Integrated Assessment in a Multi-region World with Multiple Energy Sources and Endogenous Technical Change
PRELIMINARY

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Abstract

We construct an integrated assessment model with multiple imperfectly substitutable energy sources—including fossil fuels and "green energy"—and multiple world regions. The model also incorporates fracking and endogenous technical change directed at reducing the production costs for the different energy sources. Apart from the oil price, all endogenous variables have closed-form solutions. We derive four main results. First, subsidies to green R&D is an insufficient method for combating global warming. Second, an effective carbon tax must be levied also on oil that is produced with fracking technology. Third, a coal tax that is proportional to the coal price is completely impotent. Fourth, per-unit taxes are effective in mitigating global warming. In this case, endogenous technical change reinforces the effectiveness of carbon taxes.

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1 Introduction

Climate change is often portrayed as a key challenge for the survival of mankind. In principle, the analysis of climate change and how to handle it with economic policy is straightforward and based on two pieces of century-old knowledge.

The first piece of knowledge comes from Arrhenius (1896), who described and quantified the greenhouse effect. Carbon dioxide (CO_2) is released into the atmosphere when fossil fuel is burnt. In contrast to gases where molecules have two atoms, like oxygen and hydrogen, CO_2 lets through high-frequency electromagnetic radiation, like sunlight, but it reflects low-frequency infrared heat radiation. Arrhenius examined this effect in a laboratory setting and his key quantitative findings still hold up.

The other piece of knowledge is from Pigou (1920). Emitted CO_2 mixes quickly into the atmosphere, affecting the energy budget, and therefore the climate, independently of the identity and geographic location of the emitter. Thus, if climate change has economic consequences, this creates a pure externality that implies that unregulated markets do not function perfectly. Pigou's general solution to externality problems, which is to apply a tax that is equal to the marginal societal effect that markets do not internalize, can straightforwardly be applied here: one needs to adopt a global tax on emissions equal to the marginal economic effects on the economy. This principle applies whether these effects are positive or negative; in this area, the estimates suggest that the effects are negative on average: climate change causes damages.

Neither of these pieces of knowledge can reasonably be questioned (except by members of the Flat Earth Society).[1] However, the issue of climate change is far from settled. One reason for this is arguably that the two quoted pieces of evidence are not sufficient for calculating a quantitative value for the tax with sufficient precision. To devise an implementable policy, i.e., specify the right tax on fossil fuels, we thus need to know (i) how much climate change emissions induce, (ii) how long carbon stays in the atmosphere, and (iii) how large the economic consequences are. Although there exists substantial research on these issues, significant uncertainty remains. Another reason is likely political: the economic impacts of climate change differ significantly by geographic location. For example, there are reasons to believe that countries/regions which are currently in climate zones with a very low average temperature stand to gain, thus potentially making their populations averse to carbon taxes,

[1] Joining is free of charge at `http://theflatearthsociety.org/home/index.php/about-the-society/joining-the-society`.

at least unless they are compensated by those who would gain from the tax.

In terms of the quantitative uncertainties, it is well established that the direct effect of a doubling of the CO_2 concentration implies a perturbation of the energy budget (incoming energy flows from the sun minus outgoing flows from earth to space) by around 4 W/m^2. In itself, this is not a large amount and would not cause dramatic warming. There are, however, good reasons to believe that this direct effect leads to a multitude of feedback effects, each of uncertain strength and even sign.[2] A reasonable interval of uncertainty includes both aggregate feedback mechanisms that are so weak as to not amplify the direct effect of CO_2 emissions much at all, and the opposite: very large positive aggregate feedback effects that can cause dramatic warming.

There is also substantial uncertainty about how emissions affect the CO_2 concentration over time. There is a constant large flow of carbon between a large number of reservoirs, including the atmosphere, plants in the biosphere, the top layer of earth, the surface of the oceans, and the deep oceans. The flows between these reservoirs and their respective storage capacities are affected directly by emissions and, more importantly, indirectly by climate change. Despite the substantial uncertainty regarding the natural-science mechanisms behind emission-induced climate change, the uncertainties in the social-science domain are, in fact, even greater.

This paper builds on previous work by Nordhaus (1977, 1994, 2011), Nordhaus and Boyer (2000), Hassler and Krusell (2012), and Golosov et al. (2014) in developing an *Integrated Assessment Model* that can be used to study how taxation affects global warming. Specifically, we set up a model with several regions and multiple imperfectly substitutable energy sources. We also incorporate the effects of hydraulic fracturing ("fracking") into the analysis, and allow for directed technical change where energy-producing firms may lower the marginal costs of producing the different energy sources. In the calibration, we allow for six regions that respectively represents the United States, Europe, China, India, and Africa. The considered energy inputs include conventional oil, coal, green (renewable) energy, and the output from fracking that is highly substitutable with conventional oil. Despite this relatively rich model structure, all variables except the oil price have closed-form solutions, which makes the analysis tractable and transparent.

A key question in our previous work was about the quantitative determination of the

[2] Airborne particles in combination with cloud formation is a potentially strong feedback mechanism with uncertain sign. Recent overestimates of global warming on the basis of some climate models are sometimes argued to be accounted for by this kind of feedback mechanism having a negative sign, i.e., dampening the direct warming effect.

optimal carbon tax. A result from that analysis is that the optimal carbon tax should be levied equally across regions of the world and on different fuels. The present paper has a different focus in that it draws attention to the introduction of sub-optimal taxes. How important is it that all regions, in fact, do implement the optimal tax? Suppose, instead, that only Europe introduces a carbon tax.[3] We show that while such a partial tax has some effects in mitigating global warming, they are very limited. However, if Europe adopts a high tax while other regions use a tax substantially lower than the optimal, this has important effects on global emissions.

A related question is to what extent it is quantitatively important to tax all fossil fuels. Our answer is that it is important to tax coal. Whether a global carbon tax applies to *conventional oil* or not is immaterial for the effect of the tax on climate change.[4] However, it is critical that an effective carbon tax also is implemented on the nonconventional oil that is produced with the fracking technology. The reason is that the fracking technology has the potential to dramatically increase the stock of nonconventional oil, which may generate substantial global warming.

One potentially important aspect for both global warming and the optimal policy is the fact that technical change, over time, alters the costs for extracting different energy sources. For instance, coal production per unit of labor input increased by 3.2% per year between 1949 and 2011.[5] At the same time, the technological advancement in fracking has now made large reserves of unconventional oil profitable to extract.[6] Similarly, the cost of producing renewable energy has also recently been fallen at a fast pace. These trends have both been portrayed as a great threat to the climate (cheaper fossil fuels), and as a salvation (cheaper green energy). In effect, it seems like a technology race with high stakes! As a first step, we incorporate these exogenous cost-reducing technologies into the neoclassical growth framework to quantify their effects on global warming. The results show that if the coal-extracting technology evolves relatively slowly so that the relative price of coal increases over time then, this has similar effects as the introduction of the optimal tax. Cheaper green energy, however, is not in itself sufficient for mitigating global warming.

Since the assumption of exogenous technology trends is unsatisfactory, however, we ex-

[3] Or, equivalently, adopts a well-functioning quantity regulation with emission trading and where the implied market price corresponds to the tax.

[4] In contrast to a coal tax, however, an tax on conventional oil has large consequences for the distribution of income across regions.

[5] Source: EIA (2012).

[6] For a discussion of fracking and its effects on the oil market, see Bornstein, Krusell, and Rebelo (2017).

pand the model by allowing for directed technical change. Specifically, energy-producing firms may then lower the marginal cost of producing one form of energy at the expense of the marginal costs of other energy sources.[7] This allows for an analysis of how energy taxes interact with cost-reducing technical change. The results from this exercise show that taxes that are proportional to the price of a specific energy good are completely impotent in reducing the demand for this good. The intuition for this potentially surprising result comes from the fact that a proportional tax generates two effects with different signs. On the one hand, a proportional tax increases the marginal value of cost reductions. On the other hand, the proportional tax reduces the demand for the energy good, which also reduces the marginal value of cost reductions. In our setting, these two effects exactly balances. Research and Development, thus, effectively nullifies the effects of taxes on the after-tax price.

Even though proportional taxes are ineffective is in dealing with global warming, this is not true for per-unit taxes (i.e., taxes per ton of the energy source) that, in fact, are effective in mitigating global warming. In this case, endogenous technical change actually reinforces the effectiveness of carbon taxes.

Our contribution stands on the shoulders of giants. Nordhaus was the first to construct a simple but reasonably accurate representations of the natural-science elements—the circulation between carbon reservoirs and climate change induced by the greenhouse effect—and integrate them into a growth model building on Ramsey, Solow, and Dasgupta & Heal (1974). A key feature of his analysis was the aim to make quantitative statements both on the natural-science and social-science sides of the analysis. This explains the choice of basing the economic analysis on the neoclassical growth model, which is the established framework for analyzing long-run macroeconomics, here in its global version. Nordhaus thus contributed the first *Integrated Assessment Models* with his sequence of papers. Hémous (2016) analyzes unilateral environmental policies in a world with several regions. Another recent contribution with a global growth model with several regions that analyses climate policy is Hillebrand and Hillebrand (2017). Our extension to endogenous technical change draws fundamentally on Romer (1986), but a more proximate precursor is the seminal paper on the importance of directed technical innovation for climate change is Acemoglu et al. (2012) and also Acemoglu et al. (2014).

The paper is organized as follows: Section 2 sets up the basic model with exogenous growth. Section 3 describes calibration of the model, and the benchmark results are then

[7] For a recent discussion of the importance of endogenous cost reductions in fossil fuel production, see Chapter 1 in IMF (2017). See also Hassler, Krusell, and Olovsson (2017) for a related study.

presented in Section 4. The analysis is then extended in section 5 to incorporate fracking and endogenous technical change in energy production. Section 6 discusses the main findings and concludes.

2 Model

2.1 Economy

The world consists of r regions. Region 1 is endowed with a finite amount of oil that it extracts and sells to the rest of the world. This region will be referred to as the *oil producer*. The remaining $r-1$ regions have no endowments of conventional oil. Instead, they import oil from the oil producer and pay for this import with a common final good that is identical in all countries. These regions are similar in the sense that the only difference between them is parametric (size and productivity), and we will refer to them as *oil consumers.* There are are no international capital markets.

Each region $i \in \{1, 2, ..., r\}$ features a representative consumer with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(C_{i,t}). \tag{1}$$

Each oil-consuming region also has access to an aggregate production function for the final good, $Y_{i,t}$, given by

$$Y_{i,t} = A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha} E_{i,t}^{\nu}$$

where $A_{i,t}$ is total factor productivity, $L_{i,t}$ is labor used in final-good production, $K_{i,t}$ is the capital stock, and $E_{i,t}$ energy services.

Energy services are provided by firms that act competitively with a CRS production function in the n different energy inputs

$$E_{i,t} = \mathcal{E}(e_{1,i,t}, \ldots, e_{n,i,t}) = \left(\sum_{k=1}^{n} \lambda_k \left(e_{k,i,t}\right)^{\rho} \right)^{\frac{1}{\rho}}. \tag{2}$$

Here, $e_{1,i,t}$ is country $i's$ import of oil in period t. The remaining energy sources $e_{2,i,t}, ..., e_{n,i,t}$ are all produced domestically with a production technology that is linear in the final good. Specifically, $p_{k,i,t}$ units of the final good are required to produce $e_{k,i,t}$ units of energy source $k \in \{2, ..., n\}$ in region i and period t. In the benchmark specification of the model, the productivity in energy production is exogenous, but it is made endogenous in Section 5.2.

Final goods not used for energy production are consumed or invested. The resource constraint for the final good is thus

$$C_{i,t} + K_{i,t+1} = A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha} E_{i,t}^{\nu} - p_{1,t} e_{1,i,t} - \sum_{k=2}^{n} p_{k,i,t} e_{k,i,t},$$

where we, for analytical tractability, have assumed that capital depreciates fully between periods.

Finally, region 1 produces oil without any resource cost, and the total stock of oil in the ground at time t has size R_t. With extraction $\sum_{i=2}^{r} e_{1,i,t}$, the next-period size of the resource is thus

$$\begin{aligned} R_{t+1} &= R_t - \sum_{i=2}^{r} e_{1,i,t}, \\ R_t &\geq 0, \forall t, \end{aligned} \tag{3}$$

Since the oil producer is assumed to derive all its income from oil, its budget/resource constraint is given by

$$C_{1,t} = p_{1,t} \left(R_t - R_{t+1} \right), \tag{4}$$

where $p_{1,t}$ denotes the world market price of oil. The assumption of an identical final good allows us to express this price in terms of the global final good.

2.2 Carbon circulation

Usage of certain energy sources generate CO_2 emissions. Specifically, total emissions from region i in period t are given by

$$M_{i,t} = \sum_{k=1}^{n} g_k e_{k,i,t},$$

where g_k measures how dirty energy source k is. We measure fossil energy sources by their carbon content. Thus, these energy sources all have $g_k = 1$, whereas the purely green energy sources have $g_k = 0$.[8]

[8] Allowing intermediate cases, i.e., emissions from non-fossil energy sources is straightforward.

Following Golosov et al. (2014), the law-of-motion for the atmospheric excess stock of carbon S_t is given by

$$S_t = \sum_{s=0}^{\infty}(1 - d_s) \sum_{i=2}^{r} M_{i,t-s},$$

where

$$1 - d_s = \varphi_L + (1 - \varphi_L) \varphi_0 (1 - \varphi)^s$$

measures carbon depreciation from the atmosphere.

Specifically, the share of emissions that remains in the atmosphere forever is φ_L, the share that leaves the atmosphere within a period is $1 - \varphi_0$, and the remainder $(1 - \varphi_L) \varphi_0$ depreciates geometrically at rate φ.

2.3 Climate and damages

The climate is affected by the concentration of CO_2 in the atmosphere via the greenhouse effect. Golosov et al. (2014) shows that the effect of the CO_2 concentration on productivity is well captured by a log-linear specification. We therefore assume that

$$A_{i,t} = \exp\left(z_{i,t} - \gamma_{i,t} S_{t-1}\right), \tag{5}$$

where $z_{i,t}$ is a potentially stochastic productivity trend, and $\gamma_{i,t}$ is a region-specific and possibly time-varying parameter that determines how climate-related damages depend on the level of the atmospheric CO_2 concentration. Note that this specification implies that the marginal damage per unit of excess carbon in the atmosphere is a constant share of net-of-damage output given by the parameter $\gamma_{i,t.}$. The value $\gamma_{i,t}$ is positively affected by i) the sensitivity of the global mean temperature to changes in the CO_2 concentration ii) the sensitivity of the regional climate to global mean temperature, and iii) the sensitivity of the regional economy to climate change.

We assume that the climate damage appears with a one-period lag. Given that climate change is a slow-moving process and that the evolution of the atmospheric CO_2-concentration is sluggish, this is immaterial for the dynamics of climate damages. As we will see below, however, the computation of the equilibrium allocation is much simplified by the introduction of this lag.

The climate system is borrowed from the energy budget model in DICE/RICE:

$$\begin{aligned} T_t &= T_{t-1} + \sigma_1 \left(\frac{\eta}{\ln 2} \ln \left(\frac{S_{t-1}}{S_0} \right) - \kappa T_{t-1} - \sigma_2 \left(T_{t-1} - T_{t-1}^L \right) \right) \\ T_t^L &= T_{t-1}^L + \sigma_3 \left(T_{t-1} - T_{t-1}^L \right) \end{aligned}$$

where T_t is the global mean temperature in the atmosphere (and upper layers of the oceans) and T_t^L temperature in the deep oceans. Both temperatures are measured as deviations from their pre-industrial levels.

