Intersectoral Linkages, Diverse Information, and Aggregate Dynamics in a Neoclassical Model*

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Abstract

What do firms learn from their interactions in markets, and what are the implications for aggregate dynamics? We address this question in a multi-sector real-business cycle model with a sparse input-output structure. In each sector, firms observe their own productivity, along with the prices of their inputs and the price of their output. We show that general equilibrium market-clearing conditions place heavy constraints on average expectations, and characterize a set of cases where average expectations (and average dynamics) are exactly those of the full-information model. This "aggregate irrelevance" of information can occur even when sectoral expectations and dynamics are quite different under partial information, and despite the fact that each sector represents a non-negligible portion of the overall economy. In numerical examples, we show that even when the conditions for aggregate irrelevance of information are not met, aggregate dynamics remain nearly identical to the full-information model under reasonable calibrations.

Keywords: Imperfect information, Information frictions, Dispersed information, Sectoral linkages, Strategic complementarity, Higher-order expectations

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1 Introduction

The literature on information frictions in macroeconomics focuses squarely, if not exclusively, on their consequences for new-Keynesian models. The related literature includes many results supportive of the empirical relevance of information frictions. Among other findings, the literature has found that information frictions can generate realistically smooth and hump-shaped dynamics of output in response to monetary shocks (Woodford, 2001); that mistaken expectations about aggregate productivity can impact these economies in a manner akin to the "demand disturbances" of ubiquitous importance in estimated DSGE models (Lorenzoni, 2009); and that the data strongly favor information frictions relative to other forms of stickiness commonly included in new-Keynesian models (Melosi, 2011).

The key insight behind this literature, and the main feature driving the empirical success of these models, is that environments of dispersed information and strategic complementarity can deliver highly persistent responses to shocks, even when agents' own learning about the truth is relatively fast. Strategic complementarity in these models requires that agents forecast the forecasts of others before making their own choices, while dispersed information ensures that these higher-order forecasts respond sluggishly to new shocks. While this mechanism has achieved some notable success in the context of price-setting by firms, it has received relatively little attention in more neoclassical settings. (Two exceptions are Baxter et al. (2011) and Acharya (2013).) This, despite the fact that the modern selection of medium-scale DSGE models rely heavily on habits, adjustment costs, and other *ad hoc* frictions to propagate shocks and smooth short-run volatility in real variables.

This paper asks the question whether information frictions, and in particular the consequences of sluggish movement of higher-order expectations, can provide an alternative framework for capturing the slow and delayed adjustment of real variables to macroeconomic shocks. A positive finding would satisfy the demands of Occam's razor in at least two important ways. First, it would offer a unified, arguably more realistic, microfoundation for the family of adjustment costs and other frictions that now permeate the DSGE literature. Second, it offers the potential of a unifying explanation for sluggishness across the RBC and the new-Keynesian literatures; the same information friction that is crucial to understanding the real consequences of monetary policy may explain a very different set of macroeconomic observations.

We assess this question in the context of a neoclassical model with sectoral input-output linkages, in which each firm's output is (potentially) an input in the production of other firms. Recent work in the vein of Long and Plosser (1983) has explored how intermediate production structure influences the propagation of sectoral shocks to the aggregate economy.[1] We focus attention, in contrast, on how the intermediate structure of the economy affects the flow of information through the economy.

The realistic addition of a sparse input-output structure in an otherwise neoclassical economy creates scope for the enhanced importance of incomplete information for two reasons. First, it create a situation in which firms in different sectors may have access to, and care about, different pieces of information. Second, it creates an environment of strategic interdependence, since the optimal choices in one sector depends on the actions of firms in other sectors. Specifically, our model of intersectoral linkages generates strategic complementarity in investment: if a firm's intermediate input suppliers engage in more investment, then the firm should expect relatively lower marginal costs next period and therefore higher returns to its own investment.[2] Our model of sectoral interlinkages therefore delivers the same key elements, diversity of information and strategic complementarity, that underlie the empirical success of new-Keynesian models with imperfect information.

In order to study the interaction of intersectoral linkages with information asymmetries, we consider an environment in which the information of firms is directly linked to the production structure in the economy. In particular, we assume that firms only observe their own productivity, which is idiosyncratic to their own sector, and the prices of their output and those goods that are inputs in their production. If firms use only a small subset of all intermediate inputs, as is realistically the case, then they will have only a limited local set of information about the situation of the economy. In this context, information will also be dispersed: the firms in a given sector will have information that does not fully overlap with the information of firms in all other sectors.[3]

Note that, in contrast to much of the related new-Keynesian literature, agents in our environment can learn from endogenously determined variables. Instead of relying on exogenously generated signals about the state of the economy, we take up the suggestion

[1]Papers in this line include Horvath (1998), Dupor (1999), Horvath (2000), Carvalho (2010) and Acemoglu et al. (2012).

[2]Authors including Basu (1995), Nakamura and Steinsson (2010), and Carvalho and Lee (2011) have examined the complementarities among price-setting decisions of firms generated by intersectoral linkages.

[3]The term "dispersed information" is often used to describe the situation of atomistic agents, each of whose contribution to the aggregate is negligible. When necessary to distinguish between that situation and the current one in which there is a finite set of agent types with different information, we will call the later "diverse information."

of Baxter et al. (2011) and assume that firms condition their actions on the prices directly relevant to their own choices. Including the possibility of additional noisy aggregate information is trivial, but does not change the essential features of the model. Note that even if firms in a given sector use (and therefore learn directly from) only a small portion of the overall economy, the nature of our information structure places a high bar for generating persistent informational differences across sectors: the prices of my suppliers necessarily depend on the prices of my suppliers' suppliers, and so on. This setup generates an appealing symmetry between the information conveyed by prices and the need for firms to forecast others prices.

In this environment, we find that it is extremely difficult to generate a substantial impact of information frictions when firms observe and learn from their relevant market prices. This result stands in contrast to the new-Keynesian literature, which argues that strategic interactions among information constrained agents has extremely important consequences for dynamics. It also contrasts with the relatively small literature using more traditional neoclassical "island" economies, such as Baxter et al. (2011) and Acharya (2013), which find important consequences of similar information frictions. Those authors find that aggregate prices for capital may exhibit substantial non-fundamentalness, whereas the sectoral output prices (which are the key informative endogenous variables) in our environment do not.

Analytically, we characterize situations in which incomplete information has no aggregate consequences, even though sectoral dynamics can be quite different under the alternative information assumptions. This result is driven by two modeling choices. First, we assume that firms can observe and respond contemporaneously to the prices in their information set. Second, we assume that firms know the structure of the economy, including market clearing conditions. Firms can use knowledge of prices and market clearing to back out an approximate "aggregate" view of the economy; and they know others can do the same thing. Since a sector interacts with only a subset of other sectors in the economy, this aggregate information is not sufficient to determine the optimal investment choice of that particular sector, but it is enough to ensure that the combined response of all sectors to a shock is equivalent to the full-information response.

Our results are closely related to the findings of Hellwig and Venkateswaran (2011), who also characterize cases where information dispersion is irrelevant in a new-Keynesian model with market-generated information. The logic of this irrelevance is straightforward. Conjecture that the model's full-information equilibrium obtains. If, under the conjecture, the firm's information is a sufficient statistic for the firm's optimal choice, then the full-information equilibrium must be an equilibrium of the dispersed-information model. When we exclude all sectoral linkages, this basic logic also applies to the RBC model we study here and is the basis for our first proposition.

Hellwig and Venkateswaran (2011) show that departures from their "Hayekian benchmark", described above, can occur when firms face dynamic choices or face strategic complementaries in their price setting decision. In our model with intersectoral linkages, the investment choice is both inherently dynamic and strategically related to the investment choice of other sectors, yet an *aggregate* irrelevance result still holds. One of our main contributions is to show that dispersed information may have no impact on aggregates even when it has important implications for sectoral dynamics, and despite the fact that no law of large numbers applies.

After establishing these analytical results, we extend the model to a full-fledged RBC-style multi-sector model, and show numerically that our results are quite robust. We show that under the market-consistent information hypothesis, aggregate dynamics in the full model are numerically identical to the full-information economy. Despite this, sector-level responses are generally different and individual sectors are not able to determine the sectoral distribution of shocks. Sectoral information dispersion persists for long-periods of time even as aggregate responses exactly reproduce full-information responses.

Lastly, we use our analytical results to guide our exploration of a variety of steps that break the link between firm's own market-information and the (notional) aggregate state in the economy. We show that even when the conditions for the aggregate irrelevance of information fail - and when sectoral responses are *dramatically* different under partial information - aggregate responses remains strikingly similar to the full information case, with second moments changing only in the third or fourth significant digit. Hellwig and Venkateswaran (2011) also find numerically that deviations from the full-information equilibrium are typically quite small in their new-Keynesian model, when firms have access to market-based information.

Our results lead us to conclude that dispersed information and complementarities are not sufficient to generate interesting informational dynamics. General equilibrium places important restrictions on the formation of expectations, and "adding up" constraints can often erase sectoral information differences in the aggregate, *even in the absence of a law of large numbers.* The transmission of information in our realistic production structure is not enough for firms to back out the true state of the economy;

but it is enough to guarantee that information frictions have very little aggregate effect.

2 A Multi-Sector Model

We consider a discrete-time, island economy in the vein of Lucas (1972). The economy consists of a finite number of islands, each corresponding to a sector of the economy. On each island/sector reside of a continuum of identical consumers and identical locally-owned firms. Consumers derive utility from consumption and experiences disutility from supplying labor. The output of firms in each sector is supplied either as an intermediate input to other sectors or an input into a single final-good sector, exactly as in Long and Plosser (1983) and subsequent literature. The final goods sector does not employ any labor or capital, and its output is usable both as consumption and as the capital good in intermediate production.

2.1 Households

The representative household on island $j \in \{1, 2, ...N\}$ orders sequences of consumption and labor according to the per-period utility function, $u(c, l)$. Household income consists of wages paid to labor and the dividend payouts of the firms in sector j. Workers move freely across firms within their island but cannot work on other islands. Thus, the household's budget constraint in period t is given by

$$p_t c_{j,t} \leq w_{j,t} l_{j,t} + d_{j,t}, \tag{1}$$

where p_t is the period-t price of the composite final good, $w_{j,t}$ is the sector-specific wage denominated in terms of the final-good numeraire, $c_{j,t}$ and $l_{j,t}$ are island-specific consumption and labor respectively, and $d_{j,t}$ is the dividend paid by firms in sector j in period t.

The household maximizes

$$\max_{\{c_{j,t}, l_{j,t}\}_{t=0}^{\infty}} E_t^j \sum_{t=0}^{\infty} \beta^t u(c_{j,t}, l_{j,t})$$

subject to the budget constraint in (1). The expectation operator $E_t^j[X]$ denotes the expectation of a variable X conditional on the information set Ω_t^j, available on island j at time t. The first-order (necessary) conditions for the representative consumer's problem are

$$\begin{aligned} u_{c,t}(c_{j,t}, l_{j,t}) &= E_t^j[\lambda_{j,t} p_t] && (2) \\ -u_{l,t}(c_{j,t}, l_{j,t}) &= E_t^j[\lambda_{j,t} w_{j,t}]. && (3) \end{aligned}$$

where λ_t is the (current-value) Lagrange multiplier for the household's budget constraint for period t. Under the assumption of market-consistent information, which we describe presently and maintain throughout this paper, consumers will observe both the aggregate price and their wage, so that the first order conditions in (2) and (3) always hold *ex post* (i.e. without the expectation operators) as well as *ex ante*.

2.2 Production Sector

Output in each sector $j \in \{1, 2, ..., N\}$ is produced according to the production function

$$y_{j,t} = \theta_{j,t} F\left(k_{j,t}, l_{j,t}, \{x_{ij,t}\}; \{a_{ij}\}\right), \tag{4}$$

where $\theta_{j,t}$ is the overall productivity of the representative firm on island j, $k_{j,t}$ and $l_{j,t}$ are the amounts of capital and labor used, and $x_{ij,t}$ denotes the quantity of intermediate good i used by the sector-j firm. The time-invariant parameters $\{a_{ij}\}$ describe the input-output structure in the economy. We will use the convention that $a_{ij} = 0$ to denote no intermediate linkage.

Firms in sector j take prices as given and choose all inputs, including next period's capital stock, so as to maximize the consumers' expected (with respect to island-j information) present discounted value of dividends. We assume a standard capital accumulation relation

$$k_{j,t+1} = I_{j,t} + (1 - \delta) k_{j,t}, \tag{5}$$

where $I_{j,t}$ is the investment by the representative firm in industry j. Firm j's profit maximization problem is therefore

$$\max_{\{l_{j,t}, x_{ij,t}, I_{j,t}\}_{t=0}^{\infty}} E_t^j \sum_{t=0}^{\infty} \beta^t \lambda_{j,t} \left(p_{j,t} y_{j,t} - w_{j,t} l_{j,t} - \sum_{i=1}^{N} p_{i,t} x_{ij,t} - I_{j,t} \right)$$

subject to equations (4) and (5).

