Discussion of Patir's Sync and Bias

NBER: Multiple Equilibria and Financial Crises

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Background

Learning to believe in sunspots™

- Indeterminacy and sunspot equilibria
- Equilibrium selection: stability under adaptive learning
- Correlated equilibria and sentiments
- Are sentiments stable? Good question!

A simplified model

Model

$$\begin{aligned} y_{it} &= \beta E_{it} y_t + \beta_0 E_{it} \varepsilon_{it} \\ y_t &= \int y_{it} di \end{aligned}$$

A simplified model

PLM (agent i)

$$
\begin{aligned}
y_t &= \phi^i + \xi^i \cdot z_t \\
s_{it} &= \lambda \varepsilon_{it} + (1-\lambda)(\phi^i + \xi^i \cdot z_t) \\
\varepsilon_{it} &\sim N(0, \sigma_\varepsilon^2),\ z_t \sim N(0, I_2)
\end{aligned}
$$

A simplified model

Expectations

$$E_{it}(\star) = E(\star|s_{it})$$

$$E_{it}\left(\xi^i \cdot z_t\right) = -\frac{(1-\lambda)^2\phi^i\|\xi^i\|^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\|\xi^i\|^2}+\frac{(1-\lambda)\|\xi^i\|^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\|\xi^i\|^2}s_{it}$$

$$E_{it}\varepsilon_{it} = -\frac{(1-\lambda)\lambda\phi^i\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\|\xi^i\|^2}+\frac{\lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\|\xi^i\|^2}s_{it}$$

A simplified model

ALM

$$
\begin{aligned}
s_{it} &= \lambda \varepsilon_{it} + (1-\lambda)\left(\int \phi^i di + \int (\xi^i \cdot z_t) di\right) \\
y_{it} &= \beta \phi^i + \beta E_{it}\left(\xi^i \cdot z_t\right) + \beta_0 E_{it}\varepsilon_{it} = T^\phi(\phi^i, \xi^i) + T^\xi(\xi^i) s_{it} \\
y_t &= \int y_{it} di \\
&= \int T^\phi(\phi^i, \xi^i) di + (1-\lambda) \int T^\xi(\xi^i) di \int \phi^i di \\
&+ (1-\lambda) \int T^\xi(\xi^i) di \int (\xi^i \cdot z_t) di
\end{aligned}
$$

A simplified model

REEE

- Homogeneity: $\xi^i = \xi^j = \xi$, $\phi^i = \phi^j = 0$
- $(1-\lambda)T^{\xi}(\xi)(\xi \cdot z_t) = (1-\lambda)\left(\frac{\beta\|\xi\|^2(1-\lambda)+\beta_0\lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2+(1-\lambda)^2\|\xi\|^2}\right)(\xi \cdot z_t)$
- Thus $\xi = 0$ and $\|\xi\|^2 = \frac{\lambda\sigma_\varepsilon^2(\beta_0-(\beta_0+1)\lambda)}{(1-\beta)(1-\lambda)^2}$
- Circle of stochastic equilibria! $\left(\lambda < \frac{\beta_0}{1+\beta_0}\right)$

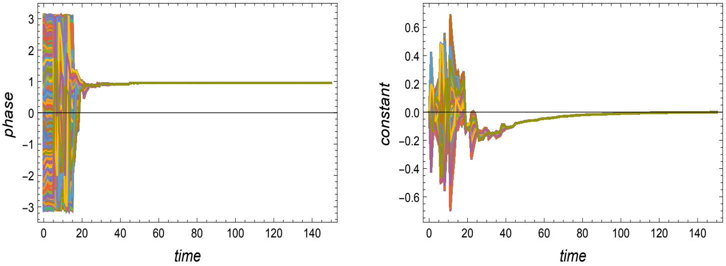
A simplified model

RTL

$$
\begin{aligned}
\hat{z}_t &= \begin{pmatrix} 1 \\ z_t \end{pmatrix} \text{ and } \zeta_t^i = \begin{pmatrix} \phi_t^i \\ \xi_t^i \end{pmatrix} \\
y_t &= \int T^{\phi}(\phi_{t-1}^i, \xi_{t-1}^i) di + (1-\lambda) \int T^{\xi}(\xi_{t-1}^i) di \int \phi_{t-1}^i di \\
&+ (1-\lambda) \int T^{\xi}(\xi_{t-1}^i) di \int (\xi_{t-1}^i \cdot z_t) di \\
R_t &= R_{t-1} + \gamma_t (\hat{z}_t \hat{z}_t' - R_{t-1}) \\
\zeta_t^i &= \zeta_{t-1}^i + \gamma_t R_t^{-1} \hat{z}_t (y_t + \Delta\phi^i - \zeta_{t-1}^i \cdot \hat{z}_t)
\end{aligned}
$$

