# Virtual Seminar on Climate Economics

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# Climate Linkers: Rationale and Pricing

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VSCE

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# Motivation

- We make a case for the emergence of a novel class of financial instruments indexed to climate-related variables (sea levels, temperatures, carbon concentrations).

- Climate Linkers...

  ... would not directly contribute to the fight against climate risks (i.e. $\neq$ green bonds);

  ... would provide, by construction, hedging against long-term global temperatures, carbon concentrations, or sea levels.

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# Contributions

1. We discuss the advantages of financial instruments (swaps, bonds, options) indexed to secular climate changes.
2. We develop a modeling framework that allows for the fast pricing of these long-term instruments ($=$ tractable Integrated Assessment Model, IAM).
   Tractability $\Rightarrow$ possibility to look for parametrizations making the model consistent with recent climate science.
3. We explore the pricing of Climate Linkers (CL) and study the *climate risk premiums* ($=$ insurance premiums) they would embed.

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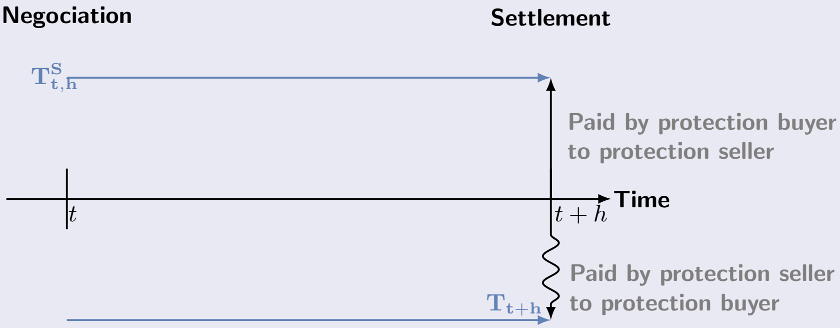
## Temperature Indexed Swap (TIS)




## Temperature Indexed Bond (TIB)

Debt instrument whose payoff at maturity $(t+h)$ is indexed to a given measure of temperature (or carbon concentration, or sea levels). Specifically:

$$1 + \chi \left[T_{t+h} - \mathbb{E}_t(T_{t+h})\right], \qquad (\chi \text{ is a "leverage factor"})$$

Note: the payoff expectation is equal to 1.

# Rationale behind Climate Linkers (CL)

**Demand**

- Growing demand for (re)insurance against weather-related disasters
  Alternative Risk Transfer solutions (insurance-linked securities, CAT bonds).
  But only specific areas and short maturities (for which climate is predictable).
- CL address long-term and global risks (e.g., index. to temperature in 2100).
- CL $\neq$ Environmental, Social, and Governance (ESG) fixed-income products, whose final payoffs are not indexed to climate.

**Supply**

- Temperature-Indexed Bonds: Widening of governments' investor basis.
- Increase in govts' exposure to climate risk. However:
  - 2nd-order compared to potential direct effect on public finances.
  - Consistent with role of "insurer of last resort" of govts (Bruggeman et al., 2010).
- Private issuance of TIBs: Natural issuers (Asset-Liability Management) = firms whose activity relates to climate-risk mitigation (e.g., renewable-energy producers).

# Rationale behind Climate Linkers (CL): Information (1/2)

**Informational content**

- CL prices would make market participants reveal their expectations regarding future climate (akin to break-even inflation rates).
- Information captured in real-time, at high frequency.
- Extraction of expected trajectories of future temperatures from market quotes of temperature-indexed swaps or bonds (as done with inflation-linked bonds, Campbell and Shiller, 1996). private under-supply
- In particular: natural way to gauge the perceived credibility and effectiveness of international commitments regarding the climate.
- Observed prices would help inform the computation of key economic variables:
  - Social Cost of Carbon (Weitzman, 2013; Nordhaus, 2017; van den Bremer and van der Ploeg, 2021);
  - Long-Term Discount Rates, and Climate Betas (Bauer and Rudebusch, 2021; Dietz et al., 2018; Gollier, 2021; Giglio et al., 2021);
  - Climate-Value-at-Risks (Dietz et al., 2016).

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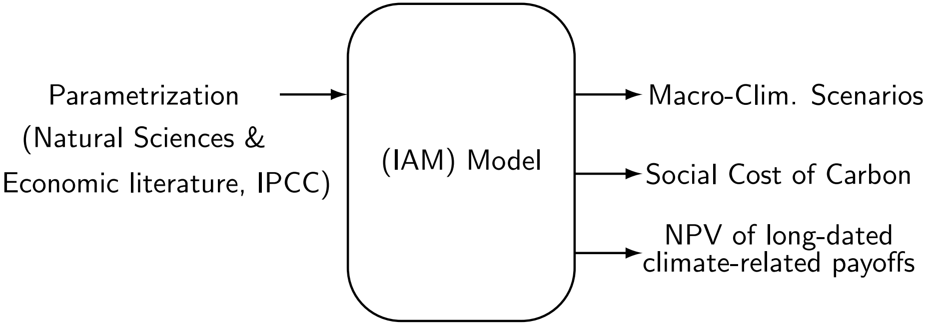
# Rationale behind Climate Linkers (CL): Information (2/2)

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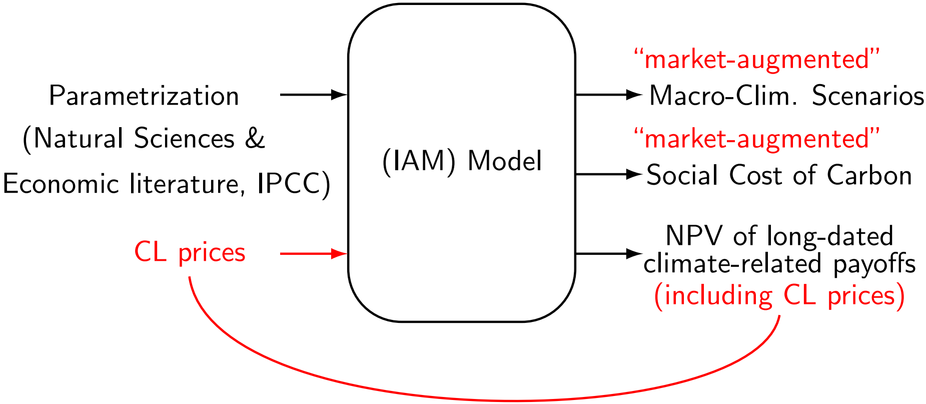


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# Model

- Joint dynamics of climate and economy: Integrated Assessment Model. Early IAM: Dynamic Integrated Climate-Economy model (DICE) of Nordhaus (1992).
- Stochastic IAMs: Jensen and Traeger (2014); Bansal et al. (2016); Cai and Lontzek (2019).
- In spite of progress in terms of numerical solution methods (Daniel et al., 2019; Barnett et al., 2020; van den Bremer and van der Ploeg, 2021), no "instant results" to solve IAMs with stochastic disasters.

