Housing and Endogenous Default*

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Abstract

This paper examines credit market imperfections in a New Keynesian model with housing. Unlike in the related literature, households may default on their debt if housing prices are sufficiently low, potentially resulting in a temporary loss of access to credit or housing markets. Default has three opposing effects on borrowers. First, the loss of access to housing markets provides borrowers with an incentive to substitute toward consumption. Second, default transfers wealth from lenders to borrowers. Third, the loss of access to credit markets prevents borrowers from smoothing their consumption, resulting in decreased consumption. We use adaptive learning to solve the model and find that borrowers respond to default by increasing their consumption. However both aggregate and lenders' consumption decrease in the default state. In addition, default distorts the housing market which causes a large amplification of the decline in housing prices that initially caused default. We thus conclude that mortgage default is not simply an effect of economic downturns, but that it is a causal factor as well.

JEL Classification: financial market frictions, credit default, housing, learning.

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1 Introduction

The recession of 2007-2009 has renewed interest in the detrimental effects of limited access to credit and mass foreclosures on business cycle fluctuations. This paper models whether or not mortgage default is simply an effect of business cycles, or whether it actually contributes to macroeconomic volatility. In a recent speech given by the Federal Reserve Bank Chairman Ben Bernanke, he claims[1]

> The multiyear boom and bust in housing prices of the past decade, together with the sharp increase in mortgage delinquencies and defaults that followed, were among the principal causes of the financial crisis and the ensuing deep recession–a recession that cost some 8 million jobs.

Bernanke's quote suggests that mortgage default is not simply a result of economic downturns, but that default itself can exacerbate a downturn. Despite this sentiment, the recent macroeconomic literature largely ignores the effects of default, instead focusing on cases where the threat of default matters, but default never actually occurs. This papers attempts to fill this gap. We find that actual default does cause reductions in aggregate output and that it amplifies the decline in housing prices that initially caused default.

This paper augments the traditional New Keynesian dynamic stochastic general equilibrium (DSGE) model with a housing market (that includes both owner occupied housing and rental housing). Housing acts as both a durable good and collateral on secured loans made by patient households to impatient households. In related settings, Bernanke and Gertler (2001) and Iacoviello (2005) examine New Keynesian models with asset markets (stocks and housing respectively) and find evidence that credit constraints act to increase the magnification and persistence of shocks. These papers do not, however, allow borrowers to actually default.

We explicitly allow for default in order to analyze the effect of insolvency on the economy. Default is characterized by a temporary loss of access to mortgage or credit markets. We find that default has three significant effects on the economy. First, default represents a transfer of wealth from lenders

[1]Taken from "Operation HOPE Global Financial Dignity Summit," Atlanta, Georgia, November 15, 2012.

to borrowers. This transfer occurs both because borrowers are unable to meet their loan obligations, and because the value of seized collateral (housing) declines as a result of default. This transfer increases the consumption of borrowers while decreasing that of lenders. Second, default creates a misallocation of housing where lenders retain most or all of the economy's housing stock. This distortion further reduces the price of housing, thereby amplifying the effects of the shock that caused lower housing prices and default. Because borrowers may lose access to housing, they are pushed into the rental housing market which may result in less utility. As a result, borrowers substitute toward consumption. Third, by losing access to credit markets, borrowers lose the ability to finance consumption. In our model, the net effects of default are that aggregate consumption, as well as that of lenders, declines, while borrowers' consumption increases.[2] The declines in consumption occur because the negative wealth effect for lenders is made severe by a substantial devaluation of housing, which they repossess in the case of default.

We consider three alternate scenarios for what happens when default occurs. In the first, excess debt is simply written off and borrowers lose access to neither credit nor mortgage markets. We show that the effects of default are small in this case, suggesting that the loss of access to financial markets is the crucial, and previously unmodeled, aspect of mortgage default. Secondly, we consider default where borrowers lose access to both markets. In this case, the effects described earlier (declining aggregate consumption and housing prices) are large. Finally, we analyze an intermediate case where borrowers lose access to bond markets but maintain access to housing markets. Here, we find similar results to the second case suggesting that the loss of access to credit markets is crucial.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 briefly reports some empirical evidence from an error correction model. These results support our theoretical model, showing feedback between both delinquency rates and housing prices, and delinquency rates and GDP. Section 4 develops the model. Section 5 discusses our approach to how expectations are formed. The model is not tractable under rational expectations. We do not, however, view this as a hindrance. A lengthy literature in macroeconomics advocates for assuming that agents use adaptive

[2]Because the model does not include capital and housing is fixed, aggregate output and consumption are the same.

learning to form expectations as opposed to rational expectations.[3] Under adaptive learning, agents use econometric estimates to make forecasts and consequently do not need to satisfy the extreme informational requirements of rational expectations, such as knowing the true model, the correct calibration, and the distribution of the stochastic shocks. Agents in our model are thus acting intelligently, but are not dramatically smarter than the readers of this paper. We believe that this concept, known as cognitive consistency, is especially appropriate given the complexity of the model. In such circumstances, a smart forecaster should turn to the data. Section 6 illustrates the causes and effects of mortgage default. Section 7 reports simulation results. Finally, Section 8 concludes.

2 Related Literature

Our paper is related to the the financial accelerator literature, first developed by Kiyotaki and Moore (1997). In this literature, financial markets act to secure debt payments by limiting borrowers' credit to an amount less than or equal to the value of their collateralized assets.[4] Consequently, the household credit caps are influenced by the price of collateralized assets. Kiyotaki and Moore (1997) find that credit constraints are a powerful transmission mechanism for the amplification and propagation of shocks. A shock that reduces the price of collateral also restricts access to credit, which reduces demand for the assets, further lowering its price, etc. This financial accelerator effect helps to explain how relatively small shocks can result in large business cycle fluctuations.

Several papers extend the financial accelerator mechanism into a New Keynesian framework. The closest paper to ours is Iacoviello (2005). He augments a New Keynesian general equilibrium model with collateral constraints and nominal debt. Borrowers in the model secure debts with housing wealth. Consistent with previous papers in the financial accelerator literature, Iacoviello (2005) finds that following a demand shock, the real economic effects are amplified and propagated due to inclusion of credit market frictions. A positive demand shock drives up consumer prices and asset prices, which relaxes the credit constraint allowing agents to increase borrowing. With the additional loans,

[3] See Evans and Honkapohja (2009) for a more detailed discussion of adaptive learning.
[4] Zeldes (1989) and Jappelli (1990) provide empirical evidence for borrowing constraints on households.

agents spend and consume more. Intensified demand raises prices, and inflation reduces the real value of debt obligations further increasing net worth.[5] Quantitatively, a one-standard deviation increase in the interest rate decreases output by 3.33 % in the absence of credit limits and 3.82 % when the credit channel is included. However, Iacoviello (2005) finds that the result is reversed when the economy experiences a supply shock. The collateral constraint acts a "decelerator" of supply shocks in that an adverse supply shock has a positive effect on borrowers net worth.

