Loss Functions for Forecasting Treasury Yields

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Background

Interest Rates Point Forecasting

Duffee (2002, JF), Ang and Piazzesi (2003, JME), Diebold and Li (2006, JE), Van Dijk, Koopman, Van der Wel and Wright (2014, JAE), Bowsher and Meeks (2008, JASA), Moench (2008, JE), Christensen, Diebold and Rudebusch (2011, JE)

Interest Rates Density Forecasting

Hong, Li and Zhao (2004, JBES), Egorov, Hong and Li (2006, JE), Shin and Zhong (2013, WP), Carriero, Clark and Marcellino (2014, WP)

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Motivation

This paper takes a different perspective and explore how the choice of loss function affects a given model's out-of-sample forecasting performance

- The specification of the loss function is critical for model estimation and evaluation
  Engle (1993, JofF), Granger (1993, JofF), Weiss (1996, JAE), Elliott and Timmermann (2008, JEL)

- Granger (1993, JofF):
  ...evaluation criteria are used twice in the modeling process, once to decide how to select the 'best' estimates of parameter values and then to evaluate the forecasts made by the model.
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Contributions

- We align the loss functions for in-sample estimation and out-of-sample evaluation of affine term structure models (ATSMs)
  - Three-Factor ATSMs with and without stochastic volatility
- We propose to estimate the ATSMs by minimizing the mean squared forecasting errors for a given forecast horizon (forecasting loss function)

Empirical Findings

- The improvement in out-of-sample forecasting performance is substantial, especially for long forecast horizons

  For the six-month forecast horizon, the improvement in the forecasting RMSEs for the $A_0(3)$ model is 12%, for the $A_1(3)$ model is 15%
- The improvement in out-of-sample forecasting performance results from the identification of different factors, especially in the case of curvature

  Cochrane and Piazzesi (2005, AER), Duffee (2011, RFS)

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Standard Loss Function

Mean-squared error (MSE) loss

- Given term structure data for months $t = 1, ..., T$ on maturities $n = 1, ..., N$, the parameters Θ are typically estimated by minimizing

$$MSE(\Theta) = \frac{1}{NT}\sum_{n=1}^{N}\sum_{t=1}^{T}(\widehat{y}_{t|t}^{n}(\Theta) - y_t^n)^2 \quad (1)$$

Out-of-sample forecasting performance

- The out-of-sample RMSE for the n-maturity yield with forecast horizon k

$$RMSE\_OS_{n,k} = \sqrt{\frac{1}{T-k}\sum_{t=1}^{T-k}(\widehat{y}_{t+k|t}^{n}(\Theta) - y_{t+k}^{n})^2} \quad (2)$$

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Forecasting Loss Function

Mean-squared forecasting error loss

- The choice of loss function at the estimation stage reflects out-of-sample forecasting purpose
- Estimate the ATSMs for a given forecast horizon k by minimizing

$$OS\_MSE_k(\Theta) = \frac{1}{N(T-k)} \sum_{n=1}^{N} \sum_{t=k+1}^{T} (\widehat{y}^{n}_{t|t-k}(\Theta) - y^{n}_{t})^2 \quad (3)$$

- Given state variables at time $t-k$, compute k-period ahead yields using Θ
- The estimation is forecast-horizon specific

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Out-of-Sample Forecasting Procedure

Forecasting loss function

- At each month t and for each forecast horizon k, we estimate the parameters Θ^k by minimizing the forecasting loss function using data up to and including t
- Subsequently, we forecast the k-period ahead yields $\widehat{y}^n_{t+k|t}(\Theta^k_t)$, $n = 1, ..., N$
- The recursion proceeds by adding one month of data, re-estimate the parameters using data up to and including $t+1$, and forecast the k-period ahead yields $\widehat{y}^n_{t+1+k|t+1}(\Theta^k_{t+1})$
- Iterate the procedure until $T-k$

Standard loss function

- At each month t, one set of parameters is estimated and used to generate forecasts for different horizons

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Model Description

Canonical form of Latent ATSMs: Dai and Singleton (2000)

$$dX_t = (K^P_{0\Delta} + K^P_{1\Delta} X_t)dt + \Sigma\sqrt{S_t}dW^P_{t+1}$$
$$dX_t = (K^Q_{0\Delta} + K^Q_{1\Delta} X_t)dt + \Sigma\sqrt{S_t}dW^Q_{t+1}$$
$$r_t = \rho_0 + \rho_1 X_t$$

- $X_t \in R^3$, r_t is the instantaneous spot interest rate
- W^P_{t+1} and W^Q_{t+1} are three-dimensional independent standard Brownian motions under P- and Q-measure
- $\Sigma S_t \Sigma'$ is the conditional covariance matrix of X_t, S_t is a 3×3 diagonal matrix with the ith diagonal element given by

$$[S_t]_{ii} = \alpha_i + \beta_i^{'} X_t$$

- Model-implied continuously compounded yields

$$\widehat{y}_t = A(\Theta^Q) + B(\Theta^Q)X_t$$

- $y_t \in R^N, N > 3$, $\Theta^Q = \{K^Q_{0\Delta}, K^Q_{1\Delta}, \rho_0, \rho_1, \Sigma, \alpha_i, \beta_i\}$