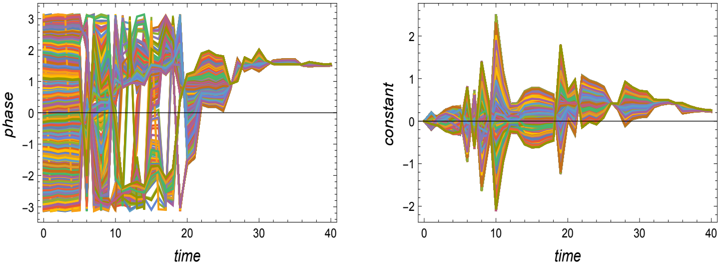
A simplified model

Figure : RTL, 1000 agents, no bias



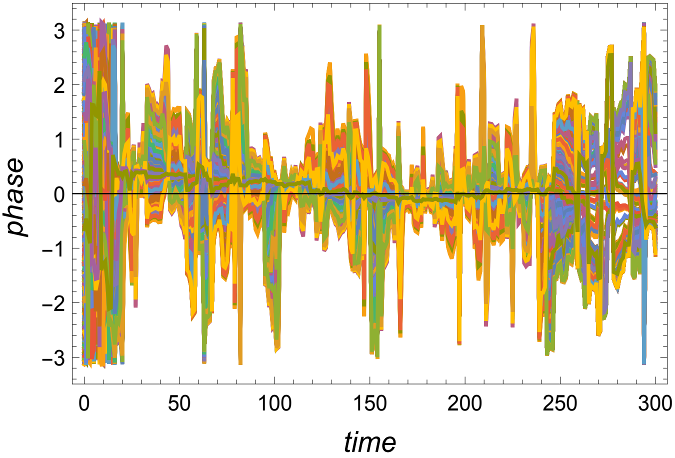
A simplified model

Figure : RTL, 1000 agents, no bias



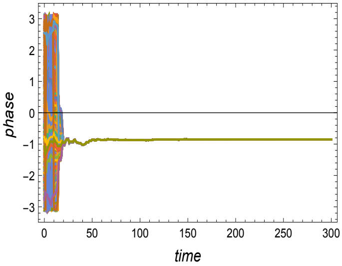
A simplified model

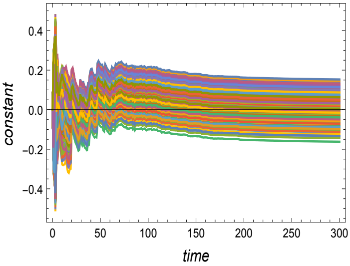
Figure : RTL, 1000 agents, normal bias, no constant



A simplified model

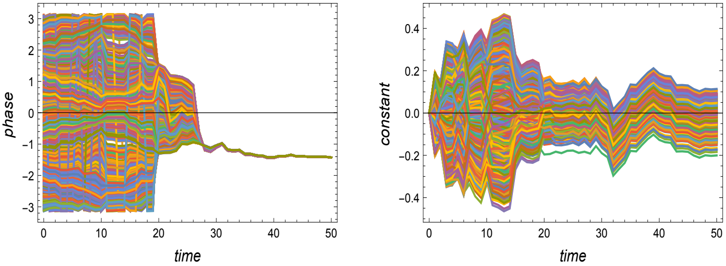
Figure : RTL, 1000 agents, normal bias





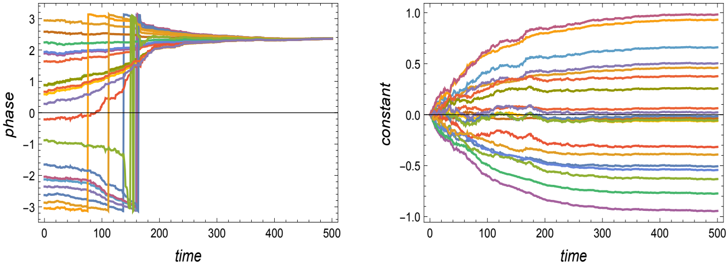
A simplified model

Figure : RTL, 1000 agents, normal bias



A simplified model

Figure : RTL, 20 agents, normal bias



A simplified model

Summary

- Patir's model and analysis are quite interesting and thought provoking.
- I have a lot to learn about coordination and sunspots