Bernanke et al. (2000) also add credit constraints to a New Keynesian setup. They use stocks as opposed to housing as the asset that serves as collateral. In this paper, the channel through which magnification and persistence of shocks occur is slightly different than the transmission mechanism described in Kiyotaki and Moore (1997), and Iacoviello (2005). The external finance premium (the difference between the cost of borrowed funds, or the interest rate, and the opportunity cost of using profits for investment activities) and net worth of borrowers are negatively related. Thus, when demand for investment rises, driving asset prices up, the net worth of borrowers also increases, which reduces the external finance premium. It is through this process that the addition of credit market frictions leads to amplification and propagation of shocks.

Each of these papers abstracts away from actual default. The potential for strategic default motivates the credit constraint, but default never actually occurs. Our paper allows for mortgage default in the New Keynesian framework. In doing so, we show that default provides an additional source of amplification that has not previously been modeled.

Our paper is also related to a separate literature that does model actual default. The bulk of this literature consists of partial equilibrium analysis that does not examine the relationship between default and the business cycle. One exception is Fiore and Tristani (2012), who allow for commercial default in a New Keynesian setup. They show that higher default risk increases firms' finance costs which acts to reduce investment. Optimal monetary policy should thus aggressively lower interest rates to reduce the impact of higher default risk. Our focus, however, is on mortgage default. To the best of our knowledge, ours is the first paper to model mortgage default in a New Keynesian setting.

[5]The net effect on demand is positive since borrowers have a higher marginal propensity to consume than lenders

Gale and Hellwig (1985) model default in a very different setting. They solve for the optimal debt contract in a framework where asymmetric information motivates costly verification in which the state is only observed if the firm is insolvent and goes bankrupt. They show that the optimal credit contract is the standard debt contract with bankruptcy. If the firm is insolvent, the lender can repossess as much of the firm's debt as possible in the form of assets; there is not, however, a credit constraint on the borrower. Thus, the lender recovers what they can of the loan, but there is no guarantee it will be equal to the full amount of the debt. We adopt this approach and apply it to our business cycle model with housing.

Several other papers model default using the approach of Gale and Hellwig (1985) Dubey et al. (2005) do not link access to credit to net wealth, but instead impose finite quantity restrictions on contractual obligations and assume the punishment is increasing in the amount of default. Goodhart et al. (2009) incorporates endogenous default in a New Keynesian framework in order to asses the implications for monetary policy and welfare. Like our paper, the penalty of default is exclusion from credit markets following bankruptcy. That paper, however, focuses on banks that loan to firms in order to finance their wage bill. It does not model mortgage default and it does not consider how credit market frictions can amplify or propagate macroeconomic shocks. Faia (2007) analyzes the impact of capital market financial frictions in a two-country DSGE model, where the loan contract is based on Gale and Hellwig (1985) and permits default. While Faia (2007) incorporates insolvency in the model, she aims to explain the relationship of business cycle fluctuations among countries based on financial structure comparison. The goal of this paper is different in that it attempts to explain one channel of output volatility through mortgage default.

By adding default, our paper creates a framework that allows for the analysis of optimal monetary policy in a setting with financial market frictions. Specifically, should the monetary authority directly respond to fluctuations in housing prices or quantities? In the standard New Keynesian model, aggressive inflation-targeting is generally accepted as optimal monetary policy, minimizing output and inflation variability.[6]. The current literature on asset prices and monetary policy assumes that inflation

[6]See Gali (2008)

and output stabilization are the only goals of the central bank and concludes that inclusion of asset prices in a simple interest rate rule yields negligible gains.

Bernanke and Gertler (2001) extend the general analysis of optimal policy to examine how an inflation-targeting rule performs in the presence of an asset price boom-and-bust cycle. Bernanke and Gertler (2001) use the model from Bernanke and Gertler (1999) to determine if central bankers should respond to changes in asset prices. The Bernanke and Gertler (1999) model is a standard New Keynesian model that includes credit market imperfections and stock bubbles. They consider technology shocks, stock price shocks to non-fundamentals, and the two together and conclude that an aggressive inflation-targeting rule reduces output and inflation volatility even when asset prices dramatically fluctuate.

Iacoviello (2005) explores how several different simple monetary policy rules affect macroeconomic volatility in a New Keynesian general equilibrium model that incorporates housing and credit constraints. He examines a simple Taylor rule in which the central bank responds to changes in current asset prices. Iacoviello (2005) finds minimal benefit in terms of output and inflation stabilization when the monetary policy authority responds directly to asset price volatility. Future work will examine the nature of optimal monetary policy in our setting. Because default has substantial effects on real variables, it is not obvious whether monetary policy should optimally respond to default risk.

Because we rely on adaptive learning to solve our general equilibrium model, our paper also contributes to the literature on learning and monetary policy. Orphanides and Williams (2008) compare optimal policy under learning and rational expectations and find that learning provides an additional incentive to manage inflationary expectations. It is therefore optimal for policymakers to more aggressively respond to inflation under learning. Xiao (ming) examines optimal policy in a New Keynesian with housing. He finds that the optimal response to housing prices is sensitive to the specific information set that agents possess. Evans and McGough (2005) examine learning in a standard New Keynesian model. They find that the condition for determinacy of equilibrium is usually, but not always, the same as the condition for stability under learning.

3 Empirical Evidence

We briefly provide some empirical evidence on the paper's central question, how mortgage delinquencies affect other key macroeconomic variables? We estimate an error correction model using U.S. data. The time series are ordered as logged GDP, the delinquency rate for all real estate loans, logged CPI (for all urban consumers), the Federal Funds Rate, and logged housing prices taken from the 20 city Case-Shiller Index.[7] The data are quarterly and run from 1987 through 2012.

Figure 1 illustrates the effects of exogenous shocks to GDP and housing prices on delinquency rates. As expected, reduced output and lower prices increase delinquencies. Figure 2, however, considers how a one standard deviation increase (1.6%) to delinquencies affect housing prices and GDP. Housing prices fall by about 3% and GDP is reduced by nearly 0.5%. These results are evidence that feedback exists where mortgage delinquencies are not simply an effect of an economic downturn, but that they help make downturns worse.

We now develop a theoretical model that illustrates the economic mechanisms behind these results.