JSZ Canonical Form

Estimation Method

Prediction of k-period ahead n-maturity yield

$$\widehat{y}^{n}_{t+k|t}(\Theta) = A_n(\Theta^Q) + B_n(\Theta^Q)\widehat{X}_{t+k|t}$$

- X_t follows VAR(1) when sampled monthly

$$\widehat{X}_{t+\Delta|t} = \underbrace{K^P_{0\Delta}\int_0^{\Delta} e^{sK^P_{1\Delta}}ds}_{K^P_0} + \underbrace{e^{\Delta K^P_{1\Delta}}}_{K^P_1} X_t, \text{ where } \Delta = 1/12$$

- K^P_0 and K^P_1 are the parameters for the VAR(1) process of X_t under P measure

$$\widehat{y}^{n}_{t+k|t}(\Theta) = A_n(\Theta^Q) + B_n(\Theta^Q)f(X_t, k; K^P_0, K^P_1)$$

$$f(X_t, k; K^P_0, K^P_1) = K^P_0(I_3 + K^P_1 + ... + (K^P_1)^{k-1}) + (K^P_1)^k X_t$$

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Out-of-Sample Forecasting Performance: $A_0(3)$

Out-of-Sample RMSEs:
$\mathbf{A_0(3)}$ with Latent Factors

Panel A: Forecasting Loss Function						
Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	38.47	55.53	68.99	78.78	88.97	99.65
6 month yield	33.51	53.37	69.85	83.52	96.27	108.70
1 year yield	37.84	59.81	78.28	92.96	105.82	117.62
2 year yield	40.41	63.36	81.95	95.63	107.89	118.75
3 year yield	37.65	59.22	76.23	88.29	99.70	109.44
4 year yield	33.77	54.00	69.40	79.79	90.30	99.13
5 year yield	30.76	50.19	66.65	78.61	87.65	96.97
10 year yield	32.10	49.40	63.15	76.06	82.02	91.09
20 year yield	28.54	44.51	52.71	61.85	70.89	80.07
Panel B: Standard Loss Function						
Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	42.22	57.51	73.06	87.73	103.87	120.70
6 month yield	36.79	55.87	70.27	85.84	101.77	119.88
1 year yield	38.75	60.28	80.46	95.58	108.62	121.45
2 year yield	41.07	65.36	82.99	96.92	110.11	127.41
3 year yield	38.60	59.48	76.86	92.61	109.65	125.54
4 year yield	35.22	58.49	72.90	88.85	104.24	118.98
5 year yield	33.92	52.85	70.46	85.83	100.68	114.68
10 year yield	33.24	50.40	63.15	77.06	89.02	101.09
20 year yield	28.44	44.51	58.71	71.85	84.89	97.07

Out-of-Sample Forecasting Performance: $A_0(3)$

Panel C: RMSE Ratio

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	0.91	0.97***	0.94***	0.90***	0.86***	0.83***
6 month yield	0.91	0.96***	0.99***	0.97***	0.95***	0.91***
1 year yield	0.98**	0.99***	0.97***	0.97***	0.97***	0.97***
2 year yield	0.98***	0.97***	0.99***	0.99***	0.98***	0.93***
3 year yield	0.98**	1.00***	0.99***	0.95***	0.91***	0.87***
4 year yield	0.96**	0.92***	0.95***	0.90***	0.87***	0.83***
5 year yield	0.91*	0.95***	0.95***	0.92***	0.87***	0.85***
10 year yield	0.97*	0.98***	1.00*	0.99***	0.92***	0.90***
20 year yield	1.00	1.00	0.90	0.86**	0.84**	0.82**

Diebold and Mariano Test: *Diebold (2015, JBES)*

- The improvements in forecasting performance for the $A_0(3)$ model are greatest for longer forecast horizons and shorter maturities
- For the six-month forecast horizon, the improvement in the forecasting RMSEs is on average across maturities approximately 12%, which corresponds to an out-of-sample R-square of 22%
- For the three-month yield, the improvement in the forecasting RMSEs is on average across forecast horizons approximately 10%, which corresponds to an out-of-sample R-square of 19%

OOS Performance JSZ Fixed

Out-of-Sample Forecasting Performance: $A_0(3)$

Panel C: RMSE Ratio

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	0.91	0.97***	0.94***	0.90***	0.86***	0.83***
6 month yield	0.91	0.96***	0.99***	0.97***	0.95***	0.91***
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2 year yield	0.98***	0.97***	0.99***	0.99***	0.98***	0.93***
3 year yield	0.98**	1.00***	0.99***	0.95***	0.91***	0.87***
4 year yield	0.96**	0.92***	0.95***	0.90***	0.87***	0.83***
5 year yield	0.91*	0.95***	0.95***	0.92***	0.87***	0.85***
10 year yield	0.97*	0.98***	1.00*	0.99***	0.92***	0.90***
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OOS Performance JSZ Fixed

JSZ Canonical Form: Joslin, Singleton and Zhu (2011)