  $\Rightarrow$ Challenging to look for model parametrizations that reproduce certain targets (moments).
- **The approach in a nutshell**:

  Make the model conditionally affine (also done by Traeger, 2021).

  NB: Different from linearization around a steady sate (large uncertainty).

  $\Rightarrow$ Closed-form solutions for conditional moments (and distributions).

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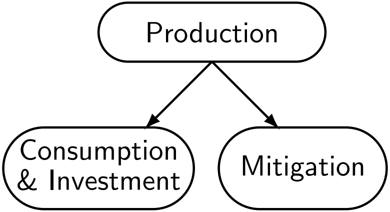
# Stochastic Affine DICE Model: Overview



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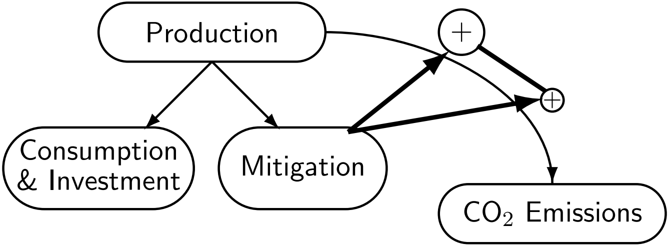

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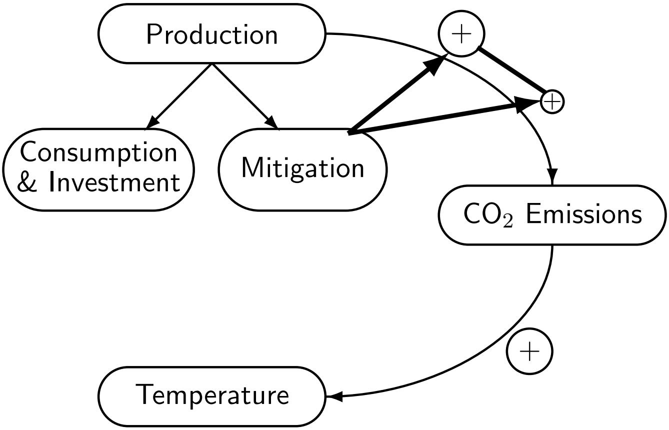

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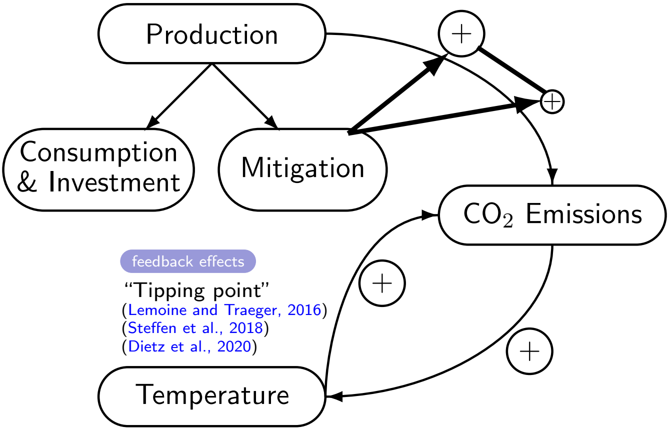

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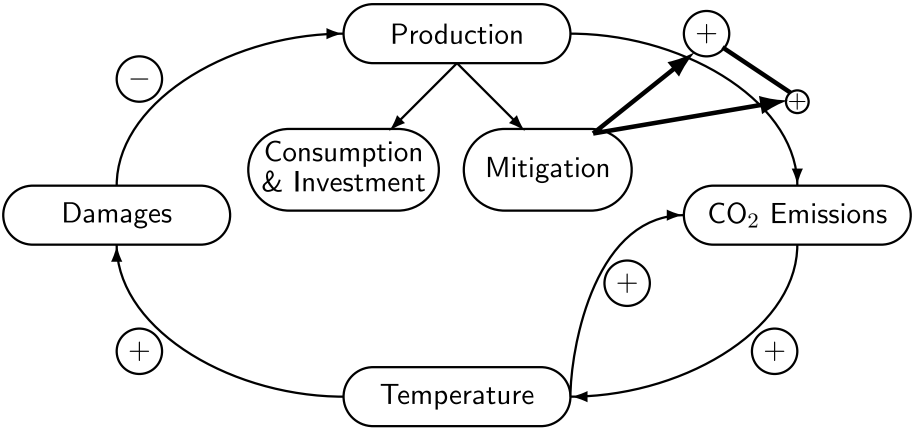

# Stochastic Affine IAM Model: Main ingredients

- Production function:

$$Y_t = A_t K_t, \quad \text{with } A_t = \bar{A} + \sigma_A \eta_{A,t},$$

- Capital dynamics (Gomes et al., 2019; Miller et al., 2020):

$$\begin{array}{lrcl} \text{Planned capital} & K_t^* & = & (1 - dep)K_{t-1} + Inv_t, \\ \text{Effective capital} & K_t & = & \exp(-D_t)K_t^*, \end{array}$$

where $D_t = 0$ (no disaster), or $D_t > 0$ (disaster).

- Budget constraint:

$$Y_t = C_t + Inv_t + \Psi_t,$$

where $\Psi_t$ is investment in mitigation technologies.

- Disaster shocks are more likely when temperature is higher:

$$D_t \sim \gamma_0(\ell_{D,0} + \ell_{D,1} T_{AT,t-1}, \mu_D),$$

$\gamma_0$ distri $\approx$ Poisson with random jumps (Monfort et al., 2017). In particular:

$$\mathbb{P}_{t-1}(D_t > 0) \approx \ell_{D,0} + \ell_{D,1} T_{AT,t-1}.$$ details

## Mitigation

- Investment in mitigation technologies:

$$\Psi_t = \mu_t^{\theta_2} BC_t Y_t.$$

$\mu_t$: mitigation (or emission control) rate; $BC_t$ exogen. $\searrow$ over time.

