Entry Costs Rise with Growth

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August 22, 2022

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Abstract

Over time and across states in the U.S., both the number of firms and the number establishments are closely tied to overall employment. By comparison, the number of businesses is at best weakly related to overall output per worker. In many models of firm dynamics, trade, and growth with a free entry condition, these facts imply that the costs of creating a new firm or plant increase sharply with productivity growth. This increase in entry costs can stem from rising cost of labor used in entry combined with weak knowledge spillovers from prior entry. How entry costs vary with growth matters for welfare. For example, our findings suggest that productivity-enhancing policies will not induce entry of firms or plants, thereby limiting the total impact on welfare.

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1 Introduction

Suppose that new businesses are created with a fixed amount of output. Then a policy which boosts productivity can generate an endogenous expansion in the number of firms, with attendant gains in variety and amplifying the total boost to productivity. This multiplier effect through entry is analogous to the multiplier effect on output from physical capital accumulation in the neoclassical growth model. If instead entry requires a fixed amount of labor, however, then policies boosting productivity will fail to generate additional entry because entry costs rise with the price of labor.

Widely used models of firm dynamics, growth, and trade make different assumptions about entry costs. Some models assume entry costs are stable or stationary (e.g. a fixed output cost to invent a new product).[1] Other models assume entry costs rise as growth proceeds, say because entry requires a fixed amount of labor and labor becomes more expensive with growth.[2] Some studies do not take a stand but emphasize that the entry technology matters for the welfare impact of policies.[3]

In the growth literature it is common to assume spillovers from previous innovation to future innovation. This includes the classic models of Romer (1990) and Aghion and Howitt (1992) as well as many successors. Jones (1995) and Bloom, Jones, Van Reenen, and Webb (2020) argue that such spillovers are limited or even negative. Positive (or negative) idea spillovers can affect the entry costs of new firms and plants carrying such innovation. Thus the extent to which entry costs rise with growth bears on whether one type of innovation

[1]Examples include Hopenhayn (1992), Hopenhayn and Rogerson (1993), Romer (1994), Foster, Haltiwanger, and Syverson (2008), Clementi and Palazzo (2016), Gutierrez, Jones, and Philippon (2019), David (2020), Boar and Midrigan (2019, 2020), and Karahan, Pugsley, and Şahin (2022a).

[2]See, for example, Lucas (1978), Grossman and Helpman (1991), Melitz (2003), Klette and Kortum (2004), Luttmer (2007), Bilbiie, Ghironi, and Melitz (2012), Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018), Atkeson and Burstein (2019), Sterk, Sedlek, and Pugsley (2021), Hopenhayn, Neira, and Singhania (2022), and Peters and Walsh (2022).

[3]See Rivera-Batiz and Romer (1991), Atkeson and Burstein (2010), Bhattacharya, Guner, and Ventura (2012), survey by Costinot and Rodríguez-Clare (2014), and Baqaee and Farhi (2021).

— that embodied in new businesses — rises with growth.

Existing evidence is limited on how entry costs change with growth and the level of development. This is perhaps why models are mixed or agnostic on the question. The evidence is mostly confined to estimates of the regulatory barriers to entry across countries, to the exclusion of the technological costs of innovating and setting up operations. Djankov, Porta, Lopez-de Silanes, and Shleifer (2002) document higher statutory costs of entry (relative to GDP per capita) in poor countries. Their pioneering effort spawned the influential *Doing Business* surveys conducted by the World Bank.

The overall distribution of employment across firms and plants provides some indirect evidence. Laincz and Peretto (2006) report no trend in U.S. average firm employment. Luttmer (2007, 2010) shows that entry costs proportional to average productivity are necessary for the existence of a stationary firm size distribution in various growth models. Across countries, however, Bento and Restuccia (2017) document higher employment per establishment in richer countries. While our paper studies secular trend in entry costs, Karahan, Pugsley, and Şahin (2022b) look at whether entry costs are fixed in the short run, and Gutiérrez and Philippon (2019) focus on the cross-industry relationship between industry entry rate and industry Tobin's Q.

In this paper, we provide evidence on how the average employment per firm or establishment varies with the level of overall labor productivity. We look over time and across states in the Business Dynamics Statistics (BDS) maintained by the U.S. Census, in particular from 1978 through 2019. We combine this Census data with U.S. Bureau of Economic Analysis (BEA) data on aggregate and state labor productivity. We argue that these simple empirical elasticities discipline the nature of entry costs in widely used models.

We find that average employment per firm or establishment varies little with the level of labor productivity, both over time and across states. These patterns imply that revenue per enterprise increases sharply with growth. Enterprises evidently need more revenue to satisfy the free entry condition in places and times with higher market-wide labor productivity. If higher revenue is associated with higher operating profits, then entry costs must be bigger for the zero profit condition to hold.[4]

We illustrate the implications of our empirical findings using a long run model of growing U.S. states with mobility of workers and firms. In this model, entry costs could rise with growth simply because entry is labor-intensive and labor becomes more expensive when productivity grows. Entry costs could also rise with growth because it is more costly for entrants to set up more technologically sophisticated operations as the economy advances (say due to limited or negative knowledge spillovers).[5] We use our empirical findings to estimate parameters governing the labor-intensity of entry costs and the relationship between entry costs and the level of technology. We find that fitting our facts requires that entry be labor-intensive and/or that knowledge spillovers are weak, thereby explaining why entry costs rise with growth.

We draw the following three conclusions for modeling and policy. First, if the choice is between fixed entry costs in terms of labor or output, our evidence favors denominating entry costs in terms of labor. Second, our evidence is consistent with at best weak knowledge spillovers for innovation embodied in entry. Third, productivity-enhancing policies have muted effects on entry, and hence are not amplified through endogenous entry.

The rest of the paper proceeds as follows. Section 2 describes a spatial growth model to illustrate why we care about the nature of entry costs. Section 3 presents evidence on how the number of businesses varies with growth over time and across states in the U.S. and draws potential implications for entry costs. Section 4 discusses the welfare implications and Section 5 concludes.

[4] We consider other possibilities, such as variable markups, firm exit rates, firm growth rates, discount rates, and industry composition. We will argue that these forces are too weak to explain the stability in average employment per firm or establishment.

[5] Our evidence is relevant for total entry costs, i.e. the sum of technological and regulatory barriers. If, as seen in the *Doing Business* surveys, regulatory entry costs increase modestly or even fall with development, then technological entry costs must be the dominant force pushing up entry costs with development.

2 A simple motivating model

We present a simple spatial equilibrium model à la Redding and Rossi-Hansberg (2017) and Redding (2020) to illustrate how the elasticity of entry costs with respect to growth matters for welfare. We also use the model to motivate the empirical patterns we use to infer whether entry costs rise with growth. While we use one model to set ideas, we show in Appendix D that entry costs matter for welfare in other workhorse models as well.

2.1 Environment

The economy consists of $s = 1, 2, \ldots, S$ states and an exogenous L mass of identical workers. Each worker chooses one state to live in and to supply one unit of labor to the firms in that state. Ex-ante identical firms choose which state to set up business. The mass of workers living in each state L_s and the mass of firms in each state N_s are therefore endogenous. States differ in their endowment of housing H_s, intermediate goods productivity A_s and entry efficiency A_s^e. Intermediate goods sent from state s to state s' incur an iceberg trade cost denoted by $d_{s',s} > 1$ if $s \neq s'$ and $d_{ss} = 1$. We assume the trade cost is symmetric or $d_{s,s'} = d_{s',s}$.

The government owns housing in each state. They set rent r_s for each unit of housing so that all available housing is used. Rents are then redistributed to each worker residing in the state as lump sum payment τ_s. The workers in state s own the firms in state s and receive same share of profit net of entry costs $\tilde{\pi}_s$.

2.2 Final goods production

In each state s, final goods are produced using CES technology

$$Y_s = \left[\sum_{s'=1}^{S} \int_0^{N_{s'}} y_{s,s'}(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

where $y_{s,s'}(j)$ is input of variety produced by firm j in state s' and $p_{s,s'}(j)$ is the price of this good in state s.

Profit maximization by perfectly competitive final goods producers implies that the price of the final good is

$$P_s = \left[\sum_{s'=1}^{S} \int_0^{N_{s'}} p_{s,s'}(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}$$

and demand for each variety in state s is given by

$$\frac{y_{s,s'}(j)}{Y_s} = \left(\frac{p_{s,s'}(j)}{P_s} \right)^{-\sigma}.$$

2.3 Worker's problem

Let h_s denote the housing used by each worker in state s and c_s denote the consumption of the final good. The worker's utility function is a Cobb-Douglas combination of consumption and housing

$$U_s = \left(\frac{c_s}{\alpha} \right)^{\alpha} \left(\frac{h_s}{1-\alpha} \right)^{1-\alpha}, \quad \alpha \in (0,1)$$

and workers in state s maximize U_s by choosing c_s and h_s subject to budget constraint

$$P_s c_s + r_s h_s \leq w_s + \tilde{\pi}_s + \tau_s \equiv v_s.$$

The worker's optimal choice is to spend α share of her income v_s on consumption and the rest on housing

$$P_s c_s = \alpha v_s, \quad r_s h_s = (1-\alpha) v_s.$$

2.4 Entry technology

To produce in state s, a firm buys an entry good that is produced using local labor l_s^e and the state's final consumption good y_s^e according to Cobb-Douglas technology:

$$N = \tilde{A}_s^e N_{s,t-1}^{\phi} \left(\frac{l_s^e}{\lambda}\right)^{\lambda} \left(\frac{y_s^e}{1-\lambda}\right)^{1-\lambda}, \quad \lambda \in (0,1)$$

In this technology, λ is the intensity of labor input and $\tilde{A}_s^e$ is the efficiency of entry goods production. The $N_{s,t-1}^{\phi}$ term captures spillover from the past stock of varieties. When $\phi > 1$, entry efficiency increases with past stock of varieties.