JSZ Canonical form of $A_0(3)$

- The state variables are observables, perfectly priced portfolio of yields $PO_t = Wy_t$, W denotes portfolio weights, which is a $3 \times N$ matrix
- PO_t is governed by the same dynamics as the latent state variable X_t
- $\Theta^P = \{K_0^P, K_1^P\}$ can be estimated separately from the parameters governing the Q-dynamics
- Estimate Θ^P through ordinary least squares (OLS)
  - $Wy_t \approx W\widehat{y}_t$, the best approximation is obtained by choosing W_0 such that $W_0 y_t = PC_t$, the first three principal components of the observed term structure of yields
- $A(\Theta^Q)$ and $B(\Theta^Q)$ are ultimately functions of $\Theta^Q = \{r_\infty^Q, \lambda^Q, \Sigma\}$
- r_∞^Q is a scalar related to the long-run mean of the short rate under risk neutral measure
- λ^Q, a 3×1 vector, represents the ordered eigenvalues of K_1^Q

Appendix: JSZ Mapping

Out-of-Sample Forecasting Performance: JSZ

Out-of-Sample RMSEs:
JSZ Canonical Form with Fixed Portfolio Weights

Panel A: Forecasting Loss Function

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	38.18	55.72	67.68	76.69	87.64	99.06
6 month yield	33.81	51.29	66.91	80.26	93.67	107.02
1 year yield	39.39	59.75	77.92	94.37	108.58	120.94
2 year yield	39.63	61.92	79.63	95.15	108.23	119.37
3 year yield	38.06	59.55	76.65	90.79	102.96	113.00
4 year yield	35.30	55.76	72.01	84.46	95.52	104.93
5 year yield	32.18	51.99	67.80	79.46	90.02	98.93
10 year yield	33.27	49.66	61.71	70.58	79.52	87.04
20 year yield	26.36	40.81	51.30	59.60	67.23	73.44

Panel B: Standard Loss Function

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	38.11	55.06	69.89	80.61	91.84	103.21
6 month yield	33.68	52.32	69.43	83.91	97.71	110.97
1 year yield	38.68	60.60	79.92	96.19	110.51	123.39
2 year yield	39.13	62.34	81.09	96.46	109.60	121.24
3 year yield	37.45	59.98	77.79	92.29	104.59	115.25
4 year yield	34.71	56.23	73.01	86.40	97.79	107.71
5 year yield	31.62	52.50	68.68	81.67	92.68	102.08
10 year yield	33.02	49.93	62.56	73.33	82.63	90.45
20 year yield	26.24	40.85	51.63	60.85	68.80	75.40

Out-of-Sample Forecasting Performance: JSZ

Panel C: RMSE Ratio

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	1.00	1.01	0.97	0.95**	0.95**	0.96***
6 month yield	1.00	0.98**	0.96*	0.96**	0.96***	0.96***
1 year yield	1.02	0.99	0.97	0.98**	0.98***	0.98***
2 year yield	1.01	0.99	0.98*	0.99***	0.99***	0.98***
3 year yield	1.02*	0.99	0.99**	0.98***	0.98***	0.98***
4 year yield	1.02	0.99	0.99**	0.98***	0.98***	0.97***
5 year yield	1.02	0.99*	0.99**	0.97***	0.97***	0.97***
10 year yield	1.01	0.99**	0.99**	0.96***	0.96***	0.96***
20 year yield	1.00*	1.00	0.99	0.98	0.98	0.97

- For the six-month forecast horizon, the improvement in the forecasting RMSEs is on average across maturities approximately 3%, which corresponds to an out-of-sample R-square of 5%

OOS Performance Latent

Out-of-Sample Forecasting Performance: JSZ

Panel C: RMSE Ratio						
Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	1.00	1.01	0.97	0.95**	0.95**	0.96***
6 month yield	1.00	0.98**	0.96*	0.96**	0.96***	0.96***
1 year yield	1.02	0.99	0.97	0.98**	0.98***	0.98***
2 year yield	1.01	0.99	0.98*	0.99***	0.99***	0.98***
3 year yield	1.02*	0.99	0.99**	0.98***	0.98***	0.98***
4 year yield	1.02	0.99	0.99**	0.98***	0.98***	0.97***
5 year yield	1.02	0.99*	0.99**	0.97***	0.97***	0.97***
10 year yield	1.01	0.99**	0.99**	0.96***	0.96***	0.96***
20 year yield	1.00*	1.00	0.99	0.98	0.98	0.97

- For the six-month forecast horizon, the improvement in the forecasting RMSEs is on average across maturities approximately 3%, which corresponds to an out-of-sample R-square of 5%

OOS Performance Latent

JSZ Canonical Form with Variable Portfolio Weights

- Allow the portfolio weights to be free parameters
  Cochrane and Piazzesi (2005, AER), Duffee (2011, RFS), Ludvigson and Ng (2009, RFS), Cooper and Priestley (2009, RFS), Joslin, Priebsch and Singleton (2014, JF), Cieslak and Povala, (2015, RFS)
- Implement iterative two-step estimation procedure to take full advantage of the computational efficiency of the JSZ method

- Use converged JSZ estimates from the standard loss function as initial values

1. For given Θ^P and Θ^Q, search for the best possible W by minimizing the forecasting loss function
2. Fix W from step 1, solve for Θ^P and Θ^Q by minimizing the forecasting loss function

- With converged Θ^P and Θ^Q from step 2, go back to step 1, the optimization goes back and forth between the two steps until it converges