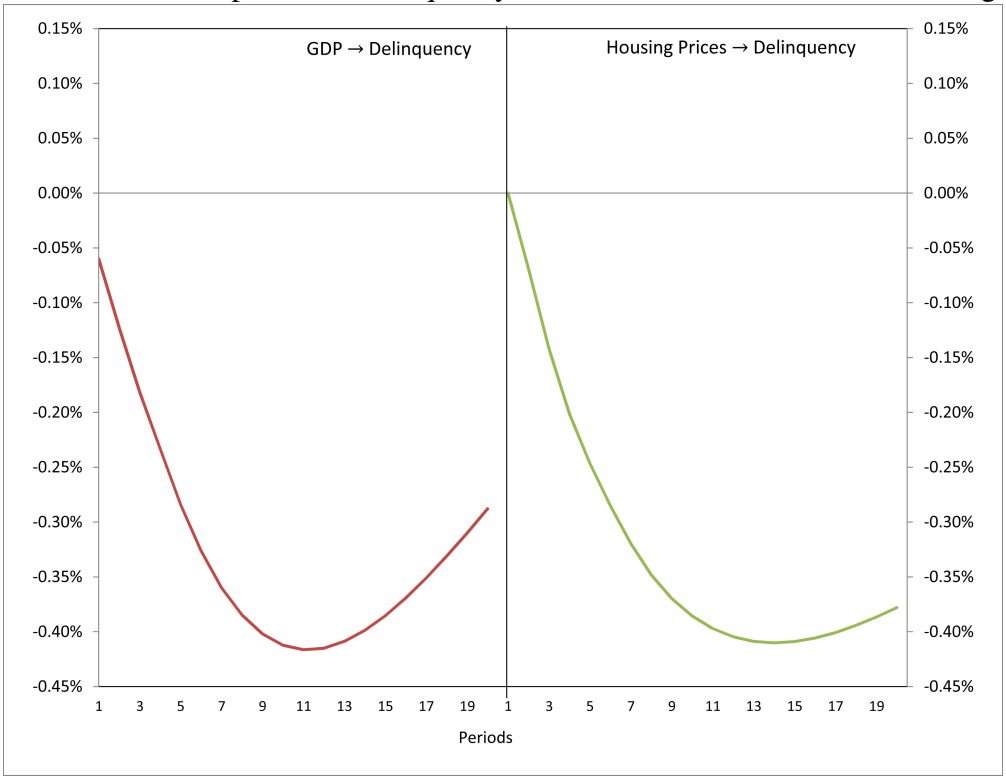
4 Model

We develop a discrete time, infinite horizon model, populated by impatient and patient households. Following Iacoviello (2005), we assume that a set of patient households have relatively high discount factors (γ) and thus typically lend to a separate set of impatient households who have lower discount factors (β).[8] Our model has four notable differences from Iaocviello's. First, we allow for actual mortgage default instead of simply assuming that the threat of default (which never occurs in Iacoviello (2005)) results in a credit constraint. Second, we add a rental housing market. This addition allows us to better examine the effects of impatient households potentially losing access to owner

[7] All data taken from the Federal Reserve Bank of St. Louis. Our fitted specification included two lags and one cointegrating vector. Alternate orderings yield similar results.

[8] Rarely, impatient households lend to patient households in equilibrium. In this case, the model is unchanged except that default risk applies to the patient households. To remain consistent with the related literature, however, we use borrowers and impatient households interchangeably hereafter.

Figure 1: Estimated Response of Delinquency Rates to Shocks to GDP and Housing Prices



occupied housing and instead being forced into the rental market exclusively. Third, to solve the model (which contains several discontinuities) we rely on adaptive learning in the non-linear model instead of linearizing the model and using rational expectations. Finally, for simplicity, we assume that housing does not act as an input in the production function.

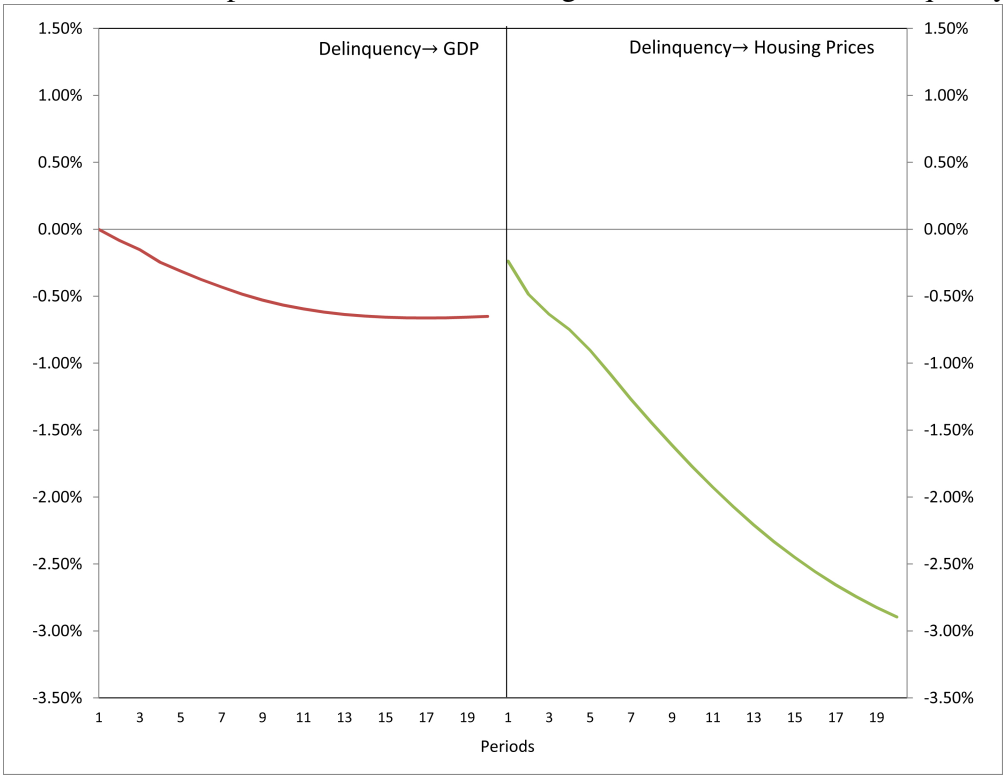
Households work, consume, demand real estate, and supply and demand rental housing.[9] Throughout this section, we denote variables that correspond to patient households with a prime symbol. We begin by assuming the following utility function for patient households,

$$u(c_t^{'}, l_t^{'}, h_t^{'}, x_t^{'}, x_t) = e_t ln(c_t^{'}) + jln(\left[(h_t^{'} - x_t)^{\epsilon} + \omega(x_t^{'})^{\epsilon}\right]^{\frac{1}{\epsilon}}) - \frac{(l_t^{'})^2}{2} \tag{4.1}$$

where e_t is an exogenous demand shock. We assume that patient households may rent housing from

[9]Note, money is not explicitly included in the utility function. Instead, we assume that the Fed can directly set the nominal interest rate. It would be trivial to add money in the utility function and derive the corresponding money demand equation. Household utility in Iacoviello (2005) depends on money balances; however, he only examines interest rate rules for which money supply will always equal money demanded in equilibrium. Given these conditions, the quantity of money does not affect the rest of the model and is disregarded.