2.4 Governments

A key purpose of the analysis is to analyze the consequences of taxing fossil fuel, and in particular, how effective partial taxation is. To this end, we allow each oil-consuming region to tax the use of fossil-fuel inputs. We will consider two cases; *ad valorem* taxes and per-unit taxes. Each region $i \in \{2, ..., r\}$ is allowed to set a carbon tax rate $\tau_{i,t}$. The cost for the energy-service provider of using energy type k in region i then becomes $(1 + \tau_{i,t} g_k) p_{k,i,t}$ or $\tau_{i,t} g_k + p_{k,i,t}$ for *ad valorem* taxes and per-unit taxes respectively. For simplicity, tax revenues are recycled within the period back to the household in the form of a negative income tax rate $\Gamma_{i,t}$ applying to total household income $w_{i,t} L_{i,t} + r_{i,t} K_{i,t}$. The government budget constraint becomes with *ad valorem* taxes

$$\Gamma_{i,t} \left(w_{i,t} L_{i,t} + r_{i,t} K_{i,t} \right) = \tau_{i,t} \sum_{k=1}^{n} g_k e_{k,i,t} p_{k,i,t}.$$

With per-unit taxes, the government budget constraint instead becomes

$$\Gamma_{i,t} \left(w_{i,t} L_{i,t} + r_{i,t} K_{i,t} \right) = \tau_{i,t} \sum_{k=1}^{n} g_k e_{k,i,t}.$$

2.5 Markets and equilibrium

All agents are price takers, and markets within regions are assumed to be perfect and complete.

2.5.1 The oil-producing region

Consider now first the oil-producing region. We assume that there are many oil producers that operate under perfect competition. The problem for the representative oil producer is then to choose how much oil to keep in the ground for next period, R_{t+1}, while taking the world market price of oil as given. Substituting (4) into (1) and taking the first order condition with respect to R_{t+1} yields

$$\frac{1}{R_t - R_{t+1}} = \mathbb{E}_t \frac{\beta}{R_{t+1} - R_{t+2}},$$

where $\mathbb{E}_t$ denotes the conditional expectations operator at time t. The above equation delivers $R_{t+1} = \beta R_t$, which implies a constant depletion rate. Consumption of the oil producers are then given by $C_{1,t} = p_{1,t}(1-\beta) R_t$. Note that even if $p_{1,t}$ is stochastic, this has no effect on oil supply. The reason is that the income and substitution effects exactly cancel with logarithmic preferences.

2.5.2 Energy providers in oil consuming regions

Consider now energy provision in the oil consuming regions. The assumption of competitive and frictionless markets imply that we can write the behavior of energy-service providers as the solution to the cost-minimization problem

$$\min_{e_{k,i,t}} \sum_{k=1}^{n} \hat{p}_{k,i,t} e_{k,i,t} - \Lambda_{i,t} \left(\left(\sum_{k=1}^{n} \lambda_k \left(e_{k,i,t}\right)^{\rho} \right)^{\frac{1}{\rho}} - E_{i,t} \right) \tag{6}$$

where $\hat{p}_{k,i.t}$ is the tax-inclusive fuel price. This price is given by $(1 + \tau_{i,t} g_k) p_{k,i,t}$ in the case of *ad valorem* taxes and by $\tau_{i,t} g_k + p_{k,i,t}$ with per-unit taxes. By construction, the Lagrange multiplier, $\Lambda_{i,t} = P_{i,t}$, defines the exact price index of energy services.

The first-order conditions with respect to $e_{k,i,t}$ are given by

$$e_{k,i,t} = E_{i,t} \left(\frac{P_{t,i} \lambda_k}{\hat{p}_{k,i,t}} \right)^{\frac{1}{1-\rho}}, \quad k \in \{2, n\}. \tag{7}$$

Similarly, the first-order condition with respect to oil consumption is given by

$$e_{1,i,t} = E_{i,t} \left(\frac{P_{t,i} \lambda_1}{\hat{p}_{1,i,t}} \right)^{\frac{1}{1-\rho}}. \tag{8}$$

Using (7) and (8) in the expenditure function of the energy-service provider, the exact price index becomes

$$P_{i,t} = \left(\sum_{k=1}^{n} \hat{p}_{k,i,t}^{\frac{\rho}{\rho-1}} \lambda_k^{\frac{1}{1-\rho}} \right)^{\frac{\rho-1}{\rho}}. \tag{9}$$

2.5.3 Final-good producers in oil consuming regions

Producers of the final good maximize profits while taking $P_{t,i}$ as given. The resulting first-order condition in region i delivers

$$P_{i,t} = \nu \frac{A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha} E_{i,t}^{\nu}}{E_{i,t}},$$

which can be rearranged and solved for energy-service use:

$$E_{i,t} = \left(\nu \frac{A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha}}{P_{t,i}} \right)^{\frac{1}{1-\nu}}. \tag{10}$$

The price-taking behavior of the final-good producing firm also implies that

$$\begin{aligned} w_{i,t} &\equiv \frac{\partial Y_{i,t}}{\partial L_{i,t}} = (1-\alpha-\nu) \frac{Y_{i,t}}{L_{i,t}} \\ r_{i,t} &\equiv \frac{\partial Y_{i,t}}{\partial K_{i,t}} = \alpha \frac{Y_{i,t}}{K_{i,t}}. \end{aligned}$$

The assumption of constant returns to scale implies that profits are zero in equilibrium, and that output net of energy expenses is given by $(1-\nu) Y_{i,t} \equiv \hat{Y}_{i,t}$. Note, however, that the shares of spending on the different energy sources is not necessarily constant unless $\rho = 0$, i.e. when the overall production function is Cobb-Douglas in all inputs.

2.5.4 Households in oil consuming regions

Households supply labor inelastically and maximize (1) subject to the budget constraint

$$C_{i,t} + K_{i,t+1} = (1 + \Gamma_{i,t}) (w_{i,t} L_{i,t} + r_{i,t} K_{i,t}) = (1 + \Gamma_{i,t}) \hat{Y}_{i,t}. \tag{11}$$

The resulting Euler equation for the households in oil-consuming countries is given by

$$\frac{C_{i,t+1}}{C_{i,t}} = \beta \left(1 + \Gamma_{i,t+1}\right) r_{i,t+1}.$$

Defining the savings rate out of net output as $s_{i,t} = \frac{\hat{Y}_{i,t} - C_{i,t}}{\hat{Y}_{i,t}}$, we get $C_{i,t} = \left(1 - s_{i,t}\right)\left(1 + \Gamma_{i,t}\right)\hat{Y}_{i,t}$ and $K_{i,t+1} = s_{i,t}\left(1 + \Gamma_{i,t}\right)\hat{Y}_{i,t}$. Using this together with $r_{i,t} = \alpha \frac{Y_{i,t}}{K_{i,t}}$ in the Euler equation yields,

$$\frac{1 - s_{i,t+1}}{1 - s_{i,t}} = \beta \frac{\alpha}{s_{i,t}\left(1 - \nu\right)}.$$

This above difference equation in $s_{i,t}$ only has one non-explosive solution, namely $s_{i,t} = \frac{\alpha\beta}{1-\nu} \equiv s$ for all t which then defines optimal household behavior.

We are now ready to summarize some results in the following proposition:

Proposition 1 *In each period, the equilibrium allocation is determined by state variables $K_{i,t}, R_t$ and S_{t-1} such that i) the capital savings rate is constant at $\frac{\alpha\beta}{1-\nu}$, ii) oil supply is given by $(1-\beta)R_t$, iii) energy prices are $P_{i,t} = \left(\sum_{k=1}^{n} \hat{p}_{k,i,t}^{\frac{\rho}{\rho-1}} \lambda_k^{\frac{1}{1-\rho}}\right)^{\frac{\rho-1}{\rho}}$, iv) energy-service demand is $E_{i,t} = \left(\nu \frac{e^{\left(z_{i,t} - \gamma_{i,t} S_{t-1}\right)} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha}}{P_{t,i}}\right)^{\frac{1}{1-\nu}}$, v) domestic fuel demand is $e_{k,i,t} = E_{i,t}\left(\frac{P_{t,i}\lambda_k}{\hat{p}_{k,i,t}}\right)^{\frac{1}{1-\rho}}$, vi) regional oil demand is $e_{1,i,t} = E_{i,t}\left(\frac{P_{t,i}\lambda_1}{\hat{p}_{1,i,t}}\right)^{\frac{1}{1-\rho}}$ and vii) net output is $\hat{Y}_{i,t} = \left(1 - \nu\right) A_{i,t} L_{i,t}^{1-\alpha-\nu} K_{i,t}^{\alpha} E_{i,t}^{\nu}$. The price of oil is determined from equilibrium in the world oil market $\sum_{i=2}^{r} e_{1,i,t} = \left(1 - \beta\right) R_t$. The laws-of-motion for the state variables are $K_{i,t} = \frac{\alpha\beta}{1-\nu}\left(1 + \Gamma_{i,t}\right)\hat{Y}_{i,t}$, $R_{t+1} = \beta R_t$ and $S_t = \sum_{v=0}^{t}(1 - d_{t-v}) \sum_i M_{i,t}$*

Two things should be noted. First, the allocation is determined sequentially without any forward-looking terms. Second, given a world market price of oil, all equilibrium conditions have closed-form solutions. Thus, in each period, finding the equilibrium is only a matter of finding the equilibrium oil price where supply is predetermined at $\left(1 - \beta\right) R_t$.

3 Calibration and simulation

Let us now calibrate the model in order to understand its quantitative implications. We set $r = 6$ to consider a world consisting of five oil-consuming regions. Specifically, these regions represent Europe, the U.S., China, India, and Africa. The number of fuel inputs into energy production n is set to three. The first fuel represents oil (as mentioned in Section 2). We then let fuel type two and three respectively represent coal and green (renewable) energy. Measuring oil and coal in carbon units imply that $g_{k,i,t} = 1$ for $k = 1, 2$ and 0 for $k = 3$.

The discount factor is set to 0.985^{10} with the understanding that a period is a decade. The production parameters α and ν respectively determine the income shares for capital and energy. We set α to 0.3, and ν to 0.055 (which implies a capital share equal to 0.3, and an energy share of 0.055). The production of energy services is calibrated in the following way. The elasticity of substitution between the three energy sources is set to 0.95 to match the unweighted mean of the oil-coal, oil-electricity and coal-electricity elasticities that is found in the metastudy by Stern (2012).[9] This implies setting ρ to -0.058 which we use as the main case. In the section on endogenous technical change, we will return to how results depend on this parameter.

To calibrate the $\lambda's$, prices and quantities of the three types of fuel are needed. Combining the demand equations (7) and (8), we derive the following relationships

$$\frac{\lambda_1}{\lambda_k} = \left(\frac{e_{1,t}}{e_{k,t}}\right)^{1-\rho} \frac{p_{1,t}}{p_{k,t}}, \quad k = 2, 3.$$

Using world market prices from Golosov at al (2014), the coal price is set to US \$74/ton and the (pre-financial crises) oil price is set to US \$70/barrel, corresponding to US \$70*7.33 per ton.[10] These prices are assumed to apply in all regions. For coal, it also represents the cost of production and transportation. The relative price between oil and coal in units of carbon is then 5.87. Using the same source for the ratio of global oil to coal use in carbon units we find that $\lambda_1/\lambda_2 = 5.348$. For green energy, we use data for the sum of nuclear, hydro, wind, waste and other renewables from Golosov et al. (2014) and stick with their (somewhat arbitrary) assumption of a unitary relative price between oil and renewables. This gives $\lambda_1/\lambda_3 = 1.527$. Together with the normalization $1 = \lambda_1 + \lambda_2 + \lambda_3$, the λ's are then given by $\lambda_1 = 0.543, \lambda_2 = 0.102$, and $\lambda_3 = 0.356$.

BP (2010) reports that global proved reserves of oil are 181.7 Gigatons. This number, however, only includes the aggregate reserves that are economically profitable to extract at current economic and technical conditions. In particular, it does not take technical progress, or that new profitable oil reserves may be discovered, into account. An alternative study is found in Rogner (1997) that does take technical progress into account and estimates global

[9] Specifically, the study is based on 47 studies of interfuel substitution.

[10] Taxes and subsidies of fossil fuel and other energy sources are disregard.

fossil reserves to be larger than 5000 Gigatons of oil equivalents (Gtoe).[11] About 16% of these reserves constitute oil, i.e., 800 Gtoe. In the benchmark calibration, we set the existing stock of oil to 400 Gtoe, i.e., somewhere well within the range of these two estimates. The carbon content of oil and coal are 84.6% and 71.% percent in weight.

The initial global GDP is set to US \$ 75 trillion, of which three quarters is produced in equal shares in the U.S., EU and China. India and Africa are set to produce 12.5% of global GDP each. The U.S. and EU are both assumed to start out on their respective balanced growth paths, where the constant technological growth rate is given by $\Delta z_{i,t} = 1.5\%$ per year. In the benchmark model, the cost of producing coal and green fuel is constant in terms of the final good. Based on the findings in Caselli and Feyrer (2007), the real interest rates are equalized across all regions. Together these assumptions imply a GDP growth rate of 2.1% per year, and they pin down initial productivities and capital stocks in all regions.[12]

Due to catching-up effects, China, India, and Africa are assumed to initially grow faster than EU and the United States. This is implemented by allowing Total Factor Productivity (TFP) in those regions to converge to a growth path that is 60% higher than the current. The implication is that China converges to a BGP with approximately twice the GDP of EU and the United States, whereas India, and Africa both converge to a BGP with the same GDP as EU and the United States. Formally, we assume that for China, India, and Africa, log (TFP) follows the following process

$$\begin{aligned} z_{i,t+1} &= z_{i,t} + 10 \ln 1.015 + \frac{1}{4} \left(\hat{z}_{i,t} - z_{i,t} \right), \qquad (12) \\ \hat{z}_{i,t+1} &= \hat{z}_{i,t} + 10 \ln 1.015, \\ \hat{z}_{i,0} &= z_{i,0} + \ln 1.6, \end{aligned}$$

where $e^{\hat{z}_{i,t}}$ is the TFP path that actual TFP converges to, and $t = 0$ is the initial period. As seen in equation (12), 1/4 of the productivity gap $(\hat{z}_{i,t} - z_{i,t})$ is removed each decade.

The region-specific damages are calibrated using the latest estimates from RICE (Nordhaus, 2010). Specifically, this study provides linear-quadratic regional damage functions that are stating the share of GDP lost due to changes in the global mean temperature. The parameters of these damage functions for each region are provided in Table 1.

The climate sensitivity, ξ, is set to $3°C$ per doubling of atmospheric CO2 concentration,

[11] By expressing quantities in oil equivalents, the difference in energy content between natural gas, oil, and various grades of coal is accounted for.

[12] See appendix for details.

Table 1: Parameters of the damage functions for the each region

	Linear ϕ_1	Quadratic ϕ_2
US	$0.000 * 10^{-2}$	$0.1414 * 10^{-2}$
EU	$0.000 * 10^{-2}$	$0.1591 * 10^{-2}$
China	$0.0785 * 10^{-2}$	$0.1259 * 10^{-2}$
Africa	$0.3410 * 10^{-2}$	$0.1983 * 10^{-2}$
India	$0.4385 * 10^{-2}$	$0.1689 * 10^{-2}$

Source: Nordhaus (2010).

and the initial stock of atmospheric carbon is set to 586 gigatons of carbon (GtC). Using the Arrhenius equation, we can then for each set of damage parameters in Table 1 express the damage elasticity as a function of the global mean temperature

$$\gamma_i = -\frac{\ln\left(1 - \left(\phi_{1,i}T + \phi_{2,i}T^2\right)\right)}{S_0\left(e^{\frac{T\ln(2)}{\xi}} - 1\right)}. \tag{13}$$

The functions implied by 13 are plotted in Figure 1. As can be seen, these functions are not very steep in the relevant range of temperatures. We therefore approximate the γ_i-functions with the respective γ_i evaluated at $3.5°C$, yielding $\gamma_{US} = 2.395$, $\gamma_{Eu} = 2.698$, $\gamma_{Ch} = 2.514$, $\gamma_{In} = 5.058$, and $\gamma_{Af} = 5.031$ and with all these numbers multiplied by 10^{-5} respectively.[13] For the carbon-cycle parameters, we again follow Golosov et al. (2014) by setting $\varphi_L = 0.2$, $\varphi_0 = 0.393$, and $\varphi = 0.0228$.

