Mortgage Default in an Estimated Model of the U.S. Housing Market

Preliminary and Incomplete *

Luisa Lambertini† EPFL

Victoria Nuguer‡ EPFL

Pinar Uysal§ EPFL

August 26, 2014

Abstract

In this paper we model endogenous default in mortgages that can explain the main features of the US macroeconomic time series. We explicitly model the housing sector, mortgages, and endogenous default as well as nominal and real rigidities in a DSGE setting. We use data for the period 1981-2006 to estimate our model using Bayesian techniques. We analyze how an increase in the mortgage default rate can spread to the rest of the economy and create a recession. Next we examine a mortgage modification policy that mitigates the effects of an increase in mortgage default rate.

Keywords: Financial Frictions; DSGE; Fiscal Policy; Bayesian Estimation.

---

*

†École Polytechnique Fédérale de Lausanne, Chair of International Finance, Station 5, Odyssea ODY 2.05, CH-1015 Lausanne, Switzerland; telephone: +(41) 21 693 0050. E-mail: luisa.lambertini@epfl.ch.

‡École Polytechnique Fédérale de Lausanne, Chair of International Finance, Station 5, Odyssea ODY 4.16, CH-1015 Lausanne, Switzerland; telephone: +(41) 21 693 0065. E-mail: victoria.nuguer@epfl.ch

§École Polytechnique Fédérale de Lausanne, Chair of International Finance, Station 5, Odyssea ODY 2.17, CH-1015 Lausanne, Switzerland; telephone: +(41) 21 693 0113. E-mail: pinar.uysal@epfl.ch

1 Introduction

The latest financial crisis has its origins in the housing sector. There was an increase in the mortgage delinquency rate that put many financial institutions under distress. The bursting of the housing bubble in the United States made it difficult for borrowers to repay their loans. As a results, the seriously delinquent mortgage rate (more than 90 days past due and in foreclosure) increased from around 2% of total loans in 2006Q3 up to almost 10% in 2010Q1, as shown in Figure 1. Banks were forced to write down several hundred billions in bad loans. A credit crunch followed that caused the failure of several financial institutions. There was a decrease in the access to credit for households, house prices fell, and so did credit, consumption, and housing investment. The turmoil in the housing sector spread over to the rest of the economy.

In this paper, we propose a model to analyze the transmission of an increase in mortgage default to the rest of the economy, the quantitative effects of the transmission and spillovers, and debt reduction policies that might mitigate these effects. First, we develop a comprehensive DSGE model with the housing sector, mortgages, and endogenous default and we estimate it using Bayesian techniques. Second, we use the estimated model to look at the effects of a reduction in mortgage payments.

This is a medium-sized model which builds on the work of Iacoviello and Neri (2010) and Forlati and Lambertini (2011). There are two types of households that differ in terms of their discount factor: patient households (lenders) save and lend to impatient ones (borrowers). Borrowers can take a loan for a fraction of the expected discounted future value of their houses. In Iacoviello and Neri's equilibrium, mortgages are always repaid. We assume instead, like in Forlati and Lambertini, that the returns to housing are sensitive to both aggregate and idiosyncratic risk. The housing investment is subject to a risk shock that determines the value of the collateral. If the value of the collateral after the risky shock is lower than the repayment of the loan, the borrower defaults on the mortgage. Lenders can observe the realized return only after paying an auditing cost. The participation constraint implied by the financial contract makes the rate of default on mortgages and the external finance premium countercyclical.

The entrance of subprime borrowers in the U.S. mortgage market between 2000 and 2006 is captured with an increase in the standard deviation of idiosyncratic housing investment risk. Because of this risk, some borrowers find themselves in a negative equity trap and default on

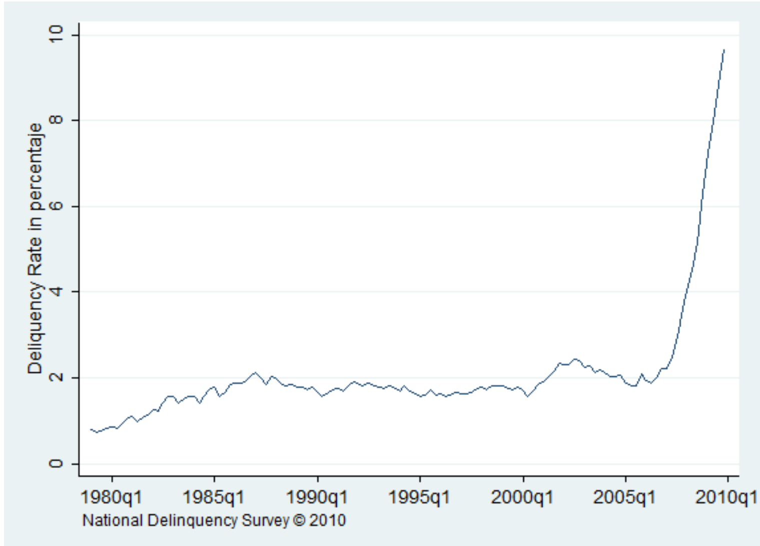

Figure 1: Delinquency Rate in Percentage Points, United States, 1979Q4-2010Q1

their mortgages. The ensuing fall in house prices generates more mortgages that are underwater and triggers a credit crunch that reduces aggregate demand for housing and non-durable goods alike. The economy experiences a recession. We estimate the model with U.S. data for the time period 1981Q1-2006Q4 using Bayesian techniques.

The Financial Stability Act of 2009 launched Making Home Affordable (MHA) in response to the increase in the mortgage default rate. Among other programs the Home Affordable Modification Program (HAMP) was launched. The objectives of this program are to help households to avoid foreclosure, to stabilize the U.S. housing market, and to improve the economy. It consists of loan modifications on the mortgage debt. The loan modifications are either permanent reductions in the mortgage payments through lowering the monthly payments or a reduction in the mortgage rate. So far, almost 1.3 million homeowners have modified their mortgages permanently through HAMP. The MHA program has transformed 3.9 million private-sector mortgages. The median reduction is 40%, which amounts to more than 520 U.S. dollars each month. The program resulted in a total of 4.5 billion U.S. dollars saving for homeowners.

In line with recent events, we use the estimated model for policy evaluation. We study a reduction in the loan payment that writes off the underwater part of mortgages. We find that this policy is effective on mitigating the macroeconomic effects of mortgage risk shocks. Moreover, the policy makes all agents better off relative to the case of no intervention.

Following the work of Kiyotaki and Moore (1997), Iacoviello (2005) and Iacoviello and Neri (2010) introduce collateral constraints in a model with housing to analyze sources and consequences of fluctuations in the U.S. housing sector. In particular, Iacoviello (2005) develops and estimates a monetary business cycle model with nominal loans and durable goods used as collateral. The main features of this framework are: (1) collateral constraint tied to real estate values (as in Kiyotaki and Moore (1997)); (2) nominal debt. These features result in a financial accelerator mechanism. Iacoviello (2005) estimates this model using U.S. data from 1974Q1 to 2003Q2. He finds that there are feedback effects from shocks on housing preferences to non-housing consumption. This effect is positive because of the collateral constraint; otherwise it would be negative.

Iacoviello and Neri (2010) develop a DSGE model of the U.S. economy that includes the housing sector and estimate their model using Bayesian techniques. The core of Iacoviello and Neri's framework, and that of ours, is a dynamic equilibrium model with neoclassical assumption and nominal and real rigidities, as the seminal works of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). These papers do not model the housing sector explicitly but they fit the U.S. data well. The housing features in Iacoviello and Neri's work builds on Iacoviello (2005). Their model is characterized by sectorial heterogeneity, as in Davis and Heathcote (2005). It features the housing sector which produces new homes using capital, labor, and land, and the non-housing sector produces consumption goods and business investment using capital and labor. On the demand side, housing can be used as collateral for borrowing, as in Iacoviello (2005). Iacoviello and Neri find that housing demand and supply shocks explain about one-quarter of the volatility of both housing investment and housing prices. They show that housing market spillovers to consumption and business investment have become more important over time.

The chapter is organized as follows. Section 2 describes the model. Section 3 discusses the methodology. Section 4 documents the characteristics of the model with the estimated parameters. In Section 5 we look at the sources of business cycle fluctuation. Section 6 we study the effects of debt reduction. Section 7 concludes.

2 Model

Our model has two sectors, durable and non-durable goods; two types of households, patient (lender) and impatient (borrower); and endogenous default on mortgage contracts. In equilibrium, some mortgages are defaulted. We assume that the returns to housing are sensitive to both aggregate and idiosyncratic risk and that lenders can observe the realized return only after paying an auditing cost. The participation constraint implied by the financial contract makes the rate of default on mortgages and the external finance premium countercyclical.

2.1 Households

There is a continuum of measure 1 of households. Households differ in terms of their discount factor: patient households (lenders) save and lend to impatient ones (borrowers).

2.1.1 Lenders

Lenders maximize their lifetime utility function:

$$\max E_0 \sum_{t=0}^{\infty} (\beta G_c)^t z_t \left[\Gamma_c \ln(c_t - \epsilon c_{t-1}) + j_t \ln h_t - \frac{\tau_t}{1+\eta} \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{1+\eta}{1+\xi}} \right], \tag{1}$$

where β is the discount factor, c_t is consumption of non-durable goods, h_t is consumption of housing services, $n_{c,t}$ and $n_{h,t}$ are hours in the durable goods sector and housing sector, respectively. The variables z_t, j_t, and τ_t denote the shocks to intertemporal preferences, housing preferences, and to labor supply.[1] The parameter ϵ measures habit persistence, η is the inverse of the Frisch labor supply elasticity, and ξ accounts for the imperfect substitution of hours in the non-durable and housing sectors. Consumption grows at the rate G_c on the balanced growth path and $\Gamma_c = (G_c - \epsilon)/(G_c - \beta\epsilon G_c)$ allows marginal utility of consumption to be equal to $1/c$ in the steady state.

[1]Please see Appendix **??** for the functional forms of these and the following shocks that are not specified in the main text.

Lenders maximize their utility subject to their budget constraint:

$$c_t + q_t h_t + b_t + \frac{k_{c,t}}{A_{k,t}} + \sum_{i=h,b} k_{i,t} + p_{l,t} l_t = \frac{R_{t-1} b_{t-1}}{\pi_t} + q_t(1-\delta_h)h_{t-1} + \sum_{i=c,h} \frac{w_{i,t} n_{i,t}}{X_{wi,t}} \tag{2}$$
$$+ [R_{h,t} z_{h,t} + 1 - \delta_{k,h}] k_{h,t-1} + \left[R_{c,t} z_{c,t} + \frac{1-\delta_{k,c}}{A_{k,t}} \right] k_{c,t-1} + (p_{l,t} + R_{l,t}) l_{t-1} + p_{b,t} k_{b,t}$$
$$+ Div_t - \phi_t - \frac{a(z_{c,t}) k_{c,t-1}}{A_{k,t}} - a(z_{h,t}) k_{h,t-1}.$$

Lenders choose non-durable goods c_t, housing services h_t, hours $n_{c,t}$ and $n_{h,t}$, land l_t, capital in the consumption sector $k_{c,t}$, capital $k_{h,t}$ and intermediate inputs $k_{b,t}$ in the housing sector, lending b_t (borrowing if positive), and capital utilization rates $z_{c,t}$ and $z_{h,t}$. The price of housing is q_t, price for intermediate inputs is $p_{b,t}$, and the price of land is $p_{l,t}$. $A_{k,t}$ could be interpreted as the investment-specific technology shock. R_{t-1} is the nominal return on the riskless bond and $\pi_t = P_t/P_{t-1}$ is the money inflation rate in the non-durable sector. Real wages are $w_{c,t}$ and $w_{h,t}$, real rental rates are $R_{c,t}$ and $R_{h,t}$, and depreciation rates are $\delta_{k,c}$ and $\delta_{k,h}$. There is monopolistic competition in the labor market, and $X_{wc,t}$ and $X_{wh,t}$ represent the markup between the wage paid by the wholesale firm and the wage paid to the households. Final goods firms and labor unions pay lump-sum profits Div_t, ϕ_t is the convex adjustment cost, and $a(z_{i,t})$ is the convex cost of setting capital utilization rate to $z_{i,t}$, which are specified in Appendix **??**.

2.1.2 Borrowers

Borrowers maximize their lifetime utility function:

$$\max E_0 \sum_{t=0}^{\infty} (\beta' G_c)^t z_t \left[\Gamma'_c \ln(c'_t - \epsilon c'_{t-1}) + j_t \ln h'_t - \frac{\tau_t}{1+\eta'} \left((n'_{c,t})^{1+\xi'} + (n'_{h,t})^{1+\xi'} \right)^{\frac{1+\eta'}{1+\xi'}} \right], \tag{3}$$

subject to their budget constraint:

$$c'_t + q_t h'_t + [1 - F_t(\bar{\omega}_t)] R_{z,t} b_{t-1} = b_t + \sum_{i=c,h} \frac{w'_{i,t} n'_{i,t}}{X'_{wi,t}} + (1-\delta_h)[1 - G_t(\bar{\omega}_t)] q_t h'_{t-1} + Div'_t. \tag{4}$$

Variables and parameters with a prime refer to the borrowers who have a lower discount rate than lenders ($\beta' < \beta$). $R_{z,t}$ is the state-contingent interest rate that non-defaulting borrowers pay at time t on the loans b_{t-1} taken at time $t-1$; it is an adjustable interest rate determined after the realization of the shocks and satisfies the participation constraint of the lenders. A fraction of borrowers default on their mortgages and, as a result, lenders seize a fraction $G_t(\bar{\omega}_t)$ of the borrowers' housing stock by paying a monitoring cost μ. The fraction of loans that is repaid to the lenders is denoted by $[1 - F_t(\bar{\omega}_t)]$. Div'_t are specified in Appendix **??**.

Each household consists of many members who are ex-ante identical. The household decides how much to invest in housing and the state-contingent mortgage rate to be paid in the next period, specified in the contract signed this period. Total housing investment is equally assigned across the members of the household. Each household signs a contract with the lender and then the idiosyncratic shock ω^i_{t+1} is realized. The shock defines the ex-post value of the house $\omega^i_{t+1} q_{t+1} h'_t$. This captures the idea that housing investment is risky. The idiosyncratic shock is i.i.d. across the household members and log-normally distributed with the cumulative distribution function $F_{t+1}(\omega^i_{t+1})$. The expected value of the ω^i_{t+1} is equal to 1 in every period, there is no shock at the household level. We assume that housing investment riskiness can change over time: the standard deviation $\sigma_{\omega,t}$ of $\ln \omega^i_t$ is subject to an exogenous shock.

Once the idiosyncratic shock is realized, the borrower decides whether to default on the mortgage or not. The borrower defaults if the idiosyncratic shock is below the threshold level $\bar{\omega}_t$ which is determined by

$$\bar{\omega}_{t+1}(1-\delta_h) q_{t+1} h'_t \pi_{t+1} = R_{z,t+1}\, b_t. \tag{5}$$

Loans greater than this threshold value are repaid at the contractual rate $R_{z,t+1}$ and loans less than the threshold value are defaulted on. The random variable ω^i_{t+1} is observed by the members of the household but lenders can only observe it after paying a cost. Then, lenders pay the monitoring cost μ and seize the collateral on the loan. This makes the borrowers truthfully reveal their idiosyncratic shock. (Townsend, 1979) Borrowers that default lose their housing stocks. There is perfect consumption insurance among household members so that non-durable consumption and housing investment are ex post equal across members of the household.