JSZ Canonical Form with Variable Portfolio Weights

- Allow the portfolio weights to be free parameters
  Cochrane and Piazzesi (2005, AER), Duffee (2011, RFS), Ludvigson and Ng (2009, RFS), Cooper and Priestley (2009, RFS), Joslin, Priebsch and Singleton (2014, JF), Cieslak and Povala, (2015, RFS)
- Implement iterative two-step estimation procedure to take full advantage of the computational efficiency of the JSZ method

- Use converged JSZ estimates from the standard loss function as initial values

1. For given Θ^P and Θ^Q, search for the best possible W by minimizing the forecasting loss function
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Out-of-Sample Forecasting Performance: JSZ Variable Weights

Out-of-Sample RMSEs:
JSZ Canonical Form with Variable Portfolio Weights

Panel A: Forecasting Loss Function

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	36.60	49.69	60.41	69.09	79.40	90.58
6 month yield	29.97	45.55	59.59	73.09	86.01	98.95
1 year yield	35.06	54.58	69.97	84.23	97.20	109.72
2 year yield	40.70	61.01	76.67	89.55	101.50	113.23
3 year yield	39.19	57.80	73.04	84.54	95.55	106.31
4 year yield	35.54	53.10	67.71	77.95	87.92	97.94
5 year yield	32.16	49.41	63.80	73.82	83.81	93.32
10 year yield	39.47	50.97	60.34	69.03	77.73	84.47
20 year yield	41.32	50.58	58.34	64.61	73.16	80.14

Panel B: Standard Loss Function

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	38.11	55.06	69.89	80.61	91.84	103.21
6 month yield	33.68	52.32	69.43	83.91	97.71	110.97
1 year yield	38.68	60.60	79.92	96.19	110.51	123.39
2 year yield	39.13	62.34	81.09	96.46	109.60	121.24
3 year yield	37.45	59.98	77.79	92.29	104.59	115.25
4 year yield	34.71	56.23	73.01	86.40	97.79	107.71
5 year yield	31.62	52.50	68.68	81.67	92.68	102.08
10 year yield	33.02	49.93	62.56	73.33	82.63	90.45
20 year yield	26.24	40.85	51.63	60.85	68.80	75.40

Out-of-Sample Forecasting Performance: JSZ Variable Weights

Panel C: RMSE Ratio

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	0.96	0.90	0.86*	0.86***	0.86***	0.88***
6 month yield	0.89	0.87***	0.86**	0.87***	0.88***	0.89***
1 year yield	0.91	0.90**	0.88**	0.88***	0.88***	0.89***
2 year yield	1.04	0.98	0.95**	0.93***	0.93***	0.93***
3 year yield	1.05**	0.96	0.94***	0.92***	0.91***	0.92***
4 year yield	1.02*	0.94*	0.93***	0.90***	0.90***	0.91***
5 year yield	1.02	0.94**	0.93***	0.90***	0.90***	0.91***
10 year yield	1.20	1.02***	0.96***	0.94***	0.94***	0.93***
20 year yield	1.57**	1.24	1.13	1.06	1.06	1.06

- The improvements in forecasting performance for the JSZ canonical form with variable portfolio weights are greatest for longer forecast horizons and shorter maturities
- For the six-month forecast horizon, the improvement in the forecasting RMSEs is on average across maturities approximately 7%, which corresponds to an out-of-sample R-square of 15%
- For short maturity yields, the improvement in the forecasting RMSEs is on average across forecast horizons approximately 11%, which corresponds to an out-of-sample R-square of 23%

Out-of-Sample Forecasting Performance: JSZ Variable Weights

Panel C: RMSE Ratio

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	0.96	0.90	0.86*	0.86***	0.86***	0.88***
6 month yield	0.89	0.87***	0.86**	0.87***	0.88***	0.89***
1 year yield	0.91	0.90**	0.88**	0.88***	0.88***	0.89***
2 year yield	1.04	0.98	0.95**	0.93***	0.93***	0.93***
3 year yield	1.05**	0.96	0.94***	0.92***	0.91***	0.92***
4 year yield	1.02*	0.94*	0.93***	0.90***	0.90***	0.91***
5 year yield	1.02	0.94**	0.93***	0.90***	0.90***	0.91***
10 year yield	1.20	1.02***	0.96***	0.94***	0.94***	0.93***
20 year yield	1.57**	1.24	1.13	1.06	1.06	1.06