Figure 2: Estimated Response of GDP and Housing Prices to a Shock to Delinquency Rates



impatient households ($x_t^{'}$) at the rental rate ($v_t^{'}$). For computational reasons, we do not allow patient households to rent housing to each other.[10] Assuming for the moment that no default occurs, the budget constraint is described by

$$\frac{A_t(l_t^{'})^{\alpha}}{m_t} - b_t + k_t^{'} + v_t x_t = c_t^{'} + q_t(h_t^{'} - h_{t-1}^{'}) + v_t^{'}x_t^{'} - R_{t-1}^{m} b_{t-1}/\pi_t + R_{t-1}k_{t-1}^{'}/\pi_t + F_t \qquad (4.2)$$

The exogenous variable A_t is a random productivity shock. We assume that patient households may borrow from each other ($k_t^{'}$) at the riskless rate $R_t - 1$. As is standard, in equilibrium, $k_t^{'} = 0$. The variable b_t represents impatient households' debt to patient households, and the variable R_t^m represents the corresponding risky interest rate. By including π_t in (4.2), we are assuming that debt is not indexed to inflation. The variable $h_t^{'}$ represents patient households' homeownership so that ($h_t^{'}$ −

[10] We also assume that impatient households may not rent to each other. These assumptions result in there being two separate rental rates.

x_t) is their level of owner occupied housing. The parameter ϵ captures the degree of substitutibility between owner occupied and rental housing, and the term $\omega \leq 1$ allows us to assume that households inherently prefer the former. The variable q_t represents the price of housing.

Finally, we assume that households produce intermediate goods which are then costlessly transformed into final goods by a retail sector which marks them up at the rate, m_t. We assume that the patient household owns the retailers and receives profits $F_t = (1 - m_t^{-1}) \left[(A_t l_t)^\sigma + (A_t l_t^{'})^\sigma\right]^{\frac{1}{\sigma}}$.

Optimization yields the following first-order conditions:

$$\frac{\alpha A_t e_t (l_t^{'})^{\alpha-1}}{c_t^{'} m_t} = l_t^{'} \tag{4.3}$$

$$\frac{j\omega (x_t^{'})^{\epsilon-1}}{v_t^{'}\left[(h_t^{'} - x_t)^\epsilon + \omega (x_t^{'})^\epsilon\right]} = \frac{e_t}{c_t^{'}} \tag{4.4}$$

$$\frac{e_t}{c_t^{'}} = \gamma E_t \left[e_{t+1} \frac{R_t}{c_{t+1}^{'} \pi_{t+1}}\right] \tag{4.5}$$

$$\frac{j(h_t^{'} - x_t)^{\epsilon-1}}{q_t\left[(h_t^{'} - x_t)^\epsilon + \omega (x_t^{'})^\epsilon\right]} + \gamma E_t \left[\frac{e_{t+1} q_{t+1}}{q_t c_{t+1}^{'} \pi_{t+1}}\right] = \frac{e_t}{c_t^{'}} \tag{4.6}$$

$$\frac{e_t v_t}{c_t^{'}} = \frac{j(h_t^{'} - x_t)^{\epsilon-1}}{\left[(h_t^{'} - x_t)^\epsilon + \omega (x_t^{'})^\epsilon\right]} \tag{4.7}$$

Equation (4.3) is the labor supply rule. Equation (4.4) is the rental demand equation. Equation (4.5) is a standard consumption Euler Equation. Equation (4.6) is the housing demand equation. Equation (4.7) is the rental supply equation and simply equates the consumption that results from renting out an additional unit of housing to the utility of using that housing as owner occupied housing.

We now consider the impatient households. Their optimization problem is similar to that of the patient households, except that they potentially default on their mortgage debt and thus borrow at a risky interest rate:

$$u(c_t, l_t, h_t, x_t, x_t^{'}) = e_t ln(c_t) + jln(\left[(h_t - x_t^{'})^{\epsilon} + \omega x_t^{\epsilon}\right]^{\frac{1}{\epsilon}}) - \frac{l_t^2}{2} \tag{4.8}$$

we assume that the demand shock affects both types of households identically. Again ignoring the potential for default, the household's budget constraint is:

$$\frac{A_t l_t^{\alpha}}{m_t} + b_t = c_t + q_t(h_t - h_{t-1}) + v_t x_t - v_t^{'} x_t^{'} + R_{t-1}^m b_{t-1}/\pi_t \tag{4.9}$$

Optimization yields:

$$\frac{j\omega x_t^{\epsilon-1}}{v_t\left[(h_t - x_t^{'})^{\epsilon} + \omega x_t^{\epsilon}\right]} = \frac{e_t}{c_t} \tag{4.10}$$

$$\frac{\alpha A_t e_t l_t^{\alpha-1}}{c_t m_t} = l_t \tag{4.11}$$

$$\frac{e_t}{c_t} = \beta E_t^* \left[\frac{e_{t+1} R_t^m}{c_{t+1}\pi_{t+1}}(1 - p(default)\right] + \beta E_t^* \left[\frac{\partial p(default)}{\partial b_t}\right] \Gamma_t \tag{4.12}$$

$$\frac{j(h_t - x_t^{'})^{\epsilon-1}}{q_t\left[(h_t - x_t^{'})^{\epsilon} + \omega x_t^{\epsilon}\right]} + \beta E_t^* \left[\frac{e_{t+1} q_{t+1}}{q_t c_{t+1}\pi_{t+1}}(1 - p(default)\right] = \frac{e_t}{c_t} + \frac{\beta}{q_t} E_t^* \left[\frac{\partial p(default)}{\partial h_t}\right] \Gamma_t \tag{4.13}$$

$$\frac{e_t v_t^{'}}{c_t} = \frac{j(h_t - x_t^{'})^{\epsilon-1}}{\left[(h_t - x_t^{'})^{\epsilon} + \omega x_t^{\epsilon}\right]} \tag{4.14}$$

where * indicates the conditional expectation in the case of no default. Equation (4.10) is the rental demand equation, and (4.11) is the labor supply equation. Equation (4.12) is the Euler Equation. Impatient households must consider both the probability of default as well as the utility loss resulting from a potential loss of access to markets, denoted Γ_t. Increased debt (b_t) increases the probability of a welfare reducing loss of access to financial markets. Due to this default risk, impatient households borrow at the risky rate $R_t^m - 1$ instead of the riskless rate $R_t - 1$.

Equation (4.13) is the housing demand equation. Once again, impatient households must factor in how their choice of housing affects default risk. Higher values of h_t increase collateral and make default less likely. Equation (4.14) is the rental supply equation.

We further suppose a regulatory constraint that requires:

$$\beta E_t^* \left[\frac{e_{t+1} R_t^m}{c_{t+1} \pi_{t+1}} (1 - p(default) \right] + \beta E_t^* \left[\frac{\partial p(default)}{\partial b_t} \right] \Gamma \leq$$

$$\frac{j(h_t - x_t^{'})^{\epsilon - 1}}{q_t \left[(h_t - x_t^{'})^{\epsilon} + \omega x_t^{\epsilon} \right]} + \beta E_t^* \left[\frac{e_{t+1} q_{t+1}}{q_t c_{t+1}} (1 - p(default) \right] - \frac{\beta}{q_t} E_t^* \left[\frac{\partial p(default)}{\partial h_t} \right] \Gamma \tag{4.15}$$

Examination of (4.12) and (4.13) shows that this condition necessarily holds (with equality) in equilibrium. By imposing it even out of equilibrium, however, we prevent high leverage ratios which allows $p(default) \rightarrow 1$, and a corresponding corner solution where $b_t \rightarrow \infty$, and $c_t \rightarrow \infty$. This condition may thus be interpreted as a credit regulation that ensures well behaved financial markets.