We assume a one-period mortgage contract as in Bernanke, Gertler and Gilchrist (1999). Lenders demand the gross rate of return R_t which is predetermined and not state contingent. The participation constraint of the lenders is

$$R_t b_t \;=\; \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}(1-\mu)(1-\delta_h) q_{t+1} h'_t \pi_{t+1} f_{t+1}(\omega) d\omega + \int_{\bar{\omega}_{t+1}}^{\infty} R_{z,t+1} b_t f_{t+1}(\omega) d\omega, \tag{6}$$

where $f(\omega)$ is the probability density function of ω. Remember that ω is subject to an exogenous shock to its standard deviation, making it time variant. The return on total loans consists of two parts: the housing stock net of monitoring costs and depreciation of the defaulting borrowers (first-term on the right-hand side of 6), and the repayment by the non-defaulting borrowers (second-term on the right-hand side of 6). After the idiosyncratic and aggregate shocks are realized, the threshold value $\bar{\omega}_t$ and the state-contingent mortgage rate $R_{z,t}$ are determined. The participation constraint holds state by state.

As in Forlati and Lambertini (2011) we define the expected value of the idiosyncratic shock for those who default multiplied by the probability of default as

$$G_{t+1}(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} f_{t+1}(\omega) d\omega \tag{7}$$

and the expected share of housing value, gross of monitoring costs that goes to lenders as

$$\Gamma_{t+1}(\bar{\omega}_{t+1}) \equiv \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f_{t+1}(\omega) d\omega + G_{t+1}(\bar{\omega}_{t+1}). \tag{8}$$

Then, the participation constraint, Equation (6) is

$$R_t b_t = \left[\Gamma_{t+1}(\bar{\omega}_{t+1}) - \mu G_{t+1}(\bar{\omega}_{t+1})\right](1-\delta_h) q_{t+1} h'_t \pi_{t+1}, \tag{9}$$

where the loan-to-value ratio is

$$\left[\Gamma_{t+1}(\bar{\omega}_{t+1}) - \mu G_{t+1}(\bar{\omega}_{t+1})\right]. \tag{10}$$

We derive Equation (9) in Appendix **??**. We also show another way of writing the borrowers' budget constraint, Equation (4). Equation (9) states that the borrower can borrow b_t up to the amount of his expected present value of his house times the loan-to-value ratio.

Borrowers maximize their utility (3) subject to the budget constraint (4) and the borrowing constraint (9). They choose c'_t, h'_t, b_t, $n'_{h,t}$, $n'_{c,t}$, and $\bar{\omega}_{t+1}$.

We follow Bernanke et al. (1999) and Forlati and Lambertini (2011) to specify the idiosyncratic risk in the housing sector. The shock ω is distributed log-normally:

$$\ln \omega \sim N\left(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2\right). \tag{11}$$

2.2 Firms

There are competitive flexible price wholesale firms that produce non-durable goods and housing using two technologies. There is also a final good firm in the non-durable sector which operates under monopolistic competition.

Wholesale firms hire labor and capital services and buy intermediate goods to produce wholesale goods Y_t and new houses IH_t to maximize their profits

$$\frac{Y_t}{X_t} + q_t IH_t - \left[\sum_{i=c,h} \left(w_{i,t}n_{i,t} + w'_{i,t}n'_{i,t} + R_{i,t}z_{i,t}k_{i,t-1}\right) + R_{l,t}l_{t-1} + p_{b,t}k_{b,t}\right], \tag{12}$$

where the production technologies are:

$$Y_t = \left[A_{ct}\left(n_{ct}^{\alpha}n_{ct}^{\prime 1-\alpha}\right)\right]^{1-\mu_c}\left(z_{ct}k_{ct-1}\right)^{\mu_c}, \tag{13}$$

$$IH_t = \left[A_{ht}\left(n_{ht}^{\alpha}n_{ht}^{\prime 1-\alpha}\right)\right]^{1-\mu_h-\mu_b-\mu_l}\left(z_{ht}k_{ht-1}\right)^{\mu_h} k_{bt}^{\mu_b} l_{t-1}^{\mu_l}. \tag{14}$$

X_t is the markup of final good relative to wholesale non-durable good. Non-durable goods are produced with labor and capital as in (13). New houses are produced with labor, capital, land, and intermediate input k_b as in (14). A_{ct} and A_{ht} denote the productivities in the non-durable and housing sectors.

2.3 Nominal Rigidities and Monetary Policy

There is price rigidity in the non-durable good sector and wage rigidities in the non-durable good and housing sectors. We assume monopolistic competition at the retail level and adjustment costs of changing nominal prices à la Calvo to introduce price stickiness (**?**). Retailers buy wholesale goods Y_t from wholesale firms at the price P_t^w in a competitive market, differentiate the goods at no cost, and sell them at a markup $X_t = P_t/P_t^w$ over the marginal cost. The CES aggregates of these goods are converted back into homogeneous consumption and investment goods by households. Each period, a fraction $1-\theta\pi$ of retailers set prices optimally, while a fraction $\theta\pi$ cannot do so, and index prices to the previous period inflation rate with an elasticity equal to $\iota\pi$. These assumptions deliver the following consumption sector Phillips curve:

$$\ln \pi_t - \iota_\pi \ln \pi_{t-1} = \beta G_c \left(E_t \ln \pi_{t+1} - \iota_\pi \ln \pi_t\right) - \varepsilon_\pi \ln\left(\frac{X_t}{X}\right) + u_{p,t}, \tag{15}$$

where $\varepsilon_\pi = (1-\theta_\pi)(1-\beta G_c \theta_\pi)/\theta_\pi$ and $u_{p,t}$ is a cost shock.

Wage setting is similar to the price setting. Unions receive homogeneous labor supply from borrowers and lenders. Then they differentiate the labor services and set wages a la Calvo. Wholesale labor packers buy these labor services and assemble them into n_c, n_h, n'_c, and n'_h. Wholesale firms hire labor from these packers. Under Calvo pricing with partial indexation to past inflation, the pricing rules set by the union imply four wage Phillips curves that are isomorphic to the price Phillips curve. These equations are in Appendix **??**.

To close the model, we assume that the central bank sets the interest rate R_t according to a Taylor rule that responds to past interest rate, inflation, and GDP growth:

$$R_t = (R_{t-1})^{r_R} \left[\pi_t^{r_\pi} \left(\frac{GDP_t}{G_c GDP_{t-1}}\right)^{r_Y} \bar{rr}\right]^{1-r_R} \frac{u_{Rt}}{s_t} \tag{16}$$

where $\bar{rr}$ is the steady-state interest rate, u_{Rt} is i.i.d. monetary policy shock, and s_t is persistent inflation objective shock.

2.4 Equilibrium

The equilibrium in the goods market is as follows:

$$C_t + \frac{IK_{c,t}}{A_{k,t}} + IK_{h,t} + k_{b,t} = Y_t - \phi_t \tag{17}$$

where $C_t = c_t + c_t^{'}$ is the aggregate consumption, $IK_{c,t} = k_{c,t} - (1-\delta_{k,c})k_{c,t-1}$, $IK_{h,t} = k_{h,t} - (1-\delta_{kh})k_{h,t-1}$, and ϕ_t is described in Appendix **??**.

New houses are produced in the housing market:

$$h_t + h_t^{'} - (1-\delta_h)\left[h_{t-1} + h_{t-1}^{'}\left(1-\mu G_t(\bar{\omega}_t)\right)\right] = IH_t. \tag{18}$$

Loan market also clears. Total land is fixed and normalized to one.

2.5 Trends and Balances Growth

To complete the model, we allow for different trends in productivity in the consumption, business, and residential sector. The processes that they follow are:

$$
\begin{aligned}
\ln A_{c,t} &= t\ln\left(1+\gamma_{AC}\right)+\ln Z_{c,t}, &&\text{where} && \ln Z_{c,t}=\rho_{AC}\ln Z_{c,t-1}+u_{C,t};\\
\ln A_{h,t} &= t\ln\left(1+\gamma_{AH}\right)+\ln Z_{h,t}, &&\text{where} && \ln Z_{h,t}=\rho_{AH}\ln Z_{h,t-1}+u_{H,t};\\
\ln A_{k,t} &= t\ln\left(1+\gamma_{AK}\right)+\ln Z_{k,t}, &&\text{where} && \ln Z_{k,t}=\rho_{AK}\ln Z_{k,t-1}+u_{K,t};
\end{aligned}
$$

The innovations are $u_{C,t}$, $u_{H,t}$, and $u_{K,t}$ and they have mean zero and standard deviations σ_{AC}, σ_{AH}, and σ_{AK}. The shocks are serially uncorrelated. The parameters γ_{AC}, γ_{AH}, and γ_{AK} are the net growth rates of technology for each of the sectors. We estimate the last six parameters.

A balance growth path exists. We follow Iacoviello and Neri (2010) to define the growth rates for the real variables; they are defined as

$$G_C = G_{IKh} = G_{q\times IH} = 1+\gamma_{AC}+\frac{\mu_C}{1-\mu_C}\gamma_{AK}; \tag{19}$$

$$G_{IKc} = 1+\gamma_{AC}+\frac{1}{1-\mu_C}\gamma_{AK}; \tag{20}$$

$$G_{IH} = 1+(\mu_h+\mu_b)\gamma_{AC}+\frac{\mu_c\left(\mu_h+\mu_b\right)}{1-\mu_c}\gamma_{AK}+(1-\mu_h-\mu_l-\mu_b)\gamma_{AH}; \tag{21}$$

$$G_q = 1+(1-\mu_h-\mu_b)\gamma_{AC}+\frac{\mu_c\left(1-\mu_h-\mu_b\right)}{1-\mu_c}\gamma_{AK}-(1-\mu_h-\mu_l-\mu_b)\gamma_{AH}. \tag{22}$$

3 Parameter Estimates

3.1 Methods and Data

We take the linearized version of the equations around the balanced growth path and estimate the model with Bayesian methods. The chosen prior distributions will be discussed in the next sub-section, and we estimate the posterior distributions using the Metropolis-Hastings algorithm. We have eleven observables: loan-to-value ratio, real consumption, real residential investment, real business investment, real house prices, nominal interest rates, inflation, hours and wage inflation in the consumption sector, hours and wage inflation in the housing sector. The variables are explained in Appendix A. We measure the loan-to-value ratio following Boz and Mendoza (2014) and define it as net credit market assets of households divided by the value of residential land from Davis and Heathcote (2007).

We use quarterly data for the United States from 1981Q1 until 2006Q4, to exclude the financial crisis period. However, we also experiment with the data until 2008Q4, before the nominal interest rate reached the zero lower bound. We plot the data that we use in the estimation in Figure **??** on page **??**.

3.2 Calibrated Parameters

We present the parameters we calibrate in Table 1 and we follow Iacoviello and Neri (2010).[2] The discount rate for borrowers is $\beta = 0.9925$, and the discount rate for lenders is $\beta' = 0.9700$. The weight on housing in the utility function j is equal to 0.12. The share of capital in the non-durable goods production μ_c is set to 0.35. In the housing production capital share μ_h, the share of land μ_l, and the share of intermediate goods μ_b are all set to 0.1.

We set the depreciation rates for capital in the non-durable goods sector $\delta_{kc} = 0.025$, capital in the housing sector $\delta_{kh} = 0.03$, and the depreciation in the housing sector $\delta_h = 0.01$. The steady state markups are equal to 15 percent, so we fix X_{ss}, X_{wc}, and X_{wh} at 1.15. The autocorrelation parameter for the cost shock ρ_s is fixed at 0.975. Finally, we set the monitoring cost μ equal to 0.12 as in Forlati and Lambertini (2011).

3.3 Prior Distributions

We present our prior distributions in Tables 2 and 3. Most of the priors have the same distributions as in Iacoviello and Neri (2010). The only difference is the parameters that specify the autoregressive part of the shocks i.e. ρ's. We set the prior mean of these parameters to 0.5.

We specify a normal distribution for the mean of standard deviation of idiosyncratic shocks in the housing sector $\bar{\sigma}_\omega$. The idiosyncratic shock itself is specified similar to the other shocks in the model, i.e. the autoregressive parameter has a beta distribution with mean equal to 0.5 and standard deviation equal to 0.1, and the standard deviation has an inverse gamma distribution

[2] Please see Iacoviello and Neri (2010) for the details of the calibration.

Parameter	Value
β	0.9925
β'	0.9700
j	0.1200
μ_c	0.3500
μ_h	0.1000
μ_l	0.1000
μ_b	0.1000
δ_{kc}	0.0250
δ_{kh}	0.0300
δ_h	0.0100
X_{ss}	1.1500
X_{wc}, X_{wh}	1.1500
ρ_s	0.9750
μ	0.1200

Table 1: Calibrated Parameters

with mean 0.001 and a standard deviation of 0.01.

Tables 2 and 3 also show the posterior distribution of the parameters and the shock processes, respectively. Figures **??** to **??** in Appendix **??** present the plots of the prior and the posterior distributions.

4 Properties of the Estimated Model

4.1 Steady-State Analysis

In this section we analyze the steady state properties of the estimated model for the different time periods. Table 4 presents the results for the estimation until 2006Q4 in the first column and for the whole time period in the second column. The estimated risk shock $\sigma_{\omega,t}$ is higher when we include the crisis period, which is not surprising. Also, the annual default rate on the mortgages is higher. However, the loan-to-value ratio is not so different in the two time periods. Housing prices are higher if we include the crisis period, and both the borrowers and lenders have lower housing services. Housing prices are higher because the prices from 2007Q1 until 2008Q4 are higher than the mean of the time series, but clearly, as we show in Figure **??**, Appendix **??**, there is a drop in house prices. Both lenders and savers also have lower non-durable good consumption which leads to a lower aggregate consumption. Loans are also lower,

	Prior Distribution			Posterior Distribution		
Parameter	Distribution	Mean	St. Dev.	Mean	2.5%	97.5%
α	beta	0.65	0.05	0.7440	0.6826	0.8014
ϵ	beta	0.5	0.075	0.4001	0.3182	0.4841
ϵ'	beta	0.5	0.075	0.5846	0.4764	0.6897
η	gamma	0.5	0.1	0.5434	0.3819	0.7166
η'	gamma	0.5	0.1	0.5024	0.3485	0.6563
ξ	normal	-1	0.1	-0.7343	-0.9816	-0.5067
ξ'	normal	-1	0.1	-0.9747	-1.1416	-0.8023
$\phi_{k,c}$	gamma	10	2.5	14.3053	11.3023	17.6198
$\phi_{k,h}$	gamma	10	2.5	10.9936	7.0095	14.9590
r_π	normal	1.5	0.1	1.6608	1.5369	1.7787
r_R	beta	0.75	0.1	0.6828	0.6308	0.7377
r_Y	normal	0	0.1	0.3739	0.2827	0.4665
θ_π	beta	0.667	0.05	0.8692	0.8421	0.8967
ι_π	beta	0.5	0.2	0.5039	0.3090	0.6910
$\theta_{w,c}$	beta	0.667	0.05	0.8645	0.8351	0.8999
$\iota_{w,c}$	beta	0.5	0.2	0.2381	0.1109	0.3583
$\theta_{w,h}$	beta	0.667	0.05	0.8867	0.8516	0.9205
$\iota_{w,h}$	beta	0.5	0.2	0.4491	0.1810	0.7061
γ_{AC}	normal	0.005	0.01	0.0048	0.0045	0.0051
γ_{AH}	normal	0.005	0.01	0.0048	0.0041	0.0054
γ_{AK}	normal	0.005	0.01	0.0011	0.0007	0.0016
ζ	beta	0.5	0.2	0.8759	0.7818	0.9774
$\bar{\sigma}_\omega$	normal	0.1	0.005	0.1012	0.0934	0.1093

Table 2: Prior and Posterior Distribution of the Structural Parameters

	Prior Distribution			Posterior Distribution		
Parameter	Distribution	Mean	St. Dev.	Mean	2.5%	97.5%
ρ_{AC}	beta	0.5	0.1	0.8648	0.8165	0.9123
ρ_{AH}	beta	0.5	0.1	0.9418	0.9026	0.9862
ρ_{AK}	beta	0.5	0.1	0.8840	0.8473	0.9177
ρ_j	beta	0.5	0.1	0.9121	0.8826	0.9435
ρ_{σ_ω}	beta	0.5	0.1	0.8965	0.8657	0.9264
ρ_τ	beta	0.5	0.1	0.7884	0.7053	0.8671
ρ_z	beta	0.5	0.1	0.9434	0.9230	0.9642
σ_{AC}	inv.gamma	0.001	0.01	0.0089	0.0076	0.0103
σ_{AH}	inv.gamma	0.001	0.01	0.0168	0.0148	0.0189
σ_{AK}	inv.gamma	0.001	0.01	0.0123	0.0095	0.0150
σ_j	inv.gamma	0.001	0.01	0.0888	0.0607	0.1149
σ_R	inv.gamma	0.001	0.01	0.0017	0.0014	0.0021
σ_z	inv.gamma	0.001	0.01	0.0162	0.0121	0.0200
σ_ω	inv.gamma	0.001	0.01	0.2006	0.1692	0.2313
σ_τ	inv.gamma	0.001	0.01	0.0412	0.0212	0.0641
σ_p	inv.gamma	0.001	0.01	0.0031	0.0025	0.0036
σ_s	inv.gamma	0.001	0.01	0.0002	0.0002	0.0003
$\sigma_{n,h}$	inv.gamma	0.001	0.01	0.0948	0.0830	0.1058
$\sigma_{w,h}$	inv.gamma	0.001	0.01	0.0055	0.0047	0.0063

Table 3: Prior and Posterior Distribution of Shock Processes

housing investment is half the value of 2006Q4 when we include the financial crisis. Business investment drops.

