# Bond Risk Premia in Consumption-based Models

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# Motivation

## Chairman Janet Yellen: large-scale asset purchases December, 2014, Press Conference

> *...we're reminding the public that we continue to hold a large stock of assets, and that is tending to push down term premiums in longer-term yields.*

## Chairman Ben Bernanke: decomposition March, 2006, New York

> *To the extent that the decline in forward rates can be traced to a decline in the term premium... the effect is financially stimulative and argues for greater monetary policy restraint... However, if the behavior of long-term yields reflects current or prospective economic conditions, the implications for policy may be quite different-indeed, quite the opposite.*

## Chairman Alan Greenspan: conundrum June, 2005, Beijing

> *That improved performance has doubtless contributed to lower inflation-related risk premiums, and the lowering of these premiums is reflected in significant declines in nominal and real long-term rates. Although this explanation contributes to an understanding of the past decade, I do not believe it explains the decline of long-term interest rates over the past year despite rising short-term rates.*

# Term premium: two models & two channels

- Gaussian ATSM:
  - benchmark model
  - time-varying term premia via price of risk
  - lack micro foundation
- structural models with recursive preferences
  - Gaussian: constant term premia
  - SV: time-varying term premia via SV
  - economic structure imposes restrictions

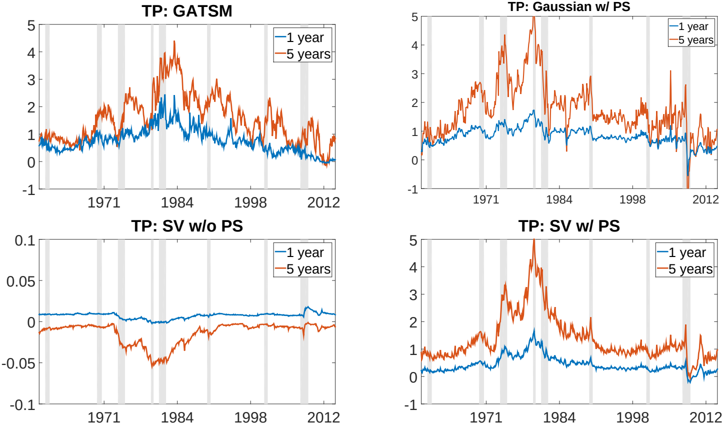
**Goal of this paper:** reconcile the two literatures

# Contributions

- Introduce a new structural model with recursive preferences
  - preference shocks $\rightarrow$ time-varying prices of risk
- Bridge two literatures on bonds
  - GATSM and consumption based models.
- Representative agent's problem must be well-posed
  - provide conditions guaranteeing a model solution
    - representative agent cannot be too patient
    - risk aversion cannot be too large
  - the geometry of the feasible regions makes estimation challenging

Literature

# Results: term premia



# Outline

1. Model
2. Estimation
3. Results
4. Model solution

# Gaussian Model

Agent's problem

$$V_t = \max \left[ (1-\beta)\, \Upsilon_t C_t^{1-\eta} + \beta \left\{ \mathrm{E}_t \left[ V_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\eta}{1-\gamma}} \right]^{\frac{1}{1-\eta}}$$

$$\text{s.t.} \quad W_{t+1} = (W_t - C_t)\, R_{c,t+1}$$

Log stochastic discount factor

$$m_{t+1} = \vartheta \ln(\beta) + \vartheta \Delta \upsilon_{t+1} - \eta\vartheta \Delta c_{t+1} + (\vartheta - 1)\, r_{c,t+1},$$

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}$$

where $\vartheta = \frac{1-\gamma}{1-\eta}$

State variables

$$\Delta c_t = Z_c' g_t, \qquad \pi_t = Z_\pi' g_t,$$

$$g_{t+1} = \mu_g + \Phi_g g_t + \Sigma_{0,g} \varepsilon_{g,t+1} \qquad \varepsilon_{g,t+1}, \sim \mathrm{N}(0, I).$$

- It's a companion form, nesting long-run risk & VARMA.