Figure 1. Implied damage elasticity

4 Results

4.1 Taxes

We are now ready to present simulation results for a set of different policies. The different policy scenarios differ in terms of coverage and in the level of the tax. The first scenario imposes a global Pigouvian tax. This implies setting the initial global tax to US $76.8 per ton of carbon, which then increases by 2.2% per year, i.e., approximately at the growth rate of GDP. This tax fully internalizes the climate externality given the chosen parameters,

[13] See Hassler, Krusell, and Olovsson (2018) for an analysis that evaluates ranges of plausible estimates for both the climate sensitivity and the sensitivity of the economy to climate change.

as is shown in Golosov et al. (2014).[14] The second assumes that only Europe implements the fossil-fuel tax. The third scenario feature a global tax, but only on coal. The fourth scenario considers a unilateral European coal tax only. The fifth scenario, considers a case when Europe implements the Pigouvian tax while the rest of the world is less ambitious and chooses a coal tax that is only a quarter of the optimal. Finally, we consider two more ambitious tax policies: one in which a global carbon tax that is twice as large as the Pigouvian tax is introduced, and one where the current Swedish carbon tax is introduced. This latter tax is close to seven times the Pigouvian tax that is imposed globally on coal.[15] In some cases, we will also present results for the *laissez faire* scenario.

Let us now first consider the effectiveness of the different policies in mitigating climate change. Figure 2 plots the increases in global mean temperature over the next 200 years that results from the different scenarios. A couple of important results are immediate from the figure. First, global fossil-fuel taxes are effective in mitigating climate change. Towards the end of this century, the difference in temperature between the *laissez faire* and global Pigouvian taxation is 0.65 degrees Celsius, and this difference increases substantially thereafter. A global Swedish tax would stop global warming almost completely and keep the global mean temperature just above 1.5 degrees Celsius at the end of century. Second, it is not effective to impose taxes only in EU. In fact, the difference in temperature between the case with only taxes in EU and the *laissez faire* are negligible. Third, what matters for the resulting change in the temperature is taxes on coal. Both the cases with global taxes and with taxes only in EU are identical in terms of climate change, i.e., regardless of whether oil is taxed or not. The intuition for this result is straightforward. The supply of oil is inelastic and does not respond to taxes, while the opposite is true for coal. Fourth, also an asymmetric coal tax where most of the world implements a quite low tax has important effects on climate change.

Figure 2. Increase in global mean temperature

Let us now compare the consumption levels that are generated in the different scenarios. Consumption is expressed relative to the consumption that is generated in the *laissez faire*. Figure 3 presents results for the EU, and shows that a global carbon tax increases welfare for EU consumers: consumption is always higher and substantially so far into the future.

[14] Compared to European taxes on gasoline, the tax is modest. Given a carbon content of around 0.65 Kg/liter for gasoline, this implies a tax of 5 cents per liter of gasoline.

[15] The Swedish CO_2-tax is SEK 1.14 per kg CO. This corresponds to approximately $1.14 * 3.67/8 * 1000 = 523$ US dollars per ton carbon. This amounts to 34 US cents per liter gasoline.

The effects are, however, relatively small in the current century. A global coal tax improves European welfare by less than as if the tax also includes oil, with the difference coming from the fact that the revenues from the oil producing region are then not redistributed. The effects of having taxes only in EU are very small. A carbon tax increases consumption marginally more due to its redistributional consequences. A global coal tax at the very high Swedish levels has a negative, but not very large effect in the current century. The long-run consequences are, however, positive and as large as those associated with the Pigouvian tax.

Figure 3. Consumption relative to no taxes – Europe.

The distributional consequences of the oil tax become clear when we consider the consumption paths for the oil producing region, which are shown in Figure 4. As can be seen, a tax that includes oil reduces consumption over most of the considered time period. Note though, that a tax on coal has a positive impact on the consumption of the oil producers towards the end of the simulation period. The reason for this is that the coal tax increases output through lower climate damages, and this leads to higher oil demand.

Figure 4. Consumption relative to no taxes – Oil Producers.

The consequences of the different scenarios for China and the U.S. are very similar to those for Europe, as seen in Figures 5 and 6.

Figure 5. Consumption relative to no taxes – U.S.

Figure 6. Consumption relative to no taxes – China.

Figures 7 and 8, finally, reveal that the stakes are highest in Africa and India. The differences between the different scenarios are quite large during the 22nd century.

Figure 7. Consumption relative to no taxes – Africa.

Figure 8. Consumption relative to no taxes – India.

4.2 Technical change

The previous section revealed that a global coal tax is an efficient way of mitigating climate change. Specifically, while coal use increases by a factor of 40 over the coming 200 years in the *laissez faire* economy, a coal tax that increases by 2% per year reduces the increase in coal use to a factor of two. Total emissions then fall over time as is shown in Figure 8. Higher taxes than the Pigouvian leads to a substantial reduction in emissions. The growth rate of green energy is, however, largely unaffected in the different scenarios.

Figure 9. Carbon emissions

If we use equation (7) to compute the relative use of coal and green, we get

$$\frac{e_{2,i,t}}{e_{3,i,t}} = \left(\frac{\lambda_2}{\lambda_3} \frac{p_{3,i,t}}{(1+\tau_{i,t})\, p_{2,i,t}} \right)^{\frac{1}{1-\rho}} .$$

The above equation shows that the relative usage of coal and green fuel is driven by the relative price including taxes. It also suggests that if technology were to develop in a way that increases the relative price of coal, this would have similar effects as a tax on coal. We therefore consider two more scenarios. First, one where green energy becomes 2% cheaper per year to produce (i.e., $p_{3,i,t}$ falls by 2% per year in all regions) and second, one that adds to the first scenario that coal becomes more expensive over time (i.e., $p_{2,i,t}$ increases over time). It may, at first, seem odd that costs increase over time, given that the growth rates of production costs are thought to come from technological change. Recall, however, that TFP in the final-good sector grows over time, implying that the production of final goods grow by about 2% per year. Since prices $p_{k,i,t}$ are measured in terms of the final goods, the price of coal, $p_{2,i,t}$, will then increase over time if the technology in the extraction of coal grows slower than that in the final good sector.

The results of the simulations are presented in Figure 10 and they show that the fact that the price of green energy falls over time by 2% is completely ineffective in mitigating global warming. Indeed, emissions are even larger in this scenario than in the base line *laissez faire* case. The reason is that lower green-energy prices implies lower energy prices in general, which increases the demand for all energy services, also for coal. The Figure also shows that the second scenario, where $p_{2,i,t}$ increases and $p_{3,i,t}$ falls produces a path of the global mean temperature that is virtually indistinguishable from that with global Pigouvian taxes.

Figure 10. Climate change with different rates of technical change.

The finding that directing technical change away from the production of fossil fuel is a powerful means of overcoming the problems associated with climate change suggests that we need to analyze the determinants of technical change. Doing this, still in a stylized a transparent way is one purpose of the next section.

5 Extensions

5.1 Energy sources with higher degree of substitutability

In our calibration above, we imposed a moderate degree of substitutability between coal and oil, and we did not allow oil to be produced outside of the oil-producing region. In recent years, however, hydraulic fracturing (fracking) has made it possible to produce oil and gas in substantial quantities in the United States, as well as in some other regions. The output from these sources is highly substitutable with conventional oil and gas, but it has fairly high production costs, thus implying that it can be considered to be something of a hybrid between oil and coal. The fact that the "fracking revolution" has had important implications for the world oil market motivates an extension of the model that includes fossil fuels that are costly to produce, and that are good substitutes for conventional fossil fuels.

To achieve this, we generalize the production function for energy services to a nested CES:

$$E_{i,t} = \mathcal{E}(e_{1,i,t}, \ldots, e_{n,i,t}) = \left(\lambda_1 l \left(\sum_{k=1}^{l} \lambda_{1,k} \left(e_{1,k,i,t}\right)^{\rho_h} \right)^{\frac{\rho}{\rho_h}} + \sum_{2}^{n} \lambda_k \left(e_{k,i,t}\right)^{\rho} \right)^{\frac{1}{\rho}}$$

where $\sum_{k=1}^{n} \lambda_k = \sum_{k=1}^{l} \lambda_{1,k} = 1$. In the above specification, the l fuels $e_{1,k,i,t}$ can be allowed to be more substitutable with each other by setting $\rho_h > \rho$. We define the oil composite

$$O_{i,t} \equiv l \left(\sum_{k=1}^{l} \lambda_{1,k} \left(e_{1,k,i,t}\right)^{\rho_h} \right)^{\frac{1}{\rho_h}},$$

and interpret $e_{1,k,i,t}$ as conventional oil imports to region i in period t. The other components, $e_{1,k,i,t}$, $k > 1$, are locally produced close substitutes of oil, i.e., output from fracking. Hence, this specification allows for both locally produced substitutes to conventional fossil fuels, as well as a higher elasticity of substitution between these objects.

The regional demand of the different components of the oil composite can be determined from the following problem

$$\min_{e_{1,k,i,t}} \sum_{k=1}^{l} \hat{p}_{1,k,i,t} e_{1,k,i,t} - P_{i,t}^{O} \left(l \left(\sum_{k=1}^{l} \lambda_{1,k} \left(e_{1,k,i,t}\right)^{\rho_h} \right)^{\frac{1}{\rho_h}} - O_{i,t} \right) \tag{14}$$

where $\hat{p}_{1,k,i,t}$ denotes the different fuel prices, and $\hat{p}_{1,1,i,t}$ is the global market price of conventional oil including the region-specific tax. As for coal, we assume that the other components of the oil composite are determined from the regional cost side.[16]

The first-order condition to the problem defined by (14) yields

$$e^*_{1,k,i,t} = \frac{O_{i,t}}{l}\left(\frac{\lambda_{1,k} l P^O_{i,t}}{\hat{p}_{1,k,i,t}}\right)^{\frac{1}{1-\rho_h}}, \quad k \in \{1, l\}.$$

Following the procedure in Section 2.5, the price index for the oil composite can be shown to be given by

$$P^O_{i,t} = l^{-1}\left(\sum_{k=1}^{l} (\lambda_{1,k})^{\frac{1}{1-\rho_h}} (\hat{p}_{1,k,i,t})^{\frac{\rho_h}{\rho_h-1}}\right)^{\frac{\rho_h-1}{\rho_h}}.$$

It then follows that the price of energy services and the demand for the oil composite, respectively, are given by

$$P_{t,i} = \left((\lambda_1)^{\frac{1}{1-\rho}} \left(P^O_{i,t}\right)^{\frac{\rho}{\rho-1}} + \sum_{k=2}^{n} (\lambda_k)^{\frac{1}{1-\rho}} (\hat{p}_{k,i,t})^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}},$$

and

$$O^*_{i,t} = \left(\frac{\lambda_1 P_{i,t}}{P^O_{i,t}}\right)^{\frac{1}{1-\rho}} E_{i,t},$$

where $E_{i,t}$ is still given by (10).

Finally, the demand for other energy sources are determined by the following equations

$$e^*_{k,i,t} = E_{i,t}\left(\frac{\lambda_k P_{i,t}}{\hat{p}_{k,i,t}}\right)^{\frac{1}{1-\rho}}, k \in \{2, n\}.$$

5.1.1 Calibration with fracking

The amount of non-conventional reserves of fossil fuel extractable by fracking and other existing or future technologies is obviously hard to assess. The calibration in this section should therefore be seen as illustrative, and a more worked through calibration is left for future work. With these caveats in mind, we set $l = 2$ and assume that fracking costs in the U.S. are US \$40 per barrel, corresponding to US \$347 per ton carbon. Also China is assumed to be able to produce fracked oil at a cost that is 50 percent higher than that for the

[16] Assuming that all regions have the same cost of producing these fuels is isomorphic to allowing a global market for "fracked" oil.

United States, while the other regions cannot or abstain from fracking. Finally, we impose a high degree of substitutability between the output from fracking and conventional fossil fuels and set the elasticity of substitution between the components of the oil composite to 10, implying $\rho_h = 0.9$.[17]

5.1.2 Results with fracking

The results from this exercise are displayed in Figure 11. A striking feature is that the addition of a fossil fuel that is a good substitute for conventional oil, but is provided without scarcity rents makes the problem of climate change substantially more severe. In the *laissez faire,* the temperature increases by 5 degrees by the end of this century, and by 12 degrees at the end of the simulation period. The good news is that taxes remain to be an effective way of dealing with climate change. Compared to the case without fracking, however, there is more global warning under the comprehensive tax scheme. An important difference compared to the case in the previous section is that a tax only on coal now results in substantially more global warning than a global carbon tax. The intuition for this is that the supply of the oil composite is highly elastic, implying that it responds strongly to a tax. A global tax on coal and fracking, however, results in global warning almost identical to the one under a global carbon tax.

As in the case without fracking, a zero tax on conventional oil leads to substantially higher welfare for the region that is producing the conventional oil. In fact, for the oil-producing region, carbon taxes excluding conventional oil is substantially better than the *laissez faire.*

Figure 11. Climate change with fracking under different tax regimes.

5.2 Endogenous technical change

We now turn to an analysis of endogenous technical change. The purpose of this section is to provide a simple framework for endogenizing the cost of producing the different sources of energy. We here, again, abstract from fracking and maintain the assumption that oil is imported, whereas the other energy sources are domestically produced at costs $p_{k,i,t}$. We allow a tax $\tau_{k,i,t}$ on each fuel. Specifically, we will consider both per-unit taxes and proportional *ad-valorem* taxes.

[17] A related study about the U.S. shale oil boom is Çakir Melek, Plante, and Yücel (2017).

As in the previous sections, there is an energy-producing representative firm selling energy services on a competitive market. The important difference is that the energy producer now also has the possibility to improve the technologies for producing the different domestic energy inputs. Specifically, all energy inputs except oil can be reduced at a costs that is determined by the constraint $RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right) \geq 0$.

The problem of the representative energy-service provider with *ad-valorem* taxes is then given by

$$\begin{aligned} &\min_{\{e_{k,i,t}\}_1^n,\{p_{k,i,t}\}_2^n} \sum_{k=1}^{n} \left(1+\tau_{k,i,t}\right) p_{k,i,t} e_{k,i,t} \\ &-P_{i,t}\left(\mathcal{E}(e_{1,i,t},\ldots,e_{n,i,t}) - E_{i,t}\right) - \Lambda_{i,t} RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right). \end{aligned} \tag{15}$$

The problem defined by (15) differs from that in (6) in that the former problem includes a set of new choice variables $\{p_{k,i,t}\}_2^n$, as well as a new constraint $RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right)$ with a Lagrange multiplier $\Lambda_{i,t}$. Hence, the energy-producer takes the oil price, $p_{1,i,t}$ as given, but can affect $p_{k,i,t}$ for $k \neq 1$. It is straightforward to verify that all features of Proposition 1 still hold up.

The first-order conditions with respect to $e_{k,i,t}$ and $p_{k,i,t}, k \neq 1$ are, respectively, given by

$$e^*_{k,i,t} = E_{i,t}\left(\frac{P_{t,i}\lambda_k}{\left(1+\tau_{k,i,t}\right)p_{k,i,t}}\right)^{\frac{1}{1-\rho}}, \tag{16}$$

and

$$\left(1+\tau_{k,i,t}\right)e^*_{k,i,t} = \Lambda_{i,t}\frac{\partial RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right)}{\partial p_{k,i,t}}. \tag{17}$$

Note that (16) is identical to (7) in Section (2.5.2).

