Average Inflation Targeting: Time Inconsistency and Intentional Ambiguity

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The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or the Board of Governors of the Federal Reserve System.
Ambiguous Communication

Chair Powell announced a new operating framework of average inflation targeting at 2020 Jackson Hole Symposium

“In seeking to achieve inflation that averages 2% over time, we are not tying ourselves to a particular mathematical formula that defines the average.”

BoC Kozicki at Fed Listens

“Average inflation… need to choose average period for inflation…”

Brunnermeier at 2021 Jackson Hole Symposium

“Average inflation targeting over how many periods?”
Research Question

Q: Is there any benefit to ambiguous communication?

A: Potentially yes!
We focus on two key issues

- **Time inconsistency**
  - AIT improves the trade-off in Phillips curve
  - Central bank has an incentive to deviate ex post

- **Ambiguous communication**
  - **Flexibility motive:**
    - Optimal horizon of AIT announcement is time dependent
  - **Credibility motive:**
    - Ambiguous communication improves central bank’s credibility
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**Contribution**

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  - **Credibility motive:**
    ambiguous communication improves central bank’s credibility
1. Full-Information Rational Expectations & Phillips Curve

2. Time Inconsistency & Flexibility Motive of Ambiguous Communication

3. Social Learning & Credibility Motive of Ambiguous Communication
Central Bank’s Problem

- **Central bank’s objective function**

\[
L_{cb}^t(L) = \frac{1}{2} \left( \frac{\pi_t + \pi_{t-1} + \ldots + \pi_{t-L+1}}{L} \right)^2 + \lambda^{cb}(L)\hat{y}_t^2 \right) + \beta \mathbb{E}_t L_{t+1}^{cb}(L) \ (1)
\]

- **Period loss of social welfare**

\[
L_t = \frac{1}{2} \left( \pi_t^2 + \lambda\hat{y}_t^2 \right).
\]

which corresponds to \( L = 1, \lambda^{cb} = \lambda \) in (1)

- **Central bank minimizes (1) subject to the forward-looking Phillips curve**

\[
\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \hat{y}_t + \nu_t.
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Central Bank’s Problem

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\[ \pi_t = \beta E_t \pi_{t+1} + \kappa\hat{y}_t + u_t. \]
Equilibrium

We start with 2-period AIT to gain some intuition

\[
\pi_t = a_\pi \pi_{t-1} + b_\pi u_t
\]

\[
\hat{y}_t = a_y \pi_{t-1} + b_y u_t
\]

\(a_\pi < 0 \Rightarrow \) inflation oscillates.

IRFs to a one-unit iid cost-push shock
The Expectations Channel implies lower positive inflation

- Forward-looking Phillips curve
  \[ \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \hat{y}_t + u_t. \]

  For AIT
  \[ \mathbb{E}_t \pi_{t+1} = a_\pi \pi_t \]

- Reduced-form Phillips curve for AIT
  \[ \pi_t = \frac{\kappa}{1 - \beta a_\pi} \hat{y}_t + \frac{1}{1 - \beta a_\pi} u_t \]

  Reduced-form Phillips curves for IT
  \[ \pi_t = \kappa \hat{y}_t + u_t. \]

  \[ a_\pi < 0 \Rightarrow 1 - \beta a_\pi > 1 \Rightarrow \text{AIT has smaller intercept and slope.} \]
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  \(<0\)

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- Forward-looking Phillips curve

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For AIT

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Reduced-Form Phillips Curves

Positive cost-push shock

AIT

- has a smaller intercept and slope
- closer to the origin ⇒ better trade-off (only quadrant II is relevant)
Time Inconsistency

- AIT has a more favorable Phillips curve, but doesn’t guarantee higher welfare
- Time inconsistency: announce AIT but implements IT (red dot)

Time inconsistency leads to two motives for ambiguous communication

lagged inflation
Time Inconsistency

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- Time inconsistency leads to two motives for ambiguous communication.

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We have discussed $L = 2$.

What’s the optimal strategy for any $L$?

- Suppose a shock hits at $t$
- One optimal strategy is to announce $L = 2$ from $t + 1$ onward
- Why? At $t + h$,

$$\pi_{t+h} = \frac{\kappa}{1 - \beta a_{\pi,1}^{(L)}} \hat{y}_{t+h} + \sum_{l=1}^{L-2} \frac{\beta a_{\pi,l+1}^{(L)}}{1 - \beta a_{\pi,1}^{(L)}} \pi_{t+h-l} + \frac{1}{1 - \beta a_{\pi,1}^{(L)}} u_{t+h}.$$

intercept is zero for $L = 2 \Rightarrow$ dual stability
Optimal Multi-Period AIT Announcements

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Intercept is zero for $L = 2 \Rightarrow$ dual stability.
Flexibility Motive of Ambiguous Communication

- What’s optimal at $t$?
- CB implements IT ex post ⇒ compare equilibria ⇔ compare Phillips curves

Optimal to announce the largest $L$ at $t$, then announce $L = 2$ afterwards
- Ambiguous communication can accommodate such flexibility

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Flexibility Motive of Ambiguous Communication

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▶ Optimal to announce the largest $L$ at $t$, then announce $L = 2$ afterwards