# Preference shocks

$$\Delta \upsilon_{t+1} = Z_{\upsilon}' g_{t+1} + \Lambda_1 (g_t) + \Lambda_2 (g_t)' \varepsilon_{g,t+1}$$
$$\Lambda_2 (g_t) = -\eta \Sigma_{0,g}^{-1} (\lambda_0 + \lambda_g g_t)$$

- $Z_{\upsilon}' g_{t+1}$: Albuquerque, Eichenbaum, & Rebelo (2014)
  - no time-varying price of risk.
- $\lambda_g \neq 0 \Rightarrow$ **time-varying price of risk/ term premium**

# Sources of risk premia

$$m^{\$}_{t+1} - \mathrm{E}_t\left[m^{\$}_{t+1}\right] = -\lambda^{\$,\prime}_{g,t}\varepsilon_{g,t+1}$$

where the price of risk $\lambda^{\$}_{g,t}$ is

$$\Sigma_{0,g}\lambda^{\$}_{g,t} = \overbrace{\Sigma_{0,g}\Sigma'_{0,g}\left(\gamma Z_c + Z_\pi\right)}^{\text{power utility}} - \overbrace{\kappa_1\frac{(\eta-\gamma)}{(1-\eta)}\Sigma_{0,g}\Sigma'_{0,g}D_g}^{\text{recursive preferences}}$$
$$\overbrace{-\vartheta\Sigma_{0,g}\Sigma'_{0,g}Z_\upsilon + \eta\vartheta\lambda_0 + \eta\vartheta\lambda_g g_t}^{\text{preference shocks}}$$

- the price of risk only varies through time if $\lambda_g \neq 0$.
- Campbell & Cochrane (1999): Risk sensitivity function $\lambda(s_t)$ creates time-varying price of risk

$$m_{t+1} - \mathrm{E}_t\left[m_{t+1}\right] = -\gamma\sigma_c\left(1 + \lambda(s_t)\right)\varepsilon_{c,t+1}$$

- Our model
  - $\Lambda_2(g_t)$ is (potentially) a function of any element of the state vector $g_t$.
  - Bond prices are known in closed-form.

# Nominal risk neutral measure $\mathbb{Q}^{\$}$

$$g_{t+1} = \mu_g^{\mathbb{Q},\$} + \Phi_g^{\mathbb{Q},\$} g_t + \Sigma_{0,g} \varepsilon_{g,t+1}^{\mathbb{Q},\$}, \qquad \varepsilon_{g,t+1}^{\mathbb{Q},\$} \sim \mathrm{N}(0, I).$$

where

$$\Sigma_{0,g} \lambda_{g,t}^{\$} = \left(\mu_g - \mu_g^{\mathbb{Q},\$}\right) + \left(\Phi_g - \Phi_g^{\mathbb{Q},\$}\right) g_t$$

Therefore,

$$\Phi_g^{\mathbb{Q},\$} = \Phi_g - \overbrace{\eta \vartheta \lambda_g}^{\textbf{preference shocks}}$$

- Same form as a Gaussian ATSM.
- the price of risk only varies through time if

$$\lambda_g \neq 0 \Rightarrow \Phi_g \neq \Phi_g^{\mathbb{Q},\$}$$

- Duffee (2002): important feature of the data

# Bond prices

Bond prices

$$P_t^{\$,(n)} = \mathrm{E}_t\left[\exp\left(m_{t+1}^{\$}\right) P_{t+1}^{\$,(n-1)}\right]$$

yields

$$y_t^{\$,(n)} \equiv -\frac{1}{n}\ln\left(P_t^{\$,(n)}\right) = a_n^{\$} + b_{n,g}^{\$,\prime} g_t$$

- Bond loadings are similar to Gaussian ATSMs.
- They are functions of $(\beta, \gamma, \psi)$, where $\psi = 1/\eta$