Let A_s^e denote the combined entry efficiency $\tilde{A}_s^e N_{s,t-1}^{\phi}$. Assuming that entry goods producers are perfectly competitive, the equilibrium price of the entry good p_s^e is related to factor prices and the combined entry efficiency A_s^e by

$$p_s^e \propto \frac{w_s^{\lambda} P_s^{1-\lambda}}{A_s^e} \tag{1}$$

such that the entry cost rises with factor prices and declines with entry efficiency. Furthermore, the labor and goods share of entry costs are

$$\frac{w_s L_s^e}{p_s^e N_s^e} = \lambda, \quad \frac{P_s Y_s^e}{p_s^e N_s^e} = 1 - \lambda$$

where L_s^e and Y_s^e are the aggregate labor and final goods used for entry goods production. Since we have an one-shot economy, the aggregate number of firms created N_s^e is also the total number of firms N_s. In the empirical section, we will show evidence for both the stock of firms and new firms.

2.5 Intermediate firm's problem

For simplicity, we assume all intermediate goods producers in state s are ex-ante identical and have the same productivity A_s after entry into state s so that they all make the same production choices. In the following, we will drop the j firm index. A firm in state s can produce y units of its variety using y/A_s units of labor. Since delivering a unit of the good from state s' to state s requires $d_{s,s'}$ units of the good, the labor input used by a firm in state s' for delivering y units of goods to state s is given by

$$l_{s,s'} = y\frac{d_{s,s'}}{A_{s'}}.$$

Given this technology and the demand function in each state s, a firm in state s' chooses prices $p_{s,s'}$ for each destination state s to maximize post-entry profit

$$\sum_{s=1}^{S}\left(p_{s,s'} - w_{s'}\frac{d_{s,s'}}{A_{s'}}\right)\left(\frac{p_{s,s'}}{P_s}\right)^{-\sigma} Y_s$$

The optimal price is a fixed markup over the marginal cost where the firm charges more for destinations with larger trade costs

$$p_{s,s'} = \frac{\sigma}{\sigma-1}\frac{d_{s,s'}w_{s'}}{A_{s'}}.$$

The profit and labor costs for selling to state s are thus

$$\pi_{s,s'} = \frac{p_{s,s'}y_{s,s'}}{\sigma}, \quad w_{s'}l_{s,s'} = \pi_{s,s'}(\sigma-1).$$

A firm enters in state s' if and only if total profits across all destinations exceed the entry cost or

$$\pi_{s'} \equiv \sum_{s=1}^{S}\pi_{s,s'} \geq p^e_{s'}.$$

2.6 Definition of equilibrium

Given L, $\{A_s, A^e_s, \{d_{s,s'}\}_{s'}, H_s\}_s$, an equilibrium consists of prices w_s, r_s, P_s, p^e_s in each location s and $p_{s,s'}$ for each trading pair s, s' and allocations $\{c_s, h_s, L_s, L^e_s, L^y_s, C_s, Y_s, Y^e_s, N_s, \tau_s, \{y_{s,s'}, l_{s,s'}\}_{s'}\}_s$ such that for all states s

1. consumption and housing per capita $(c_s,\ h_s)$ solve the worker's problem given prices and transfers

2. $l_{s,s'}, y_{s,s'}, p_{s,s'}$ solve the intermediate good's firms problem

3. L_s^e, Y_s^e solve the entry goods producers problem
4. the zero profit condition holds:
   $N_s(\pi_s - p_s^e) = 0, \quad \pi_s - p_s^e \geq 0, \quad N_s \geq 0$
5. land markets clear: $H_s = L_s h_s$
6. labor markets clear: $L_s = L_s^e + L_s^y$ and $L = \sum_s L_s$
7. final goods markets clear: $Y_s = C_s + Y_s^e$, where $C_s = L_s c_s$
8. government budgets are balanced: $r_s H_s = \tau_s L_s$
9. workers are indifferent between locations.

Since the model is standard, we relegate the solution of the model to Appendix A and focus next on the welfare implication and inference of the entry cost parameters λ and ϕ.

2.7 Entry technology and amplification of shocks

Next, we will use the simple model to illustrate the importance of the labor share of entry costs λ for the impact of shocks to welfare. From the utility function of workers, welfare depends on consumption per capita and housing per capita in each location. At the steady state, consumption per capita in each state is equal to the real wage in each location

$$c_s = \frac{w_s}{P_s}.$$

However, from the goods market clearing condition, the entry goods price function and the free entry condition, consumption also satisfies

$$c_s = \frac{Y_s - Y_s^e}{L_s} = \frac{Y_s - (1-\lambda)\frac{p_s^e}{P_s} N_s}{L_s} = \frac{Y_s}{L_s}\left(1 - \frac{w_s}{P_s}\frac{1-\lambda}{\sigma - 1 + \lambda}\right)$$

Therefore consumption and real wage are proportional to output per worker

$$c_s = \frac{w_s}{P_s} = \frac{Y_s}{L_s}\left(\frac{\sigma - 1 + \lambda}{\sigma}\right).$$

on the other hand, from the final goods production function with symmetric intermediate goods producers, final output Y_s satisfies

$$Y_s = \left[\sum_{s'=1}^{S} N_{s'} y_{s,s'}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = N_s^{\frac{\sigma}{\sigma-1}} y_{s,s}\left[\sum_{s'=1}^{N} \frac{N_{s'}}{N_s}\left(\frac{y_{s,s'}}{y_{s,s}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

The term in the brackets is just the expenditure on goods from state s' relative to domestic goods. Hence real wages is related to intermediate goods production by

$$\frac{w_s}{P_s} = \frac{N_s}{L_s} N_s^{\frac{1}{\sigma-1}} y_{s,s}\left[\frac{1}{b_{s,s}}\right]^{\frac{\sigma}{\sigma-1}}\left(\frac{\sigma - 1 + \lambda}{\sigma}\right)$$

where $b_{s,s}$ is the expenditure share in state s on local goods. Substitute in $y_{s,s} = A_s l_{s,s}$, real wage depends on the number of firms, local productivity, the share of production labor used to produced domestically consumed goods $n_{s,s}$ and the share of expenditure on domestically produced goods

$$\frac{w_s}{P_s} = N_s^{\frac{1}{\sigma-1}} A_s n_{s,s}\left[\frac{1}{b_{s,s}}\right]^{\frac{\sigma}{\sigma-1}}\left(\frac{\sigma - 1}{\sigma}\right)$$

From the free entry condition (12), we have

$$N_s = \frac{1}{\sigma - 1 + \lambda}\left(\frac{w_s}{p_s}\right)^{1-\lambda} A_s^e L_s \tag{2}$$

Substituting this into the previous expression for real wage yields

$$\left(\frac{w_s}{P_s}\right)^{\sigma+\lambda-2} = \frac{1}{\sigma - 1 + \lambda}\left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} A_s^e L_s A_s^{\sigma-1} n_{s,s}^{\sigma-1}\left[\frac{1}{b_{s,s}}\right]^{\sigma} \tag{3}$$

and the change in real wage is

$$\Delta \ln \frac{w_s}{P_s} = \frac{\Delta \left(\ln A_s^e + \ln L + (\sigma - 1) \ln A_s + \ln \frac{L_s}{L} + (\sigma - 1) \ln(n_{s,s}) - \sigma \ln(b_{s,s}) \right)}{\sigma + \lambda - 2} \tag{4}$$

Consider a case of symmetric states all experiencing the same A_s, A_s^e or H_s shock or L changes. In this case, $\frac{L_s}{L}$, $n_{s,s}$ and $b_{s,s}$ do not change. The elasticity of real wage in every state to the A_s shock is

$$\frac{\sigma - 1}{\sigma + \lambda - 2}$$

while the elasticity with respect to L and A_s^e shocks is

$$\frac{1}{\sigma + \lambda - 2}.$$

Since the consumption share of utility is α and housing does not change for symmetric A_s and A_s^e shocks, the elasticity of welfare to the shock is

$$\alpha \frac{\sigma - 1}{\sigma + \lambda - 2}$$

for A_s shocks and

$$\frac{\alpha}{\sigma + \lambda - 2}$$

for A_s^e shocks. Consumption increases when total population increases. However, housing per capita also declines. Hence the welfare impact of a shock to total population L is

$$\frac{\alpha}{\sigma + \lambda - 2} - (1 - \alpha).$$

Looking at the above equations, a smaller labor share in entry (lower λ) amplifies the positive effects of higher productivity, entry efficiency and population on welfare.

