Discussion on "Stagnation Traps"

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- Existence and Persistence of Stagnation Trap in a Monetary Endogenous Growth Model with Quality Ladders

  $\Rightarrow$ Coexistence of Positive Unemployment, Low Growth, and Liquidity Trap

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- The Key Mechanism

  (1) Unemployment and Weak Aggregate Demand $\Rightarrow$ Reduces Firms' Investment in Innovation $\Rightarrow$ Low Growth

  (2) Low Growth $\Rightarrow$ Reduces Real Interest Rate $\Rightarrow$ Pushes Nominal Interest Rate to Zero

- Two Steady States in Baseline Model

(1) Full Employment $y^f = 1$, High Growth g^f, Positive Nominal Interest Rate $i^f > 0$, and Positive/Negative Inflation Rate $\pi^f \gtrless 1$

(2) Unemployment $y^u < 1$, Low Growth $g^u < g^f$, Zero Nominal Interest Rate $i^u = 0$, and Negative Inflation Rate $\pi^u < 1$

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- Two Extensions: Precautionary Savings and Time-Varying Inflation Rate

- Constant or Countercyclical Subsidy to Firms' Investment in Innovation $\Rightarrow$ Removal of Low-Growth Steady State

- Two Steady States: $y^f = 1$ and $y^u < 1$

  $\Rightarrow$ y Denotes the Level of Actual Output

  $\Rightarrow 1 - y =$ Output Gap

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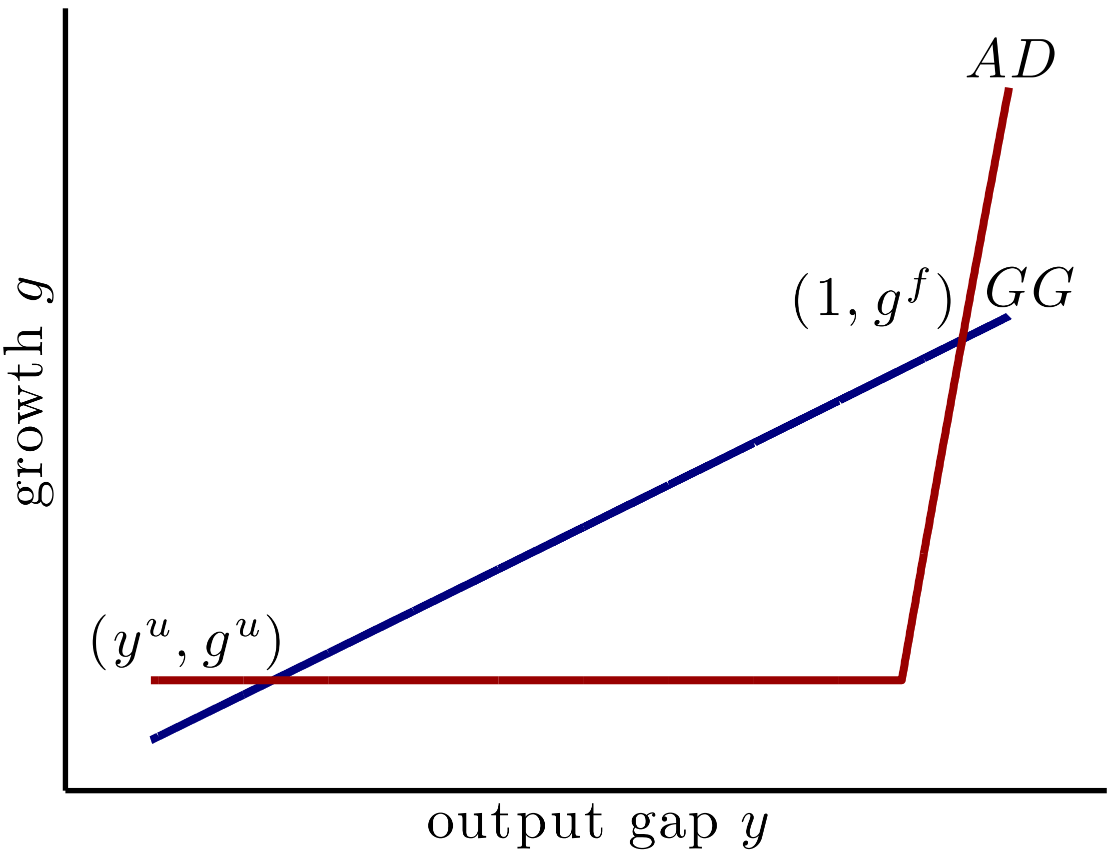
- Figure 1 $\Rightarrow$ Local Stability Property of Each Steady State: Saddle, Sink or Source

- Possibility of Global Indeterminacy $\Rightarrow$ Various Forms of Bifurcations



AD
$(1, g^f)$
GG
growth g
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- This Paper

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma}, \quad 0 < \beta < 1$$

$$C_t = \exp\left(\int_0^1 \ln q_{jt} c_{jt} dj\right) \quad \text{and} \quad Q_t = \exp\Big(\int_0^1 \ln q_{jt} dj\Big)$$