We close the model with the following equations:

$$h_t + h_t^{'} = 1 \tag{4.16}$$

which assumes a fixed housing stock, normalized to one:

$$m_t = E_t \left[\frac{\pi_{t+1}^{\frac{\gamma}{\lambda}} \pi_t^{\frac{-1}{\lambda}}}{\sigma} \right] \tag{4.17}$$

which may be derived using the Calvo mechanism, where $\lambda = \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta(1-\alpha(1-\frac{1}{1-\sigma}))}$, and where θ is the fraction of firms that do not re-optimize their price each period.[11]

$$R_t = \bar{\pi} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{q_t}{\bar{q}} \right)^{\phi_q} \tag{4.18}$$

which is a monetary policy reaction function that potentially responds to asset prices.

[11] See Woodford (2003) for details.

The model thus includes the following 16 endogenous variables:

$c_t, c_t^{'}, x_t, x_t^{'}, h_t, h_t^{'}, l_t, l_t^{'}, b_t, z_t^{'}, q_t, v_t, R_t, R_t^m, m_t, \pi_t$. It consists of ten first-order conditions, one budget constraint, (4.16)-(4.18), interest rate arbitrage:

$$E_t\left[\frac{e_{t+1}R_t}{c_t^{'}}\right] = (1-p(default))E_t^*[\frac{e_{t+1}R_t^m}{c_t^{'}}] + p(default)E_t^{**}[\frac{e_{t+1}R_t^m rec_t}{c_t^{'}}] \tag{4.19}$$

where ** indicates the conditional expectation in the case of default and rec_t is the perceived rate of recovery in the case of default:

and a combined production function:

$$c_t + c_t^{'} = A_t\left[l_t^{\sigma} + (l_t^{'})^{\sigma}\right]^{\frac{1}{\sigma}} \tag{4.20}$$

4.1 Default

We assume that credit markets work as follows. At the start of each period, the credit market clears which requires that the household sell off assets in order to fully pay off its debt. If $q_t h_{t-1}\pi_t \geq R_{t-1}^m b_{t-1}$, then it is able to do so and no default occurs. The household then makes its labor supply, consumption, and savings choices. If $q_t h_{t-1}\pi_t < R_{t-1}^m b_{t-1}$, however, then default occurs. When the default condition binds, we consider several cases:

1. No access to housing or bond markets. In this case, impatient households may neither purchase housing nor access credit markets. In this case, $b_t = h_t = x_t^{'} = 0$, and R_t^m and $v_t^{'}$ are undefined. Equations (4.2), (4.4), (4.12)-(4.14), and (4.19) no longer describe the model's equilibrium. The following, condition, however, must hold.

$$\frac{A_t l_t}{m_t} = c_t + v_t x_t \tag{4.21}$$

Equation (4.21) is simply the budget constraint for impatient households when they lack access to

housing and credit.

2. No access to bond markets. Here, impatient households may purchase housing, but may not borrow using the bond market.[12] Here, $b_t = 0$, and (4.2) and (4.14) no longer bind. The following budget constraint, however, holds in equilibrium.

$$\frac{A_t l_t}{m_t} + v_t^{'} x_t^{'} = c_t + v_t x_t + q_t h_t \tag{4.22}$$

3. Writedown. Here, impatient households may access both housing markets and credit markets. We further assume that unpaid debt is written off, equivalent to imposing $h_{t-1} = b_{t-1} = 0$, and the model is otherwise unchanged.

4. No default. Here we ignore the default condition and continue the model unaffected.

5 Learning and Expectations Formation

The most common approach for modeling expectations is rational expectations. Under rational expectations, agents are assumed to use the model's reduced form solution in order to form mathematically optimal forecasts. Rational expectations has been criticized for requiring agents to possess implausibly high amounts of information. They likely must, for example, know the exact model generating the data despite the field's disagreement over which model is best and an innumerable list of candidates. They most likely must also know the model's correct calibration and the true nature of its stochastic shocks.

The most prominent alternative to rational expectations is adaptive learning.[13] Adaptive learning is motivated by the principle of cognitive consistency, which suggests that agents in a model be neither much dumber nor much smarter than the people modeling them. It thus assumes that agents use econometric algorithms (ordinary least squares in this paper) to form expectations. Perhaps the most

[12] We do not consider the case where impatient households may access bond, but not housing markets, because such a scenario ensures that default will occur in the next period.

[13] For a detailed treatment of adaptive learning, see Evans and Honkapohja (2001).

compelling defense for learning is that the reader, if asked for form forecasts of variables such as consumption and housing prices, is most likely to rely on econometrics rather than simply conjuring up a rational expectation based on minimal data (lagged productivity, housing prices and debt, along with current shocks, and whether the economy is in default). He would thus behave like an adaptive learner.

Our general equilibrium model is not tractable under rational expectations. We argue that, based on cognitive consistency, that this strengthens the case for assuming adaptive learning. To endogenize expectations, we thus rely on the adaptive learning algorithm of this section.

We assume a simple type of learning where agents fit most variables to AR(1) processes. Furthermore, to simplify the analysis, we assume that agents do not consider whether or not the economy is in its default state when fitting the model. For $w_t = c_t, c_t^{'}, \pi_t, q_t$, we assume that agents form expectations using:

$$w_t = a_w + b_w(w_{t-1} - a_w) + u_t \tag{5.1}$$

where a_w and b_w are regression coefficients obtained through recursive least squares:

$$\begin{bmatrix} a_{w_t}^{'} \\ b_{w_t} \end{bmatrix} = \begin{bmatrix} a_{w_{t-1}}^{'} \\ b_{w_{t-1}} \end{bmatrix} + t^{-1}R_t^{-1} \begin{bmatrix} 1 \\ w_{t-1} \end{bmatrix} (w_t - a_{w_{t-1}}^{'} - b_{w_{t-1}} w_{t-1}) \tag{5.2}$$

$$R_t = R_{t-1} + t^{-1} \left(\begin{bmatrix} 1 \\ w_{t-1} \end{bmatrix}^2 - R_{t-1} \right) \tag{5.3}$$

$$a_w = \frac{a_w^{'}}{1 - b_w} \tag{5.4}$$

It then follows that agents use (5.1) to form expectations according to:

$$E_t[w_{t+1}] = a_w + b_w(w_t - a_w) \tag{5.5}$$

Agents also use this algorithm to estimate the default distribution. We assume that rely on point expectations so that:

$$E_t[q_{t+1}\pi_{t+1}] = E_t[q_{t+1}]E_t[\pi_{t+1}] \tag{5.6}$$

Agents obtain an estimate of the expectational error from (5.6) using the following process:

$$\sigma_{q\pi,t} = \sigma_{q\pi,t-1} + t^{-1}|(E_{t-1}[q_t]E_{t-1}[\pi_t] - q_t\pi_t)| \tag{5.7}$$