2.8 Why do entry costs matter?

Why is λ important for the welfare effect of shocks? The intuition behind this is akin to the capital multiplier in the neoclassical growth model. Consider the special case of only one region (and normalize $P = 1$). In the equilibrium, the real wage is

$$w = \frac{\sigma - 1}{\sigma} A N^{\frac{1}{\sigma - 1}}.$$

Hence the impact of a change in aggregate productivity (holding fixed L) on welfare is

$$\frac{\partial \ln w}{\partial \ln A} = 1 + \frac{1}{\sigma - 1} \frac{\partial \ln N}{\partial \ln A}.$$

That is, an increase in A not only raises welfare directly, but also has the potential to improve welfare indirectly through variety expansion.

From the free entry condition, the equilibrium number of varieties N satisfies

$$N \propto \frac{wL}{p^e} \propto w^{1-\lambda} A^e L$$

so that the number of varieties depends on the value of labor relative to the entry cost. Using this relationship, we arrive at

$$\frac{\partial \ln N}{\partial \ln A} = (1 - \lambda) \frac{\partial \ln w}{\partial \ln A} + \frac{\partial \ln A^e}{\partial \ln A} = (1 - \lambda) \frac{\partial \ln w}{\partial \ln A}$$

where the last equality follows from A^e being exogenous to A. That is, the elasticity of the number of varieties with respect to A is larger when the share of output used in producing varieties $1 - \lambda$ is bigger. Higher A means more output per unit of labor input, and some of this output is devoted to producing more varieties if final goods are used in entry production ($\lambda < 1$). Substituting $\frac{\partial \ln N}{\partial \ln A}$ into the equation for $\frac{\partial \ln w}{\partial \ln A}$ yields the compounding impact of A on welfare

$$\frac{\partial \ln w}{\partial \ln A} = 1 + \frac{1 - \lambda}{\sigma - 1 - (1 - \lambda)} \tag{5}$$

with the second term capturing the amplification from variety expansion.

Lower labor share in entry implies more amplification.

The amplification of an increase in productivity depends on σ, the degree of substitutability of intermediate goods, because varieties are more valuable when substitutability is low. To illustrate the potential importance of variety expansion, consider the Broda and Weinstein (2006) estimates of $\sigma \approx 4$ at the 3-digit to 4-digit product level. For this value of σ, the amplification (ratio of amplified impact to direct impact) can range from 50% when $1-\lambda=1$ to 0% when $1-\lambda=0$. Thus, for a plausible value of σ, the nature of entry costs matters immensely for the welfare impact of changes in production technology A.

2.9 Endogenous growth

The above model describes an one-shot economy where state productivity A_s is given. A simple way to introduce endogenous growth in A_s is to let each firm in state s chooses its productivity $A_{st}(j)$ given productivity $A_{s,t-1}$. As we show below, both the labor share in entry λ and the spillover of past varieties ϕ are important for the effect of shocks on the *growth* rate of the economy.

Let the entry efficiency $\tilde{A}^e_{st}$ depend on the productivity the firm chooses relative to past aggregate productivity and a shock to entry efficiency that is common to all firms in state s

$$\tilde{A}^e_{s,t} = e^{-\mu \frac{A_{s,t}(j)}{A_{s,t-1}} + \epsilon_{st}}.$$

A positive μ means that entry costs increase with $A_{s,t}(j)/A_{s,t-1}$. In each period, the firms observe the entry efficiency shock ϵ_{st} and then decide $A_{st}(j)$. As before, entry cost in the equilibrium is given by

$$\frac{p^e_{st}}{P_{st}} = e^{-\epsilon_{st}} e^{\mu \frac{A_{st}(j)}{A_{s,t-1}}} N^{-\phi}_{s,t-1} \left(\frac{w_{st}}{P_{st}}\right)^{\lambda} =: \frac{\left(\frac{w_{st}}{P_{st}}\right)^{\lambda}}{A^e_{st}(A_{st}(j), A_{s,t-1}, N_{s,t-1}, \epsilon_{st})}.$$

Profit maximization by intermediate goods producers and free entry imply that the choice of $A_{st}(j)$ by firm j satisfies[6]

$$\frac{\partial \ln \pi_{st}(A_{st}(j))}{\partial \ln A_{st}(j)} = \frac{\partial \ln p^e_{st}}{\partial \ln A_{st}(j)}.$$

Since variable profits $\pi_{st}(A_{st}(j))$ is proportional to $A_{st}(j)^{\sigma-1}$, the firm's optimal choice of $A_{st}(j)$ is given by

$$\sigma - 1 = \mu \frac{A_{st}(j)}{A_{s,t-1}}$$

and all regions have the same growth in A

$$g^A_t := \ln \frac{A_{st}(j)}{A_{s,t-1}} = \ln \frac{\sigma - 1}{\mu}.$$

which increases with the elasticity of substitution and declines with the elasticity of entry costs with growth in A. Following from this, the entry efficiency at the equilibrium is $\tilde{A}^e_{st} = e^{-(\sigma-1)+\epsilon_{st}}$.

With the equilibrium $A_{st}(j)$ and $\tilde{A}^e_{st}$, the model solves as in the level model where the number of varieties in each state grows at rate

$$g^N_{st} = \frac{g^L_t + (1-\lambda) g^{w/p}_t + \Delta \epsilon_{st}}{1-\phi} \tag{6}$$

[6]Firms choose $A_{s,t}(j)$ to maximize profit post entry costs. Hence, $A_{s,t}(j)$ satisfies the first order condition

$$\frac{\partial \pi_{st}(A_{st}(j))}{\partial A_{st}(j)} = \frac{\partial p^e_{st}(A_{st}(j))}{\partial A_{st}(j)}.$$

At the equilibrium, we also have $\pi_{st}(A_{st}(j)) = p^e_{st}(A_{st}(j))$ and hence

$$\frac{\partial \ln \pi_{st}(A_{st}(j))}{\partial \ln A_{st}(j)} = \frac{\partial \pi_{st}(A_{st}(j))}{\partial A_{st}(j)} \frac{A_{st}(j)}{\pi_{st}(A_{st}(j))} = \frac{\partial p^e_{st}(A_{st}(j))}{\partial A_{st}(j)} \frac{A_{st}(j)}{p^e_{st}(A_{st}(j))} = \frac{\partial \ln p^e_{st}(A_{st}(j))}{\partial \ln A_{st}(j)}.$$

which implies that the real wage grows at rate[7]

$$g_{st}^{w/p} = \frac{(\sigma-1)g_t^A + \frac{g_t^L + \Delta\epsilon_{st}}{1-\phi}}{\sigma - 1 - \frac{1-\lambda}{1-\phi}}. \tag{7}$$

The trend growth of real wage in each state is driven by the growth of productivity A and national population L that is common to the states. States deviate from the common trend growth rate due to entry efficiency shock $\Delta\epsilon_{st}$ that differs across states. As in the levels model, the wage effects of these driving forces are amplified through entry when λ is less than 1. In addition, the wage effects are also amplified when when there is positive spillover of past variety stock to the efficiency of creating new varieties $(0 < \phi < 1)$.

The intuition is similar to the multiplier effect we detailed previously for the λ channel. Consider again the special case of one state. An increase in the *growth* rate of population $\Delta \ln L$ raises real wage *growth* by raising the growth rate of varieties $\Delta \ln N$

$$\frac{\partial \Delta \ln w}{\partial \Delta \ln L} = \frac{1}{\sigma - 1}\frac{\partial \Delta \ln N}{\partial \Delta \ln L}. \tag{8}$$

The effect is larger when consumers care more about varieties (lower σ).

Through the entry cost, the change in the growth rate of varieties is

$$\frac{\partial \Delta \ln N}{\partial \Delta \ln L} = 1 + (1-\lambda)\frac{\partial \Delta \ln w}{\partial \Delta \ln L} + \frac{\partial \Delta \ln A^e}{\partial \Delta \ln L} \tag{9}$$

Faster population growth directly increases the growth rate of varieties. It can also indirectly increase the growth rate of varieties through two channels. The first channel is through the goods share of entry, as we discussed previously for the levels model. The power of this channel is governed by λ. The second channel is the effect on entry efficiency because $\frac{\partial \Delta \ln A^e}{\partial \Delta \ln L} = \frac{\phi \partial \Delta \ln N_{t-1}}{\partial \Delta \ln L}$. This channel was muted in the levels model where the lagged number of varieties is

[7] Population and domestic expenditure shares by state also affect the level of real wage. These capture heterogeneous entry efficiency ϵ_s, trade costs $d_{s,s'}$ or amenities H_s. For our illustration purpose, we assume these factors do not experience persistent changes and hence do not include changes in population and domestic expenditure shares in (7).

not affected by a change in the current level of population. However, higher population *growth* raises the growth rate of varieties and raises the growth of entry efficiency if there is positive knowledge spillover through varieties ($\phi > 0$).

Combining all direct and indirect channels, the effects of a change in the growth rate of population on the growth rate of varieties and real wages are

$$\frac{\partial \Delta \ln N}{\partial \Delta \ln L} = \frac{1}{1-\phi}\left(1 + (1-\lambda)\frac{\partial \Delta \ln w}{\partial \Delta \ln L}\right) \tag{10}$$

and

$$\frac{\partial \Delta \ln w}{\partial \Delta \ln L} = \frac{1}{1-\phi}\frac{1}{\sigma - 1 - \frac{1-\lambda}{1-\phi}}. \tag{11}$$

Hence, the effect of population growth on real wage growth is amplified when entry uses goods or when there is positive spillover from past varieties to creating entry.