▸ Loadings

Consumption-inflation representation

$$y_t^{\$,(n)} = -\ln(\beta) + \frac{\eta}{n}\sum_{j=0}^{n-1}\mathrm{E}_t^{\mathbb{Q},\$}\Delta c_{t+j+1} + \frac{1}{n}\sum_{j=0}^{n-1}\mathrm{E}_t^{\mathbb{Q},\$}\pi_{t+j+1} - \frac{1}{n}\sum_{j=0}^{n-1}\mathrm{E}_t^{\mathbb{Q},\$}\Delta \upsilon_{t+j+1} + J.I.$$

# SV model: dynamics

Stochastic volatility process from Creal & Wu (2015 JEconometrics)

$$
\begin{aligned}
g_{t+1} &= \mu_g + \Phi_g g_t + \Phi_{gh} h_t + \Sigma_{gh}\varepsilon_{h,t+1} + \Sigma_{g,t}\varepsilon_{g,t+1} \qquad \varepsilon_{g,t+1} \sim \mathrm{N}(0, I) \\
\Sigma_{g,t}\Sigma'_{g,t} &= \Sigma_{0,g}\Sigma'_{0,g} + \sum_{i=1}^{H} \Sigma_{i,g}\Sigma'_{i,g} h_{it} \\
h_{t+1} &\sim \mathrm{NCG}(\nu_h, \Phi_h, \Sigma_h) \\
\varepsilon_{h,t+1} &= h_{t+1} - \mathrm{E}_t[h_{t+1}|h_t]
\end{aligned}
$$

Long run risk

$$
\begin{aligned}
\pi_{t+1} &= \bar{\pi}_t + \varepsilon_{\pi_1,t+1} & \varepsilon_{\pi_1,t+1} &\sim \mathrm{N}(0, h_{t,\pi_1}) \\
\Delta c_{t+1} &= \bar{c}_t + \varepsilon_{c_1,t+1} & \varepsilon_{c_1,t+1} &\sim \mathrm{N}(0, h_{t,c_1}) \\
\bar{\pi}_{t+1} &= \mu_\pi + \phi_\pi \bar{\pi}_t + \phi_{\pi,c}\bar{c}_t + \varepsilon_{\pi_2,t+1} & \varepsilon_{\pi_2,t+1} &\sim \mathrm{N}(0, h_{t,\pi_2}) \\
\bar{c}_{t+1} &= \mu_c + \phi_{c,\pi}\bar{\pi}_t + \phi_c \bar{c}_t + \sigma_{c,\pi}\varepsilon_{\pi_2,t+1} + \varepsilon_{c_2,t+1} & \varepsilon_{c_2,t+1} &\sim \mathrm{N}(0, h_{t,c_2})
\end{aligned}
$$

# SV model: preference shock

Preference shock

$$\begin{aligned} \Delta \upsilon_{t+1} &= Z_\upsilon' g_{t+1} + \Lambda_1(g_t, h_t) + \Lambda_2(g_t, h_t)' \varepsilon_{g,t+1} + \Lambda_3(h_t)' \varepsilon_{h,t+1} \\ \Lambda_2(g_t, h_t) &= -\eta \Sigma_{g,t}^{-1}\left(\lambda_0 + \lambda_g g_t + \lambda_{gh} h_t\right), \qquad \Lambda_3(h_t) = -\lambda_h. \end{aligned}$$

pricing kernel

$$m_{t+1}^{\$} - \mathrm{E}_t\left[m_{t+1}^{\$}\right] = -\lambda_{g,t}^{\$,\prime} \varepsilon_{g,t+1} - \lambda_{h,t}^{\$,\prime} \tilde{\varepsilon}_{h,t+1}$$