We assume that firms always observe the current period price of their inputs and output (we discuss this assumption below). Thus, the firm sets the marginal value product of labor and the relevant intermediate inputs equal to their price. Let $w_{j,t}$ and $p_{i,t}$ be the wage rate and the price of good i in period t. Then, we have the following intratemporal optimality conditions for firms in sector j:

$$w_{j,t} = p_{j,t} \frac{\partial y_{j,t}}{\partial l_{j,t}}, \tag{6}$$

$$p_{i,t} = p_{j,t} \frac{\partial y_{j,t}}{\partial x_{ij,t}}, \qquad \forall i \text{ s.t. } a_{ij} > 0 \tag{7}$$

Finally, firm j's first order condition with respect to the investment choice is

$$p_t = \beta E_t^j \left[\frac{\lambda_{t+1}^j}{\lambda_t^j} \left(p_{j,t+1} \frac{\partial y_{j,t+1}}{\partial k_{j,t+1}} + p_{t+1}(1-\delta) \right) \right], \tag{8}$$

where p_t denotes the price of the aggregate good used for investment.

Our assumption that firms, rather than consumers, choose future capital stands in contrast typical practice in the RBC literature. When firms and consumers have the same information, as we will assume, this choice is inconsequential: firms that maximize the discounted value of profits according to the consumer's own discount factor arrive at the capital accumulation decisions that would be preferred by consumers.

2.2.1 Final Goods Sector

Competitive firms in the final goods sector simply aggregate intermediate goods using a standard CES technology,

$$y_t = \left\{ \sum_{i=1}^{N} a_{j,t}^{\frac{1}{\zeta}} z_{j,t}^{1-\frac{1}{\zeta}} \right\}^{\frac{1}{1-1/\zeta}}, \tag{9}$$

where y_t is the output of the final good, $z_{j,t}$ is the usage of inputs from industry j, and the $a_{j,t}$ represent exogenous and potentially time-varying weights in the CES aggregator. In this case, input demands are given by

$$z_{j,t} = a_{j,t} \left(\frac{p_{j,t}}{p_t} \right)^{-\zeta} y_t. \tag{10}$$

2.3 Equilibrium

The equilibrium of the economy is described by equations (1) through (10), exogenous process for $\theta_{j,t}$ and $a_{j,t}$, and the set of island-specific market clearing conditions,

$$y_{j,t} = z_{j,t} + \sum_{i=1}^{N} x_{ji,t}. \tag{11}$$

By Walras' law, we have ignored the aggregate condition $y_t = \sum_{j=1}^{N} c_{j,t} + \sum_{j=1}^{N} I_{j,t}$. Thus, we have 1+8N+N^2 equations in the same number of unknowns: y_t, $\{p_{j,t}\}_{i=1}^N$, $\{w_{j,t}\}_{i=1}^N$, $\{c_{j,t}\}_{i=1}^N$, $\{\lambda_{j,t}\}_{i=1}^N$, $\{y_{j,t}\}_{i=1}^N$, $\{l_{j,t}\}_{i=1}^N$, $\{I_{j,t}\}_{i=1}^N$, $\{k_{j,t}\}_{i=1}^N$, and $\{x_{ij,t}\}_{i,j=1}^N$. Depending on the number of the zeros in the input-output matrix, some of the unknown $x_{ij,t}$ and corresponding first-order conditions in equation (7) will drop out reducing the size of the system. Since p_t is observed by all islands, it is common knowledge and we treat it as the numeraire.

2.4 Information

In this paper, we follow the suggestion of Graham and Wright (2010) that agents should learn about the economy based on "market-consistent" information. That is, firms' information set should include, as a minimal requirement, those prices that are generated by the markets they trade in. In our context, this means that firms see and learn from the prices of their output and any inputs into their own production. In addition to these prices, we also take as a baseline assumption that firms observe their own productivity.[4] We will denote the information set containing (full-histories) of market-relevant prices and own-productivity by $\Omega_t^{j,MC}$.

Our key observation is that, under the assumption of market-consistent information, the nature of intersectoral trade will be a crucial determinant of the information available to firms. In particular, the existence of a relatively sparse input-output structure, which is the empirically relevant case, will imply that firms will have direct observations on a very small portion of the overall economy. The macroeconomic literature on intersectoral linkages has traditionally focused on how intersectoral linkages affect the economy-wide propagation of sectoral shocks; our goal is to study how such linkages affect the propagation of information throughout the economy.

Assumptions about information are susceptible to "Lucas critique" objections that what agents choose to learn about should be influenced by policy and other non-information features of the economic environment. The assumption of market-consistent information also represents a compromise between assuming an exogenous fixed information structure (as much of the previous literature on information frictions does) and the assumption that agents endogenously design an optimal signaling mechanism according to a constraint or cost on information processing (as suggested by the literature on rational inattention initiated by Sims (2003).) Because agents form expectations based on prices, the information content of which depends on agents' actions, there is scope for an endogenous response of information to the fundamental parameters governing the environment. Thus, the assumption of market-consistent information offers at least a partial response to a "Lucas critique" argument. If one accepts that hypothesis that agents face a discretely lower marginal cost of learning from variables which they must anyways observe in their market transactions, then comparative statics for small changes in parameters can be entirely valid.

[4]Since firms know that they are identical, observing any endogenous island-specific variable (firm profits, for example) would be sufficient to infer own productivity.

3 Diverse Information: Irrelevance Results

In this section, we develop three propositions that provide important benchmarks for assessing when information frictions matter for the dynamics of the model. The first proposition establishes conditions on production and preferences that guarantee that market consistent information leads to the full-information (and therefore optimal) allocations in the economy. The final two propositions follow the tradition in the RBC and information-friction literature and focus on a linearized version of our model.

The first proposition establishes conditions under which market-consistent information is sufficient to ensure that *all* allocations are those of the full-information model.

Proposition 1 (Long and Plosser (1983) equilibrium)**.** *Suppose that capital depreciates fully each period, that the production function is cobb-douglas in all inputs, and that the time-separable utility function is given by*

$$u(c,l) = \log(c) + v(l).$$

Then the model with market consistent information replicates the full-information equilibrium of the economy.

Proof. Under cobb-douglas production and log utility, the optimality conditions of the firm in equations (7) and (8) become

$$p_{i,t}x_{i,t} = \alpha_{ij}p_{j,t}y_{j,t} \tag{12}$$

$$\frac{k_{j,t+1}}{c_{j,t}} = \beta E_t^j\left[\frac{\alpha_{kj}}{c_{j,t+1}}p_{j,t+1}y_{j,t+1}\right] \tag{13}$$

Combining this result for each intermediate sector with the island resource constraint yields

$$p_{j,t}y_{j,t} = \frac{1}{\left(1-\sum_{i=1}^{N}\alpha_{ij}\right)}\left(c_{j,t}+k_{j,t+1}\right). \tag{14}$$

Substituting this expression into the intertemporal condition of the firm,

$$\frac{k_{j,t+1}}{c_{j,t}} = \beta\frac{\alpha_{kj}}{1-\sum_{i=1}^{N}\alpha_{ij}}E_t^j\left[1+\frac{k_{j,t+2}}{c_{j,t+1}}\right]$$

Recursively substituting, the law of iterated expectations and the transversality condition yields

$$\frac{k_{j,t+1}}{c_{j,t}} = \frac{1-\sum_{i=1}^{N}\alpha_{ij}}{1-\beta\alpha_{kj}-\sum_{i=1}^{N}\alpha_{ij}},$$

which is independent of the information assumption we made. ∎

Proposition 1 therefore establishes a very strong “irrelevance” result: the presence of incomplete, diverse information of the kind considered here has no impact on either aggregate *or* sectoral quantities or prices. Market consistent information is all that is needed for the firm to back out its own optimal action. This is true even though firms may not know (and indeed generally have a very inaccurate perceptions of) what is happening in other sectors. The essence of proposition 1 is thus rather clear: aggregate outcomes are the same as under full information because individual choices do not depend on the “missing” information.

This proposition bears a close relationship to the finding of Long and Plosser (1983), which is further discussed by King et al. (1988). These authors show that, under full-information and the conditions above, income and substitution effects cancel so that the capital choice becomes essentially static and is disconnected from the stochastic nature of the underlying shock. This unravelling of the dynamic choice leads our result to closely resemble the first proposition in Hellwig and Venkateswaran (2011). Those authors show in a static model of monopolistic price-setting that market-generated information is sufficient for firms to infer their own (full-information) optimal pricing choice. As discussed in the introduction, when this is true, the full-information outcome must be an equilibrium of the partial information model.

An important difference arises, however, because in our model the investment choice becomes static only after imposing market-clearing at *all future dates.* It is only because the firm knows the model, and in particular that future choices will be also be based on the relevant prices, that it can infer its current optimal choice. Thus, the proof above highlights the fact that although the full-information optimal action is independent of expectations *ex-post*, this result depends crucially on the information assumption made and remains fundamentally driven by the formation of rational expectations about future firms choices.

Outside of the special case discussed in proposition 1, it is impossible to make generic statements about the consequences of information for the non-linear model. We hereafter focus on a symmetric, linearized version of the model in which labor is supplied inelastically, preferences take a CRRA-form with an elasticity of intertemporal substation equal to τ, and capital fully depreciates each period. While symmetry and linearization are important, the results do not depend on the latter assumptions. Moreover, while these results are exact for the linearized model, more generally they should be thought of as holding only approximately in the fully non-linear model. Nevertheless, they too provide crucial guidance on what features of the economic environment serve to limit the consequences of heterogenous information in our environment.

In order to further simplify the analysis, we take a stylized "circle" view of production interlinkages, such that firm j requires as an input the output of only one other firm. Without further loss of generality, we can order the firms such that firm j uses the output of firm $j+1$ as its input (with obvious adjustment that firm N uses the output of firm one.) As we demonstrate shortly, the details of the intermediate production structure will turn out to be irrelevant for our results, *so long as islands are symmetric in the appropriate sense.*[5]

Let $\hat{X}_t$ be the log-deviation from steady-state for any variable X_t. Moreover, let the price of the aggregate good, which is commonly observed, be fixed as the numeraire so that $\hat{p}_t = 0$ for all t. Then the linearized first order condition of the consumer is

$$\hat{c}_{j,t} = -\tau \hat{\lambda}_{j,t}$$

Intermediate production is characterized by the linearized production function

$$\hat{y}_{j,t} = \hat{\theta}_{j,t} + \varepsilon_k \hat{k}_{j,t} + \varepsilon_x \hat{x}_{j+1j,t}, \tag{15}$$

where the parameters ε_k, ε_x, $\varepsilon_k + \varepsilon_x < 1$, are functions of the model parameters. The firm's optimal choice of input $\hat{x}_{j+1j,t}$ is given by

$$\hat{p}_{j+1,t} = \hat{p}_{j,t} + \hat{\theta}_{j,t} + \varepsilon_k^x \hat{k}_{j,t} + \varepsilon_x^x \hat{x}_{j+1j,t}. \tag{16}$$

Linearizing the intertemporal equation of the firm, and using the consumer's first order condition to substitute out $\hat{\lambda}_t$ yields

$$E_t^j \left[\hat{c}_{j,t} - \hat{c}_{j,t+1}\right] = -\tau E_t^j \left[\hat{p}_{j,t+1} + \hat{\theta}_{j,t+1} + \varepsilon_k^k \hat{k}_{j,t+1} + \varepsilon_x^k \hat{x}_{j+1j,t+1}\right]. \tag{17}$$

Final goods aggregation implies that

$$\hat{y}_t = \frac{1}{N} \sum_{j=1}^{N} \left(\hat{z}_{j,t} + \hat{a}_{j,t}\right), \tag{18}$$

with $\hat{z}_{j,t}$ demanded according to

$$\hat{z}_{j,t} = \hat{a}_{j,t} - \zeta \hat{p}_{j,t} + \hat{y}_t. \tag{19}$$

[5] These details would be more important if we were concerned, as is the previous literature on intersectoral linkages, about the rate at which sectoral shocks "die out" through aggregation. The focus of our analytical results is on how information is transmitted for a given N, however, and this production structure serves only as a convenient way to capture intuition.

Sectoral and aggregate market clearing imply that

$$\hat{y}_{j,t} = \varepsilon_x \hat{x}_{j-1,t} + (1-\varepsilon_x)\hat{z}_{j,t} \tag{20}$$

$$\hat{y}_t = s_c \hat{c}_t + \frac{1-s_c}{N}\sum_{j=1}^{N} \hat{k}_{j,t+1} \tag{21}$$

where s_c is the steady-state share of consumption in output, and we have used the fact that capital depreciates fully each period. Finally, for the remainder of this section, we will assume that the coefficients $a_{j,t}$ are equal and time-invariant and that the process for $\theta_{j,t}$ is independent and identically distributed across firms according to an AR(1) process in logs. That is, we assume

$$\hat{a}_{j,t} = 0 \tag{22}$$

$$\hat{\theta}_{j,t+1} = \rho\hat{\theta}_{j,t} + \epsilon_{j,t+1}. \tag{23}$$

Equations (15) through (23) fully characterize the linearized model.