4.2 Impulse Responses

Mortgage Risk Shock.- Figure 2 plots impulse responses to the estimated mortgage risk shock.[3] The magnitude of the shock is equal to the mode of the estimated distribution, $\sigma_\omega = 0.1927$. The mortgage risk shock is captured by an unanticipated increase in $\sigma_{\omega,t}$, the standard deviation of the distribution of the idiosyncratic housing investment risk. An increase in mortgage risk implies an increase in the default rate of borrowers and an increase in the monitoring costs. Lenders require a higher interest rate from the borrowers who do not default on their loans, because the participation constraint needs to be satisfied, and external finance premium increases. Additionally, the loan-to-value of the borrowers decreases which in turn leads to a tightening of the borrowing constraint of the borrowers. The worsening of financial condi-

[3]Figure **??** in Appendix **??** plots additional variables.

	2006	2008
Loans	1.5690	1.3315
Consumption Savers	1.4650	1.4273
Consumption Borrowers	0.3792	0.3192
Aggregate Consumption	1.8442	1.7465
Housing Investment	0.1426	0.0765
Business Investment	0.6123	0.5633
Housing Consumption Savers	11.9518	6.5545
Housing Consumption Borrowers	2.3057	1.0907
House prices	0.8936	1.6016
Default rate	2.7196	2.7280
$\sigma_{\omega,t}$	0.2006	0.2225
Loan-to-value Ratio	0.7523	0.7529

Table 4: Steady States Comparison

tions leads borrowers to cut back on non-durable consumption, housing investment, and loans. The housing demand of borrowers falls substantially; this drives the large fall in house prices. Savers, who are consumption smoothers, can take advantage of the lower prices and increase housing demand. Nevertheless, the large drop in the housing demand of borrowers dominates. The consumption of lenders falls less than the consumption of borrowers because the borrowers bear all the risk in this type of contract. Savers also reduce lending due to lower interest rates. Overall, total consumption and output fall.

The wages in both sectors are lower both for borrowers and lenders. Wages for borrowers fall more than lenders. Savers and borrowers work less.

Housing Preference Shock.- Next, we analyze impulse responses to a housing preference shock in Figure 3.[4] It is, as Iacoviello and Neri (2010) call it, a housing demand shock. A higher housing demand leads to an increase in housing prices and in housing investment. The collateral capacity of the borrowers increases, and also the wages for borrowers in both sectors increase and they work more. These together allow the borrowers to consume more. The default rate of the borrowers and the monitoring costs of the lenders are lower. Lenders require a lower external finance premium. The nominal interest rate is higher and lenders give out more loans. Savers decrease housing demand and consumption. The decrease in housing is too small to

[4] Figure **??** in Appendix **??** plots additional variables.

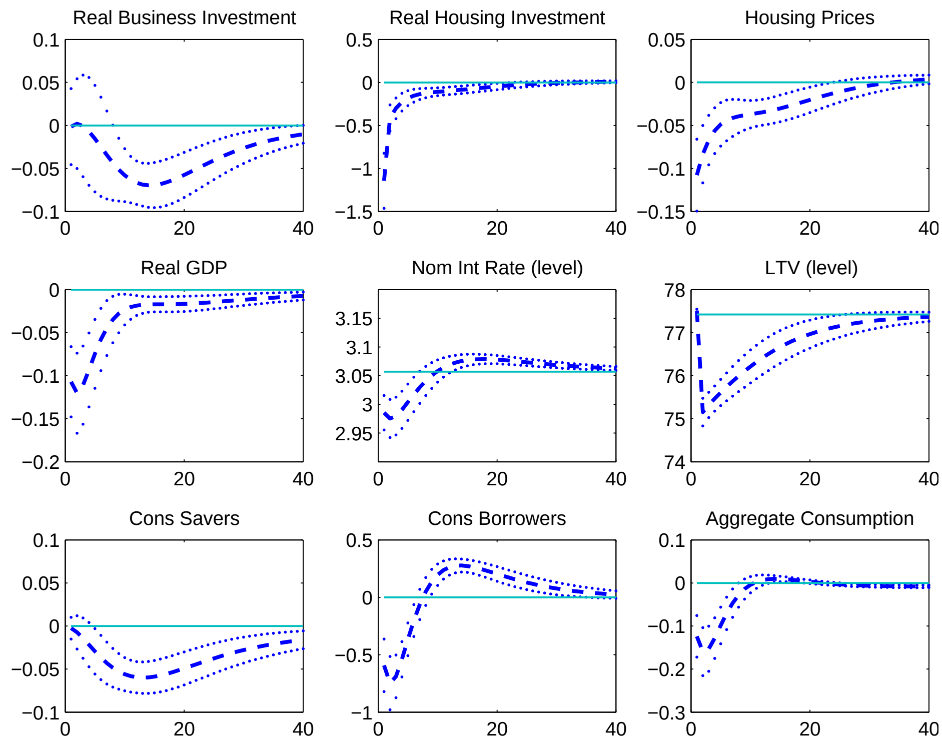

Figure 2: Impulse Responses to Mortgage Risk Shock
Dashed lines represent the mean of the posterior impulse responses; dotted lines indicate the 2 standard deviations confidence intervals.

change the sign of the total demand. Aggregate consumption increases because it is driven mainly by the borrowers' non-durable consumption.

Housing Technology Shock.- Impulse responses to a housing technology shock are presented in Figure 4.[5] A positive shock to housing technology results in higher housing investment and lower housing prices. The fall in house prices decreases the collateral capacity of the borrowers. The default rate is higher, so are the monitoring costs and the external finance premium (not on impact because the risk free rate of return is predetermined). Lenders cut back their loans. They increase their housing demand and decrease their non-durable good consumption. Borrowers consume less non-durable goods.

Savers' wages in the housing sector go up and they work more; their wages in the non-durable goods sector decrease and they work less (on impact). Borrowers' wages in the housing sector also go up and so do hours; in the consumption sector they move on different direction. Total hours of savers and borrowers increase in the housing sector and decrease in the non-durable

[5]Figure **??** in Appendix **??** plots additional variables.

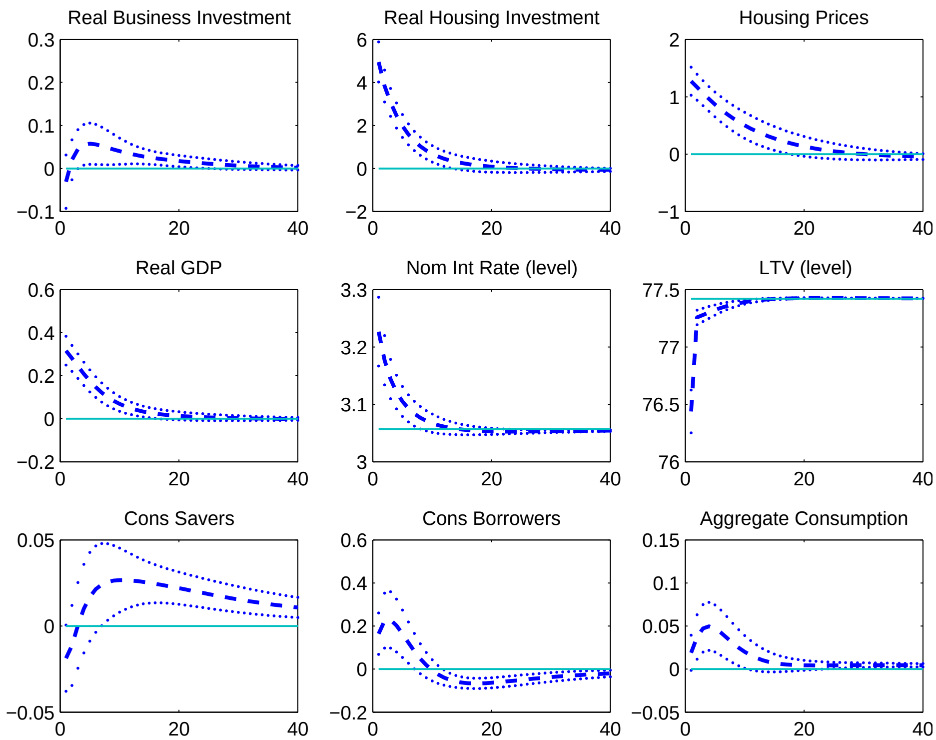

Figure 3: Impulse Responses to Housing Preference Shock
Dashed lines represent the mean of the posterior impulse responses; dotted lines indicate the 2 standard deviations confidence intervals.

sector; investment in the different sectors co-moves with total hours. More resources go into the durable sector. Aggregate output in the non-durable sector falls. Overall there is an increase in housing and real GDP goes up.

An interesting characteristic of the impulse responses is the high persistence of the housing prices to the estimated shock; this is due to the high posterior mean of the autocorrelation parameter of the housing technology shock, $\rho_{AH} = 0.9474$. However, our estimated value is lower than Iacoviello and Neri's, $\rho_{AH} = 0.997$, prompting a faster return to the steady state.

Monetary Policy Shock.- Figure 5 plots the impulse responses to an adverse i.i.d. monetary policy shock.[6] The nominal interest rate goes up. Non-durable good consumption, real business and housing investment fall. Housing investment falls the most and is followed by business investment and consumption, as in Iacoviello and Neri (2010). The consumption of savers falls because they are consumption smoothers. Consumption of borrowers drops due to two reasons. First, credit is more expensive. Second, the price of consumption goods falls and real debt is higher. Borrowers also have to reduce housing demand and hours worked. There is a negative

[6]Figure **??** in Appendix **??** plots additional variables.

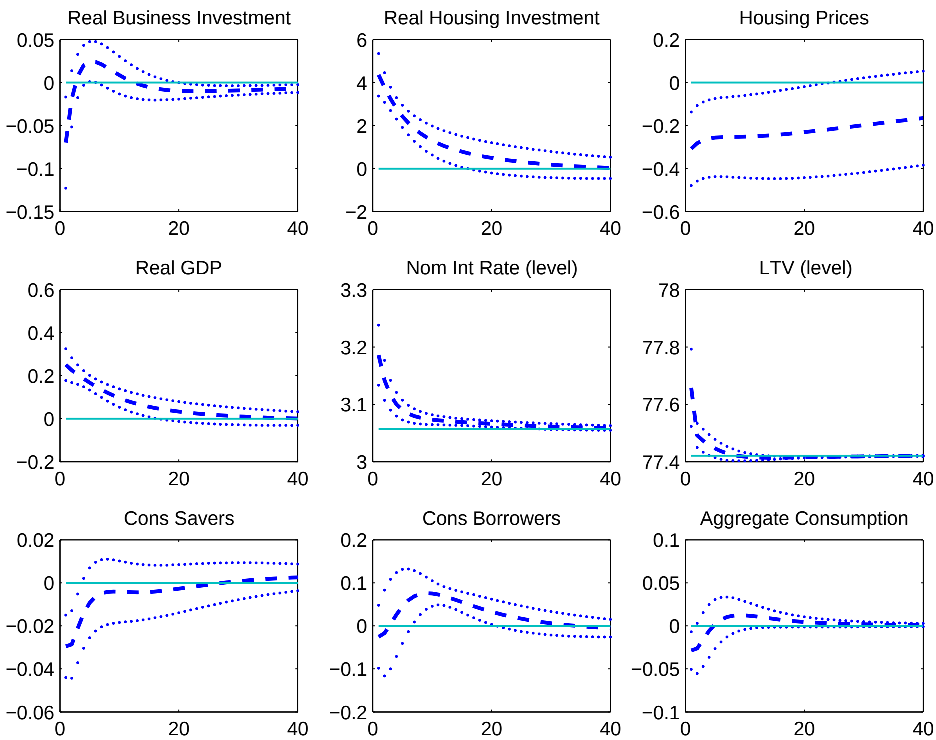

Figure 4: Impulse Responses to Housing Technology Shock
Dashed lines represent the mean of the posterior impulse responses; dotted lines indicate the 2 standard deviations confidence intervals.

wealth effect for the borrowers.

Total demand in the housing sector falls and housing prices drop, too. More borrowers default: for a larger fraction of members of the households the value of their house is lower than the value of their debt. Monitoring costs increase and so does the interest rate that borrowers pay. Lenders substitute loans with housing demand. The loan-to-value rate increases, as expected.

As aggregate demand falls, the wages in all sectors go down and both borrowers and lenders work less.

4.3 Business Cycle Properties

In Table 5, we present the business cycle statistics of our estimated model. Panel A reports the standard deviations, and Panel B reports the correlations. All of the standard deviations of the data are within the 95% of the probability interval computed from the model. The comovements of the variables are also well-replicated with our model.

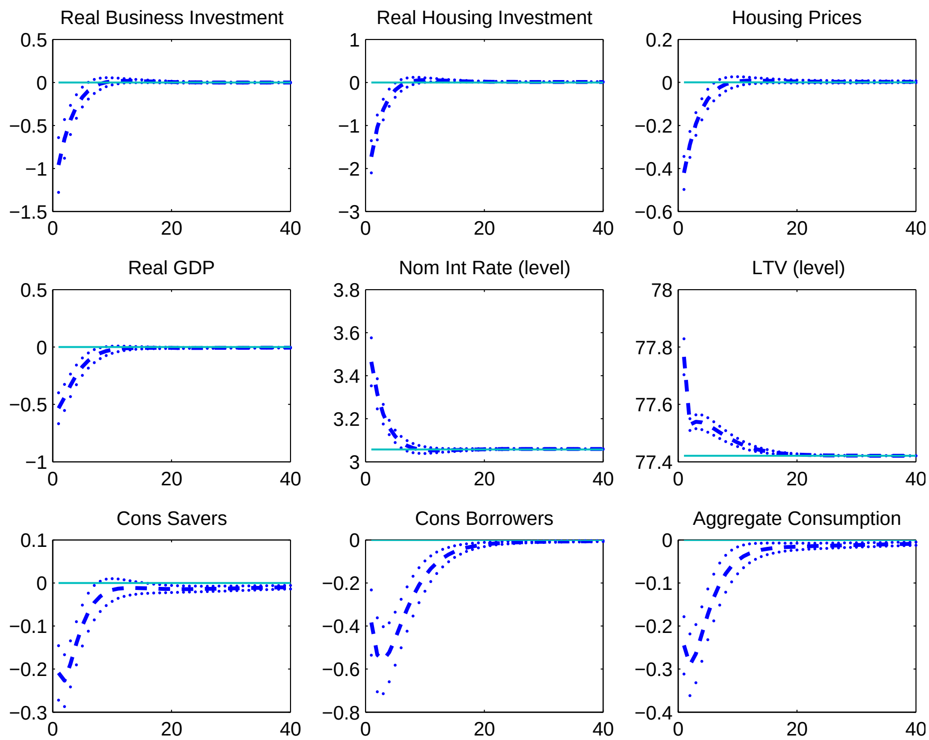

Figure 5: Impulse Responses to Monetary Policy Shock

Dashed lines represent the mean of the posterior impulse responses; dotted lines indicate the 2 standard deviations confidence intervals.