- Agents dynamically decide on $C_t/Inv_t$, but decide on future $\mu_t$s on date $t = 0$. Parametric trajectory: ▸ chart

$$\mu_t = \min\left[\exp\left(-\theta_a + \theta_b \times t\right); 1\right].$$

- Up to mitigation $(1 - \mu_t)$, industrial emissions grow as planned capital:

$$\mathcal{E}_{Ind,t} = (1 - \mu_t) \exp\left[\sum_{i=1}^{t} (\mu_{k,i} + \sigma_{k,i}\eta_{A,i})\right],$$

proxied by [using $\exp(\mu + \sigma\varepsilon) \approx \exp(\mu + \sigma^2/2)(1 + \sigma\varepsilon)$]:

$$\mathcal{E}_{Ind,t} = (1 - \mu_t) \exp\left[\sum_{i=1}^{t} \left(\mu_{k,i} + \frac{\sigma_{k,i}^2}{2}\right)\right]\left[1 + \sum_{i=1}^{t} \sigma_{k,i}\eta_{A,i}\right].$$

# Climate block (1/2)

- Atmospheric temperature depends on radiative forcings:

$$T_{AT,t} = T_{AT,t-1} + \xi_1 \left( F_t - \frac{\tau}{\nu} T_{AT,t-1} - \xi_2 \left[ T_{AT,t-1} - T_{LO,t-1} \right] \right).$$

- Radiative forcings depend on atmospheric carbon concentration: Linearization

$$F_t \quad = \quad \tau \log_2(m_0) + \frac{\tau}{\log(2) m_0} \left( \frac{M_{AT,t}}{M_{PI}} - m_0 \right) + F_{EX,t} + \sigma_f \eta_{f,t}.$$

- Carbon concentrations flow between reservoirs (AT, atmosphere, LO, lower ocean, UP, upper ocean), and depend on emissions ($\mathcal{E}_t$):

$$M_t = \begin{bmatrix} M_{AT,t} \\ M_{UP,t} \\ M_{LO,t} \end{bmatrix} = \begin{bmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet \\ 0 & \bullet & \bullet \end{bmatrix} M_{t-1} + \frac{\Delta t}{3.666} \begin{bmatrix} \mathcal{E}_{t-1} \\ 0 \\ 0 \end{bmatrix}.$$

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## Climate block (2/2)

- Positive feedback loop: details on FL Model schematic view

$$\begin{array}{ccccc} \mathcal{E}_t & \rightarrow & M_t & \rightarrow & T_t \\ & \nwarrow & & \swarrow & \\ & & N_t & & \end{array}$$

- More precisely:

$$\mathcal{E}_t = \mathcal{E}_{Ind,t} + \mathcal{E}_{Land,t} + N_t.$$

- The higher the temperature, the more likely feedback effects:

$$N_t \sim \gamma_0(\rho_N N_{t-1} + \ell_{0,N} + \ell_{1,N} T_{AT,t-1}, \mu_N). \tag{1}$$

- If one of these loops is triggered, the probability of triggering the next one jumps ⇒ "tipping point" mechanisms (Lemoine and Traeger, 2016; Steffen et al., 2018; Dietz et al., 2020).

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## IAM extension: Sea level rise (SLR)

- One of the most critical climate change's dangers (e.g., Hauer et al., 2016; Desmet et al., 2021).
- Relationship between temperatures and sea level: Rahmstorf (2007), Rahmstorf (2010), Kopp et al. (2016), and Mengel et al. (2018).
- Two principal channels of SLR: (a) melting of ice sheets, (b) the volume of the ocean is expanding as the water warms.
- "Semi-empirical" models: dynamic response of SLR to temperatures.
- Specification from Vermeer and Rahmstorf (2009):

$$H_t = H_{t-1} + \Delta t \times a_{SAT}(T_{AT,t} - T_{0,S}) + b_{SAT}\Delta T_{AT,t}, \tag{2}$$

where $H_t$ measures global mean sea level, and $T_{0,S}$ is the average atmospheric temperature for the period from 1951 to 1980.

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# Stochastic Affine IAM Model: Solution

- Repr. agent with Epstein-Zin preferences (unit EIS, risk aversion $\gamma > 1$).
- Simple solution. Resulting consumption growth process:

$$\Delta c_t = \mu_{c,t} + \sigma_{c,t}\eta_{A,t} - D_t,$$

where $\mu_{c,t}$ and $\sigma_{c,t}$ are deterministic functions of time.

**Dynamics of the state vector $X_t$ (macro + climate variables)**

Stochastic affine dynamics around a deterministic trend. Laplace transform:

$$\mathbb{E}_t[\exp(u'X_{t+1})] = \exp(a_t(u) + b_t(u)'X_t),$$

where $a_t$ and $b_t$ are available in closed form.

$\Rightarrow$ Tractability of the model.

In particular, simple derivation of conditional distributions, at any horizon (based on Fourier transforms, as in, e.g., Duffie et al., 2000).

# Calibration

- Calibration approach exploits the tractability of the model.
- Closed-form solutions to:
  - utility and Stochastic Discount Factor (s.d.f., $\mathcal{M}_{t,t+1}$);
  - first- and second-order (un/conditional) moments of state variables.

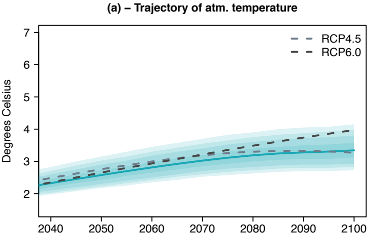
Table 1: Targeted and model-implied moments (in 2100)

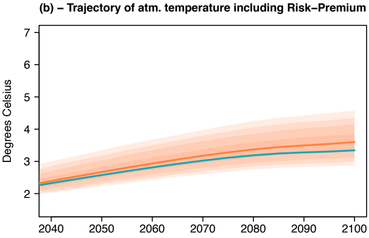
| Moment | Target | Model-implied | Source |
|---|---|---|---|
| $\mathbb{E}(T_{AT,2100})$ | $3.50°$C | $3.34°$C | RCP4.5+RCP6.0 |
| Std($T_{AT,2100}$) | $0.25°$C | $0.34°$C | RCP4.5+RCP6.0 |
| $\mathbb{E}$(contribution of FL to GMST) | $0.25°$C | $0.27°$C | Burke et al. (2012) |
| $\mathbb{E}$(increase in $Cum_E$ due to FL) | $188$Gt$CO_2$ | $190$Gt$CO_2$ | Burke et al. (2012) |
| Slope of $Cum_D$ on GMST | $-0.12$ | $-0.12$ | Burke et al. (2015) |
| Long-term rate target | $1.00\%$ | $0.99\%$ | US Treasury |
| $\mathbb{E}(H_{2100})$ | $0.45$m | $0.53$m | RCP4.5+RCP6.0 |
| Standard Deviation of $H_{2100}$ | $0.10$m | $0.05$m | Mengel et al. (2016) |

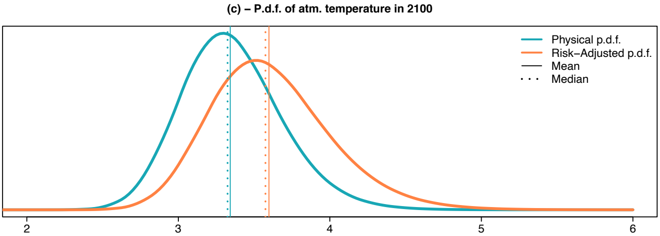
Note: RCP stands for Representative Concentration Pathway.

details on "Slope" details on "increase in $CumE$"