Given the linearized equations and the assumed symmetry, it is straightforward to demonstrate that, under full information, aggregates in the model can be represented without reference to sector-specific variables. To see this, let $\hat{\theta}_t \equiv \frac{1}{N}\sum_{j=1}^{N}\hat{\theta}_{j,t}$, and define $\hat{y}_t, \hat{c}_t, \hat{k}_t$ and $\hat{x}_t$ analogously. Then aggregate dynamics are captured by the following equations:

$$\hat{y}_t = \hat{\theta}_t + \varepsilon_k \hat{k}_t + \varepsilon_x \hat{x}_t \tag{24}$$

$$\hat{\theta}_t = \varepsilon_k^x \hat{k}_t + \varepsilon_x^x \hat{x}_t \tag{25}$$

$$E_t^f\left[\hat{c}_t - \hat{c}_{t+1}\right] = -\tau E_t^f\left[\hat{\theta}_{t+1} + \varepsilon_k^k \hat{k}_{t+1} + \varepsilon_x^k \hat{x}_{t+1}\right] \tag{26}$$

$$\hat{y}_t = s_c\hat{c}_t + (1-s_c)\hat{k}_{t+1} \tag{27}$$

$$\hat{\theta}_{t+1} = \rho\hat{\theta}_t + \epsilon_{t+1} \tag{28}$$

Only equation (26) is potentially affected by imperfect information of the kind we consider here; the remaining equations (24), (25), (27), and (28) will all hold under the market-consistent information assumption as well.

3.1 Irrelevance in the Linearized Model

Typically, very little can be said about environments of incomplete information, even with a linearized model, without resorting to numerical solution methods. In this case, however, we can establish some important properties of the model without fully solving the firm's inference problem. We assume throughout this section that model is parameterized so that it has a unique stationary equilibrium under full-information.

Proposition 2 establishes that when there are no interlinkages in the model, market consistent information is sufficient to reproduce the full-information equilibrium of the model.

Proposition 2. *Suppose that the share of intermediates is zero (so that, $\varepsilon_x = \varepsilon_x^k = 0$), and that the histories $\Omega_t^j \equiv \{\hat{\theta}_{j,t-h}, \hat{p}_{j,t-h}\}_{h=0}^{\infty}$. Then the equilibrium of the full information model is also an equilibrium of the diverse-information model.*

Proof. In this case, $\hat{z}_{j,t} = \hat{y}_{j,t} = -\zeta \hat{p}_{j,t} + \hat{y}_t$, implying that observations of sector j's own price and production are sufficient to determine aggregate output $\hat{y}_t$ in each period. Under the full-information equilibrium, the history of $\hat{y}_t$ is sufficient to infer $\hat{\theta}_t$ and $\hat{k}_t$, and therefore to optimally predict future $\hat{y}_t$. But the forecast of $\hat{y}_t$ is the only piece of non-local information that is required to forecast $\{\hat{p}_{j,t+i}\}_{i=1}^{\infty}$. And so, if aggregate dynamics follow the full-information path, forecasts of future $\hat{p}_{j,t+i}$ are equivalent to full-information forecasts. Each sector can thus infer its optimal investment choice under full information, and therefore sectoral allocations are consistent with the full-information equilibrium. ∎

Proposition 2 is analogous to the second proposition in Hellwig and Venkateswaran (2011) which considers the choice of price-setters who must take into account future, as well as current, conditions, and characterize conditions under which market-generate information leads to an equilibrium identical to that under full-information. Because demand and aggregate output are integrally linked, market consistent information is a powerful force for learning about aggregates, pushing the model towards it full information equilibrium.

Proposition 3 considers the consequences of our information friction, after reintroducing a strategic interaction among sectors in the form of intersectoral linkages. It is proved in the appendix.

Proposition 3. *Suppose that $\Omega_t^j = \{\hat{\theta}_{j,t-h}, \hat{p}_{j,t-h}, , \hat{p}_{j+1,t-h}, \hat{\theta}_{t-h}\}_{h=0}^{\infty}$. Then the equilibrium of the diverse information economy has the same aggregate dynamics as the full information equilibrium.*

Corollary 1. *Any equilibrium of the model with $\Omega_t^j = \{\hat{\theta}_{j,h}, \hat{p}_{j,t-h}, , \hat{p}_{j+1,t-h}, \hat{\theta}_{t-h}\}_{h=0}^{\infty}$ is also an equilibrium of the model with $\Omega_t^j = \{\hat{\theta}_{j,t-h}, \hat{p}_{j,t-h}, , \hat{p}_{j+1,t-h}, \hat{X}_{t-h}\}_{h=0}^{\infty}$, where X_{t-h} is an aggregate endogenous variable that is sufficient to infer $\hat{\theta}_t$ in the economy under full information.*

Proposition 3 is a bit more startling given earlier results in the literature. First, the presence of complementarities in decisions means that higher-order expectations matter for the decisions of individual firms. In the context of price-setting firms, such complementarity typically leads to large aggregate consequence of information frictions, and increased persistence in particular. Second, the result on aggregates holds even though sectoral expectations and choices can be substantially different under market-consistent information. Sectoral mistakes cancel each other out, despite the fact that *no law of large numbers is being invoked*, nor does any apply in our economy.

Technically, the key to the results above is that agents have some means of inferring the average state of productivity from their information set. In fact, the above result is generic so long as this "notional" aggregate state exists, as is ensured by the symmetry in the model, and when agents are able to infer that state. When these conditions hold, agents can track aggregates in the economy quite independently of their ability to track the idiosyncratic conditions relevant to their choices. Since average expectations must then be consistent with the common knowledge aggregate dynamics, expectational mistakes and, therefore mistakes in actions, must cancel out and the economy exhibits a bifurcation between what is happening in aggregate and what is happening at the sectoral level. In principal this bifurcation allows very large implications of limited information at the sectoral level while perfectly imitating the aggregate dynamics of the full-information model; in practice we will find that it is hard to generate such cases for reasonable calibrations and specifications of the information structure.

While the ability to forecast aggregates is essential to exact result in the propositions, in practice the consequences for aggregates of removing the aggregate variable $\hat{X}_t$ from the information set is not that large. In the calibration section, we show that relative prices, in conduction with the observation of own-sector productivity, do a nearly perfect job at revealing the aggregate state despite the fact that, with intermediate inputs, the firm can no longer use its prices and market clearing condition in its sector to determine aggregate output. Any movement in relative prices must be explained by a change in overall productivity in the economy. While many constellations of idiosyncratic shocks can lead to same observed relative price movements (among the two prices each sector observes) they all share roughly the same overall change in average productivity.

Both proposition 2 and 3 require a caveat at this point: we have not established that the diverse information equilibrium we describe in each proposition is the unique diverse information equilibrium of the model. Numerical experimentation, however, confirms in both cases that the equilibria in each case are unique, insofar as the full-information economies also display uniqueness.

4 Diverse Information: a Calibrated Example

Having demonstrated in a stylized version of our model that market-consistent information is sufficient to reproduce full-information aggregate dynamics, we now examine the degree to which this result generalizes in a more fully-fledged RBC model, with the nested CES production function described below, along with gradual depreciation of capital and a labor-leisure choice on the part of agents. In this section we demonstrate that the result in proposition 3, that aggregate dynamics are invariant to market-consistent limited information, holds exactly (to numerical precision) in our baseline calibration of the model with only i.i.d. sectoral productivity shocks. Moreover, although sectoral dynamics are impacted in this case, the impact of the restriction to market-consistent information is extremely small. Finally, we add a common aggregate productivity shock to sectoral productivity and show that, when the persistence of this common aggregate component is different than that of the i.i.d. sectoral component, aggregate dynamics can indeed be slightly different under limited information. But once again the effect of this information friction on aggregate dynamics is extremely small. In what follows, we label these as the case *without-aggregate-shock* and the case *with-aggregate-shock*.

Solving this model poses a technical challenge because agents must "forecast the forecasts of others" (Townsend, 1983) and because they must condition these expectations on the information embodied in endogenous variables. A recent literature has developed a number of techniques for addressing one or both of these challenges, including Kasa et al. (2004); Hellwig and Venkateswaran (2009); Baxter et al. (2011); Nimark (2011) and Rondina and Walker (2012). These techniques are not applicable here because they assume information symmetry across all agent types and a large number (or continuum) of agents. In these environments, agents are shown to care only about their own expectation of the states, the economy-wide average expectation of the same states, the average expectation of the average expectation, and so on. In contrast, with a finite number of sectors, we must keep track of complete structure of each agent-type's expectation of other agent-type's expectation, *for each level of expectation.* Concretely, firms in sector one must follow the expectations of firms in sector two and firms in sector three separately, as the dependence of their optimal choice on these two sectors is not identical.

The linearized equations in our model can be rearranged to take the form

$$0 = \sum_{j=0}^{N} \left(\left[\begin{array}{cc} \mathbf{A}_1^i & \mathbf{A}_2^i \end{array} \right] E_t^j \left[\begin{array}{c} \mathbf{x}_{t+1} \\ \mathbf{y}_{t+1} \end{array} \right] + \left[\begin{array}{cc} \mathbf{B}_1^i & \mathbf{B}_2^i \end{array} \right] E_t^j \left[\begin{array}{c} \mathbf{x}_t \\ \mathbf{y}_t \end{array} \right] \right).$$

where $j = 0$ denotes the full information set. The vector of endogenous choice variables, $\mathbf{y}_t$, has dimension $n_y \times 1$ and the vector predetermined state, $\mathbf{x}_t$, is of dimension $n_x \times 1$. The state vector $\mathbf{x}_t$ is decomposed into a vector $\mathbf{x}_t^1$ of endogenous state variables and a vector $\mathbf{x}_t^2$ of exogenous state variables which follow the autoregressive process

$$\mathbf{x}_{t+1}^2 = \rho \mathbf{x}_t^2 + \tilde{\eta} \boldsymbol{\epsilon}_{t+1} \tag{29}$$

where ρ is a square matrix of dimension n_{x^2}. The n_ϵ vector of exogenous shocks ϵ_t is assumed to be i.i.d. with identity covariance matrix.

In general, the solution to such a model is an MA(∞) process. Chahrour (2010) shows how to approximate the solution to such models as an ARMA(1,K) under the assumption that past shocks become common knowledge in period $K + 1$. Nimark (2011) discusses some theoretical requirements for a related approach to approximation to be valid, although such theoretical details have yet to be fully expounded for our current environment. The (approximate) solution to the model can then be written

$$\mathbf{x}_{t+1} = \mathbf{h}_x \mathbf{x}_t + \sum_{\kappa=0}^{K} \mathbf{h}_\kappa \epsilon_{t-\kappa} + \eta \epsilon_{t+1} \tag{30}$$

$$\mathbf{y}_t = \mathbf{g}_x x_t + \sum_{\kappa=0}^{K} \mathbf{g}_\kappa \epsilon_{t-\kappa}. \tag{31}$$

One observation, which has not been made previously, is that the matrices $\mathbf{h}_x$ and $\mathbf{g}_x$ here do not depend on the information assumption: they are the transition and observation matrices delivered by a first-order solution to the full information model. Thus the presence of incomplete (and heterogeneous) information is captured completely by the MA terms in equations (30) and (31). Chahrour (2010) provides a numerical approach for finding the matrices $\mathbf{h}_\kappa$ and $\mathbf{g}_\kappa$, which we employ in our calibration exercise below.

4.1 A Basic Calibration

In solving the calibrated model, we use the per-period utility function

$$u(c, l) = \frac{\left(c(1-l)^\varphi\right)^{1-\frac{1}{\tau}} - 1}{1 - \frac{1}{\tau}}. \tag{32}$$

Here, τ has the usual interpretation as the elasticity of intertemporal substitution and the Frisch elasticity of labor supply is given by

$$\frac{1-\bar{l}}{\bar{l}}\frac{1}{1+\varphi(1-\tau)},$$

where $\bar{l}$ is the average fraction of overall hours dedicated to production.

On the firm side of the economy, we assume that the production function $F(\cdot)$ takes the form of a nested-CES technology:

$$F\left(k_{j,t}, l_{j,t}, \{x_{ij,t}\}\right) = \left[b_1 \left\{\sum_{i=1}^{N} a_{ij} x_{ij,t}^{1-\frac{1}{\xi}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}} + b_2 \left\{a_{lj} l_{j,t}^{1-\frac{1}{\kappa}} + a_{kj} k_{j,t}^{1-\frac{1}{\kappa}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}}\right]^{\frac{1}{1-1/\sigma}} \tag{33}$$

where ξ is the elasticity of substitution between intermediate inputs, κ is elasticity of substitution between capital and labor, and σ is the elasticity of substitution between the composite intermediate input and the composite capital-labor input. Finally, a_l,a_k, b_1 and b_2 are production parameters that are set to match the cost shares of various inputs. Without loss of generality, we normalize $b_1 = b_2 = 1$.

The full calibration is summarized in table 1. We take the number of sectors to be six. Although this is a small number, we show below that this is sufficient to generate significant inference problems for the firms in our model regardless of the input-output structure. A other parameters choices warrant special attention. First, we calibrate the elasticity between the composite input, σ, to 0.20, well below unity. This calibration is important because it is a crucial parameter in determining the degree of complementarity in the model, as we discuss in the next section. This value is in line with the estimates discussed in the working-paper version of Moro (2012). Additionally, we set the final goods elasticity $\zeta = 1.5$, which is higher than the value used Horvath (2000) and somewhat less than is typically assumed in the new-Keynesian literature (which focus on the markups generated by imperfect competition). Finally, we take $\phi = 15$, which implies a frisch-elasticity in our model of slightly under two. Other parameters are set at "standard" values.