5 Sources of Business Cycle Fluctuations

In Table 6, we present results from the forecast error variance decomposition. We find that housing demand (housing preference) shocks explain more than half of the variance in house prices. However, housing supply (housing technology) does not explain much of the variation in house prices. Each of these shocks explains more than a third of housing investment. Technology shocks in the non-durable sector explain investment in that sector.

Mortgage risk shocks explain about 88% of the variation in loan-to-value ratio, however it does not explain most of the other variables. This result is a limitation of our estimation. We are adding one observed variable and one shock, and the shock explains the variables. However, this result is not very surprising because the mortgage default rate does not really fluctuate at the business cycle frequency.

Consumption is mainly driven by inter-temporal preference and cost-push shocks; output is driven by technology shocks in the non-durable sector and cost-push shocks.

Figure 6 provides a visual representation of the forecast error variance decomposition for

	Model			
	Median	2.5%	97.5%	Data
Panel A. Standard deviations				
C	1.13	0.82	1.65	0.90
IH	7.52	5.95	10.03	7.16
IK	4.00	3.12	4.91	4.51
LTV	3.86	3.13	5.01	1.36
q	1.88	1.42	2.58	1.59
π	0.31	0.24	0.38	0.26
R	0.24	0.18	0.31	0.27
GDP	1.79	1.19	2.36	1.63
Panel B. Correlations				
C, GDP	0.73	0.37	0.85	0.83
IH, GDP	0.57	0.27	0.77	0.64
IK, GDP	0.87	0.75	0.94	0.80
q, GDP	0.57	0.18	0.83	0.45
q, C	0.34	-0.08	0.69	0.43
q, IH	0.64	0.34	0.81	0.49

Table 5: Business Cycle Properties of the Model

	u_c C tech.	u_h H tech.	u_k IK tech.	u_j H pref.	u_R i.i.d. monetary
C	12.41	0.16	1.56	0.38	12.35
IH	1.15	38.70	0.27	43.40	4.53
IK	5.07	0.05	57.76	0.05	8.09
LTV	0.16	0.51	0.04	8.21	1.05
q	4.43	3.18	0.58	66.04	5.51
π	5.01	0.06	1.13	0.25	1.94
R	4.68	1.91	15.31	3.79	20.59
GDP	10.78	3.67	21.85	5.90	15.31
	u_z Intert. Pref.	u_τ L supply	u_p Cost-push	u_s Infl. Objt.	$u_{\bar{\omega}}$ Default
C	30.07	12.90	19.74	6.01	4.42
IH	3.82	4.57	1.95	1.41	0.20
IK	7.98	4.16	13.06	3.75	0.03
LTV	0.45	0.05	0.23	0.44	88.86
q	7.45	2.50	8.13	1.84	0.33
π	4.37	1.98	69.42	15.43	0.42
R	8.87	3.42	26.32	13.33	1.80
GDP	0.91	12.68	21.08	6.84	0.98

Table 6: Asymptotic Variance Forecast Error Decomposition

loan-to-value ratio. The solid black line represents the detrended historical data, the other lines show the historical decomposition under our estimated parameters. We see that the loan-to-value ratio was in decline since the beginning of the 1980s until the beginning of the 1990s. This was followed by an increase in loan-to-value ratios until 2003.

As we expressed in Equations (19) to (22), business investment, residential investment, and

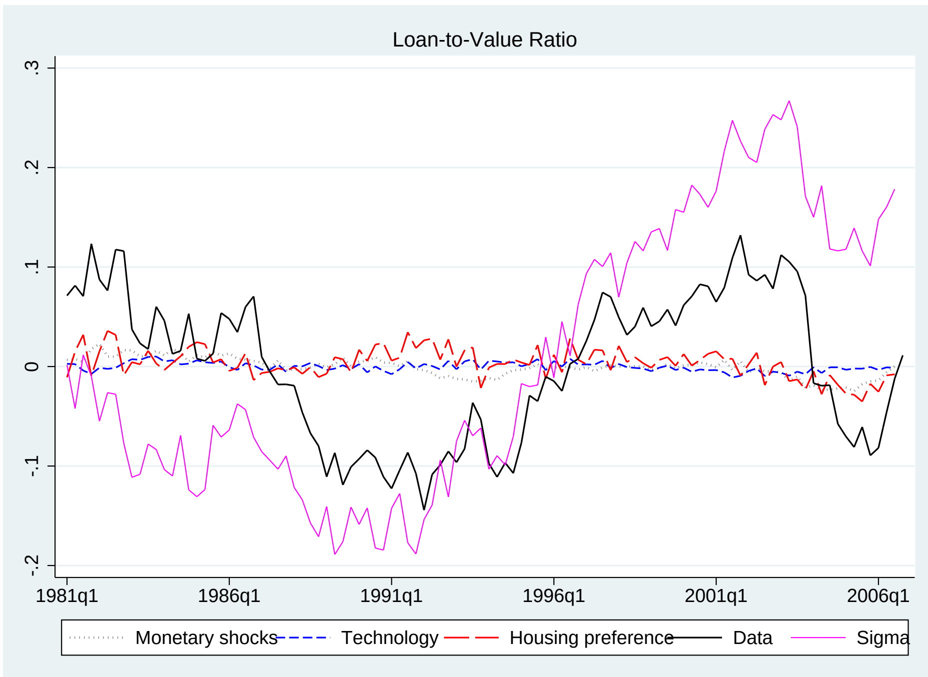

Figure 6: Historical Decomposition of Loan-To-Value Ratio

consumption have different trends. We estimate these trends and we compare them with the data. Figure 7 shows the results. In line with the data, we find that real business investment has a faster technological progress than real consumption and real residential investment. The lower rate in the construction sector can be explained by the trend in real house prices. One of the reasons for the increase in house prices is the price of land which is a factor that limits the construction of new homes.

Finally, we do two robustness checks. First, we estimate the model for two different subsamples: including and excluding the financial crisis; the two subsamples are 1981Q1-2006Q4 and 1981Q1-2008Q4. We stop at the end of 2008 because in December the monetary policy rate reached the zero lower bound. As Table 7 shows, the estimation is robust to the different periods. We note that the standard deviation of the mortgage risk shock, σ_ω, increases and the standard deviation of the labor supply shock, σ_τ, falls. These two shocks are important when

Figure 7: Estimated Trends

Dashed lines represent the median, 2.5 percentiles, and 97.5 percentile of the posterior distribution of the estimated trends. The solid line is the data.

explaining the financial crisis. Second, we estimate the model without including the housing preference shock in the estimation. We do this because this shock gains relevance in comparison to the results presented by Iacoviello and Neri (2010), and because the mortgage risk shock has similar effects to the housing demand shock. Table 8 provides the estimation results for the period 1981Q1-2006Q4. The two estimations reveal very similar results. There are very little changes in the standard deviations of the shocks. However, the mean of the idiosyncratic shock, $\bar{\sigma}_\omega$, increases almost 9 times. At the steady state, the default rate is higher and is capturing part of the volatility of the housing demand shocks.

By including the mortgage risk shock and an endogenous loan-to-value ratio, we want to describe the effects of the recent financial crisis. In contrast with the work of Iacoviello and Neri, we are able to match the qualitative dynamics of the model with the data after a shock that simulates the recent financial crisis. This is the main difference between the two works. In terms of the estimation, we verify Iacoviello and Neri's results and we estimate a richer model.

	Structural parameters			Shocks and measurement errors	
	1981Q1-2006Q4 Post. Mean	1981Q1-2008Q4 Post. Mean		1981Q1-2006Q4 Post. Mean	1981Q1-2008Q4 Post. Mean
α	0.7440	0.7716	ρ_{AC}	0.8648	0.9081
ϵ	0.4001	0.4002	ρ_{AH}	0.9418	0.9776
ϵ'	0.5846	0.5507	ρ_{AK}	0.8840	0.8789
η	0.5434	0.4409	ρ_j	0.9121	0.9003
η'	0.5024	0.5122	ρ_z	0.9434	0.9341
$\phi_{k,c}$	14.3053	14.0674	ρ_{σ_ω}	0.8965	0.9115
$\phi_{k,h}$	10.9936	10.0974	ρ_τ	0.7884	0.8956
ι_π	0.5039	0.3415	σ_{AC}	0.0089	0.0103
$\iota_{w,c}$	0.2381	0.3087	σ_{AH}	0.0168	0.0180
$\iota_{w,h}$	0.4491	0.4115	σ_{AK}	0.0123	0.0120
ξ	-0.7343	0.0345	σ_j	0.0888	0.1001
ξ'	-0.9747	-1.0605	σ_R	0.0017	0.0018
r_π	1.6608	1.6269	σ_z	0.0162	0.0138
r_R	0.6828	0.6615	σ_ω	0.2006	0.2225
r_Y	0.3739	0.3809	σ_τ	0.0412	0.0229
θ_π	0.8692	0.8677	σ_p	0.0031	0.0031
$\theta_{w,c}$	0.8645	0.8121	σ_s	0.0002	0.0002
$\theta_{w,h}$	0.8867	0.7659	$\sigma_{n,h}$	0.0948	0.1380
γ_{AC}	0.0048	0.0040	$\sigma_{w,h}$	0.0055	0.0038
γ_{AH}	0.0048	0.0042			
γ_{AK}	0.0011	0.0020			
ζ	0.8759	0.8656			
$\bar{\sigma}_\omega$	0.1012	0.1009			

Table 7: Samples Estimates. Different Time Periods, 1981Q1-2006Q4 and 1981Q1-2008Q4

6 Policy Analysis

We use the model estimated until 2006Q4 to evaluate a mortgage modification policy carried out in response to a mortgage risk shock. We focus on policies that reduce the rate of default by targeting marginal mortgages, namely the mortgages the smallest (in absolute value) negative equity. In the absence of policy all homeowners experiencing an idiosyncratic shock below $\bar{\omega}_t$ default on their mortgages. When the policy is active $\bar{\omega}_t$, as still determined by Equation (9), can be interpreted as the *natural* threshold level at which homeowners would start defaulting. The policy we consider lowers default by choosing $\underline{\omega}_t < \bar{\omega}_t$ and reducing mortgage payment for households between these thresholds so as to eliminate their negative equity exposure. The

	Structural parameters			Shocks and measurement errors	
	With a_j Post. Mean	Without a_j Post. Mean		With a_j Post. Mean	Without a_j Post. Mean
α	0.7440	0.7462	ρ_{AC}	0.8648	0.8691
ϵ	0.4001	0.4010	ρ_{AH}	0.9418	0.9247
ϵ'	0.5846	0.5875	ρ_{AK}	0.8840	0.8858
η	0.5434	0.5417	ρ_j	0.9121	
η'	0.5024	0.4973	ρ_z	0.9434	0.9399
$\phi_{k,c}$	14.3053	14.2919	ρ_{σ_ω}	0.8965	0.9369
$\phi_{k,h}$	10.9936	11.2468	ρ_τ	0.7884	0.7838
ι_π	0.5039	0.5095	σ_{AC}	0.0089	0.0087
$\iota_{w,c}$	0.2381	0.2421	σ_{AH}	0.0168	0.0167
$\iota_{w,h}$	0.4491	0.4353	σ_{AK}	0.0123	0.0123
ξ	-0.7343	-0.7422	σ_j	0.0888	
ξ'	-0.9747	-0.9663	σ_R	0.0017	0.0017
r_π	1.6608	1.6671	σ_z	0.0162	0.0159
r_R	0.6828	0.6873	σ_ω	0.2006	0.2035
r_Y	0.3739	0.3624	σ_τ	0.0412	0.0432
θ_π	0.8692	0.8717	σ_p	0.0031	0.0031
$\theta_{w,c}$	0.8645	0.8667	σ_s	0.0002	0.0002
$\theta_{w,h}$	0.8867	0.8870	$\sigma_{n,h}$	0.0948	0.0944
γ_{AC}	0.0048	0.0048	$\sigma_{w,h}$	0.0055	0.0055
γ_{AH}	0.0048	0.0046			
γ_{AK}	0.0011	0.0012			
ζ	0.8759	0.8912			
$\bar{\sigma}_\omega$	0.1012	0.8959			

Table 8: Samples Estimates. With and Without Housing Preference Shock, 1981Q1-2006Q4

payment reduction for household ω_t, conditional on $\underline{\omega}_t < \omega_t < \bar{\omega}_t$, is

$$(\bar{\omega}_t - \omega) q_t h'_{t-1} \pi_t (1 - \delta_h)$$

and the total policy intervention is

$$P_t = \int_{\underline{\omega}_t}^{\bar{\omega}_t} (\bar{\omega}_t - \omega) q_t h'_{t-1} \pi_t (1 - \delta_h)(1 + \mu') f_t(\omega) d(\omega).$$

This policy can be interpreted as a targeted mortgage modification with principal reduction. Notice that principal is reduced just enough to make homeowner ω prefer repayment over default. Smaller values of $\underline{\omega}_t$ generate lower default rates in equilibrium but imply larger intervention costs. The parameter $\mu' \geq 0$ captures the costs of the mortgage modification program. These are the costs of monitoring and processing the applications.

The participation constraint of lenders, Equation (6), becomes:

$$R_t b_t \;=\; \int_0^{\underline{\omega}_{t+1}} \omega_{t+1}(1-\mu)(1-\delta)q_{t+1}h_t'\pi_{t+1}f_{t+1}(\omega)d\omega + \int_{\underline{\omega}_{t+1}}^{\infty} R_{z,t}b_t f_{t+1}(\omega)d\omega.$$

Since $\underline{\omega}_{t+1}$ is smaller than the *natural* threshold $\bar{\omega}_{t+1}$, default is lower than it would have been in the absence of policy. The budget constraint of borrowers, Equation (4), now becomes:

$$c_t' + q_t h_t' + [1 - F_t(\underline{\omega}_t)]R_{z,t}b_{t-1} = b_t + \sum_{i=c,h} \frac{w_{i,t}' n_{i,t}'}{X_{wi,t}'} + (1-\delta_h)[1 - G_t(\underline{\omega}_t)]q_t h_{t-1}' + Div_t'.$$

Let

$$\tilde{\Gamma}_t(\bar{\omega}_t, \underline{\omega}_t) \equiv \bar{\omega}_t \left[1 - F_t(\underline{\omega}_t)\right] + G_t(\underline{\omega}_t).$$

Using this definition and substituting the incentive-compatibility constraint, Equation (9), the participation constraint can be rewritten as

$$R_t b_t \;=\; \left[\tilde{\Gamma}_t(\bar{\omega}_t, \underline{\omega}_t) - \mu G(\underline{\omega}_t)\right](1-\delta_h)q_{t+1}h_t'\pi_{t+1}. \tag{23}$$

We consider policies that respond to a mortgage risk shock; for the choice of $\underline{\omega}_t$ we assume

$$\underline{\omega}_t = \bar{\omega}_t - \phi_0 \epsilon_{\sigma_\omega,t} - \phi_1(\sigma_{\omega,t} - \bar{\sigma}_\omega), \tag{24}$$

where ϕ_0, ϕ_1 are positive parameters, $\epsilon_{\sigma_\omega,t}$ is the mortgage risk shock, and $\sigma_{\omega,t}$ is the time-varying standard deviation of housing investment risk. Intuitively, our mortgage policy remains active as long as the volatility in the mortgage market is high. The marginal threshold $\underline{\omega}_t$ is kept below its natural counterpart as long as the standard deviation of housing risk $\sigma_{\omega,t}$ is above its steady-state level. Our policy allows for an additional, contemporaneous response ϕ_0 to the mortgage risk shock; this feature helps reducing the default rate at time t but it is not necessary. The policy is unexpected but once it starts at t agents correctly anticipate that the policy will be effective until the volatility in the mortgage market subsides.