# Expected Atmospheric Temperature Path and Distributions

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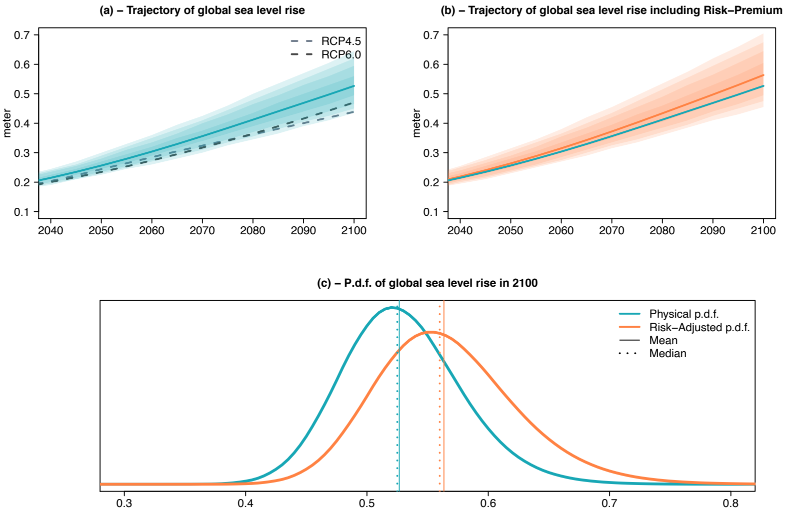
# Expected Global Mean Sea Level

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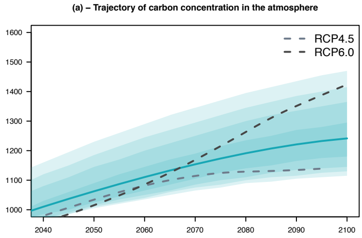
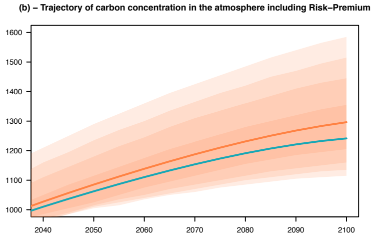
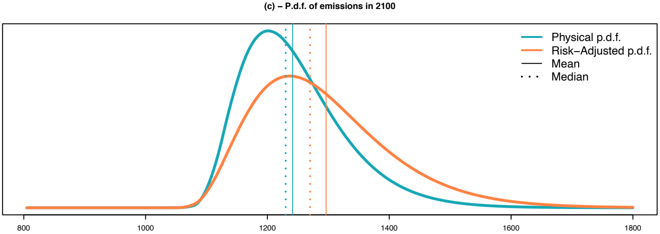
# Expected Carbon Concentration Path and Distributions

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# Closed-Form Solutions

- Tractability of the model exploited at the calibration stage (supra).
- Asset-pricing analysis also benefits from closed-form formulas.
- Formulas for date-$t$ prices of generic payoffs (settled on $t+h$):

$$\omega' X_{t+h} \quad \text{and} \quad (\omega' X_{t+h}) \mathbb{1}_{\{a' X_{t+h} < b\}},$$

where $X_t$ is the state vector.

$\Rightarrow$ Building on these prices, closed-form solutions to temperature-indexed swaps, bonds, and options (and social cost of carbon, SCC)

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# Temperature Indexed Bond (TIB)

$$\text{Payoff at maturity } (t+h): \quad 1+\chi\left[T_{t+h}-\mathbb{E}_t(T_{t+h})\right],$$

where $\chi$ is a "leverage factor".

- If agents were not risk-averse, TIB prices would satisfy (maturity $= h$):

$$P^{rf}_{t,h} = P^{TIB}_{t,h},$$

with $P^{rf}_{t,h} = \mathbb{E}_t(\mathcal{M}_{t,t+h})$, price of riskfree bond (same expected payoffs: 1).

- General case:

$$P^{TIB}_{t,h} = P^{rf}_{t,h} + \chi prem_{t,h}, \quad \text{with} \quad prem_{t,h} = \frac{\mathbb{C}ov_t\left[T_{t+h}, \mathcal{M}_{t,t+h}\right]}{\mathbb{E}_t\left[\mathcal{M}_{t,t+h}\right]}.$$

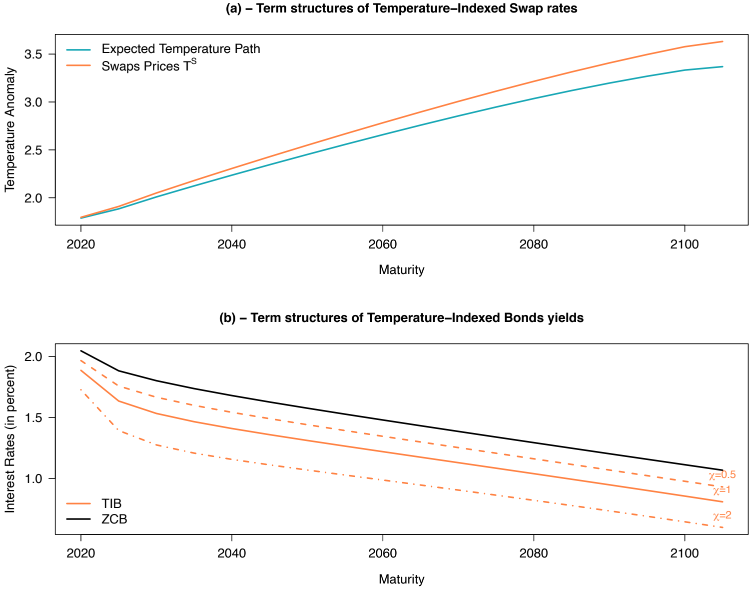
$\Rightarrow$ If higher temperature = "bad states of the world" (high marginal utility), then

$$\underbrace{P^{TIB}_{t,h} - P^{rf}_{t,h}}_{\text{climate risk premium}} > 0.$$

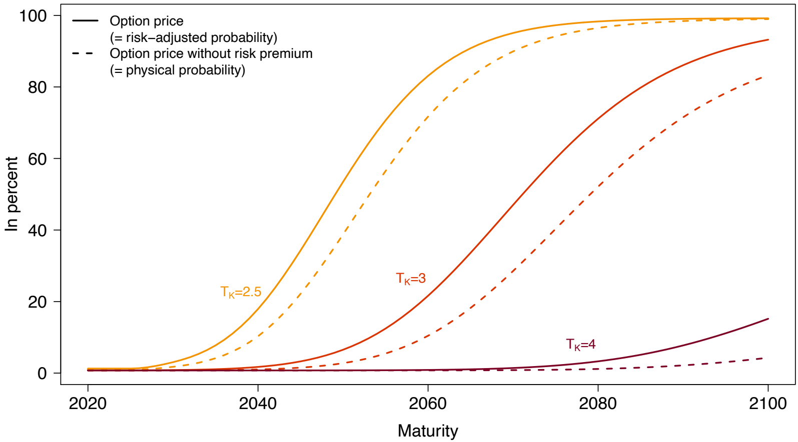
- "Risk-adjusted distributions" are right-shifted.