risk premium

$$\begin{aligned} \Sigma_{g,t}\lambda_{g,t}^{\$} = \; & \Sigma_{0,g}\Sigma_{0,g}'\left(\gamma Z_c + Z_\pi\right) - \kappa_1 \frac{(\eta-\gamma)}{(1-\eta)} \Sigma_{0,g}\Sigma_{0,g}' D_g - \vartheta \Sigma_{0,g}\Sigma_{0,g}' Z_\upsilon + \eta\vartheta\lambda_0 + \eta\vartheta\lambda_g g_t \\ & + \overbrace{([\gamma Z_c + Z_\pi] \otimes I_G)' \tilde{S}_g h_t}^{\text{power utility}} - \overbrace{\kappa_1 \frac{(\eta-\gamma)}{(1-\eta)} (D_g \otimes I_G)' \tilde{S}_g h_t}^{\text{recursive preferences}} \\ & \overbrace{-\vartheta (Z_\upsilon \otimes I_G)' \tilde{S}_g h_t + \eta\vartheta\lambda_{gh} h_t}^{\text{preference shocks}} + \text{ leverage effects} \end{aligned}$$

$$\Sigma_{h,t}^{\prime,-1}\lambda_{h,t}^{\$} = \overbrace{\Sigma_{gh}'\left(\gamma Z_c + Z_\pi\right)}^{\text{power utility}} - \overbrace{\kappa_1 \frac{(\eta-\gamma)}{(1-\eta)} \left(\Sigma_{gh}' D_g + D_h\right)}^{\text{recursive preferences}} - \overbrace{\vartheta\left(\Sigma_{gh}' Z_\upsilon - \lambda_h\right)}^{\text{preference shocks}}$$

# Data

Monthly data from Feb. 1959 to June 2014

Yields

- Fama-Bliss zero-coupon yields from CRSP
- maturities: 3m, 1y, 2y, 3y, 4y, 5y

Inflation + Population

- FRED database at St. Louis FRB
- CPI inflation
- Civilian population over 16

Consumption

- U.S. Bureau of Economic Analysis
- non-durables + services

# Estimation approach

Step 1: estimate the time series dynamics $(\theta^{\mathbb{P}}, g_t, h_t)$

- use MCMC + particle filters $\rightarrow$ Particle Gibbs sampler. Details

Step 2: estimate $\theta^u = (\beta, \gamma, \psi)$ and $\theta^\lambda$

- Run a cross-sectional regression on filtered estimates $\hat{g}_t$ and $\hat{h}_t$

$$y_t^\$ = A^\$ \left(\theta^u, \theta^\lambda\right) + B_g^\$ \left(\theta^u, \theta^\lambda\right) \hat{g}_t + B_h^\$ \left(\theta^u, \theta^\lambda\right) \hat{h}_t + \eta_t \qquad \eta_t \sim \mathrm{N}\left(0, \Omega\right)$$

to estimate $\hat{\zeta}^r = \left(\hat{A}^{\$,r,\prime} \ \ \mathrm{vec}\left(\hat{B}_g^{\$,r}\right)' \ \ \mathrm{vec}\left(\hat{B}_h^{\$,r}\right)'\right)'$

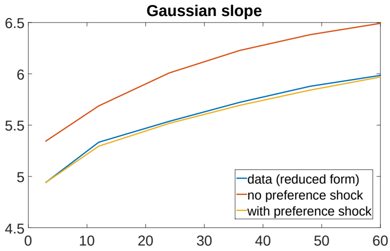
- We use minimum-$\chi^2$ estimation of Hamilton & Wu (2012 JEconometrics)

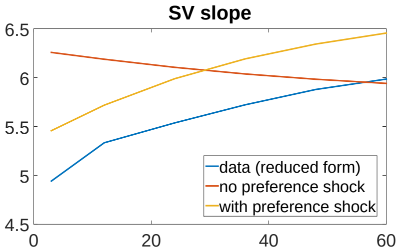
$$\underset{\theta^u, \theta^\lambda}{\mathrm{argmin}} \ T\left(\hat{\zeta}^r - \zeta\left(\theta^u, \theta^\lambda\right)\right)' W \left(\hat{\zeta}^r - \zeta\left(\theta^u, \theta^\lambda\right)\right)$$