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta (1 + r_t) g_{t+1}^{1-\sigma}, \quad \text{where} \quad g_{t+1} = \frac{Q_{t+1}}{Q_t}$$

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- Need $\sigma > 1$ such that

(1) Positive Relationship between Present Consumption and Innovation Growth

(2) Existence of Unemployment Steady State

(3) $i^f > 0$ at Full-Employment Steady State

- Alternative Specification (Footnote 14)

$$\max \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \quad 0 < \beta < 1$$

$$y_t = f\left(\int_0^1 q_{jt} X_{jt} dj\right) = f(Q_t)$$

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$\Rightarrow$ Isomorphic Formulations Only When $\sigma = 1$

- This Paper

$$\text{Euler:} \quad \left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta \frac{(1+i_t)}{\bar{\pi}} g_{t+1}^{1-\sigma}$$

$$\text{Growth} \quad : \quad 1 = \beta\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} g_{t+1}^{1-\sigma}\left(\chi\frac{\gamma-1}{\gamma}y_{t+1} + 1 - \frac{\ln g_{t+2}}{\ln \gamma}\right)\right]$$

When $\sigma > 1 \Rightarrow$ Positive Relationship between y_{t+1} and g_{t+1}

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When $\sigma > 1 \Rightarrow$ Positive Relationship between y_{t+1} and g_{t+1}

$$\text{Market Clearing:} \quad c_t + \frac{\ln g_{t+1}}{\chi \ln \gamma} = y_t$$

$$\text{Monetary Policy: } 1 + i_t = \max \left\{ (1 + \bar{\imath})\, y_t^{\phi},\ 1 \right\}$$

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$$\text{Supply for } X_{jt}\text{:} \; X_{jt} = L_{jt}, \;\; \text{where} \quad \int_0^1 L_{jt}dj + L_t^{RD} + U_t = L$$

$$\text{R\&D Firms' Profits:} \; \pi_{jt} = (P_{jt} - W_t)X_{jt}, \;\; \frac{W_t}{W_{t-1}} = \bar{\pi}$$

$$\text{Monopoly Pricing:} \quad P_{jt} = \frac{W_t}{\alpha}$$

$$\text{Equilibrium Quantity:} \quad X_{jt} = \left(\frac{A\alpha^2 q_{jt}^{\alpha}}{W_t} \right)^{\frac{1}{1-\alpha}}$$

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$$\text{Aggregate Output:} \quad Y_t = A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} W_t^{\frac{-\alpha}{1-\alpha}} Q_t,$$

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$$\text{Equilibrium Profit:} \quad \pi_{jt} = \alpha(1-\alpha) q_{jt}^{\frac{\alpha}{1-\alpha}} \frac{Y_t}{Q_t}$$

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$$\text{Growth: } 1 = \left(\beta\bar{\pi}^{\frac{\sigma\alpha}{1-\alpha}}\right)g_{t+1}^{-\sigma}\left[\alpha(1-\alpha)q_{j(t+1)}^{\frac{\alpha}{1-\alpha}}\frac{\chi Y_{t+1}}{LW_tQ_{t+1}} + \bar{\pi}(1-\frac{g_{t+2}}{\gamma^{\frac{\alpha}{1-\alpha}}})\right]$$