Temperature p.d.f. Global Sea Level p.d.f. Carbon concentration p.d.f.

# Swaps and Temperature-Indexed Bonds



# Digital Options

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# Concluding remarks

- Climate Linkers (CL) = long-term financial instruments whose payoffs are indexed to climate-related variables.
- Because agents are averse to climate risks, the pricing of CL would embed climate-risk premiums.
- CL would offer a public good by making market participants reveal their (risk-adjusted) expectations regarding future climate; akin to inflation-linked products.

- Necessary condition for development of a CL market: initial issuance of TIBs by governments (as for inflation-linked markets).
- First issuances: prices affected by "novelty premium."

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# — Appendix —

# New Sovereign Bonds as Public Goods back

## Back to 1996: A Scorecard for Indexed Government Debt

*"It is widely acknowledged that the proper role of the government is to provide public goods, and the demonstration by example of the potential for new financial markets and instruments is really a public good. [...]*

*Any firm which took on the public relations effort needed to first issue private indexed bonds would not be able to appropriate much of the societal benefits to doing so."* (Campbell and Shiller, 1996)

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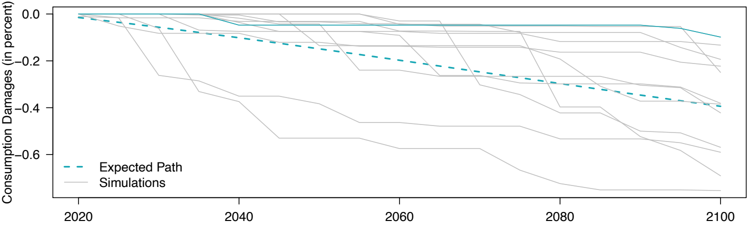
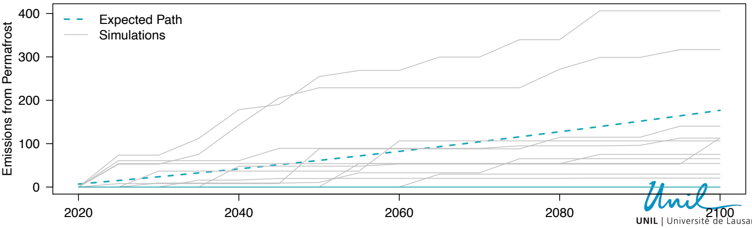
# Simulations of Shocks $D_t$ and $N_t$ back

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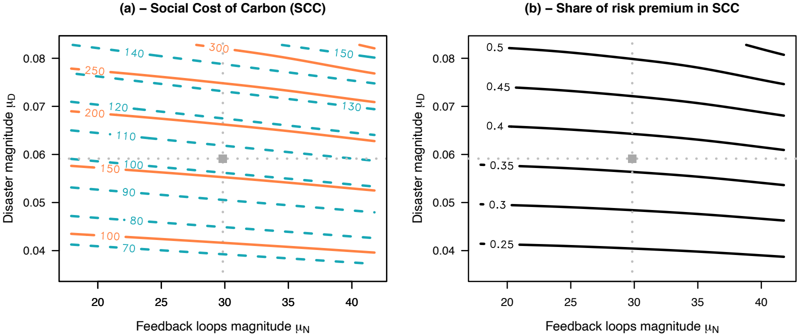
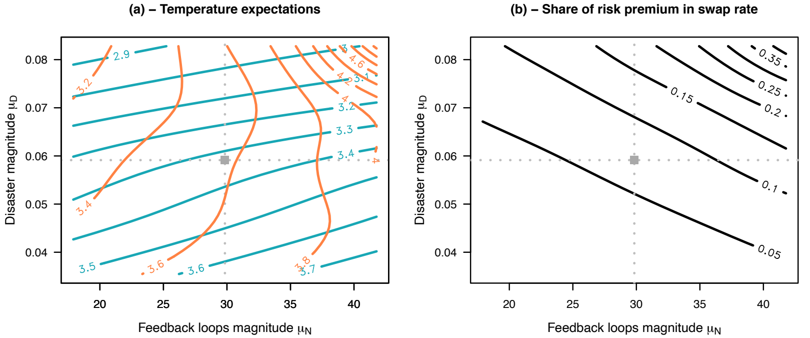
# SCC and Risk Premium back

- Social Cost of carbon = Willingness to pay to reduce carbon emission by one ton.
- Marginal rate of substitution between atm. carbon concentration and $C_t$:

$$SCC_t = -\frac{\partial U_t}{\partial M_{AT,t}} \Big/ \frac{\partial U_t}{\partial C_t}. \tag{3}$$



# Risk Premium



# Risk Premium

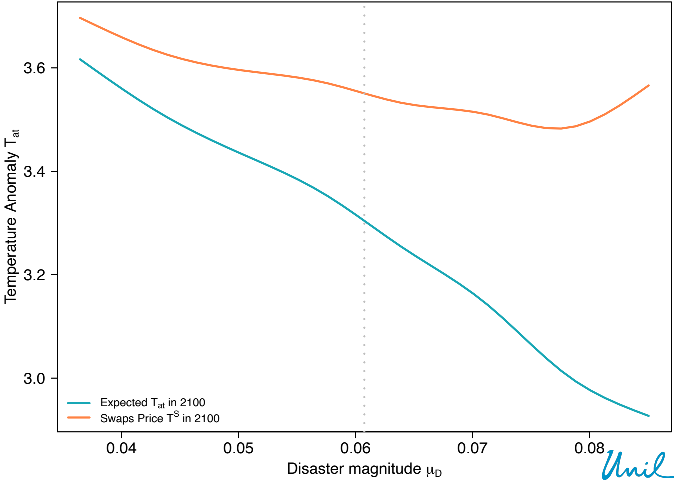


Table 2: SCC comparison

| Study | SCC (U.S. $ per tC) | Tipping points | Stoch |
|---|---|---|---|
| Nordhaus (2017) | 113 | | |
| Stern (2007) | 312 | | |
| Jensen and Traeger (2014) | [40;70] | | |
| Barnett et al. (2020) | [240;411] | | |
| Cai and Lontzek (2019) | [40;100] | ✓ | |
| Bansal et al. (2016) | [4;104] | ✓ | |
| Lemoine and Traeger (2014) | [37;55] | ✓ | |
| van den Bremer and van der Ploeg (2021) | 146 | | |
| **This paper** | 167 | ✓ | |

Note: This table reports different SCC estimates. Cited studies differ along many dimensions, the last three columns highlight particularly important ones.

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# Carbon-release feedback effects back1 back2

- Earth is composed of an unknown number of feedback loops (FL), positive or negative.
  - Negative FL: absorption of greenhouse gases.
  - Positive FL = amplificiation of positive imbalances in radiative forcings.
- Positive FL: (a) release of tons of methane trapped in the permafrost, and (b) acidification of oceans.
- If one of these loops is triggered, the probability of triggering the next one jumps ⇒ "tipping point" mechanisms (Lemoine and Traeger, 2016; Steffen et al., 2018; Dietz et al., 2020).
- In our econometric specification: the probability of having a non-zero $N_t$ is typically small, but if it happens (i.e. when $N_t$ jumps): $\nearrow$ carbon concentration ⇒ $\nearrow$ temperature ⇒ ...

