if they are not, then the ECB is subject to the same risk as the Federal Reserve: when the recovery comes, and asset prices fall, the capital losses leads to negative net income.

Figure 9 assumes instead that the ECB is not automatically recapitalized but can draw on its D account, which again has an upper bound of 2 percent of GDP. If banks always honor their repurchase agreement, the ECB is in less danger than the Fed. After exiting the

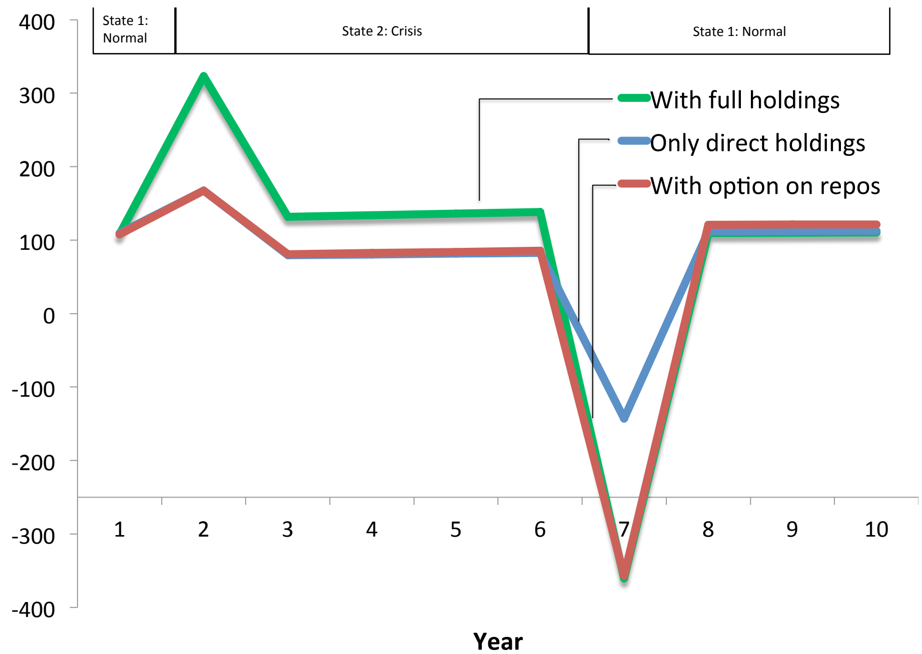

Figure 8: Net income under alternative scenarios

crisis, the losses on the securities that the ECB holds would raise reserves by €143 billion. This is below the 2% limit so the D account absorbs the full amount. Moreover, it would take only one year of positive income to run down the account and retire the excess reserves. If instead the repos are not honored, then the ECB is in significant trouble. When asset prices fall, the holders of the repurchase agreements hand in the collateral to the ECB. The losses rise to €357 billion, well above the limit faced by the central bank on its ability to retain future dividends. Therefore, reserves stay forever higher at €145 billion. The ECB would need to ask the member countries for this amount to be recapitalized and return to the pre-crisis level of reserves. The ECB's solvency would be at risk.

5 Concluding Remarks

We find that the Fed is in no danger of failing to meets its obligations or requiring recapitalization in an economy subject to occasional crises resembling the one that began in 2008. The answer to the question posed at the outset of this paper—is the Fed at risk for losing the ability to stabilize prices because it becomes insolvent?—is an unambiguous no. Given the historical volatility of interest rates and bond prices, the Fed is quite safe under a fully effective price stabilization policy. The Fed is quite bulletproof as long as the Treasury does

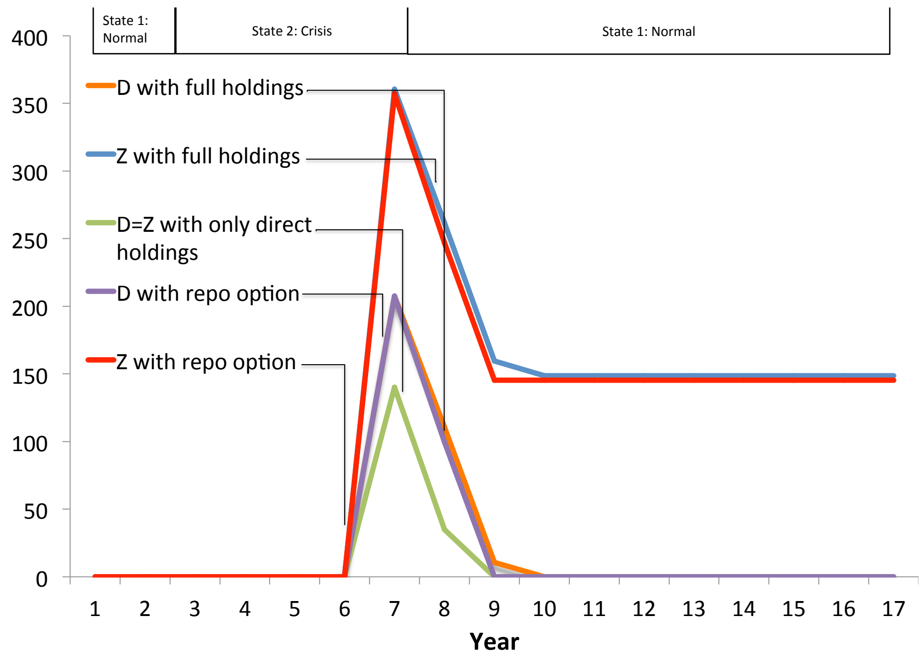

Figure 9: Balance in D account and extra reserves Z

not increase its demands for current income. The central bank of Argentina has suffered from cash drains forced by the central government that imperil its solvency, but there are no signs of similar problems for the Fed.

The ECB has an important potential advantage over the Fed because it holds a substantial fraction of its assets as collateralized loans to banks rather than as outright ownership of bonds. As long as the banks meet their obligations to repay the loans even when the collateral value declines, the ECB's repo positions amount to safe short-term borrowing, insulated from interest-rate fluctuations. Whereas the Fed suffers capital losses on its entire portfolio when interest rates rise in a recovery, the ECB's losses only occur among its direct holdings of bonds. But the ECB is at risk for a meltdown in which the collateral in its repos loses substantial value and the counterparty banks are unable to repay.

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