4.2 Intersectoral Linkages and Complementarities

Before proceeding to our numerical results, it is helpful to understand the nature and strength of the strategic interactions generated by the introduction of a intermediate production structure. In new-Keynesian environments, strategic complementarities pertain to the static price-setting decision of firms.[6] In contrast, investment decisions are inherently dynamic, complicating any discussion of complementarities. In order to maintain tractability, we therefore consider the strategic interactions in investment occurring in steady-state in the two-sector/two-island version of our model. Specifically, we consider the steady-state investment choice of sector one, and examine the sector one's response to a percentage deviation, Δ, of sector two investment from its steady-state equilibrium value relative.[7]

In appendix B, we show that the investment choice of sector one is given by

$$\hat{k}_1^* = \hat{k}_{1,ss} + \frac{\phi_k}{1+\phi_k}\Delta.$$

The parameter ϕ_k, therefore, measures the relative strength of the response to a change in other firm's capital choice, and therefore is the most relevant strategic complementarity in the model.

In order to do some simple comparative statics, it is helpful to specialize to the cobb-douglas production function. Specifically, assume that

$$F(k,l,x) = k^{\alpha_k(1-\mu)} l^{(1-\alpha_k)(1-\mu)} x^{\mu}.$$

In this formulation, α_k represents the shares of capital in *value-added* output in the economy and μ is the economy-wide share of intermediates in production. In this special case, we have that

$$\phi_k = \frac{1}{2}\left(\frac{\alpha_k}{1-\alpha_k}\right)\left(\frac{(1+\mu)^2}{(\mu-1)^2\zeta + 4\mu}\right). \tag{34}$$

It follows immediately that complementarity increases in the model when (1) the share of capital in value added is very large (2) the share of intermediates is large and (3) the input elasticity in the final-goods sector is relatively low. Notice, in particular, the contrast of comparative static (3) relative to the standard new-Keynesian environments where higher elasticities lead to greater, rather than smaller, pricing complementarities.

Since complementarity is increasing in μ, the limit as $\mu \to 1$ delivers an upper bound on the degree complementarity:

$$\lim_{\mu\to 1} \phi_k = \frac{1}{2}\frac{\alpha_k}{1-\alpha_k}.$$

[6] This is true even if prices are sticky, as in Angeletos and La'O (2009) , as the optimal price can be viewed as a weighted average of future target prices.

[7] The decentralized first-order conditions of sector one can be interpreted as the first order conditions of a price-taking planner, that is a planner who does not internalize her effect on the equilibrium prices faced by the sector.

Thus, under a standard calibration with a capital share of one-third, a one-percentage exogenous increase in sector two's capital choice can deliver no more than a $\frac{1/4}{1+1/4} = 0.2$-percentage increase in sector ones own capital choice, a relatively weak complementarity by the standard of the new-Keynesian literature.

In the more general version of the model, the steady-state investment complementarity may differ from the value in the fixed-labor/cobb-douglas version of the model. Figure 1 plots the value of $\frac{\phi_k}{1+\phi_k}$ against the share of intermediates under the baseline calibration of the model. Although the comparative statics derived about are robust, the bound under cobb-douglas production turns out to be quite conservative. This difference is driven primarily by the introduction of an endogenous labor choice, and by our calibration of a much-lower-than-one elasticity of substitution between the intermediate good and the capital-labor composite. Under our baseline calibration of an intermediate share of 0.6, the value of the this complementary is roughly $\frac{\phi_k}{1+\phi_k} = 0.78$. Though slightly lower than the standard new-Keynesian calibration[8], this value of complementarily is sufficient to generate a strong role for higher-order expectations in equilibrium dynamics, as we demonstrate shortly.

4.3 Sector-specific Shocks Only

We begin by considering the model with only sectoral shocks. The first row of table 2 summarizes the aggregate moments of the full-information model. The model does a relatively good job of capturing the relative variances of output, consumption, and investment. The model shows somewhat low volatility of hours, which is a well known challenge for the basic neoclassical model. However, we are primarily concerned with how the information friction may change the dynamics of the model, in particular responses over time to shocks, and to this question we turn now.

4.3.1 Exogenous Information

We first consider the dynamics of the full-information model, when productivity is determined by independent sectoral shocks only. This is our without-aggregate-shock case mentioned above. As a baseline, we begin by examining the consequences of the information friction based on an exogenous information set. In particular, we assume that investment choices are based on the information set

$$\Omega^j = \{\theta_{j,t}, s_{j,t}, \}$$

[8]See Woodford (2003) for a detailed discussion how this parameter has been calibrated in new-Keynesian models.

where $s_{j,t} = \frac{1}{N} \sum \theta_{j,t} + \nu_{j,t}$ is a signal on average productivity in the economy.

Figure 2 shows impulse responses of investment for the exogenous information and full-information economy under different assumptions about the share of intermediates in the economy. Under exogenous information, other sectors learn gradually about the shock hitting sector one. However, as the right-panel shows, the average sector has nearly completely learned the nature of the shock after five quarters. With low intermediate share, the dynamics of these first-order expectations essentially determine the investment response, which follows a very similar pattern. As the intermediate share increases, however, sluggish higher-order expectations take an increasingly important role. With an intermediate share of 0.9, complementarities lead to an extremely muted and gradual response of investment to the shock

Figure 3 shows that both output and labor supply inherit the hump-shaped dynamics of investment, while consumption does not. Consistent with these impulse responses, table 2 shows that over volatility is much lower in the baseline model. In short, the model with exogenous information generates very different dynamics than the full-information model and realistically hump-shaped responses for at least investment, output and labor.

4.3.2 Market-Consistent Information

Next, we solve the model in which agent's information contains market-based information. In particular, we consider two cases. In the first, we assume that firms observe not only their own productivity and relevant market prices but also aggregate GDP. Consistent with our theoretical results in proposition 3, we find that aggregate dynamics are identical under this restricted information assumption. This result is an exact result - it is true to the numerical tolerances we set in the algorithm - and it holds regardless of the number of periods for which we assume information remains dispersed. Despite this result, sectoral dynamics are not exactly the same under the restricted information assumption. Table 2 shows, as an example, that sectoral investment is different at the fifth decimal place. While this difference is tiny in our example, it highlights the point that - conceptually - sectoral dynamics can be quite different under market-consistent information without any impact at all on aggregate dynamics.

What explains these results? Figure 4 show the inference of a firm in sector three to the shock in sector one. While the firm's inference about the sectoral shocks faced by other sectors is imperfect (and indeed quite so!) it has perfectly inferred the movement in average productivity in the economy (the last panel.) All other firms have done the same, leaving no room for any dynamics induced by higher-order expectations (or indeed any sort of imperfect information) in the aggregate.

Next, we consider the consequence of removing GDP from the information set of firms, so that the only endogenous variables they learn from are prices. Table 2 shows that moments, both aggregate and sectoral, are very little changed. Figure 4 again show that, despite somewhat large sectoral mistake agents back out the average change in productivity so well that their inference is visually identical to that in the full-information case. The aggregate consequences of the information friction remain virtually nill.

Next, we consider the case where firms observe only the price of their own good, and not that of their supplier's good. Recall that in the case of the model without sectoral linkages this price was enough to infer the aggregate state, and therefore to generate both aggregate and sectoral irrelevance. In this case, the restricted information is not enough to infer the aggregate state exactly, but it still does a very good job at revealing it, as demonstrated by figure 6. Thus, while the demand market clearing condition is no longer available to directly infer the aggregate state of productivity in the economy, the combination of relative price and own productivity remains immensely informative about aggregates.

Finally, we consider the (perhaps unrealistic) case where firms observe their own market consistent information with a one-period lag, while observing GDP contemporaneously and compare this to the case that GDP is not observed. Table 3 shows that the addition of the GDP, which again is a sufficient statistic for the state of aggregate productivity, once again generates identical aggregate moments in the economy. In this case, however, sectoral quantities are dramatically different as demonstrated by the much-greater volatility of sectoral investment in the table. This result highlights the bifurcation that can occur in the economy between aggregate outcomes, and the makeup of sectoral movements that generate them.

4.4 Disentangling Aggregate and Sectoral Disturbances

So far we have followed the earlier literature on sectoral interlinkages in explicitly excluding aggregate productivity shocks from our consideration. Indeed the goal of most of the literature has been to argue that such linkages allow the RBC model to explain aggregate fluctuations without recourse to (implausibly large) aggregate shocks. In contrast, much the literature on the consequences of information frictions emphasizes the difficulty agents may face in disentangling aggregate and idiosyncratic shocks. For some examples, see Lorenzoni (2009); Graham and Wright (2010); Acharya (2013). While we are sympathetic to the goal of explaining aggregate fluctuations without aggregate shocks, we now turn to the question of whether adding such shocks might "reinstate" the importance of the information friction in out model.

To do this, we decompose the process for $\theta_{j,t}$ into aggregate and sectoral components, A_t and $\varsigma_{j,t}$, so that, in this with-aggregate-shock case, we have

$$\log(\theta_{j,t}) = \log(A_t) + \log(\varsigma_{j,t}). \tag{35}$$

We assume that each component follows an AR(1) process with potentially different persistence

$$\log(A_{t+1}) = \rho_A \log(A_t) + \epsilon_{t+1} \tag{36}$$

$$\log(\varsigma_{j,t+1}) = \rho_\varsigma \log(\varsigma_{j,t}) + \epsilon_{j,t+1} \tag{37}$$

where the shocks ϵ_t and $\varsigma_{j,t}$ have variances σ_A and σ_ς respectively. We calibrate the aggregate shock so that it is somewhat more persistent than the idiosyncratic shock ($\rho_\varsigma = 0.70, \rho_A = 0.95$) and so that it accounts for around 50% of aggregate fluctuations in the economy. As an aside, note that if we assume that aggregate and idiosyncratic shocks had identical persistence, as do Graham and Wright (2010), we will once again recover the result that the information assumption has zero consequence for aggregate dynamics.

Figure 7 shows that the restricted information assumption has a modest effect on aggregate dynamics, at least in response to the aggregate shock.[9] But this effect is precisely the opposite effect one might expect using the intuition from a model with exogenous information. In fact, the investment response is greater than the full-information investment response for a natural reason and one that is not linked to dispersed information at all. Since each sector sees prices they can once again infer average productivity in the economy. However they are uncertain about whether that average productivity is driven by a coincidence of (more temporary) sectoral shocks or by a (more permanent) aggregate shock. As the model is calibrated, short lived shocks lead to a relatively greater increase in optimal investment due to the standard permanent income logic. To the extent that agents perceive the aggregate shock as more temporary than it really is, they will tend to overreact to the shock leading to a larger initial change in investment.

Moreover, the presence of price information in the information set has completely killed any role for higher-order expectations in this version of the model. Figure 8 shows

[9] In fact, overall moments change very little, since the "over reaction" in response to aggregate shocks is offset somewhat by "under reaction" to sectoral shocks.

that in response to the aggregate shock, first-order and higher-order expectations of the shock are perfectly aligned, i.e. that there is no disagreement about the aggregate in the economy. As a consequence, the aggregate quantities in the economy look identical the quantities delivered by a representative agent model in which productivity has two components; one with higher persistence than the other. Figure 9 shows that, in response to a sector-specific shock, agreement is once again achieved regarding the aggregate state in the economy but disagreement about the sectoral distribution of those changes can indeed lead to large difference in higher-order expectations with respect to first-order expectations. In short, prices transmit all aggregate information, but can leave behind substantial residual disagreement about the distribution of sectoral disturbances.

5 Conclusions and Future Work

Here we have explored an environment of dispersed information and strategic interactions among firms in the which the information friction typically has no or very little effect on aggregate outcomes. This is true even though sectoral dynamics can change, sometimes substantially, and no law of large numbers is available. In one respect, this paper makes the cautionary point that informational asymmetries and strategic interdependence, the two key ingredients in nearly all the related literature, do not guarantee an important role for information. We believe that the key assumption driving this difference - that firms condition their investment choices on their market-based information - is realistic. More generally, we have argued that general equilibrium places important restrictions on expectations conditioned on endogenous information, many of which are independent of the precise details of the agent's information set. Our analytical results offer some avenues for "breaking" these results, and thereby generate an important role for information frictions. However, our preliminary quantitative results suggest even when exact irrelevance fails to hold, the plausible quantitative consequences are quite small.