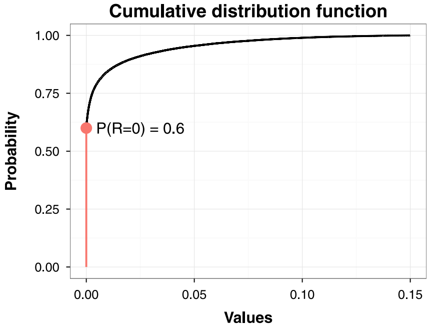
= "Self-excitation", as in Hawkes (1971) processes (applications to financial contagion, e.g., Aït-Sahalia et al., 2015).

# Gamma-zero distribution ▸ back

## Definition

The non-negative r.v. $X \sim \gamma_0(\lambda, \mu)$, $\lambda > 0$ and $\mu > 0$, if

$$X \mid Z \sim \gamma_Z(\mu) \quad \text{with} \quad Z \sim \mathcal{P}(\lambda)$$
$$\Rightarrow \qquad \mathbb{P}(X = 0) = \mathbb{P}(Z = 0) = \exp(-\lambda)\,.$$

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# Calibrating damages ▸ back

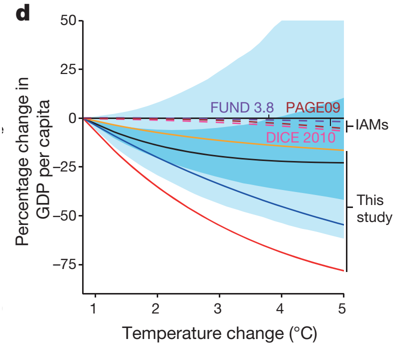


Figure 1: Burke et al. (2015, Figure 5.d)

- Decrease in consumption of about 50% for a 4-degree increase in $T$.

$\Rightarrow$ Regression slope of cumulated effects of disasters on consumption of $-50\%/4$.

- Model-implied equivalent: Population slope $= \frac{\mathbb{C}ov_t(CumD_{t+H}, T_{t+H})}{\mathbb{V}ar_t(T_{t+H})}$.

# Calibrating feedback effects ▸ back

- Possibilities of feedback loops (FL) amplifying the positive imbalances in radiative forcings. (May give rise to "tipping points".)
- Examples of FL: (a) release of tons of methane trapped in the permafrost, and (b) acidification of oceans.
- Some studies aim at estimating the specific effect of FL on carbon release and temperatures (Burke et al., 2012).
- In our model, we can compare model-implied expected emissions with FL ($\mu_N > 0$ in eq. 1) and without FL ($\mu_N = 0$ in eq. 1):

$$\begin{aligned}\text{Emission effect of FL} &= \mathbb{E}_t(CumE_{t+H}) - \mathbb{E}_t^{noFL}(CumE_{t+H}) \\ \text{Temperature effect of FL} &= \mathbb{E}_t(T_{t+H}) - \mathbb{E}_t^{noFL}(T_{t+H})\end{aligned}$$

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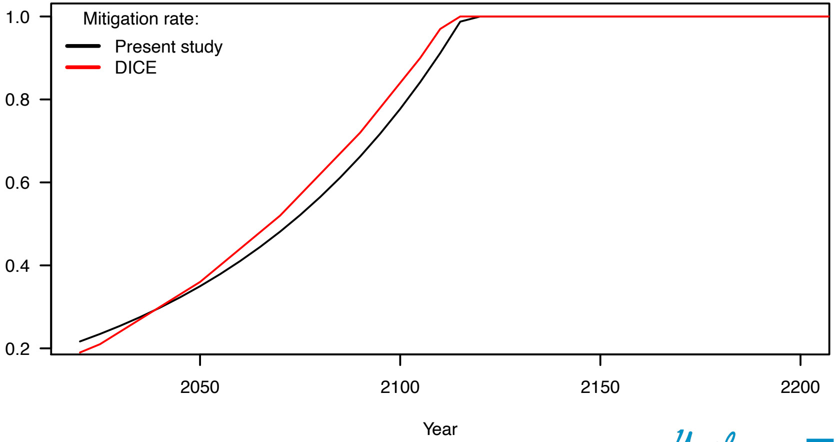
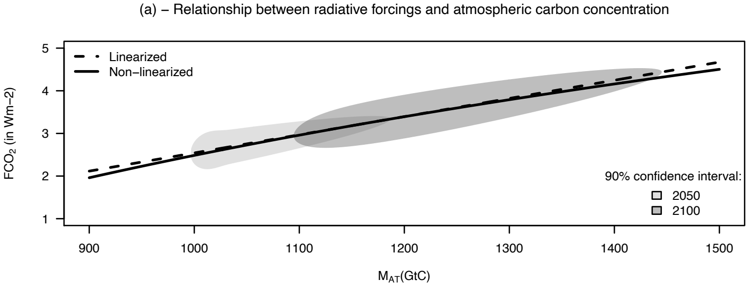
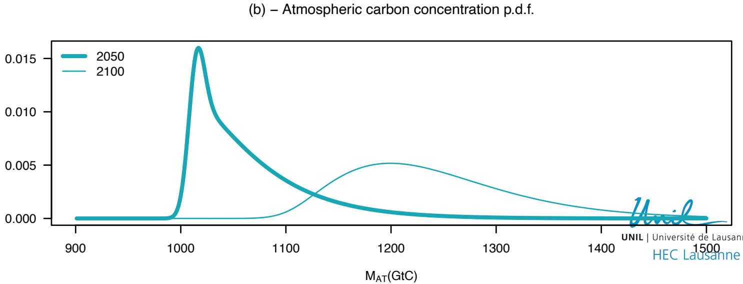
# Mitigation rate $\mu_t$ ▸ back



# Linearization of radiative forcings ▸ back



# Equilibrium Climate Sensitivity (ECS) uncertainty ▸ back

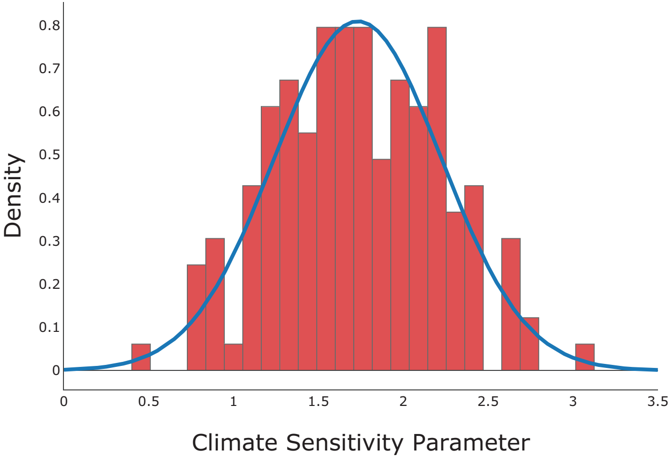



**Figure 2**
Climate sensitivity uncertainty. Histogram (red) and normal density approximation (blue) for the climate sensitivity parameter $\beta$ across models. The climate sensitivity parameter is in units of degrees centigrade per teraton carbon. Figure based on evidence reported in Figure 3A by MacDougall, Swart, and Knutti (2017) (© American Meteorological Society, used with permission) and constructed with data provided by the authors.