# Structural parameters

| | Gaussian | | Gaussian+PS | | SV | | SV+PS | |
|---|---|---|---|---|---|---|---|---|
| $\beta$ | 1.00 | | 1.00 | | 1.00 | | 1.00 | |
| $\psi$ | 0.59 | | 1.09 | | 0.61 | | 0.97 | |
| $\gamma$ | 84.4 | | 416.6 | | 1e-5 | | 1.70 | |
| $\bar{pc}$ | 7.78 | | 5.54 | | 7.06 | | 8.16 | |
| $\Phi_g$ | 0.95 | 0.03 | 0.95 | 0.03 | 0.96 | 0.05 | 0.96 | 0.05 |
| | -0.02 | 0.87 | -0.02 | 0.87 | 0.00 | 0.91 | 0.00 | 0.91 |
| $\Phi_g^Q$ | | | 0.97 | 0.93 | | | 0.98 | -0.02 |
| | | | 0.02 | 0.36 | | | -0.00 | 0.99 |
| MCS | 1330 | | 477 | | 3794 | | 1027 | |

# Unconditional yield curves

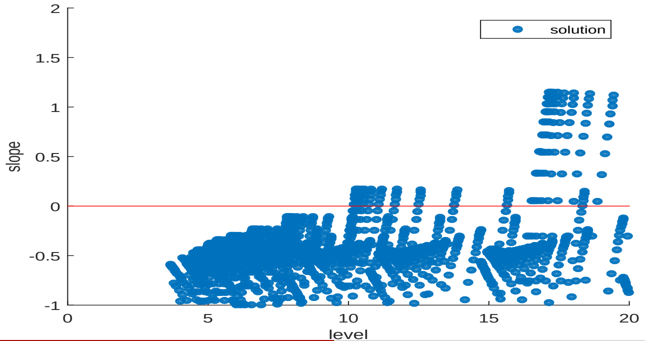


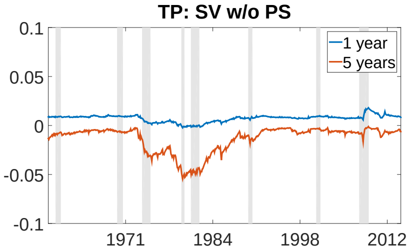
# Why SV is doing worse?

- SV seems to be more flexible with $h_t$
- But there are more moments to match
- There are only 3 free parameters to match all
- It's difficult to match both the average level, and slope

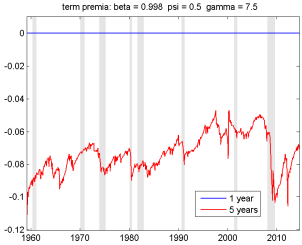
$\psi = [0, 20]; \beta = [0.6, 1]; \gamma = [0.001, 1100]$

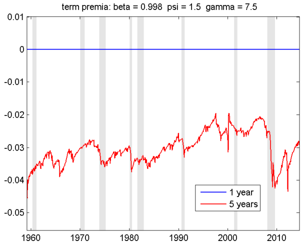


# Term premia: SV without preference shock




parameters from Bansal & Yaron (2004)



# Comparison with the literature

Tension between fitting macroeconomic variables and yields

- Macro variables: $\mathbb{P}$ parameters
- Cross section of yields: $\mathbb{Q}$ parameters
- Term premia: the difference between $\mathbb{P}$ and $\mathbb{Q}$

In consumption-based models with recursive preferences, $\Phi_g = \Phi_g^{\mathbb{Q}}$

- If we force the model to fit macro variables (*ours*), then
  - $\Phi_g$ determines the slope is downward
- If we force the model to fit the upward slope (*literature*), then
  - $\Phi_g^{\mathbb{Q}}$ determines factor dynamics
  - macro factors mimic level, slope and curvature of yields

Autocovariance of volatility, $\Phi_h \neq \Phi_h^{\mathbb{Q}}$, is not enough to break the tension