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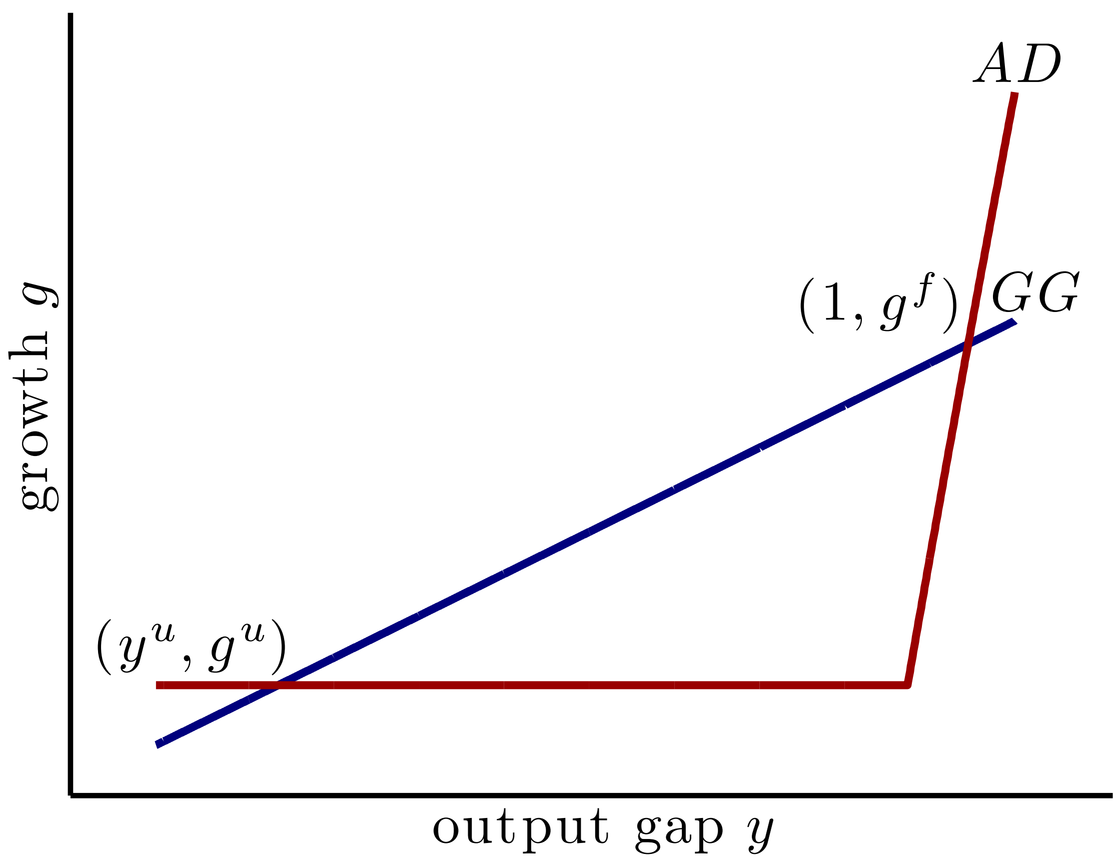
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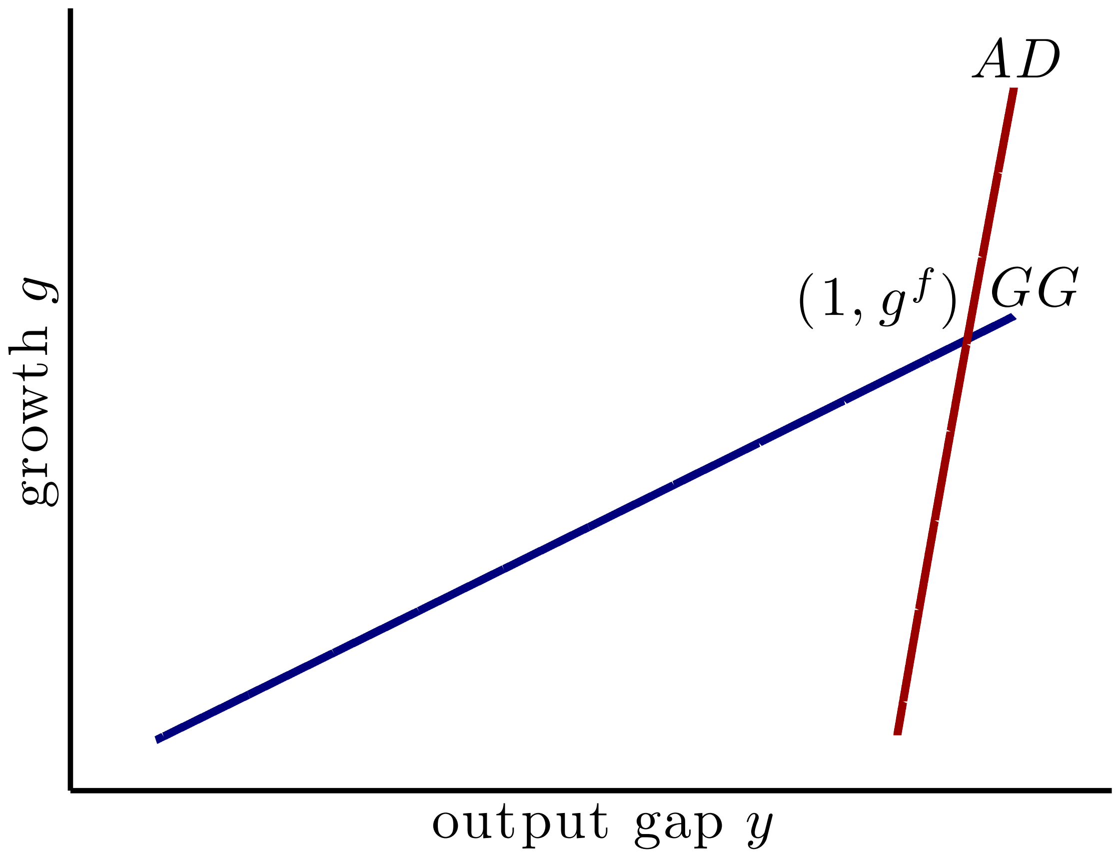
(2) Zero Nominal Interest Rate $i^u = 0$

Negative Nominal Interest Rates Observed in Europe: ECB's Deposit Rate of -0.2%, and Swiss National Bank's Deposit Rate of -0.75%

$$\Rightarrow 1 + i_t = \max\left\{(1+\bar{\imath})\, y_t^{\phi},\ \underline{\mathrm{i}}\right\}, \text{ where } \underline{\mathrm{i}} < 1$$



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