With per-unit taxes imposed on the energy sources, the objective function (15) of the energy-service provider becomes

$$\sum_{k=1}^{n}\left(\tau_{k,i,t}+p_{k,i,t}\right)e_{k,i,t}.$$

The resulting first-order conditions are then given by

$$e^*_{k,i,t} = E_{i,t}\left(\frac{P_{t,i}\lambda_k}{\tau_{k,i,t}+p_{k,i,t}}\right)^{\frac{1}{1-\rho}}$$

and

$$e_{k,i,t}^{*}=\Lambda_{i,t}\frac{\partial RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right)}{\partial p_{k,i,t}}. \tag{18}$$

Comparing (17) and (18), we find an important difference. The left-hand side of both equations represents the value of reducing costs of producing a particular fuel. In the case of *ad-valorem* taxes, this value increases in the tax for given quantity $e_{k,i,t}^{*}$. This is not the case with taxes per unit. We will return to this feature below.

5.2.1 Specializing R&D technology

We now make specific assumptions on the technology that is available to the energy-service producers. As with exogenous technologies, we still consider three different energy sources, i.e., oil, coal, and green energy. Now, however, production and extraction costs of coal and green fuel can be reduced subject to a constraint on the weighted average of the relative improvements. Formally, the constraint is given by

$$\begin{aligned} RD_{i,t}\left(p_{2,i,t},p_{3,i,t}\right) &= \min\left(\varepsilon_{2,i}\ln\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}},0\right)+\min\left(\varepsilon_{3,i}\ln\frac{p_{3,i,t}}{\bar{p}_{3,i,t-1}},0\right)+a\geq 0, && (19)\\ \sum_{k}\varepsilon_{k,i} &= 1, && (20)\end{aligned}$$

where $\bar{p}_{k,i,t-1}$ denotes costs if no cost-reductions occur. These costs are determined by aggregate innovation decisions in the previous period, either in the region or globally. Cost-reductions then spill over with a one-period lag, which implies that firms are myopic in their choices of technology. It is natural to think that $\bar{p}_{k,i,t-1}$ is higher than average costs in the previous period in line with the discussion in the previous section. However, it appears natural to assume that $p_{k,i,t}$ cannot be increased by making $p_{j,i,t}$ larger than the cost when no cost-reductions occur. This motivates the min operator in (19).

The R&D production technology defined by (19) can be thought of as an interesting starting point in the spirit of Romer (1986), where the number of researchers that are active in R&D determines the rate of technological change. Our specification can then be seen as an extension to directed technical change where the number of R&D workers is fixed, and the productivity in improving the technology differs between the energy sources.

The specification in (19) now reads for $k=2,3$

$$\frac{\partial RD_{i,t}\left(p_{2,i,t},...,p_{n,i,t}\right)}{\partial p_{k,i,t}}=\frac{\varepsilon_{k}}{p_{k,i,t}},$$

which, in turn, implies

$$\left(1+\tau_{k,i,t}\right) p_{k,i,t}^{*} e_{k,i,t}^{*}=\varepsilon_{k} \Lambda_{i,t}. \tag{21}$$

Proposition 2 *When first-order conditions for the technology choice are satisfied, and taxes are* ad-valorem*, spending on green energy is a fixed fraction of all spending on domestically produces energy sources, i.e.,*

$$\frac{\left(1+\tau_{3,i,t}\right) p_{3,i,t}^{*} e_{3,i,t}^{*}}{P_{t,i} E_{i,t}-\left(1+\tau_{1,i,t}\right) p_{1,i,t} e_{1,i,t}^{*}}=\varepsilon_{3}.$$

Proof. Follows directly from noting that $\left(1+\tau_{1,i,t}\right) p_{1,i,t} e_{1,i,t}^{*}+\sum_{k=2}^{3}\left(1+\tau_{k,i,t}\right) p_{k,i,t}^{*} e_{k,i,t}^{*}=P_{i,t} E_{i,t}$ and using (21). ■

Furthermore, using the expression (16) in (21) for $k=2$ and 3, yields the following two conditions

$$\left(1+\tau_{k,i,t}\right) p_{k,i,t}^{*}=\left(\frac{\varepsilon_{k} \Lambda_{i,t}}{E_{i,t}}\right)^{\frac{\rho-1}{\rho}}\left(P_{t,i} \lambda_{k}\right)^{\frac{1}{\rho}}, \tag{22}$$

and

$$\frac{\left(1+\tau_{2,i,t}\right) p_{2,i,t}^{*}}{\left(1+\tau_{3,i,t}\right) p_{3,i,t}^{*}}=\left(\frac{\varepsilon_{2}}{\varepsilon_{3}}\right)^{\frac{\rho-1}{\rho}}\left(\frac{\lambda_{2}}{\lambda_{3}}\right)^{\frac{1}{\rho}}. \tag{23}$$

Since the right-hand-side of (23) only contains technological constants, it follows that taxes are unable to change the after-tax relative price of domestic fuels - provided that the solutions to the first-order conditions all are interior. Furthermore, the growth rates of the two production costs are identical.

With both prices and spending being independent of taxes, also volumes are independent of taxes, i.e.,

$$\frac{e_{2,i,t}^{*}}{e_{3,i,t}^{*}}=\left(\frac{\lambda_{3}}{\lambda_{2}} \frac{\varepsilon_{2}}{\varepsilon_{3}}\right)^{\frac{1}{\rho}}.$$

We can get an intuition for this result by considering the first-order condition for the choice $p_k, k \neq 1$ given by (17). The left-hand side of the equation represents the marginal value of cost reductions. As can be seen, this value is increasing in the tax rate $\tau_{k,i,t}$ for a given quantity $e_{k,i,t}^{*}$. The fact that taxes are proportional to the cost of production implies that costs reductions are worth more the higher is the tax, *ceteris paribus.* However, an increased tax also reduces $e_{k,i,t}^{*}$ and this reduces the value of cost reductions. With the log-linear specification of R&D costs, the two effects exactly balances. Hence, R&D effectively nullifies the effect of taxes on after-tax prices.

This intuition also suggests that per-unit taxes should, in fact, lead to higher costs simply because the positive effect of taxes on the value of cost reductions just described disappears. Let us now turn to the formal analysis on the effects of per-unit taxes.

In this case, the first-order condition are given by

$$p_{k,i,t}e^*_{k,i,t} = \Lambda_{i,t}\varepsilon_k,$$

where

$$e^*_{k,i,t} = E_{i,t}\left(\frac{P_{t,i}\lambda_k}{\tau_{k,i,t}+p_{k,i,t}}\right)^{\frac{1}{1-\rho}}.$$

Combining the conditions for $k = 2$ and 3, we get the following condition

$$\frac{p_{2,i,t}\left(\tau_{2,i,t}+p_{2,i,t}\right)^{\frac{-1}{1-\rho}}}{p_{3,i,t}\left(\tau_{3,i,t}+p_{3,i,t}\right)^{\frac{-1}{1-\rho}}} = \frac{1-\varepsilon_3}{\varepsilon_3}\left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{\rho-1}}. \tag{24}$$

We can now consider the effects of changes in the coal tax. Total differentiation of the previous expression, noting that the R&D constraint implies $\frac{dp_{3,i,t}}{dp_{2,i,t}} = -\frac{1-\varepsilon_3}{\varepsilon_3}\frac{p_{3,i,t}}{p_{2,i,t}}$, and evaluating at both taxes being zero, yields

$$\left.\frac{dp_{2,i,t}}{d\tau_{2,i,t}}\right|_{\tau_{3,i,t}=\tau_{2,i,t}=0} = -\frac{\varepsilon_3}{\rho}.$$

Hence, when ρ is negative but close to zero, an increase in coal taxes leads to a large increase in the relative price of coal.

5.2.2 Calibration with endogenous technical change

In this section, we add to the calibration with exogenous technologies, the assumptions that the choice of extraction and production costs for green energy and coal is interior and that taxes are zero. In other words, we assume that current production prices of coal and green energy are optimally chosen, satisfy $\ln\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}} > 0$ and $\ln\frac{p_{3,i,t}}{\bar{p}_{3,i,t-1}} > 0$ and that taxes are zero. These assumptions, in particular the last one, are not necessarily satisfied in reality where both taxes and subsidies to energy production as well as to R&D can coexists. Our calibration, however, serves the purpose of illustrating the quantitative implications of the model.

Expressed per ton of carbon, the price of coal is calibrated to US \$103.4 and the price of green in carbon oil equivalents to US \$600. Equipped with our previously calibrated values of for λ_2 and λ_3, we can use these prices to calibrate the R&D parameter ε_3 from the following relationship

$$\left[\frac{p_{2,i,t}\left(0+p_{2,i,t}\right)^{\frac{-1}{1-\rho}}}{p_{3,i,t}\left(0+p_{3,i,t}\right)^{\frac{-1}{1-\rho}}}=\frac{1-\varepsilon_3}{\varepsilon_3}\left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{\rho-1}}\right]_{p_{2,i,t}=103.4,p_{3,i,t}=600,\lambda_2=0.102,\lambda_3=0.356,\rho=-0.058}$$
$$\rightarrow \quad \varepsilon_3=0.782.$$

The implication of this calibration is that it is easier to improve the productivity in coal production than in green-energy production. If that would not be the case, the model would predict that, currently, all R&D efforts should be directed toward reducing the relatively high cost of green energy.

Consider now the implications of introducing a carbon tax on coal. Using the R&D constraint (19), and initially disregarding the constraint on cost increases ($p_{2,i,t} \leq \bar{p}_{2,i,t-1}$), the relation between the two domestic fuel prices must imply that $\left(p_{2,i,t}\right)^{(1-\varepsilon_3)}\left(p_{3,i,t}\right)^{\varepsilon_3}$ is constant. Using the calibrated value for ε_3, and the initial prices, this constant is 409.0, yielding $p_{3,i,t}$ as a function of $p_{2,i,t}$

$$p_{3,i,t}=p_3\left(p_{2,i,t}\right)=409.0^{\frac{1}{\varepsilon_3}}\left(p_{2,i,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}. \tag{25}$$

Using this relation in (24) delivers

$$\frac{p_{2,i,t}\left(\tau_{2,i,t}+p_{2,i,t}\right)^{\frac{-1}{1-\rho}}}{p_{3,i,t}\left(p_{2,i,t}\right)^{\frac{\rho}{\rho-1}}}=\frac{1-\varepsilon_3}{\varepsilon_3}\left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{\rho-1}},$$

which implicitly defines $p_{2,i,t}$ as a function of the coal tax. Having found $p_{2,i,t}$, equation (25) provides $p_{3,i,t}$.

5.2.3 Results with endogenous technical change

The solid curves in Figure 12 depicts the resulting relations between taxes and prices in an interior solution. As can be seen, the coal price is an increasing function of the tax, whereas the green-energy price is a decreasing function. Thus, R&D strongly amplifies the effect of a tax. The fact that $\varepsilon_3 > 1/2$ implies that it requires more R&D resources to reduce the cost of producing green energy than reducing the cost of producing coal. Both curves are, however, quite steep. Already for a coal tax of around US \$ 35 per ton carbon, the coal price becomes as high as the price of green energy. For the coal tax used in the previous section, i.e., US \$76.8 per ton carbon the coal price is substantially higher than the price of green energy (US \$680 vs. US \$400 per ton carbon) rather than only a fourth of the green price without taxes (see Figure 12).

The large effect of taxes on equilibrium prices is due to the fairly high elasticity of substitution. Already a moderately lower elasticity makes taxes a much less potent driver of the R&D direction. The dashed curves are computed under the assumption of an elasticity of substitution of 1/2, i.e., $\rho = -1$, with $\lambda's$ and $\varepsilon's$ recalibrated. Clearly, the coal and green-energy prices are then fairly insensitive to taxes. A coal tax of US \$100 per ton leads to an increase in the coal price before taxes of approximately US \$50. Thus, the R&D feedback mechanism is still positive, but much weaker than with an elasticity of substitution that is close to unity. The intuition for this result is straightforward. The direct effect of a tax is to reduce the volume demanded, the more the larger is the elasticity of substitution. From (18), it follows that the marginal value of directing R&D to reduce costs is given by the volume. Thus, the reduction in the marginal value of R&D falls more the higher is the elasticity of substitution.

Figure 12. Coal and green energy prices in interior R&D optimum as function of coal tax.

Let us now return to the constraint $p_{2,i,t} \leq \bar{p}_{2,i,t-1}$. As discussed above, it seems reasonable to assume that technology cannot regress. Without technical advances in the coal industry, it may be assumed that costs in terms of the final good increases at about the rate of GDP-growth. This would amount to somewhat more than 20% over a decade, which can be seen as an upper bound on how much the coal price can increase within a period. The conclusions are then that even a very modest tax on coal would lead to a stagnant coal technology, and that R&D would be directed towards improving the green energy. Let us therefore, finally, consider the implications of a global coal tax of the same size as in the previous section, but noting that this will lead all R&D to focus on reducing the cost of green energy. We use the R&D-parameter calibrated for Europe, and endogenize the growth rates of coal and green-energy prices from an initial situation where both of them are constant in terms of the final good (the same rate of technical advances as in the final good sector). Then, redirecting R&D towards green, implies (by assumption) that coal prices growth at 2% per year. The price of green energy instead falls by $((1-\varepsilon_3)/\varepsilon_3) * 2\% = 0.56\%$ per year. It may also be noted that the calibration implies that if R&D instead would be fully directed towards reducing production costs for coal, the relative price of coal would fall by $(\varepsilon_3/(1-\varepsilon_3)) * 2 = 7.2\%$ per year. This is a large number but, in fact, not completely out of line with data. On average, the relative price of coal in units of GDP[18] fell by 4.8% per year over the period 1950 to 2000. For most of the decades, the reduction was much faster. If we exclude the the 1970s, i.e., the decade with the two major oil-price shocks, the average decline in the relative coal price was 6.93% per year.

In Figure 13, we show the paths for the global mean temperature with taxes and endogenous technical change, as well as the *laissez faire* scenario (which, by assumption, remains identical to the case with exogenous technical change). As can be seen, the temperature rises substantially less with endogenous technical change due to the fact that coal use immediately starts falling. Recall that with exogenous technical change and a global coal tax, coal use increased over the coming century and peaked around 2090.

Figure 13. Increase in global mean temperature global coal tax with and without endogenous technical change and laissez faire.

Figure 14 plots the paths for consumption with a global coal tax and endogenous technical change relative to the *laissez faire*, where we assume neutral technological change (i.e., no change in relative prices). In comparison to the case of exogenous technical change, Europe and Africa now gains even more by the introduction of the global coal tax, while the consumption of all other regions remains largely unaffected.

Figure 14. Consumption under global coal tax and endogenous technical change relative to laissez faire.

So far, results have been derived under the assumption that interior solutions to the R&D problem define the optimum. However, if the elasticity of substitution $1/(1-\rho)$ is large enough, the energy-producing firm may specialize in some fuels. The purpose of the next sub-section is to analyze this case.

[18] This is calculated as the difference between the growth of nominal GDP and the rate of change of the nominal coal price. The GDP data is from the FRED database, and the data on coal prices from EIA (2012).