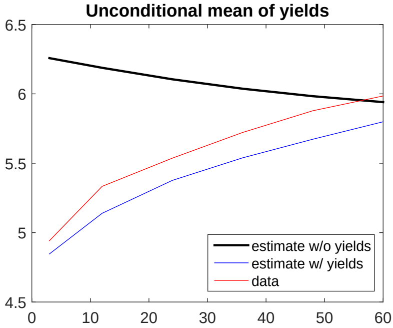
**Preference shock allows $\Phi_g \neq \Phi_g^{\mathbb{Q}}$**

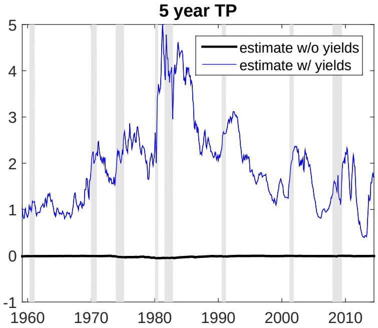
# Macro factors and yields

Regression $R^2$s of macro factors on yields

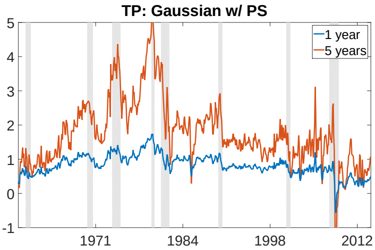
| | estimates w yields | | estimates w/o yields |
|---|---|---|---|
| | w/o m.e. | w/ m.e. | |
| inflation | 1 | 0.90 | 0.28 |
| consumption | 1 | 0.99 | 0.58 |
| inflation V | 1 | 0.66 | 0.47 |
| consumption V | 1 | 0.87 | 0.23 |

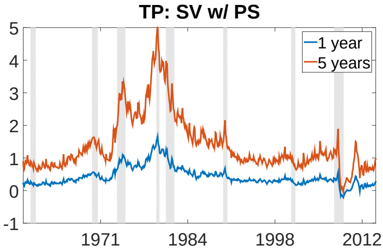
# Slope and term premium in SV model



# Adding preference shocks




- Preference shock breaks the strong tie between $\mathbb{P}$ and $\mathbb{Q}$
- With the preference shock, SV does not add more flexibility for term premia.

# Model solution

- Step #1: log-linearize $r_{c,t+1}$ via Campbell & Shiller (1989)

$$r_{c,t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}$$

- Step #2: guess a solution

$$pc_t = D_0 + D_g' g_t + D_h' h_t$$

- Step #3: solve the fixed point problem.

$$\bar{pc} = D_0 (\bar{pc}) + D_g (\bar{pc})' \bar{\mu}_g + D_h (\bar{pc})' \bar{\mu}_h$$

- Step #4: plug the solution into the SDF.

- Problem: a solution to the fixed point problem does not always exist.

# Solution to the fixed point problem

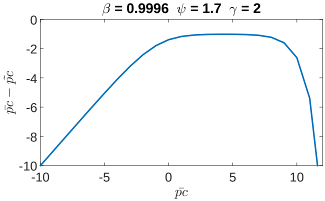
## Proposition

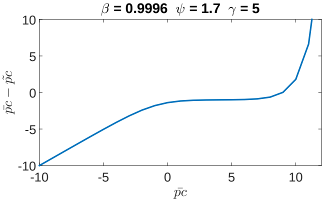
*There is a value $\bar{\beta}(\psi, \gamma, \theta^r)$ such that if $\beta < \bar{\beta}$, then there exists a solution for the fixed point problem.*

## Sketch of proof

Define $\tilde{pc}\left(\bar{pc}\right) = D_0\left(\bar{pc}\right) + D_g\left(\bar{pc}\right)' \bar{\mu}_g + D_h\left(\bar{pc}\right)' \bar{\mu}_h$

The fixed point problem has a solution if $\bar{pc} - \tilde{pc}\left(\bar{pc}\right) = 0$