Agents then fit (5.7) to a truncated normal distribution so that:

$$E_t\left[\frac{\partial p(default)}{\partial b_t}\right] = \frac{R_t^m}{h_t} g(\frac{E_t[q_{t+1}\pi_{t+1}] - \frac{b_t R_t^m}{h_t}}{\sigma_{q\pi}}) \tag{5.8}$$

$$E_t\left[\frac{\partial p(default)}{\partial h_t}\right] = -\frac{b_t R_t^m}{h_t^2} g(\frac{E_t[q_{t+1}\pi_{t+1}] - \frac{b_t R_t^m}{h_t}}{\sigma_{q\pi}}) \tag{5.9}$$

where $(\frac{E_t[q_{t+1}\pi_{t+1}] - \frac{b_t m_t}{h_t}}{\sigma_{q\pi}})$ is the truncated normal probability density function.[14]

Agents must obtain an estimate for Γ_t, the utility loss from losing access to housing and credit markets. We assume they do so by comparing equilibrium in the default state with the hypothetical equilibrium had they been allowed access to credit markets. This parameter is only updated in the default state:

$$\Gamma_t = (1 - (t^*)^{-1})\Gamma_{t-1} + (t^*)^{-1} E_t\left[u(\hat{c}_t, \hat{h}_t, \hat{x}_t, \hat{l}_t) - u(c_t, h_t, x_t, l_t) + \frac{q_{t+1}\hat{h}_t - \hat{R}_t^m \hat{b}_t}{c_{t+1}}\right] \tag{5.10}$$

where "hats" indicate the values of variables in a version of the model where impatient households maintain access to all markets and t^* is the sample size of default periods. Individual households solve this problem taking all prices as given.

[14] We truncate the normal distribution at two standard deviations. For values below -2 standard deviations, the distribution is then linear until where $b_t = 0$ where $g(\dot{}) = 0$. For values above 2 standard deviations, it is linear until 12 standard deviations where $g(\dot{}) = 0$.

Finally, agents also update their estimate of the recovery rate in the default state only. This is obtained by:

$$rec_t = rec_{t-1} + (t^*)^{-1} \left(\frac{q_t \pi_t h_{t-1}}{b_{t-1} R^m_{t-1}} - rec_{t-1} \right) \tag{5.11}$$

We now simulate the model until the learning process converges. Table 1 reports our calibration of exogenous parameters:

Table 1: Calibration

j	weight on housing	0.1
ϵ	substitutibility of housing types	0.6
α	labor's share in production function	0.67
β	impatient households' discount factor	0.90
γ	patient households' discount factor	0.99
ω	weight on rental housing	0.9
ϕ_π	policy response to inflation	2
σ	substitutibility of consumer goods	5/7
θ	degree of price stickiness	2/3
σ_a	st. dev. of innovations to productivity	0.01
σ_e	st. dev. of innovations to demand	0.02
ρ_a	AR(1) coefficient for productivity shocks	0.95
ρ_e	AR(1) coefficient for demand shocks	0.00

We consider three different values for the policy response to asset prices: $\phi_q = -1, 0, 0.5$. Table 2 reports the associated learning coefficients:

Table 2: Learning Coefficients [PRELIMINARY]

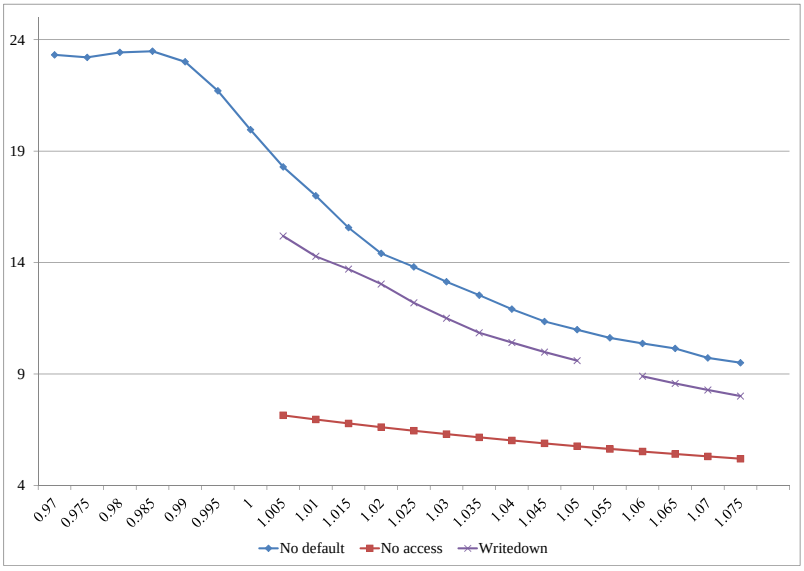
[h!]	$\phi_q = -1$	$\phi_q = 0$	$\phi_q = 0.5$
a_c	0.66	0.84	0.85
b_c	0.72	0.90	0.63
$a_{c'}$	1.49	1.49	1.47
$b_{c'}$	0.90	0.93	0.91
a_q	9.76	10.94	10.38
b_q	0.89	0.92	0.89
a_π	0.72	0.99	0.69
b_π	0.96	0.98	0.97
$\sigma_{q\pi}$	2.81	2.81	2.05
Γ	2.46	2.14	2.0
rec_t	0.91	0.79	0.84

6 Causes and Effects of Default

To examine what causes default and how default affects equilibrium, we consider the model at a fixed point in time for $\phi_q = 0$. We impose the initial conditions, $h_{t-1} = 0.15$, $b_t = 0.81$, and $A_{t-1} = 1$, chosen so that the mean value of shocks ($A_t = e_t = 1$) is close to the default cutoff. As shown in Section 7, the results that follow are representative of the model's systematic behavior.

We begin by simulating this scenario for different values of the demand shock holding A_t constant at 1.04. Figure 3 shows the results for housing prices.

Figure 3: Behavior of q_t



The demand shock increases the marginal utility of consumption. Higher values thus induce agents to substitute from housing toward consumption, reducing q_t. In this simulation, default occurs when $e_t \geq 1.005$.

The top line ignores the default constraint and continues the model unaffected and the middle line shows the case where debt is written off.[15] The bottom line shows the case where impatient households loss access to housing and credit markets. This causes a discrete amplification of the decline in $q_t$that caused default to occur.