5.3 Corner solutions with $\rho > 0$

Cost-minimization amounts to minimizing the price index $P_{i,t}$ as defined by (9), subject to the relevant constraints. Formally, the problem is

$$\min_{p_{2,i,t} \le \bar{p}_{2,i,t-1}, p_{3,i,t} \le \bar{p}_{3,i,t-1}} P_{i,t}$$
$$s.t. \left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}}\right)^{\varepsilon_2} \left(\frac{p_{3,i,t}}{\bar{p}_{3,i,t-1}}\right)^{\varepsilon_3} = \exp\left(-a\right).$$

Using the constraint to replace $p_{3,i,t}$ and maximizing over $p_{2,i,t}$, the second derivative evaluated at the first-order condition is

$$\frac{\rho}{\rho-1} \frac{\varepsilon_2 + \varepsilon_3}{\varepsilon_3} \frac{\varepsilon_2}{\varepsilon_3} \frac{1}{(p_{2,i,t})^2} \left((1 + \tau_{3,i,t}) \bar{p}_{3,i,t-1} \left(\frac{p_{2,i,t}}{\bar{p}_{2,i,t-1}}\right)^{-\frac{\varepsilon_2}{\varepsilon_3}} \exp\left(-\frac{a}{\varepsilon_3}\right)\right)^{\frac{\rho}{\rho-1}} \lambda_3^{\frac{1}{1-\rho}},$$

which is negative iff $\rho > 0$. In this case, the first-order condition identifies a maximal cost. Minimum energy cost then arises at one of the corners[19]

$$\left(\bar{p}_{2,i,t-1} e^{\frac{-a}{\varepsilon_2}}, \bar{p}_{3,i,t-1}\right) \text{ and } \left(\bar{p}_{2,i,t-1}, \bar{p}_{3,i,t-1} e^{\frac{-a}{\varepsilon_3}}\right).$$

It is immediate that iff

$$X_{2/3} \equiv \frac{\hat{p}_{2,3}^{\frac{\rho}{\rho-1}} \exp\left(\frac{a}{\varepsilon_2}\frac{\rho}{1-\rho}\right) + 1}{\hat{p}_{2,3}^{\frac{\rho}{\rho-1}} + \exp\left(\frac{a}{\varepsilon_3}\frac{\rho}{1-\rho}\right)} < 1, \tag{26}$$

where $\hat{p}_{2,3} \equiv \frac{(1+\tau_{2,i,t})\bar{p}_{2,i,t-1}}{(1+\tau_{3,i,t})\bar{p}_{3,i,t-1}} \left(\frac{\lambda_2}{\lambda_3}\right)^{-\frac{1}{\rho}}$, then costs are lower if the first corner is used (i.e., when all resources are employed reducing the cost for producing fuel of type 2). The cut-off where energy firms are indifferent between the corners is

$$\bar{p}_{2,3} \equiv \left(\frac{\exp\left(\frac{\rho}{1-\rho}\frac{a}{\varepsilon_2}\right) - 1}{\exp\left(\frac{\rho}{1-\rho}\frac{a}{\varepsilon_3}\right) - 1} \right)^{\frac{\rho-1}{\rho}}.$$

Since $X_{2/3}$ is increasing in $\hat{p}_{2,3}$ when $\rho > 0$, costs are lower (higher) in the corner where costs of fuel 2 are reduced whenever $\hat{p}_{2,3} < (>)\, \bar{p}_{2,3}$.

[19] We have not yet proved global concavity, but that seems straightforward.

A key difference between the result here and in the previous subsection is that the direction of R&D can permanently be shifted by a temporary tax. To see this, note that the base-line costs enter in the optimality condition (26). A temporary tax may be required to make R&D switch to renewables. Over time, this leads $\frac{\bar{p}_{2,i,t-1}}{\bar{p}_{3,i,t-1}}$ to increase and eventually, no tax is required to keep R&D directed towards reducing the cost of renewables. This is the mechanism in Acemoglu et al. (2012).

6 Discussion and concluding remarks

We set up an integrated assessment model with multiple imperfectly substitutable energy sources and multiple world regions to study the effects of sub-optimal taxation. The model also incorporates hydraulic fracturing - fracking - of nonconventional oil, and endogenous technical change directed at reducing the production costs for the different energy sources. In the calibration, we allow for six regions that respectively represents the United States, Europe, China, India, and Africa. The considered energy inputs include conventional oil, coal, green (renewable) energy, and the output from fracking that is highly substitutable with conventional oil. Despite this relatively rich model structure, all variables except the oil price have closed-form solutions. This makes it straightforward to increase the number of regions as well as the energy sources in future extensions of the analysis.

Four main findings stand out from our analysis. First, we find that optimal taxes that only are implemented in Europe are insufficient in mitigating global warming. Consequently, it is necessary that the other regions also impose, at least some, carbon taxes to reduce global warming.

Second, the fracking technology has the potential to dramatically increase the stock of nonconventional oil which, in the absence of policy, can be expected to generate substantial global warming. It is therefore crucial that an effective carbon tax also is implemented on the nonconventional oil that is produced with the fracking technology.

Third, taxes that are proportional to the price of a specific energy good are completely impotent in reducing the demand for this good. The intuition for this potentially surprising result comes from the fact that a proportional tax generates two effects with different signs. On the one hand, a proportional tax increases the marginal value of cost reductions. On the other hand, the proportional tax reduces the demand for the energy good, which also reduces the marginal value of cost reductions. In our setting, these two effects exactly balances. The net effect of R&D is then to effectively nullify the effects of taxes on the after-tax price.

The disappointing result that proportional taxes are ineffective is, however, overturned by our fourth finding that shows that per-unit taxes, in fact, are effective in mitigating global warming. In this case, endogenous technical change actually reinforces the effectiveness of carbon taxes.

Our results can be contrasted to those in Acemoglu et al. (2012) that show that the policy implications depend critically on whether the elasticity of substitution between the different energy sources is below or above unity. We also note these differences, but find them to be somewhat mitigated. In particular, we find that a tax can be effective in moving the direction of R&D even with an elasticity that is below unity. In this case, a permanent tax is needed to achieve the optimum level of global warming.

The model we provide in this paper builds on well-established macroeconomic principles, which is why we think that its quantitative implications contain valuable information. In particular, the model results can complement those from more complicated existing integrated assessment models. At the same time, we have made some simplifying assumptions to gain tractability and closed-form solutions. One such convenient assumption is the non-existence of an international bond market. Allowing bond trade between the oil-consuming countries would not change the results as long as these countries all are on a common balanced growth path. We, however, realistically assumed that China, India and Africa currently are growing faster than the other regions. In this setting, the possibility to trade in bonds would induce the faster growing regions to run, potentially very large, current account deficits. This is not something that we observe in reality, but rather we observe the opposite. Allowing for trade in bonds between oil consumers and oil producers would change the analysis in that it would complicate the oil-supply decision. In particular, the oil price would then obey the Hotelling rule, which also is also a feature that does not seem to have any empirical support. Against this background, it is not clear that allowing for trade in bonds would make the model more realistic.

Another simplifying assumption that we made concerns the way we model the R&D decision. Specifically, we assumed that the production of all energy inputs (except oil) can be reduced at a cost that is determined by a log-linear constraint. The result that proportional taxes are impotent directly relies on that specific functional form. Future research needs to evaluate the sensitivity of this and other results with respect to this assumption. In particular, microeconomic evidence on the R&D process as in Popp (2002) and Aghion et al. (2016) may be useful here

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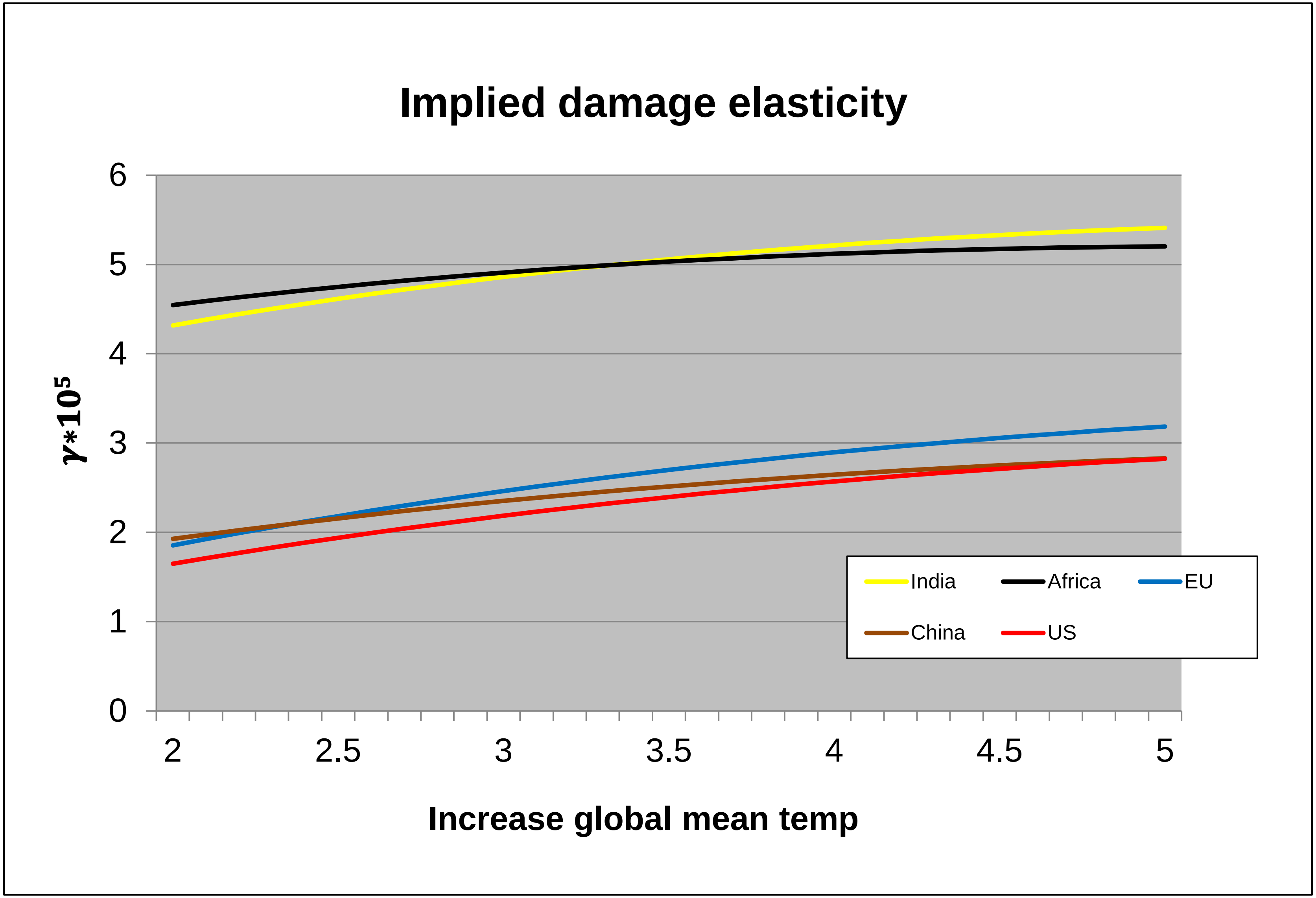
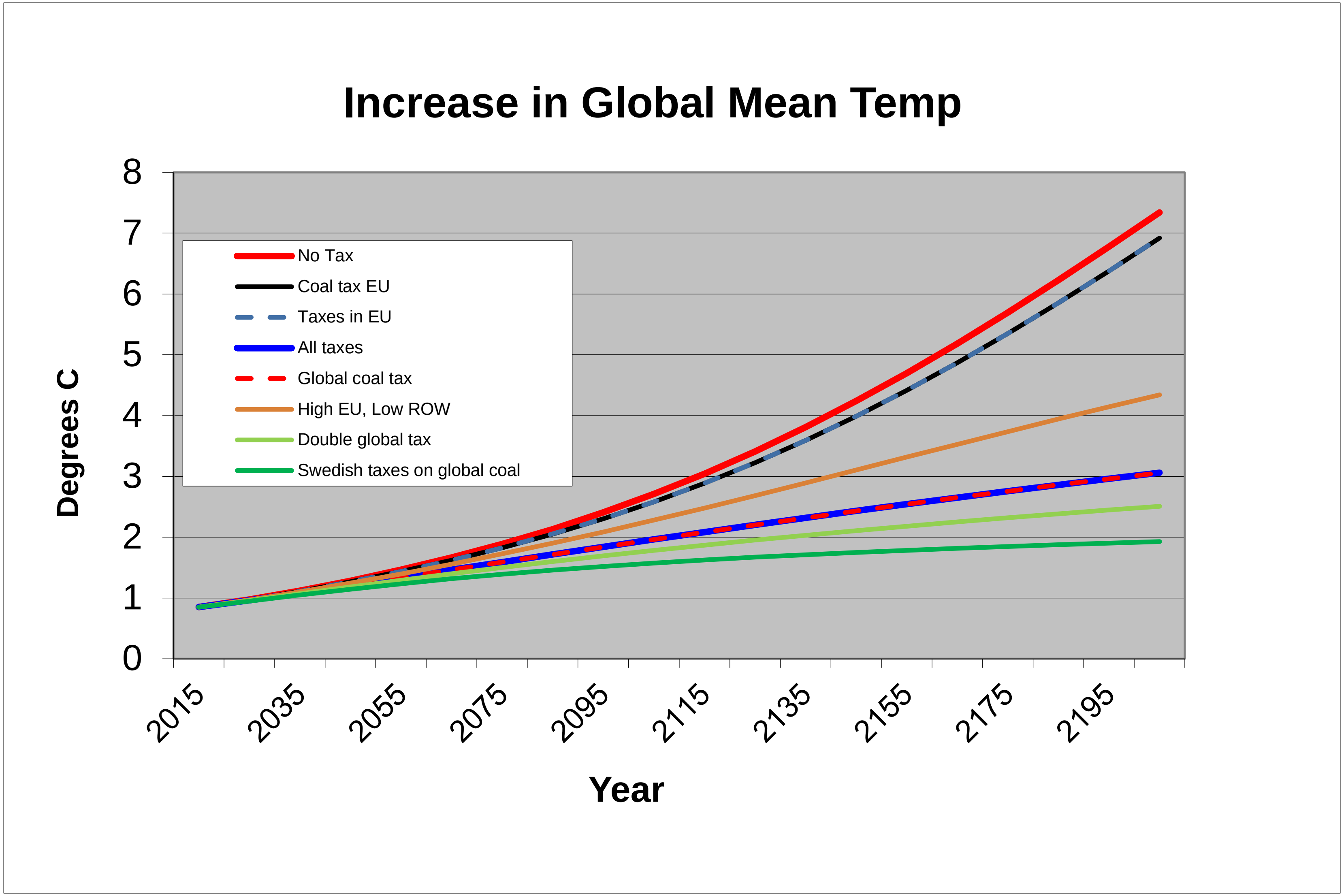
Figure 1.