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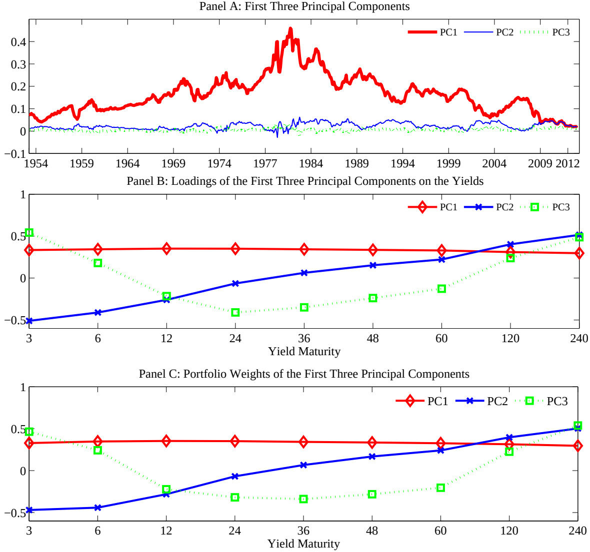
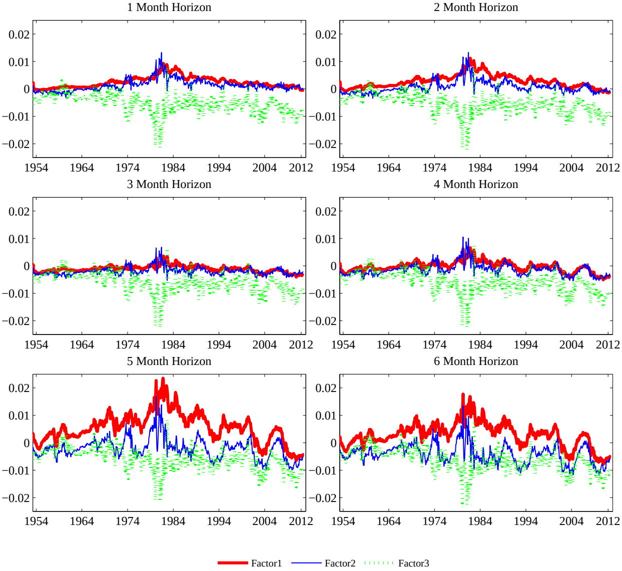
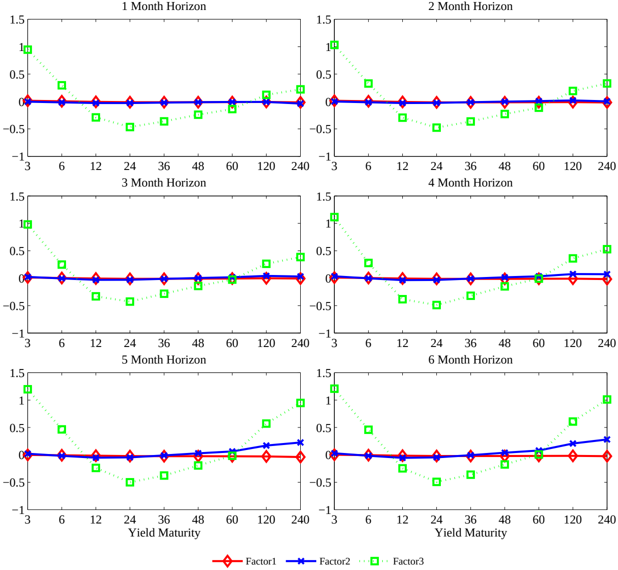
State Variables: JSZ Canonical Form with Standard Loss Function



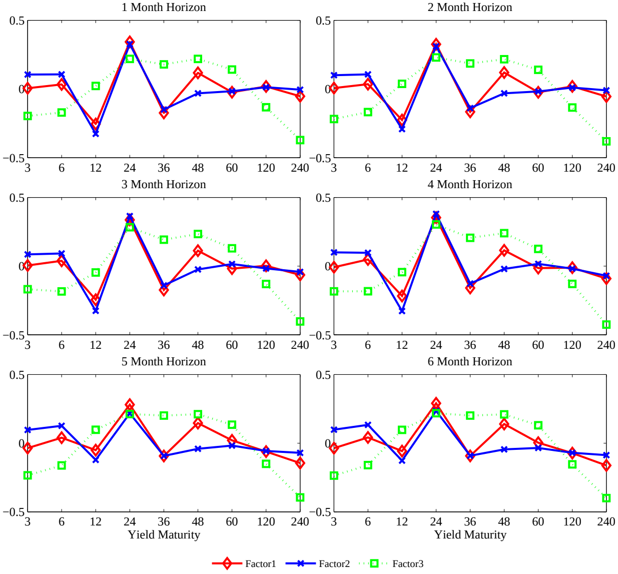
State Variables Difference: Forecasting VS. Standard Loss Function