Our benchmark scenario assumes that savers bear the cost of mortgage modification, which is paid in the form of a lump-sum tax. Their budget constraint, Equation (2), becomes:

$$
\begin{aligned}
c_t + q_t h_t + b_t + \frac{k_{c,t}}{A_{k,t}} + \sum_{i=h,b} k_{i,t} + p_{l,t} l_t = \frac{R_{t-1} b_{t-1}}{\pi_t} + q_t (1-\delta_h) h_{t-1} + \sum_{i=c,h} \frac{w_{i,t} n_{i,t}}{X_{wi,t}} \\
+ \left[R_{h,t} z_{h,t} + 1 - \delta_{k,h} \right] k_{h,t-1} + \left[R_{c,t} z_{c,t} + \frac{1-\delta_{k,c}}{A_{k,t}} \right] k_{c,t-1} + (p_{l,t} + R_{l,t}) l_{t-1} + p_{b,t} k_{b,t} \\
+ Div_t - \phi_t - \frac{a(z_{c,t}) k_{c,t-1}}{A_{k,t}} - a(z_{h,t}) k_{h,t-1} - P_t.
\end{aligned}
$$

In Figure 8 we present the impulse responses to same mortgage risk shock analyzed in Figure 2.[7] We assume that $\mu' = \mu = 0.12, \phi_0 = 0.05, \phi_1 = 1$ in this exercise. We do not have estimates for the value of μ' upon which to rely, so we set the cost of monitoring applications for mortgage reduction equal to the monitoring cost we estimated. Although plausible, we regard this value as high so that our results can be interpreted as the lower bound on the effectiveness of the policy. The first subplot displays the response of the threshold $\bar{\omega}$ in the absence of policy (dashed blue line) and of $\underline{\omega}$ with policy (solid green line). Reducing the principal for marginal homeowners has strong effects on the mortgage market: on impact default jumps above 4 percentage points on annual basis instead of 10 percentage points. Lower default reduces the adjustable mortgage rate R_z and the external finance premium; the contraction of the LTV is also contained so that borrowers do not need to deleverage as much. Overall borrowers' financial conditions worsen far less when the policy is in place, which in turn mitigates the sharp fall in their demand for houses. The policy has important effects on the real economy as aggregate consumption, housing and business investment, and GDP display far smaller reductions. By reducing default, the policy reduces the losses stemming from foreclosure and monitoring. These costs are borne ultimately by borrowers via higher mortgage rates and a reduced capacity to borrow. The key reason for the milder recession is that financial conditions of borrowers do not deteriorate as much under the policy.

Interestingly, savers' consumption falls less under the policy even though it is these agents that bear its cost. There are two reasons for this effect. First, the size of the intervention is very small. Initially the policy reaches 5.2 percent (on an annual basis) of the outstanding mortgages and, over the first 40 quarters, less than 9 percent of mortgages are modified. More importantly, loan modifications are relatively small because only the underwater part of each

[7] Figure **??** in Appendix **??** plots additional variables.

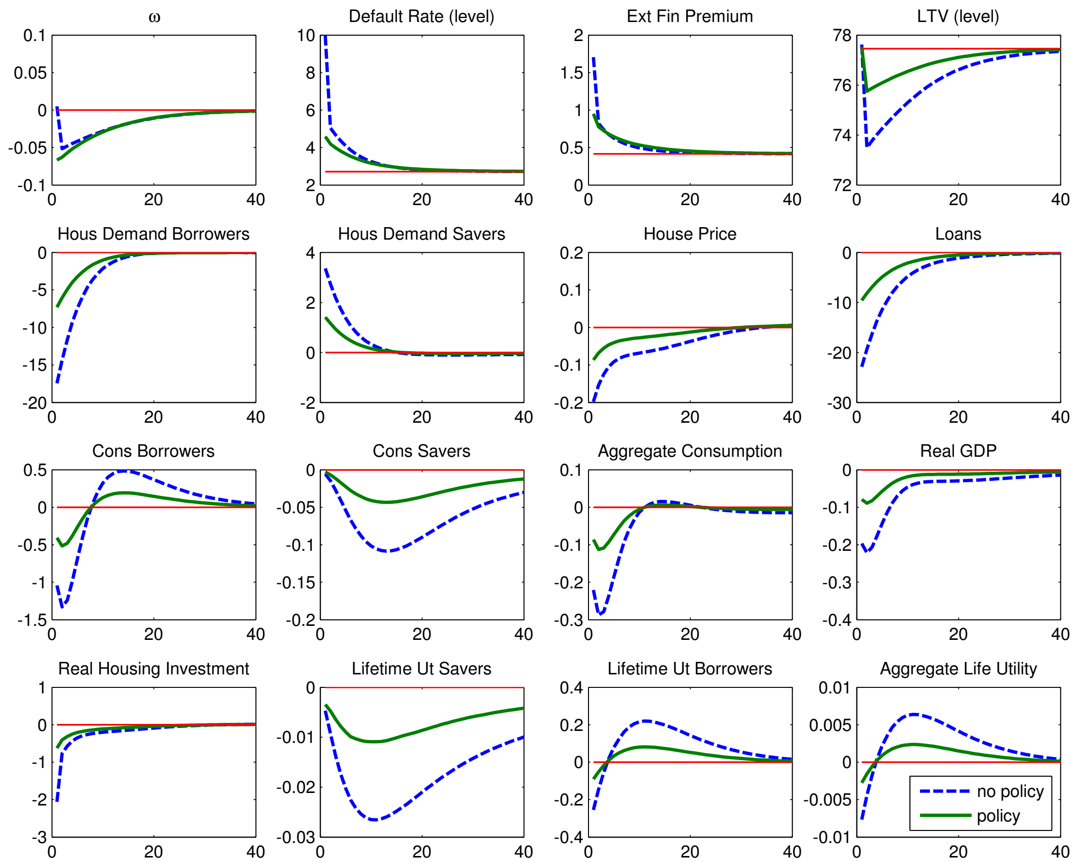

Figure 8: Impulse Responses to Mortgage Risk Shock with Policy Intervention

mortgage is forgiven. In the first quarter, the cost of the policy in terms of GDP, P_t/Y_t, reaches 0.043 percentage points; the total (not discounted) cost over the first 10 years is 0.19 percentage points of GDP.

The last three subplots of Figure 8 display the impulse response of the lifetime utility of savers, Equation (1), the lifetime utility of borrowers, Equation (3), and social or aggregate lifetime utility calculated as

$$\tilde{u}_t = (1-\beta)u_t + (1-\beta')u'_t.$$

These impulse responses are obtained from the second-order approximation of the model. Both borrowers' and savers' expected lifetime utility are higher at the time of shock under the policy. Hence the policy is Pareto-improving over the lack of policy response. If given the choice, both agents would therefore choose to undertake the policy when the mortgage risk shock arises. Four quarters after the shock, however, borrowers' lifetime utility with policy intersects and becomes lower than lifetime utility without policy. On the other hand, savers are strictly better off with the policy over the entire horizon. These welfare effects stem from the rebound of the housing sector without policy, which leads to an increase in borrowers' wages[8] and a more pronounced increase in non-durable consumption for these agents. Impatient agents' welfare is affected more than patient ones' by a mortgage risk shock, as attested by the responses in Figure 8. This is not surprising as impatient agents face a constraint constraint and allocate a larger fraction of their income to housing. This explains why aggregate lifetime utility mimics that of borrowers even though there is an identical mass of either type of agents in our model.

6.1 Making Home Affordable?

In response to the crisis in the housing market the Financial Stability Act of 2009 launched Making Home Affordable (MHA), which included Home Affordable Refinance Program (HARP), Home Affordable Modification Program (HAMP), and Home Affordable Foreclosure Alternatives (HAFA). Overall, the goals of MHA are to help households to avoid foreclosure, to stabilize the U.S. housing market, and to improve the economy. HARP allows borrowers to refinance their mortgages even if they have insufficient equity to qualify for traditional refinance. To qualify for HARP borrowers must be current on their payments; mortgages must be owned or guaranteed by Fannie Mae or Freddie Mac; and loans had to be originated before June 2009. In response to low participation in the program, loans of 20 years or less and with LTVs greater than 125 percentage points were made eligible for the program, and some fees were eliminated. HAMP helps homeowners to avoid foreclosure by facilitating permanent modifications to their mortgages by extending the term, reducing the mortgage rate, or reducing the principal. HAMP reduces monthly payments for qualifying borrowers to 31 percent of their income; these modifications are made possible by paying incentives to servicers and lenders. HAFA provides two alternative to foreclosure (short sale or Deed-in-Lieu of foreclosure). The application deadline for MHA programs has been set to the end of 2015.

The U.S. Department of the Treasury publishes MHA Program Performance Reports and MHA Data Files on a monthly basis. The reports summarize basic figures for HAMP and HAFA but give no information on HARP; MHA Data Files consists of three sets of loan-level

[8] See Figure **??**.

mortgage modification data: First Lien, Second Lien, and HAFA. Since the policy analyzed in the previous section consists of principal reduction, we focus on the results of HAMP, First and Second Lien Modifications.

There are important differences between model and data that make a comparison of policies difficult. In particular, mortgages last one quarter in our model while their duration is typically 20 to 30 years in the data. A permanent modification to a 30-year contract however cannot be compared to a modification that lasts just one quarter in the model. For this reason we consider the cumulated modification measures, both in our model and in the data. In addition to this, permanent modifications under HAMP follow a series of waterfall steps that include interest rate adjustment, term extension and principal forbearance. According to the MHA Program Performance Report Through May 2014[9], about one-third of all permanent modifications feature principal forbearance, 64 percent term extension and 96 percent interest rate reduction. Our policy, on the other hand, uses only principal forbearance. For this reason we compare HAMP and our policy in terms of unpaid principal after modification.

According to the Performance Report through May 2014 and the related data files:

1. 954,694 mortgages have received permanent modification through HAMP. This is to say that 21 quarters after its inception, the policy has reached almost 2 percent of total outstanding mortgages. The corresponding number in our policy, namely the cumulated fraction of modified mortgages up to 21 quarters after the shock, is 8.1 percent.

2. There were 228,802 HAMP modifications with principal reduction; the total outstanding principal balance reduction amounted to USD 14,209,424,479. This represents 0.1 percent of total mortgage debt outstanding as of 2014 Q1. After 21 quarters our policy reduced the principal balance by 0.3 percent of mortgage debt.

3. According to the data, the median unpaid balance after modification is USD 223,517 against USD 242,443 before modification, which implies an 8 percent reduction in the unpaid principal. In our model, the median reduction in the unpaid principal after 21 quarter is 27 percent of the balance before modification.

The policy in our model reaches a larger fraction of mortgages and it reduces the principal

[9] 2014 MHA Program Performance Reports are available at the U.S. Department of the Treasury web site.

of underwater mortgages more aggressively, both in terms of total and median principal forbearance, relative to HAMP. Agarwal, Amromin, Ben-David, Chomsisengphet, Piskorski and Seru (2012) examine the impact of HAMP on various margins of renegotiation rates and on broader outcomes such as house prices. This study finds a fairly negligible increase in the rate of permanent modifications and reduction in the rate of foreclosure as a result of HAMP; it is also finds no evidence that house prices and other macroeconomic variables were affected in the regions more exposed to the program and cite lack of responsiveness by some large servicers to the financial incentives as the main reason for this shortfall. Further analysis of the details of modifications could shed light on the limited success of the program and provide guidance in designing such policies in the future.

7 Conclusion

We have presented a plausible model with housing and endogenous default on mortgages that we estimate using Bayesian techniques. We first evaluate the estimated model through the analysis of the first order approximation of the model to different shocks. Second, we compare the results of the estimated model with the data. The estimated model explains several features of the data. The model matches the volatility of the real variables and the trend of the housing sector. Third, given recent events, we simulate the model to an increase in the mortgage risk and we study the spillover to the rest of the economy. Finally, we analyze a policy that reduces the principal balance so as to eliminate its underwater part. We find that this policy is effective in reducing the macroeconomic consequences of a mortgage risk shock and it makes all agents better off.

References

Agarwal, Sumit, Gene Amromin, Itzhak Ben-David, Souphala Chomsisengphet, Tomasz Piskorski, and Amit Seru, "Policy Intervention in Debt Renegotiation: Evidence from the Home Affordable Modification Program," *NBER Working Paper 18311*, August 2012.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist, "The financial accelerator in a quantitative business cycle framework," in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Vol. 1 of Handbook of Macroeconomics, Elsevier, April 1999, chapter 21, pp. 1341–1393.

Boz, Emine and Enrique G. Mendoza, "Financial innovation, the discovery of risk, and the U.S. credit crisis," *Journal of Monetary Economics*, 2014, *62* (0), 1 – 22.

Calvo, Guillermo, "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, September 1983, *12* (3), 383–398.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, February 2005, *113* (1), 1–45.

Davis, Morris A. and Jonathan Heathcote, "Housing And The Business Cycle," *International Economic Review*, 08 2005, *46* (3), 751–784.

_ and _ , "The price and quantity of residential land in the United States," *Journal of Monetary Economics*, November 2007, *54* (8), 2595–2620.

Forlati, Chiara and Luisa Lambertini, "Risky Mortgages in a DSGE Model," *International Journal of Central Banking*, 2011, *7* (1), 285–335.

Iacoviello, Matteo, "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle," *American Economic Review*, 2005, *95* (3), 739–764.

_ and Stefano Neri, "Housing Market Spillovers: Evidence from an Estimated DSGE Model," *American Economic Journal: Macroeconomics*, 2010, *2* (2), 125–64.

Kiyotaki, Nobuhiro and John Moore, "Credit Cycles," *Journal of Political Economy*, 1997, *105* (3), 211–248.

Smets, Frank and Rafael Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, June 2007, *97* (3), 586–606.

Townsend, Robert M., "Optimal contracts and competitive markets with costly state verification," *Journal of Economic Theory*, October 1979, *21* (2), 265–293.

A Appendix: Data and Sources

Aggregate Consumption Real Personal Consumption Expenditure (seasonally adjusted, billions of chained 2000 dollars), divided by the Civilian Noninstitutional Population Source: Bureau of Economic Analysis (BEA) and Bureau of Labor Statistics (BLS).

Business Fixed Investment Real Private Nonresidential Fixed Investment (seasonally adjusted, billions of chained 2000 dollars, per capita. Source: BEA.

Residential Investment Real Private Residential Fixed Investment (seasonally adjusted, billions of chained 2000 dollars) per capita. Source: BEA.

Inflation Quarter on quarter log differences in the implicit price deflator for the non-farm business sector, demeaned. Source: Bureau of Labor Statistics (BLS).