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A Model Steady-State

Computing in closed-form the non-stochastic steady of the model with a nested-CES production structure is non-trivial. In this appendix, we detail the steps required. Recall that we take $p = 1$ to be the numeraire in the economy. In steady state, the following sector-specific equations must hold for each sector j:

$$\lambda_j = c_j^{-\frac{1}{\tau}}(1-l_j)^{\varphi(1-\frac{1}{\tau})} \tag{38}$$

$$\lambda_j w_j = \varphi c_j^{1-\frac{1}{\tau}}(1-l_j)^{\varphi(1-\frac{1}{\tau})-1} \tag{39}$$

$$w_j = p_j F_{l,j} \tag{40}$$

$$p_i = p_j F_{x_{ij},j} \quad \forall i \text{ s.t. } a_{ij} > 0 \tag{41}$$

$$1 = \beta(p_j F_{k,j} + 1 - \delta) \tag{42}$$

$$z_j = p_j^{-\zeta} y \tag{43}$$

$$y_j = z_j + \sum_i x_{ji} \tag{44}$$

$$p_j y_j = c_j + i_j + \sum_i p_i x_{ij} \tag{45}$$

$$y_j = F(k_j, l_j, \{x_{ij}\}) \tag{46}$$

$$I_j = \delta k_j \tag{47}$$

where

$$F(k_j, l_j, \{x_{ij}\}) = \left[\left\{\sum_i a_{ij} x_{ij}^{1-\frac{1}{\xi}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}} + \left\{a_{lj} l_j^{1-\frac{1}{\kappa}} + a_{kj} k_j^{1-\frac{1}{\kappa}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}}\right]^{\frac{1}{1-\frac{1}{\sigma}}} \tag{48}$$

and

$$F_{l,j} = y_j^{\frac{1}{\sigma}} \left\{a_{lj} l_j^{1-\frac{1}{\kappa}} + a_{kj} k_j^{1-\frac{1}{\kappa}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}-1} a_{lj} l_j^{-\frac{1}{\kappa}} \tag{49}$$

$$F_{k,j} = y_j^{\frac{1}{\sigma}} \left\{a_{lj} l_j^{1-\frac{1}{\kappa}} + a_{kj} k_j^{1-\frac{1}{\kappa}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}-1} a_{kj} k_j^{-\frac{1}{\kappa}} \tag{50}$$

$$F_{x_{ij},j} = y_j^{\frac{1}{\sigma}} \left\{\sum_i a_{ij} x_{ij}^{1-\frac{1}{\xi}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}-1} a_{ij} x_{ij}^{-\frac{1}{\xi}}. \tag{51}$$

Moreover, the following aggregate conditions must also hold

$$y = \left\{ \sum_{j=1}^{N} z_j^{1-\frac{1}{\zeta}} \right\}^{\frac{1}{1-\frac{1}{\zeta}}} \tag{52}$$

$$y = c + \sum_{j=1}^{N} i_j. \tag{53}$$

We proceed by fixing the share of good j in final production, the capital share of *value added output* in sector j, and the share of sector j's revenue dedicated to purchasing inputs from sector i. Call these shares ϕ_{jy}, ϕ_{kj}, and ϕ_{ij} respectively. Note that $\sum_{j=1}^{N} \phi_{jy}$ must equal one. These values, along with the normalization of aggregate output, $y = 1$, fix the production parameters a_{ij}, a_{kj}, a_{lj}. Since we have little a priori guidance on the value of φ, we calibrate φ to match a value for the steady-state Frisch elasticity, which we denote ϵ_{fr}.

To solve for p_j, multiply both sides of the demand function in equation (43) by p_j and rearrange to get

$$p_j = \phi_{jy}^{\frac{1}{1-\zeta}}.$$

It immediately follow that

$$z_j = \phi_{jy}/p_j.$$

Substitute the shares of revenue devoted to intermediate intermediate inputs into the market clearing condition in (44), we have that

$$y_j = z_j + \sum_i \alpha_{ji} \frac{p_i y_i}{p_j}.$$

Combing the N equations, yields a matrix expression for the values of $p_j y_j$,

$$\mathbf{py} = (I_n - IO)^{-1} \mathbf{pz} \tag{54}$$

where boldface letter represent the vector of sector values (e.g. $\mathbf{p} = [p_1, p_2, ..., p_n]'$.) Having solved for the vector $\mathbf{py}$, we can directly back out the values of sectoral production, y_j.

It follows from the definition of $\phi_{ij} \equiv \frac{p_i x_{ij}}{p_j y_j}$ that

$$x_{ij} = p_j y_j \frac{\phi_{ij}}{p_i}.$$

Multiply the intermediate input first order condition in equation (41) by x_{ij}, and sum sectors i for which $a_{ij} > 0$ to get

$$\sum_i p_i x_{ij} = p_j y_j^{\frac{1}{\sigma}} \left\{ \sum_i a_{ij} x_{ij}^{1-\frac{1}{\xi}} \right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}-1} \sum_i a_{ij} x_{ij}^{1-\frac{1}{\xi}} \tag{55}$$

$$= p_j y_j^{\frac{1}{\sigma}} \left\{ \sum_i a_{ij} x_{ij}^{1-\frac{1}{\xi}} \right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}}, \tag{56}$$

which can easily be solved for $\Omega_{1,j} \equiv \sum_i a_{ij} x_{ij}^{1-\frac{1}{\xi}}$. Plugging this value back into equation (41), yields a solution for a_{ij}

$$a_{ij} = \frac{p_i}{p_j} x_{ij}^{\frac{1}{\xi}} y_j^{-\frac{1}{\sigma}} \Omega_{1,j}^{1-\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}}.$$

Using a similar procedure, we can now solve a_{kj} and a_{lj}. First, use the production function to solve for $\Omega_{2,j} \equiv a_{lj} l_j^{1-\frac{1}{\kappa}} + a_{kj} k_j^{1-\frac{1}{\kappa}}$:

$$\Omega_{2,j} = \left(y_j^{1-\frac{1}{\sigma}} - \Omega_{1,j}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}} \right)^{\frac{1-\frac{1}{\kappa}}{1-\frac{1}{\sigma}}}$$

To back out k_j, note that

$$\phi_{kj} \equiv \frac{p_j F_{k,j} k_j}{p_j y_j - \sum_i p_i x_{ij}} \tag{57}$$

$$= \frac{F_{k,j} k_j / y_j}{1 - \sum_i \phi_{ij}}. \tag{58}$$

Rearranging equation (42) gives the following expression for capital in sector j:

$$k_j = \frac{p_j \phi_{k,j} y_j}{\beta^{-1} - 1 + \delta} \left(1 - \sum_i \phi_{ij} \right).$$

Sectoral investment is now simply $i_j = \delta k_j$. To solve for a_{kj}, use the above result and the expression for $F_{k,j}$, to find

$$a_{kj} = \phi_{k,j} \left(1 - \sum_i \phi_{ij} \right) y_j^{1-\frac{1}{\sigma}} \Omega_{2,j}^{1-\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}} k_j^{\frac{1}{\kappa}-1}.$$

From this, we can also easily determine

$$a_{lj} l_j^{1-\frac{1}{\kappa}} = \Omega_{2,j} - a_{kj} k_j^{1-\frac{1}{\kappa}}. \tag{59}$$

Using island market clearing in equation (45), sectoral output and investment can be used to compute consumption on each island. Finally, to determine sectoral labor, use consumer equations (38) and (39) to derive the relation $\varphi = \frac{w_j}{c_j}(1 - l_j)$, which implies that

$$w_j = c_j\varphi + w_j l_j. \tag{60}$$

From the labor choice condition in equation (40) we have,

$$w_j l_j = p_j y_j^{\frac{1}{\sigma}} \Omega_{2,j}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}-1} a_{lj} l_j^{1-\frac{1}{\kappa}},$$

which can be plugged back into equation (60) to determine the wage. The steady-state value of l_j follows directly. Finally, equation (59) can be used to solve for a_{lj} and consumer equations (38) can be used to determine λ_j.

B Derivation of Steady-State Investment Complementarities

In this section, I derive the expression for the steady-state complementarities in capital for the two sector model. In steady-state, equations (15), (16), (19), and (20) become respectively,

$$\hat{y}_j = \varepsilon_k \hat{k}_j + \varepsilon_x \hat{x}_{ij} \tag{61}$$

$$\hat{p}_i = \hat{p}_j + \varepsilon_k^x \hat{k}_j + \varepsilon_x^x \hat{x}_{ij} \tag{62}$$

$$\hat{z}_j = -\zeta \hat{p}_j + \frac{1}{2}\sum_i \hat{z}_i \tag{63}$$

$$\hat{y}_j = \varepsilon_x \hat{x}_{ji} + (1 - \varepsilon_x)\hat{z}_j. \tag{64}$$

Moreover, since we are considering steady-state, consumption drops from the intertemporal relation in equation (17) to yield

$$\hat{p}_1 = -\varepsilon_k^k \hat{k}_j - \varepsilon_x^k \hat{x}_{ij}. \tag{65}$$

Now, combine equations (62) and (63) to find,

$$(\hat{z}_j - \hat{z}_i) = \zeta \left(\varepsilon_k^x \hat{k}_j + \varepsilon_x^x \hat{x}_{ij}\right). \tag{66}$$

Since the above equation holds for all i and j, we have that

$$2(\hat{z}_j - \hat{z}_i) = \zeta \left[\varepsilon_k^x(\hat{k}_j - \hat{k}_i) + \varepsilon_x^x(\hat{x}_{ij} - \hat{x}_{ji})\right] \tag{67}$$

Equations (61) and (64) can be combined to yield

$$\hat{z}_j = \frac{\varepsilon_k}{1-\varepsilon_x} k_j + \frac{\varepsilon_x}{1-\varepsilon_x}(\hat{x}_{ij} - \hat{x}_{ji}), \tag{68}$$

which implies that

$$(\hat{z}_i - \hat{z}_j) = \frac{\varepsilon_k}{1-\varepsilon_x}(\hat{k}_1 - \hat{k}_2) + \frac{2\varepsilon_x}{1-\varepsilon_x}(x_{ij} - \hat{x}_{ji}). \tag{69}$$

Combine equations (67) and (69) to find

$$(\hat{z}_i - \hat{z}_j) = \phi_1(\hat{k}_i - \hat{k}_j), \tag{70}$$

where $\phi_1 \equiv \frac{\epsilon_x^x \epsilon_k - 2\epsilon_x \epsilon_k^x}{\epsilon_x^x - \epsilon_x(\epsilon_x^x - 4)}$. Plugging equation (70) back into equation (67) yields

$$\hat{p}_i = -\frac{1}{2\zeta}\phi_1(\hat{k}_i - \hat{k}_j). \tag{71}$$

Now, combine equations (65) and (62) to get

$$\hat{p}_1 = \phi_2 \hat{k}_1, \tag{72}$$

where $\phi_2 \equiv \frac{\epsilon_x^k \epsilon_k^x - \epsilon_x^x \epsilon_k^k}{\epsilon_x^x - \epsilon_x^k}$. Finally, combining equations (71) and (72), yields the expression,

$$\hat{k}_1 = \frac{1}{2\zeta}\frac{\phi_1}{\phi_2}(\hat{k}_2 - \hat{k}_1), \tag{73}$$

so that $\phi_k \equiv \frac{1}{2\zeta}\frac{\phi_1}{\phi_2}$. Evaluating the linearization coefficients for the cobb-douglas case yields the expression in equation (34).

C Proof of Aggregate Irrelevance in the Island Model with Fixed Labor

Lemma 1. *Relative prices do not depend on the aggregate shock θ_t.*

Proof. In any equilibrium, the price of the good in sector j can be written as a weighted sum of past sectoral shocks:

$$\begin{aligned}
\hat{p}_{j,t} &= \sum_{\tau=0}^{\infty}\sum_{i=1}^{N} \alpha_{ij,\tau}\hat{\theta}_{i,t-\tau} \\
&= \sum_{\tau=0}^{\infty}\sum_{i=1}^{N} \alpha_{ij,\tau}(\hat{\theta}_{i,t-\tau} - \hat{\theta}_{t-\tau}) + \sum_{\tau=0}^{\infty}\hat{\theta}_{t-\tau}\left(\sum_{i=1}^{N}\alpha_{ij,\tau}\right),
\end{aligned} \tag{74}$$

where the coefficients $\alpha_{i,j,\tau}$ are generic coefficients in the MA representation of $\hat{p}_{j,t}$. Summing this expression across the (symmetric) sectors and dividing by N yields

$$0 \equiv \hat{p}_t = \sum_{\tau=0}^{\infty} \sum_{i=1}^{N} (\hat{\theta}_{i,t-\tau} - \hat{\theta}_{t-\tau}) \left(\frac{1}{N} \sum_{j=1}^{N} \alpha_{ij,\tau} \right) + \sum_{\tau=0}^{\infty} \hat{\theta}_{t-\tau} \left(\frac{1}{N} \sum_{i=1}^{N} \alpha_{ij,\tau} \right)$$
$$= 0 + \sum_{\tau=0}^{\infty} \hat{\theta}_{t-\tau} \left(\frac{1}{N} \sum_{i=1}^{N} \alpha_{ij,\tau} \right) \quad (75)$$

where the second line follows from the fact that, by symmetry, $\left(\frac{1}{N} \sum_{j=1}^{N} \alpha_{ij,\tau} \right)$ is constant for all i and from the definition of θ_t. Since the last equation must hold for any sequence of $\theta_{t-\tau}$, however, it immediately follows that $\left(\frac{1}{N} \sum_{j=1}^{N} \alpha_{ij,\tau} \right) = 0, \forall \tau$, so that $p_{j,t}$ may only depend only the deviations of productivity from the average, $\theta_{i,t-\tau} - \theta_{t-\tau}$ and not independently on the average. ∎

Corollary 2. *Suppose that the information set of firms in sector j consists of market consistent information and $\hat{\theta}_t$. Then, sector j's expectations of any price at any future horizon must be a function only of the histories of $(\hat{\theta}_{j,t} - \hat{\theta}_t), p_{j,t}$, and $p_{j+1,t}$.*

Proof. This holds because relative prices and aggregate outcomes are orthogonal at all horizons. ∎

Corollary 3. *Suppose that the information set of firms in sector j consists of market consistent information and $\hat{\theta}_t$. Then the average expectations of any future price, $\sum_{j=1}^{N} E_t^j[\hat{p}_{j,t+\tau}] = \sum_{j=1}^{N} E_t^j[\hat{p}_{j+1,t+\tau}] = \sum_{j=1}^{N} E_t^j[\hat{p}_{j+2,t+\tau}] = ... = 0, \forall \tau$.*

We now prove proposition 3:

Proof. Our goal is to prove that

$$\frac{1}{N} \sum_{j=1}^{N} E_t^j[\hat{x}_{j,t+1}] = E_t^f[\hat{x}_{t+1}], \quad (76)$$

for any variable $\hat{x}_{j,t+1}$. If this is true, then individual Euler equations can be summed to yield the aggregate full-information Euler in equation (26) and the conclusion follows.