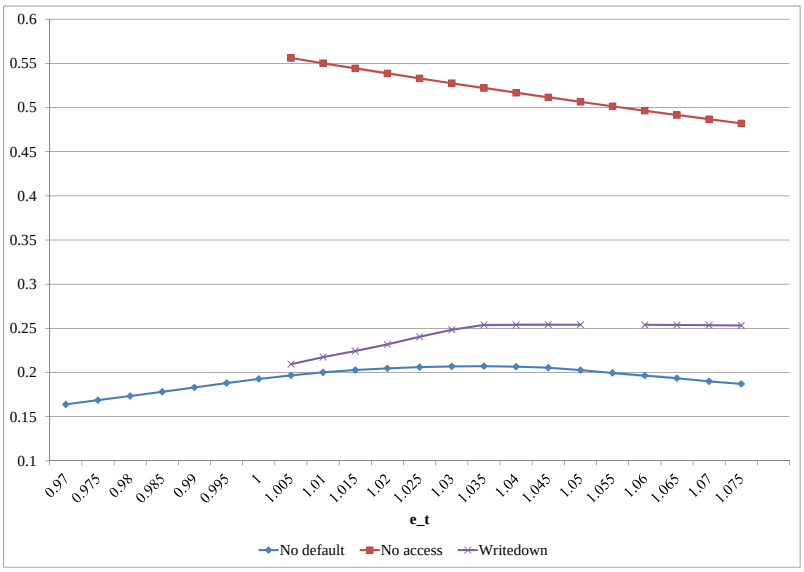
[15]The writedown and no-default cases assume an unexpected one-time deviation from the model's usual default behavior. If debt is always written down then $\Gamma = 0$, and a well behaved equilibrium does not exist.

The further reduction of housing prices occurs because default causes a misallocation of housing. Patient households own all housing when default occurs, resulting in a decreased marginal utility of housing and lower housing prices.

We also simulated the case where households lose access to bond, but not housing markets. Without access to credit, impatient households purchased little housing and the results are thus similar to the case where access to both markets is lost. We thus exclude these results from all the figures in this section.

Figure 4 illustrates the effects on impatient household's consumption.

Figure 4: Behavior of c_t

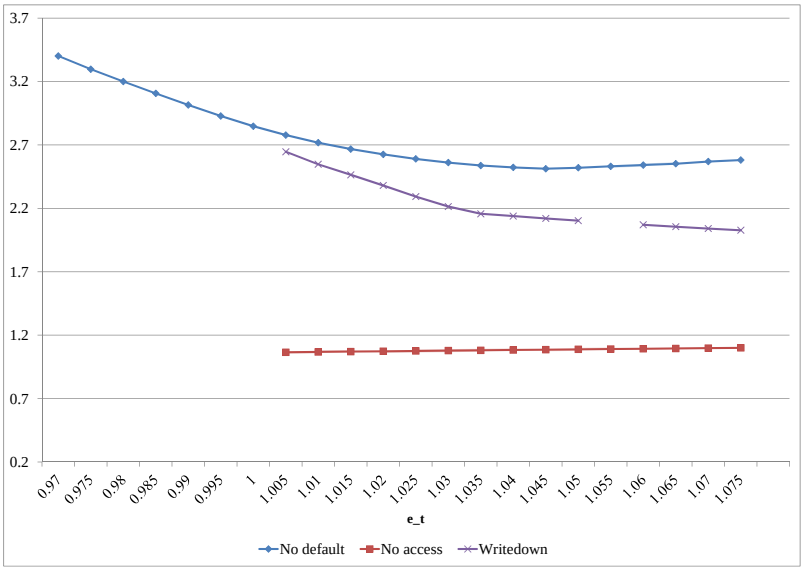


There are competing effects from default. The lack of access to credit markets reduces the ability of impatient households to borrow to finance consumption. Default also transfers wealth from from patient to impatient households which increases consumption. The loss of access to housing also provides an incentive to substitute toward consumption. The latter effects dominates and c_t increases when default occurs.

Figure 5 illustrates the effects of default on patient households' consumption.

Default transfers wealth from patient households to impatient households. Because impatient households recover housing as collateral when default occurs, the severe decline in housing prices

Figure 5: Behavior of $c_t^{'}$



shown in Figure 3 amplifies the scope of this wealth effect. As a result, default causes a discrete decrease in patient households' consumption.

Comparing Figures 4 and 5, default causes a decrease in aggregate output (which equals aggregate consumption). Our theoretical model thus matches the empirical findings from Section 3, default is not simply a result of lower housing prices, it instead causes significant decreases to both aggregate output and housing prices.

Figures 6 and 7 show the effects on housing that is rented from patient households to impatient households.

When impatient households lose access to owner occupied housing, they are forced into the rental housing market. As a result, x_t increases dramatically when default occurs. The effect on the rental rate is theoretically ambiguous because there is both increased supply and demand. In this simulation, the rental rate decreases further amplifying the adverse wealth effect that patient households experience.

Adverse productivity shocks may also induce default in the model. As A_t falls, so does output. Households wish to substitute from housing toward consumption which reduces both housing prices and rents. In this simulation default occurs if $A_t \leq 1.03$, holding e_t constant at 1. Figure 8 plots

Figure 6: Behavior of x_t

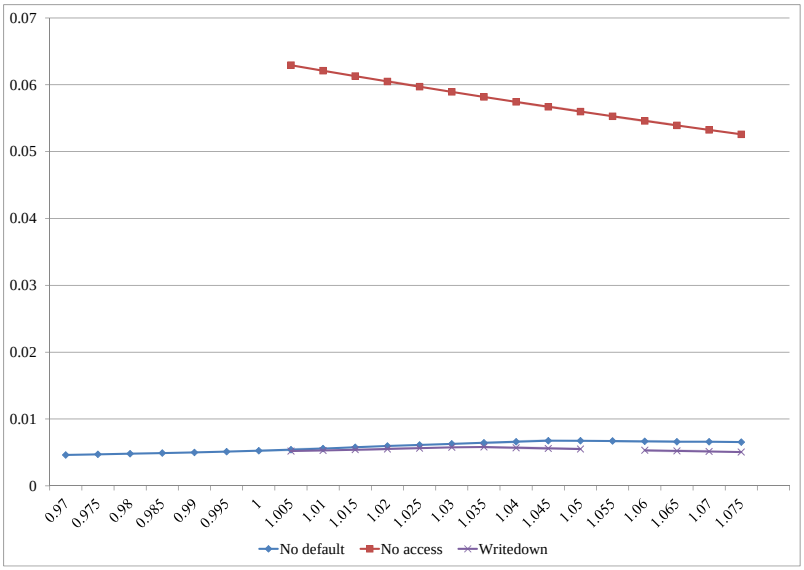
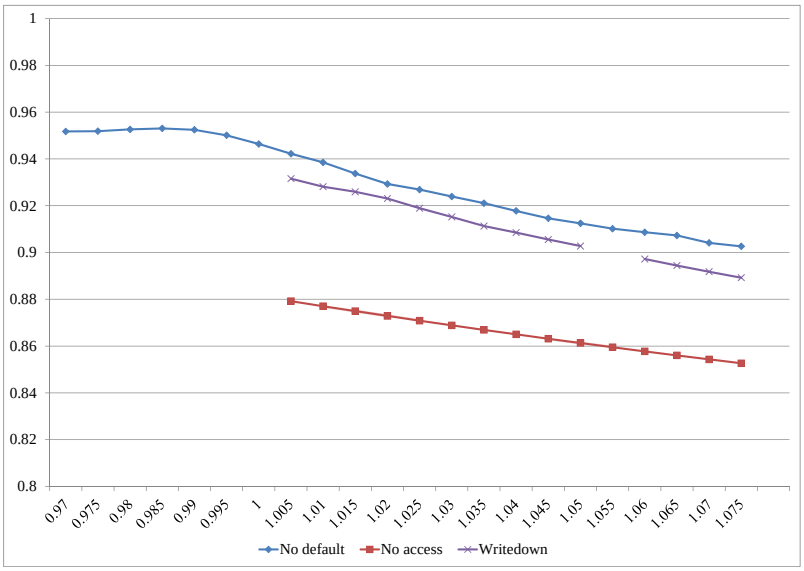