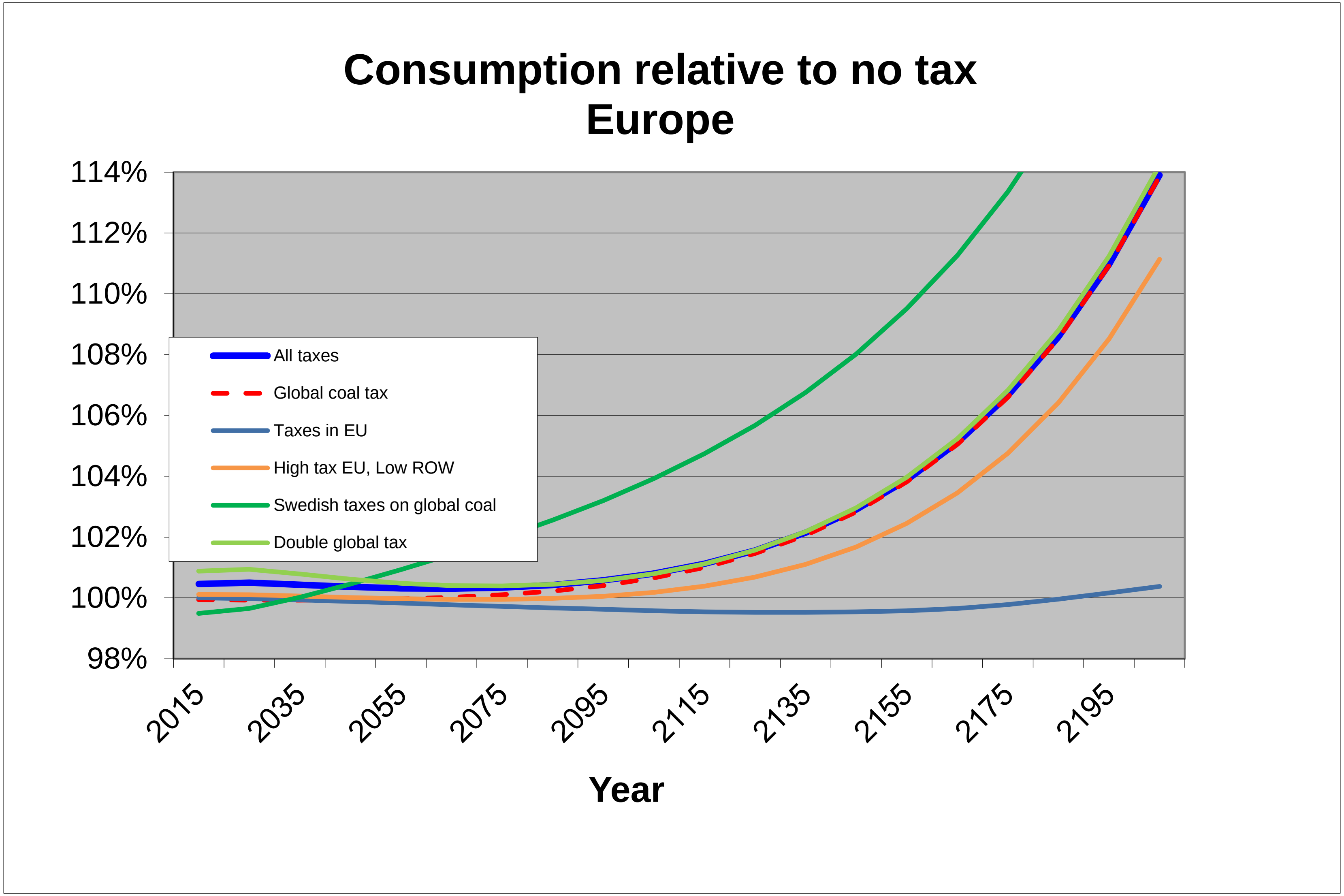
Implied damage elasticity
6
5
4
3
2
1
0
γ*10^5
2
2.5
3
3.5
4
4.5
5
Increase global mean temp
India
Africa
EU
China
US

Figure 2.



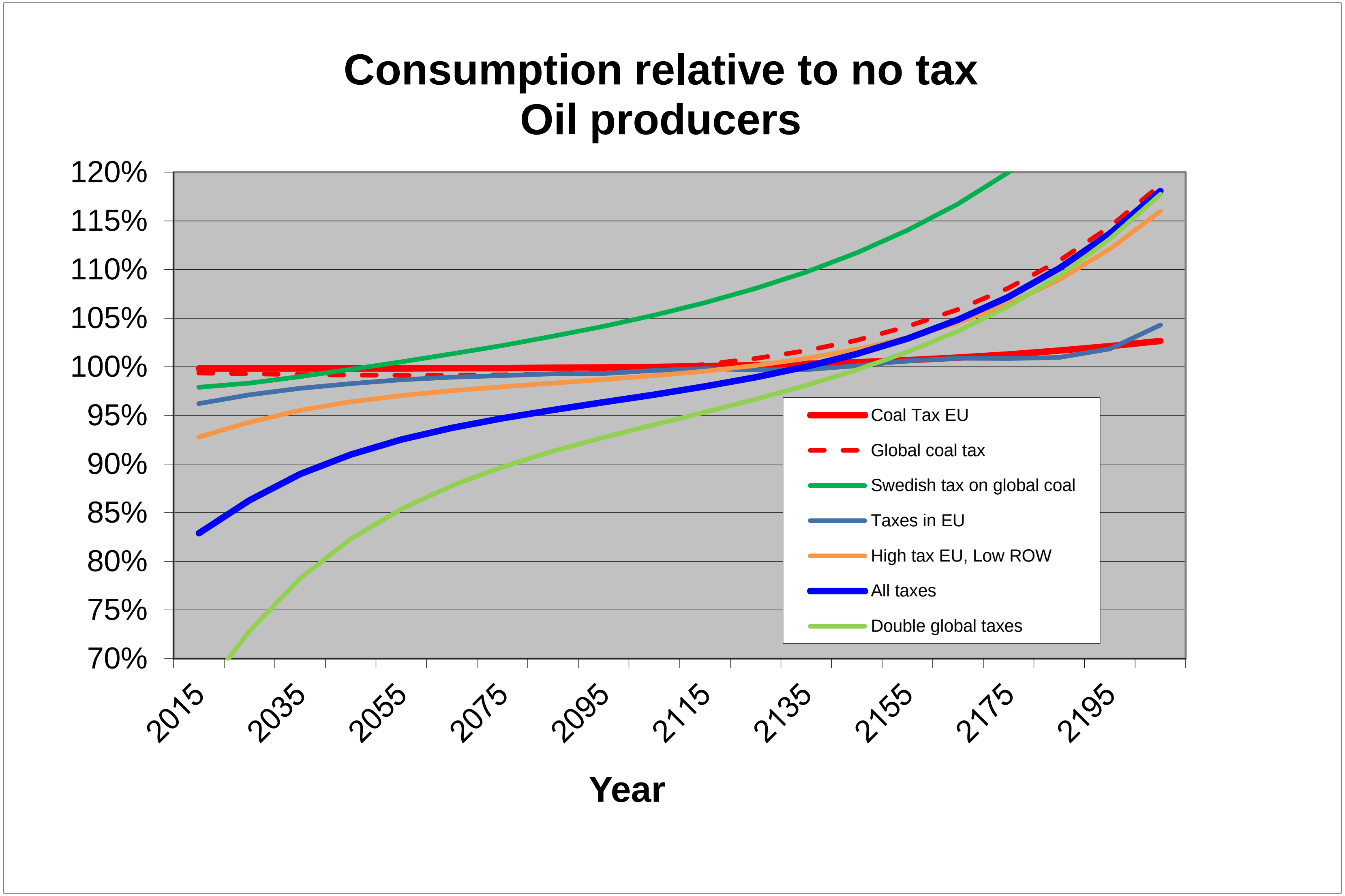
Increase in Global Mean Temp
Degrees C
0
1
2
3
4
5
6
7
8
No Tax
Coal tax EU
Taxes in EU
All taxes
Global coal tax
High EU, Low ROW
Double global tax
Swedish taxes on global coal
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year

Figure 3.



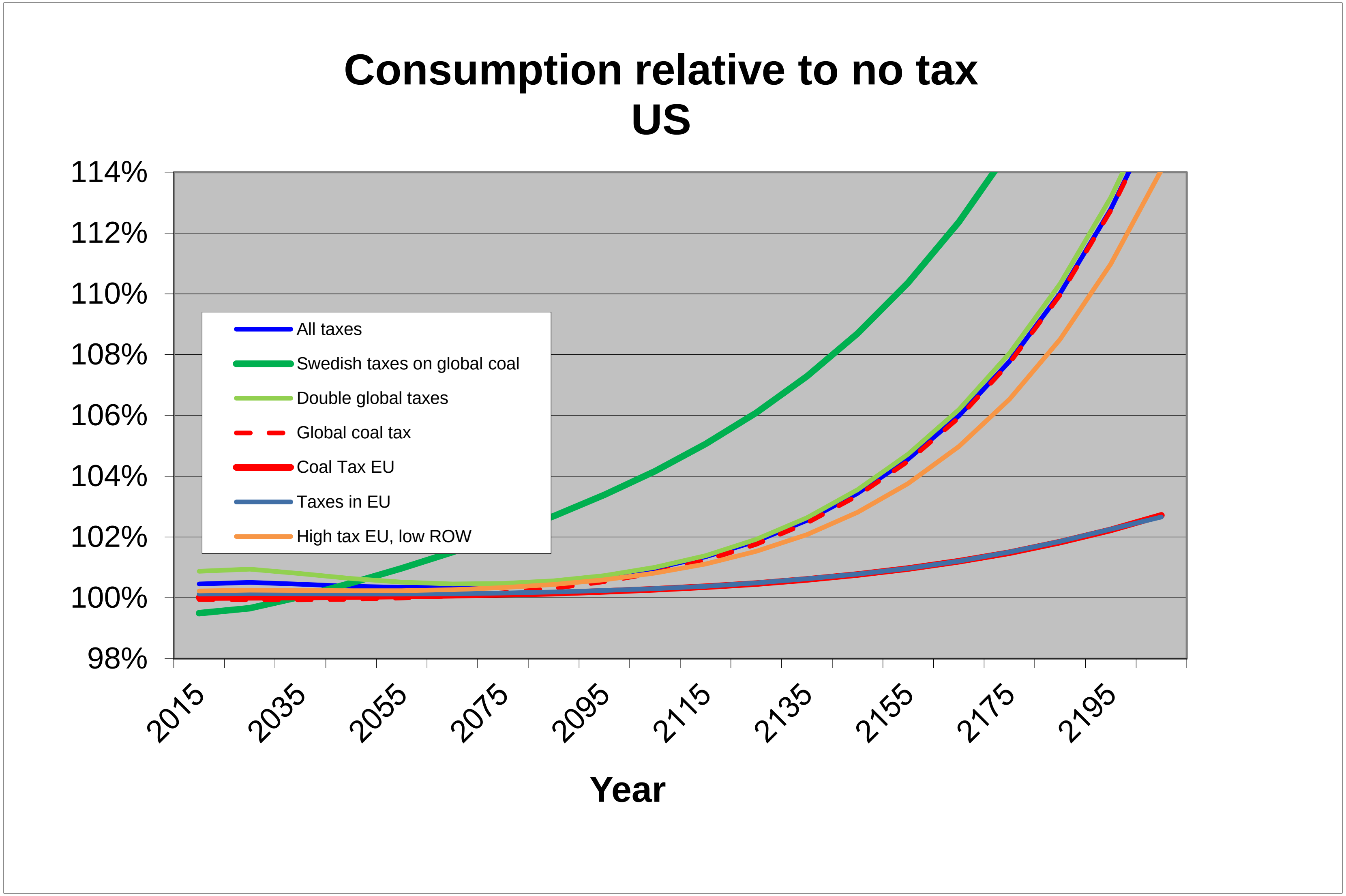
Consumption relative to no tax
Europe
114%
112%
110%
108%
106%
104%
102%
100%
98%
All taxes
Global coal tax
Taxes in EU
High tax EU, Low ROW
Swedish taxes on global coal
Double global tax
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year

Figure 4.



Consumption relative to no tax
Oil producers
120%
115%
110%
105%
100%
95%
90%
85%
80%
75%
70%
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year
Coal Tax EU
Global coal tax
Swedish tax on global coal
Taxes in EU
High tax EU, Low ROW
All taxes
Double global taxes

Figure 5.



Consumption relative to no tax
US
114%
112%
110%
108%
106%
104%
102%
100%
98%
All taxes
Swedish taxes on global coal
Double global taxes
Global coal tax
Coal Tax EU
Taxes in EU
High tax EU, low ROW
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year

Figure 6.

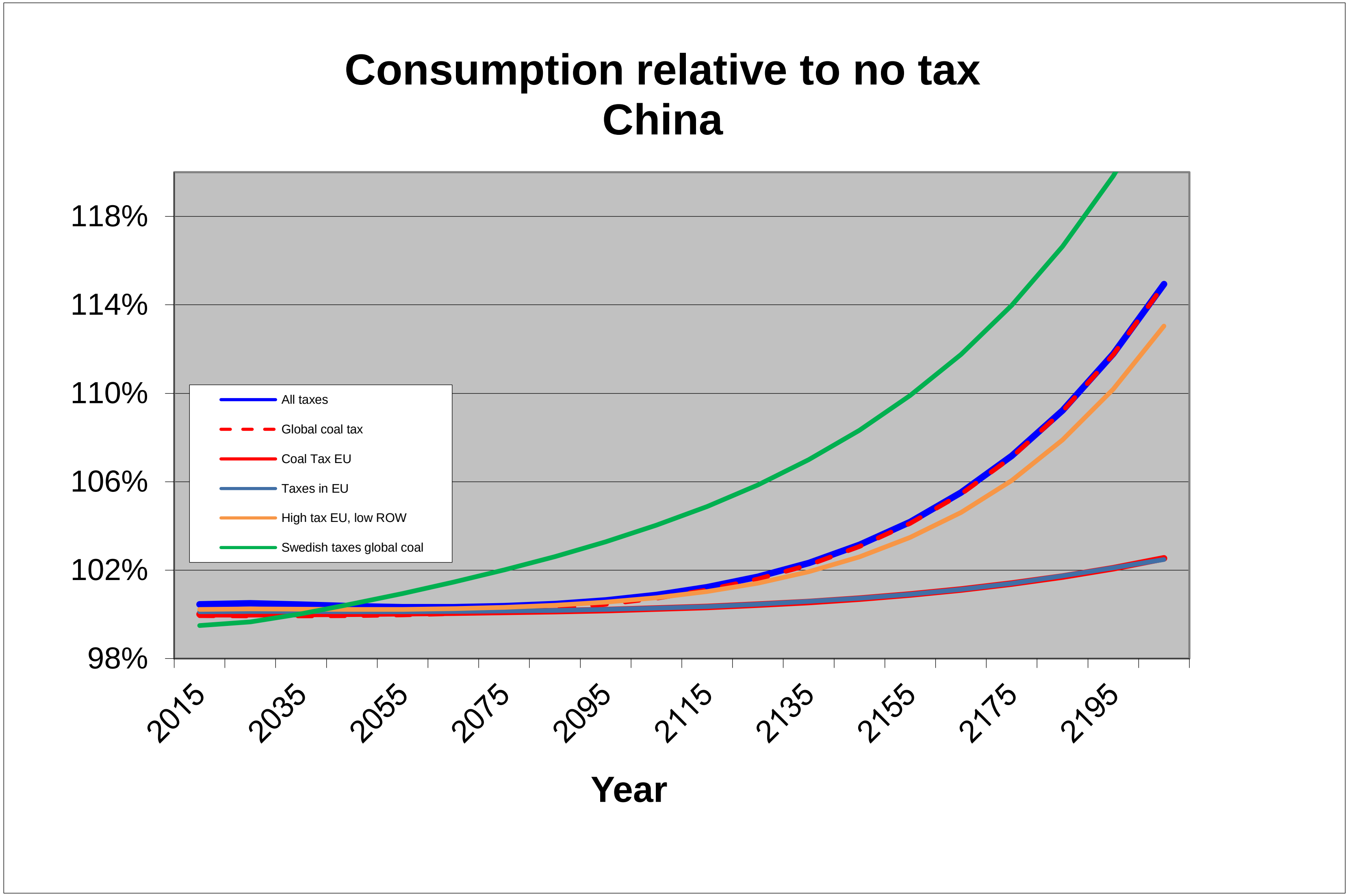

Consumption relative to no tax
China
118%
114%
110%
106%
102%
98%
All taxes
Global coal tax
Coal Tax EU
Taxes in EU
High tax EU, low ROW
Swedish taxes global coal
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year

Figure 7.