Factor Loadings Difference: Forecasting VS. Standard Loss Function



Portfolio Weights Difference: Forecasting VS. Standard Loss Function



JSZ with Variable Portfolio Weights: Parameter Estimates

Panel A: Forecasting Loss Function

Forecast Horizon	K_0^P	P-Dynamics K_1^P			Eigenvalues	K_0^Q	Q-Dynamics K_1^Q			Eigenvalues
1 month										
	-0.0016	0.9993	0.0631	0.6416	0.9938	0.0004	0.9982	0.0949	-0.6988	0.9991
	0.0005	0.0059	0.9339	0.3664	0.9259	-0.0004	-0.0006	0.9492	0.6744	0.9593
	0.0005	-0.0024	-0.0032	0.7770	0.7906	0.0002	0.0006	0.0031	0.8143	0.8034
2 month										
	-0.0017	1.0000	0.0668	0.7077	0.9942	0.0004	0.9976	0.0970	-0.7441	0.9992
	0.0004	0.0061	0.9383	0.3870	0.9287	-0.0003	-0.0002	0.9524	0.6930	0.9608
	0.0005	-0.0022	-0.0036	0.7833	0.7986	0.0001	0.0005	0.0028	0.8082	0.7981
3 month										
	-0.0014	1.0003	0.0668	0.6341	0.9940	0.0005	0.9987	0.0880	-0.7480	0.9993
	0.0004	0.0062	0.9423	0.4072	0.9324	-0.0004	-0.0009	0.9606	0.7453	0.9605
	0.0005	-0.0029	-0.0045	0.7507	0.7669	0.0002	0.0006	0.0008	0.7760	0.7755
4 month										
	-0.0014	0.9988	0.0757	0.7156	0.9942	0.0005	0.9993	0.0849	-0.8361	0.9992
	0.0004	0.0059	0.9451	0.4420	0.9303	-0.0004	-0.0018	0.9691	0.7976	0.9616
	0.0004	-0.0023	-0.0061	0.7428	0.7621	0.0002	0.0009	-0.0008	0.7641	0.7717
5 month										
	-0.0021	0.9974	0.1140	0.8687	0.9966	0.0004	0.9999	0.0919	-0.7866	0.9993
	0.0003	0.0043	0.9549	0.3965	0.9322	-0.0003	-0.0027	0.9765	0.6517	0.9691
	0.0004	-0.0011	-0.0089	0.7626	0.7861	0.0001	0.0011	-0.0003	0.8030	0.8110
6 month										
	-0.0021	1.0001	0.1242	0.8707	0.9969	0.0004	0.9991	0.0884	-0.7991	0.9995
	0.0003	0.0047	0.9555	0.3872	0.9306	-0.0003	-0.0016	0.9806	0.6556	0.9694
	0.0004	-0.0017	-0.0107	0.7559	0.7840	0.0001	0.0008	-0.0017	0.7951	0.8059

Panel B: Standard Loss Function

K_0^P	P-Dynamics K_1^P			Eigenvalues	K_0^Q	Q-Dynamics K_1^Q			Eigenvalues
-0.0021	0.9940	0.0549	0.3129	0.9948	0.0004	1.0052	0.1039	-0.2569	1.0000
0.0004	0.0017	0.9337	0.1538	0.9274	-0.0003	-0.0073	0.9370	0.2717	0.9648
0.0012	-0.0002	-0.0042	0.8084	0.8139	0.0003	0.0042	0.0136	0.8685	0.8458

Source of Improvement

- Estimation using forecasting loss function reveals a different set of factors, especially in the case of the third factor
  - Time series of state variables
  - Factor loadings
  - Portfolio weights
- Parameter estimates show that the third factor behaves differently under the two loss functions
  - The persistence of the third factor
  - How the third factor affects other factors

Conclusion

- We propose estimating ATSMs by aligning the loss functions for in-sample estimation and out-of-sample evaluation
- Aligning loss functions provides substantial improvements in out-of-sample forecasting performance, especially for long forecast horizons
- The improvements in out-of-sample forecasting performance results from identification of the third factor

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JSZ Mapping

$$K_1^Q = WB(\Theta_L^Q)diag(\lambda^Q)(WB(\Theta_L^Q))^{-1} + I_M$$
$$K_0^Q = -WB(\Theta_L^Q)diag(\lambda^Q)(WB(\Theta_L^Q))^{-1}WA(\Theta_L^Q)$$
$$\rho_0 = r_\infty^Q - \tau(WB(\Theta_L^Q))^{-1}WA(\Theta_L^Q)$$
$$\rho_1 = \tau(WB(\Theta_L^Q))^{-1}$$

- $A(\Theta_L^Q)$ and $A(\Theta_L^Q)$ are functions of Θ_L^Q through a set of Ricatti ODEs, where $\Theta_L^Q = \{r_\infty^Q, \lambda^Q, \Sigma\}$

Back to JSZ Canonical Form

Check of Diebold-Mariano Assumption

Augmented Dickey-Fuller Test (Critical Value: -3.44 at 1% Significance Level)						
Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	-20.01	-11.21	-9.71	-8.07	-6.78	-6.46
6 month yield	-17.08	-7.92	-6.94	-5.14	-4.69	-4.75
1 year yield	-17.71	-7.88	-6.59	-5.63	-4.90	-4.72
2 year yield	-11.81	-7.70	-6.83	-5.96	-5.52	-5.06
3 year yield	-12.54	-7.91	-7.14	-6.24	-6.09	-5.37
4 year yield	-12.49	-8.84	-7.91	-6.80	-6.53	-5.91
5 year yield	-12.17	-8.88	-8.04	-7.01	-6.96	-6.13
10 year yield	-11.40	-9.52	-8.53	-8.24	-7.28	-7.03
20 year yield	-8.99	-9.44	-7.26	-7.41	-6.60	-6.19

Back to OOS Forecasting Performance

Related Literature

- The estimation of the ATSMs is challenging due to high level of nonlinearity in the parameters and badly behaved likelihood surfaces
  Duffee (2011, WP), Duffee and Stanton (2012, QJF), Hamilton and Wu (2012, JE)
- Innovative estimation approaches to address the identification issues in the estimation of ATSMs
  Joslin, Singleton and Zhu (2011, RFS), Hamilton and Wu (2012, JE), Bauer, Rudebusch and Wu (2012, JBES), Duffee (2011, WP), Adrian, Crump and Moench (2013, JFE), Diez de los Rios (2014, JBES), Creal and Wu (2015, JE)
- Estimate the Gaussian-ATSMs using an objective function that takes into account excess returns for different horizons
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Data