# Solution to the fixed point problem: Gaussian

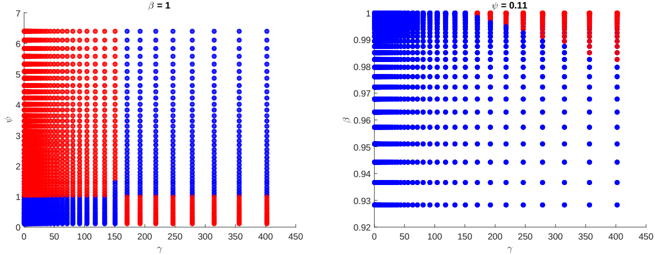
## Corollary

*If there is no preference shock, then*

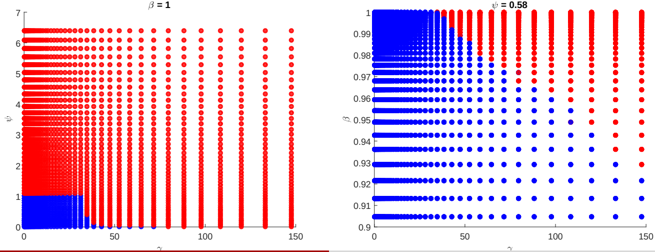
1. *If* $Z_1^{\infty\prime}\mu_g \leq 0$, *then* $\vartheta = \frac{1-\gamma}{1-\eta} < 0$ *guarantees the existence of a solution.*
2. *If* $\beta = 1$, *there is a value* $\bar{\gamma}(\theta^r)$ *such that* $\frac{\bar{\gamma}-\gamma}{1-\eta} < 0$ *guarantees a solution.*
3. *For any* $\psi$, $\bar{\beta}$ *is monotonic in* $\gamma$*: for* $\psi > 1$, *then* $\frac{d\bar{\beta}}{d\gamma} > 0$; *for* $\psi < 1$, *then* $\frac{d\bar{\beta}}{d\gamma} < 0$.

# Model #1: long-run risk, no SV, no preference shock

## Gaussian model




## SV model



# Conclusion

- Build a new consumption-based asset pricing model
  - capture realistic dynamics for risk premia
  - preference shocks $\rightarrow$ flexible time-varying prices of risk
  - stochastic volatility $\rightarrow$ time-varying quantity of risk
- Empirical results:
  - Flexible time-varying price of risk is key
    - term premia fluctuate with expected inflation
  - Term premia from recursive preferences with stochastic volatility are implausible
- Provide conditions guaranteeing a solution for these models.

# Literature:

## Consumption-based models

- recursive preferences: *Piazzesi Schneider (07), Le Singleton (10)*
- recursive preferences + SV: *Bansal Yaron (04), Bansal Gallant Tauchen (07), Bansal Shaliastovich (13),*
- habit formation: *Wachter (06)*
- recursive preferences + SV in ICAPM: *Campbell Giglio et al. (14)*
- recursive preferences + preference shocks: *Albuquerque Eichenbaum Rebelo (14) Schorfheide Song Yaron (14)*

## DSGE models

- habit formation: *Rudebusch Swanson (08)*
- recursive preferences: *Rudebusch Swanson (08), van Binsbergen et al. (12), Dew-Becker (14)*
- solution methods: *Caldara et al. (12)*