We used the love-of-variety spatial model in this section to illustrate the importance of entry costs for welfare. In Appendix D, we show that entry costs also matter for welfare in models with span-of-control (no love-of-variety), congestion of entry and growing variety within a firm.

3 Evidence on entry costs rising with growth

Motivated by the previous section, we next consider what values of λ and ϕ are consistent with data on U.S. firms and establishments.

3.1 Inference strategy

From the free entry condition and the solution to the firm's problem, we can derive the following relationship between employment per firm and the entry cost in a state

$$\frac{L_s^y}{N_s} = (\sigma-1)\frac{\pi_s}{w_s} = (\sigma-1)\frac{p_s^e}{w_s} = (\sigma-1)\left(\frac{w_s}{P_s}\right)^{\lambda-1}\frac{1}{A_s^e} \tag{12}$$

The free-entry condition is a zero-profit condition which equalizes firm profit with the entry cost. In the simple frame work, firm profit is proportional to employment per firm. Therefore, the relationship between employment per firm with real wages across states is informative of how entry costs vary with real wages. For example, if all states have the same entry efficiency, then the model predicts that employment per firm does not vary with real wages across states if entry production uses only labor $(\lambda = 1)$ but declines with real wages if entry is denominate in output $(\lambda = 0)$.

As we will discuss later, we have data on all workers and gross state product. The model predicts a similar relationship between these data variables as it does for production workers and real wages in (12). If σ and λ are the same across states, then production worker per firm is proportional to total employment per firm. Namely, from the labor market clearing condition, we have

$$L_s = L_s^e + L_s^y = \frac{\lambda N_s p_s^e}{w_s} + L_s^y = \left(\frac{\lambda}{\sigma-1}+1\right)L_s^y.$$

Real wage can be measured using real local output per worker GSP since

$$\frac{GSP_s}{L_s} = \frac{N_s}{L_s}\sum_{s'}^{S}\frac{p_{s',s}y_{s',s}}{P_s} = \frac{N_s}{L_s}\frac{w_s}{P_s}\sum_{s'}^{S}\frac{\sigma}{\sigma-1}l_{s',s} = \frac{\sigma}{\sigma-1}\frac{w_s}{P_s}\frac{L_s^y}{L_s} = \frac{w_s}{P_s}\frac{\sigma}{\sigma-1+\lambda}.$$

Substituting the expressions for L_s and GSP_s/L_s into (12) yields the following equations in level and growth that can be taken to the data

$$\ln\frac{L_s}{N_s} = \text{constant} + (\lambda-1)\ln\frac{GSP_s}{L_s} - \phi\ln N_{s,t-1} - \ln\tilde{A}_s^e \tag{13}$$

and

$$\Delta\ln\frac{L_{st}}{N_{st}} = (\lambda-1)\Delta\ln\frac{GSP_{st}}{L_{st}} - \phi\Delta\ln N_{s,t-1} - \Delta\epsilon_{st} \tag{14}$$

We will first use OLS regression of the form in (13) and (14) to show that employment per firm is stable relative to the variations in GSP per worker and lagged number of firms. From the perspective of our model, these patterns suggest that entry costs rise with labor productivity across states and over time within states. However, the OLS regression coefficient do not correctly identify the mechanism for entry costs rising with growth (the values of λ and ϕ) because GSP_s and the growth of N_s are endogenous to entry efficiency $\tilde{A}_s^e$ and $\Delta\epsilon_{st}$. We will calibrate the model to data on L_s, N_s, $N_{s,t-1}$, GSP_s, P_s and bilateral trade shares to infer the values of λ and ϕ.

3.2 Empirical patterns

We use the Business Dynamics Statistics (BDS) data from the Census Bureau to calculate employment L_{st} and number of firms or establishments N_{st}. We use real gross value added by state from the Bureau of Economic Analysis (BEA) to calculate GSP_{st}. These variables allow as to run the OLS regression in (13) and (14). We also use the Commodity Flow Survey to calculate bilateral trade shares $b_{s,s'}$ when we calibrate the model to infer values for ϕ and λ. The BDS and BEA data combined extends from 1978 to 2019 and is available yearly while the Commodity Flow Survey is available every five years from 1997 to 2017. In addition to looking at average employment per firm, we also look at new firm and establishments. We describe the data in more detail in Appendix B.

Table 1 displays the result of regressing log employment per firm or plant in the U.S. on log real GDP per worker and lagged log number of firms or establishment, also at the national level. The data is yearly from 1979 to 2019.[8] The first column displays the result of using employment per firm and lagged number of firms while the second column displays the result of using employment per plant and lagged number of plants. This OLS regression yields λ^{OLS} = 1.222 (s.e. 0.066) and ϕ^{OLS} = 0.237 (s.e. 0.087) for firms and λ^{OLS} =

[8] The BDS data starts in 1978. Our regression starts in 1979 because we need one year lagged firms or plants.

Table 1: Employment per firm vs GDP per worker and lagged number of firms, national 1979–2019

Dep variable	All firms	All plants	New firms	New plants
λ^{OLS}	1.222	0.996	0.885	0.959
	(0.066)	(0.077)	(0.024)	(0.031)
ϕ^{OLS}	0.237	0.207	-0.049	0.342
	(0.087)	(0.081)	(0.107)	(0.126)
R^2	0.872	0.647	0.434	0.250
N	41	41	41	41

Source: Employment, firms and establishment data are from the Business Dynamics Statistics of the Census Bureau and County Business Patterns. Real output is from the BEA. λ^{OLS} is equal to one plus the regression coefficient on log output per worker and ϕ^{OLS} is equal to -1 times the coefficient on log lagged number of firms or establishments.

0.996 (s.e. 0.024) and ϕ^{OLS} = 0.207 (s.e. 0.107) for plants. Over the past 40 decades in the U.S., average employment per plant has been stable and average employment per firm has been increasing while labor productivity grew. The free-entry condition in our baseline model interprets this pattern as a rise in entry cost with labor productivity. These regressions using data on all firms and plants do not control for the aging of firms and establishments as documented by Karahan et al. (2022a) and Hopenhayn et al. (2022). To control for aging, the third and fourth columns of Table 1 runs the same regression but using average employment of *new* firms or plants as regressors, while keeping the explanatory variables the same. We find that average employment of new firms and plants are stable relative to the rise in output per worker implying OLS λ that are close to 1.

In addition to over time in the U.S., our spatial model also has predictions for the cross-state relationship between changes in state level average firm or plant size with the growth in real state output per worker across (regression equation (14)). Table 2 displays the OLS regression results when we regress change in log employment per firm or establishment in a state on change in log real GSP per worker and change in log lagged number of firms for different horizons of change (1, 5, 10 and 40 years). The 1 year horizon is the state counterpart to the national firm level regression in column 1 of Table 1. We use first difference rather than levels to control for state fixed effects such as state specific markups, entry cost shifter etc. We find that average employment per firm does not vary strongly with output per worker which implies $\hat{\lambda}$ in the range of 0.678 to 0.921, depending on the horizon we use. For the 40 years horizon, which perhaps corresponds the best to our long run framework, the implied OLS $\hat{\lambda}$ is 0.921 (s.e. 0.093). We do not find a statistically significant relationship between average employment per firm and lagged firms for most horizons. Table A2 in Appendix E shows similar results when we run the regression using establishments instead of firms.

Table 2: Change in average firm size on change in GSP per worker and lagged number of firms

Horizon	40 years	10 years	5 years	1 year
λ^{OLS}	0.921	0.741	0.698	0.678
	(0.093)	(0.054)	(0.048)	(0.015)
ϕ^{OLS}	0.074	0.155	0.010	-0.047
	(0.060)	(0.043)	(0.038)	(0.021)
N	51	153	306	2040
R^2	0.042	0.237	0.115	0.199

Source: Employment, firms and establishment data are from the Business Dynamics Statistics of the Census Bureau and County Business Patterns. Real output is from the BEA. λ^{OLS} is equal to one plus the regression coefficient on log output per worker and ϕ^{OLS} is equal to -1 times the coefficient on log lagged number of firms or establishments.

The stability of average employment in state with respect to state labor productivity may also be partly coming from states having different pace of aging for firms. Table 3 displays the results when we use average new firm employment instead of average employment for all firms as the left hand side variable (see Table A3 in Appendix E for results using new plants). The OLS point estimate of λ is still close to 1 while the point estimate of $\hat{\phi}$ is higher than using all firms.

Table 3: Change in average *new* firm size on change in GSP per worker and lagged number of firms

Horizon	40 years	10 years	5 years	1 year
λ^{OLS}	0.995	0.768	0.888	0.754
	(0.120)	(0.191)	(0.140)	(0.127)
ϕ^{OLS}	0.281	0.367	-0.033	0.206
	(0.077)	(0.152)	(0.109)	(0.177)
N	51	153	306	2040
R^2	0.219	0.058	0.003	0.002

Source: Employment, firms and establishment data are from the Business Dynamics Statistics of the Census Bureau and County Business Patterns. Real output is from the BEA. λ^{OLS} is equal to one plus the regression coefficient on log output per worker and ϕ^{OLS} is equal to -1 times the coefficient on log lagged number of firms or establishments.