Summary Statistics							
	Central Moments				Autocorrelation		
	Mean	St.Dev	Skewness	Kurtosis	Lag 1	Lag 12	Lag 30
3 month yield	0.0450	0.0290	0.8938	4.3247	0.9773	0.7944	0.5197
6 month yield	0.0479	0.0305	0.8717	4.2283	0.9850	0.8126	0.5359
1 year yield	0.0516	0.0306	0.6980	3.6594	0.9857	0.8317	0.5891
2 year yield	0.0536	0.0301	0.6734	3.4957	0.9878	0.8509	0.6485
3 year yield	0.0554	0.0294	0.6703	3.4460	0.9884	0.8611	0.6782
4 year yield	0.0569	0.0288	0.6903	3.4142	0.9882	0.8655	0.7000
5 year yield	0.0579	0.0282	0.7270	3.3717	0.9890	0.8739	0.7183
10 year yield	0.0617	0.0275	0.9148	3.5853	0.9890	0.8739	0.7183
20 year yield	0.0638	0.0265	0.9158	3.5373	0.9930	0.8936	0.7724

- Fama CRSP Treasury Bill files, zero coupon files and Federal Reserve Database
- Monthly zero coupon bond yields (continuously compounded)
- With maturities of 3 months, 6 months, 1-5 years, 10 and 20 years
- April 1953 to Dec 2012

Out-of-Sample Forecasting Performance: Stochastic Volatility Models

Out-of-Sample Forecasting RMSE Ratio						
Panel A: $A_1(3)$ with Latent Factors						
Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	0.92*	0.90***	0.91***	0.88***	0.88***	0.89***
6 month yield	0.90*	0.92***	0.91***	0.90***	0.90***	0.90***
1 year yield	1.02	0.94**	0.90***	0.88***	0.87***	0.87***
2 year yield	1.24***	1.00	0.92***	0.88***	0.86***	0.85***
3 year yield	1.20***	0.98	0.90***	0.86***	0.84***	0.83***
4 year yield	1.10***	0.94***	0.88***	0.84***	0.82***	0.82***
5 year yield	0.97	0.89***	0.85***	0.83***	0.82***	0.81***
10 year yield	0.95	0.87***	0.84***	0.82***	0.80***	0.79***
20 year yield	1.73***	1.23***	1.04	0.94	0.88***	0.84***
Panel B: $A_2(3)$ with Latent Factors						
Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	0.92	0.91***	0.91***	0.89***	0.89***	0.90***
6 month yield	0.91**	0.92***	0.92***	0.90***	0.90***	0.90***
1 year yield	1.02	0.94**	0.89***	0.88***	0.87***	0.87***
2 year yield	1.24***	0.99	0.91***	0.88***	0.85***	0.85***
3 year yield	1.20***	0.97*	0.89***	0.86***	0.84***	0.83***
4 year yield	1.09**	0.93***	0.87***	0.84***	0.82***	0.81***
5 year yield	0.96*	0.88***	0.85***	0.83***	0.81***	0.81***
10 year yield	0.93	0.86***	0.83***	0.81***	0.80***	0.79***
20 year yield	1.69***	1.19**	1.00	0.91*	0.86***	0.82***

Out-of-Sample Forecasting Performance: Stochastic Volatility Models

Panel C: $A_3(3)$ with Latent Factors						
Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	0.91	0.93***	0.94***	0.89***	0.90***	0.90***
6 month yield	0.94**	0.94**	0.93***	0.91***	0.91***	0.92***
1 year yield	1.03	0.94*	0.90***	0.89***	0.88***	0.88***
2 year yield	1.24***	1.00	0.92***	0.88***	0.86***	0.85***
3 year yield	1.20***	0.97	0.9***	0.86***	0.84***	0.83***
4 year yield	1.10***	0.93***	0.87***	0.84***	0.82***	0.82***
5 year yield	0.97	0.89***	0.85***	0.83***	0.82***	0.81***
10 year yield	0.94	0.86***	0.83***	0.81***	0.80***	0.80***
20 year yield	1.69***	1.18**	1.00	0.91*	0.86***	0.83***

- The improvements in forecasting performance for the stochastic volatility models are greatest for longer forecast horizons
- For the six-month forecast horizon, the improvement in the forecasting RMSEs of the $A_1(3)$ model is on average across maturities approximately 15%, which corresponds to an out-of-sample R-square of 28%
- The improvements of the $A_2(3)$ model and the $A_3(3)$ model are very similar to that of the $A_1(3)$ model

Out-of-Sample Forecasting Performance: Stochastic Volatility Models

Panel C: $\mathbf{A_3(3)}$ with Latent Factors						
Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	0.91	0.93***	0.94***	0.89***	0.90***	0.90***
6 month yield	0.94**	0.94**	0.93***	0.91***	0.91***	0.92***
1 year yield	1.03	0.94*	0.90***	0.89***	0.88***	0.88***
2 year yield	1.24***	1.00	0.92***	0.88***	0.86***	0.85***
3 year yield	1.20***	0.97	0.9***	0.86***	0.84***	0.83***
4 year yield	1.10***	0.93***	0.87***	0.84***	0.82***	0.82***
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Trade-off between In-Sample and Out-of-Sample Fit