The action of a firm in sector j can be written

$$\hat{x}_{j,t} = \sum_{\tau=0}^{\infty} \tilde{\varphi}_{1,\tau} \hat{\theta}_{j,t-\tau} + \tilde{\varphi}_{2,\tau} \hat{p}_{j,t-\tau} + \tilde{\varphi}_{3,\tau} \hat{p}_{j+1,t+\tau} + \tilde{\varphi}_{4,\tau} \hat{\theta}_{t-\tau}.$$

The average action is thus given by

$$\hat{x}_t = \sum_{\tau=0}^{\infty} \left(\tilde{\varphi}_{1,\tau} + \tilde{\varphi}_{4,\tau} \right) \hat{\theta}_{t-\tau}.$$

and the one-period ahead full information expectation is given by

$$E_t^f[\hat{x}_{t+1}] = \left(\sum_{\tau=1}^{\infty} \left(\tilde{\varphi}_{1,\tau} + \tilde{\varphi}_{4,\tau}\right) \hat{\theta}_{t+1-\tau}\right) + \left(\tilde{\varphi}_{1,0} + \tilde{\varphi}_{4,0}\right) \rho\hat{\theta}_t.$$

The one period ahead expectation of a firm in sector j is given by $E_t^j[\hat{x}_{j,t+1}]$ is then given by

$$E_t^j[\hat{x}_{j,t+1}] = \left(\sum_{\tau=1}^{\infty} \tilde{\varphi}_{1,\tau}\hat{\theta}_{j,t+1-\tau} + \tilde{\varphi}_{2,\tau}\hat{p}_{j,t+1-\tau} + \tilde{\varphi}_{3,\tau}\hat{p}_{j+1,t+1-\tau} + \tilde{\varphi}_{4,\tau}\hat{\theta}_{t-\tau}\right) + \tilde{\varphi}_{1,0}\rho\hat{\theta}_{j,t} + \tilde{\varphi}_{1,0}E_t^j[\hat{p}_{j,t+1}] + \tilde{\varphi}_{1,0}E_t^j[\hat{p}_{j+1,t+1}] + \tilde{\varphi}_{4,0}\rho\hat{\theta}_t \tag{77}$$

Averaging across sectors yields

$$\frac{1}{N}\sum_{j=1}^{N} E_t^j[\hat{x}_{j,t+1}] = \left(\sum_{\tau=1}^{\infty} \left(\tilde{\varphi}_{1,\tau} + \tilde{\varphi}_{4,\tau}\right) \hat{\theta}_{t+1-\tau}\right) + \left(\tilde{\varphi}_{1,0} + \tilde{\varphi}_{4,0}\right) \rho\hat{\theta}_t = E_t^f[\hat{x}_{t+1}].$$

∎

Table 1: Baseline parameterization of the model.

Parameter	Concept (Target)	Vaue
N	Number of sectors	6.00
δ	Capital depreciation	0.05
κ	Capital-labor elasticity	0.99
ξ	Elasticity among intermediates (when used)	0.33
σ	Elasticity between composite inputs	0.20
ζ	Final goods elasticity	1.50
Φ_x	Share of intermediate inputs (when used)	0.00
Φ_k	Capital share of value-added	0.34
β	Discount factor	0.99
τ	Intertemporal elasticity	0.50
φ^{-1}	Implied Frisch elasticity = 1.9	15.00
ρ_ς	AR coeff. sectoral prod. shocks	0.90
ρ_d	AR coeff. sectoral demand shocks (when used)	0.00
ρ_A	AR coeff. agg shock (when used)	0.95

Table 2: Unconditional standard deviations for circle production structure under different information assumptions, with sectoral shocks only.

	Output	Cons.	Inv.	Hours	Sect. Inv.
Full Information	1.00000	0.69781	1.88524	0.35229	1.999461
Market-consistent + GDP	1.00000	0.69781	1.88524	0.35229	1.999506
Market-consistent	1.00001	0.69781	1.88527	0.35230	1.999512
Own-price only	1.03151	0.70732	1.98956	0.38088	2.046435
Exogenous	0.77456	0.64663	1.16956	0.17471	1.262126

Table 3: Unconditional standard deviations for circle production structure under different information assumptions, with sectoral shocks only.

	Output	Cons.	Inv.	Hours	Sect. Inv.
Full Information	1.000	0.698	1.885	0.352	1.999
Lagged M-C + GDP	1.000	0.698	1.885	0.352	2.565
Lagged M-C	0.939	0.660	1.850	0.277	2.700

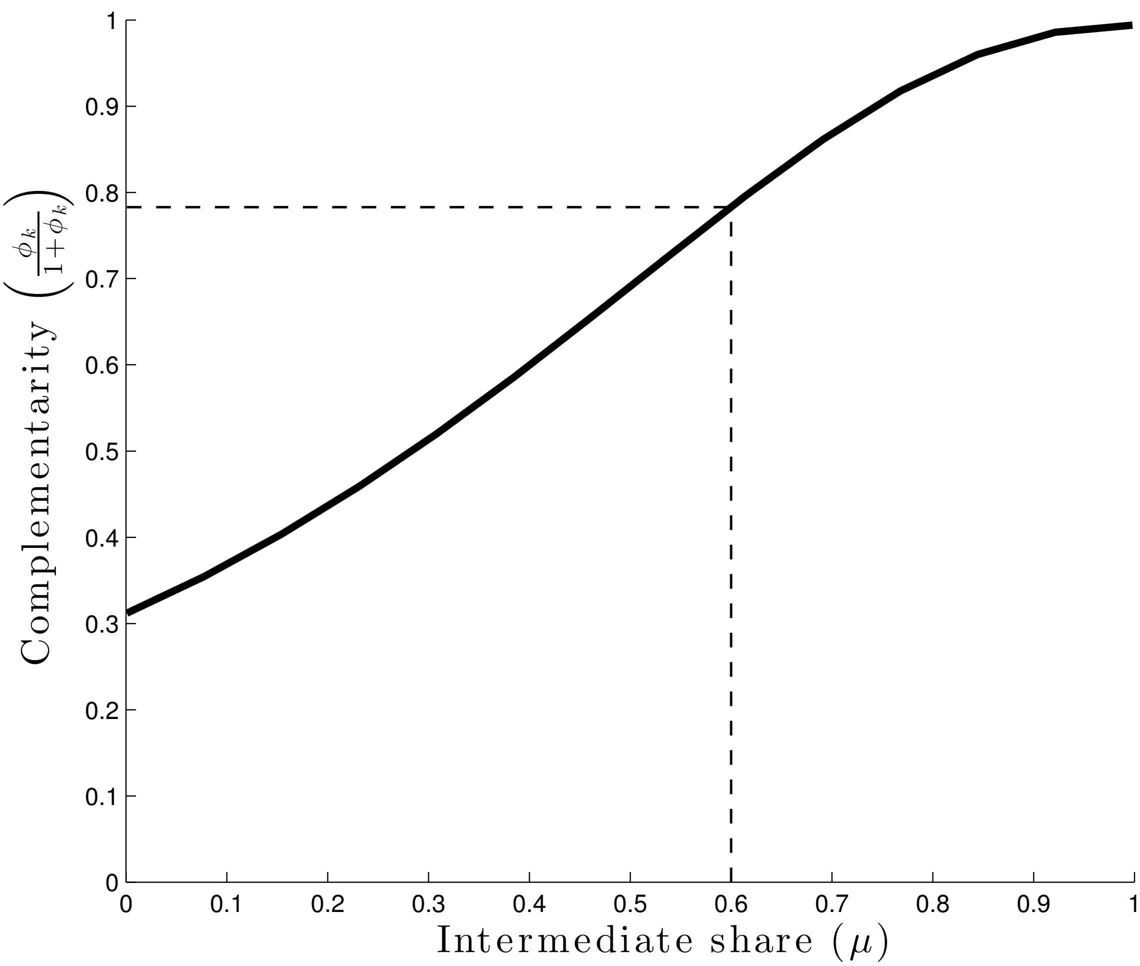

Figure 1: Steady-state complementarities for the general model, with different intermediate shares in production.

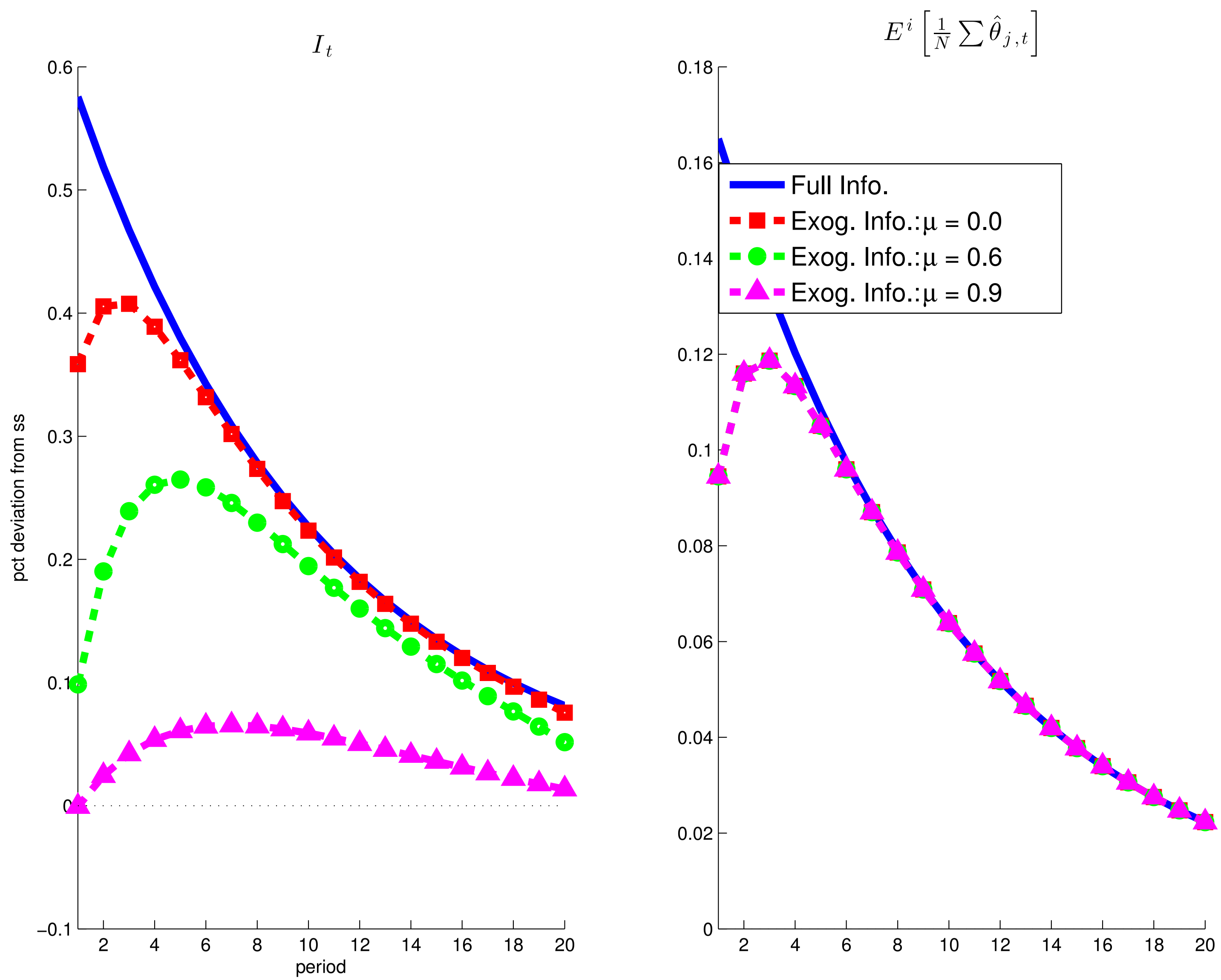

Figure 2: Aggregate investment impulse responses to a sector specific technology shock (in sector one) for the full and exogenous information economies. When the intermediate share is high, complementarities lead to a much slower "hump-shaped" response of investment.