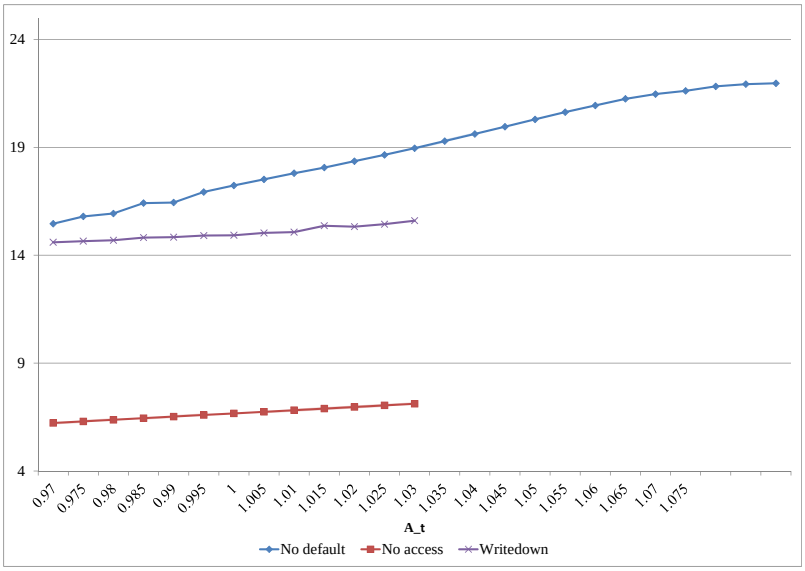
Figure 7: Behavior of v_t



housing prices under each scenario:

As before, default creates a misallocation of housing that results in an amplification of the decline in housing prices. The remaining effects of default, including a decline in aggregate consumption, are similar to the case where default results from a demand shock.

Figure 8: Behavior of q_t in Response to Productivity Shocks



7 Simulation Results

We now simulate the model to quantify its dynamics and systematically examine the effects of default. We consider three values of ϕ_q because there is considerable interest in whether or not the monetary authority would target asset prices. Iacoviello (2005) concludes that the advantages of attempting to stabilize asset prices are very small. Because our model allows for actual default, however, it is far from obvious whether this result applies to our model. Table 3 reports the result for 300 periods for the learning coefficients from Table 2.

Table 3: Equilibrium Dynamics for Different Values of ϕ_q [PRELIMINARY]

	$\phi_q = -1$		$\phi_q = 0$		$\phi_q = 0.5$	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
c_t	0.53	0.24	0.62	0.14	0.52	0.14
$c_t^{'}$	1.15	0.26	1.11	0.23	1.10	0.24
h_t	0.35	0.30	0.18	0.17	0.22	0.24
q_t	8.73	1.75	8.91	2.21	8.97	1.99
x_t	0.02	0.02	0.03	0.02	0.02	0.02
$x_t^{'}$	0.15	0.13	0.05	0.07	0.08	0.09
u_t	-1.02	0.78	-0.63	0.63	-0.93	0.47
$u_t^{'}$	0.77	0.51	0.78	0.34	0.81	0.38
$default$	14.0%		5.0%		21.7%	

The case where $\phi_q = -1$ represents a counterintuitive monetary policy where interest rates are lowered when housing prices rise. Unsurprisingly, this adds volatility and results in default more frequently than when monetary policy does not respond to asset prices ($\phi_q = 0$). Both types of households do poorly under this policy and it is clearly suboptimal.

Surprisingly, the case where $\phi_q = 0.5$ leads to the highest rate of default. Default occurs from high demand shocks and low productivity shocks. Both types of shock simultaneously increase inflation and reduce housing prices. A positive value of ϕ_q thus works against the standard monetary policy of raising interest rates in response to inflation. We speculate that this results in a generally weak monetary policy that destabilizes the system. Patient households do about as well as when $\phi_q = 0$. Impatient households, however, do worse. Among these three calibrations, the best policy appears to be ignoring fluctuations in asset prices.

We now quantify the effects of default by conducting a similar exercise as Section 6. When the model is in the default state, we compare the actual equilibrium versus the alternate case where the default condition is ignored.[16] The results are similar to the one-period analysis conducted in Section 6. Default continues to cause discrete declines in output and housing prices. Table 4 reports the results.[17]

Table 4: Mean Values with and without Default

	No Default	Default
c_t	0.41	0.68
c_t	1.45	1.03
h_t	0.28	0.00
q_t	10.68	7.83
x_t	0.007	0.078
$x_t^{'}$	0.13	0.00

[16] We are not comparing equilibrium in the default state versus that in the no default state. In Table 4, No Default means that the default condition is ignored when it would ordinarily imply default.

[17] Results are the average across all periods where default occurs for all three values of ϕ_q.

8 Conclusion

This papers adds default to a New Keynesian model with housing. We find that allowing for actual default, as opposed to just the threat of default, has important implications for the model's behavior. Default creates a misallocation of housing that results in a discrete drop in housing prices that amplifies the initial decline that caused default to occur. Furthermore, borrowers increase their consumption due to a beneficial wealth effect and an incentive to substitute toward consumption due to their limited or non-existent ability to purchase housing. Aggregate consumption, as well as lenders' consumption, however has the opposite effect.

We also show that the nature of default matters. If unpaid debt is simply written off, without an accompanying loss of access to either housing or credit markets, then default does not have large discrete effects. It is the lack of borrower access to credit markets that makes default especially interesting.

Despite the recent volatility in housing markets, this paper, to the best of our knowledge, is the first to examine the effects of actual default in a New Keynesian framework. Our model has made numerous simplifications. It would be of interest to add some additional important features of housing markets to our model. We conclude by briefly discussing three. First, it is obviously not the case that all borrowers in the economy either default or do not. It would thus be of interest to add an idiosyncratic shock to the model that allows for a default rate between 0 and 1. Second, many governments subsidize home ownership through the use of tax incentives. These could be added to the model to examine how they affect aggregate volatility. Finally, this paper treats the housing stock as constant. When default occurs, lower housing prices might incentivize producers to produce less new housing. Endogenizing the housing stock could thus yield larger effects on output than in the present paper.

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