# Literature:

## Term premium

- ATSM: *Duffee (02), Ang Piazzesi (03), Wright (11), Bauer Rudebusch Wu (12)*

## Model solution

- *Hansen Scheinkman (12), Campbell Giglio et al. (14)*

Back

# Gaussian bond loadings

$a_n = -\frac{1}{n}\bar{a}_n$, $b_{n,g} = -\frac{1}{n}\bar{b}_{n,g}$

$$\begin{aligned}
\bar{a}_n^{\$} &= \bar{a}_{n-1}^{\$} + \bar{a}_1^{\$} + \left(\mu_g - \eta\vartheta\lambda_0 + \Sigma_{0,g}\Sigma_{0,g}' \left[Z_2 - Z_\pi\right]\right)' \bar{b}_{n-1,g}^{\$} \\
&+ \frac{1}{2}\bar{b}_{n-1,g}^{\$,\prime}\Sigma_{0,g}\Sigma_{0,g}'\bar{b}_{n-1,g}^{\$} \\
\bar{b}_{n,g}^{\$} &= \left(\Phi_g - \eta\vartheta\lambda_g\right)' \bar{b}_{n-1,g}^{\$} + \bar{b}_{1,g}^{\$}
\end{aligned}$$

where the initial conditions are

$$\begin{aligned}
\bar{a}_1^{\$} &= \ln(\beta) + \bar{\Lambda} + \left(Z_\upsilon - \eta Z_c - Z_\pi\right)' \left(\mu_g - \eta\vartheta\lambda_0\right) - \frac{(\vartheta - 1)\vartheta}{2} Z_1'\Sigma_{0,g}\Sigma_{0,g}' Z_1 \\
&+ \frac{1}{2} Z_2'\Sigma_{0,g}\Sigma_{0,g}' Z_2 + \frac{1}{2} Z_\pi'\Sigma_{0,g}\Sigma_{0,g}' Z_\pi - Z_2'\Sigma_{0,g}\Sigma_{0,g}' Z_\pi \\
\bar{b}_{1,g}^{\$} &= \left(\Phi_g - \eta\vartheta\lambda_g\right)' \left(Z_\upsilon - \eta Z_c - Z_\pi\right)
\end{aligned}$$

where

$$\begin{aligned}
Z_1 &= Z_\upsilon + (1 - \eta) Z_c + \kappa_1 D_g \\
Z_2 &= \vartheta Z_\upsilon - \gamma Z_c + (\vartheta - 1)\kappa_1 D_g
\end{aligned}$$

Back

# Particle Gibbs sampler

For $j = 1, \ldots, M$

$$\begin{aligned} (g_{1:T}, h_{0:T})^{(j)} &\sim p\left(g_{1:T}, h_{0:T} \middle| Y_{1:T}, \theta^{\mathbb{P},(j-1)}\right) \\ \theta^{\mathbb{P},(j)} &\sim p\left(\theta^{\mathbb{P}} \middle| Y_{1:T}, g_{1:T}^{(j)}, h_{0:T}^{(j)}\right) \end{aligned}$$

- Draw the state variables using the particle filter, see Andrieu, Doucet, Holenstein (10).
- Use independence Metropolis-Hastings to draw the parameters $\theta^{\mathbb{P}}$.

# Particle Gibbs sampler

For $t = 1, \ldots, T$, run:

- For $j = 2, \ldots, J$, draw from a proposal: $(h_t)^{(j)} \sim q\left(h_t | h_{t-1}^{(j)}, y_t, \theta\right)$.
- For $j = 1, \ldots, J$, calculate the importance weight:

$$w_t^{(j)} \quad \propto \quad \frac{p\left(y_t | h_t^{(j)}, \theta\right) p\left(h_t^{(j)} | h_{t-1}^{(j)}, \theta\right)}{q\left(h_t^{(j)} | h_{t-1}^{(j)}, y_t, \theta\right)}$$

- For $j = 1, \ldots, J$, normalize the weights: $\hat{w}_t^{(j)} = \frac{w_t^{(j)}}{\sum_{j=1}^{J} w_t^{(j)}}$.
- Conditionally resample the particles $\left\{h_t^{(j)}\right\}_{j=1}^{J}$ with probabilities $\left\{\hat{w}_t^{(j)}\right\}_{j=1}^{J}$. In this step, the first particle $h_t^{(1)}$ always gets resampled and may be randomly duplicated.

Key point: the particle approximation: $\{\hat{w}_{1:T}, h_{0:T}\}_1^J \approx p\left(h_{0:T} | y_{1:T}, \theta\right)$

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