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| Parameter | Notation | Equation | Value | Unit/Note | Reference |
|---|---|---|---|---|---|
| Average TFP | $\bar{A}$ | (I.7) | 0.4 | | |
| Standard deviation of the TFP shock | $\sigma_A$ | (I.7) | 0.05 | | |
| Average for approximation term | $m_0$ | (32) | $\frac{1168}{607}-1$ | | CDICE + IPCC |
| Rate of preference for present | $\delta$ | (37) | 0.95 | | |
| Risk aversion | $\gamma$ | (37) | 7 | | |
| Carbon emissions from land 2015 | $\varepsilon_0$ | (21) | 2.6 | $GtCO_2$ per year | DICE2016 |
| Decline rate in land emissions (Eq. 21) | $\rho$ | (21) | 0.115 | per period | DICE2016 |
| Equilibrium concentration in atmosphere | $mateq$ | (33) | 607 | GtC | CDICE |
| Equilibrium concentration in upper strata | $mueq$ | (33) | 600 | GtC | CDICE |
| Equilibrium concentration in lower strata | $mleq$ | (33) | 1772 | GtC | CDICE |
| 2015 forcings of non-$CO_2$ GHG | $\phi_0$ | (22) | 0.5 | Wm-2 | DICE2016 |
| 2100 forcings of non-$CO_2$ GHG | $\phi_1$ | (22) | 1 | Wm-2 | DICE2016 |
| Preindustrial concentration of carbon in the atmosphere | $M_{PI}$ | (32) | 607 | GtC | CDICE |
| Carbon cycle parameter between atmosphere and upper ocean | $\varphi_{12}$ | (33) | 0.053 | | CDICE |
| Carbon cycle parameter between upper and lower ocean | $\varphi_{23}$ | (33) | 0.0042 | | CDICE |
| Climate equation coefficient for upper level | $\xi_1$ | (34) | $\Delta t \times 0.137$ | | CDICE |
| Transfer coefficient upper to lower stratum | $\xi_2$ | (34) | $\Delta t \times 0.10001$ | | CDICE |
| Transfer coefficient for lower level | $\xi_3$ | (35) | $\Delta t \times 0.00689$ | | CDICE |
| Forcings of equilibrium $CO_2$ doubling | $\tau$ | (32)+(34) | 3.45 | Wm-2 | CDICE |
| Equilibrium temperature impact | $\nu$ | (34) | 3.25 | °C per doubling $CO_2$ | CDICE |
| Decline rate of decarbonization | $\delta_\sigma$ | (17) | $-0.001$ | per period | DICE2016 |
| Carbon intensity 2010 | $\sigma_0$ | (17) | $\frac{e_0}{q_0(1-\mu_0)}$ | $kgCO_2$ per output 2005 USD 2010 | DICE2016 |
| Industrial emissions in 2015 | $e_0$ | $(\sigma_0)+(\mathscr{E}_{2015})$ | 35.85 | $GtCO_2$ per year | DICE2016 |
| Initial world gross output in 2015 | $q_0$ | (29)+($\sigma_0$) | 105.5 | trillions of 2010 USD | DICE2016 |
| Initial emision control rate in 2015 | $\mu_0$ | (18) | 0.03 | | DICE2016 |
| Initial growth of sigma | $g_{\sigma,1}$ | (17) | $-0.0152$ | per year | DICE2016 |
| Initial cost decline backstop cost | $gback$ | (19) | 0.025 | per period | DICE2016 |
| Exponent of control cost function | $\theta_2$ | (20)+(23) | 2.6 | | DICE2016 |
| Persistence of the radiative forcings shock | $\Phi_{[2,2]}$ | (26) | 0.95 | | |
| Global surface temperature weights $[T_{AT}, T_{LO}]$ | $weights_T$ | | $[0.6, 0.4]$ | | IPCC |
| Base Temperature (sea level equilibrium) | $T_{0,S}$ | (36) | $-0.375$ | °C, Baseline [1951-1980] | Vermeer and Rahmstorf (2009) |
| Coefficient attached to $\Delta_{(T_{AT},T_{0,S})}$ | $a_{SAT}$ | (36) | 0.0015 | m per °C per year | |
| Coefficient attached to $\Delta T_{AT,t}$ | $b_{SAT}$ | (36) | 0.025 | m | Vermeer and Rahmstorf (2009) |
| Capital depreciation rate | $dep$ | (I.7) | 0.27 | | |
| Time step | $\Delta t$ | | 5 | | |

Note: This table presents the parameters used in our baseline model. DICE16 refers to Nordhaus (2017). IPCC refers to IPCC (2014, Table 2.1). CDICE refers to Folini et al. (2021)

back

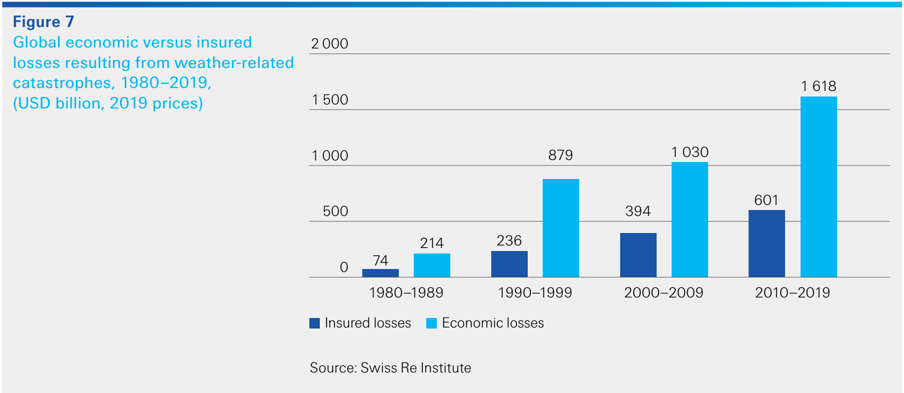


Figure 2: Insurance gap – Source: Swiss Re, 2020.