In-Sample RMSEs: JSZ Canonical Form

Panel A: Forecasting Loss Function with Variable Portfolio Weights

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	10.10	10.15	10.65	10.80	10.84	10.81
6 month yield	13.43	13.93	14.84	13.37	13.64	14.89
1 year yield	12.88	13.35	12.42	13.02	14.33	14.60
2 year yield	15.34	16.02	17.07	17.62	16.92	19.12
3 year yield	16.95	12.63	15.54	17.72	19.49	21.27
4 year yield	17.95	15.41	16.51	16.82	17.68	17.53
5 year yield	16.38	15.49	15.47	16.74	17.62	17.35
10 year yield	28.09	29.20	28.85	28.29	28.86	27.96
20 year yield	33.08	34.99	30.94	30.48	32.25	28.70

Panel B: Forecasting Loss Function with Fixed Portfolio Weights

Forecast Horizon k	1 month	2 month	3 month	4 month	5 month	6 month
3 month yield	15.45	15.30	15.35	15.55	15.37	15.62
6 month yield	14.48	14.52	14.58	14.97	14.78	14.90
1 year yield	15.32	15.34	15.42	15.76	16.06	15.92
2 year yield	9.71	9.79	9.88	10.00	10.00	10.98
3 year yield	8.10	8.13	8.18	8.32	8.15	8.31
4 year yield	11.93	11.94	11.98	11.95	11.96	12.07
5 year yield	12.73	12.73	12.75	12.75	12.78	12.77
10 year yield	14.24	14.24	14.26	14.31	14.39	14.39
20 year yield	13.52	13.54	13.57	13.76	13.99	13.92

Panel C: Standard Loss Function with Fixed Portfolio Weights

3 month yield	15.18
6 month yield	13.99
1 year yield	15.04
2 year yield	8.55
3 year yield	6.87
4 year yield	9.86
5 year yield	10.68
10 year yield	11.62
20 year yield	11.54

JSZ with Fixed Portfolio Weights: Parameter Estimates

Panel A: Forecasting Loss Function										
	P-Dynamics					Q-Dynamics				
Forecast Horizon	K_0^P	K_1^P			Eigenvalues	K_0^Q	K_1^Q			Eigenvalues
1 month										
	-0.0022	0.9950	0.0547	0.3116	0.9956	0.0004	1.0051	0.1033	-0.2582	0.9995
	0.0004	0.0017	0.9274	0.1542	0.9196	-0.0003	-0.0073	0.9353	0.2699	0.9634
	0.0012	-0.0002	-0.0041	0.8302	0.8374	0.0003	0.0042	0.0137	0.8647	0.8423
2 month										
	-0.0021	0.9945	0.0544	0.3023	0.9951	0.0004	1.0051	0.1023	-0.2581	0.9995
	0.0004	0.0017	0.9324	0.1543	0.9243	-0.0003	-0.0073	0.9354	0.2703	0.9636
	0.0012	-0.0002	-0.0041	0.8376	0.8451	0.0003	0.0042	0.0138	0.8643	0.8417
3 month										
	-0.0021	0.9942	0.0546	0.2793	0.9950	0.0004	1.0051	0.1016	-0.2590	0.9994
	0.0004	0.0017	0.9315	0.1540	0.9248	-0.0003	-0.0073	0.9352	0.2705	0.9637
	0.0012	-0.0002	-0.0042	0.8145	0.8205	0.0003	0.0042	0.0138	0.8636	0.8409
4 month										
	-0.0019	0.9947	0.0537	0.2595	0.9955	0.0004	1.0051	0.1007	-0.2553	0.9995
	0.0004	0.0018	0.9306	0.1547	0.9243	-0.0003	-0.0073	0.9351	0.2706	0.9634
	0.0012	-0.0002	-0.0042	0.8019	0.8074	0.0003	0.0043	0.0138	0.8622	0.8396
5 month										
	-0.0020	0.9954	0.0512	0.2727	0.9962	0.0004	1.0052	0.0994	-0.2581	0.9995
	0.0004	0.0017	0.9325	0.1494	0.9275	-0.0003	-0.0072	0.9350	0.2701	0.9639
	0.0014	-0.0002	-0.0041	0.7731	0.7772	0.0003	0.0043	0.0142	0.8618	0.8386
6 month										
	-0.0018	0.9947	0.0523	0.2703	0.9955	0.0004	1.0052	0.1002	-0.2573	0.9994
	0.0004	0.0017	0.9303	0.1534	0.9252	-0.0003	-0.0073	0.9348	0.2702	0.9631
	0.0014	-0.0002	-0.0042	0.7684	0.7727	0.0003	0.0042	0.0138	0.8616	0.8392
Panel B: Standard Loss Function										
	P-Dynamics					Q-Dynamics				
	K_0^P	K_1^P			Eigenvalues	K_0^Q	K_1^Q			Eigenvalues
	-0.0021	0.9940	0.0549	0.3129	0.9948	0.0004	1.0052	0.1039	-0.2569	1.0000
	0.0004	0.0017	0.9337	0.1538	0.9274	-0.0003	-0.0073	0.9370	0.2717	0.9648
	0.0012	-0.0002	-0.0042	0.8084	0.8139	0.0003	0.0042	0.0136	0.8685	0.8458