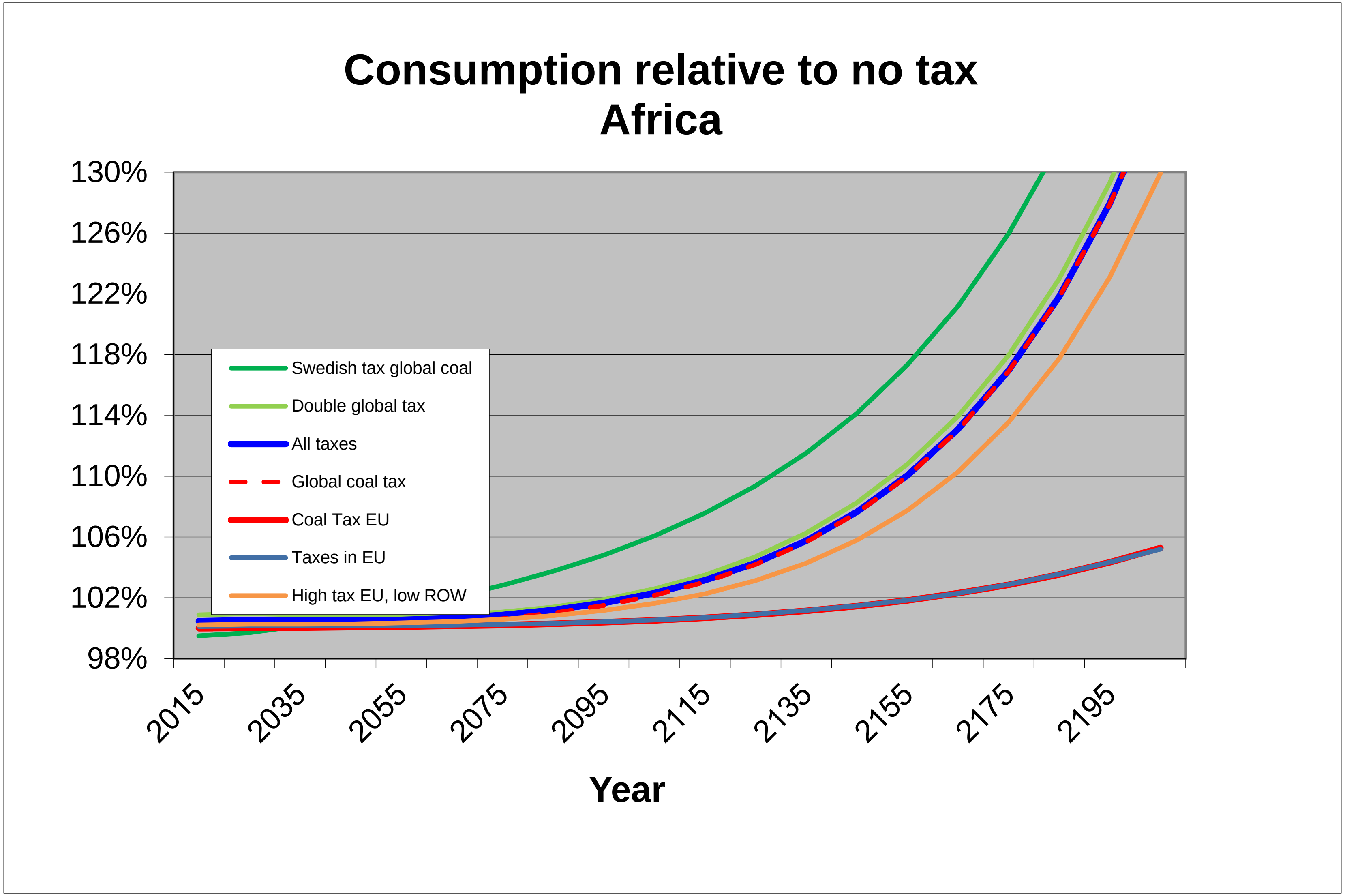
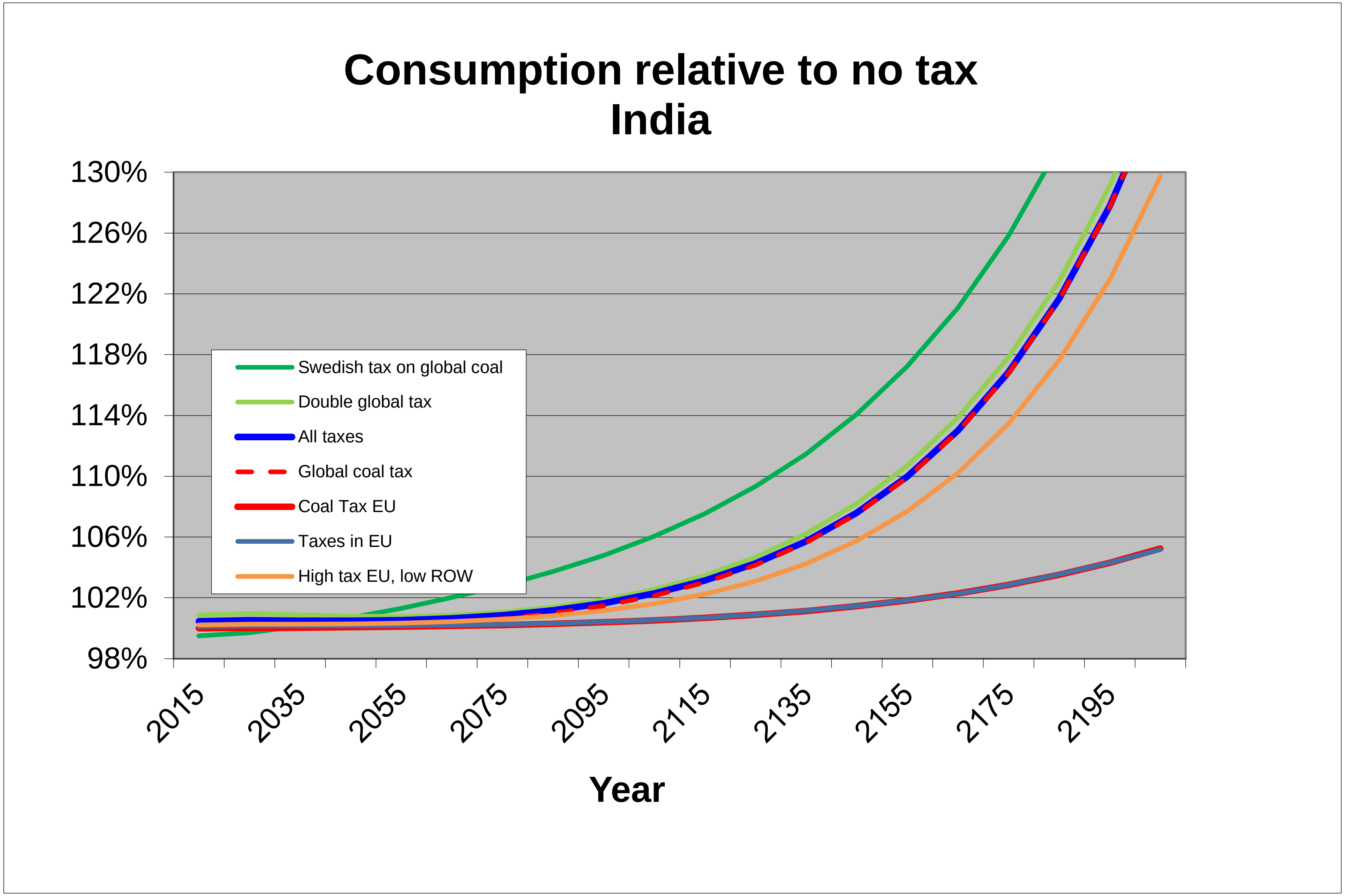
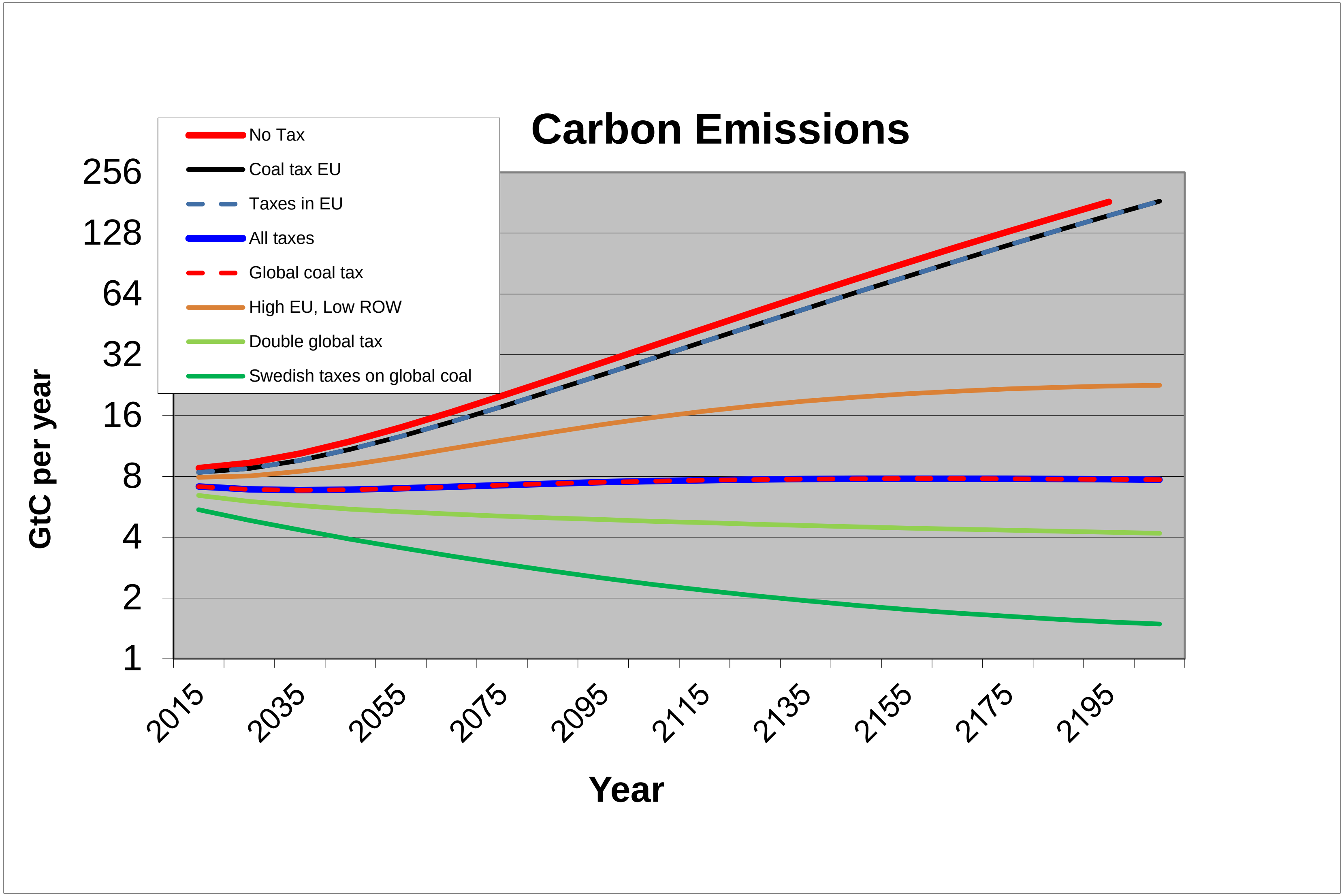

Consumption relative to no tax
Africa
130%
126%
122%
118%
114%
110%
106%
102%
98%
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year
Swedish tax global coal
Double global tax
All taxes
Global coal tax
Coal Tax EU
Taxes in EU
High tax EU, low ROW

Figure 8.



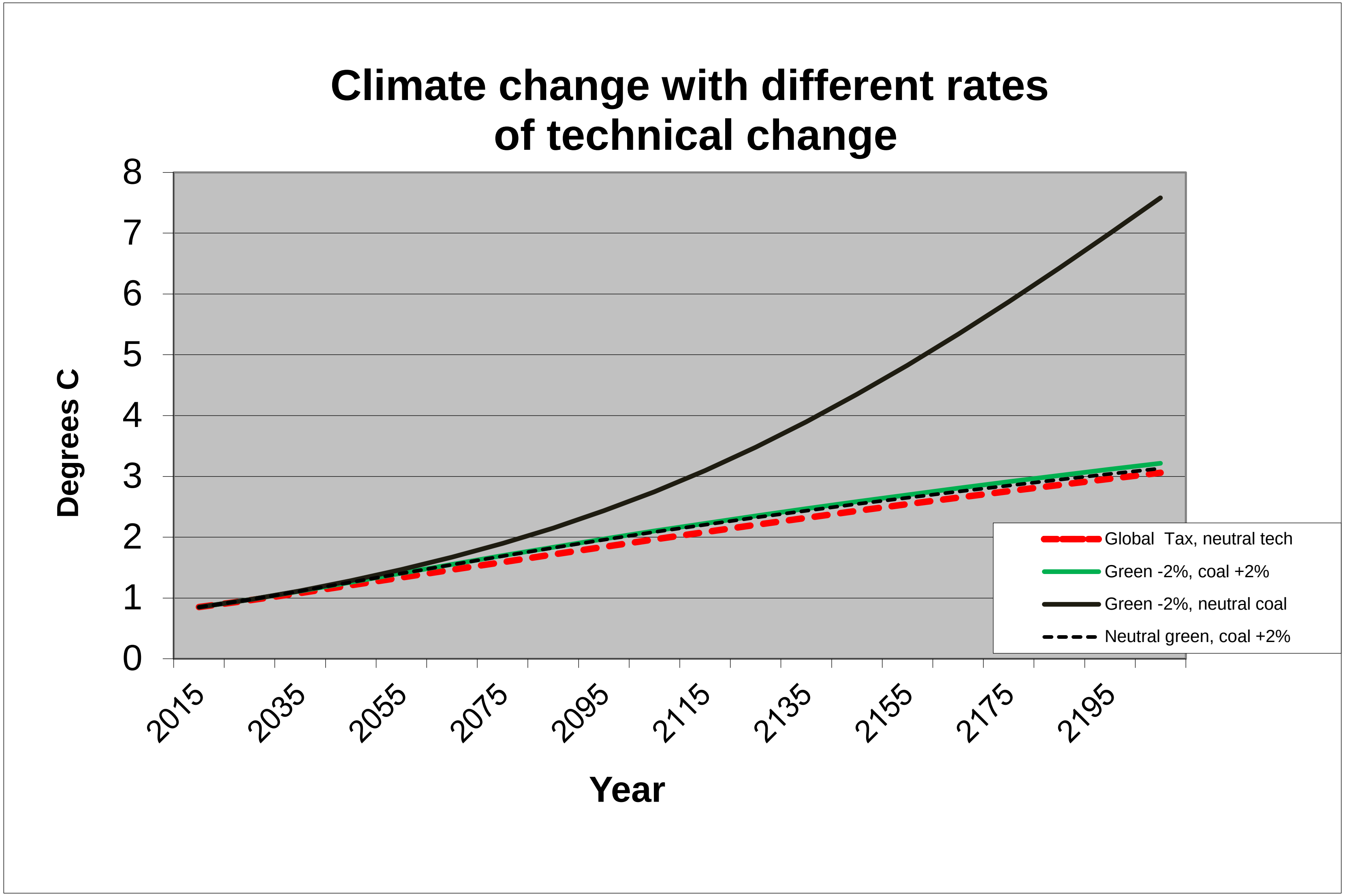
Consumption relative to no tax
India
130%
126%
122%
118%
114%
110%
106%
102%
98%
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year
Swedish tax on global coal
Double global tax
All taxes
Global coal tax
Coal Tax EU
Taxes in EU
High tax EU, low ROW

Figure 9.



Carbon Emissions
No Tax
Coal tax EU
Taxes in EU
All taxes
Global coal tax
High EU, Low ROW
Double global tax
Swedish taxes on global coal
256
128
64
32
16
8
4
2
1
GtC per year
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year

Figure 10.



Climate change with different rates of technical change
Degrees C
8
7
6
5
4
3
2
1
0
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year
Global Tax, neutral tech
Green -2%, coal +2%
Green -2%, neutral coal
Neutral green, coal +2%

Figure 11.