Nominal Short-term Interest Rate 3-month Treasury Bill Rate (Secondary Market Rate), expressed in quarterly units, demeaned. Source: Board of Governors of the Federal Reserve System.

Real House Prices Census Bureau House Price Index (new one-family houses sold including value of lot) deflated with the implicit price deflator for the non-farm business sector. Source: Census Bureau.

Hours in Consumption Sector Total Non-farm Payrolls less all employees in the construction sector, times Average Weekly Hours of Production Workers, per capita. Demeaned. Source: BLS.

Hours in Housing Sector All Employees in the Construction Sector, times Average Weekly Hours of Construction Workers, per capita. Demeaned. Source: BLS.

Wage Inflation in Consumption-good Sector Quarterly changes in Average Hourly Earnings of Production/Nonsupervisory Workers on Private Nonfarm Payrolls, Total Private. Demeaned. Source: BLS.

Wage Inflation in Housing Sector Quarterly changes in Average Hourly Earnings of Production/ Non-supervisory Workers in the Construction Industry. Demeaned. Source: BLS.

Loan-to-value ratio Net credit market assets of US household divided by Value of residential land. Source: Flow of Funds Accounts of the US and Davis and Heathcote (2007).

B Appendix: Model Equations

Budget constraint for lenders:

$$c_t + q_t h_t + b_t + \frac{k_{c,t}}{A_{k,t}} + \sum_{i=h,b} k_{i,t} + p_{l,t} l_t = \frac{R_{t-1} b_{t-1}}{\pi_t} + q_t(1-\delta_h)h_{t-1} + \sum_{i=c,h} \frac{w_{i,t} n_{i,t}}{X_{wi,t}} \tag{B1}$$
$$+ [R_{h,t} z_{h,t} + 1 - \delta_{k,h}]\, k_{h,t-1} + \left[R_{c,t} z_{c,t} + \frac{1-\delta_{k,c}}{A_{k,t}}\right] k_{c,t-1} + (p_{l,t} + R_{l,t}) l_{t-1} + p_{b,t} k_{b,t}$$
$$+ Div_t - \phi_t - \frac{a(z_{c,t}) k_{c,t-1}}{A_{k,t}} - a(z_{h,t}) k_{h,t-1}$$

First order conditions for lenders:

$$u_{c,t} q_t = u_{h,t} + \beta G_C E_t (u_{c,t+1} q_{t+1} (1-\delta_h)) \tag{B2}$$

$$u_{c,t} = \beta G_C E_t (u_{c,t+1} R_t / \pi_{t+1}) \tag{B3}$$

$$u_{c,t} \left(\frac{1}{A_{k,t}} + \frac{\partial \phi_{c,t}}{\partial k_{c,t}} \right) = \beta G_C E_t u_{c,t+1} \left(R_{c,t+1} z_{c,t+1} - \frac{a(z_{c,t+1}) + 1 - \delta_{kc}}{A_{k,t+1}} - \frac{\partial \phi_{c,t+1}}{\partial k_{c,t}} \right) \tag{B4}$$

$$u_{c,t} \left(1 + \frac{\partial \phi_{h,t}}{\partial k_{h,t}} \right) = \beta G_C E_t u_{c,t+1} \left(R_{h,t+1} z_{h,t+1} - a(z_{h,t+1}) + 1 - \delta_{kh} - \frac{\partial \phi_{h,t+1}}{\partial k_{h,t}} \right) \tag{B5}$$

$$
\begin{aligned}
u_{c,t} w_{c,t} &= u_{nc,t} X_{wc,t} && \text{(B6)}\\
u_{c,t} w_{h,t} &= u_{nh,t} X_{wh,t} && \text{(B7)}\\
u_{c,t}(p_{bt} - 1) &= 0 && \text{(B8)}\\
R_{ct} A_{kt} &= a'(z_{c,t}) && \text{(B9)}\\
R_{ht} &= a'(z_{h,t}) && \text{(B10)}\\
u_{c,t} p_{l,t} &= \beta G_C E_t u_{c,t+1}(p_{l,t+1} + R_{l,t+1}) && \text{(B11)}
\end{aligned}
$$

The budget and borrowing constraint for the borrowers are:

$$
c'_t + q_t h'_t + [1 - F_t(\bar{\omega}_t)] R_{z,t} b_{t-1} = b_t + \sum_{i=c,h} \frac{w'_{i,t} n'_{i,t}}{X'_{wi,t}} + (1 - \delta_h)[1 - \mu G_t(\bar{\omega}_t)] q_t h'_{t-1} + Div'_t. \quad \text{(B12)}
$$

$$
b_t = [\Gamma_{t+1}(\bar{\omega}_{t+1}) - \mu G_{t+1}(\bar{\omega}_{t+1})] E_t \left[(1 - \delta_h) \frac{q_{t+1} h'_t \pi_{t+1}}{R_t} \right], \quad \text{(B13)}
$$

the first order conditions are:

$$
\begin{aligned}
u_{c',t} q_t &= u_{h',t} + \beta' G_C E_t (u_{c,t+1} q_{t+1} (1 - \delta_h)(1 - \mu G(\omega_{t+1}))) \\
&\quad + E_t \left(\frac{\lambda_{t+1}(1 - \delta_h) q_{t+1} \pi_{t+1}}{R_t} [\Gamma_{t+1}(\bar{\omega}_{t+1}) - \mu G_{t+1}(\bar{\omega}_{t+1})] \right) && \text{(B14)}
\end{aligned}
$$

$$
u_{c',t} = \beta' G_C E_t (u_{c',t+1} R_t / \pi_{t+1}) + \lambda_{t+1} \quad \text{(B15)}
$$

$$
\begin{aligned}
u_{c',t} w'_{c,t} &= u_{nc',t} X'_{wc,t} && \text{(B16)}\\
u_{c',t} w'_{h,t} &= u_{nh',t} X'_{wh,t} && \text{(B17)}
\end{aligned}
$$

The threshold value $\bar{\omega}_t$ is determined by:

$$
\bar{\omega}_{t+1}(1 - \delta_h) q_{t+1} h'_t \pi_{t-1} = R_{z,t-1}\, b_t. \quad \text{(B18)}
$$

The lenders' participation constraint is:

$$R_t b_t = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}(1-\mu)(1-\delta)q_{t+1}h'_t\pi_{t+1}f(\omega)d\omega + \int_{\bar{\omega}_{t+1}}^{\infty} R_{z,t}b_t f(\omega)d\omega \tag{B19}$$

Production technologies are:

$$Y_t = \left[A_{ct}\left(n_{ct}^{\alpha}n_{ct}^{\prime 1-\alpha}\right)\right]^{1-\mu_c}(z_{ct}k_{ct-1})^{\mu_c} \tag{B20}$$

$$IH_t = \left[A_{ht}\left(n_{ht}^{\alpha}n_{ht}^{\prime 1-\alpha}\right)\right]^{1-\mu_h-\mu_b-\mu_l}(z_{ht}k_{ht-1})^{\mu_h}k_{bt}^{\mu_b}l_{t-1}^{\mu_l} \tag{B21}$$

The first order conditions for wholesale goods firms are:

$$(1-\mu_c)\alpha Y_t = X_t w_{c,t}n_{c,t} \tag{B22}$$

$$(1-\mu_c)(1-\alpha)Y_t = X_t w'_{c,t}n'_{c,t} \tag{B23}$$

$$(1-\mu_h-\mu_l-\mu_b)\alpha q_t IH_t = w_{h,t}n_{h,t} \tag{B24}$$

$$(1-\mu_h-\mu_l-\mu_b)(1-\alpha)q_t IH_t = w'_{h,t}n'_{h,t} \tag{B25}$$

$$\mu_c Y_t = X_t R_{c,t}z_{c,t}k_{c,t-1} \tag{B26}$$

$$\mu_h q_t IH_t = R_{h,t}z_{h,t}k_{h,t-1} \tag{B27}$$

$$\mu_l q_t IH_t = R_{l,t}l_{t-1} \tag{B28}$$

$$\mu_b q_t IH_t = p_{b,t}k_{b,t} \tag{B29}$$

Phillips curve is:

$$\ln\pi_t - \iota_\pi \ln\pi_{t-1} = \beta G_c\left(E_t\ln\pi_{t+1} - \iota_\pi\ln\pi_t\right) - \varepsilon_\pi \ln\left(\frac{X_t}{X}\right) + u_{p,t} \tag{B30}$$

Nominal wage inflation is $\omega_{i,t} = (w_{i,t}\pi_t)/(w_{i,t-1})$ for each sector/household. Wage equations are as follows:

$$\ln\omega_{c,t} - \iota_{wc}\ln\pi_{t-1} = \beta G_C(E_t \ln\omega_{c,t+1} - \iota_{wc}\ln\pi_t) - \varepsilon_{wc}\ln(X_{wc,t}/X_{wc}) \tag{B31}$$

$$\ln\omega'_{c,t} - \iota_{wc}\ln\pi_{t-1} = \beta' G_C(E_t \ln\omega'_{c,t+1} - \iota_{wc}\ln\pi_t) - \varepsilon'_{wc}\ln(X_{wc,t}/X_{wc}) \tag{B32}$$

$$\ln\omega_{h,t} - \iota_{wh}\ln\pi_{t-1} = \beta G_C(E_t \ln\omega_{h,t+1} - \iota_{wh}\ln\pi_t) - \varepsilon_{wh}\ln(X_{wh,t}/X_{wh}) \tag{B33}$$

$$\ln\omega'_{h,t} - \iota_{wh}\ln\pi_{t-1} = \beta' G_C(E_t \ln\omega'_{h,t+1} - \iota_{wh}\ln\pi_t) - \varepsilon'_{wh}\ln(X_{wh,t}/X_{wh}) \tag{B34}$$

where $\varepsilon_{wc} = (1-\theta_{wc})(1-\beta G_C\theta_{wc})/\theta_{wc}$, $\varepsilon'_{wc} = (1-\theta_{wc})(1-\beta' G_C\theta_{wc})/\theta_{wc}$, $\varepsilon_{wh} = (1-\theta_{wh})(1-\beta G_C\theta_{wh})/\theta_{wh}$, and $\varepsilon'_{wh} = (1-\theta_{wh})(1-\beta' G_C\theta_{wh})/\theta_{wh}$.

The Taylor rule is:

$$R_t = (R_{t-1})^{r_R}\left[\pi_t^{r_\pi}\left(\frac{GDP_t}{G_c GDP_{t-1}}\right)^{r_Y}\bar{rr}\right]^{1-r_R}\frac{u_{Rt}}{s_t} \tag{B35}$$

Two market clearing conditions are:

$$C_t + \frac{IK_{c,t}}{A_{k,t}} + IK_{h,t} + k_{b,t} = Y_t - \phi_t \tag{B36}$$

$$h_t + h'_t - (1-\delta_h)\left[h_{t-1} + h'_{t-1}(1-\mu G_t(\bar{\omega}_t))\right] = IH_t \tag{B37}$$

Total land is normalized to unity $l_t = 1$. Dividends paid to households are:

$$Div_t = \frac{X_t - 1}{X_t}Y_t + \frac{X_{wc,t}-1}{X_{wc,t}}w_{c,t}n_{c,t} + \frac{X_{wh,t}-1}{X_{wh,t}}w_{h,t}n_{h,t} \tag{B38}$$

$$Div'_t = \frac{X'_{wc,t}-1}{X'_{wc,t}}w'_{c,t}n'_{c,t} + \frac{X'_{wh,t}-1}{X'_{wh,t}}w'_{h,t}n'_{h,t} \tag{B39}$$

Finally, the functional forms for the capital adjustment cost and the utilization rate are:

$$\phi_t = \frac{\phi_{kc}}{2G_{IK_c}}\left(\frac{k_{c,t}}{k_{c,t-1}}G_{IK_c}\right)^2\frac{k_{c,t-1}}{(1-\gamma_{AK})^t} + \frac{\phi_{kh}}{2G_{IK_h}}\left(\frac{k_{h,t}}{k_{h,t-1}}G_{IK_h}\right)^2 k_{h,t-1} \tag{B40}$$

$$a(z_{c,t}) = R_c(\varpi z_{c,t}^2/2 + (1-\varpi)z_{c,t} + (\varpi/2 - 1)) \tag{B41}$$

$$a(z_{h,t}) = R_h(\varpi z_{h,t}^2/2 + (1-\varpi)z_{h,t} + (\varpi/2 - 1)). \tag{B42}$$

The shocks are the following:

$$\ln z_t = \rho_z \ln z_{t-1} + u_{z,t} \tag{B43}$$

$$\ln \tau_t = \rho_\tau \ln \tau_{t-1} + u_{\tau,t} \tag{B44}$$

$$\ln j_t = (1-\rho_j)\ln j + \rho_j \ln j_{t-1} + u_{j,t} \tag{B45}$$

$$\ln s_t = \rho_s \ln s_{t-1} + u_{s,t} \tag{B46}$$

$$\ln \frac{\sigma_{\omega,t}}{\bar{\sigma}_\omega} = \rho_{\sigma_\omega} \ln \frac{\sigma_{\omega,t-1}}{\bar{\sigma}_\omega} + u_{\sigma_\omega,t}, \tag{B47}$$

where $u_{z,t}$, $u_{\tau,t}$, $u_{j,t}$, $u_{\sigma_\omega,t}$ and $u_{R,t}$ from Equation (16), are independently and identically distributed, with variances σ_z^2, σ_τ^2, σ_j^2, σ_ω^2 and σ_R^2, respectively. And, $u_{s,t}$ is normally distributed with mean zero and variance σ_s^2.

C Appendix: Lenders' Participation Constraint and Borrowers' Budget Constraint

We start with Equation 6, the participation constraint of the lenders:

$$R_t b_t = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}(1-\mu)(1-\delta_h) q_{t+1} h'_t \pi_{t+1} f(\omega) d\omega + \int_{\bar{\omega}_{t+1}}^{\infty} R_{z,t} b_t f(\omega) d\omega,$$

and we use the definitions of $G_{t+1}(\bar{\omega}_{t+1})$ and $\Gamma_{t+1}(\bar{\omega}_{t+1})$, Equations 7 and 8, respectively. Rewriting the participation constraint yields

$$\begin{aligned}
R_t b_t &= (1-\mu)(1-\delta_h)q_{t+1}h'_t\pi_{t+1}\int_0^{\bar{\omega}_{t+1}}\omega_{t+1}f(\omega)d\omega + R_{z,t}b_t\int_{\bar{\omega}_{t+1}}^{\infty}f(\omega)d\omega \\
&= (1-\mu)(1-\delta_h)q_{t+1}h'_t\pi_{t+1}\int_0^{\bar{\omega}_{t+1}}\omega_{t+1}f(\omega)d\omega + \bar{\omega}_{t+1}(1-\delta_h)q_{t+1}h'_t\pi_{t+1}\int_{\bar{\omega}_{t+1}}^{\infty}f(\omega)d\omega \\
&= (1-\mu)(1-\delta_h)q_{t+1}h'_t\pi_{t+1}\int_0^{\bar{\omega}_{t+1}}\omega_{t+1}f(\omega)d\omega \\
&\quad +(1-\delta_h)q_{t+1}h'_t\pi_{t+1}\left\{\bar{\omega}_{t+1}\int_{\bar{\omega}_{t+1}}^{\infty}f(\omega)d\omega + G_{t+1}(\bar{\omega}_{t+1}) - G_{t+1}(\bar{\omega}_{t+1})\right\} \\
&= (1-\mu)(1-\delta_h)q_{t+1}h'_t\pi_{t+1}G_{t+1}(\bar{\omega}_{t+1}) + (1-\delta_h)q_{t+1}h'_t\pi_{t+1}\left[\Gamma_{t+1}(\bar{\omega}_{t+1} - G_{t+1}(\bar{\omega}_{t+1})\right] \\
&= (1-\delta_h)q_{t+1}h'_t\pi_{t+1}\left[(1-\mu)G_{t+1}(\bar{\omega}_{t+1}) + \Gamma_{t+1}(\bar{\omega}_{t+1} - G_{t+1}(\bar{\omega}_{t+1})\right] \\
R_t b_t &= (1-\delta_h)q_{t+1}h'_t\pi_{t+1}\left[\Gamma_{t+1}(\bar{\omega}_{t+1} - \mu G_{t+1}(\bar{\omega}_{t+1})\right]
\end{aligned}$$

where from the first to the second step we use the definition of $\bar{\omega}_{t+1}$, Equation 5.