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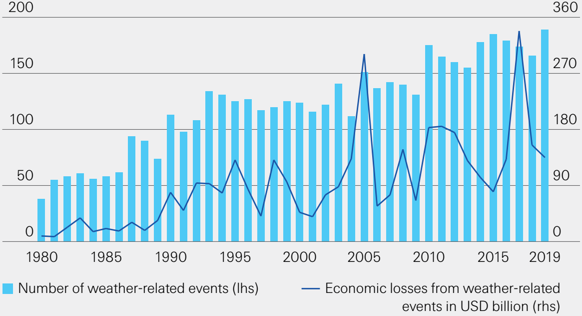
**Figure 2**
Number of weather-related events and associated economic losses, 1970-2019
(USD billion, 2019 prices)




Source: Swiss Re Institute

Figure 3: Weather-related events and losses – Source: Swiss Re, 2020.

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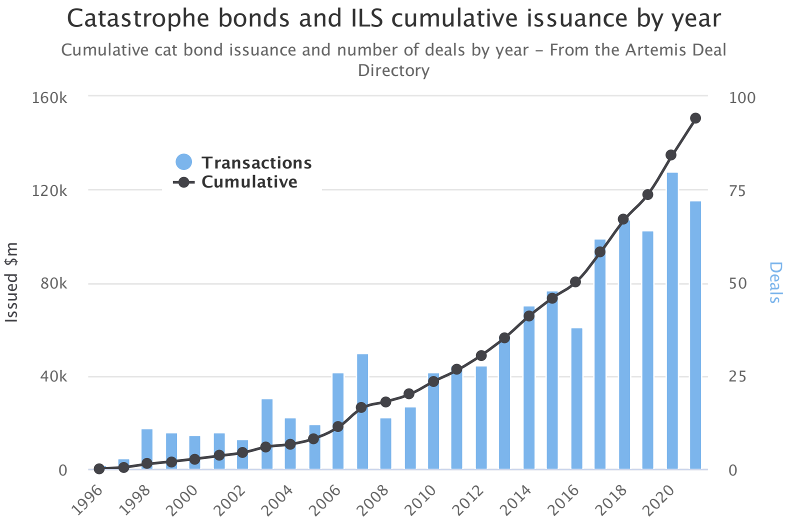


Source: www.Artemis.bm Deal Directory

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## Temperature Indexed Swap (TIS)

Protection buyer and protection seller exchange cash flows at $t+h$ (maturity). On date $t$ (negotiation date):

$$\underbrace{\text{Market value of protection leg}}_{\mathbb{E}_t(\mathcal{M}_{t,t+h}T_{t+h})} = \underbrace{\text{Market value of premium leg}}_{\mathbb{E}_t(\mathcal{M}_{t,t+h})T^S_{t,h}}.$$

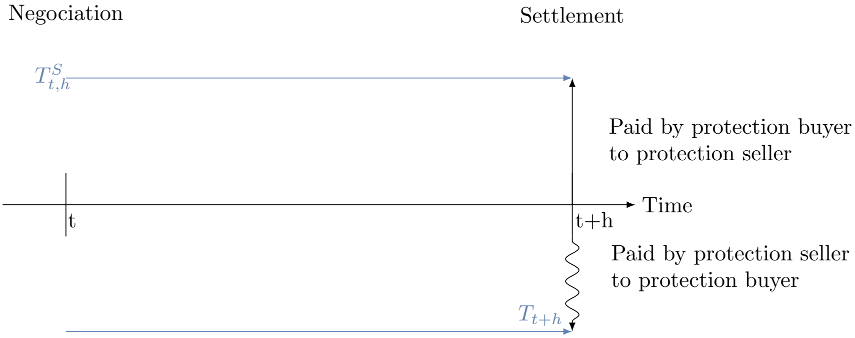



Figure 4: TIS

## Temperature Indexed Bond (TIB)

Debt instrument whose payoff at maturity $(t+h)$ is indexed to a given measure of temperature (or carbon concentration). Specifically:

$$1 + \chi \left[ T_{t+h} - \mathbb{E}_t(T_{t+h}) \right],$$

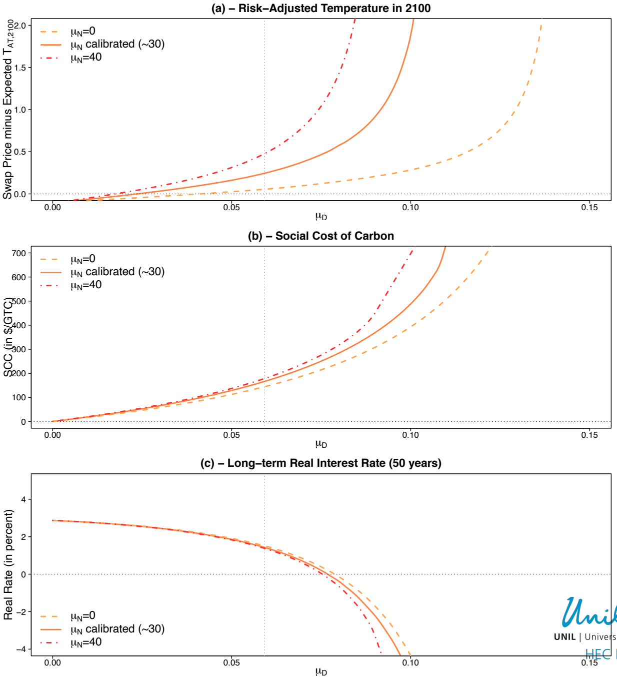
where $\chi$ is a "leverage factor".

Note: the payoff expectation is equal to 1.

## Temperature Options

Nonlinear payoffs.

| Option type | Price (notation) | Payoff (settled on maturity date $t+h$) |
|---|---|---|
| Digital | $Dig_{t,h}(T_K)$ | $\mathbb{1}_{\{T_{t+h} > T_K\}}$ |
| Call | $Call_{t,h}(T_K)$ | $(T_{t+h} - T_K)^{+} = \mathbb{1}_{\{T_{t+h} > T_K\}}(T_{t+h} - T_K)$ |
| Put | $Put_{t,h}(T_K)$ | $(T_{t+h} - T_K)^{-} = \mathbb{1}_{\{T_{t+h} < T_K\}}(T_K - T_{t+h})$ |


(a) – Risk–Adjusted Temperature in 2100
Swap Price minus Expected $T_{AT,2100}$
2.0
1.5
1.0
0.5
0.0
$\mu_N$=0
$\mu_N$ calibrated (~30)
$\mu_N$=40
0.00
0.05
0.10
0.15
$\mu_D$
(b) – Social Cost of Carbon
SCC (in $/(GTC))
700
600
500
400
300
200
100
0
$\mu_N$=0
$\mu_N$ calibrated (~30)
$\mu_N$=40
0.00
0.05
0.10
0.15
$\mu_D$
(c) – Long–term Real Interest Rate (50 years)
Real Rate (in percent)
4
2
0
−2
−4
$\mu_N$=0
$\mu_N$ calibrated (~30)
$\mu_N$=40
0.00
0.05
0.10
0.15
$\mu_D$