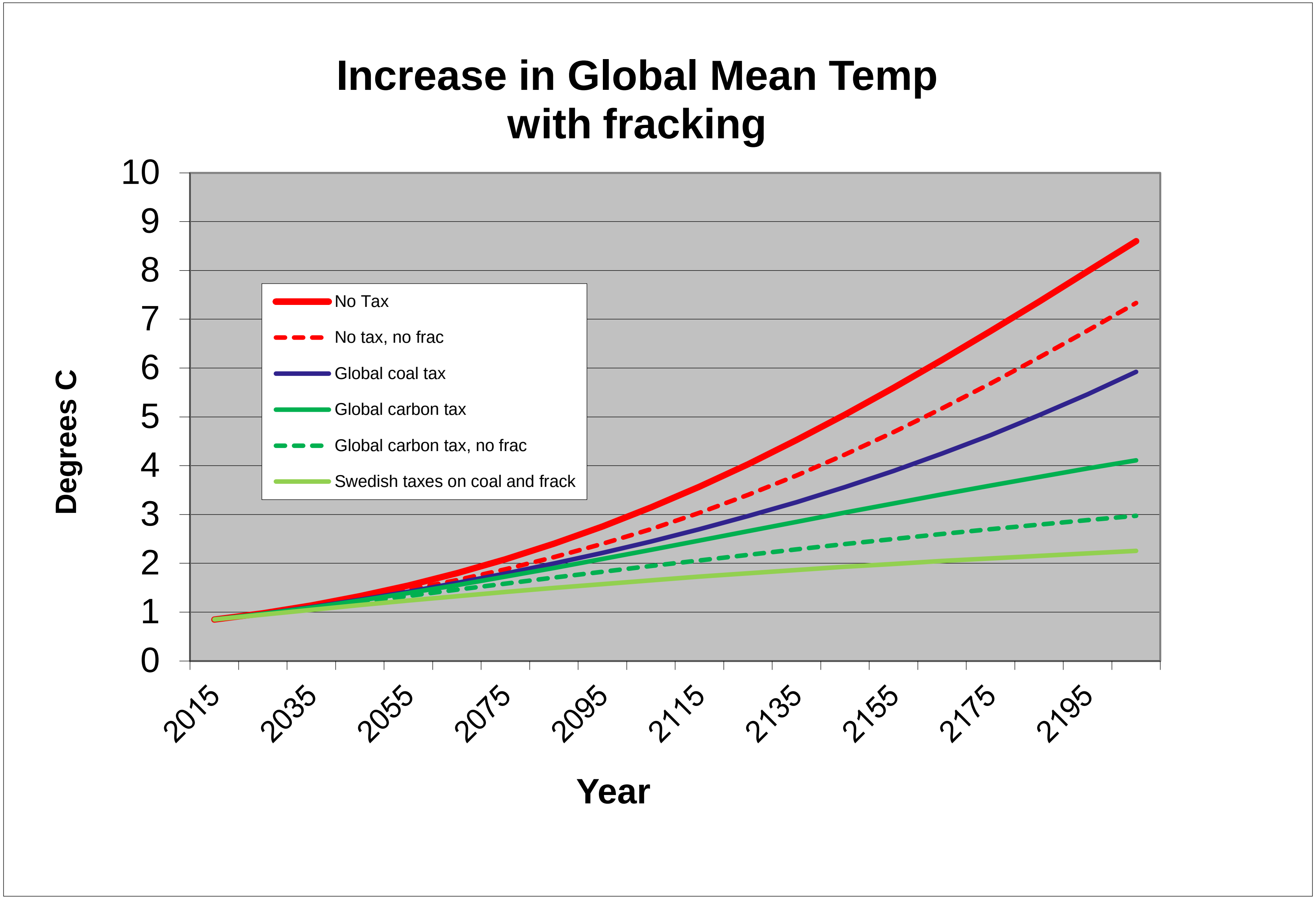

Increase in Global Mean Temp
with fracking
Degrees C
10
9
8
7
6
5
4
3
2
1
0
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year
No Tax
No tax, no frac
Global coal tax
Global carbon tax
Global carbon tax, no frac
Swedish taxes on coal and frack

Figure 12.

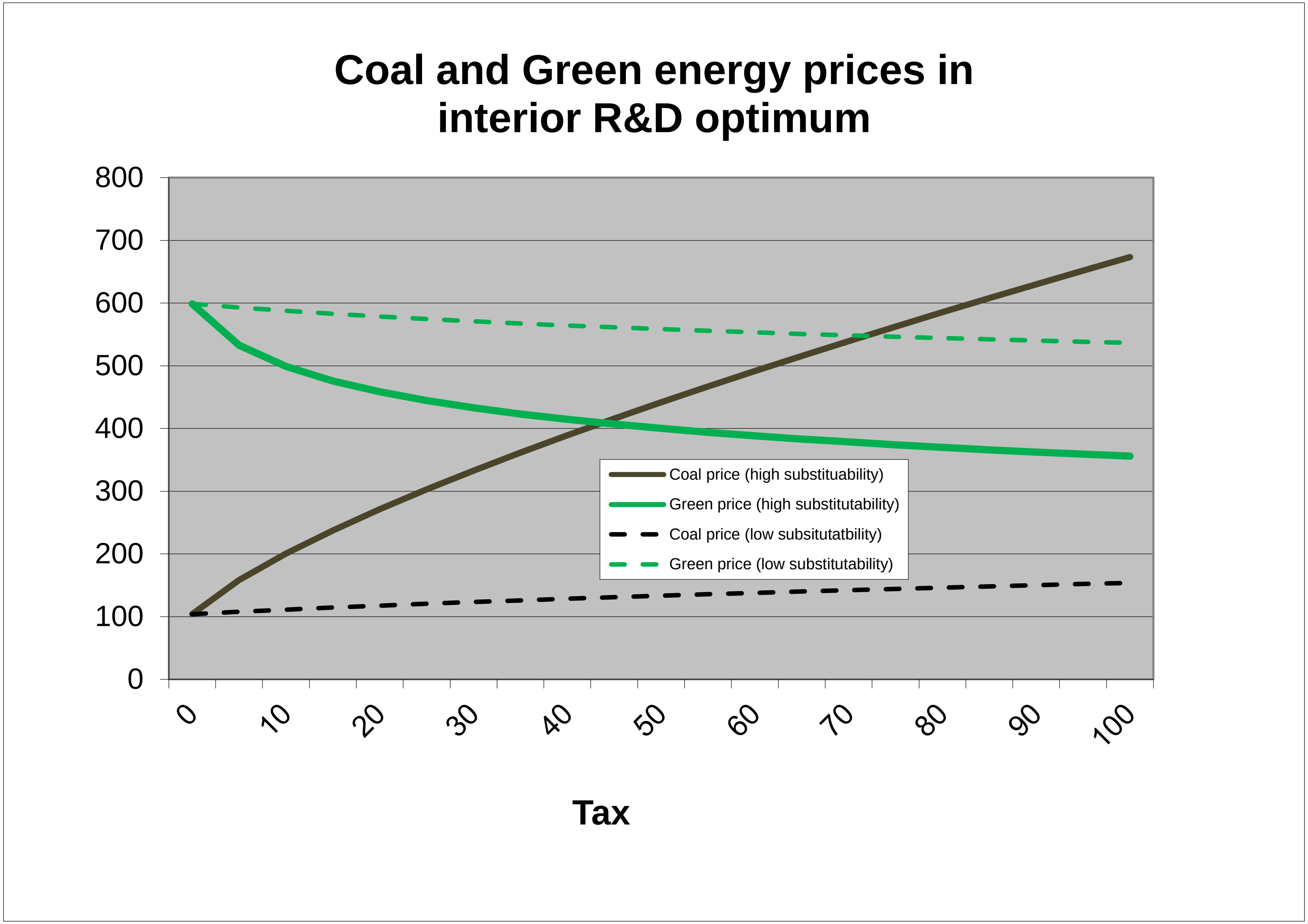
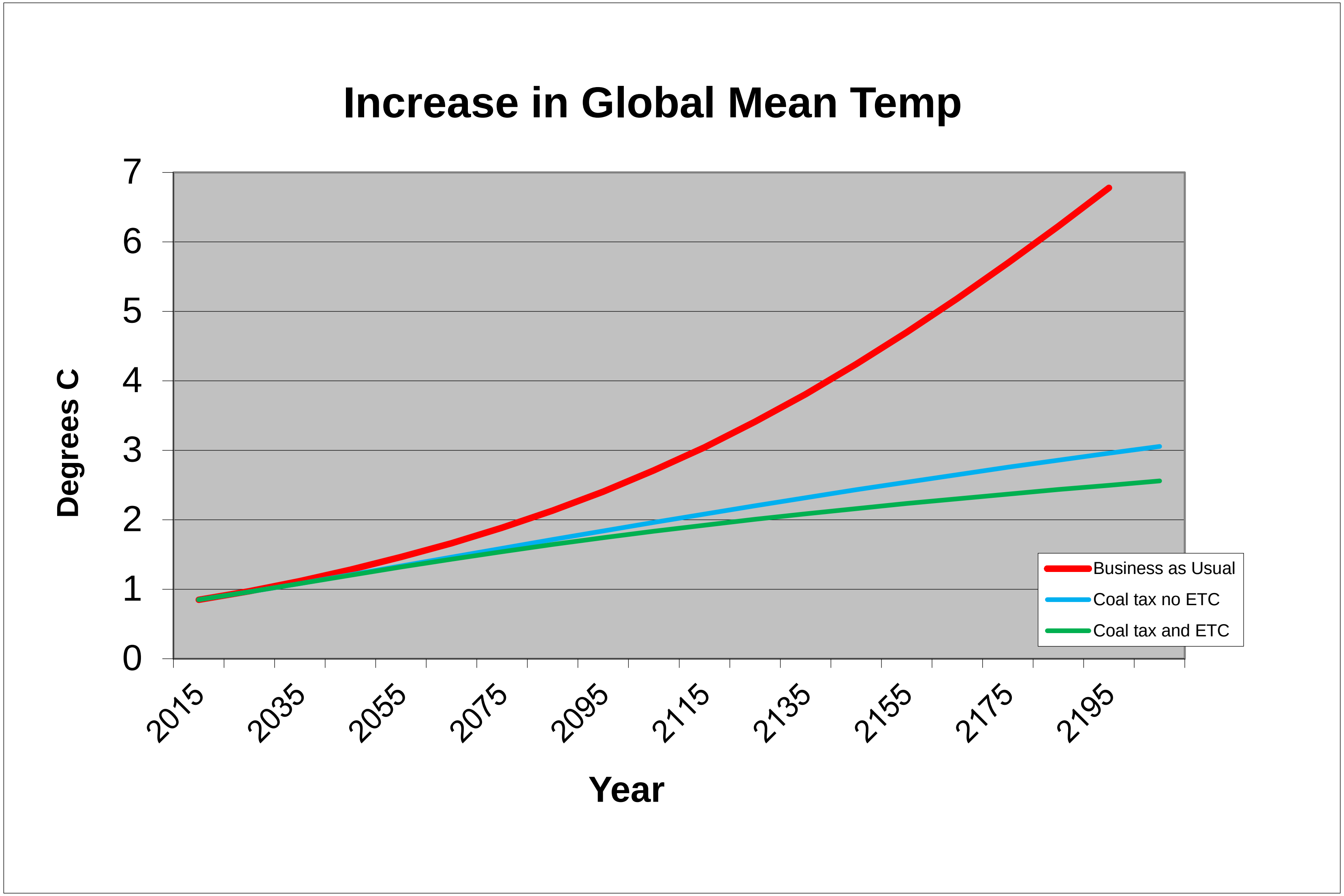
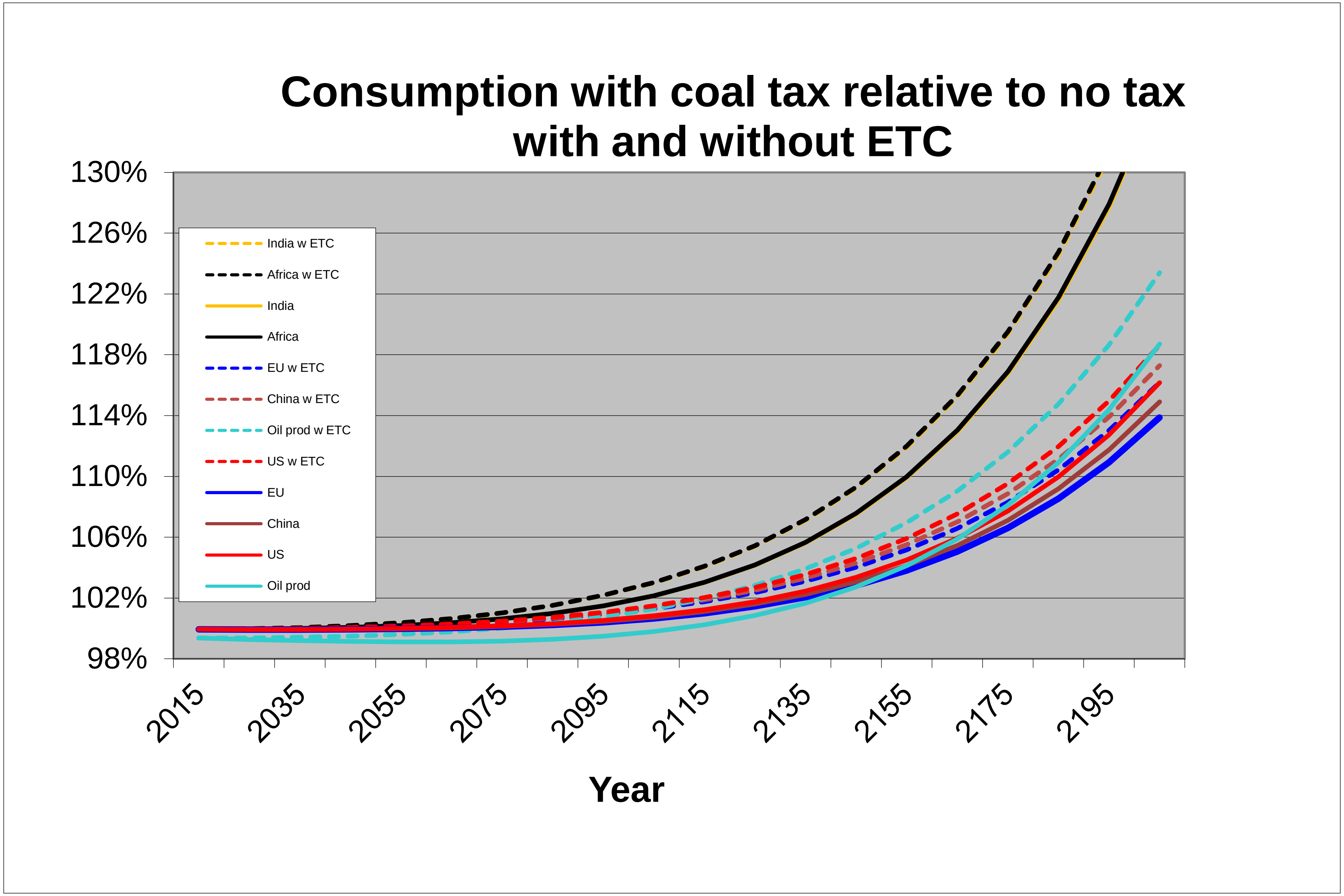

Coal and Green energy prices in interior R&D optimum
800
700
600
500
400
300
200
100
0
0
10
20
30
40
50
60
70
80
90
100
Tax
Coal price (high substituability)
Green price (high substitutability)
Coal price (low subsitutatbility)
Green price (low substitutability)

Figure 13.



Increase in Global Mean Temp
Degrees C
7
6
5
4
3
2
1
0
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year
Business as Usual
Coal tax no ETC
Coal tax and ETC

Figure 14.



Consumption with coal tax relative to no tax with and without ETC
130%
126%
122%
118%
114%
110%
106%
102%
98%
India w ETC
Africa w ETC
India
Africa
EU w ETC
China w ETC
Oil prod w ETC
US w ETC
EU
China
US
Oil prod
2015
2035
2055
2075
2095
2115
2135
2155
2175
2195
Year