For the budget constraint of the borrowers, we can apply a similar transformation to eliminate $R_{z,t}$. We get:

$$c'_t + q_t h'_t + R_t b_{t-1} = b_t + \sum_{i=c,h}\frac{w'_{i,t}n'_{i,t}}{X'_{wi,t}} + (1-\delta_h)[1-\mu G_t(\bar{\omega}_t)]q_t h'_{t-1} + Div'_t.$$

D Appendix: Additional Graphs

Figure 9: Data, 1981Q1-2008Q4

Note: The red vertical line corresponds to 2006Q4, the break for the two samples that we use. Consumption and investment are divided by population and log-transformed. Consumption, investment, and house prices are normalized to zero in the first period. Inflation, nominal interest rate, hours, wage inflation and loan-to-value ratio are demeaned.

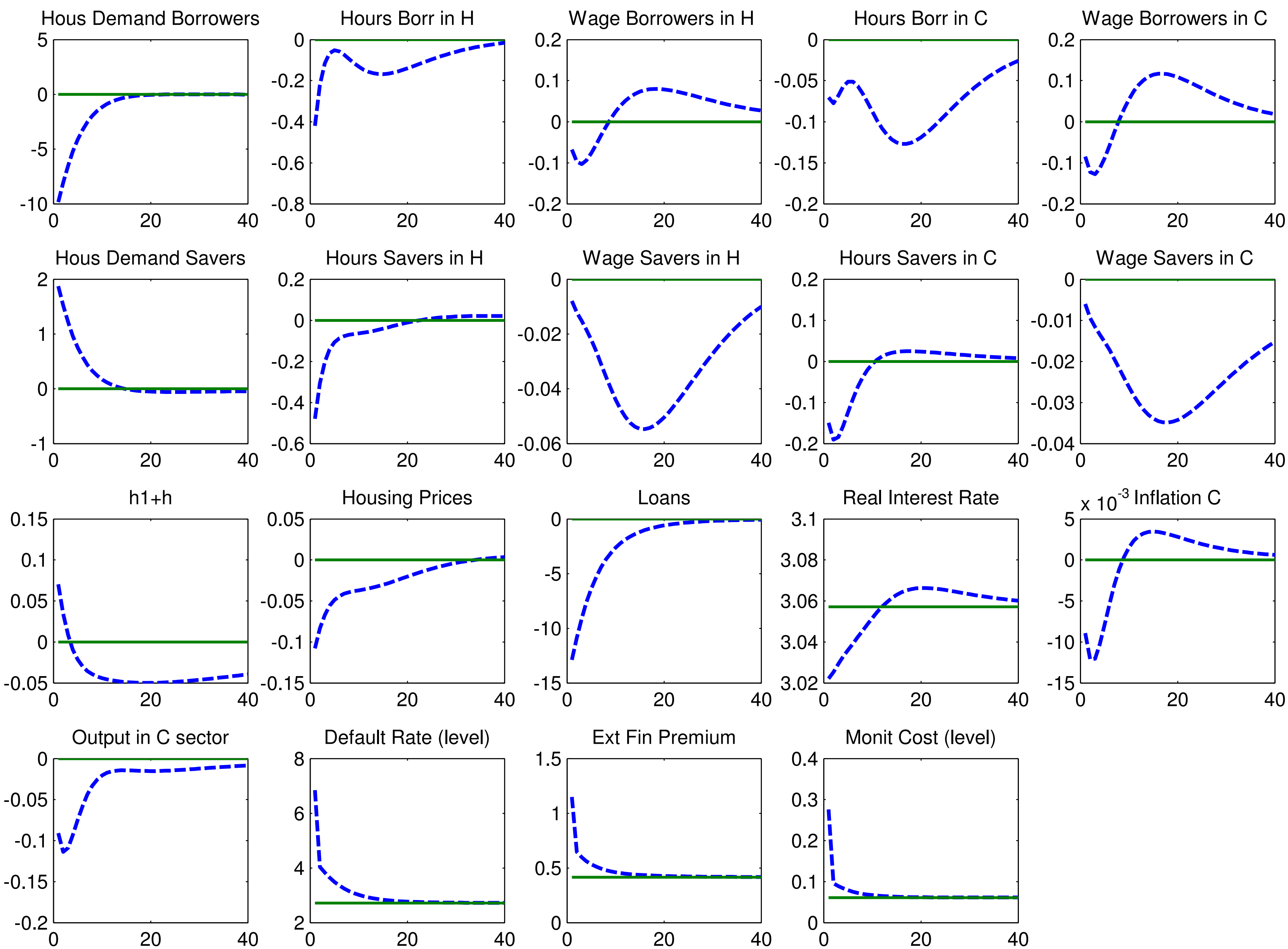

Figure 10: Impulse Responses to Mortgage Risk Shock, Large Set of Variables

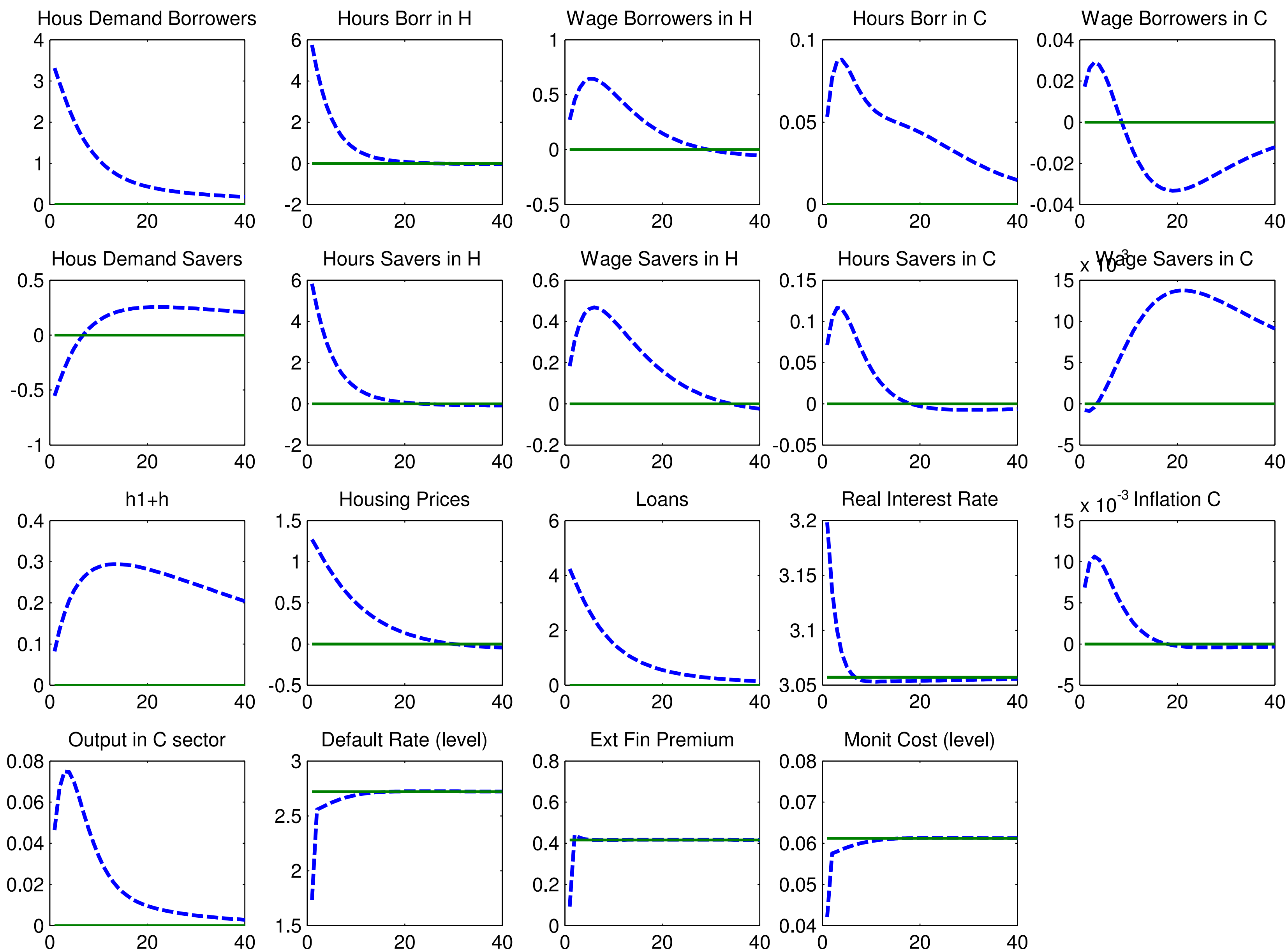

Figure 11: Impulse Responses to Housing Preference Shock, Large Set of Variables

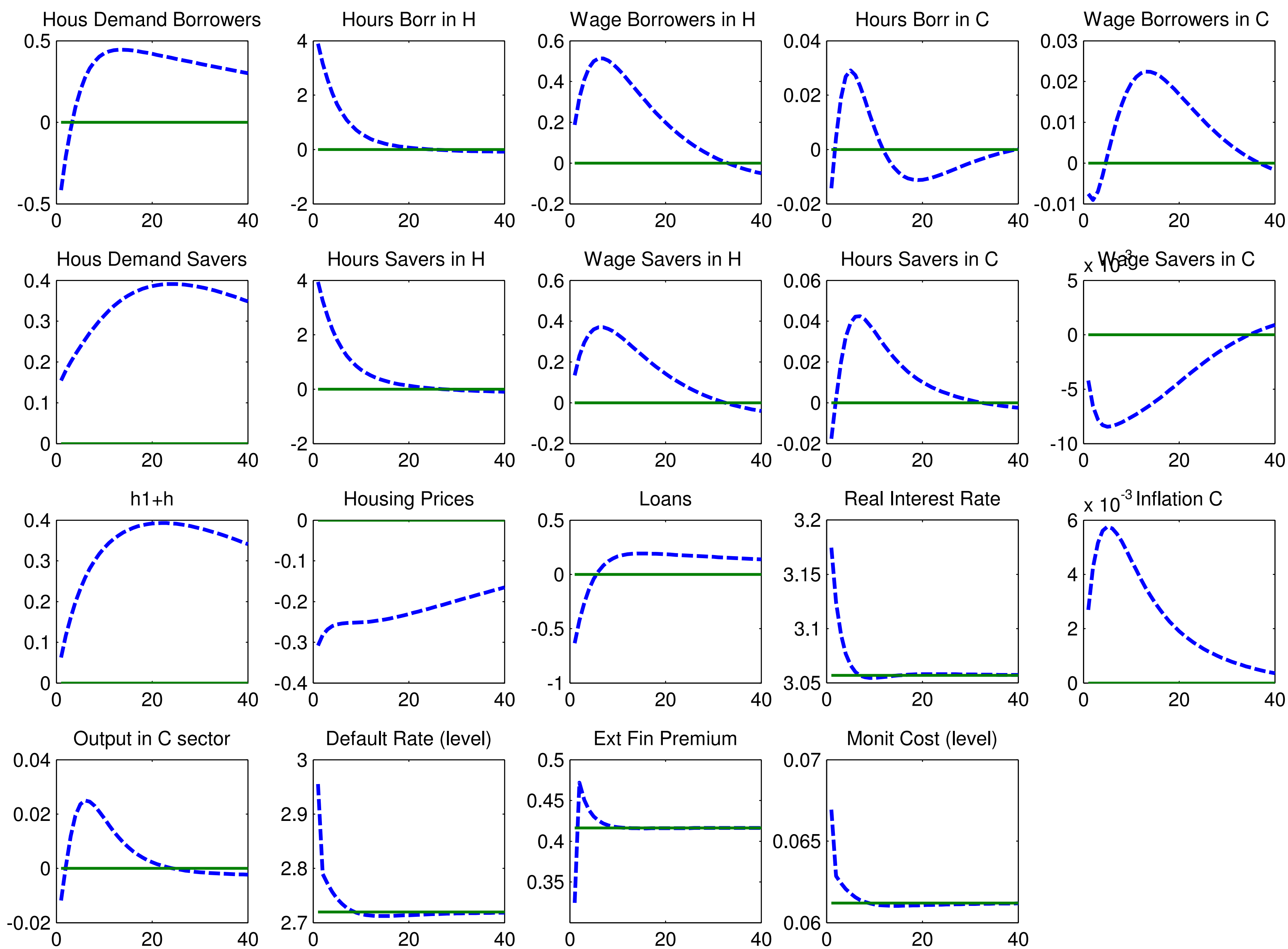

Figure 12: Impulse Responses to Housing Technology Shock, Large Set of Variables

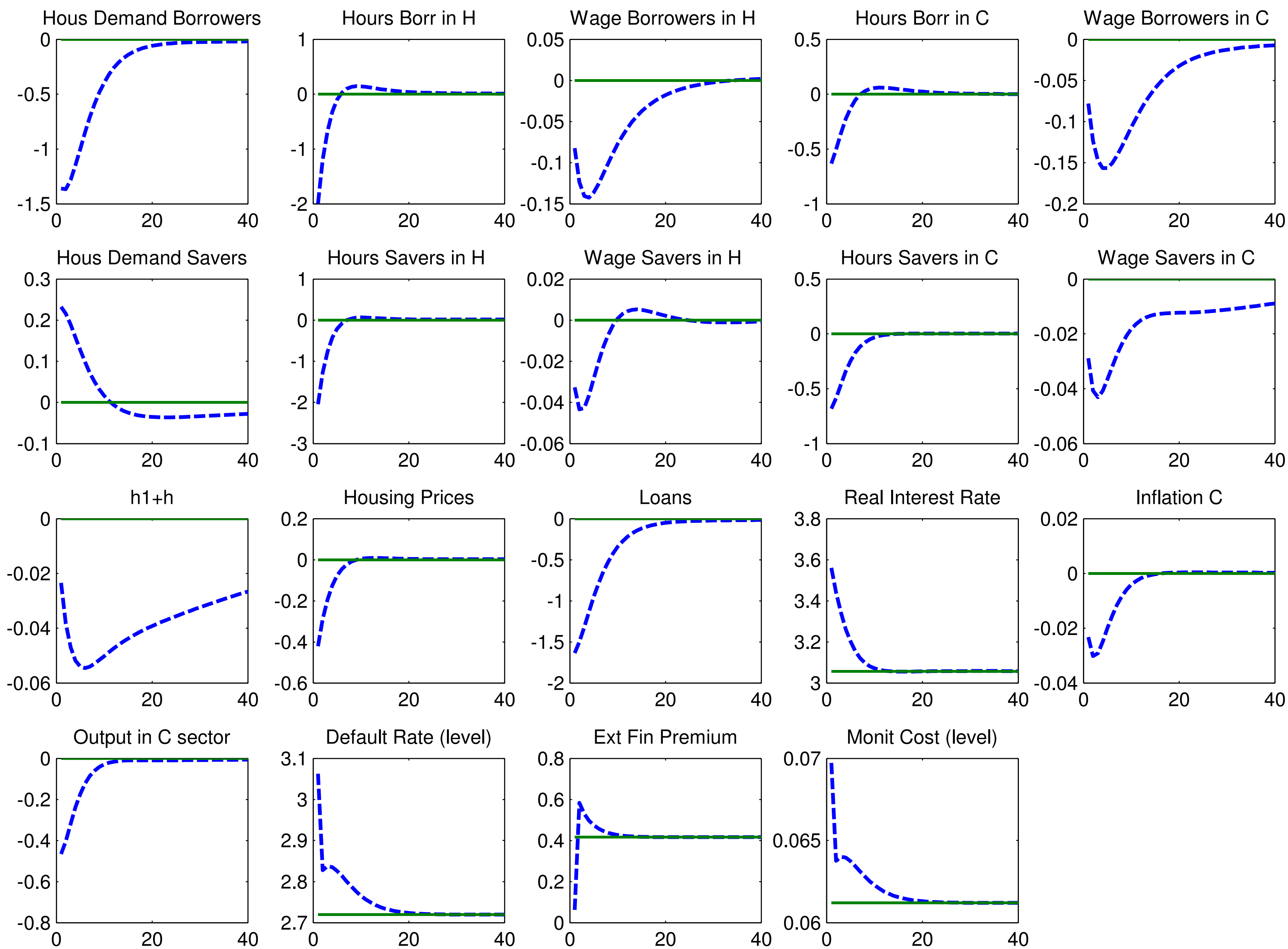

Figure 13: Impulse Responses to Monetary Policy Shock, Large Set of Variables

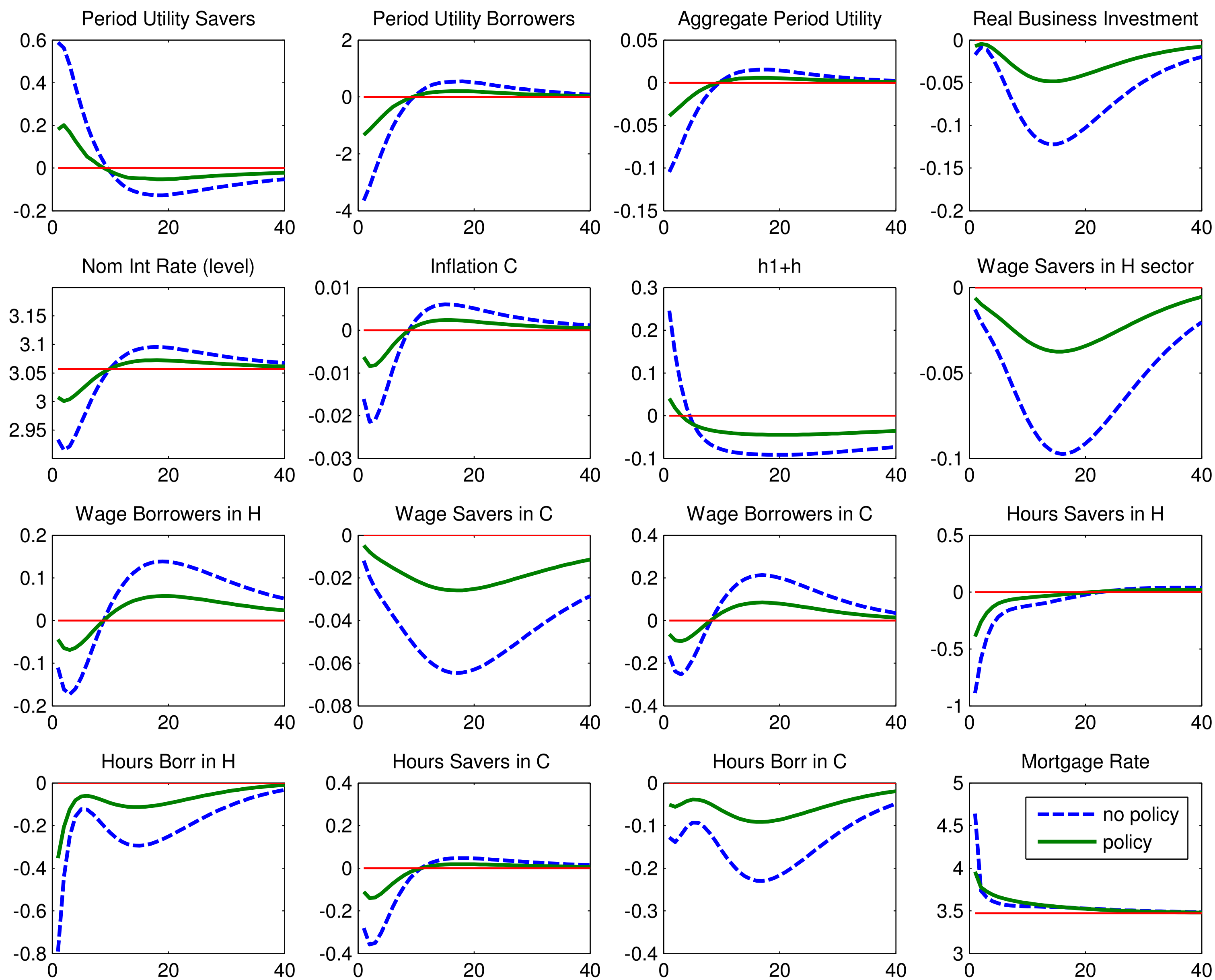

Figure 14: Impulse Responses to Mortgage Risk Shock with Policy Intervention

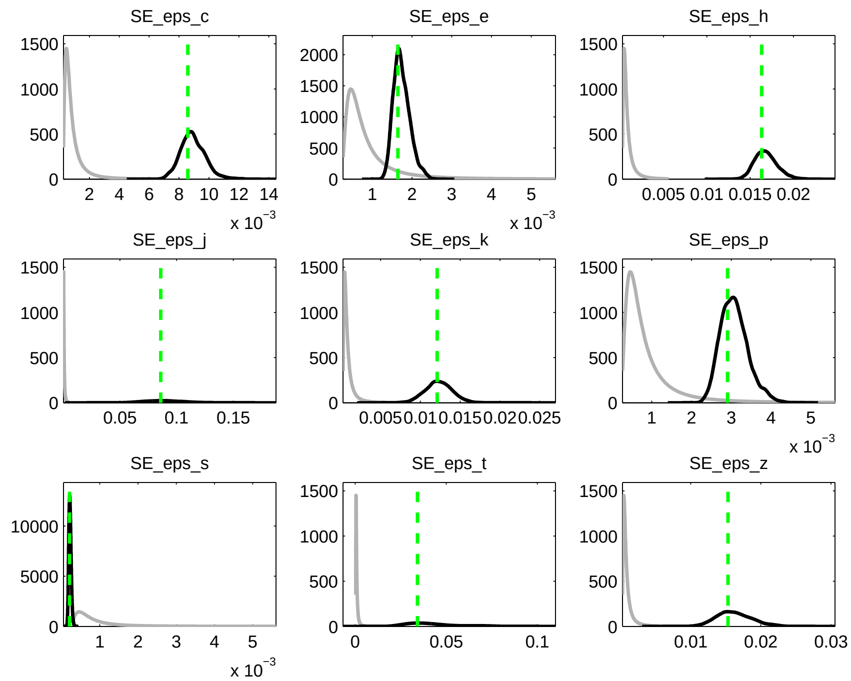

Figure 15: Prior and Posterior Distributions of Estimated Parameters