3.3 Empirical robustness checks

In this section we check the robustness of entry costs rising with growth by considering alternative explanations for the stability of employment per firm with respect to output per worker.

3.3.1 Discount rate, post-entry growth rate and exit rate

Suppose that entrants in each period enter with productivity A_0, incumbents' productivity grows at rate g after entry and incumbents exit at exogenous rate δ, and firms discount future profits at rate r. Also assume that g is small enough relative to δ and r so that the present discount value of profit exists. The free entry condition in this model then equalizes the entry cost with the expected sum of discounted profits:

$$\frac{p_e}{w} = \frac{\Pi A_0^{\sigma-1}}{w} \sum_{a=0}^{\infty} \left(\frac{(1+g)^{\sigma-1}(1-\delta)}{1+r} \right)^a = (\sigma - 1)\frac{L_0}{N_0} \sum_{a=0}^{\infty} \left(\frac{(1+g)^{\sigma-1}(1-\delta)}{1+r} \right)^a$$

where L_0/N_0 is the average employment of *new* firms or establishments. The regression results in Table 3 say that L_0/N_0 is stable relative to changes in output per worker. However, entry costs may still decline relative to output per work due to heavier discounting (r rising with output per worker), or when revenue is back-loaded (g declining with output per worker) or higher exit rate (δ rising with output per worker). Hence discount rates, post-entry growth rates, and exit rates could potentially explain why employment per firm is stable relative to growth in output per worker over time or across states even when the entry cost declines relative to output per worker.

However, we do not see significantly higher interest rates over time in the US (Farhi and François (2018)) and we do not expect interest rate to vary across states as capital flows freely across states. Furthermore, studies document that firm exit rate by age has been stable over time while employment growth rate by age has been stable or increasing for older firms (see Karahan, Pugsley, and Şahin (2022a), Hopenhayn, Neira, and Singhania (2022)). This suggests that the present discounted value of profit may have increase faster with growth than our estimate using new firm employment. Hence, we infer that entry costs rise with growth even after considering post-entry dynamics.

3.3.2 Measurement error in labor

The modest relationship we find between average employment per firm or plant and labor productivity across time and states could be biased downward by measurement error in labor L. We check whether our results is driven by this division bias by using alternative measures of labor for the left hand side variable L/N and right hand side variable Y/L of the regression. More precisely, we use employment from the County Business Patterns (CBP) to construct gross state product per worker but employment from the Business Dynamics Statistics to construct employment per firm or establishment. Table 4 displays the regression results. The longest horizon is shorter than the longest horizon in the baseline regression due to availability of the CBP data. The regression coefficients are similar to the baseline results in Table 2. Therefore, our results do not appear to be due to measurement error in L.

Table 4: Change in employment per firm vs change in GSP per CBP employment measure and lagged number of firms

Horizon	33 years	10 years	5 years	1 year
λ^{OLS}	0.928	0.894	0.697	0.778
	(0.096)	(0.058)	(0.048)	(0.016)
ϕ^{OLS}	0.123	0.209	-0.041	-0.018
	(0.067)	(0.048)	(0.039)	(0.023)
N	51	102	255	1683
R^2	0.073	0.248	0.144	0.111

Source: Employment, firms and establishment data are from the Business Dynamics Statistics of the Census Bureau and County Business Patterns. Real output is from the BEA. λ^{OLS} is equal to one plus the regression coefficient on log output per worker and ϕ^{OLS} is equal to -1 times the coefficient on log lagged number of firms or establishments.

3.3.3 Markup trends

Our baseline model assumes the elasticity of substitution σ is either constant over time or homogenous across states. Suppose entry costs are in fixed units of output but σ and hence the price/cost markup varies over time or across states. In our baseline model, the relationship between average employment per firm and entry costs becomes

$$\left(\frac{L}{N}\right)_{st} = (\sigma_{st} - 1)\left(\frac{w_{st}}{P_{st}}\right)^{-1}$$

Firm employment may raise with labor productivity over time or across states because markups shrink with labor productivity (σ rise with labor productivity). Our regression of within state changes in (13) controls for markups heterogeneity across states that can be picked up by cross-state fixed effects, i.e., σ varies across states but not over time. For over time in the US, the literature tend to find rising or stable markups Autor, Dorn, Katz, Patterson, and Van Reenen (2020) and De Loecker, Eeckhout, and Unger (2020). We also ran (13) with a time fixed effect to control for changes in markup over time. Note that we cannot run this regression for the longest horizon in Table 5 because we only have one period in that case. For the horizons we can run the regression, we find similar coefficients to our baseline regression in Table 2.

Table 5: Change in average firm size on change in GSP per worker, lagged number of firms and a time fixed effect

Horizon	10 years	5 years	1 year
λ^{OLS}	0.734	0.813	0.777
	(0.181)	(0.143)	(0.027)
ϕ^{OLS}	0.137	0.104	0.100
	(0.053)	(0.086)	(0.041)
N	153	306	2040
R^2	0.167	0.075	0.123

Source: Employment, firms and establishment data are from the Business Dynamics Statistics of the Census Bureau and County Business Patterns. Real output is from the BEA. λ^{OLS} is equal to one plus the regression coefficient on log output per worker and ϕ^{OLS} is equal to -1 times the coefficient on log lagged number of firms or establishments.

3.3.4 Selection on entry

Our inference strategy assumes the entrants do not know their productivity before entering and hence entry costs is proportional to average firm employment. However, if entrants know their productivity before entering, then under free entry, entry costs is proportional to the employment of the marginal entrant rather than average employment across all entrants. The Business Dynamics Data provides firms and employment counts by firm employment bins. We use the smallest bin of 1 to 4 employees to proxy for marginal entrants. There are about 2 employees per firm in this smallest bin. Table 6 shows the result of regressing average employment of firms in this bin on GSP per worker and lagged total number of firms. This is the same regression as that in Table 2 except for the the left hand side variable. We find that average employment of firms in the smallest bin does not vary significantly with output per worker and lagged total number of firms, which is consistent with entry costs rising with growth when entry costs equals the profit of the marginal entrant.

Table 6: Change in average employment for firms with 1 to 4 employees on change in GSP per worker and lagged number of firms

Horizon	40 years	10 years	5 years	1 year
λ^{OLS}	1.054	0.968	1.058	0.967
	(0.025)	(0.015)	(0.010)	(0.008)
ϕ^{OLS}	0.140	0.100	-0.121	0.145
	(0.018)	(0.016)	(0.015)	(0.014)
N	51	153	306	2040
R^2	0.584	0.330	0.216	0.057

Source: Employment, firms and establishment data are from the Business Dynamics Statistics of the Census Bureau and County Business Patterns. Real output is from the BEA. λ^{OLS} is equal to one plus the regression coefficient on log output per worker and ϕ^{OLS} is equal to -1 times the coefficient on log lagged number of firms or establishments.

3.3.5 Industry composition

Our inference is based on a single industry model. We can easily extend the inference to multiple industries. Suppose aggregate output is $Y = F(Y_i)$ where F is a production function with industry output Y_i. Industry output is produced using the CES structure as in our baseline model. Entry into an industry uses c_i units of the entry good. Then free entry into each industry implies that average employment in an industry is equal to $(\sigma-1+\lambda)c_i\frac{p^e}{w}$ and aggregate employment per firm is the industry weighted average of entry costs relative to wages

$$\frac{L}{N} = (\sigma - 1 + \lambda)\frac{p^e \sum_i(\frac{N_i}{N}c_i)}{w}.$$

Therefore, the empirical pattern of stable L/N relative to output-per-worker still implies that entry costs $p^e\sum_i(\frac{N_i}{N}c_i)$ rises with growth. However, in addition to the entry technology channel (ϕ and λ) that works through p^e, the rise in entry costs can also be explained by reallocation towards industries with higher entry costs c_i. We can distinguish between the reallocation versus the entry technology channels by using a measure of average employment that is not affected by reallocation across industries. Namely, let s_i be some fixed weight on an industry, the free entry condition holding in each industry implies that

$$\sum_i s_i \frac{L_i}{N_i} = (\sigma - 1 + \lambda)\frac{p^e}{w}\left(\sum_i s_i c_i\right).$$

Changes in this fixed weight average comes purely from changes in p^e/w

$$d\ln\left(\sum_i s_i \frac{L_i}{N_i}\right) = d\ln\left(\frac{p^e}{w}\right).$$

Table 7 shows the same regression as Table 2 but with a measure of average employment per firm on the left hand side of the regression that controls for industry composition by setting s_i in each state to the 1978–2019 average industry share of firms.[9] We find similar coefficients as the baseline results in Table 2.

Table 7: Change in average firm size on change in GSP per worker and lagged number of firms, controlling for industry composition

Horizon	40 years	10 years	5 years	1 year
λ^{OLS}	0.815	0.835	0.716	0.802
	(0.081)	(0.047)	(0.026)	(0.012)
ϕ^{OLS}	0.065	0.072	-0.196	-0.193
	(0.059)	(0.051)	(0.039)	(0.022)
N	51	153	306	2040
R^2	0.120	0.143	0.290	0.148

Source: Employment, firms and establishment data are from the Business Dynamics Statistics of the Census Bureau and County Business Patterns. Real output is from the BEA. λ^{OLS} is equal to one plus the regression coefficient on log output per worker and ϕ^{OLS} is equal to -1 times the coefficient on log lagged number of firms or establishments.