▶ Ambiguous communication can accommodate such flexibility
Social Learning

- So far, we have assumed the central bank has full credibility.
- We assess this assumption with social learning e.g. Hachem and Wu (2017)
- Sequence of events

\[
\begin{align*}
  & t - 1: L_t^i \text{ formed} \\
  & t: u_t \text{ realized, } u_t^i \text{ observed, nowcast formed} \\
  & \text{forecast formed } \pi_t \text{ and } \hat{y}_t \text{ realized} \\
  & \text{Social learning } L_{t+1}^i \text{ updated}
\end{align*}
\]

- The central bank minimizes the welfare loss subject to

\[
\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa \hat{y}_t + u_t.
\]

where

\[
\bar{E}_t \pi_{t+1} = \frac{1}{N} \sum_i E^i_t \pi_{t+1} \quad \text{Details}
\]

- Agents meet in pairs and compare nowcast errors

\[
\varepsilon^i_t = |\mathbb{E}(\pi_t|\mathcal{I}_{t-1}, L_t^i, u_t^i) - \pi_t| + w |\mathbb{E}(\hat{y}_t|\mathcal{I}_{t-1}, L_t^i, u_t^i) - \hat{y}_t|.
\]

In each pair, the agent with larger error switches belief about \( L \).
Social Learning

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**Sequence of events**

1. **t − 1**: $L_t^i$ formed
2. **t**: $u_t$ realized, $u_t^i$ observed, nowcast formed
3. **t**: Forecast formed $\pi_t$ and $\hat{y}_t$ realized
4. **Social learning**: $L_{t+1}^i$ updated

- The central bank minimizes the welfare loss subject to

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa \hat{y}_t + u_t.$$ 

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- Agents meet in pairs and compare nowcast errors

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In each pair, the agent with larger error switches belief about \( L \)
Credibility Motive of Ambiguous Communication

- **Simulation**: random shock each period, average over 1000 draws
- **Clear communication**: 2-period AIT has most believers
- **Ambiguous communication** improves credibility

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Welfare

Clear communication:

- IT is universally the worst
- 4-period is best initially: best Phillips curve (PC)
- But loses credibility quickly (Cred)

Ambiguous communication is associated with the smallest loss
Why Can AIT Perform Better When the Truth is IT?

AIT believers

- have a smaller error on the output gap $\lambda^*$
- hence a smaller overall error if $w$ is large

\[ \varepsilon^i_t = |\mathbb{E}_t^i \pi_t - \pi_t| + w|\mathbb{E}_t^i \hat{y}_t - \hat{y}_t| \]
Different weights in the nowcast error

- $w = 0$
- $w \to \infty$

- AIT has a larger following with larger $w$.
- Ambiguous communication is always better.
Initial Credibility

- We have assumed the central bank starts with full credibility
- Coibion et al. (2020) use survey data to show little public understanding

Total fraction of AIT believers with ambiguous communication

- Initial credibility does not affect long-run credibility
Conclusion

- AIT improves the trade-off in Phillips curve
- Central bank has the incentive to deviate and implement IT
- Optimal horizon of AIT varies over time
- Ambiguous communication helps the central bank gain credibility
Zero Lower Bound

AIT has

- flatter PC
- downward sloping IS
- an equilibrium with negative inflation and negative output gap (so does IT)
- a better equilibrium
Positive lagged inflation

- Dual stability is achievable
- AIT has flatter PC
- IT = AIT + IT > AIT
Optimal $\lambda^{cb}(L)$

Minimize the unconditional welfare loss

$$\mathbb{E}_0 \mathcal{L}_t = \frac{1}{2} (\text{var}[\pi_t] + \lambda \text{var}[\hat{y}_t])$$

where the unconditional variances are calculated using the AIT equilibrium.
Social Learning

▶ FIRE equilibrium

\[ \pi_t = a_{\pi,1} \pi_{t-1} + \ldots + a_{\pi,L-1} \pi_{t-L+1} + b_{\pi} u_t \]

\[ \hat{y}_t = a_{y,1} \pi_{t-1} + \ldots + a_{y,L-1} \pi_{t-L+1} + b_{y} u_t. \]

▶ Nowcast

\[ \mathbb{E}(\pi_t | \mathcal{I}_{t-1}, L_t, u_t^i) = a_{\pi,1} \pi_{t-1} + \ldots + a_{\pi,L_t^i} \pi_{t-L_t^i+1} + b_{\pi}^{(L_t^i)} u_t^i \]

\[ \mathbb{E}(\hat{y}_t | \mathcal{I}_{t-1}, L_t, u_t^i) = a_{y,1} \pi_{t-1} + \ldots + a_{y,L_t^i} \pi_{t-L_t^i+1} + b_{y}^{(L_t^i)} u_t^i. \]

▶ Forecast

\[ \mathbb{E}_t^i \pi_{t+1} = a_{\pi,1} \pi_t + \ldots + a_{\pi,L_t^i} \pi_{t-L_t^i+2}. \]
Uncertainty does not change results qualitatively but quantitatively

- Uncertainty increases, more AIT believers
- But AIT has followers even with no uncertainty
- Ambiguous communication has more credibility (not shown)