Note: The gray line is the prior distribution of the parameter, while the black line is the posterior distribution. The dashed green line corresponds to the mode of the posterior distribution.

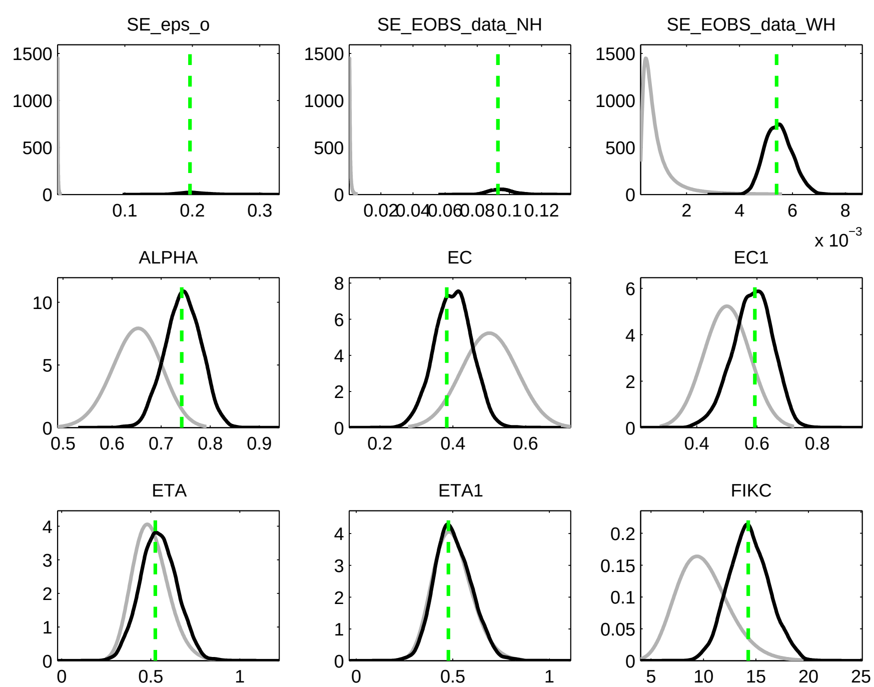

Figure 16: Prior and Posterior Distributions of Estimated Parameters (cont. 1)
Note: The gray line is the prior distribution of the parameter, while the black line is the posterior distribution. The dashed green line corresponds to the mode of the posterior distribution.

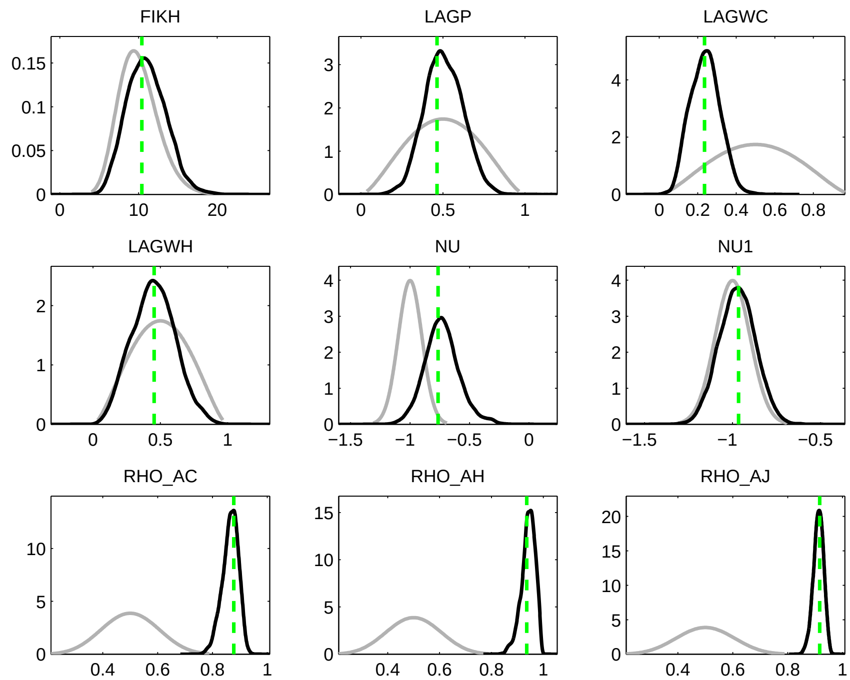

Figure 17: Prior and Posterior Distributions of Estimated Parameters (cont. 2)
Note: The gray line is the prior distribution of the parameter, while the black line is the posterior distribution. The dashed green line corresponds to the mode of the posterior distribution.

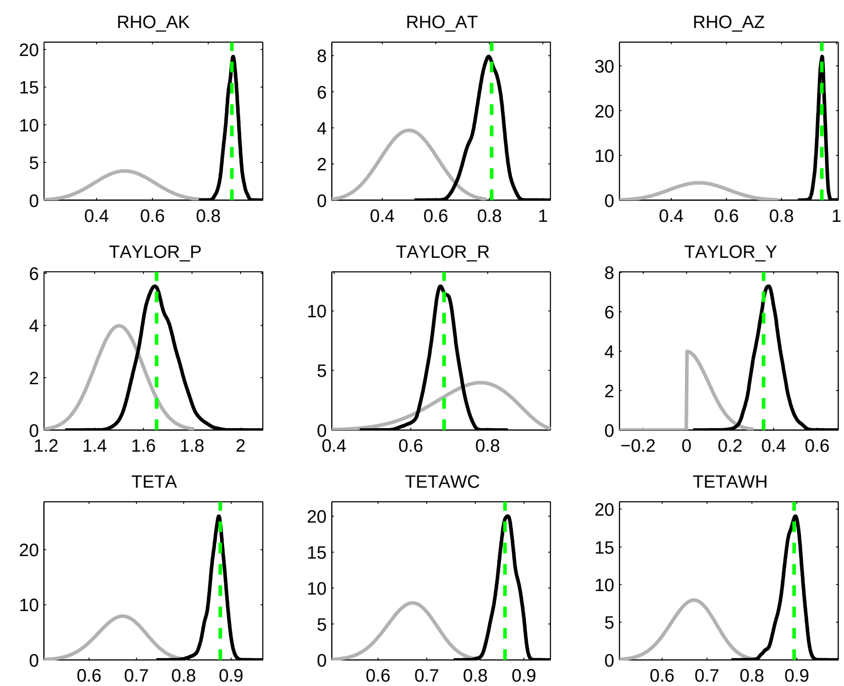

Figure 18: Prior and Posterior Distributions of Estimated Parameters (cont. 3)
Note: The gray line is the prior distribution of the parameter, while the black line is the posterior distribution. The dashed green line corresponds to the mode of the posterior distribution.

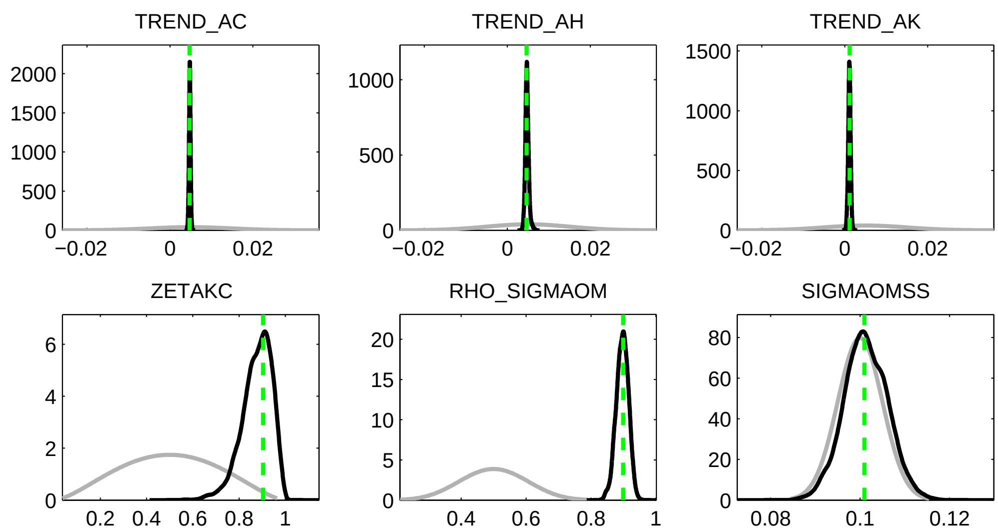

Figure 19: Prior and Posterior Distributions of Estimated Parameters (cont. 4)
Note: The gray line is the prior distribution of the parameter, while the black line is the posterior distribution. The dashed green line corresponds to the mode of the posterior distribution.