4 Inference of λ and ϕ

The previous section shows the robust empirical pattern that average employment per firm or establishment is stable relative to changes in the output per worker and number of firms or plants. From the lens of the free-entry condition, this pattern is consistent with entry costs rising with growth. What could be driving the rise in entry costs?

To shed light on the mechanisms, this section considers what value of λ and ϕ could explain the empirical patterns in the previous section. The OLS estimates λ^{OLS} and ϕ^{OLS} are not consistent estimates of the structural λ and ϕ

[9] The BDS data reports employment and firm by NAICS 2 digit in each state-year. The sectors cover the entire non-farm private economy.

because real wage is endogenous to the residual entry efficiency in the regressions. For the national model, we impose that $\tilde{A}^e$ is independent of productivity A and population growth. For the spatial model, we impose two orthogonality conditions that requires the residual $\tilde{A}^e_s$ to be independent of local productivity A_s and local amenities H_s and then calibrate the model to data to find values of λ and ϕ that are consistent with the regression results and the orthogonality conditions. The spatial model has four channels for entry costs to rise with growth: labor share in entry $\lambda > 0$, negative spillover $\phi < 0$, higher productivity requiring more setup costs and areas with high amenities have lower entry efficiency. By imposing the two orthogonality conditions, we essentially shuts down the third and fourth channel and quantify the first and second channels.

Table 8 displays the inferred structural λ and ϕ for the national and spatial model. The national model fits the model to the same sample as the regression in column 1 of Table 1 and yields $\lambda = 1$ and $\phi = -0.57$. Average firm size increases with productivity growth in this sample. Since λ is bounded about by 1, the model needs negative knowledge spillover to fit the rise in firm size.

For the spatial model, we use the bilateral trade data from latest Commodity Flow Survey data in 2017 to discipline the bilateral trade share $b_{s,s'}$. Therefore, we fits the model to the cross-state relationship between average employment and labor productivity and lagged number of firms in 2017. The OLS λ is about three-quarters but the model needs λ equal to 1. Entry efficiency pushes up real wages and pushes down employment per firm, generating the downward bias in this the OLS λ. The calibrated ϕ is -0.11 and implies slightly negative knowledge spillover.

Finally, we consider the welfare implications of the calibrated values of λ and ϕ. The first row of Table 9 displays the general formula from Section 2 for the real wage effect of shock to growth in productivity A, population and entry efficiency. The second row shows the effect when $\lambda = 1$ and $\phi = 0$ and there is not amplification through entry. Changes to A has an one-for-one effect on the

Table 8: Calibrated values of λ and ϕ

Model	Assumption	λ^{OLS}	λ	ϕ^{OLS}	ϕ
National	$\ln \tilde{A}_t^e \perp \ln A_t, \ln \tilde{A}_t^e \perp \ln Pop_t$	1.22	1.00	-0.24	-0.57
Spatial	$\ln \tilde{A}_s^e \perp \ln A_s, \ln \tilde{A}_s^e \perp \ln H_s$	0.74	1.00	-0.10	-0.11

real wage while the elasticity of real wage with respect to population or entry efficiency is $\frac{1}{\sigma-1}$, which is 0.33% when $\sigma = 4$. This row is the benchmark we use to define amplification.

The third row calculates the real wage effect when there entry uses only goods but there is no knowledge spillover. At $\sigma = 4$, the effect of change in A is now 1.5% which is 50% larger than the no amplification case–a 50% amplification. For the population and entry efficiency shocks, the effect increases from 0.33% to 0.5%, also a 50% amplification. On top of the λ channel, the fourth row adds knowledge spillover with $\phi = 0.5$. In this case, the effect of A changes increases to 3% while the effect of population and entry efficiency changes increases to 2%.

The last row calculates the wage effects under the calibrated λ and ϕ for the spatial model. Since the calibrated λ is one and ϕ is close to zero, the wage effects are very close the no amplification scenario. This suggests that in models calibrated to U.S. data, changes in productivity, population and entry efficiency may not affect entry. Their effects on real wages are not amplified through a rise in the number of businesses.

Table 9: Response of real wage growth to shocks, spatial model

Parameters	g^A shock	g^L or $\Delta\epsilon$ shocks
General case	$\frac{\sigma-1}{\sigma-1-\frac{1-\lambda}{1-\phi}}$	$\frac{\frac{1}{1-\phi}}{\sigma-1-\frac{1-\lambda}{1-\phi}}$
$\lambda = 1$, $\phi = 0$ (no amplification)	1%	0.33%
$\lambda = 0$, $\phi = 0$ (λ amplification)	1.5%	0.5%
$\lambda = 0$, $\phi = 0.5$ (and ϕ amplification)	3%	2%
$\lambda = 1$, $\phi = -0.11$ (our point estimates)	1%	0.3%

Notes: Entries show the % response in real wages to a 1% shock to productivity (A), employment (L), or entry efficiency (ϵ). The last row provides the responses using our point estimates for λ and ϕ over time within U.S. states (i.e., our "spatial" estimates). We assume $\sigma = 4$ throughout.

5 Conclusion

In the U.S., the number of plants or firms per worker increases modestly with output per worker both over time and across states. The number of businesses is more closely tied to the number of workers.

These facts can be explained by a model in which entry costs rise with labor productivity. Entry costs can rise with productivity for multiple reasons. First, if entry is labor-intensive then higher wages that go along with higher labor productivity raise the cost of entry. Second, the costs of setting up operations could be increasing with the level of technology, worker skill, or physical capital per worker. We leave it for future research to try to distinguish between these explanations.

We draw out several implications for policy and modeling. First, policies that boost productivity need not increase the number of firms or plants. Second, if the choice is between denominating entry costs in terms of labor or output, the more realistic choice is fixed entry costs in terms of labor. Third, we empirically corroborate the common assumption in endogenous growth models that the cost of innovation rises with the level of technology attained. Fourth, technological entry costs appear to be at least as important as the government-imposed costs captured by the World Bank's *Doing Business* surveys.

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A Solving the spatial model

This appendix describes the solution to the spatial model in Section 2. From the government budget balancing and zero profit condition of the firms, the income of each worker in a state satisfies

$$v_s = w_s + (1-\alpha)v_s = \frac{w_s}{\alpha}$$

and hence housing demand per worker is

$$h_s = \frac{(1-\alpha)v_s}{r_s} = \frac{w_s}{r_s}\frac{1-\alpha}{\alpha}.$$

Substituting this into the land market clearing condition pins down rent r_s given H_s, L_s and w_s:

$$r_s = \frac{w_s}{\alpha}\frac{1-\alpha}{h_s}, \quad h_s = \frac{H_s}{L_s}.$$

Furthermore, since the marginal cost of a unit of utility in each location is $P_s^{\alpha} r_s^{1-\alpha}$, the worker's indifference between states implies that there exists $\bar{V}$ such that

$$\frac{v_s}{P_s^{\alpha} r_s^{1-\alpha}} = \bar{V} \quad \forall s. \tag{A1}$$

Combining this condition with the relationship between r_s and v_s above, we can derive the following expression for welfare:

$$\bar{V} = \left(\frac{H_s}{L_s}\right)^{1-\alpha} \left(\frac{w_s}{P_s}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha}. \tag{A2}$$

This says that areas with higher wages have smaller (quality-adjusted) dwellings per worker. Since population across states must sum to the exogenous total population L, we have

$$L = \sum_{s=1}^{S} H_s \left\{ \frac{1}{\bar{V}} \left(\frac{w_s}{P_s}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \right\}^{1/(1-\alpha)} .$$

This solves for $\bar{V}$ given real wages across states

$$\bar{V} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left\{ \sum_{s=1}^{S} \frac{H_s}{L} \left(\frac{w_s}{P_s}\right)^{\alpha/(1-\alpha)} \right\}^{1-\alpha} . \tag{A3}$$

We follow the method of Allen and Arkolakis (2014) to solve for real wages. First, rearranging (A2), we have

$$\frac{w_s}{P_s} \left(\frac{H_s}{L_s}\right)^{\frac{1-\alpha}{\alpha}} = \overline{V}^{\frac{1}{\alpha}} \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} \equiv \overline{W}. \tag{A4}$$

Then substituting in the expression for P_s in terms of equilibrium $p_{s,s'}$ and $N_{s'}$, we arrive at

$$w_s \left(\frac{H_s}{L_s}\right)^{\frac{1-\alpha}{\alpha}} = \overline{W} \left(\sum_{s'} N_{s'} p_{s,s'}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} . \tag{A5}$$

The number of firms N_s is related to the population in the state s through the free entry and labor market clearing condition:

$$L_s = L_s^e + L_s^y = \frac{\lambda N_s p_s^e}{w_s} + (\sigma - 1)\frac{N_s p_s^e}{w_s} = (\sigma - 1 + \lambda)\frac{N_s p_s^e}{w_s}.$$