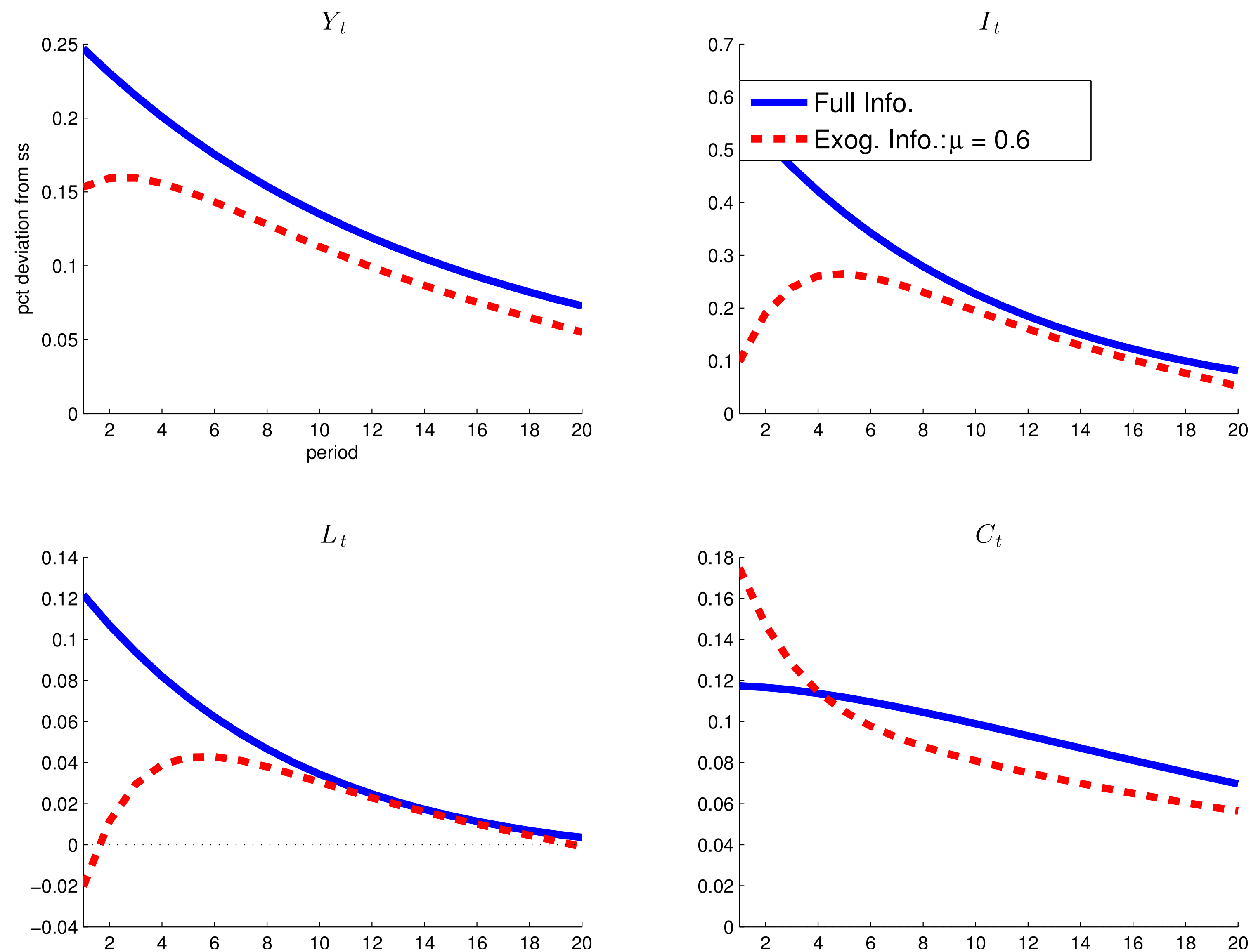

Figure 3: Aggregate impulse responses to a sector specific technology shock (in sector one) for the full and exogenous information economies under the baseline parameterization.

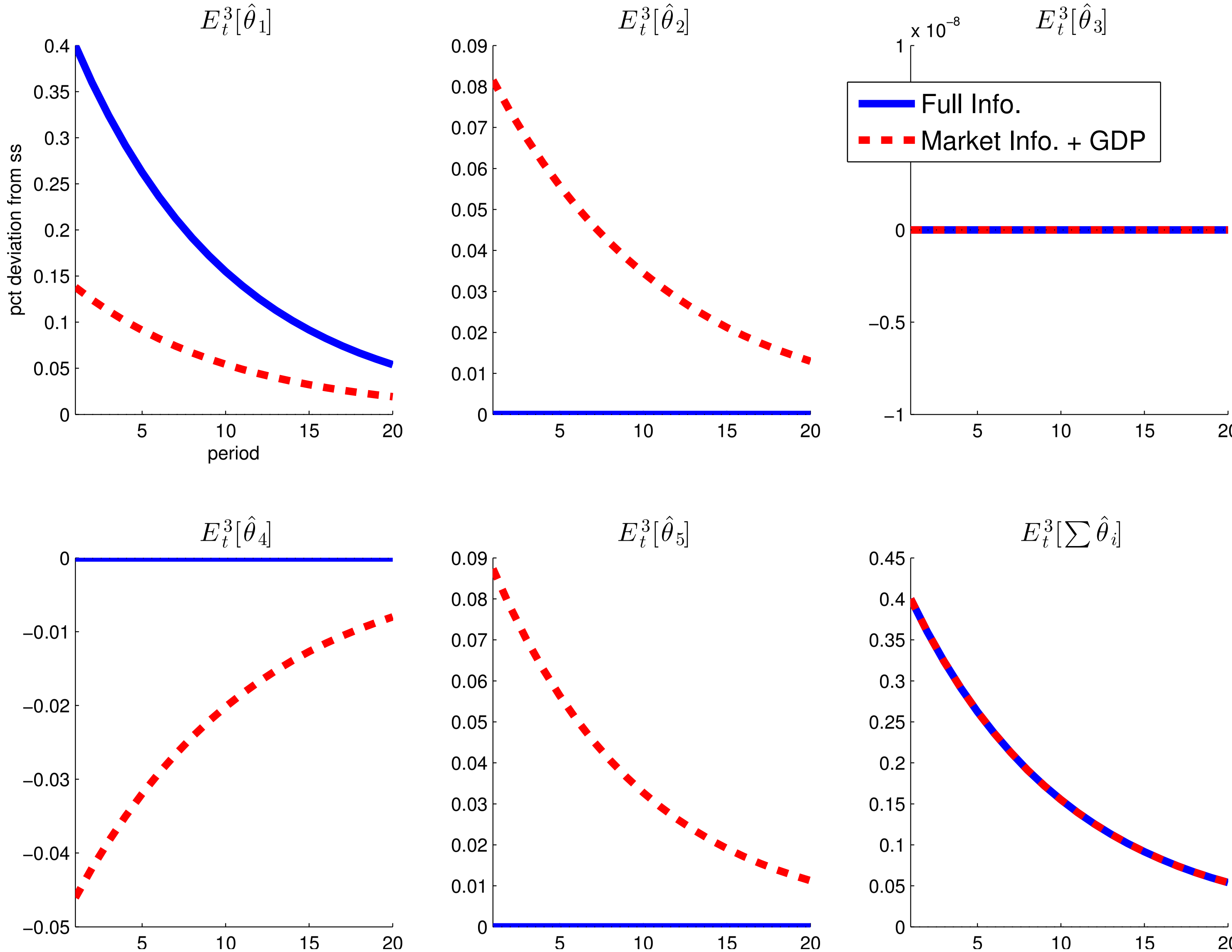

Figure 4: The inference of sector three in response to a sector-one productivity shock under the market consistent + GDP information assumption.

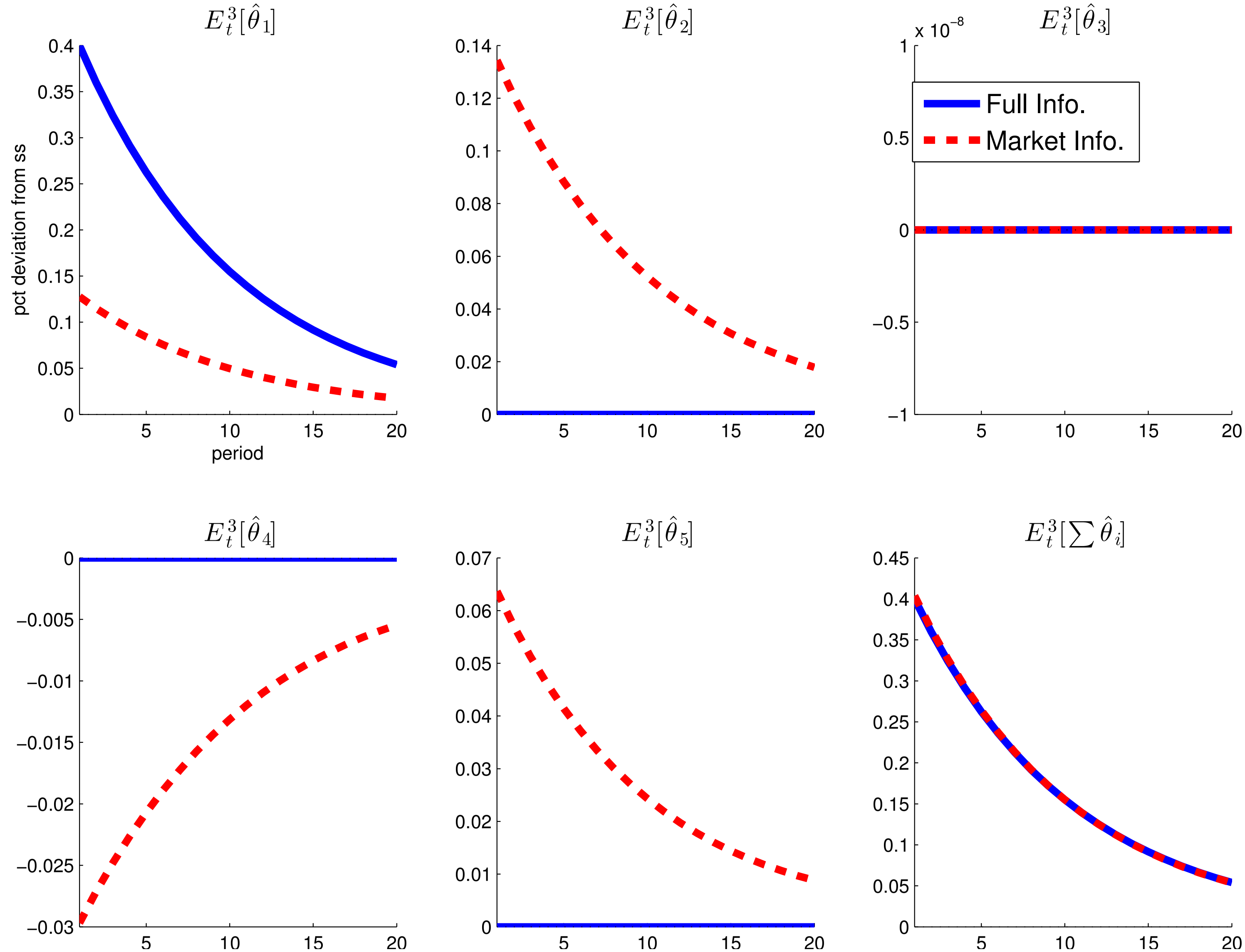

Figure 5: The inference of sector three in response to a sector-one productivity shock under the market consistent information assumption.

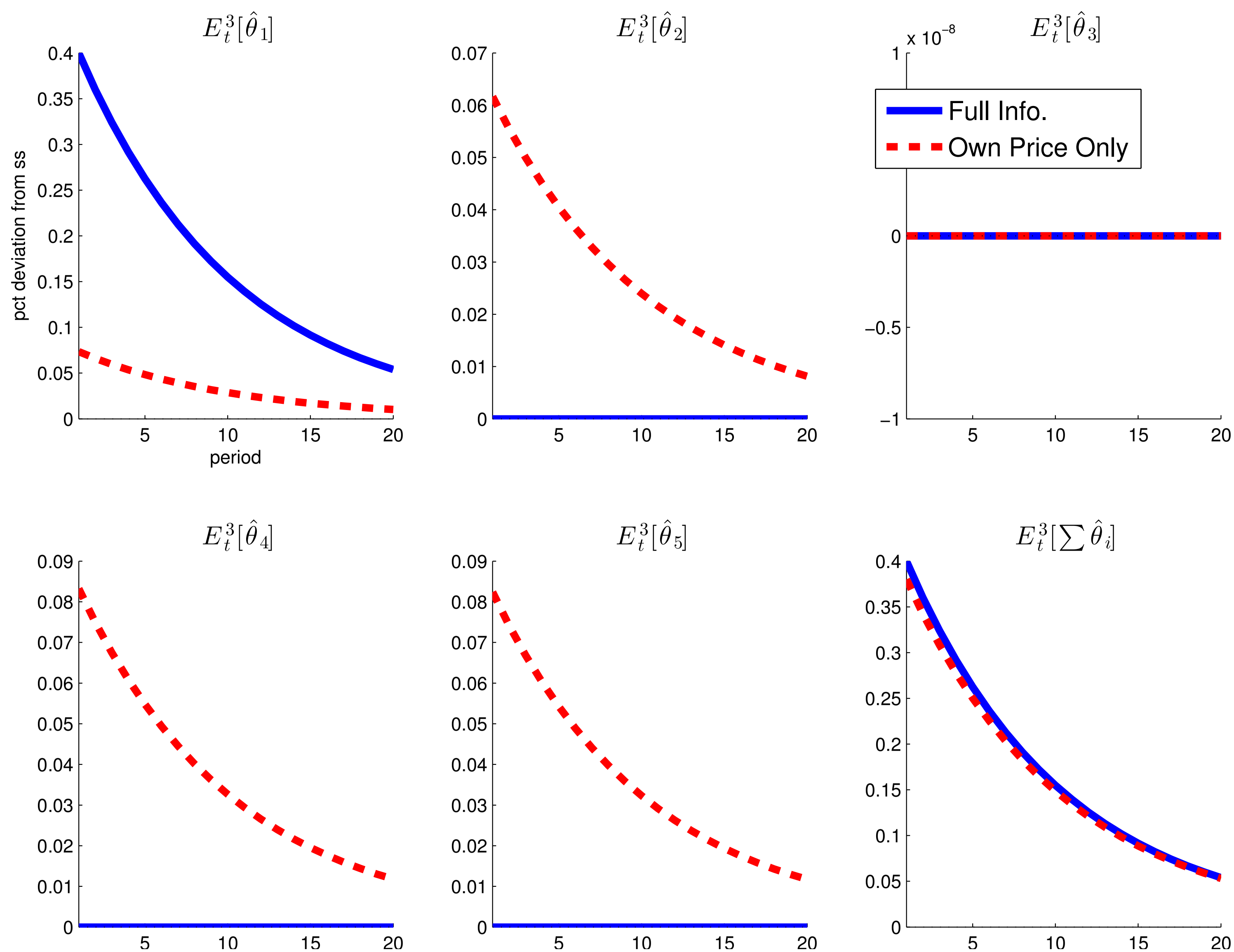

Figure 6: The inference of sector three in response to a sector-one productivity shock when only own price and productivity are observed.