Substitute this and $p_{s,s'}$ into (A5), we have

$$w_s \left(\frac{H_s}{L_s}\right)^{\frac{1-\alpha}{\alpha}} = \overline{W} \frac{\sigma}{\sigma - 1} \left(\frac{1}{\sigma - 1 + \lambda} \sum_{s'} L_{s'} \frac{w_{s'}}{p_{s'}^e} \left(\frac{w_{s'} d_{s,s'}}{A_{s'}}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} . \tag{A6}$$

Alternatively, substituting in the demand function for each variety and

$\frac{P_s Y_s}{w_s L_s} = \frac{\sigma}{\sigma-1}\frac{\sigma-1}{\sigma-1+\lambda}$ into the free entry condition yields

$$p_s^e = \frac{1}{\sigma}\sum_{s'}\left(\frac{p_{s',s}}{P_{s'}}\right)^{1-\sigma} P_{s'}Y_{s'} = \sum_{s'} p_{s',s}^{1-\sigma}\left(\frac{w_{s'}}{P_{s'}}\right)^{1-\sigma}\frac{w_{s'}^{\sigma}L_{s'}}{\sigma-1+\lambda} \tag{A7}$$

Substituting in the equilibrium value for $p_{s',s}$ and (A4) into equation (A7) yields

$$(p_s^e)^{\frac{1}{1-\sigma}}\frac{A_s}{w_s} = \overline{W}\frac{\sigma}{\sigma-1}\left(\frac{1}{\sigma-1+\lambda}\sum_{s'} L_{s'}d_{s',s}^{1-\sigma}w_{s'}^{\sigma}\left(\frac{H_{s'}}{L_{s'}}\right)^{-\frac{1-\alpha}{\alpha}(1-\sigma)}\right)^{\frac{1}{1-\sigma}}. \tag{A8}$$

Following Allen and Arkolakis (2014), we can show that there exists ζ such that equilibrium wage satisfies

$$\zeta = w_s^{1-2\sigma}\frac{w_s}{p_s^e}A_s^{\sigma-1}\left(\frac{H_s}{L_s}\right)^{\frac{1-\alpha}{\alpha}(1-\sigma)}. \tag{A9}$$

Under this wage function, equilibrium conditions (A6) or (A8) are equivalent when trade costs are symmetric. We can solve for the equilibrium value of L_s for each s using either condition. For example, substituting the wage function (A9) with ζ normalized to 1, entry cost function (1) and (A4) into (A6) yields

$$\begin{aligned}&(A_s^e\lambda^\lambda(1-\lambda)^{1-\lambda})^{\frac{1-\sigma}{2\sigma-1}}A_s^{-\frac{(1-\sigma)^2}{2\sigma-1}}\left(\frac{H_s}{L_s}\right)^{\frac{1-\alpha}{\alpha}\frac{(\sigma-1+\lambda)(1-\sigma)}{2\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}(\sigma-1+\lambda)\\ =\ &\overline{W}^{1-\sigma+1-\lambda}\sum_{s'}L_{s'}(A_{s'}^e\lambda^\lambda(1-\lambda)^{1-\lambda})^{\frac{\sigma}{2\sigma-1}}d_{s,s'}^{1-\sigma}\left(\frac{H_{s'}}{L_{s'}}\right)^{\frac{1-\alpha}{\alpha(2\sigma-1)}((1-\sigma)^2-\sigma(1-\lambda))}A_{s'}^{\frac{(\sigma-1)\sigma}{2\sigma-1}}.\end{aligned} \tag{A10}$$

The exponent on L_s in the left hand side can be written as $\tilde{\sigma}\gamma_1$ where $\tilde{\sigma} = \frac{\sigma-1}{2\sigma-1}$ and $\gamma_1 = \frac{1-\alpha}{\alpha}(\sigma-1+\lambda)$. The exponent on the right hand side can be expressed as $\tilde{\sigma}\gamma_1\frac{\gamma_2}{\gamma_1}$ where $\gamma_2 = 1+\frac{\sigma}{\sigma-1}+\left(\frac{\sigma(1-\lambda)}{\sigma-1}-(\sigma-1)\right)\frac{1-\alpha}{\alpha}$. Applying Fujimoto and Krause (1985), one can shown that as long as $\frac{\gamma_2}{\gamma_1}\in(0,1)$, iterating on (A10) from any initial $\{L_s^0\}_s$ will converge to the equilibrium $\{L_s^*\}_s$. That is, from (A10), let

T denote operator

$$T(\{L_s\}) = \left(\frac{\overline{W}^{1-\sigma+1-\lambda} \sum_i (A^e_{s'} \lambda^\lambda (1-\lambda)^{1-\lambda})^{\frac{\sigma}{2\sigma-1}} d_{s,s'}^{1-\sigma} \left(H_{s'}\right)^{\frac{1-\alpha}{\alpha(2\sigma-1)}((1-\sigma)^2-\sigma(1-\lambda))} L_{s'}^{\tilde{\sigma}\gamma_1 \frac{\gamma_2}{\gamma_1}} A_{s'}^{\frac{(\sigma-1)\sigma}{2\sigma-1}}}{(A^e_s \lambda^\lambda (1-\lambda)^{1-\lambda})^{\frac{1-\sigma}{2\sigma-1}} A_s^{-\frac{(1-\sigma)^2}{2\sigma-1}} \left(H_s\right)^{-\tilde{\sigma}\gamma_1} \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} (\sigma - 1 + \lambda)} \right)^{\frac{1}{\tilde{\sigma}\gamma_1}}.$$

For any $\{L_s^0\} \neq 0$ and $\frac{T^k\{L_s^0\}}{|T^k\{L_s^0\}|} \longrightarrow \{\tilde{L}_s^*\}$ and $L_s^* = L\frac{\tilde{L}_s^*}{\sum \tilde{L}_s^*}$.

B Data

Table A1 displays the data source we use to construct the variables in the regressions. The BDS data is available from 1978 to 2019 while the CPS data is available from 1986 to 2019. The raw data for GSP by state comes from the BEA. Real GSP from the BEA has a break in 1997 where the pre-1997 data is constructed using SIC industry level data in constant 1997 dollars while the post 1997 data is constructed using NAICS industry data in constant 2012 dollars. Haver calculates a chained 2012 data for all industries prior to 1997 by using SIC chained quantity indexes (1977-1997). First, the NAICS quantity index (1997-present) is rebased to 1997=100 (base year of the SIC chained quantity index). Second, the SIC chained quantity index and the rebased NAICS quantity index are combined and then rebased to 2012=100. The SIC chained quantity index rebased to 2012=100 is then multiplied by the GDP value in 2012 to get Chained 2012 data for 1977 through 1996.

The 1 year horizon regressions regresses the change in employment per firm between year $t-1$ and t and the change in output per worker between year $t-1$ and t and the change in the number of firms between year $t-2$ and $t-1$. The 40 year horizon specification regresses the change in employment per firm between 1979 and 2019 on the change in output per worker between 1979 and 2019 and the change in the number of firms between year 1978 and 2018. We construct non-overlapping averages for the 5 and 10 year horizons. For the 5 year horizon, we divide the data into six periods: 1978–1984, 1985–1991,...,2013-

Table A1: Data sources

Data set	Source	Variables obtained
Business Dynamics Statistics (BDS)	U.S. Census	Firms (N), Establishments (N), Employment (L)
Gross State Product (GSP), Gross Domestic Product (GDP)	Haver/ BEA	Y In 2012 constant $
County Business Patterns (CBP)	U.S. Census	Alternative L
Commodity Flows Survey (CFS)	U.S. Census	$b_{s,s'}$ bilateral trade shares
Population	U.S. Census	Pop_t for calibration

2019. The subperiods for the 10 year horizon are 1984–1995, 1996–2007, and 2008–2019.

C Calibration method

We guess values of λ and ϕ and used assigned values of α and σ. Then $d_{s,s'}$ can be written as a function of observables L_s, w_s, $b_{s,s'}$ and unobservable A_s

$$b_{s,s'} = \frac{N_{s'}\, p_{s,s'}^{1-\sigma}}{\sum_k N_k\, p_{s,k}^{1-\sigma}} = \frac{N_{s'}\left(d_{s,s'}\frac{w_{s'}}{A_{s'}}\right)^{1-\sigma}}{\sum_k N_k\left(d_{s,k}\frac{w_k}{A_k}\right)^{1-\sigma}}.$$

Since $d_{s,s} = 1$, we can rewrite the above as

$$d_{s,s'} = \left(\frac{b_{s,s'}}{b_{s,s}}\right)^{\frac{1}{1-\sigma}} \left(\frac{N_s}{N_{s'}}\right)^{\frac{1}{1-\sigma}} \frac{w_s/A_s}{w_{s'}/A_{s'}}. \tag{A11}$$

Next from (12) and (A9) and , H_s is a function of observables L_s, w_s, N_s, and unobservable A_s

$$H_s = L_s \left(\zeta w_s^{2\sigma-1} \frac{1}{\sigma - 1 + \lambda} \frac{L_s}{N_s} A_s^{1-\sigma}\right)^{\frac{\alpha}{(1-\alpha)(1-\sigma)}}. \tag{A12}$$

Furthermore, we can use the wage equation (3) to back out A_s^e from data on $L_s, w_s/P_s, N_{s,t-1}, b_{s,s'}$. Since trade is balanced and markups are the same, the share of labor used for producing domestically consumed goods is equal to the share of expenditure on domestically consumed goods

$$n_{s,s} = \frac{N_s l_{s,s}}{L_s^y} = \frac{N_s w_s l_{s,s}}{w_s L_s^y} = \frac{N_s p_{s,s} y_{s,s}}{P_s GSP_s} = \frac{N_s p_{s,s} y_{s,s}}{P_s Y_s} = b_{s,s}$$

Therefore

$$\ln \frac{w_s}{P_s} = \text{constant} + \frac{(\ln A_s^e + \ln L_s + (\sigma - 1)\ln A_s - \ln b_{s,s})}{\sigma + \lambda - 2} \tag{A13}$$

Finally, we can use back out A_s given data on $L_s, P_s, w_s, N_s, b_{s,s'}$ using (A10). Once we have A_s, we can calculate $\ln A_s^e$ and the theoretical residual in (13)

$$\ln \tilde{A}_s^e(\lambda, \phi) = \ln A_s^e - \phi \ln N_{s,t-1}$$

We then find the values of λ and ϕ that satisfy the orthogonality conditions where $\ln \tilde{A}_s^e(\lambda, \phi)$ is independent of H_s and A_s.