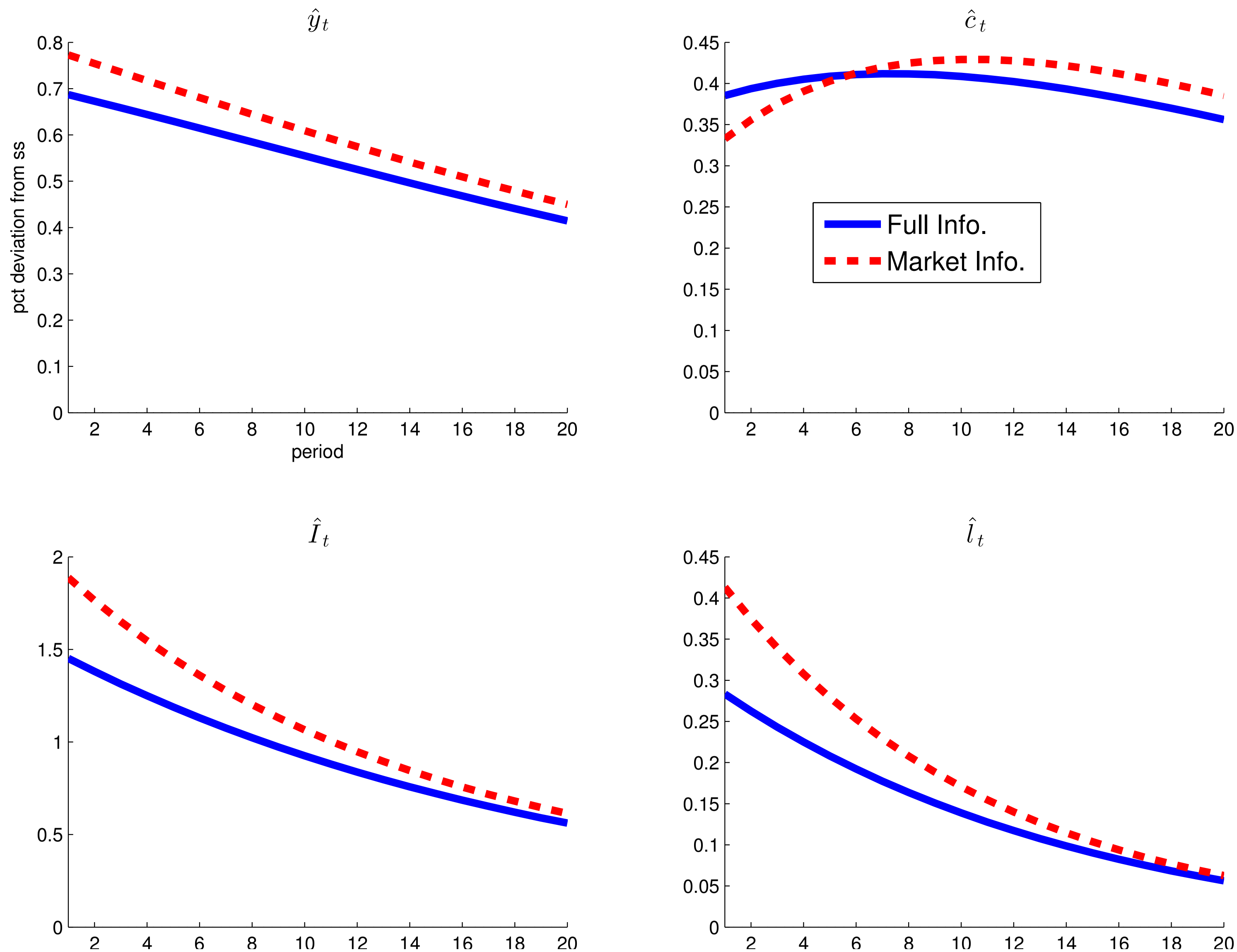

Figure 7: Aggregate impulse responses to an aggregate technology shock for the full and market-consistent information economies. Aggregate dynamics are somewhat different for market-consistent information, in contrast to the without-aggregate-shock calibration.

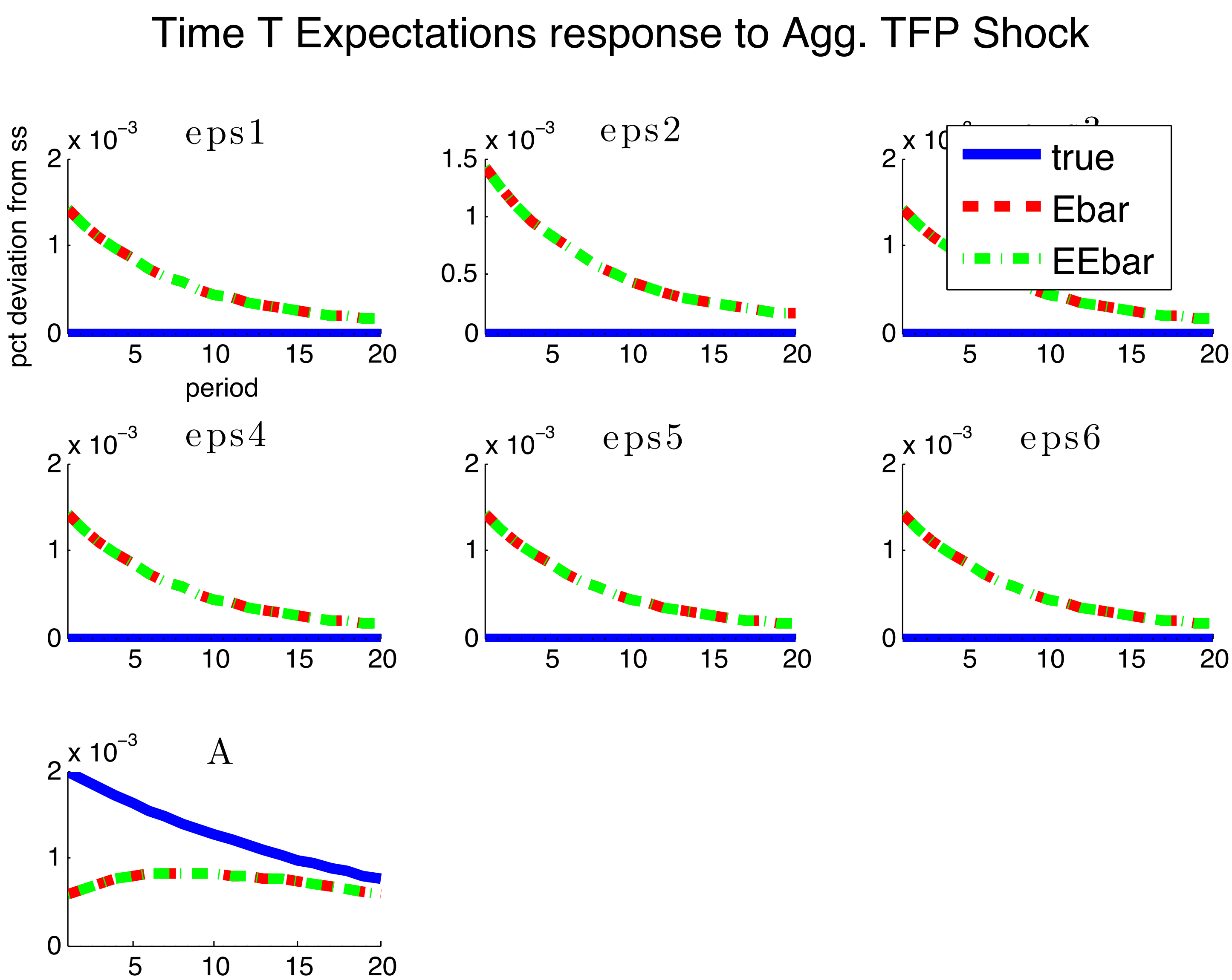

Figure 8: Average first and second order expectations in response to aggregate productivity shock. First and second order expectations are identical, indicating common knowledge among sectors.

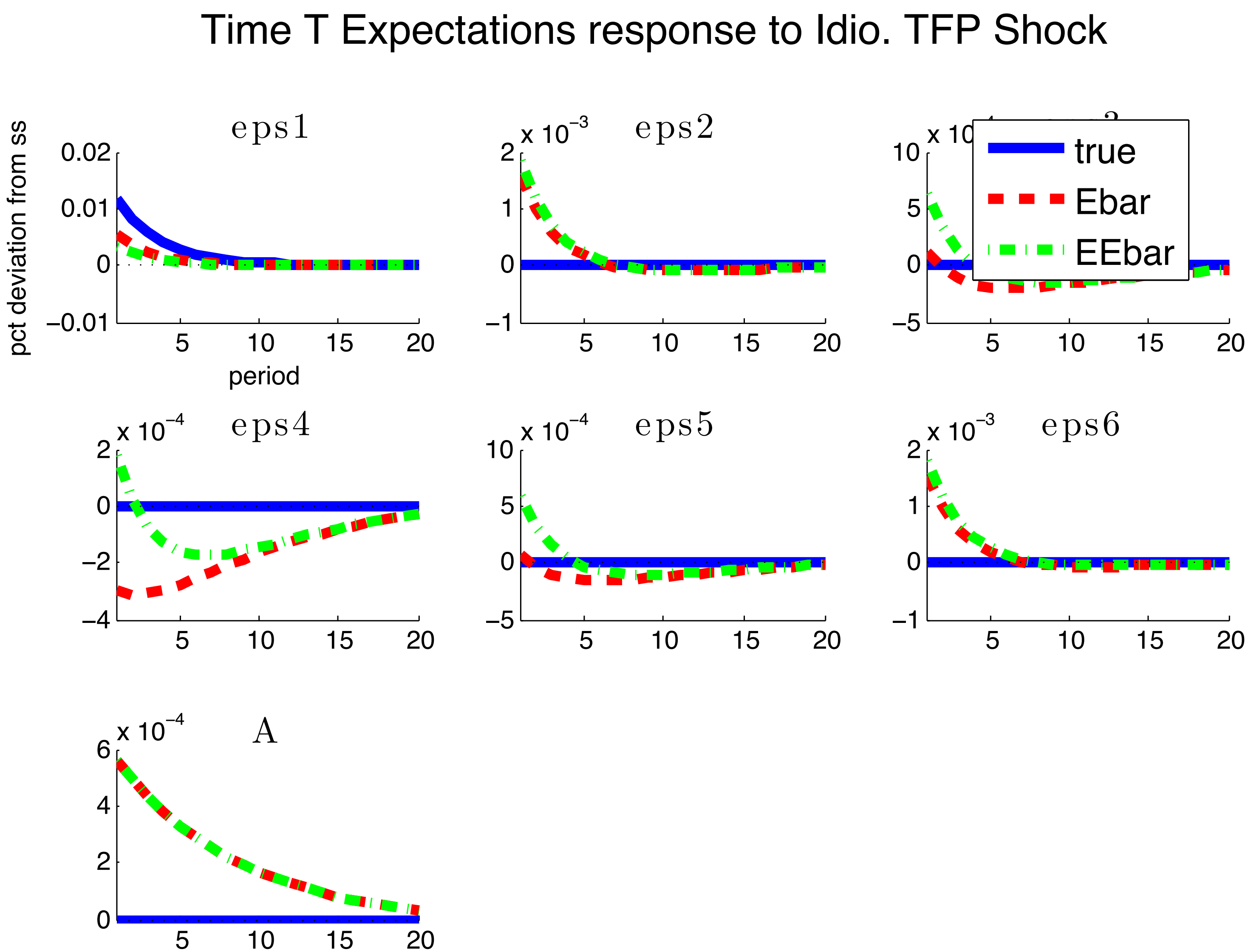

Figure 9: Average first and second order expectations in response to sectoral productivity shock. First and second order expectations diverse for sectoral shocks, indicating strong dispersed information at the sectoral level.