D Welfare and entry costs in other models

In the main text, we showed that entry costs rising with growth matters in the love-of-variety model. In this section, we show that it matters for welfare in several other models as well.

D.1 Static span-of-control model

The entry technology matters for welfare even in a Lucas span-of-control model in which there is no love-of-variety. Consider the environment

$$Y = \sum_{i=1}^{N} Y_i$$

$$Y_i = AL_i^{\gamma}$$

$$N = A^e Y_e^{1-\lambda} L_e^{\lambda}$$

The first equation says aggregate output is the simple sum of firm output levels. The second equation specifies diminishing returns to production labor for each firm ($\gamma < 1$). The third equation is the technology for entry. Whereas Lucas (1978) specified overhead costs due to a single manager's time, we allow for the possibility that overhead involves goods as well as labor. Bloom, Eifert, Mahajan, McKenzie, and Roberts (2013) for example, argue that overhead costs include some information technology equipment. Variable profits are then

$$\pi_i = (1-\gamma)Y_i = A^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}}.$$

As in the love-of-variety model, free entry implies

$$\pi_i = p_e \propto \frac{1}{A^e} w^{\lambda} P^{1-\lambda}.$$

In general equilibrium

$$\ln w \;=\; \frac{\frac{1}{1-\gamma}\ln A + \ln A^e}{\frac{1}{1-\gamma} - (1-\lambda)} + \text{constant}$$

The welfare impact of a change in A here is the same as in the love-of-variety model when $1-\gamma = \frac{1}{\sigma-1}$. If better production technology boosts entry, then production labor is spread more thinly across firms, limiting scale diseconomies. Thus entry can amplify the welfare impact of better technology, just as in the love-of-variety model. Unlike in the love-of-variety model, however, changes in L do not affect welfare. A bigger population increases the number of firms proportionately, but leaves aggregate productivity unchanged.

D.2 Static love-of-variety model with congestion

Consider the static version of our baseline model but with only one region. Suppose that the entry technology is now

$$N_e = \frac{A_e}{N_e^{\psi}} Y_e^{1-\lambda} L_e^{\lambda} \tag{A14}$$

The terms L_e, Y_e and A_e are the same as the baseline model but the new term N_e^{ψ} allows for entry costs to depend on the number of entrants in the equilibrium N_e. It captures congestion effects in Gutierrez, Jones, and Philippon (2019) and Boar and Midrigan (2019, 2020). A positive ψ means that the resources needed per entry rise with the number of entrants in the equilibrium.

Real wage in this economy is given by

$$\ln w = \frac{(\sigma-1)\ln A + \frac{1}{1+\psi}\ln L + \frac{1}{1+\psi}\ln A_e}{\sigma - 1 - \frac{1-\lambda}{1+\psi}} + \text{constant} \tag{A15}$$

Thus, the impact of A on variety and welfare is dampened when entry costs rise with productivity, either through higher labor costs (λ close to 1) or congestion (positive ψ).

D.3 A growth model with expanding varieties within firms

Consider our baseline growth model with only one region extended to allow for each firm to produce multiple varieties. In addition to choosing its quality A_t, each entering firm can also choose the number of varieties v_t it will produce.

In each period t, the past pool of knowledge A_{t-1} improves the current entry technology and producing more varieties in a firm raises the entry cost of setting up the firm through $f(v_t, A_t)$:

$$p_t^e \propto e^{\mu \frac{A_t}{A_{t-1}}} f(v_t, A_t) w_t^{\lambda} =: \frac{w_t^{\lambda} P_t^{1-\lambda}}{A_t^e}.$$

Profit maximization and free entry imply that

$$\frac{\partial \ln \pi_t(A_t, v_t)}{\partial \ln A_t} = \frac{\partial \ln p_t^e}{\partial \ln A_t}$$

and

$$\frac{\partial \ln \pi_t(A_t, v_t)}{\partial \ln v_t} = \frac{\partial \ln p_t^e}{\partial \ln v_t}.$$

Variable profits are $\pi_t(A_t, v_t) = \pi_t A_t^{\sigma-1} v_t$, so the firm's optimal choice of A_t satisfies

$$\sigma - 1 = \mu \frac{A_t}{A_{t-1}} + \frac{f_A(v_t, A_t)}{f(v_t, A_t)} A_t$$

and its optimal choice of v_t is given by

$$1 = \frac{f_v(v_t, A_t)}{f(v_t, A_t)} v_t.$$

Assume

$$f(v, A) = e^{\frac{v^{\rho}}{A}}, \rho > 1$$

so that the marginal cost of producing an additional variety in a firm is increasing in the number of varieties produced in the firm, and choosing a higher technology level lowers the overall cost of producing varieties in a firm.[10] This particular functional form implies that the growth rate of quality between $t-1$ and t is

$$g_t^A := \ln \frac{A_t}{A_{t-1}} = \ln \frac{\sigma - 1 + \frac{1}{\rho}}{\mu}$$

and the number of varieties per firm grows at

$$g_t^v := \ln \frac{v_t}{v_{t-1}} = \frac{1}{\rho} g_t^A$$

The equilibrium number of firms per worker is

$$\ln \frac{N_t}{L_t} = (1-\lambda) \ln \frac{Y_t}{L_t} - \ln f(v_t, A_t) + \text{constant}$$

where N_t is the number of firms. The number of varieties produced in the economy is $M_t := N_t v_t$. The real wage and hence welfare in this economy is

$$\ln w_t = \frac{\sigma - 1}{\sigma - 1 - (1-\lambda)} \left(\ln A_t + \frac{\ln L_t v_t - \ln f(v_t, A_t)}{\sigma - 1} \right) + \text{constant}$$

and the growth rate of the real wage is

$$g_t^w := \frac{g^L + g^A(\sigma - 1) + g^v}{\sigma - 1 - (1-\lambda)}.$$

Similar to the static love-of-variety model, a higher λ implies a smaller welfare effect of changes in the level and growth rate of A_t and L_t. This model illustrates that amplification through entry can occur in an endogenous growth model with rising quality, expanding variety, and population growth — and in which firms produce multiple varieties. In particular, amplification is from variety expansion through an increase in the number of firms, whether or

[10] We want to allow higher quality to facilitate growing variety per firm because there is evidence of variety growth in the U.S. See Bernard, Redding, and Schott (2010) and Broda and Weinstein (2010).

not there are multiple or even growing varieties per firm.

E Additional empirical results

Table A2: Change in average plant size on change in GSP per worker and lagged number of plants

Horizon	40 years	10 years	5 years	1 year
λ^{OLS}	0.946	0.711	0.698	0.675
	(0.083)	(0.054)	(0.049)	(0.015)
ϕ^{OLS}	-0.012	0.137	0.011	-0.037
	(0.056)	(0.044)	(0.041)	(0.023)
N	51	153	306	2040
R^2	0.011	0.247	0.114	0.194

Source: Employment, firms and establishment data are from the Business Dynamics Statistics of the Census Bureau and County Business Patterns. Real output is from the BEA. λ^{OLS} is equal to one plus the regression coefficient on log output per worker and ϕ^{OLS} is equal to -1 times the coefficient on log lagged number of firms or establishments.

Table A3: Change in average new plant size on change in GSP per worker and lagged number of plants

Horizon	40 years	10 years	5 years	1 year
λ^{OLS}	0.912	0.856	0.501	0.517
	(0.132)	(0.186)	(0.162)	(0.115)
ϕ^{OLS}	0.236	-0.128	-0.777	-0.214
	(0.090)	(0.153)	(0.136)	(0.175)
N	51	153	306	2040
R^2	0.128	0.007	0.126	0.010

Source: Employment, firms and establishment data are from the Business Dynamics Statistics of the Census Bureau and County Business Patterns. Real output is from the BEA. λ^{OLS} is equal to one plus the regression coefficient on log output per worker and ϕ^{OLS} is equal to -1 times the coefficient on log lagged number of firms or establishments.